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Demand-Based Asset Pricing in General Equilibrium*

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Abstract

I develop a general equilibrium macro-finance model that integrates *demand-based asset pricing*. Assets are held by financial intermediaries ("funds") with *investment mandates* that induce downward-sloping demand curves. A representative household seeks out profitable investment opportunities by shifting its savings across funds, but it does so only *gradually* due to frictions in adjusting its portfolio. The aggregate demand for assets in this economy is *inelastic* and depends on the distribution of net worth across funds. Consequently, shocks to asset supply and unanticipated financial flows have meaningful effects on asset prices. The framework is general enough to accommodate an arbitrary set of intermediaries and assets, so it can be applied to several questions in macro-finance. Analytically, I characterize sufficient statistics to construct counterfactual asset price responses to shocks and show how these statistics relate to estimates of asset demand elasticity in the literature. Quantitatively, I demonstrate that the model can account for the "excess volatility" in asset prices.

JEL codes: E44, E50, E58

Keywords: Asset Pricing, Macro-Finance, Financial Intermediation, Slow-Moving Capital

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1 Introduction

Macro-finance is full of important and hotly debated economic questions. Why is the aggregate stock market so volatile (Shiller, 1981)? Why do shocks to the supply of assets, such as large-scale asset purchases (LSAPs) by central banks, have large and persistent effects on asset prices (Krishnamurthy and Vissing-Jorgensen 2012; Haddad, Moreira, and Muir, 2021)? Do flows into financial intermediaries, such as mutual funds and ETFs, move markets (Edelen and Warner, 2001; Coval and Stafford, 2007)?

Recent approaches to these questions have employed a *demand-based asset pricing* framework in which financial intermediaries choose their portfolios based on simple rules-of-thumb rather than choosing an optimal portfolio based on risk and return considerations. Koijen and Yogo (2019) and Koijen et al. (2021) emphasize that intermediaries' demand depends on asset characteristics (such as market/book ratios for stocks or maturity for bonds), whereas Gabaix and Koijen (2021) assume that intermediaries have rigid portfolio weights on certain asset classes. These assumptions, which receive striking empirical support in the data, have a key implication: shocks to intermediary asset demand and the supply of assets matter for asset prices. Up until this point, the demand-based asset pricing framework has shown promise in addressing stylized asset pricing facts, estimating price responses to exogenous financial flows, and studying policy counterfactuals primarily in *partial equilibrium* models of the asset market.[1]

To address many interesting questions in macro-finance and intermediary-based asset pricing, however, one needs a *general equilibrium* model that can endogenize flows of savings into the financial sector and provide a link between asset prices and household preferences, at least in the long run. Macro-finance puzzles are rooted in the difficulty of explaining asset prices in light of observed macroeconomic data and household preferences. A satisfactory theory of how financial flows affect asset prices over the long run must take account of what caused those flows in the first place.

In this paper, I develop a model that integrates demand-based asset pricing into a general equilibrium macro-finance setting. The model is flexible enough to accommodate a wide range of financial intermediaries and assets. Analytically, the model can be used to understand asset prices' response to supply, demand, and flow shocks both in the short run and the long run. On the one hand, the model can illustrate which deep structural parameters are identified in the existing empirical literature, which is helpful in understanding exactly what those estimates imply about asset price responses to shocks. On the other hand, the model also provides new sufficient statistics to estimate counterfactual asset price responses, highlighting

[1]Gabaix and Koijen (2021) also suggest a method to integrate demand-based asset pricing into general equilibrium with *behavioral* households. I focus on a setting with rational households.

whether aggregate macro data or detailed micro data are needed. The model can also address quantitative questions, such as whether demand-based asset pricing can account for the level of risk premia and whether non-fundamental shocks to the financial sector account for a large share of asset price volatility.

Model. The backbone of the model is a continuous-time endowment economy featuring stochastic growth and a representative household with recursive preferences, as in Bansal and Yaron (2004). There are two types of shocks: permanent innovations in output and temporary innovations in the trend growth rate.

The Lucas tree that produces the aggregate endowment is tranched into an arbitrary set of assets, which may be stocks, risk-free bonds, risky bonds (e.g., corporate bonds), or other types of claims. Households do not invest directly in these assets: instead, assets are held by several financial intermediaries called *funds*, which are meant to represent a variety of financial market participants, such as mutual funds, pension funds, or hedge funds. Households decide how to allocate their savings across funds, and then funds choose their portfolios to maximize returns with respect to the household's stochastic discount factor (subject to the frictions discussed below).

Of course, if funds are free to choose an optimal portfolio on the household's behalf, or if the household can frictionlessly reallocate its savings across funds, the financial sector will simply be a "veil" that prices assets according to the household's intertemporal marginal rate of substitution. The main departure from standard macro-finance models lies in the frictions I introduce in the financial sector. The model's financial sector has three key ingredients:

1. **Investment mandates**: Each fund has a target set of portfolio weights (its "investment mandate") and pays a larger cost the further it deviates from that target. The investment mandate is meant to represent institutional or contractual constraints, such as a 60/40 stock/bond mandate for a mutual fund, value-at-risk constraints, or a portfolio manager's incentives to keep returns close to a benchmark. This assumption implies that funds' asset demand is downward-sloping: an increase in an asset's expected returns induces funds to place a greater portfolio weight on that asset, but this portfolio response is not nearly as aggressive as in standard frictionless models.

2. **Household portfolio frictions**: I assume that the household cannot frictionlessly reallocate its savings across funds – instead, it pays a quadratic cost to do so, which is meant to capture inertial forces such as transaction costs (Gârleanu and Pedersen, 2013) or inattention (Reis, 2006). In line with the data on mutual fund flows (e.g., Chevalier and Ellison, 1997; Gabaix et al., 2022), households are "slow-moving": they *gradually* reallocate their savings to funds that are expected to earn high risk-adjusted

returns going forward.

3. **No-arbitrage**: As in Vayanos and Vila (2021), I introduce a risk-averse arbitrageur to ensure the absence of arbitrage, which is not guaranteed when funds demand assets based partially on payoff-irrelevant characteristics.

Taken together, investment mandates and household portfolio frictions imply that the aggregate demand for assets is *inelastic*, so that asset supply and demand shocks can have large effects, unlike in standard models (Petajisto, 2009). For example, an increase in the supply of bonds requires either a fall in the price of bonds or that funds increase their portfolio weight on bonds in the aggregate. With inelastic intermediary demand, bond prices must fall at least some before funds will be willing to allocate a larger portfolio share to bonds. In response, households will shift some of their savings into funds that hold bonds, eventually undoing the effect of the initial supply shock. Hence, the model provides a substantially weakened version of Wallace's (1981) "Modigliani-Miller theorem of macro," which states that if endowments are held fixed, asset supply is irrelevant to asset prices. Here, asset supply and demand shocks are neutral *in the long run*: inelastic intermediary demand determines asset prices in the short run, but in the long run, asset prices are determined by the household's preferences, as usual.

The absence of arbitrage implies that what matters for asset prices is not the elasticity of demand for individual assets or characteristics, but rather the elasticity of demand for *risk factors* (Peng and Wang, 2021; An, 2024). Even if funds' investment mandates lead them to prize assets with a particular characteristic, if that characteristic is unrelated to cash flows, the arbitrageur will simply take the other side of trades by those funds, preventing funds' mandates from influencing prices. Funds' demand for assets with particular characteristics matters only to the extent that those characteristics are correlated with cash flows (Daniel and Titman, 1997; Kelly, Pruitt, and Su, 2019), resolving a theoretical conundrum in demand-based asset pricing literature (Fuchs, Fukuda, and Neuhann, 2023).

In equilibrium, the *distribution of wealth* is a key state variable that determines asset prices above and beyond fundamentals. When funds that hold mostly stocks are well-capitalized relative to their long-run average, for instance, stock prices will tend to be higher than usual: these funds are reluctant to tilt their portfolios away from stocks unless expected stock returns have fallen substantially. Unexpected inflows into funds tend to mechanically increase the demand for the assets they hold, since those funds try to keep their portfolio weights close to their mandates. Therefore, flow shocks affect asset prices. Moreover, the role of the wealth distribution implies that asset prices will exhibit "excess volatility" relative to fundamentals: a positive fundamental shock moves prices, which then has second-order effects by redistributing wealth towards the intermediaries that hold risky assets, amplifying the initial price dislocation.

Applications. I highlight the model's potential to address both analytical and quantitative questions. Analytically, I derive a log-linear solution to the model that permits a closed-form characterization of the effects of shocks on asset prices. I show that the response of asset prices to *any* shock can be decomposed into three components: (1) the *fundamental* effect of the shock on prices (i.e., the response of prices in a model without asset market frictions), (2) the *direct demand* effect, which depends on how the shock directly influences the supply of and demand for assets, and (3) the *redistributive* effect, capturing how the asset demand curve shifts in response to the shock's initial effect on the wealth distribution. I derive (in principle measurable) sufficient statistics to measure the strength of each channel.

This decomposition is instrumental in understanding what, exactly, previous estimates of intermediaries' asset demand elasticity imply about the response of asset prices to various shocks. I show that the "macro elasticity" of asset prices estimated by Gabaix and Koijen (2021) – the response of the *aggregate* stock market to an increase in the supply of stocks – can be found by simply aggregating the "micro elasticities" estimated by Koijen and Yogo (2019), which capture *individual* intermediaries' price sensitivities. However, the model also highlights that the macro elasticity captures only the direct demand effect – the redistributive effect, which results from a shift in the aggregate demand curve, can be just as important (especially in the short run).

The analytical model solution also suggests an empirical method to account for the effect of financial shocks on asset prices. I propose a *demand-based accounting method* that uses vector autoregressions (VARs) to back out how much variation in asset prices is attributable to fund flow shocks, asset supply and demand shocks, and amplification of fundamental shocks through the redistributive channel. The theoretical restrictions implied by the model allow for the identification of the effects of structural shocks. Furthermore, a byproduct of this accounting method is a new estimate of aggregate demand elasticity based on macroeconomic data rather than instrumental variables estimation of individual funds' demand curves.

I also solve the model numerically using Chebyshev polynomials to assess its quantitative potential. Although I focus on a numerical example rather than a formal calibration, the results are enlightening. First, for realistic levels of macroeconomic volatility, the first-order approximation is highly accurate, suggesting that the analytical applications of the model can provide realistic quantitative conclusions. The basic idea is that even though the log-linear solution misses the *level* of asset prices (since risk premia are second-order), the approximation performs reasonably well for *impulse responses*. Second, the quantitative model appears suited to answer questions about the predictability, volatility, and level of risk premia. I show that price-dividend ratios closely track the distribution of intermediary wealth (i.e., the model can match the volatility and predictability of returns), but the level of returns is pinned down by the long-run volatility of the *household's* stochastic discount factor (which, in turn, is determined by preferences and endowments).

Literature review. In addition to the seminal contributions of Koijen and Yogo (2019) and Gabaix and Koijen (2021) to demand-based asset pricing, my model is related to large empirical literatures that have focused on estimating intermediary demand elasticities and the effects of financial flows across various asset classes. Prior to the emergence of demand-based asset pricing, Shleifer (1986), Harris and Gurel (1986), and Wurgler and Zhuravskaya (2002) estimated micro elasticities of demand for individual stocks. More recently, a vast literature has attempted to formally estimate asset demand curves using tools drawn from industrial organization. Bretscher et al. (2022) and Fang (2022) apply demand-based asset pricing tools to the bond market. Darmouni, Siani, and Xiao (2022) and Azarmsa and Davis (2024) estimate "two-layer" asset demand systems in which funds have inelastic demand and households allocate savings across funds, as in my model. The literature on mutual funds studies how flows affect asset prices (e.g., Warther, 1995) and the extent to which flows can explain the cross-section of equity returns (e.g., Frazzini and Lamont, 2008). My model integrates these insights into a general equilibrium framework.

Theoretically, my model also relates to a literature on segmented markets, starting with the asset demand systems of Brainard and Tobin (1968) and Friedman (1977). Papers such as Greenwood and Hanson (2013), Greenwood and Vayanos (2014), Ray (2019), and Lenel (2020) have studied asset pricing in segmented bond markets, whereas Alvarez, Atkeson, and Kehoe (2002), Gabaix and Maggiori (2015), Greenwood et al. (2023), and Itskhoki and Mukhin (2023) instead study the implications of segmented markets for exchange rates. Related to the household side of my model, Mitchell, Pedersen, and Pulvino (2007) and Duffie (2010) study asset pricing when funds flow slowly to the most profitable investment opportunities. Relative to this literature, I combine segmented markets on the intermediary side with slow-moving household capital in a general model that can capture arbitrary sets of assets and intermediaries.

Finally, this paper is related to a literature on intermediary-based asset pricing. Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999) share the foundational insight that fluctuations in intermediary capitalization can account for persistent downturns in asset prices and economic activity. He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) solve similar frameworks globally, showing that asset prices behave quite differently in "crises" far away from the steady state. Adrian, Etula, and Muir (2014) and Haddad and Muir (2021) empirically study the relationship between intermediary capitalization and asset prices. The distribution of wealth matters for asset prices in my model as well, but I am focused on business-cycle fluctuations rather than rare financial crises.

Organization. The remainder of the paper is organized as follows. Section 2 presents the full model and defines an equilibrium. The model's log-linear solution is derived in Section 3, and Section 4 derives applications of the model based on this approximation. Section 5 presents the full non-linear solution and studies its properties. Section 6 concludes. All proofs are in the Appendix.

2 Model

I consider a continuous time, infinite-horizon environment, $t \in [0, \infty)$. The economy is populated by a representative household, a representative firm, and $I+1$ competitive financial intermediaries called *funds* indexed by $i \in \{0, 1, \ldots, I\}$, which are owned by the household and maximize returns with respect to its stochastic discount factor, and an arbitrageur. Fund $i = 0$ is a bond fund that will hold only short-term risk-free bonds, and $i \geq 1$ are "mixed funds" with specific investment mandates. The arbitrageur ensures the absence of arbitrage opportunities, as in Vayanos and Vila (2021).[2] The firm owns a Lucas tree that produces output and finances itself by issuing claims in financial markets. The household invests its wealth in funds, which in turn purchase the financial assets issued by the firm.

Technology and shocks: The Lucas tree produces an exogenous stream of output Y_t. As in Bansal and Yaron (2004), aggregate output evolves according to a geometric Brownian motion with a mean-reverting stochastic growth rate g_t:

$$\frac{dY_t}{Y_t} = g_t dt + \sigma^Y dZ_t^Y, \tag{1}$$

$$dg_t = \kappa(\overline{g} - g_t)dt + \sigma^g dZ_t^g, \tag{2}$$

where $(\kappa, \overline{g}, \sigma^Y, \sigma^g)$ are parameters, and $d\mathbf{Z}_t = (dZ_t^Y, dZ_t^g)$ are independent Brownian motions, referred to as "output shocks" and "growth shocks," respectively. These shocks are the only sources of uncertainty in the economy.

