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Polarized Contributions but Convergent Agendas*

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Abstract

In a canonical model of policy formation, campaign contributions, and electoral competition, we show that, despite donor polarization, candidates' agendas converge. If purely office-motivated candidates move away from the centrist agenda, they increase their opponents' contributions more than their own. An extension that introduces a "job ladder" for the candidates leads to candidates caring about absolute levels of campaign contributions and generates divergence of political agendas in equilibrium. We provide empirical evidence of campaign contributions affecting candidates' chances of "promotion," and characterize key comparative statics of the extended model. In the model, caps on campaign contributions lower polarization in equilibrium.

Keywords: Polarization; Campaign Contributions; Agendas

JEL Codes: D72, H41

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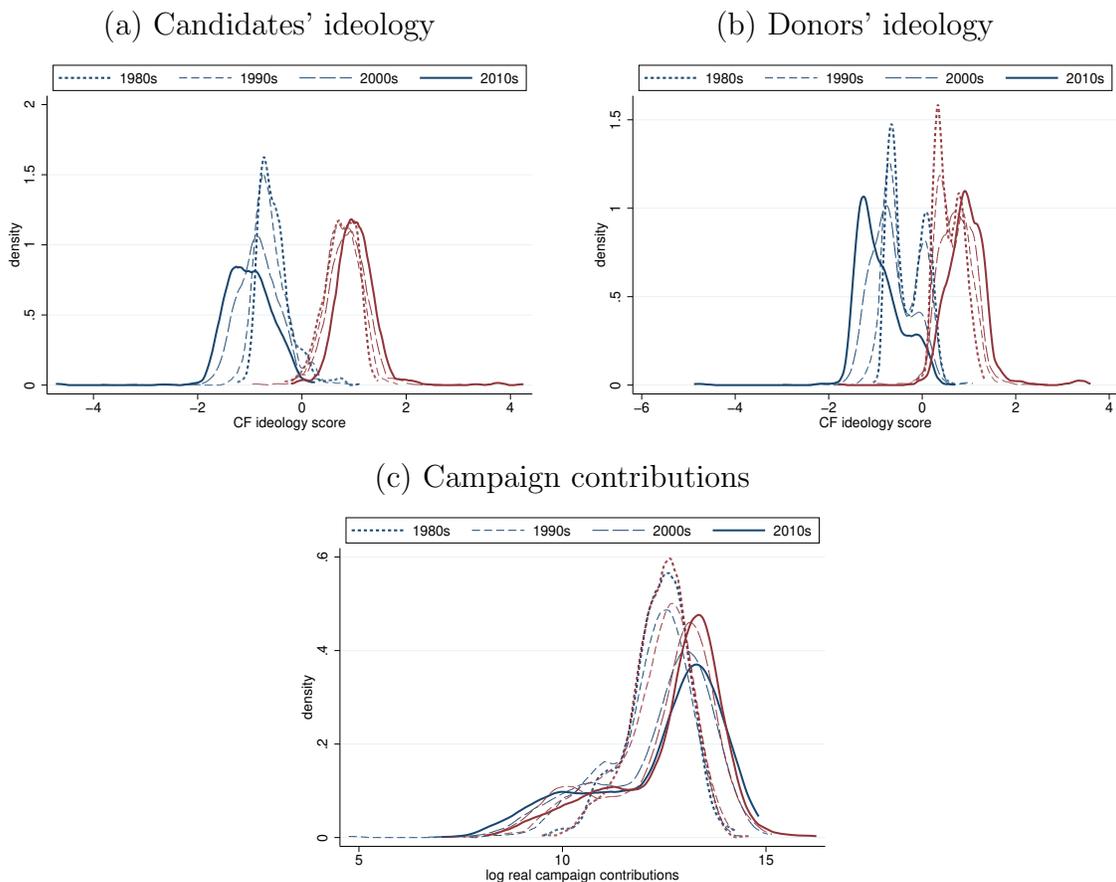
1 Introduction

The political process in the United States has become increasingly polarized in recent decades. Legislators are adopting increasingly different political positions (e.g., [McCarty et al. \(2006\)](#), [Bonica \(2014\)](#), [Gentzkow et al. \(2019\)](#)). The increased polarization of political candidates has also coincided with the increased polarization of donors and with increased campaign contributions. This is illustrated in panels (a) through (c) in [Figure 1](#), which uses data on the distribution of campaign finance scores from [Bonica \(2016\)](#) to show that distributions for both candidates and donors have shifted out over time.¹ Work on the role of ideologically extreme campaign donors in influencing the polarization of elected politicians (e.g., [La Raja and Wiltse \(2012\)](#), [Bonica et al. \(2013\)](#), [Barber \(2016\)](#), [La Raja and Schaffner \(2015\)](#), [Broockman and Malhotra \(2020\)](#)) suggests a possible connection between campaign donations and polarization. This, in turn, has implications for debates about the desirability of campaign finance reform ([Mann and Corrado \(2014\)](#), [Pildes \(2020\)](#), [Vandewalker \(2020\)](#)).

Motivated by these observations, we present a theoretical model of policy formation, campaign contributions, and electoral success. Our theory can be used to study the connection between campaign contributions and political polarization. The model builds on the analysis of the canonical model of collectivist policies with uninformed voters in [Baron \(1994\)](#). In our model, two political candidates who only value winning office first commit to a policy agenda. Lobby groups who differ in their preference for policies then decide whether and how much to contribute to each candidate’s campaign. Candidates use the contributions to attract the votes of uninformed citizens. As extreme interest groups value more extreme policies, there may be an incentive for candidates to commit to more polarized agendas in order to elicit larger contributions from more polarized interest groups.

In the model, tailoring policy positions to match (extreme) contributors’ preferences increases the amount of contributions received by the politician. However, polarizing one’s agenda affects the opposing candidate’s contributions as well. Un-

¹Figure 1 plots densities of the campaign finance scores from [Bonica \(2016\)](#) for the four decades since 1980, separately for candidates (panel (a)) and donors (panel (b)), and separately for Republicans and Democrats. Panel (c) plots the corresponding distribution of real campaign contributions, the mass of which has shifted to the right over time.



Data sources: [Bonica \(2016\)](#); FRED. We use data on all district-regular general election pairs with both a Democratic and a Republican candidate with at least eight donors in the data set. Campaign contributions are deflated by the core CPI index. See Appendix A for details. Red (blue) denotes the distribution for Republicans (Democrats).

Figure 1: The distribution of donors' and candidates' ideology scores has shifted away from the center over time, as campaign contributions increased: U.S. House of Representatives general election candidates, 1980–2014.

der the standard assumption that the donors' loss functions are convex, the increase in the opponent's contributions is larger than the increase in the candidate's own campaign chest in any symmetric equilibrium.² Hence, candidates choose to moderate their policy agenda so as to improve the *relative* size of their campaign contributions.

²In Appendix A, we show that the main mechanism in our model – that contributions are a centripetal force – is consistent with the data: Using the data for U.S. House races from [Bonica \(2016\)](#), which also underlies Figure 1, we show that a candidate's share of contributions is falling in the extremity of her own agenda, but rising in the extremity of her opponent's agenda.

In the canonical model, politicians care only about the probability of winning office. If this probability is symmetric in both their own and their rival’s campaign contributions, the result is complete convergence of agendas to the midpoint between the ideal policies of the two extreme lobby groups. With identical policies, lobby groups are indifferent between candidates, and equilibrium campaign contributions are zero. The key mechanism is that candidates care as much about their opponent’s contributions as they do about their own.