Preferences: The household has standard recursive preferences (Epstein and Zin, 1989; Duffie and Epstein, 1992). The household's lifetime utility starting from time t, U_t, is defined by

$$U_t = \mathbb{E}_t \left[\int_0^\infty f(C_{t+s}, U_{t+s}) ds \right], \tag{3}$$

[2] Including an arbitrageur is important to guarantee the existence of a stochastic discount factor in financial markets that prices all assets.

where C_t is consumption and the aggregator $f(C,U)$ is defined as

$$f(C,U) = \frac{1}{1-\psi}\left(\frac{\rho C^{1-\psi}}{\big((1-\gamma)U\big)^{(\gamma-\psi)/(1-\gamma)}} - \rho(1-\gamma)U\right).$$

Here, ρ is the time rate of preference, ψ^{-1} is the household's intertemporal elasticity of substitution (IES), and γ is the risk aversion coefficient. Recursive preferences allow for a separation between the household's IES and its risk aversion, which is typically necessary to quantitatively match asset price data. Standard CRRA preferences instead require $\psi^{-1} = \gamma$.

In equilibrium, the household's consumption is equal to aggregate output,

$$C_t = Y_t. \tag{4}$$

Therefore, it is possible to derive the household's stochastic discount factor (SDF) between times t and $t+s$, $\frac{\Lambda_{t+s}}{\Lambda_t} = \frac{\partial U_{t+s}/\partial C_{t+s}}{\partial U_t/\partial C_t}$, directly from the dynamics of output. The SDF follows

$$d\log\Lambda_t = \Big((\theta-1)\frac{Y_t}{W_t} - \theta\rho\Big)dt - \theta\psi d\log Y_t + (\theta-1)d\log W_t, \tag{5}$$

where $\theta \equiv \frac{1-\gamma}{1-\psi}$ and W_t is the present value of aggregate output, as evaluated by the household:

$$0 = \Lambda_t Y_t dt + \mathbb{E}_t[d(\Lambda_t W_t)]. \tag{6}$$

Generally, the household will not be marginal in the market for each asset, so the SDF Λ_t does not necessarily price financial assets.

Assets: There are $J+1$ assets $j \in \{0,1,\ldots,J\}$ that the firm can issue to finance its holdings of the Lucas tree. Asset $j=0$ corresponds to a short-term risk-free bond that pays an instantaneous real interest rate $r_t dt$. Asset J is the firm's equity, which pays all residual cash flows not paid out to other claimants. Each asset $j \geq 1$ pays a (possibly stochastic and endogenous) cash flow z_t^j and trades at an endogenous price Q_t^j. Later, in the quantitative exercise, we will specialize to an environment in which the assets are a risk-free short-term bond, a safe long-term bond, a risky corporate bond, and equity.

The return process $dR_t^j = z_t^j/Q_t^j dt + dQ_t^j/Q_t^j$ is conjectured to follow an Ito process in equilibrium. The (endogenous) vector of returns for assets $j \geq 1$, $d\boldsymbol{R}_t = (dR_t^1, \ldots, dR_t^j)$, therefore satisfies

$$d\boldsymbol{R}_t = \boldsymbol{r}_t dt + \boldsymbol{\Sigma}_t^r d\boldsymbol{Z}_t, \tag{7}$$

where $\boldsymbol{r}_t = (r_t^1, \ldots, r_t^j)$ is the vector of expected asset returns and $\boldsymbol{\Sigma}_t^r$ is the matrix of return loadings on the exogenous shocks, with row j being equal to $(\sigma_t^{j,Y}, \sigma_t^{j,g})$.

Having described the environment, I next turn to individual optimization problems, market-clearing conditions, and the definition of equilibrium.

2.1 Asset supply: The firm

The firm holds a Lucas tree whose value is Q_t^K and finances its asset holdings by issuing the assets listed above. For now, assume that the firm finances a *fixed* fraction of its asset holdings with each type of asset: when the firm purchases a share of the tree worth Q_t^K, it funds that purchase by issuing a quantity of asset j worth $\zeta_j Q_t^K$, with $\sum_{j=0}^{J} \zeta_j = 1$. That is, for simplicity I assume that the *supply* of each asset is just a fixed fraction of the capital stock, so asset j's supply is inelastic in the sense that it does not respond to the price of asset j. It is possible, however, to extend the model to permit the firm to issue assets more flexibly. From the firm's debt issuance policy, it is possible to derive aggregate dividends dz_t^E (the cash flow paid by firm equity):

$$dz_t^E = Y_t dt + dQ_t^K - Q_t^K \big(\zeta_0 r_t dt + \sum_{j=1}^{J-1} \zeta_j dR_t^j\big).$$

2.2 Financial markets and asset demand

Funds: Each fund i enters time t with financial net worth N_{it} and chooses its portfolio weights $\boldsymbol{\omega}_{it} = \{\omega_{ijt}\}$ on each asset $j \geq 1$. The bond fund, $i = 0$, is constrained to invest its entire portfolio in bonds, so it chooses $\omega_{0jt} = 0$ for each j. Mixed funds $i \geq 1$ have more flexibility in their investment strategies: each mixed fund faces an *investment mandate* that makes it costly, but not impossible, to deviate from some *target portfolio composition* $\boldsymbol{\omega}_i^* = \{\omega_{ij}^*\}$. Specifically, fund i pays a cost $\varphi_i(\boldsymbol{\omega}_{it} - \boldsymbol{\omega}_i^*)N_{it}$, where $\varphi_i(\cdot)$ is a concave, differentiable, and non-negative function satisfying $\varphi_i(\mathbf{0}) = 0, \nabla\varphi_i(\mathbf{0}) = \mathbf{0}$. That is, the fund pays a larger cost the further it strays from its mandate. A fund's mandate can be thought of as a formal requirement to keep a certain fraction of its portfolio in an asset class, such as a pension fund with a target weight on equities, or other institutional or contracting frictions that restrict intermediaries' portfolio choice, such as value-at-risk constraints or incentives to keep returns close to a benchmark.

Fund i pays out an exogenous (but possibly stochastic) fraction of its net worth $\delta_{it} dt$ at each instant[3] and receives *endogenous* inflows dF_{it} from the household, so its net worth

[3]This assumption is typical in the macro-finance literature, see, e.g., Gertler and Kiyotaki (2015) or Maggiori (2017).

evolves according to

$$dN_{it} = N_{it}\Big(\underbrace{r_t dt + \boldsymbol{\omega}_{it}\cdot(d\boldsymbol{R}_t - \mathbf{1}r_t dt)}_{\text{fund returns } dR_t^i} - \underbrace{\varphi_i(\boldsymbol{\omega}_{it} - \boldsymbol{\omega}_i^*)dt}_{\text{mandate}} - \underbrace{\delta_{it}N_{it}dt}_{\text{payout}}\Big) + \underbrace{dF_{it}}_{\text{flow}}. \tag{8}$$

The fund's problem is to maximize the present value of its payouts to the household:

$$\max_{\boldsymbol{\omega}_{it}} \ \mathbb{E}_0\left[\int_0^\infty \Lambda_t \delta_{it} N_{it} dt\right] \ \text{ s.t. (8), } \ N_{i0} \text{ given.} \tag{9}$$

The next section demonstrates how this optimization problem gives rise to a downward-sloping asset demand curve for each intermediary.

No-arbitrage: I introduce an arbitrageur to rule out arbitrage opportunities. This is necessary because funds may value assets for cash flow-irrelevant characteristics. The arbitrageur enters each instant with zero wealth, chooses a portfolio, and then pays out any profits or losses to the household. The arbitrageur therefore cares about maximizing returns with respect to the household's SDF, but it also dislikes risk: as in Gabaix and Maggiori (2015) or Itskhoki and Mukhin (2023), the arbitrageur suffers a cost that is quadratic in the variance of its returns (which could represent, for example, an agency problem between the household and the arbitrageur). It chooses the real quantities $\boldsymbol{X}_t = (X_{0t}, \dots, X_{Jt})$ it wishes to hold to statically maximize the objective function

$$\max_{\boldsymbol{X}_t} \ \mathbb{E}_t\left[\boldsymbol{X}_t \cdot \frac{d(\Lambda_t \boldsymbol{R}_t)}{\Lambda_t}\right] - \frac{1}{2}\alpha \ Var_t\big(\boldsymbol{X}_t \cdot d\boldsymbol{R}_t\big) \text{ s.t. } \sum_{j=0}^{J} X_{jt} = 0, \tag{10}$$

where α is the arbitrageur's risk aversion parameter.

The absence of arbitrage implies the existence of a *financial market SDF* M_t such that

$$\frac{dM_t}{M_t} = -r_t dt - \boldsymbol{\eta}_t \cdot d\boldsymbol{Z}_t, \tag{11}$$

where $\boldsymbol{\eta}_t = (\eta_t^Y, \eta_t^g)$ represent the prices of risk for output and growth shocks, respectively. The arbitrageur's optimality conditions do not necessarily pin down the quantity of each asset that the arbitrageur chooses to hold. Instead, they pin down the *quantity of risk* borne by the arbitrageur.

Lemma 1. *The arbitrageur's optimal portfolio* $\boldsymbol{X}_t$ *satisfies*

$$\boldsymbol{\Sigma}_t^{rT}\boldsymbol{X}_t = \frac{1}{\alpha}(\boldsymbol{\eta}_t + \boldsymbol{\sigma}_t^\Lambda), \tag{12}$$

where $\boldsymbol{\sigma}_t^\Lambda = (\sigma_t^{\Lambda,Y}, \sigma_t^{\Lambda,g})$ *denotes the loading of the household's SDF on the risk factors.*

The arbitrageur chooses to hold assets that load on a particular shock when the risk price for that shock exceeds the volatility of the household's SDF. In the limit of infinite risk aversion ($\alpha \to \infty$), the arbitrageur does not bear any risk – it simply eliminates pure arbitrages.

Market clearing and asset pricing: The SDF M_t can be used to price financial assets. Let $\mu_t^j = \mathbb{E}_t[dQ_t^j/Q_t^j]$ denote the drift of asset j's price, and let $\boldsymbol{\sigma}_t^j = (\sigma_t^{j,Y}, \sigma_t^{j,g})$ denote its volatility. Then asset j's price satisfies

$$\underbrace{r_t + \boldsymbol{\eta}_t \cdot \boldsymbol{\sigma}_t^j}_{\text{exp. return } r_t^j} = \frac{z_t^j}{Q_t^j} + \mu_t^j. \tag{13}$$

As usual, the expected return on asset j is equal to the risk-free rate plus a risk premium that depends on the price of risk and j's loading on the risk factors. Note that this pricing equation holds even for Q_t^K, the value of the Lucas tree.

As the next section will show, the household's consumption stream will determine the risk-free rate r_t, whereas risk prices are pinned down by equilibrium in financial markets. The market clearing conditions state that for each source of risk, the quantity of risk demanded by funds and the arbitrageur must equal the total supply of risk (i.e., the volatility of the capital stock with respect to that risk times its value Q_t^K). These conditions can be written as

$$\boldsymbol{\Sigma}_t^{rT}\big(\boldsymbol{X}_t + \sum_{i=1}^{I} \boldsymbol{\omega}_{it} N_{it}\big) = Q_t^K \boldsymbol{\sigma}_t^K. \tag{14}$$

2.3 Fund flows: The household's problem

The last block of the model is the household's consumption-savings problem. The household enters each instant t with savings B_t in the bond fund and $\boldsymbol{S}_t = (S_{1t}, \ldots, S_{It})$ in mixed funds. It chooses its consumption C_t, flows $\boldsymbol{F}_t = (F_{1t}, \ldots, F_{It})$ into mixed funds, and the change in its bond holdings dB_t. Savings already invested in a fund i increase at that fund's rate of return net of its dividend payout rate δ_{it}, so S_{it} evolves according to

$$dS_{it} = S_{it}(dR_{it} - \delta_{it}dt) + F_{it}dt \quad \forall\, i \in \{1, \ldots, I\}, \tag{15}$$

where $dR_{it} = \boldsymbol{\omega}_{it} \cdot d\boldsymbol{R}_t$ denotes the returns on fund i. Of course, in equilibrium, the household's saving in each fund must equal that fund's net worth, $S_{it} = N_{it}$.

The only departure from a standard model of household portfolio choice is that the household faces frictions in reallocating its portfolio across funds, which could reflect, for example, transaction costs or inattention (Garleânu and Pedersen 2013; Reis 2006). Specifically, the household must pay a quadratic cost $\frac{1}{2}\chi_{it}F_{it}^2$ when choosing flows F_{it} into fund i, where $\chi_{it} = \frac{\chi_i}{Y_t}$ and χ_i is an exogenous parameter capturing the sluggishness of adjustment of savings in fund i.[4] On the other hand, the household can frictionlessly adjust its bond holdings, which evolve according to

$$dB_t = \big(r_t B_t + \boldsymbol{\delta}_t \cdot \boldsymbol{S}_t - C_t - \sum_{i=1}^{I}(F_{it} + \frac{1}{2}\chi_{it}F_{it}^2)\big)dt + dT_t, \tag{16}$$

where $\boldsymbol{\delta}_t = (\delta_{1t}, \ldots, \delta_{It})$ is the vector of payout rates and dT_t includes all transfers received by the household. To maintain tractability of a typical endowment economy with resource constraint (4), I assume that all portfolio adjustment costs faced by households and funds are rebated back to the household lump-sum. Hence, transfers dT_t include these adjustment costs as well as the arbitrageur's profits.

The household's problem is

$$\max_{C_t, \boldsymbol{F}_t} U_0 \;\; \text{s.t. } (15),\; (16),\; (B_0, \boldsymbol{S}_0) \text{ given.} \tag{17}$$

We postpone a full analysis of the household's problem to the next section, but one important result is immediate: since the household faces no cost to adjust its bond holdings, it is marginal in the bond market. Then, the risk-free rate satisfies a standard Euler equation.

Lemma 2. *The risk-free rate r_t satisfies*

$$r_t = -\frac{1}{dt}\,\mathbb{E}_t\big[\frac{d\Lambda_t}{\Lambda_t}\big]. \tag{18}$$

The household's consumption therefore pins down the risk-free rate, whereas the price of risk in financial markets $\boldsymbol{\eta}_t$ is pinned down by the financial market equilibrium conditions (14), as claimed.

2.4 Equilibrium

The equilibrium definition is standard.

Definition 1. *An **equilibrium** of this economy consists of household decisions and state variables $\{C_t, \boldsymbol{F}_t, B_t, \boldsymbol{S}_t\}$, fund net worth and portfolio decisions $\{N_{it}, \boldsymbol{\omega}_{it}\}_{i=1}^{I}$, arbitrageur*

[4] I assume that portfolio adjustment costs scale with the size of the economy for convenience: this assumption guarantees the existence of a balanced growth path along which the price-dividend ratio of capital is stationary.

portfolio decisions $\{\boldsymbol{X}_t\}$*, aggregate variables* $\{r_t, \boldsymbol{\eta}_t, d\boldsymbol{R}_t, \Lambda_t, z_t^E, dT_t\}$*, and individual asset prices* $\{\{Q_t^j\}_{j=1}^K, Q_t^K\}$ *such that*

- *Household decisions solve (17);*
- *Fund portfolio decisions solve (9) and arbitrageur decisions solve (10);*
- *Markets clear, and the consistency conditions for aggregate variables* (Λ_t, z_t^E, dT_t) *are satisfied;*
- *Asset prices satisfy (13).*

3 Analytical results: Log-linear equilibrium

This section derives a first-order log-linear approximation to the model's equilibrium conditions. This approximation serves to highlight the model's key mechanisms and to derive sharp analytical characterizations of the economy's response to various shocks. Of course, a first-order approximation will fail to match asset price *levels* because risk premia are a second-order phenomenon. However, the next section demonstrates that the first-order approximation provides sufficient statistics that fare reasonably well in matching *impulse responses* in the full non-linear model. Hence, the model can be informative about the structural parameters that determine aggregate asset demand elasticities.

Point of expansion: I approximate the economy's dynamics in the limit where volatilities of the shocks (σ^Y, σ^g) go to zero. In this limit, the equilibrium is a *balanced growth path* (BGP) in which funds' net worth N_{it} and the price of capital Q_t^K grow linearly with output Y_t, flows F_{it} and the arbitrageur's holdings $\boldsymbol{X}_t$ are equal to zero, and the interest rate r_t is constant. Importantly, to ensure that the arbitrageur does not bear all of the risk in the economy as shocks become small, I also take risk aversion α to infinity so that $\alpha\sigma^2$ remains finite in the limit, as in Itskhoki and Mukhin (2023).

For small shocks, the *share* of net worth held by each fund,

$$\nu_{it} \equiv \frac{N_{it}}{Q_t^K},$$

will be a stationary, endogenous state variable. The only exogenous state variable is the expected growth rate g_t of output. Hence, it will be possible to solve for all outcomes in terms of the vector of states $(g_t, \nu_{1t}, \ldots, \nu_{It})$. To make things simple, in this section I also assume that the payout rate of each fund is equal to the aggregate dividend-price ratio, $\delta_{it} = Y_t/Q_t^K$, but this assumption is not essential for the main results. I guess and verify that in a log-linear equilibrium, the volatilities of all quantities are constant, e.g., $\boldsymbol{\Sigma}_t^r = \boldsymbol{\Sigma}^r$.

Notation: I denote BGP levels with stars, e.g., ν_i^* is the share of wealth held by fund i along the BGP. All assets earn the same rate of return,

$$r^* = \rho + (\psi + \theta - 1)\overline{g},$$

so the price of capital satisfies

$$Q_t^K = \frac{Y_t}{r^* - \overline{g}}.$$

Since the arbitrageur does not hold any assets along the BGP, and all funds hold their target portfolio weights on capital, a steady-state distribution of net worth $\boldsymbol{\nu}^* = (\nu_1^*, \ldots, \nu_I^*)$ must satisfy

$$\boldsymbol{\Omega}^{*T}\boldsymbol{\nu}^* = \boldsymbol{\zeta},$$

where $\boldsymbol{\Omega}^*_{[i,j]} = \omega^*_{ij}$ and $\boldsymbol{\zeta} = (\zeta_1, \ldots, \zeta_J)$.

For most quantities, I denote *log*-deviations from steady-state values with a hat. So, for example, $\hat{\nu}_{it} = \frac{\nu_{it} - \nu_i^*}{\nu_i^*}$. The only exceptions are rates of return $(r_t, \boldsymbol{\eta}_t)$, portfolio weights $\boldsymbol{\omega}_i$, and flows f_{it}: for these variables, hats represent *level* deviations from the BGP, e.g., $\hat{r}_t = r_t - r^*$. Expected rates of change are denoted with a dot, e.g., $\dot{\hat{\nu}}_{it} = \frac{1}{dt}\,\mathbb{E}_t[d\hat{\nu}_{it}]$.

Approach: The model consists of three blocks that I solve in turn.

1. *Financial block*: The optimal portfolio choices made by funds and the arbitrageur can be used to derive *risk prices* as a function of the *distribution of intermediary wealth*. Portfolio decisions and rates of return can also be used to derive the dynamics of intermediary wealth given household flows.

2. *Household block*: The household's consumption-saving problem yields the risk-free rate (via an Euler equation) and the *flow-return relationship* that dictates how fund flows depend on anticipated future returns.

3. *Asset pricing block*: Given the risk prices derived from the financial block and the fund flows derived from the household block, it is possible to derive the price-dividend ratio of any asset in the model.

3.1 Financial market block

The assumption of no-arbitrage implies that expected excess returns on assets must be determined by the price of risk $\hat{\boldsymbol{\eta}}$,

$$\hat{\boldsymbol{r}}_t - r_t = \boldsymbol{\Sigma}^r \hat{\boldsymbol{\eta}}_t.$$

To first order, excess returns equal risk-adjusted excess returns, since risk premia vanish to first order.

Equilibrium in financial markets is shaped by funds' inelastic demand curves, which can be derived from their optimization problem (9). To first order, fund i's optimal portfolio weights satisfy

$$\hat{\boldsymbol{\omega}}_{it} \equiv \boldsymbol{\omega}_{it} - \boldsymbol{\omega}_i^* = \boldsymbol{\varepsilon}_i \boldsymbol{\Sigma}^r \hat{\boldsymbol{\eta}}_t, \tag{19}$$

where $\boldsymbol{\varepsilon}_i$ is a $J \times J$ matrix with entries

$$\boldsymbol{\varepsilon}_{i,[j,j']} = \left(\frac{\partial^2 \varphi_i}{\partial \omega_{ij} \partial \omega_{ij'}} \bigg|_{\boldsymbol{\omega}_i = \mathbf{0}} \right)^{-1}.$$

Funds would like to invest in assets that earn excess returns, but they find it costly to deviate from their mandates, so they do not stray too far from their target portfolio weights $\boldsymbol{\sigma}_i^*$. The matrix $\boldsymbol{\varepsilon}_i$ represents the *elasticity* of fund i's asset demand to excess returns. It depends on the convexity of the portfolio adjustment cost function φ_i: intuitively, the more costly it is for fund i to deviate from its mandate, the less sensitive its demand will be to changes in returns. For example, if a fund faces no cost of deviating from its mandate, the entries of the elasticity matrix diverge to infinity: the fund will respond to excess returns on asset j by investing as aggressively as possible in that asset. It could also be that fund i faces a mandate but finds two assets (say, j and j') to be perfect substitutes. In this case, the fund will arbitrage away any return differential between assets j and j' but will not necessarily respond to excess returns on other assets.

The arbitrageur's portfolio decision (12) can be log-linearized to obtain its asset demand

$$\boldsymbol{\Sigma}^r \hat{\boldsymbol{x}}_t = \frac{1}{\alpha} \hat{\boldsymbol{\eta}}, \tag{20}$$

where $\hat{\boldsymbol{x}}_t \equiv \boldsymbol{X}_t / Q_t^K$. Then, combining portfolio decisions with the market clearing condition (14),

$$\boldsymbol{\Sigma}^{rT} \underbrace{\sum_{i=1}^{I} \hat{\nu}_{it} \times \nu_i^* \boldsymbol{\omega}_i^*}_{\hat{\boldsymbol{d}}_t} + \underbrace{\left(\frac{1}{\alpha} \boldsymbol{I} + \boldsymbol{\Sigma}^{rT} (\sum_{i=1}^{I} \nu_i^* \boldsymbol{\varepsilon}_i) \boldsymbol{\Sigma}^r \right)}_{\boldsymbol{\varepsilon}_{agg}} \hat{\boldsymbol{\eta}}_t = 0. \tag{21}$$

These conditions describe market clearing for aggregate risk in terms of linear demand curves.

The first term is the matrix of asset risk loadings times an *aggregate demand shifter* $\hat{\boldsymbol{d}}_t$, which depends on the distribution of net worth across funds. Intuitively, an increase in fund i's net worth mechanically increases demand for the assets that it is mandated to invest in (i.e., those that appear in its target portfolio weights $\boldsymbol{\omega}_i^*$). For example, suppose an equity fund that is mandated to invest only in stocks receives an inflow of net worth. The fund then increases the aggregate demand for whichever risk factors are correlated with stock returns.

The second term is an *aggregate demand elasticity* $\boldsymbol{\varepsilon}_{agg}$ times the vector of risk prices $\hat{\boldsymbol{\eta}}_t$. The aggregate elasticity matrix depends on a weighted sum of funds' individual demand elasticities, where the weight on each fund is related to its steady-state share of asset holdings. There is also a term $\frac{1}{\alpha}\boldsymbol{I}$ coming from arbitrageur demand, since the arbitrageur has some capacity to bear risk as well.

It is worth noting that in this model, what matters is demand for aggregate risk rather than demand for specific assets: any asset demand imbalances that are unrelated to risk will simply be absorbed by the arbitrageur. Hence, even if funds demand assets with specific characteristics (as in Koijen and Yogo, 2019), inelastic demand matters for asset prices only to the extent that those characteristics are related to risk loadings ("covariances," as in Kelly, Pruitt, and Su, 2019).

It remains to specify the dynamics of the demand shifter (i.e., net worth shares). Log-linearizing funds' budget constraint (8),

$$\dot{\hat{\nu}}_{it} = (\boldsymbol{\omega}_i^* - \boldsymbol{\zeta})^T \boldsymbol{\Sigma}^r \hat{\boldsymbol{\eta}}_t + \hat{f}_{it}, \tag{22}$$

where $\hat{f}_{it} \equiv F_{it}/N_{it}$ denotes flows into fund i normalized by net worth. The dynamics of net worth are simple: fund i's net worth tends to increase as a share of total wealth if either (1) the fund is receiving inflows from the household, or (2) the fund loads on aggregate risk factors that pay an excess return to a greater extent than the capital price (which has risk loadings $\boldsymbol{\zeta}^T \boldsymbol{\Sigma}^r$).

3.2 Household block

The household block will determine flows into the fund, which will in turn pin down the dynamics of net worth and risk prices, as well as the risk-free rate. With both the risk-free rate and risk prices, it will be possible to price assets.

The household's Euler equation for bonds (18) is standard. When log-linearized, it implies that the risk-free rate is a function of the growth rate of output $\hat{g}_t$ only:

$$\hat{r}_t = (\psi + \theta - 1)\hat{g}_t. \tag{23}$$

The novel element of this model is the household's portfolio adjustment frictions, which imply a "slow-moving" response of capital flows to changes in expected returns (Duffie, 2010). The household must pay a large marginal cost to drastically adjust its portfolio following a shock, which is not optimal. Instead, the household smoothly adjusts its savings in response to the *full path* of expected excess returns on funds. Optimal flows satisfy

$$r^* \hat{f}_{it} = \frac{1}{\chi_i} \boldsymbol{\omega}_i^* \boldsymbol{\Sigma}^r \hat{\boldsymbol{\eta}}_t + \dot{\hat{f}}_{it}. \tag{24}$$

The behavior of flows parallels the behavior of inflation in New Keynesian models with price adjustment costs (Rotemberg, 1982). In New Keynesian models, inflation is equal to the present value of future increases in marginal costs. In this model, likewise, flows into fund i are equal to the present value of that fund's risk-adjusted excess returns, $\boldsymbol{\omega}_i^* \boldsymbol{\Sigma}^r$, divided by the cost χ_i of transferring savings to that fund.

Importantly, the household's portfolio decision implies a long-run neutrality result: short-run changes in the supply of assets or the distribution of fund net worth, which can distort risk prices, are undone by the household in the long run. If, for example, a decrease in risk-bearing intermediaries' net worth temporarily depresses asset prices, risk premia will be high. The household will then choose to transfer savings to intermediaries that hold risky assets and earn excess returns, gradually recapitalizing the financial sector. This long-run neutrality parallels the short-run neutrality present in frictionless models: if the household faces no adjustment costs, then it can immediately neutralize the effects of changes in the distribution of wealth by transferring savings across funds, and changes in asset supply are irrelevant (Wallace, 1981).

Together, the household block and the financial block pin down the dynamics of risk prices. Specifically, (21), (22), and (24) are $2I+2$ equations in the $2I+2$ unknowns $(\hat{\boldsymbol{\eta}}_t, \hat{\boldsymbol{\nu}}_t, \hat{\boldsymbol{f}}_t)$. These equations fully determine the dynamics of those variables independently of the exogenous state $\hat{g}_t$.

3.3 Asset pricing

Along a balanced growth path, asset prices may not be stationary – for example, the price of capital Q_t^K grows with output Y_t. However, price-dividend ratios are stationary. Let $pd_t^j = Q_t^j / z_t^j$ denote the price-dividend ratio of asset j, and let $g_t^j = \frac{1}{dt} \mathbb{E}_t[dz_t^j / z_t^j]$ denote the expected growth rate of dividends. Then the asset pricing condition (13) can be log-linearized to obtain the Campbell-Shiller relationship

$$(r^* - g^{j*}) \hat{pd}_t^j = \hat{g}_t^j - \hat{r}_t - \boldsymbol{\sigma}^j \cdot \hat{\boldsymbol{\eta}}_t + \dot{\hat{pd}}_t^j. \tag{25}$$

An asset's price-dividend ratio is the discounted value of future dividend growth rates minus discount rates. Discount rates, in turn, depend on the asset's risk loadings and the price of risk in financial markets. If demand for the risk factors that asset j loads on is depressed, then the discount rate on asset j will be high, implying a low price-dividend ratio.

The asset pricing equation will be central to understanding the economy's response to shocks. Shocks transmit to aggregate asset demand by changing initial asset prices and redistributing wealth. Following a shock at $t = 0$, the return on an intermediary's portfolio is

$$dR_0^i \approx \boldsymbol{\omega}_i^* \cdot (d\hat{\boldsymbol{z}}_0 + d\hat{\boldsymbol{pd}}_0), \tag{26}$$

where $\boldsymbol{pd}_t = (pd_1, \ldots, pd_J)$ denotes the vector of price-dividend ratios. The initial shock to fund i's wealth share is

$$d\hat{\nu}_{i0} = dR_0^i - (\sigma^Y dZ_0^Y + d\hat{pd}_0^K), \tag{27}$$

where $\hat{pd}_0^K$ denotes the price-dividend ratio of the capital stock. This equation links changes in asset prices to fund net worth via a two-way feedback loop: asset price shocks redistribute net worth across funds, which in turn generates a persistent shift in the asset demand curve that affects discount rates via changes in future risk prices.