Two extensions illustrate the robustness of the main result. In the first extension, we endogenize the policy preferences of the contributing lobbies by introducing many potential donors whose preferred policies span the policy spectrum. We show that the model embeds a natural incentive for campaign contributions to become completely polarized. As more extreme lobby groups care more intensely about policies than do moderate lobby groups, moderate lobby groups find it optimal to “free ride” on the campaign contributions of more extreme lobby groups. This prediction that free-riding leads to a polarization of political contributors is also consistent with the data. Several papers support this notion: [La Raja and Schaffner \(2015, ch. 2, ch. 3\)](#) and [Bafumi and Herron \(2010\)](#) show that individual donors tend to come from ideological extremes. [Broockman and Malhotra \(2020\)](#) show not only that party donors hold more extreme views than party voters, but also that these differences are more pronounced among the top 1% of donors.³ Notably, this endogenous polarization of the lobbies does not lead to any polarization of the agendas in our benchmark model.

In a second, separate extension of the benchmark model, we follow [Baron \(1994\)](#) and add informed voters to the model. Unsurprisingly, the equilibrium of this enriched model features both candidates choosing identical agendas in between the median voter’s preferred policy and the mid-point of the two extreme lobbies.

In order to explain the significant amounts of observed polarization of both political agendas and donors, we then consider an intuitive departure from the canonical model, which makes the candidates care about the absolute level of their contributions. This preference for the *absolute level* of contributions emerges natu-

³It is unclear how the ideology of institutional donors compares with that of the underlying population. However, [La Raja and Schaffner \(2015, ch. 3\)](#) show also that issue groups and, taken together, unions and business groups, tend to donate funds to ideologically polarized candidates, while party committees tend to support centrist candidates.

rally from a two-period extension of our baseline model, where campaign spending in the first period affects the candidate’s probability of running for a higher office in the second period. We borrow the basic idea from the theory of “progressive political ambition” (Shafer 2018). This theory, which has its roots in the work of Schlesinger (1966), asserts that office holders strive for higher office. We label this mechanism a “political job ladder”. Focusing on the probability of U.S. Representatives to be “promoted” to run for Senate, we find that this probability is robustly related to funds raised in the current period: Given a baseline probability of promotion of 3%, doubling the fundraising receipts would increase the probability of a Senate run by about 1 percentage point.

Extended in this way, the model generates divergence of candidates’ agendas in equilibrium and yields predictions that are in line with three key empirical facts: (1) Candidates strategically set agendas to mobilize contributions (La Raja and Wiltse 2012). (2) Increased contribution limits for individual donors lead to more polarized candidates in U.S. states (Barber 2016). (3) Polarization of donors and candidates has risen in parallel, as shown in Figure 1, which is based on data from Bonica (2016).

It is worth noting that making the candidates’ positions “sticky” across the two elections can act as an additional centripetal force. A sufficient condition for this to happen is that the candidates are so polarized, that the lobbies would prefer more centrist agendas in order to enhance their candidate’s winning probability. This reinforces the logic that increasing one’s own *absolute* level of contributions comes at a cost of increasing the opponent’s contributions even more, thus damaging one’s chances of winning the election.

Literature. In an important early contribution, Austen-Smith (1987) considered competition between two rival lobbying groups and established a convergence result for candidate agendas. We generalize the convergence result to a situation in which the identity and number of contributors is endogenous, and where free-riding leads to divergence of donor groups. We also establish the limits of this result once candidates are allowed to value the absolute level of their contributions

— independent of whether there are two or more lobbies.⁴ The intuitive idea that donors and their campaign contributions may lead to polarization is present in [Baron \(1994\)](#) and [Persson and Tabellini \(2000\)](#) among others. Unlike in [Baron \(1994\)](#), candidates in our model recognize that their polarization increases their opponent’s contributions *more* than their own. While [Baron \(1994\)](#) attributes the convergence of agendas to the presence of informed voters, we show that the convergence arises even in the absence of informed voters.

An active literature analyzes models that can explain the empirically documented polarization of politicians. Our divergence result, which is grounded in a preference for the absolute level of contributions as an incentive to please extreme donors, is distinct from other explanations that rely on divergent candidate policy preferences ([Roemer \(1991\)](#), [Lindbeck and Weibull \(1993\)](#), [Herrera et al. \(2008\)](#), or [Krasa and Polborn \(2014\)](#)). Apart from preferences, the literature has proposed uncertainty about candidate type ([Bernhardt and Ingberman \(1985\)](#)), or the threat of a third-party candidate’s entry ([Palfrey \(1984\)](#) as explanations for divergent agendas; see also the survey by [Osborne \(1995\)](#)). More closely related are papers that analyze polarization and campaign finance. [Le and Yalcin \(2018\)](#) argue that lobbying can pull candidates’ agendas away from the median voter, but they do so in a model with the opposite timing, where the donor(s) move first, offering contributions in exchange for a specified policy. In contrast, we show that when candidates choose their platforms and recognize the effect of this choice on both their own and their opponent’s contributions, then lobbying becomes a centripetal force. In the context of particularistic policies in a two-dimensional policy space, [Konishi and Pan \(2020\)](#) show that campaign contributions by a single interest group can lead to polarization. Note that [Dekel et al. \(2008, 2009\)](#) study campaign spending with fixed agendas, while [Jackson et al. \(2007\)](#) establish the possibility of agenda divergence in a model where candidates spend their own resources in the absence of lobbyists.

The remainder of the paper is organized as follows. Section 2 describes the benchmark model. Section 3 establishes the convergence of the agendas in the equilibrium of that model, highlights its robustness to two separate extensions of the model, and discusses the key driving force behind it. Section 4 then in-

⁴The number of lobbies in our framework does not affect the results. The key is only whether candidates have a preference for absolute level of contributions.

troduces the job-ladder model, where candidates value contributions above and beyond their effect on the probability of winning the current election, offers empirical support for this micro-foundation and characterizes the equilibrium with policy divergence, analyzes comparative statics of the extended model, and derives implications for the effects of contribution limits. Section 5 concludes. All proofs are in the appendix.

2 Environment

We begin by presenting the simplest model that captures the key mechanism. The baseline game consists of two politicians (candidates), indexed by $i \in \{1, 2\}$, who choose their policy agendas, and two lobbies (potential donors, who are identified by their policy positions $j \in \{L, R\}$), who contribute to candidates' campaigns, thus influencing the probabilities of the election outcome. Specifically, the policy space is the unit interval. In the first stage of the game the two office-motivated candidates simultaneously choose their respective policy positions (agendas) $a \in [0, 1]$ and commit to them. In the second stage, having observed the agenda choices of the two candidates, the two donors simultaneously choose their campaign contributions c . Lastly, the "election" takes place, with probabilities of winning p determined by the respective campaign spending of the two candidates. Figure 2 illustrates the timeline of the game and the respective problems of the agents.

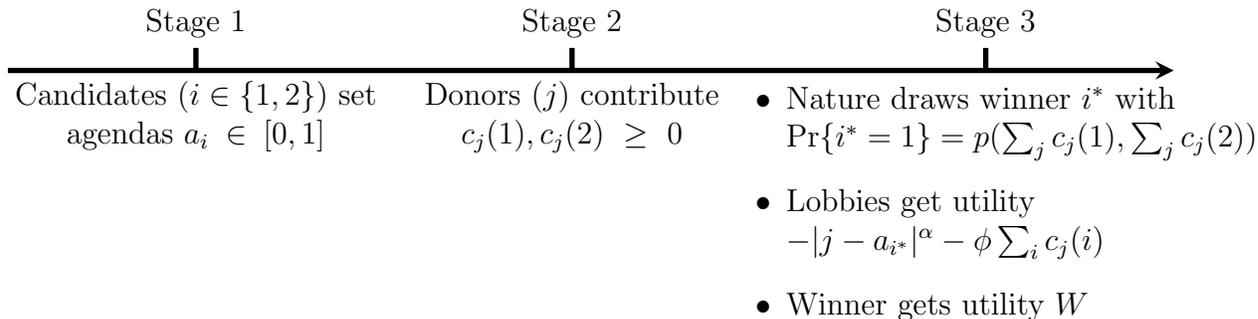


Figure 2: Timeline, actions, and payoffs.