3.4 The economy's response to shocks

The main benefit of the log-linear approximation is that it generates sharp analytical predictions about the economy's response to shocks. The model itself has two exogenous stochastic processes – output shocks and growth shocks. However, since certainty equivalence holds under a first-order approximation, the model can actually lend itself to the analysis of shocks outside the model that occur with probability zero, such as "net worth" shocks that exogenously increase a fund's net worth.[5]

The mechanics of output and growth shocks in this model are simple: only growth shocks affect the exogenous state variable $\hat{g}_t$, whereas both types of shocks affect net worth shares $\hat{\boldsymbol{\nu}}_t$, the key endogenous state variables. Changes in the growth rate $\hat{g}_t$ affect asset prices via both cash flows (since they affect the growth rate of aggregate dividends) and the risk-free rate (an increase in $\hat{g}_t$ increases $\hat{r}_t$ via Equation 23). Changes in wealth shares, by contrast, affect asset prices via shifts in the asset demand curve that affect risk prices $\hat{\boldsymbol{\eta}}_t$.

Figure 1 illustrates the model's dynamics in a benchmark calibration with a single representative mixed fund and two assets: equity and long-term, risk-free bonds that pay coupons decaying at a deterministic rate τ (as in Hatchondo and Martinez, 2009, or Chatterjee and Eyigungor, 2012). I denote the representative fund's net worth share simply by $\hat{\nu}_t$. The cali-

[5] Such shocks are common in the macro-finance literature (e.g., Gertler and Karadi, 2011). Furthermore, in this model a net worth shock is equivalent to the "flow shocks" analyzed by Gabaix and Koijen (2021), who study the effects of an exogenous inflow of investor capital into a fund.

bration is described in further detail in Section 5, where I analyze the model's full non-linear solution.

A positive output shock does not change future dividend growth rates or the risk-free rate. However, it increases the price of capital, generating a capital gain for the fund that increases its wealth share $\hat{\nu}_t$. The increase in the fund's wealth share mechanically boosts demand for equity and long-term bonds, pushing up the price of both types of assets. Since the returns of both assets load positively on output shock risk, this shift in demand pushes down the price of output risk $\hat{\eta}_t^Y$. Bonds will load negatively on growth shock risk (since growth shocks increase the interest rate). Hence, an increase in the fund's net worth actually decreases the demand for assets that load on the growth factor, which leads to an increase in the risk price $\hat{\eta}_t^g$.

The increase in the fund's net worth therefore pushes down the expected future returns on the assets it holds in its portfolio. The household, anticipating low future returns, begins to shift wealth out of the fund. A positive net worth shock hence initially increases the fund's net worth but induces persistent outflows going forward. In fact, outflows persist until the fund's net worth returns to its steady-state level, bringing asset valuations back to their long-term averages.

A positive growth shock, on the other hand, has the opposite effect. An increase in expected output growth raises the interest rate and lowers long-term bond prices. The representative fund therefore takes a loss, causing a further reduction in bond demand that amplifies the initial shock and spills over to equity markets. Expected asset returns increase going forward, leading to an inflow from households until the fund's net worth has recovered.

3.5 The quality of the approximation

The next section will illustrate several potential quantitative applications of the log-linearized model. Of course, the relevance of these applications is highly dependent on the quality of the log-linear approximation. Here I briefly demonstrate that in the calibrated model, the log-linearized economy's response to shocks is quantitatively very close to the non-linear economy's response to shocks.

Figure 2 illustrates this point. The non-linear dynamics of the state variables are extremely close to those in the linearized model, at least for quantitatively reasonable shock volatilities – the model does display significant non-linearities far away from the steady state, as discussed later. Due to the fact that (1) the state dynamics are well-approximated, and (2) volatilities do not vary much in the vicinity of the steady state, the linear model also closely matches the volatility of risk premia.

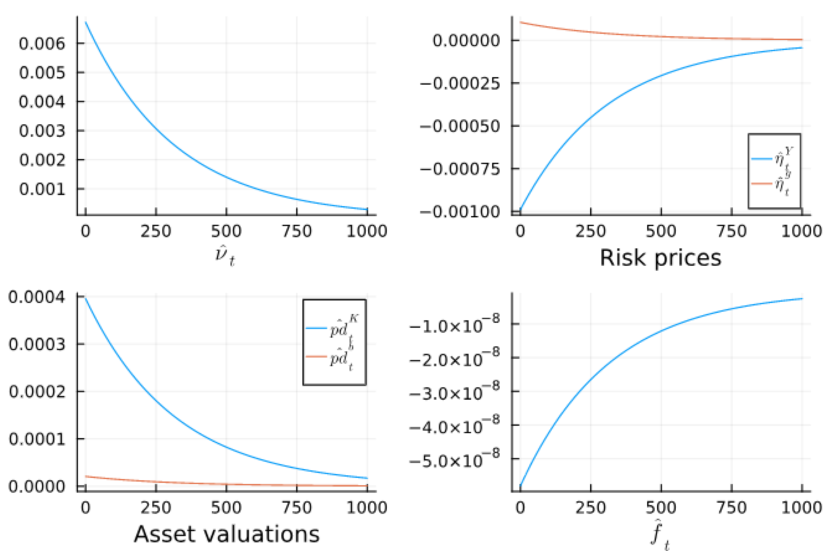

Figure 1: The log-linearized economy's response to an output shock. The top-left panel shows the response of the representative fund's wealth share $\hat{\nu}_t$, the top-right shows the response of risk prices $\hat{\boldsymbol{\eta}}_t$, the bottom-left shows the response of asset valuation ratios $(\hat{pd}_t^K, \hat{pd}_t^b)$, and the bottom-right shows the response of flows $\hat{f}_t$ into the representative fund.

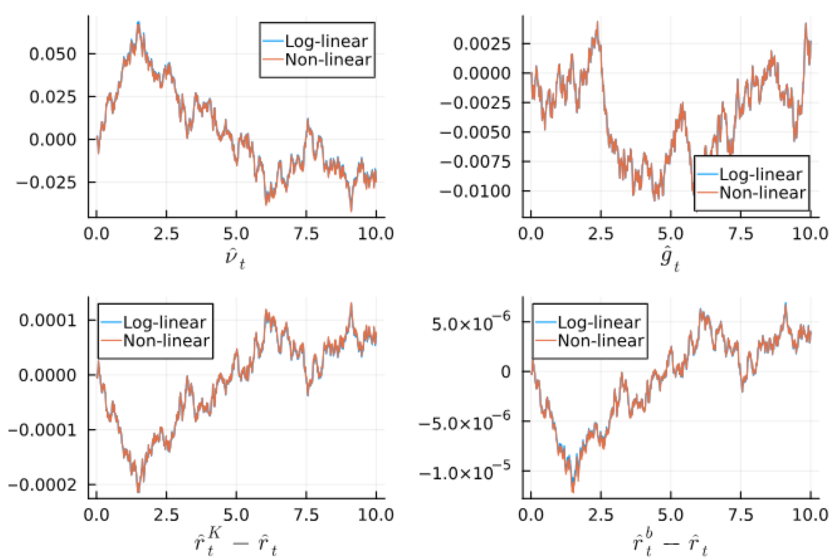

Figure 2: A comparison of dynamics in the log-linearized model and the full non-linear model. The blue (red) lines represent log deviations from steady-state levels in the linear (non-linear) model. The top-left panel shows the dynamics of the fund's wealth share, the top-right shows the dynamics of the growth rate of output, and the bottom panels show the dynamics of the risk premia on capital and bonds, respectively.

The linear model's ability to match *impulse responses* of asset prices and returns does not necessarily mean that it can match the *level* of asset prices. In particular, the household's required compensation for risk generates a risk premium that affects the level of asset prices in the full nonlinear solution. However, this is a second-order phenomenon, so it is absent from the linearized model. I postpone an analysis of the level of asset prices to Section 5, which studies the full non-linear model.

4 Applications

In this section, I develop some applications of the linearized model. In the first application, I study how structural parameters determine the transmission of shocks, including a discussion of how popular methods of identifying aggregate asset demand elasticities can be interpreted through the lens of the model. The second application studies the properties of the stochastic discount factor and cross-sectional asset pricing tests implied by the model.

4.1 The transmission of shocks and the macro elasticity of asset demand

In the Appendix, I extend the model to incorporate additional probability-zero shocks (i.e., "MIT" shocks) to flows, funds' target portfolio weights, and asset supply. The empirical literature often studies such shocks: Gabaix and Koijen (2021) study the relationship between exogenous flows into mutual funds and asset prices, a large body of empirical work estimates asset demand elasticities using shocks to the set of assets that intermediaries are permitted to invest in (e.g. Shleifer, 1986; Koijen and Yogo, 2019), and the literature on Quantitative Easing estimates the effect of large-scale asset purchases (LSAPs) by exploiting shocks to central bank portfolio holdings.

I show that in the extended model, it is possible to decompose the impulse response to *any* shock into three intuitively interpretable components.

Proposition 1. *The impulse response of any asset valuation* $\hat{pd}_t^j$ *to a sequence of shocks can be written as*

$$\mathbb{E}_0[\hat{pd}_t^j] = \hat{pd}_t^{j,frictionless} + \hat{pd}_t^{j,demand} + \hat{pd}_t^{j,redist.}, \tag{28}$$

where

$$\hat{pd}_t^{j,frictionless} = \int_0^\infty e^{-(r^*-g^{j,*})s}(\hat{g}_{t+s}^j - \hat{r}_{t+s})ds,$$

$$\hat{pd}_t^{j,demand} = -\int_0^\infty e^{-(r^*-g^{j,*})s}\boldsymbol{\sigma}^{jT}\boldsymbol{\Sigma}^r\boldsymbol{\varepsilon}_{agg}^{-1}\boldsymbol{\Sigma}^{rT}\hat{\boldsymbol{\epsilon}}_{t+s}^d ds,$$

where $\hat{\boldsymbol{\epsilon}}_t^d$ is the vector of asset demand shocks, and $\hat{pd}_t^{j,redist.}$ is the asset price change induced by redistribution: it is the price that would have arisen in an equilibrium without shocks if initial net worth shares are

$$\hat{\boldsymbol{\nu}}_0 = \left(\boldsymbol{I} - (\boldsymbol{\Omega}^T - \boldsymbol{1}\boldsymbol{\zeta}^T)\boldsymbol{P}\right)^{-1} (\boldsymbol{\Omega} - \boldsymbol{1}\boldsymbol{\zeta}^T)\left(\hat{\boldsymbol{z}}_0 + \hat{\boldsymbol{pd}}_0^{frictionless} + \hat{\boldsymbol{pd}}_0^{demand}\right),$$

where $\boldsymbol{\Omega}$ is the $J \times I$ matrix of target portfolio weights and $\boldsymbol{P}$ is the matrix relating asset prices to wealth shares in equilibrium, so that $\hat{\boldsymbol{pd}}_t = \boldsymbol{P}\hat{\boldsymbol{\nu}}_t$.

This decomposition shows that there are three channels through which shocks affect asset prices:

1. The effect in a *frictionless* model: a shock can affect asset valuations simply because it exogenously changes expectations of dividend growth or future interest rates, such as a growth shock. This channel does not require the frictions that generate inelastic aggregate demand for assets.

2. The *direct demand* effect: an increase in an asset's supply, or a decrease in a particular fund's target portfolio weight on that asset, increases the residual supply that the rest of the market must absorb, causing a shift along the demand curve and increasing the price of risk associated with that asset. This effect persists as long as the asset supply shock.

3. The *redistributive* effect: Shocks change expectations of future cash flow growth, interest rates, and risk prices. The resulting change in asset prices redistributes wealth across intermediaries, changing the demand shifter $\hat{\boldsymbol{d}}_t$. The redistributive effect of any shock eventually dissipates as funds' wealth shares converge back to their steady-state levels.

This decomposition is immediately useful in understanding the relationship between (1) the model's structural parameters and (2) common methods used to estimate aggregate demand elasticities in the literature. For instance, Gabaix and Koijen (2021) distinguish between a "micro" elasticity – the price sensitivity of an individual intermediary's demand – and a "macro" elasticity – the effect of an increase in an asset's supply on its market price. They estimate the macro elasticity using mutual fund flow shocks, arguing that such a shock effectively functions as an exogenous shift in demand for the assets held by a mutual fund. This decomposition clarifies that there is a subtle difference between flow shocks and direct asset supply shocks: a flow shock is effectively a redistributive shock that eventually dissipates as wealth shares converge back to steady state. On the other hand, an asset demand shock can be permanent, at least in principle. As a result, the two shocks do not necessarily induce identical asset price responses.

The decomposition also highlights that there is some ambiguity in the definition of the "macro elasticity." Here, the price effects of an asset supply shock depend crucially on the persistence of that shock: for example, a central bank asset purchase that is expected to unwind soon after will affect risk prices only for a short period of time, limiting the response of asset prices. On the other hand, a permanent asset purchase would have a much larger effect. Given this ambiguity, one may want to focus on estimating the aggregate demand elasticity matrix $\boldsymbol{\varepsilon}_{agg}$, but this is a difficult task as well: at the time of an asset purchase, the direct demand effect (the price movement induced by a shift *along* the demand curve) is amplified by the redistributive effect (the *shift* in the demand curve induced by the change in asset prices). It is generally not possible to identify $\boldsymbol{\varepsilon}_{agg}$ with high-frequency event studies of the sort used in the literature estimating the effects of QE.

Nevertheless, as shown earlier, the aggregate elasticity can be estimated by aggregating individual intermediaries' micro elasticities, as in Koijen and Yogo (2019). In particular, their preferred instrument – an exogenous, permanent shock to the investment universe of another intermediary – can be introduced into the model to derive the relationship between a fund's portfolio holdings and a shock to another fund's demand.

4.2 Demand-based return accounting

The model can also be used to study the extent to which intermediaries' inelastic asset demand drives the seemingly "excess" volatility in valuations studied by a large empirical literature in finance (e.g., Shiller, 1981). The model implies a tight relationship between flows, intermediary capitalization and portfolio weights, and asset risk premia.

The classical approach to account for variation in asset returns is to run a vector autoregression (VAR) of the form[6]

$$\begin{pmatrix} \hat{\boldsymbol{R}}_{t+1} \\ \hat{\boldsymbol{g}^d}_{t+1} \\ \hat{\boldsymbol{pd}}_{t+1} \end{pmatrix} = A \begin{pmatrix} \hat{\boldsymbol{R}}_{t} \\ \hat{\boldsymbol{g}^d}_{t} \\ \hat{\boldsymbol{pd}}_{t} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon}_t^R \\ \boldsymbol{\epsilon}_t^d \\ \boldsymbol{\epsilon}_t^{pd} \end{pmatrix}, \tag{29}$$

where $\boldsymbol{R}_t$ is a vector of asset returns, $\boldsymbol{g^d}_t$ is a vector of dividend growth rates, $\boldsymbol{pd}_t$ is a vector of price-dividend ratios, and A is a matrix. Usually, such regressions find that the majority of variation in returns comes from variation in discount rates rather than expected cash flow

[6]Of course, there are more complex variants of this approach, which use additional control variables to account for variation in dividend growth rates and discount rates. This is simply a representative example.

growth (Campbell and Shiller, 1988). However, the classical accounting method does not take a stance on the fundamental source of discount rate variation.

This model of inelastic intermediary demand can be used to make some progress on this question. In the model, discount rates vary with the supply and demand for assets. The demand for assets, in turn, depends on (1) intermediary capitalization, which is driven by inflows and asset returns, and (2) any exogenous variation in intermediaries' investment mandates (such as the investment mandate shocks studied in the previous section). Therefore, the model suggests a method to account for the extent to which discount rate variation is driven by

1. Return shocks that transmit to intermediary capitalization,
2. Fund flow shocks, and
3. Exogenous shocks to intermediary demand.

To account for the sources of return variation, it is necessary to expand the VAR to include the key variables that drive discount rates in the model. Let $\boldsymbol{s}_t$ denote any variation in the *supply* of assets, and let

$$\hat{\boldsymbol{d}}_t^n \equiv \sum_{i=1}^{I} \nu_i^* \boldsymbol{\omega}_i^* \times \hat{\nu}_{it} - \hat{\boldsymbol{s}}_t$$

denote the *net demand* for assets. Then, run the VAR

$$\begin{pmatrix} \hat{\boldsymbol{R}}_{t+1} \\ \hat{\boldsymbol{g}^d}_{t+1} \\ \hat{\boldsymbol{pd}}_{t+1} \\ \hat{\boldsymbol{d}}_{t+1}^n \\ \sum_{i=1}^{I} \nu_i^* \hat{\boldsymbol{\omega}}_{i,t+1} \\ \hat{\boldsymbol{f}}_{t+1} \end{pmatrix} = B \begin{pmatrix} \hat{\boldsymbol{R}}_t \\ \hat{\boldsymbol{g}^d}_t \\ \hat{\boldsymbol{pd}}_t \\ \hat{\boldsymbol{d}}_t^n \\ \sum_{i=1}^{I} \nu_i^* \hat{\boldsymbol{\omega}}_{it} \\ \hat{\boldsymbol{f}}_t \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon}_t^R \\ \boldsymbol{\epsilon}_t^d \\ \boldsymbol{\epsilon}_t^{pd} \\ \boldsymbol{\epsilon}_t^n \\ \boldsymbol{\epsilon}_t^\omega \\ \boldsymbol{\epsilon}_t^f \end{pmatrix}, \tag{30}$$

which includes net demand, intermediary portfolio weights, and flows. As usual, theoretical restrictions are needed to deduce the effects of *structural* shocks. The theory provides useful guidance: intermediaries' portfolio choices and flows are both forward-looking variables that depend only on expected future returns. Hence, portfolio weights and flows can contemporaneously load on dividend growth rates and price-dividend ratios, but they should be independent of all past variables that cannot be used to predict future returns. Moreover, intermediaries' current capitalization is determined entirely by returns and flows in the previous period via a budget constraint. These restrictions are sufficient to identify the model's structural shocks.

Once the structural shocks have been identified, it is possible to answer two types of questions. First, how much can inelastic intermediary demand account for variation in discount rates? That is, are shocks to intermediary capitalization, portfolio weights, asset supply, and flows responsible for a large share of asset price volatility? Or does most of that volatility remain unexplained by the model?

Second, what is the aggregate elasticity of asset demand? The VAR specifies how intermediaries' portfolio choices react to changes in expected returns, so the VAR coefficients can be used to back out the aggregate elasticity ε_{agg}. Here, identification comes from the theoretical restriction that intermediary asset demand is forward-looking rather than from an instrumental variables strategy of the type used in the literature. This strategy permits the estimation of asset demand elasticity from data aggregated at a high level, since it is not necessary to find a shock that identifies each individual intermediary's demand. With an estimated elasticity, it is possible to answer classic questions, such as: how much do Treasury prices respond to (announced or unannounced) central bank asset purchases? To what extent do changes in Treasury prices spill over to the corporate bond and mortgage markets?

5 The full non-linear model

In this section, I solve the full non-linear model and study its properties. The exercise should be thought of as a numerical example, as the parameters are chosen for convenience rather than quantitative realism. The exercise is meant to illustrate the model's potential to capture an environment with a large number of assets, although the number of funds must be limited to avoid the curse of dimensionality. I show that the non-linear model may be able to address asset price excess volatility and return predictability puzzles in a variety of markets.

5.1 Setting and parameter values

I study an economy with a single mixed fund, which I refer to as the "representative fund." This fund should be interpreted as an aggregate of financial market participants that hold both risky and safe assets, such as mutual funds, pension funds, and banks. The key endogenous state variable, ν_t, will denote the share of wealth held by the representative fund (while the remaining share $1 - \nu_t$ is held by the bond fund).

Parameter	Description	Value
y	Output per unit capital	1
σ^Y	Volatility of output	0.02
$\overline{g}$	Average growth rate	0.02
κ	Mean reversion rate of g_t	0.2
σ^g	Volatility of g_t	0.005
ρ	Rate of time preference	0.05
γ	Risk aversion	1.0
ψ^{-1}	IES	1.0

Table 1: Standard values for the model's preference and technology parameters.

There are two assets that the fund and the arbitrageur can hold: equity and safe, long-term bonds that pay exponentially decaying coupons, as in Hatchondo and Martinez (2009). Specifically, the bond's coupon z_t^b evolves according to

$$dz_t^b = -\tau z_t^b dt,$$

so that the bond's duration is roughly τ^{-1}. This is the simplest setting that can capture the fact that assets may have different correlations with the underlying shocks: in a model without portfolio frictions, the long-term bond's price declines after a positive growth shock, whereas the price of equity may increase or decline (depending on the value of the intertemporal elasticity of substitution ψ^{-1}).

The parameters governing preferences and technologies take standard values. I assume log utility ($\gamma = \psi = 1$) and a discount rate of $\rho = 0.05$. For the technological parameters, I choose the normalization $y = 1$ and take standard parameter values $\sigma^Y = 0.02$, $\overline{g} = 0.02$. The mean-reversion rate of growth and its volatility are set to $\kappa = 0.2$ and $\sigma^g = 0.005$, respectively. Table 1 lists the preference and technology parameters.

The model's key parameters govern the elasticity of intermediary asset demand and the household's portfolio adjustment costs. I assume that the representative fund's investment mandate takes a quadratic form,

$$\varphi(\boldsymbol{\omega} - \boldsymbol{\omega}^*) = \frac{1}{2}\Big(\varphi_e(\omega_e - \omega_e^*)^2 + \varphi_b(\omega_b - \omega_b^*)^2\Big),$$

where ω_e, ω_b represent the portfolio weights on equity and long-term bonds, respectively. I set $\varphi_e = \varphi_b = 0.02$, so that a 1% increase in an asset's excess returns causes the fund to increase its portfolio weight on that asset by 50%. I set the fraction of bonds issued by the representative firm to $\zeta = 0.5$, so that the firm finances half its assets with long-term bonds

Parameter	Description	Value
ζ	Long-term bond supply/Total asset supply	0.5
ω_e^*	Target portfolio weight on equity	1.0
ω_b^*	Target portfolio weight on bonds	1.0
ϕ_e^{-1}	Fund elasticity of equity demand	50
ϕ_b^{-1}	Fund elasticity of bond demand	50
χ	Household portfolio adjustment cost	1000
α	Arbitrageur risk aversion	1.0

Table 2: The parameter values used in the numerical solution of the model.

and the remaining half with equity. Correspondingly, I set the fund's target portfolio weights to $\omega_e^* = \omega_b^* = 1$, so that at a steady state, the fund will be levered up two-to-one and will hold equal quantities of bonds and equity.

For the household's portfolio adjustment cost parameter, I set $\chi = 1000$. I also set the arbitrageur's risk aversion parameter to $\alpha = 1$. These parameters control the predictability of returns and the maximum Sharpe ratio in financial markets, respectively. The parameter values are reported in Table 2.

5.2 The model's solution

I look for a recursive equilibrium in two state variables: the growth rate g_t and the representative fund's net worth share ν_t. All equilibrium quantities are thus represented as functions of (g, ν). I solve the model using Chebyshev polynomials.

The key state variable that determines risk prices in the model is the fund's net worth share ν. When the fund's net worth share is high, it bids up the prices of the assets it holds. Equity prices load positively on output shocks, since equity holders absorb all dividend risk. Long-term bond prices, by contrast, load *negatively* on growth shocks: an increase in the growth rate pushes up interest rates, reducing long-term bond prices. As a result, when the fund bids up asset prices, the price of output risk, η_Y, decreases and the price of growth risk, η_g, increases. The left panel of Figure 3 plots risk prices as a function of ν when the growth rate is at its mean g_{avg}.

The dynamics of the fund's wealth share ν depend on asset returns and flows. When the fund's wealth share is high, the returns on the assets it holds are depressed, causing ν to drift back down to its steady-state level. This effect is reinforced by flows from the household: the low returns on the fund incentivize the household to shift its wealth out of the fund and into short-term, risk-free bonds. Conversely, when the fund's wealth share is low, asset prices are depressed and it earns high returns going forward, which are reinforced by flows into the

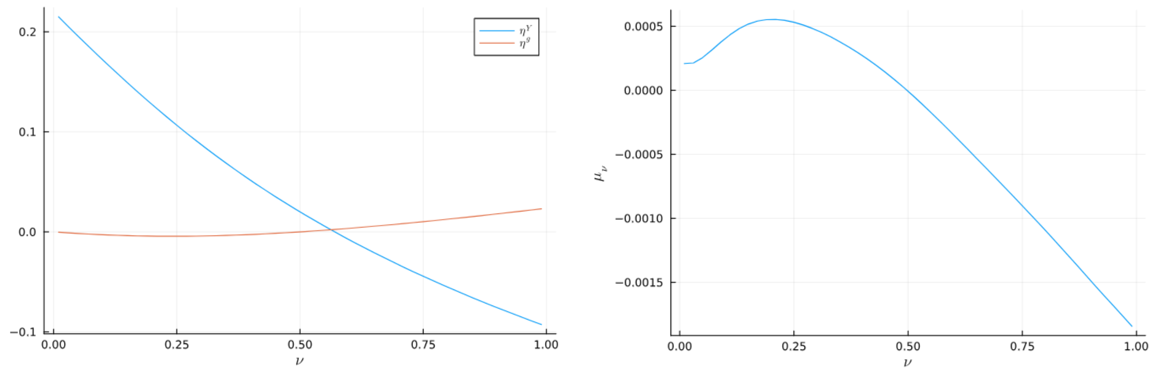

Figure 3: The left panel illustrates how risk prices η^Y and η^g vary as a function of ν, the fund's wealth share, when the growth rate is held at its average level g_{avg}. The right panel shows the drift of ν with $g = g_{avg}$.

fund. The right panel of Figure 3 plots the drift of ν when the growth rate is at its mean.

Investment mandates and households' sluggish portfolio adjustment cause asset prices to diverge from their frictionless values. Figure 4 illustrates how asset prices vary with ν and g. The price of capital (left panel) is essentially invariant to the growth rate, as it would be in a frictionless model with log preferences: changes in the growth rate of future cash flows are exactly offset by changes in the risk-free rate, keeping the price-dividend ratio fixed. However, changes in the fund's wealth share result in changes in discount rates (i.e., risk prices) that are unrelated to cash flows. When the fund's wealth share is high (low), the price-dividend ratio is above (below) its frictionless level $1/\rho$.

By contrast, bond prices do not vary much with the fund's wealth share. In this numerical example, interest rates and bond prices are not sufficiently volatile for the risk premium on bonds to be quantitatively large. Instead, bond prices mostly vary with the level of interest rates as they would in a frictionless model.

5.3 Risk premia and return predictability

I conclude the analysis of the non-linear model with a remark about the level and variability of risk premia. In this model, the *average* level of risk premia is roughly equal to that in the corresponding frictionless economy (i.e., an economy with the same underlying endowment process but no frictions in household portfolio allocation). If risk premia are above the price of risk demanded by the household, the fund will receive inflows until the household is indifferent between investing in the fund and investing in bonds. Hence, with log utility, the model will generally be unable to generate realistic risk premia. However, a calibration with recursive preferences as in Bansal and Yaron (2004), which the model can

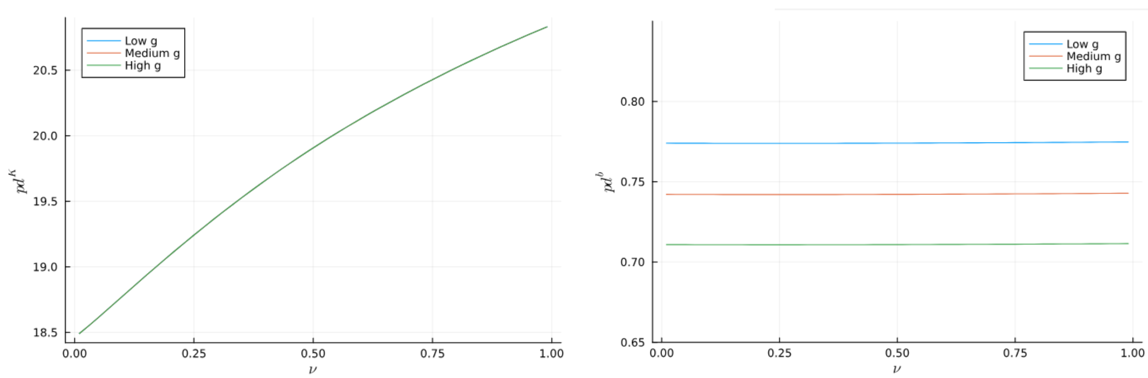

Figure 4: The left panel plots the price-dividend ratio of capital as a function of ν for several values of g: a low value three standard deviations below the mean (blue line), the mean value (red line), and a high value three standard deviations above the mean. The right panel similarly plots the price-dividend ratio of long-term bonds.

easily accommodate, would be able to overcome this issue.