The candidates care only about winning the election (we relax this assumption

in Section 4). We denote the value of a win by $W > 0$. The winning probability p depends only on campaign spending C_i of the candidates, which is equal to the sum of contributions they receive from the lobbies $C_i = \sum_j c_j(i)$, i.e., it does not depend on the agendas directly (we relax this assumption in Appendix C, where we analyze the case with both informed and uninformed voters as in Baron (1994)). Winning probabilities are symmetric, sum to 1, and follow the following functional form:

Assumption 1 *The winning probability function is given by:*

$$p(C_i, C_{-i}) = (1 - \psi) \frac{1}{2} + \psi \frac{g(\bar{C}, C_i)}{g(\bar{C}, C_i) + g(\bar{C}, C_{-i})}, \quad (2.1)$$

where $g(\bar{C}, C_i) = \left((1 - \omega)^{1/\rho} \bar{C}^{1-1/\rho} + \omega^{1/\rho} C_i^{1-1/\rho} \right)^{\frac{\rho}{\rho-1}}$, $\bar{C} > 0$, $\rho > 1$, $\omega \in (0, 1)$, and $\psi \in (0, 1]$.

The parameter \bar{C} makes the winning probability a continuous function of contributions. Without the \bar{C} , this contest function would be a Tullock function (Levine and Mattozzi 2022) with a discontinuity at 0 contribution level — when the opponent’s contributions are 0, infinitesimal contribution to a candidate moves their probability of winning from $\frac{1}{2}$ to $\frac{1}{2}(1 + \psi)$.⁵ Here, ψ represents the share of swing voters, i.e., those affected by campaign spending.⁶

There are two lobbies with opposite policy preferences. In slight abuse of notation, we identify the lobbies by their preferred policy positions — the left lobby is $j = L$ and the right lobby is $j = R$, with $0 \leq L < R \leq 1$. Section 3.3 points out that the game with two extreme contributors is a reduced form of an environment with multiple potential donors and free-riding, as only the most motivated (i.e., extreme) donors possibly donate in any subgame. A lobby j ’s preferences over agendas, a , are represented by

$$U_j(a) = -|a - j|^\alpha, \quad (2.2)$$

with a common $\alpha > 1$. Lobbies have a linear utility cost of funds $\phi \sum_i c_j(i)$, $\phi > 0$.

⁵We thank John Duggan for pointing out the undesirable behavior of the Tullock function in this context.

⁶In mapping the model of Baron (1994) into our framework in Appendix C, ψ represents the share of uninformed voters.

3 Benchmark result: Convergent agendas

We focus on pure strategy subgame perfect equilibria and solve the model by backward induction. We first establish that lobbies contribute in a subgame equilibrium if and only if candidates' agendas are not identical. We then show that, despite this fact, the equilibrium agendas of the candidates converge at the midpoint between the two (extreme) lobbies. To streamline the exposition, we start by assuming that lobbies can contribute to only one candidate. We later show that it is never optimal to give to both candidates (Lemma 2).

3.1 Preliminary analysis: Campaign contributions

Consider the problem of a lobby j who is considering contributing to their preferred candidate i . The lobby solves the following problem, taking as given the opponent's campaign fund C_{-i} :

$$\max_{c \geq 0} -p_i(c, C_{-i})|a_i - j|^\alpha - (1 - p_i(c, C_{-i}))|a_{-i} - j|^\alpha - \phi c,$$

which can be restated as

$$\max_{c \geq 0} p_i(c, C_{-i})\Delta_j(a_i, a_{-i}) - \phi c,$$

where $\Delta_j(a_i, a_{-i}) = U_j(a_i) - U_j(a_{-i}) = |a_{-i} - j|^\alpha - |a_i - j|^\alpha$ is the added benefit to lobby j of policy a_i over policy a_{-i} .

Under Assumption 1, the marginal impact of the first dollar of contributions on the candidate's probability of winning $\frac{\partial p_i}{\partial C}|_{C=0}$ is infinite, which means that in any subgame where candidates agendas are not identical, the equilibrium levels of contributions are pinned down by the lobbies' first order conditions:

Lemma 1 *Contributions are strictly positive if and only if there is divergence of agendas. For any subgame with $a_l < a_r$, there is a pure strategy equilibrium with contributions given by*

$$\frac{\partial p_l(C_l, C_r)}{\partial C_l} \Delta_L(a_l, a_r) = \phi, \quad \frac{\partial p_r(C_r, C_l)}{\partial C_r} \Delta_R(a_r, a_l) = \phi. \quad (3.1)$$

Note that choosing an agenda closer to the contributing lobby's bliss point increases the intensity of its preference for the candidate (i.e., increases Δ) and thus results in larger level of contributions (i.e., lower marginal impact $\frac{\partial p_i}{\partial C}$ in equation (3.1)) around the symmetric equilibrium.

And yet, as we show next, the candidates choose the same centrist agenda in equilibrium. The key is that what matters for electoral success is not the absolute level of one's contributions, but rather the gains relative to the opponent's. These gains combine the given name recognition and the endogenous contributions. This can be illustrated by combining the two equations in (3.1) into

$$\frac{(1-\omega)^{1/\rho}\bar{C}^{1-1/\rho}C_l^{1/\rho} + \omega^{1/\rho}C_l}{(1-\omega)^{1/\rho}\bar{C}^{1-1/\rho}C_r^{1/\rho} + \omega^{1/\rho}C_r} = \frac{\Delta_L(a_l, a_r)}{\Delta_R(a_r, a_l)}, \quad (3.2)$$

which highlights that the key determinant of the contributions is the *relative* gains for the lobbies.

3.2 Main theorem

One key insight from the analysis above is that the contributions to the candidates are pinned down by the intensity of the preferences of the extreme lobbies for their proposed policies, $\frac{\Delta_L(a_l, a_r)}{\Delta_R(a_r, a_l)}$. The main result of our paper is that this generates a centripetal force on the candidates' agendas. Taking a more extreme position increases the intensity of the contributing lobby's preferences and thus their contributions. However, moving toward the extreme increases the opposing lobby's dislike for the candidate even more and thus increases the opposing candidate's contributions more than the polarizing candidate's own contributions.

Theorem 1 *In the unique pure strategy subgame perfect equilibrium, both candidates choose the policy agenda in the middle of the two extreme lobbies, $j_m = \frac{L+R}{2}$.*

The formal proof is in Appendix B, but the intuition for this key result is very straightforward: polarizing one's agenda (moving towards the extreme lobby) increases one's contributions, but it increases the opponent's contributions more.

This can be seen from the candidates' first order condition:

$$\frac{\partial p_i}{\partial C_i} \frac{\partial C_i}{\partial \Delta_j} \frac{\partial \Delta_j}{\partial a_i} + \frac{\partial p_i}{\partial C_{-i}} \frac{\partial C_{-i}}{\partial \Delta_{-j}} \frac{\partial \Delta_{-j}}{\partial a_i} = 0,$$

where j is the identity of the lobby contributing to candidate i . Thus, in any symmetric pure strategy equilibrium, we must have⁷

$$\frac{\partial p_i}{\partial C_i} \frac{\partial C_i}{\partial \Delta_j} \left(\frac{\partial \Delta_j}{\partial a_i} - \frac{\partial \Delta_{-j}}{\partial a_i} \right) = 0. \quad (3.3)$$

But note that the term in the parentheses is negative for any positive level of polarization, since we assumed $\alpha > 1$.

The key mechanism behind the centripetal result of Theorem 1 is that the candidates understand the impact of their policy position on both their and their opponents' campaign contributions (and thus winning probability) and that picking an extreme agenda increases the opponents' contributions more than the candidates' own. Importantly, we find empirical support for this mechanism by considering the contributions to US Congressional election campaigns between 1980 and 2014 (see Appendix A.2).