While the frictions in the model cannot explain the level of interest rates, they could potentially resolve empirical puzzles related to return predictability and the seeming excess variability of discount rates (Shiller 1981, Campbell and Shiller 1988). As discussed above, changes in the fund's wealth share move discount rates while leaving cash flows unchanged. Shifts in discount rates caused by shocks to the fund's wealth share are persistent: they last until portfolio flows from the household undo the initial shock to net worth. Moreover, this model will generally generate pro-cyclical valuation ratios, as in the data: positive cash flow shocks will usually increase the fund's net worth, decreasing discount rates and amplifying the initial shock to asset prices.

Figure 5 illustrates this point. The price-dividend ratio of the capital stock varies in response to shocks, unlike in a frictionless model: with log utility, the price-dividend ratio would be equal to $1/\rho$ at all times. In particular, the price-dividend ratio closely tracks the fund's wealth share ν, meaning that variation in valuation ratios, in this model, is almost entirely attributable to changes in discount rates rather than changes in expectations of cash flow growth.

The model suggests that the capitalization of financial intermediaries may be an important predictor of returns and could potentially explain some of the return predictability puzzles in the data. It is particularly well suited to address any return predictability attributable to "slow-moving capital" in the cross section of asset classes, as suggested by Duffie (2010).

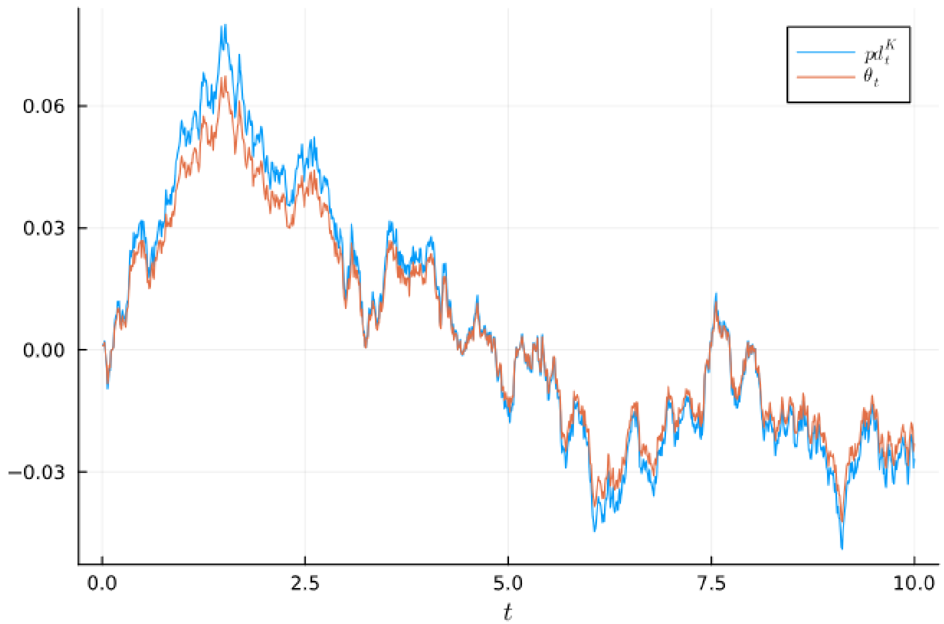

Figure 5: The price-dividend ratio of capital is plotted alongside the fund's wealth share over time for a series of simulated shocks.

6 Conclusion

I integrate demand-based asset pricing in a general equilibrium framework. The model is tractable and admits an arbitrary set of intermediaries and assets. The model delivers several key insights. First, when intermediary demand is inelastic, the distribution of wealth is central to the transmission of shocks to asset prices. Second, while intermediary demand matters for asset prices in the short run, asset prices are determined by household prices in the long run, so although demand-based asset pricing has the potential to explain excess asset price volatility, it cannot explain the puzzling *level* of many risk premia. Third, in the absence of arbitrage, what matters is not the elasticity of demand for assets with particular characteristics, but rather the elasticity of demand for risk factors.

I develop both analytical and quantitative applications of the model. Analytically, I demonstrate how existing estimates of asset demand elasticity can be used to derive counterfactual responses of asset prices to shocks and propose new methods as well. I also demonstrate how to solve the model quantitatively to assess the importance of inelastic intermediary demand for aggregate asset prices. The model's potential applications, however, are not limited to those studied here. The model could also be applied to study the term structure of bond yields or exchange rates, for instance. I leave these tasks to future research.

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A Optimization problems and equilibrium

In this section, I derive the full non-linear solution to all agents' optimization problems.

A.1 The fund's problem

Fund i's problem (9) can be written as a Hamilton-Jacobi-Bellman (HJB) equation. The fund's value function depends on net worth N_{it}, the only individual state variable, as well as (possibly) the entire history of aggregate shocks $\boldsymbol{h}^t = \{d\boldsymbol{Z}_s\}_{s=0}^t$. For simplicity, we denote the value function by $V_{it}(N)$, with the understanding that the subscript t captures all dependence on the prior history.

The HJB equation can be written as

$$0 = \max_{\boldsymbol{\omega}_i} \ \delta_{it} N dt + \mathbb{E}_t \left[\frac{d(\Lambda_t V_{it}(N))}{\Lambda_t} \right] \ \text{s.t. (8)}. \tag{31}$$

The first step in solving the fund's problem is to prove that the value function is affine in net worth N.

Proposition 2. *The fund's value function $V_{it}(N)$ has the form*

$$V_{it}(N) = \xi_{it} N + \zeta_{it} \tag{32}$$

for some exogenous processes $\{\xi_{it}, \zeta_{it}\}$.

Proof. Consider the problem of a fund that receives no future inflows. The fund's budget constraint and objective function are linear in net worth N, so the value function clearly must be linear in N as well, $\hat{V}_{it}(N) = \xi_{it} N$ for some ξ_{it}.

Now consider the full problem with flows. Flows are exogenous, so the fund's decisions do not affect future flows. Hence, decisions are independent of future flows, and the value obtained by the fund is equal to $\xi_{it} N$ plus the present value of flows. The full value function is then $V_{it}(N) = \xi_{it} N + \zeta_{it}$ for some process ζ_{it}. □

With this proposition, it is possible to solve the fund's problem. Suppose that ξ_{it} and ζ_{it} follow Ito processes,

$$d\xi_{it} = \mu_{it}^{\xi} dt + \boldsymbol{\sigma}_{it}^{\xi} d\boldsymbol{Z}_t,$$

$$d\zeta_{it} = \mu_{it}^{\zeta} dt + \boldsymbol{\sigma}_{it}^{\zeta} d\boldsymbol{Z}_t.$$

Then the HJB equation is

$$0 = \max_{\boldsymbol{\omega}_i} \Big(\delta_{it} + \mu_t^\Lambda \xi_{it} + \mu_{it}^\xi + \xi_{it}\big(r_t + (\boldsymbol{r}_t - r_t)\cdot \boldsymbol{\omega}_i - \delta_{it} - \varphi_i(\boldsymbol{\omega}_i - \boldsymbol{\omega}_i^*)\big) \\ + \boldsymbol{\omega}_i^T \boldsymbol{\Sigma}_t^r (\boldsymbol{\sigma}_{it}^\xi + \xi_{it}\boldsymbol{\sigma}_t^\Lambda) + \boldsymbol{\sigma}_t^\Lambda \cdot \boldsymbol{\sigma}_{it}^\xi\Big) N + \mu_t^\Lambda \zeta_{it} + \xi_{it} F_{it} + \mu_{it}^\zeta + \boldsymbol{\sigma}_t^\Lambda \cdot \boldsymbol{\sigma}_{it}^\zeta.$$

The optimal portfolio weights are independent of net worth N_{it}. They satisfy

$$\nabla\varphi_i(\boldsymbol{\omega}_{it} - \boldsymbol{\omega}_i^*) = \boldsymbol{r}_t + \boldsymbol{\Sigma}_t^r (\frac{\boldsymbol{\sigma}_{it}^\xi}{\xi_{it}} + \boldsymbol{\sigma}_t^\Lambda) - r_t. \tag{33}$$

Then, the processes ξ_{it}, ζ_{it} are defined by

$$0 = \delta_{it} + \xi_{it}\big(r_t + \boldsymbol{\omega}_{it}\cdot\nabla\varphi_i(\boldsymbol{\omega}_{it} - \boldsymbol{\omega}_i^*) - \varphi_i(\boldsymbol{\omega}_{it} - \boldsymbol{\omega}_i^*) - \delta_{it}\big) + \frac{1}{dt}\,\mathbb{E}_t\big[\frac{d(\Lambda_t \xi_{it})}{\Lambda_t}\big], \tag{34}$$

$$0 = \xi_{it} F_{it} + \frac{1}{dt}\,\mathbb{E}_t\big[\frac{d(\Lambda_t \zeta_{it})}{\Lambda_t}\big]. \tag{35}$$

It is possible to solve for ξ_{it} and ζ_{it} in terms of the household's SDF, asset returns, and the fund's portfolio decision. Let H_{it} denote the quantity of savings left in the fund per unit invested at $t = 0$, so

$$\frac{dH_{it}}{H_{it}} = \big(r_t - \delta_{it} - \varphi_i(\boldsymbol{\omega}_{it} - \boldsymbol{\omega}_i^*)\big)dt + \boldsymbol{\omega}_{it}\cdot(d\boldsymbol{R}_t - r_t dt).$$

Then

Lemma 3. *The exogenous processes ξ_{it}, ζ_{it} in the fund's value function $V_{it}(N) = \xi_{it} N + \zeta_{it}$ satisfy*

$$\xi_{it} = \mathbb{E}_t\Big[\int_0^\infty \frac{\Lambda_{t+s} H_{i,t+s}}{\Lambda_t H_{it}} \delta_{it} ds\Big], \tag{36}$$

$$\zeta_{it} = \mathbb{E}_t\Big[\int_0^\infty \frac{\Lambda_{t+s}}{\Lambda_t} \xi_{i,t+s} F_{i,t+s} ds\Big]. \tag{37}$$

A.2 The arbitrageur's problem

The risk-adjusted mean and variance of the arbitrageur's payoffs are

$$\mathbb{E}_t \big[X_t \cdot \frac{d(\Lambda_t \boldsymbol{R}_t)}{\Lambda_t}\big] = \boldsymbol{X}_t \cdot (\boldsymbol{r}_t - r_t + \Sigma_t^r \boldsymbol{\sigma}_t^\Lambda)dt$$

$$= \boldsymbol{X}_t \cdot \boldsymbol{\Sigma}_t^r(\boldsymbol{\eta}_t + \boldsymbol{\sigma}_t^\Lambda)dt,$$

$$Var_t(X_t \cdot d\boldsymbol{R}_t) = \boldsymbol{X}_t^T \boldsymbol{\Sigma}_t^r \boldsymbol{\Sigma}_t^{rT} \boldsymbol{X}_t dt.$$

Then the arbitrageur's problem can be written as

$$\max_{\boldsymbol{X}_t} \boldsymbol{X}_t \cdot \boldsymbol{\Sigma}_t^r(\boldsymbol{\eta}_t + \boldsymbol{\sigma}_t^\Lambda) - \frac{1}{2}\alpha_t \boldsymbol{X}_t^T \boldsymbol{\Sigma}_t^r \boldsymbol{\Sigma}_t^{rT} \boldsymbol{X}_t.$$

The first-order condition is

$$\boldsymbol{\Sigma}_t^{rT} \boldsymbol{X}_t = \frac{1}{\alpha_t}(\boldsymbol{\eta}_t + \boldsymbol{\sigma}_t^\Lambda), \tag{38}$$

as claimed.

A.3 The household's problem

This section solves the household's problem. Before doing so, I derive some useful properties of Duffie-Epstein utility.

Lemma 4. *Let f_{Ct}, f_{Ut} denote the derivatives of the aggregator $f(\cdot,\cdot)$ with respect to its first and second arguments, respectively. Then the household's stochastic discount factor Λ_t satisfies*

$$\Lambda_t = \exp\big(\int_0^t f_{Us} ds\big) f_{Ct}. \tag{39}$$

Proof. Consider a perturbation of the path of consumption, $\{C_{t+s}\} \to \{C_{t+s} + dC_{t+s}\}$. Then, taking the total derivative of (3),

$$dU_t = \int_0^\infty \big(f_{C,t+s} dC_{t+s} + f_{U,t+s} dU_{t+s}\big) ds$$

$$= \int_0^\infty \exp\big(\int_0^s f_{U,t+u} du\big) f_{C,t+s} dC_{t+s} ds$$

Hence, the marginal increase in utility U_0 given a one-unit increase in consumption C_t is

$$\Lambda_t = \exp\big(\int_0^t f_{Us} ds\big) f_{Ct}.$$

□

The household's value function $U(B, \boldsymbol{S}, Y, g)$ depends on the aggregate state variables (Y_t, g_t), bond holdings B_t, and savings $\boldsymbol{S}_t$ in funds. Write aggregate transfers dT_t to the household as

$$dT_t = \mu_t^T dt + \boldsymbol{\sigma}_t^T \cdot d\boldsymbol{Z}_t.$$

Then, the loadings of the relevant state variables $(B, S_1, S_2, \ldots, S_I, Y, g)$ on the exogenous shocks can be written as

$$\boldsymbol{\Sigma}_t^H(\boldsymbol{S}) = \begin{pmatrix} \boldsymbol{\sigma}_t^T \\ \cdots \\ S_{it}\boldsymbol{\sigma}_{it} \\ \cdots \\ (\sigma^Y, 0) \\ (0, \sigma^g) \end{pmatrix},$$

where $\boldsymbol{\sigma}_{it}$ denotes the return volatility for fund i. Denote the columns of this matrix by $\boldsymbol{\Sigma}_{Yt}^H, \boldsymbol{\Sigma}_{gt}^H$. The HJB equation can then be written as

$$\begin{aligned} 0 = \max_{C, \boldsymbol{F}} \ & f(C, U) + \big(r_t B + \boldsymbol{\delta}_t \cdot \boldsymbol{S} - C - \sum_{i=1}^{I} (F_i + \frac{1}{2}\chi_{it} F_i^2) + \mu_t^T\big) U_B \\ & + \sum_{i=1}^{I} \Big(\big(r_{it} - \delta_{it} - \varphi_i(\boldsymbol{\omega}_{it} - \boldsymbol{\omega}_i^*)\big) S_i + F_i\Big) U_i + g_t U_Y + \kappa(\overline{g} - g_t) U_g \\ & + \frac{1}{2}\boldsymbol{\Sigma}_{Yt}^H(\boldsymbol{S})^T \Delta U \boldsymbol{\Sigma}_{Yt}^H(\boldsymbol{S}) + \frac{1}{2}\boldsymbol{\Sigma}_{gt}^H(\boldsymbol{S})^T \Delta U \boldsymbol{\Sigma}_{gt}^H(\boldsymbol{S}), \end{aligned}$$

where the t subscript denotes dependence on aggregate states and ΔU denotes the Hessian of the value function.

The first-order conditions are

$$\begin{aligned} (C): \ & U_{Bt} = f_{Ct} = \frac{\rho C_t^{-\psi}}{\big((1-\gamma) U_t\big)^{(\gamma - \psi^{-1})/(1-\gamma)}}, \\ (F_i): \ & U_{it} = (1 + \chi_{it} F_{it}) U_{Bt}. \end{aligned}$$

The envelope conditions for the individual state variables are

$$\begin{aligned}
(B):\ & 0 = f_{Ut}U_{Bt} + r_t U_{Bt} + \frac{1}{dt}\,\mathbb{E}_t[dU_{Bt}] \\
(S_i):\ & 0 = f_{Ut}U_{it} + \delta_{it}U_{Bt} + \big(r_{it} - \delta_{it} - \varphi_i(\boldsymbol{\omega}_{it} - \boldsymbol{\omega}_i^*)\big)U_{it} \\
& + \sum_{k\in\{Y,g\}} \sigma_t^{i,k}\Big(\sigma_t^{B,k}U_{Bit} + \frac{1}{2}\sigma_t^{i,k}U_{iit} + \sum_{i'\neq i}\sigma_t^{i',k}U_{ii't}\Big) \\
& + \sigma_t^{i,Y}\sigma^Y U_{iYt} + \sigma_t^{i,g}\sigma^g U_{igt} + \frac{1}{dt}\,\mathbb{E}_t[dU_{it}].
\end{aligned}$$

The envelope condition for B combined with the first-order condition for C yields

$$r_t dt = -f_{Ut}dt - \mathbb{E}_t\,\Big[\frac{df_{Ct}}{f_{Ct}}\Big] = -\,\mathbb{E}_t\,\Big[\frac{d\Lambda_t}{\Lambda_t}\Big], \tag{40}$$

where the second equality comes from (39).

The envelope condition for S_i can be solved forward to obtain

$$\begin{aligned}
U_{it} &= \mathbb{E}_t\Bigg[\int_0^\infty \exp\Big(\int_0^s f_{Uu}du\Big)\frac{H_{i,t+s}}{H_{it}} f_{C,t+s}\delta_{it}ds\Bigg] \\
&= \mathbb{E}_t\Bigg[\int_0^\infty \frac{\Lambda_{t+s}}{\Lambda_t}\frac{H_{i,t+s}}{H_{it}}\delta_{it}ds\Bigg] = \xi_{it}f_{Ct}
\end{aligned}$$

where H_{it} is the cumulative return on fund i and ξ_{it} is the marginal value of wealth for fund i, as defined in the previous section. Hence, combining this envelope condition with the first-order condition for flows into fund i,

$$F_{it} = \frac{\xi_{it} - 1}{\chi_{it}}. \tag{41}$$

A.4 Equilibrium dynamics

I now derive the dynamics of the key endogenous state variables: the fraction of aggregate net worth $\nu_{it} \equiv \frac{N_{it}}{Q_t^K}$ held by each intermediary. By Ito's lemma, ν_{it} evolves according to

$$\begin{aligned}
\frac{d\nu_{it}}{\nu_{it}} &= \frac{dN_{it}}{N_{it}} - \frac{dQ_t^K}{Q_t^K} + \frac{(dQ_t^K)^2}{Q_t^K} - \frac{dN_{it}}{N_{it}}\frac{dQ_t^K}{Q_t^K} \\
&= \frac{dN_{it}}{N_{it}} - dR_t^K + \frac{Y_t}{Q_t^K}dt + (dR_t^K)^2 - \frac{dN_{it}}{N_{it}}dR_t^K
\end{aligned}$$

Then, replacing terms,

$$\frac{d\nu_{it}}{\nu_{it}} = \left(\frac{Y_t}{Q_t^K} - \delta_{it} + (\boldsymbol{\omega}_{it} - \boldsymbol{\zeta}) \cdot \boldsymbol{\Sigma}_t^r \boldsymbol{\eta} - \varphi_i(\boldsymbol{\omega}_{it} - \omega_i^*) + \boldsymbol{\sigma}_t^Q \cdot (\boldsymbol{\sigma}_t^Q - \boldsymbol{\Sigma}_t^{rT} \boldsymbol{\omega}_{it}) + \frac{F_{it}}{N_{it}}\right) dt + (\boldsymbol{\omega}_{it}^T \boldsymbol{\Sigma}_t^r - \boldsymbol{\sigma}_t^{QT}) d\boldsymbol{Z}_t \quad (42)$$

Combining (36) and (41), flows satisfy

$$\begin{aligned} f_{it} &= \frac{1}{\chi_i} \mathbb{E}_t \left[\int_0^\infty \frac{\Lambda_{t+s} H_{i,t+s}}{\Lambda_t H_{it}} \left(\delta_{i,t+s} ds + \mathbb{E}_{t+s} \left[\frac{d(\Lambda_{t+s} H_{i,t+s})}{\Lambda_{t+s} H_{i,t+s}} \right] \right) \right] \\ &= \frac{1}{\chi_i} \mathbb{E}_t \left[\int_0^\infty \frac{\Lambda_{t+s} H_{i,t+s}}{\Lambda_t H_{it}} \left(\boldsymbol{\omega}_{i,t+s}^T \boldsymbol{\Sigma}_{t+s}^r (\boldsymbol{\eta}_{t+s} + \boldsymbol{\sigma}_{t+s}^\Lambda) - \varphi_i(\boldsymbol{\omega}_{i,t+s} - \boldsymbol{\omega}_i^*) \right) ds \right] \end{aligned}$$

where $f_{it} \equiv \frac{F_{it}}{Y_t}$ denotes flows normalized by aggregate output. Taking derivatives of both sides,

$$\left(\delta_{it} - rx_{it}\right) f_{it} dt + \left(\boldsymbol{\sigma}_t^\Lambda + \boldsymbol{\omega}_{it}^T \boldsymbol{\Sigma}_t^r\right) \boldsymbol{\sigma}_t^{f_i} dt = \frac{rx_{it}}{\chi_i} dt + \mathbb{E}_t[df_{it}], \quad (43)$$

where $\boldsymbol{\sigma}_t^{f_t}$ denotes the volatility of f_{it}, and

$$rx_{it} \equiv \boldsymbol{\omega}_{it}^T \boldsymbol{\Sigma}_t^r (\boldsymbol{\eta}_t + \boldsymbol{\sigma}_t^\Lambda) - \varphi_i(\boldsymbol{\omega}_{it} - \boldsymbol{\omega}_i^*)$$

denotes the risk-adjusted expected excess return on fund i.

B Log-linear equilibrium

This section derives the balanced growth path and the log-linear equilibrium conditions. Throughout, I conjecture that volatilities of variables remain constant over time and drop all terms of order $O(\sigma^2)$ or higher, where both σ^Y and σ^g are assumed to be of order $O(\sigma)$. Importantly, risk prices $\hat{\boldsymbol{\eta}}_t$ will be O(1), so that terms of the form $\boldsymbol{\Sigma}^r \hat{\boldsymbol{\eta}}_t$ are $O(\sigma)$.

B.1 The balanced growth path

I conjecture a balanced growth path along which

- All assets deliver the same return $r_t = r^* \equiv \rho + (\psi + \theta - 1)\overline{g}$, and risk prices $\boldsymbol{\eta}_t$ are equal to zero;
- The price of capital grows at rate $\overline{g}$, $Q_t^K = e^{\overline{g}t} Q_0^K$, and the price-dividend ratio of capital is constant at some level $pd_t^K = pd^{K*}$

- Funds' net worth grows at rate $\overline{g}$, $N_{it} = e^{\overline{g}t} N_{i0}$, and net worth shares remain constant at $\nu_i^* \equiv \frac{N_{it}}{Q_{it}^K}$;

- The marginal value of wealth in funds is equal to one, $\xi_{it} = 1$, and funds' portfolio weights are at their target values, $\boldsymbol{\omega}_{it} = \boldsymbol{\omega}_i^*$;

- Household flows into funds are equal to zero, $F_{it} = 0$, and the arbitrageur holds no assets, $\boldsymbol{X}_t = 0$.

Since no-arbitrage holds, and returns are riskless along the balanced growth path, all assets must yield the same return r^*. The steady-state interest rate is equal to $\rho + (\psi + \theta - 1)\overline{g}$ by the household's Euler equation (18).

The price-dividend ratio of capital obeys the Gordon growth formula

$$pd^{K*} = \frac{1}{r^* - \overline{g}}.$$

This implies that the price of capital grows at $\overline{g}$, the rate of output growth.

Funds earn a constant return r on their assets and pay out dividends at rate $\delta_i^* = \frac{1}{pd^{K*}} = r^* - \overline{g}$. Hence, under the conjecture that funds do not receive inflows from the household, their net worth grows at rate $\overline{g}$, just like the price of capital, so net worth shares ν_i^* are constant. From the previous section, funds' value function satisfies

$$\xi_{it} = \mathbb{E}_t \left[\int_0^\infty \exp(-(r^* - \overline{g})s) \delta_i^* ds \right] = 1.$$

Since $\xi_{it} = 1$, (41) immediately implies that flows are zero, $F_{it} = 0$. Moreover, since no asset earns excess returns, (33) implies that all funds hold their target portfolios, $\boldsymbol{\omega}_{it} = \boldsymbol{\omega}_i^*$.

Finally, it is necessary to find the market-clearing net worth shares given that the arbitrageur holds no assets. The market-clearing equation (14) becomes

$$\sum_{i=1}^{I} \nu_i^* \boldsymbol{\omega}_i^* = \boldsymbol{\zeta}.$$

Hence,

$$\boldsymbol{\nu}^* = \boldsymbol{\Omega}^{T^{-1}} \boldsymbol{\zeta},$$

where $\boldsymbol{\Omega}_{[i,j]} = \omega_{ij}^*$ represents the matrix of target portfolio weights.

B.2 The fund's problem

The first step is to linearize the fund's first-order condition (33). Taking a first-order Taylor expansion and dropping all terms of order $O(\sigma^2)$,

$$\big(\Delta\varphi_i\big)\big|_{\boldsymbol{\omega}_i=\boldsymbol{\omega}_i^*}\hat{\boldsymbol{\omega}}_{it} = \hat{\boldsymbol{r}}_t - \hat{r}_t, \tag{44}$$

where $\Delta\varphi_i$ is the Hessian matrix of φ_i. Inverting the Hessian matrix and using the no-arbitrage relationship $\hat{\boldsymbol{r}}_t - \hat{r}_t = \boldsymbol{\Sigma}^r\hat{\boldsymbol{\eta}}_t$, I obtain (19).

Next, consider the fund's marginal value of wealth ξ_{it}. Note that along a balanced growth path, $\xi_{it} = 1$, so $\hat{\xi}_{it} \equiv \xi_{it} - 1$ is $O(\sigma)$. Linearize the fund's HJB equation (34):

$$\begin{aligned}0 &= \delta_i^* + \hat{\delta}_{it} + (1+\hat{\xi}_{it})\big(r^* + \hat{r}_t + (\boldsymbol{\omega}_i^* + \hat{\boldsymbol{\omega}}_{it})\cdot(\nabla\varphi_i(\mathbf{0}) + \Delta\varphi_i(\mathbf{0})\hat{\boldsymbol{\omega}}_{it}) \\ &\quad - (\varphi_i(\mathbf{0}) + \nabla\varphi_i(\mathbf{0})\cdot\hat{\boldsymbol{\omega}}_{it}) - \delta_i^* - \hat{\delta}_{it}\big) + \mathbb{E}_t[\frac{d\Lambda_t}{\Lambda_t}] + \mathbb{E}_t[d\xi_{it}] \\ &= (r^* - \delta_i^*)\hat{\xi}_{it} + \boldsymbol{\omega}_i^*\Delta\varphi_i(\mathbf{0})\hat{\boldsymbol{\omega}}_{it} + \dot{\hat{\xi}}_{it}\end{aligned}$$

Rearranging,

$$(\delta_i^* - r^*)\hat{\xi}_{it} = \boldsymbol{\omega}_i^*\cdot\boldsymbol{\Sigma}^r\hat{\boldsymbol{\eta}}_t + \dot{\hat{\xi}}_{it}. \tag{45}$$

The equation uses the fund's optimality condition (19).

B.3 The arbitrageur's problem and financial market clearing

Log-linearization of the arbitrageur's problem is simple. I take $\alpha = \frac{\overline{\alpha}}{\sigma}$ so that the arbitrageur's risk aversion goes to infinity as σ goes to zero. I let $\hat{\boldsymbol{x}}_t \equiv \frac{\boldsymbol{X}_t}{Q^{K*}}$ denote the arbitrageur's normalized net position. The optimality condition (12) can be linearized as

$$\boldsymbol{\Sigma}^{rT}\hat{\boldsymbol{x}}_t = \frac{1}{\alpha}\hat{\boldsymbol{\eta}}_t.$$

The left-hand side is $O(\sigma)$, since $\frac{1}{\alpha}$ is $O(\sigma)$ and $\hat{\boldsymbol{\eta}}$ is $O(1)$.

The market-clearing equation (14) can then be linearized as

$$
\begin{aligned}
0 &= \boldsymbol{\Sigma}^{rT}\hat{\boldsymbol{x}}_t + \boldsymbol{\Sigma}^{rT}\sum_{i=1}^{I}\nu_i^*(\boldsymbol{\omega}_i^* + \hat{\boldsymbol{\omega}}_{it})(1+\hat{\nu}_{it}) - \boldsymbol{\sigma}^K \\
&= \boldsymbol{\Sigma}^{rT}\hat{\boldsymbol{x}}_t + \boldsymbol{\Sigma}^{rT}\sum_{i=1}^{I}\nu_i^*(\boldsymbol{\omega}_i^*\hat{\nu}_{it} + \hat{\boldsymbol{\omega}}_{it}) \\
&= \boldsymbol{\Sigma}^{rT}\sum_{i=1}^{I}\nu_i^*\boldsymbol{\omega}_i^*\hat{\nu}_{it} + \left(\frac{1}{\alpha}\boldsymbol{I} + \boldsymbol{\Sigma}^{rT}\sum_{i=1}^{I}\nu_i^*\boldsymbol{\varepsilon}_i\boldsymbol{\Sigma}^r\right)\hat{\boldsymbol{\eta}}_t \\
&= \boldsymbol{\Sigma}^{rt}\hat{\boldsymbol{d}}_t + \boldsymbol{\Sigma}^{rT}\boldsymbol{\varepsilon}_{agg}\boldsymbol{\Sigma}^r\hat{\boldsymbol{\eta}}_t.
\end{aligned}
$$

The second line uses the fact that the market-clearing condition holds along the balanced growth path. The third line uses the expressions for the arbitrageur's and the fund's optimal portfolios.