3.3 Robustness of the Main Result: Key Extensions

Our central finding of the convergence of agendas in equilibrium is robust to two important extensions of our model — incorporating informed voters, as in Baron (1994), and endogenizing the policy preferences of the contributing lobbies.

Incorporating informed voters (those not influenced by campaign spending) into the model means that the probability of winning the election depends directly on the candidates' policy positions. This creates an additional centripetal force pulling agendas towards the median voter's preferred policy. We show in Appendix C that the resulting equilibrium of the model with both informed and uninformed voters is policy convergence to a point between the median voter's preferred policy and the mid-point j_m of the two lobbies.

We next turn to the model extension that effectively endogenizes the policy

⁷In addition to symmetry, we use that $\frac{\partial p_i}{\partial C_{-i}} = -\frac{\partial p_i}{\partial C_i}$ from Lemma D.4 in the Appendix.

preferences of the contributing lobbies in the above model.

This model extension shows that having two extreme lobbies arises as the equilibrium outcome of an environment with multiple lobbies that may donate, as long as these lobbies have a constant marginal cost of funds. This is the result of free-riding amongst the potential contributors, as only the most motivated lobbies end up making contributions; and due to the convexity of the lobbies' preferences, the most motivated lobbies are the ones with the most extreme policy preferences (bliss points).

Consider a version of our benchmark model with a finite (and possibly large) number $N > 2$ of potential lobbies. Each lobby is identified with (and indexed by) its preferred agenda $j \in [0, 1]$. We will denote the left-most lobby by L and the right-most lobby by R , with $L < R$. Lobby j 's preferences over agendas, a , are represented by

$$U_j(a) = -|a - j|^\alpha, \quad (3.4)$$

with a common $\alpha > 1$. Donors have a linear utility cost of funds $\phi \sum c_i$.

We now establish that, in any subgame where candidates' agendas are not identical, the contributions are completely polarized — only the most extreme lobbies provide contributions. Consider the problem of a lobby j who is considering contributing to candidate 1. The lobby solves the following problem, taking as given the opponent's campaign fund $C_2 = \sum_k c_k(2)$ and the total contributions $C_1(-j) = \sum_{k \neq j} c_k(1)$ of other lobbies to candidate 1's campaign:

$$\max_{c \geq 0} p_1(c + C_1(-j), C_2) \Delta_j(a_1, a_2) - \phi c,$$

where $\Delta_j(a_1, a_2) = U_j(a_1) - U_j(a_2) = |a_2 - j|^\alpha - |a_1 - j|^\alpha$ is the added benefit to lobby j of policy a_1 over policy a_2 . This is a well-behaved convex problem, with the first-order condition for an optimum given by

$$\frac{\partial p_1(C_1, C_2)}{\partial C_1} \Delta_j(a_1, a_2) \leq \phi, \quad (3.5)$$

and symmetrically for contributions to candidate 2

$$\frac{\partial p_2(C_2, C_1)}{\partial C_2} \Delta_j(a_2, a_1) \leq \phi. \quad (3.6)$$

Each of these conditions holds with equality if the contribution by lobby j to the corresponding candidate ($i = 1$ or $i = 2$) is positive.

Unsurprisingly, since lobbies contribute only because they care about the difference between candidates, they contribute to at most one candidate, the one whose platform is closer to their bliss point j .

Lemma 2 *No lobby ever makes positive contributions to both candidates.*

Next, we establish that there is “free riding”: Centrist lobbies have weaker preferences over agendas than the more extreme lobbies, and thus derive a smaller benefit from influencing election probabilities via donations. Since they have the same cost of funds as the extremists, the centrist lobbies don’t contribute. This logic leads to the following key result:⁸

Theorem 2 *Only the extreme lobbies possibly contribute in any subgame. That is, in every subgame, $C_\ell = c_L$ and $C_r = c_R$, where $a_\ell \leq a_r$.*

In the statement of the theorem and from now on, we index the candidates’ agendas and contributions by ℓ and r , since their identity does not matter. In other words, in slight abuse of notation, we switch from $i \in \{1, 2\}$ to $i \in \{\ell, r\}$.

This means that the environment with multiple lobbies that could donate is observationally equivalent to that with just the two most extreme lobbies. In addition, arbitrarily small divergence of candidates is consistent with full polarization of donating lobbies. Contributions are polarized, but agendas are convergent.

4 Preference for contribution levels: Political job ladder

In order to generate policy divergence in equilibrium, we need to alter the environment so that candidates care not only about their *relative* contributions but

⁸The stark nature of this result is softened under a more general cost function for lobbies, as more than just the most extreme lobbies may contribute. However, free riding amongst lobbies remains a centrifugal force, as more extreme lobbies contribute more. As we show in a previous working paper version, for a convex cost of funds $\phi c^\varepsilon, \varepsilon \geq 1$, all lobbies who are not indifferent contribute in equilibrium, but contributions are increasing in Δ_j .

also about the *absolute* level of contributions.⁹ We do so by introducing dynamics: Candidates aspire to higher office during the next election. Candidates can increase the probability of being “promoted” to run for the higher office by increasing their contributions. We first present empirical evidence in support of this phenomenon and then present the augmented two-period model incorporating this mechanism.

4.1 Evidence of political job ladders

Building on the foundational work on “Ambition and Politics: Political Careers in the United States” (Schlesinger 1966), the literature on “progressive political ambition” assumes that current office holders strive for higher office and analyzes, for example, how this ambition affects current behaviors (Shafer 2018). In line with this reasoning, we analyze whether current fundraising affects a candidate’s probability of running for higher office in the future.

We show that data on the U.S. House of Representatives is consistent with the hypothesis that current fundraising increases the probability of running for higher office in a future general election. We focus on the U.S. Senate as the more desirable office, and we focus on members of the House as potential candidates.¹⁰ We use data on fundraising and elections from DIME (Bonica 2016).

We focus on candidates who raised positive amounts of funds and whose state had a Senate election in two years.¹¹ Our sample begins in 1980 and ends in 2014. We have up to 4,500 observations (Representative-years), depending on the covariates.¹²

Figure 3 shows the data in the form of binned scatter plots of the probability

⁹In a previous working paper version, we showed that breaking the symmetry by introducing differences in \bar{C} could also yield policy divergence.

¹⁰Borchert and Copeland (2003) call membership in Congress the “most highly prized office” in the U.S. and stress the importance of potential campaign contributors as gatekeepers. Their descriptive empirical work focuses on career paths leading up to becoming a Representative and they document heterogeneous backgrounds. We focus on transitions from the House to the Senate, between two bodies of professional legislators.

¹¹We do this to account for the fact that aside from special elections only about one third of Senate seats and thus about two thirds of U.S. states have a Senate election every two years.

¹²If we had perfect data and all states were of equal size, we would expect to have $435 \times \frac{2}{3} \times \left(\frac{2014-1980}{2} + 1\right) \approx 10150$ observations. The actual number can be higher because of special elections and lower because Senate elections from small states are available only to a few Representatives as well as missing data.

of a Senate run against the log of real receipts (in \$1,000) on the horizontal axis. Overall, the probability of a Senate run averages 3% and real receipts average about \$450,000, corresponding to a natural log of six after transforming to \$1,000. The probability of a Senate run for a Representative in the bottom 10% (left-most two markers) of the fundraising distribution is around 1% with contributions around \$90,000. It rises to 4.5% for a Representative in the top 10% (two right-most markers) with contributions across the two bins averaging around \$1.2 million.¹³

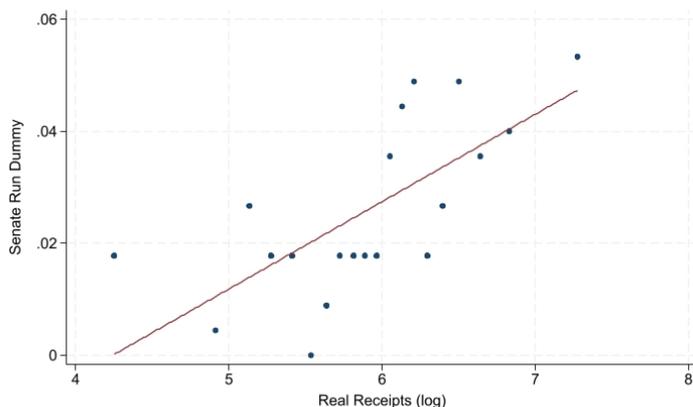


Figure 3: Campaign donations and Senate Run Probability: Binned scatter plot

The results remain economically and statistically significant across different and more complex specifications. To account for the fact that the ratio of House districts to Senate seats is much larger in big states than in small states, we control for the number of districts in most specifications. Moreover, we control for candidate party, gender, and incumbency status.

Table 1 shows the estimates of a linear probability model. There, the first row lists the point estimates of the semi-elasticity. It ranges from 0.014 to 0.016 for most specifications, implying that doubling the fundraising receipts would increase the probability of a Senate run by 1.0 percentage point to 1.1 percentage points ($\ln(2) \times 0.014 \approx 0.01$ and $\ln(2) \times 0.016 \approx 0.011$). This is a large effect relative to unconditional Senate run probability of 3%. This effect is statistically significant across all specifications.

The results are robust to different specifications and subsamples. One plausible concern is that the linear probability model is inappropriate for modeling near-zero

¹³Figure D.1 shows results with logit and probit transforms, and with state fixed effects.

Table 1: Campaign donations & Senate run probability: Linear probability model

	(1)	(2)	(3)	(4)	(5)
Real Receipts (log)	0.014*** (0.004)	0.016*** (0.005)	0.016*** (0.005)	0.015*** (0.005)	0.015*** (0.005)
# districts in state			-0.001 (0.001)	-0.001 (0.001)	-0.001* (0.001)
Republican dummy				0.014** (0.006)	0.017* (0.009)
District partisanship					0.001 (0.002)
District partisanship \times party					-0.008 (0.008)
R-Squared	0.04	0.05	0.05	0.05	0.05
Inside [0,1]	0.91	0.83	0.83	0.81	0.80
Fixed effects	State	St, Yr	St, Yr	St, Yr	St, Yr
States	49	49	49	49	49
Observations	4500	4500	4500	4500	4487

probabilities. While the baseline model predicts proper probabilities for 91% of the sample, this share falls to 80% for the most complex model in Table 1. By making functional form assumptions, Tables D.1 and D.2 in Appendix A.2 show that analogous results hold when using logit or probit models. The effects are slightly weaker in years without presidential elections, but after conditioning on year fixed effects the difference largely disappears (Table D.3). Last, the results are driven by current fundraising rather than fundraising in prior periods (Table D.4).¹⁴

4.2 Environment

To incorporate this political job-ladder mechanism into our framework and to generate divergence of agendas in equilibrium, consider the following two-period repeated version of the benchmark model. The game in the first period is exactly as described in Section 2. The winner of the election in the first period is promoted to an election for a higher office in period 2 with probability $\pi(C_i)$. In that event, the

¹⁴Adding disbursements (expenditures) as an additional control to the models in Table 1 strengthens the effect on donations. However, the two are nearly collinear, making the coefficients hard to interpret. Taken at face value, disbursements lower the “promotion” probability while donations raise them. Our regressions show the net effect of increasing both donations and disbursements.

candidates are committed to their period-1 agenda with probability λ , but get to choose a new agenda with complementary probability. This assumption is meant to capture the idea that the key issue in the election may change dramatically from one electoral cycle to the next, making previous policy pledges of candidates less relevant.¹⁵ In what follows, we assume that the identity of the (extreme) lobbies remains unchanged across the two periods, and that both the lobbies and the politicians discount the future at the rate β .¹⁶ The value of the election victory may vary over time (with natural interpretation that $W_2 \geq W_1$), as well as the intensity of policy preferences of the lobbies (which is scaled up by a factor s in period 2) and their marginal cost of funds ϕ , which becomes ϕ' . For completeness, we need to specify what the lobbies expect to happen if their candidates do not get promoted to the period-2 elections. We assume that in this case the candidates are promoted from another ex ante identical electoral district. In our analysis, we focus on symmetric equilibria.

Naturally, we require that the probability of promotion is strictly increasing in contributions:

Assumption 2 *The probability of promotion is strictly increasing in contributions: $\frac{\partial \pi(c)}{\partial c} > 0$.*

To simplify the exposition, we impose $\alpha = 2$ in what follows.

4.3 Analysis

We begin with observations that the problems of both the lobbies and the candidates in the second period are identical to the ones in our benchmark model above (unless the candidates' agendas are fixed and carry over from period 1, in which case the candidates simply do not make any choices). Thus, Lemmata 1 and 2

¹⁵In just the last few years, there are plenty of examples of such changes in the dominant theme of elections: with Canada's election swinging from immigration and environmental policy discussions to debating the response to the Trump administration's policies instead, with US midterm elections swinging from economic policy to the abortion debate after Roe v Wade was overturned, and with the European political debate switching from green policies to the response to Russian aggression in Ukraine, just to name a few.

¹⁶Alternative assumption that the set of lobbies in the second election is a distinct one would only simplify the analysis, but not change the key results.

apply in period 2, and Theorem 1 applies if the candidates get to choose their agendas.

We thus focus on the problems of the lobbies and candidates in period 1. The respective problems are as follows. Taking the candidates' positions (and the opposing lobbies' contributions) as given, an extreme lobby chooses the level of contribution to maximize the following value function:

$$\begin{aligned}
V_1 = \max_{C_i} & p_i(C_i, C_{-i}) (U_i(a_i) + \beta\lambda (\pi(C_i)V_2(a_i, \bar{a}_{-i}) + (1 - \pi(C_i))V_2(\bar{a}_i, \bar{a}_{-i}))) \\
& + (1 - p_i(C_i, C_{-i})) (U_i(a_{-i}) + \beta\lambda (\pi(C_{-i})V_2(\bar{a}_i, a_{-i}) \\
& + (1 - \pi(C_{-i}))V_2(\bar{a}_i, \bar{a}_{-i}))) - \phi C_i, \tag{4.1}
\end{aligned}$$

where \bar{a}_i denotes the period-1 policy positions of candidates from other electoral districts and $V_2(a_l, a_r)$ is the expected value to the lobby of the subgame in period 2, when candidates' agendas are a_l and a_r :

$$V_2(a_i, a_{-i}) = \max_{C'_i \geq 0} p_i(C'_i, C'_{-i}) sU_i(a_i) + (1 - p_i(C'_i, C'_{-i})) sU_i(a_{-i}) - \phi' C'_i \tag{4.2}$$

In turn, taking the lobbies' response functions into account, the candidates choose their period-1 agendas to solve the following problem:

$$\max_{a_i \in [0,1]} p_i(C_i(a_i, a_{-i}), C_{-i}(a_i, a_{-i})) \left(W_1 + \beta\pi(C_i(a_i, a_{-i})) \left(\lambda p'_i W_2 + (1 - \lambda) \frac{W_2}{2} \right) \right),$$

where $p'_i = p(C'_i(a_i, \bar{a}_{-i}), C'_{-i}(a_i, \bar{a}_{-i}))$ is the probability of winning the election in period 2, with the contributions C' being affected by the period 1 agenda choice a_i in the event that the candidates don't get to re-optimize their policy positions.