B.4 The household's problem and the dynamics of net worth

The Euler equation simplifies considerably under the first-order approximation, since all second-order (Ito) terms drop out. The interest rate satisfies

$$\hat{r}_t = (\psi + \theta - 1)\hat{g}_t.$$

Define normalized flows into fund i as

$$\hat{f}_{it} \equiv \frac{F_{it}}{N_{it}} \approx \frac{F_{it}}{\nu_i^* pd^{K*} Y_t}.$$

Per (41),

$$\hat{f}_{it} = \frac{\hat{\xi}_{it}}{\frac{1}{Y_t}\chi_i \times \nu_i^* pd^{K*} Y_t} = \frac{\hat{\xi}_{it}}{\nu_i^* pd^{K*}\chi_i}.$$

Using (45),

$$(\delta_i^* - r^*)\hat{f}_{it} = \frac{\boldsymbol{\omega}_i^{*T}\boldsymbol{\Sigma}^r\hat{\boldsymbol{\eta}}_t}{\nu_i^* pd^{K*}\chi_i} + \dot{\hat{f}}_{it}. \tag{46}$$

Then, with this equation for flows, it is possible to derive the dynamics of wealth shares. Linearizing (42), dropping second-order terms, and taking expectations,

$$
\begin{aligned}
\dot{\hat{\nu}}_{it} &\approx \frac{Y_{it}}{Q_t^K} - \delta_{it} + \left((\boldsymbol{\omega}_i^* + \hat{\boldsymbol{\omega}}_{it})^T - \boldsymbol{\zeta}^T\right)\boldsymbol{\Sigma}^r\hat{\boldsymbol{\eta}}_t + \hat{f}_{it} \\
&= (\boldsymbol{\omega}_i^* - \boldsymbol{\zeta})^T\boldsymbol{\Sigma}^r\hat{\boldsymbol{\eta}}_t + \hat{f}_{it}.
\end{aligned}
$$

B.5 Asset pricing

Consider an asset that pays cash flows z_t^j, and define its price-dividend ratio as $pd_t^j \equiv \frac{Q_t^j}{z_t^j}$. Note that

$$\frac{dpd_t^j}{pd_t^j} = \frac{dQ_t^j}{Q_t^j} - \frac{dz_t^j}{z_t^j} + \frac{(dz_t^j)^2}{z_t^j} - \frac{dz_t^j dQ_t^j}{z_t^j Q_t^j}.$$

Linearizing and dropping second-order terms, the asset pricing equation (13) can be written as

$$\begin{aligned}(r^* + \hat{r}_t + \boldsymbol{\sigma}^j \cdot \hat{\boldsymbol{\eta}}_t) pd^{j*}(1 + \hat{pd}_t^j) &= 1 + \frac{1}{dt} pd^{j*}(1 + \hat{pd}_t^j)\,\mathbb{E}_t[\frac{dQ_t^j}{Q_t^j}] \\ &\approx 1 + \frac{1}{dt} pd^{j*}(1 + \hat{pd}_t^j)\,\mathbb{E}_t[\frac{dpd_t^j}{pd_t^j} + \frac{dz_t^j}{z_t^j}] \\ (r^* - g^{j*})\hat{pd}_t^j &= \hat{g}_t^j - \hat{r}_t - \boldsymbol{\sigma}^j \cdot \hat{\boldsymbol{\eta}}_t + \dot{\hat{pd}}_t^j.\end{aligned}$$

B.6 The economy's response to shocks

I now derive the economy's response to an arbitrary shock. I extend the model to accommodate three additional types of shocks: *flow shocks* in which flows deviate from those dictated by the household's optimization problem, *demand shocks* in which intermediary asset demand departs from (19), and *supply shocks* in which the supply of each asset changes. There may also be arbitrary shocks $\hat{\boldsymbol{g}}_t^z$ to asset cash flow growth rates.

Let $\boldsymbol{\epsilon}_{it}^{\omega}$ denote the deviation of fund i's portfolio weight from the optimal choice,

$$\hat{\omega}_{it} = \hat{\omega}_{it}^{opt} + \epsilon_{it}^{\omega} = \varepsilon_i \boldsymbol{\Sigma}^r \hat{\boldsymbol{\eta}}_t + \epsilon_{it}^{\omega}.$$

Let $\boldsymbol{\epsilon}_t^s$ denote the log deviation of asset supply (in units of goods) from the firm's benchmark issuance decision. Then the market clearing condition can be linearized as

$$\boldsymbol{\Sigma}^{rT}\big(\sum_{i=1}^{I} \nu_i^*(\hat{\nu}_{it}\boldsymbol{\omega}_i^* + \epsilon_i^{\omega}) - \boldsymbol{\epsilon}_t^s\big) + \boldsymbol{\varepsilon}_{agg}\hat{\boldsymbol{\eta}}_t = 0. \tag{47}$$

The equation for flows becomes

$$r^* \hat{f}_{it} = \frac{\boldsymbol{\omega}_i^T \boldsymbol{\Sigma}^r \hat{\boldsymbol{\eta}}_t}{\chi_i} + \epsilon_{it}^f + \dot{\hat{f}}_{it}. \tag{48}$$

Per the linearized asset pricing equation (25), asset prices in this model depend on three factors: the shock to cash flow growth rates $\hat{\boldsymbol{g}}_t^z$, the shock to interest rates $\hat{r}_t$, and the shock to risk prices $\hat{\boldsymbol{\eta}}_t$. In a frictionless version of the model (either $\chi_i = 0$ for all i or $\boldsymbol{\varepsilon}_{agg}^{-1} = \mathbf{0}$), risk prices are equal to zero, so all variation in price-dividend ratios is due to variation in dividend growth rates or risk-free interest rates (caused by shocks to the aggregate growth rate $\hat{g}_t$). Shocks to dividend growth rates or interest rates can transmit to risk prices by changing asset prices and wealth shares, but shocks to asset supply or flows cannot transmit back to dividend growth rates or interest rates.

In what follows, it will be useful to write

$$\hat{\boldsymbol{d}}_t = \boldsymbol{\Omega}_\nu^T \hat{\boldsymbol{\nu}}_t,$$

where $\boldsymbol{\Omega}_{\nu,[i,j]} = \nu_i^* \omega_{ij}^*$, and to define

$$\hat{\boldsymbol{f}}_t^d \equiv \boldsymbol{\Omega}_\nu^T \hat{\boldsymbol{f}}_t.$$

Correspondingly, let

$$\boldsymbol{\epsilon}_t^{fd} \equiv \boldsymbol{\Omega}_\nu^T \boldsymbol{\epsilon}_t^f, \boldsymbol{\epsilon}_t^{\omega d} \equiv \sum_{i=1}^{I} \nu_i^* \boldsymbol{\epsilon}_{it}^{\omega}.$$

The equilibrium system determining asset prices can be written in matrix form:

$$\begin{pmatrix} \dot{\hat{\boldsymbol{pd}}}_t \\ \dot{\hat{g}}_t \\ \dot{\hat{\boldsymbol{d}}}_t \\ \dot{\hat{\boldsymbol{f}}}_t^d \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \hat{\boldsymbol{pd}}_t \\ \hat{g}_t \\ \hat{\boldsymbol{d}}_t \\ \hat{\boldsymbol{f}}_t^d \end{pmatrix} + \boldsymbol{B} \begin{pmatrix} \hat{\boldsymbol{g}}_t^z \\ \boldsymbol{\epsilon}_t^{\omega d} \\ \boldsymbol{\epsilon}_t^s \\ \boldsymbol{\epsilon}_t^{fd} \end{pmatrix}, \tag{49}$$

where

$$\boldsymbol{A} = \begin{pmatrix} r^*\boldsymbol{I} - diag(\boldsymbol{g}^{z*}) & \psi + \theta - 1 & -\boldsymbol{\Sigma}^r \boldsymbol{\varepsilon}_{agg}^{-1} \boldsymbol{\Sigma}^{rT} & \boldsymbol{0} \\ \boldsymbol{0} & -\kappa & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & 0 & -\boldsymbol{\Omega}_\nu(\boldsymbol{\Omega} - diag(\boldsymbol{\zeta}))^T \boldsymbol{\Sigma}^r \boldsymbol{\varepsilon}_{agg}^{-1} \boldsymbol{\Sigma}^{rT} & \boldsymbol{I} \\ \boldsymbol{0} & 0 & \boldsymbol{\Omega}_\nu diag(\boldsymbol{\chi})^{-1} \boldsymbol{\Omega}^T \boldsymbol{\Sigma}^r \varepsilon_{agg}^{-1} \boldsymbol{\Sigma}^{rT} & r^*\boldsymbol{I} \end{pmatrix},$$

$$\boldsymbol{B} = \begin{pmatrix} -\boldsymbol{I} & -\boldsymbol{\Sigma}^r \boldsymbol{\varepsilon}_{agg}^{-1} \boldsymbol{\Sigma}^{rT} & \boldsymbol{\Sigma}^r \boldsymbol{\varepsilon}_{agg}^{-1} \boldsymbol{\Sigma}^{rT} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{\Omega}_\nu(\boldsymbol{\Omega} - diag(\boldsymbol{\zeta}))^T \boldsymbol{\Sigma}^r \boldsymbol{\varepsilon}_{agg}^{-1} \boldsymbol{\Sigma}^{rT} & \boldsymbol{\Omega}_\nu(\boldsymbol{\Omega} - diag(\boldsymbol{\zeta}))^T \boldsymbol{\Sigma}^r \boldsymbol{\varepsilon}_{agg}^{-1} \boldsymbol{\Sigma}^{rT} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Omega}_\nu diag(\boldsymbol{\chi})^{-1} \boldsymbol{\Omega}^T \boldsymbol{\Sigma}^r \varepsilon_{agg}^{-1} \boldsymbol{\Sigma}^{rT} & -\boldsymbol{\Omega}_\nu diag(\boldsymbol{\chi})^{-1} \boldsymbol{\Omega}^T \boldsymbol{\Sigma}^r \varepsilon_{agg}^{-1} \boldsymbol{\Sigma}^{rT} & -\boldsymbol{I} \end{pmatrix},$$

and $diag(\mathbf{v})$ denotes the diagonal matrix formed from the entries of a vector $\mathbf{v}$.

This is a system of the form

$$\dot{\boldsymbol{v}}_t = \boldsymbol{A}\boldsymbol{v}_t + \boldsymbol{B}\epsilon_t, \tag{50}$$

which has a unique solution for a given initial value $\boldsymbol{v}_0$. As usual for first-order linear ODEs of this form, the solution can be composed as

$$\boldsymbol{v}_t = \boldsymbol{v}_t^I + \boldsymbol{v}_t^H,$$

where $\boldsymbol{v}_t^I$ is an inhomogeneous solution with $\boldsymbol{v}_0^I = \mathbf{0}$ that depends only on the shocks ϵ_t and $\boldsymbol{v}_0^H$ is a homogeneous solution that depends only on the initial state.

Proof. Solving (25) forward, the vector of price-dividend ratios can be written as

$$\begin{aligned}
\hat{\boldsymbol{pd}}_t &= \int_0^\infty \exp\big((r^* - \boldsymbol{g}^{z*})s\big)\Big(\hat{\boldsymbol{g}}^z_{t+s} - \hat{r}_{t+s} - \boldsymbol{\Sigma}^r\hat{\boldsymbol{\eta}}_{t+s}\Big)ds \\
&= \int_0^\infty \exp\big((r^* - \boldsymbol{g}^{z*})s\big)\Big(\hat{\boldsymbol{g}}^z_{t+s} - \hat{r}_{t+s} - \boldsymbol{\Sigma}^r\boldsymbol{\varepsilon}_{agg}\boldsymbol{\Sigma}^{rT}(\hat{\boldsymbol{d}}_{t+s} + \boldsymbol{\epsilon}^d_{t+s})\Big)ds \\
&= \int_0^\infty \exp\big((r^* - \boldsymbol{g}^{z*})s\big)\big(\hat{\boldsymbol{g}}^z_{t+s} - \hat{r}_{t+s}\big)ds - \int_0^\infty \exp\big((r^* - \boldsymbol{g}^{z*})s\big)\boldsymbol{\Sigma}^r\boldsymbol{\varepsilon}_{agg}\boldsymbol{\Sigma}^{rT}\boldsymbol{\epsilon}^d_{t+s}ds \\
&\quad - \int_0^\infty \exp\big((r^* - \boldsymbol{g}^{z*})s\big)\boldsymbol{\Sigma}^r\boldsymbol{\varepsilon}_{agg}\boldsymbol{\Sigma}^{rT}\hat{\boldsymbol{d}}_{t+s}ds
\end{aligned}$$

The first two terms are $\hat{\boldsymbol{pd}}_t^{frictionless}$ and $\hat{\boldsymbol{pd}}_t^{demand}$, defined in Proposition 1. It remains to show that the third term has the desired form. Clearly, the last term is the homogeneous solution to (50), since it depends only on the endogenous state variable $\hat{\boldsymbol{\nu}}_t$ and not the exogenous shocks. Therefore, it suffices to show that $\hat{\boldsymbol{\nu}}_0$ has the desired form.

The vector of unanticipated asset returns at $t = 0$ is

$$\begin{aligned}
d\boldsymbol{R}_0 &= \hat{\boldsymbol{z}}_0 + \hat{\boldsymbol{pd}}_0 \\
&= \hat{\boldsymbol{z}}_0 + \hat{\boldsymbol{pd}}_0^{frictionless} + \hat{\boldsymbol{pd}}_0^{demand} + \hat{\boldsymbol{pd}}_0^{redist.} \\
&= \hat{\boldsymbol{z}}_0 + \hat{\boldsymbol{pd}}_0^{frictionless} + \hat{\boldsymbol{pd}}_0^{demand} + \boldsymbol{P}\hat{\boldsymbol{\nu}}_0
\end{aligned}$$

and the change in net worth shares at $t = 0$ satisfies

$$\hat{\boldsymbol{\nu}}_0 = \big(\boldsymbol{\Omega} - \mathbf{1}\boldsymbol{\zeta}^T\big)d\boldsymbol{R}_0. \tag{51}$$

Combining these two relationships, we obtain

$$\hat{\boldsymbol{\nu}}_0 = \big(\boldsymbol{I} - \big(\boldsymbol{\Omega} - \mathbf{1}\boldsymbol{\zeta}^T\big)\boldsymbol{P}\big)^{-1}\big(\boldsymbol{\Omega} - \mathbf{1}\boldsymbol{\zeta}^T\big)\big(\hat{\boldsymbol{z}}_0 + \hat{\boldsymbol{pd}}_0^{frictionless} + \hat{\boldsymbol{pd}}_0^{demand}\big),$$

as desired. □