Incorporating the lobbies' first order conditions in both periods into the candidates' first order condition in period 1 and imposing symmetry (see Lemma D.4) yields the following equilibrium equation:

$$\frac{\partial p_i}{\partial C_i} \left(\frac{\partial C_i}{\partial a_i} - \frac{\partial C_{-i}}{\partial a_i} \right) \left(W_1 + \frac{\beta\pi W_2}{2} \right) + \frac{\beta}{4} \frac{\partial \pi}{\partial C} \frac{\partial C_i}{\partial a_i} W_2 + \frac{\beta\pi\lambda W_2}{2} \frac{\partial p'_i}{\partial C'_i} \left(\frac{\partial C'_i}{\partial a_i} - \frac{\partial C'_{-i}}{\partial a_i} \right) = 0 \tag{4.3}$$

This equation implies that the probability of promotion rules out a convergent equilibrium. Note that the first and the third term in equation (4.3) resemble

the static equilibrium equation (3.3). When $\lambda = 0$, the third term drops out and the first term equals its static counterpart from Section 2 and is negative (see Lemma D.1). In contrast, the second term in equation (4.3) is positive by Assumption 2. The equation thus captures the key tradeoff — polarizing lowers the chances of winning the election but it increases the chances of being “promoted” to the second election. An additional centrifugal force arises when $\lambda > 0$. At small amounts of polarization, the lobby also prefers the candidate to move away from the center in case the candidate is promoted and the agenda is sticky. Lemma D.2 implies that the first term is then also positive: The centrifugal motive for the lobby (the first term) reinforces the centrifugal motive for the candidate (the second term). This immediately yields our desired outcome — we can rule out convergent agendas in the symmetric pure-strategy equilibrium, proving the following Theorem.

Theorem 3 *Under Assumption 2, policy convergence is not a subgame-perfect equilibrium.*

To prove uniqueness, we first focus on the case of $\lambda = 0$. We further simplify equation (4.3) by dividing by (the strictly positive) $\frac{\partial p_r}{\partial C_r} \frac{\partial C_r}{\partial a_r}$ and substituting in for the ratio of marginal contributions, which is given by Lemma D.1: $\frac{d\Delta_\ell/da_r}{d\Delta_r/da_r} = \frac{a_r - L}{R - a_r}$ when $\lambda = 0$.

$$\left(W_1 + \beta\pi(C_r) \frac{W_2}{2} \right) \left(1 - \frac{a_r - L}{R - a_r} \right) + \beta p_r \frac{W_2}{2} \frac{\pi'(C_r)}{\partial p_r / \partial C_r} = 0. \quad (4.4)$$

To make further progress, we need to specify more properties of the promotion probability function. Since promotion is also a contest, we use the same functional form for the promotion contest function as for the probability of being elected.

Assumption 3 *There are $N \geq 2$ districts whose candidates compete to run for the higher office in the second period. Let C_n denote candidate n 's contribution, with $n = 1, \dots, N$. The promotion probability of candidate m is:*

$$\pi(C_m) = \frac{1}{N}(1 - \kappa) + \kappa \frac{g(\bar{C}, C_m)}{\sum_{n=1}^N g(\bar{C}, C_n)},$$

where g is defined as in Assumption 1 and $\kappa \in (0, 1]$.

In the symmetric equilibrium across districts ($C_n = C$), it follows that

$$\pi'(C) = \kappa \frac{(N-1)g(\bar{C}, C)}{(Ng(\bar{C}, C))^2} \left(\frac{\omega g(\bar{C}, C)}{C} \right)^{\frac{1}{\rho}}.$$

Comparing this to the expression for the marginal probability of election win

$$\frac{\partial p_i}{\partial C_i} = \psi \frac{g_{-i}}{(g_i + g_{-i})^2} \left(\frac{\omega g_i}{C_i} \right)^{1/\rho},$$

we observe that in the symmetric equilibrium $\pi'(C) = \bar{\kappa} \frac{\partial p}{\partial C}$ where $\bar{\kappa} = 4 \frac{\kappa}{\psi} \frac{N-1}{N^2}$. Combining this with $p_r(C) = \frac{1}{2}$ and $\pi(C) = \frac{1}{N}$, we get that the equilibrium equation (4.4) in the fully symmetric case reduces to

$$\left(W_1 + \beta \frac{1}{N} \frac{W_2}{2} \right) \left(1 - \frac{a_r - L}{R - a_r} \right) + \beta \frac{W_2}{4} \bar{\kappa} = 0. \quad (4.5)$$

Note that the first term is zero when $a_r = \frac{L+R}{2}$ and becomes strictly negative when there is polarization. It is also strictly decreasing in a_r , while the second term is constant and positive. Thus there is a unique solution $a_r > \frac{L+R}{2}$. This proves the following theorem:

Theorem 4 *If $\lambda = 0$, there is a unique symmetric pure strategy subgame-perfect equilibrium.*

Note that Lemma D.1 implies that the more general equilibrium condition in equation (4.3) is continuous in λ . Uniqueness for $\lambda = 0$ thus implies that for sufficiently small λ the equilibrium remains unique.

Corollary 1 *If λ is sufficiently small, there is a unique symmetric pure strategy subgame-perfect equilibrium.*

Theorem D.1 in the Appendix proves existence for general λ . A sufficient condition for existence is that the probability of winning elections is not overly sensitive to contributions. This prevents the marginal continuation value of a more polarized aligned agenda ($\frac{\partial V_{2r}}{\partial a_r}$) from turning too negative. This, in turn, ensures that

the centripetal force coming from the contributions does not dominate the centrifugal force coming from the candidates via the direct effect of the probability of promotion on candidates' payoffs.

4.4 Implications

We now consider comparative statics of the unique symmetric pure strategy subgame-perfect equilibrium. We state the comparative statics for the case of $\lambda = 0$. However, since the equilibrium condition is continuous in stickiness and agendas, the findings extend to the more general model for sufficiently small values of λ .

4.4.1 Value of Holding Office, Polarization, and Contributions

First, we establish that the extent of polarization is decreasing in the relative weight of the first period election win in the candidates' objective:

Theorem 5 *If $\lambda = 0$, then increasing the relative value of holding current office $\frac{W_1}{W_2}$ (a) reduces polarization $|a_r - a_\ell|$, and (b) lowers contributions.*

The implied co-movement of candidates' polarization and amounts of campaign donations is consistent with the data presented in Figure 1.

4.4.2 Contribution Limits and Polarization

Our model framework offers a tractable way of analyzing the effects of contribution limits on political outcomes. Barber (2016) estimates that when U.S. states had relatively higher individual contribution limits in state elections, they had relatively more ideologically polarized legislators. Seen through the lenses of our model, limiting how much any given lobby is permitted to contribute changes the identity of the marginal lobby. The free-rider problem still renders most of the lobbies inactive, but as the most extreme lobbies max out their contributions, the next most extreme contributors become the deciding agents in the contribution subgame. As a result, imposing contribution limits in our model is akin to compressing the distribution of contributors, i.e., increasing L and decreasing R .

Unsurprisingly, restricting campaign contributions limits the extent of polarization obtained in the equilibrium of our general model.

Theorem 6 *If $\lambda = 0$, then a mean preserving spread in the preferences of the most extreme lobbies (a) increases polarization, and (b) increases contributions.*

An increase in contribution limits can thus potentially account for the simultaneous polarization of (donating) lobbies and candidates, and the increase in contributions shown in Figure 1.

4.4.3 Dynamic Commitment to Agendas

When agendas become stickier, there are two (potentially) opposing forces. First, statically, more polarized agendas lower marginal contributions because of the centripetal force. Second, lobbies could dynamically benefit from more polarized agendas and prefer more polarized agendas in the first period to enhance their continuation value in case the candidates wins and is promoted. As we now formally state, if the marginal value of a more polarized agenda to a lobby is negative, this is sufficient to guarantee that the static centripetal force dominates and is reinforced by higher λ .

Theorem 7 *If the marginal continuation value of a more polarized aligned agenda to a lobby is weakly negative, i.e., $\frac{\partial V_{2r}}{\partial a_r} \leq 0$, then increasing λ reduces polarization.*

The marginal continuation value may turn negative in equilibrium because it reflects two forces: The direct effect of a more polarized agenda, which is positive, and the negative effect due to a lower winning probability as the opposing lobby's contributions increase.¹⁷

5 Conclusion

Most basic models of electoral competition predict that candidate policies should converge. Yet in practice, we observe a great deal of polarization both in candidate

¹⁷Note that the existence proof for $\lambda > 0$ (Theorem D.1 in the Appendix) relies on a sufficient condition that makes the coefficient on $\frac{d\Delta_\ell}{da_r}$ in (B.2) bigger than 1. However, $\frac{dV_{2r}}{da_r} \leq 0$ can still hold in equilibrium if agendas are sufficiently polarized so that $\frac{d\Delta_\ell/da_r}{d\Delta_r/da_r}$ is sufficiently large.

policies and in the identity of the lobbies that support them. We have shown, in contrast to a widely held intuition, that polarization of donating lobbies is consistent with arbitrarily close policy convergence (albeit to a midpoint or average lobby rather than a median voter). Moreover, polarization in the form of campaign contributions from extreme lobbies arises naturally in our model as a result of the public-good characteristic of campaign contributions.

In order to generate policy divergence in our framework, we need the candidates to care about the absolute level of contributions they are raising – not just the relative amount that affects the probability of the election win. We offer an extension of our model that delivers polarization in equilibrium based on a job-ladder model, where the probability of promotion to the higher-office election is affected by the candidates’ campaign funds. This mechanism is consistent with “progressive political ambition” (Shafer 2018) and we provide empirical support for this mechanism. Comparative statics in the augmented model show that polarization of candidates is increasing in the strength of their career concern and the degree of polarization of the lobbies. Campaign contribution limits can lead to a decline in the agenda polarization in the model.

References

- AUSTEN-SMITH, D. (1987): “Interest Groups, Campaign Contributions, and Probabilistic Voting,” *Public Choice*, 54, 123–139.
- BAFUMI, J. AND M. C. HERRON (2010): “Leapfrog Representation and Extremism: A Study of American Voters and Their Members in Congress,” *American Political Science Review*, 104, 519–542.
- BARBER, M. J. (2016): “Ideological Donors, Contribution Limits, and the Polarization of American Legislatures,” *Journal of Politics*, 78, 296–310.
- BARON, D. P. (1994): “Electoral Competition with Informed and Uninformed Voters,” *American Political Science Review*, 88, 33–47.
- BERNHARDT, M. D. AND D. E. INGBERMAN (1985): “Candidate Reputations and the ‘Incumbency Effect’,” *Journal of Public Economics*, 27, 47–67.
- BONICA, A. (2014): “Mapping the Ideological Marketplace,” *American Journal of Political Science*, 58, 367–386.
- (2016): “Database on Ideology, Money in Politics, and Elections: Public version 2.0,” <https://data.stanford.edu/dime>, stanford, CA: Stanford University Libraries.
- BONICA, A., N. MCCARTY, K. T. POOLE, AND H. ROSENTHAL (2013): “Why Hasn’t Democracy Slowed Rising Inequality?” *Journal of Economic Perspectives*, 27, 103–24.
- BORCHERT, J. AND G. COPELAND (2003): “United States: A Political Class of Entrepreneurs,” in *The Political Class in Advanced Democracies: A Comparative Handbook*, Oxford University Press.
- BROCKMAN, D. AND N. MALHOTRA (2020): “What Do Partisan Donors Want?” *Public Opinion Quarterly*, 84, 104–118.
- DEKEL, E., M. O. JACKSON, AND A. WOLINSKY (2008): “Vote Buying: General Elections,” *Journal of Political Economy*, 116, 351–380.

- (2009): “Vote Buying: Legislatures and Lobbying,” *Quarterly Journal of Political Science*, 4, 103–128.
- GENTZKOW, M., J. M. SHAPIRO, AND M. TADDY (2019): “Measuring Group Differences in High-Dimensional Choices: Method and Application to Congressional Speech,” *Econometrica*, 87, 1307–1340.
- HERRERA, H., D. K. LEVINE, AND C. MARTINELLI (2008): “Policy Platforms, Campaign Spending and Voter Participation,” *Journal of Public Economics*, 92, 501–513.
- JACKSON, M. O., L. MATHEVET, AND K. MATTES (2007): “Nomination Processes and Policy Outcomes,” *Quarterly Journal of Political Science*, 2, 67–94.
- KONISHI, H. AND C.-Y. PAN (2020): “Silent Promotion of Agendas: Campaign Contributions and Ideological Polarization,” *Public Choice*, 182, 93–117.
- KRASA, S. AND M. POLBORN (2014): “Social Ideology and Taxes in a Differentiated Candidates Framework,” *American Economic Review*, 104, 308–22.
- LA RAJA, R. J. AND B. F. SCHAFFNER (2015): *Campaign Finance and Political Polarization: When Purists Prevail*, University of Michigan Press.
- LA RAJA, R. J. AND D. L. WILTSE (2012): “Don’t Blame Donors for Ideological Polarization of Political Parties: Ideological Change and Stability Among Political Contributors, 1972-2008,” *American Politics Research*, 40, 501–530.
- LE, T. AND E. YALCIN (2018): “Lobbying, Campaign Contributions, and Electoral Competition,” *European Journal of Political Economy*, 55, 559–572.
- LEVINE, D. K. AND A. MATTOZZI (2022): “Success in contests,” *Economic Theory*, 73.
- LINDBECK, A. AND J. W. WEIBULL (1993): “A Model of Political Equilibrium in a Representative Democracy,” *Journal of Public Economics*, 51, 195–209.
- MANN, T. E. AND A. CORRADO (2014): “Party Polarization and Campaign Finance,” Tech. rep., Center for Effective Public Management at Brookings.

- MCCARTY, N. M., K. T. POOLE, AND H. ROSENTHAL (2006): *Polarized America: The Dance of Ideology and Unequal Riches*, MIT Press.
- OSBORNE, M. J. (1995): “Spatial Models of Political Competition Under Plurality Rule: A Survey of Some Explanations of the Number of Candidates and the Positions They Take,” *Canadian Journal of Economics*, 28, 261–301.
- PALFREY, T. R. (1984): “Spatial Equilibrium with Entry,” *Review of Economic Studies*, 51, 139–156.
- PERSSON, T. AND G. TABELLINI (2000): *Political Economics*, MIT Press.
- PILDES, R. H. (2020): “Participation and Polarization,” *Journal of Constitutional Law*, 22, 341–408.
- ROEMER, J. E. (1991): “A Theory of Class-Differentiated Politics in an Electoral Democracy,” UC Davis - Institute of Governmental Affairs, Paper 384.
- SCHLESINGER, J. A. (1966): *Ambition and Politics: Political Careers in the United States*, Chicago: Rand McNally, topics: Political parties.
- SHAFER, K. (2018): “Political Ambition,” *Oxford Bibliographies*.
- VANDEWALKER, I. (2020): “The Benefits of Public Financing and the Myth of Polarized Small Donors,” <https://www.brennancenter.org/our-work/research-reports/benefits-public-financing-and-myth-polarized-small-donors>, brennan Center.

Appendix

A Campaign finance data

A.1 Data sources

We use the recipient level data from [Bonica \(2016\)](#). To convert contributions to real dollars, we use the core consumer price index (CPI), retrieved from <https://fred.stlouisfed.org/series/CPILFESL>.

Of all recipients, we keep only Republican and Democratic candidates for the House of Representatives, and also keep only those who are explicitly flagged as having run in the general election, or those who are incumbents, or those with general election information (either flagged as general election winners or losers, or with a positive number of votes in the general election). We also drop those without a dynamic recipient campaign finance score or with missing information on the number of givers, or with negative individual contributions. Next, we reshape the dataset into an election level dataset, keeping only observations with exactly two candidates – as some elections are uncontested, and on some occasions more than one candidate from one party may run in a general election, for example in several 2006 Texas House races. We use total receipts as our measure of campaign contributions, divided by the core CPI. For the regressions we use the logarithm of the resulting real variable. We define districts for the purpose of computing fixed effects to become a new district in 1982, 1992, 2002, and 2012 – thus allowing for redistricting.

A.2 Campaign Finance and Agendas in the Data

The key economic mechanism behind the results above is that candidates are strategic, and internalize that by setting their agenda they influence not only their own donations, but also their opponent’s donations. We now turn to the Database on Ideology, Money in Politics, and Elections (DIME) by [Bonica \(2016\)](#) to assess this mechanism. This dataset is well-suited for our purposes, because it provides information on campaign contributions along with a measure of ideology.

Specifically, we examine how the share of donations received by candidates in an election relates to their own agenda and that of their opponents.¹⁸ As in Figure 1, we focus on the general elections to the U.S. Congress when we can identify both a Republican and a Democratic candidate. Table A.1 reports estimates of how the Republican candidate’s share of campaign contributions relates to her own campaign finance score and that of her opponent.¹⁹ We estimate the regressions using ordinary least squares and cluster standard errors at the state level. In the first column, we consider all observations, in the other columns we restrict the sample to those observations where the campaign finance (CF) score is estimated using at least eight donors, a number Bonica (2016) considers the threshold for reliable estimates of the ideological position. We then add year (election) fixed effects (column (3)), district by decade fixed effects that account for redistricting (column (4)), or both fixed effects (column (5)). To interpret the magnitudes, note that Republican candidates tend to have campaign finance scores around 0.9, whereas the average Democratic campaign finance score is about -0.8.²⁰ Standard errors are clustered by state to allow for correlation between districts and across elections.

All regressions imply that candidates receive a higher share of contributions when their opponent moves away from them. Similarly, candidates receive a higher share of contributions when they move to the center. For example, the coefficient of -0.272 in column (2) for the “opponent’s CF score” implies that when the Democratic candidate’s CF score is one standard deviation (0.43) further to the left, then the Republican’s share of contributions rises by 11.7 percentage points (-0.272×-0.43). If the Republican candidate herself moves one standard deviation to the center, she could expect a 9.4 percentage point higher contribution (-0.249×-0.38). This coefficient varies little across the first three specifications. Thus, the results are not driven by some underlying time trend. Instead, the results also hold within years across districts. Including district fixed effects, which change every ten years to allow for redistricting, the coefficient size is cut in half (columns (4) and (5)), but the coefficients remain significant. Thus, we observe the

¹⁸We construct our sample such that the Democratic and Republican share sum to 1.

¹⁹Because we define the shares so that they add up to 1 and because the regressors are identical, using the Democratic share on the LHS yields numerically identical results.

²⁰We report the regressions only for the Republican share, because the results for the Democratic share are identical up to flipping the signs and labels.

	(1) All obs	(2) ≥ 8 givers	(3) +year FE	(4) +district FE	(5) +both FE
own CF score	-0.243*** (0.03)	-0.249*** (0.04)	-0.229*** (0.05)	-0.127*** (0.04)	-0.129*** (0.04)
opponent's CF score	-0.278*** (0.02)	-0.272*** (0.02)	-0.297*** (0.04)	-0.119*** (0.03)	-0.123*** (0.03)
R-squared	0.24	0.25	0.27	0.83	0.85
R-sq, within	0.24	0.25	0.25	0.05	0.06
Observations	3230	2367	2367	1716	1716
States	51	51	51	48	48
Years	18	18	18	17	17
Fixed effects	no	no	year	district	district, year
Rep CF score: average	0.89	0.90	0.90	0.90	0.90
Rep CF score: st.dev.	0.42	0.38	0.38	0.38	0.38
Dem CF score: average	-0.78	-0.82	-0.82	-0.84	-0.84
Dem CF score: st.dev.	0.47	0.43	0.43	0.42	0.42

Standard errors clustered at the state level in parentheses. *** p-value<0.01.

Table A.1: Republican share of campaign contributions within district, own ideology, and opponent's ideology: U.S. House of Representatives, general election candidates, 1980 to 2014.

same patterns within (rezoned) districts over time and across districts for a given election. All estimates are highly significant across specifications. The regressions explain about 25% of the variation in the data without district fixed effects, and about 6% of the within-district, within-year variation in column (5).

These regressions are consistent with the basic mechanism we are emphasizing: Campaign contributions are a centripetal force. Candidates who move toward an extreme position increase the share of total contributions going to their opponent. Moving to the center increases a candidate's share of contributions.

However, such a regression result could not be generated by a literal interpretation of our model under standard (convex) donor preferences. The equilibrium of this benchmark model features complete convergence of agendas and zero contributions. We now enrich the model by introducing a motive for agents to care about the absolute level of their contributions. This generates separation of candidates' agendas and positive campaign contributions in equilibrium, while preserving the mechanism that the agendas of both a candidate and her opponent determine contributions.

B Proofs

B.0.1 Proof of Lemma 1

Proof. \Rightarrow : Suppose $C_i > 0$, but $\Delta_i = 0$. Then the lobby incurs costs of contributions without gains in terms of expected policies, a contradiction. Thus there must be polarization $\Delta_i > 0$.

\Leftarrow : Suppose there is polarization, $\Delta_i, \Delta_{-i} > 0$. To establish that donors' FOC has an interior solution, consider the marginal winning probability: $\frac{\partial p_i}{\partial C_i} = \psi \frac{g(\bar{C}, C_{-i})}{(g(\bar{C}, C_i) + g(\bar{C}, C_{-i}))^2} \omega^{1/\rho} g(\bar{C}, C_i)^{1/\rho} C_i^{-1/\rho}$. As C_i approaches zero, $g(\bar{C}, C)$ converges to a finite number for $\rho > 1$, and thus $C_i^{-1/\rho}$ dominates. I.e., $\lim_{C_i \rightarrow 0} \frac{\partial p_i}{\partial C_i} \rightarrow \infty$. Conversely, for large C_i , $g(\bar{C}, C_i)^{1/\rho} C_i^{-1/\rho} \rightarrow 1$, while $\frac{1}{(g(\bar{C}, C_i) + g(\bar{C}, C_{-i}))^2} \rightarrow 0$. By continuity, for $i = 1, 2$ there is thus a $C_i > 0$ that solves $\frac{\partial p_i}{\partial C_i} \Delta_i = \phi$.

To establish the *existence* of pure strategy equilibrium, we simply apply Brouwer's fixed point theorem to the best response function. First, observe that the lobby's best response correspondence existence is single-valued and continuous in C_{-i} , since the left-hand side of (3.1) is decreasing in C_i and continuous in (C_i, C_{-i}) , while the right-hand side is constant. To establish the compact interval on which Brouwer's theorem applies, note that the best response to any opposing lobby's strategy lies in $[0, \frac{\psi \Delta}{4\phi}]$, where $\Delta := \max\{\Delta_i, \Delta_{-i}\}$. To see that, note that the right-hand side of (3.1) as a function of C_{-i} is maximized when $g(C_{-i}) = g(C_i)$ and thus $\phi = \frac{\psi \Delta_i g(C_{-i})}{(g(C_i) + g(C_{-i}))^2} g'(C_i) \leq \frac{\psi \Delta_i g'(C_i)}{4g(C_i)}$. Next, note that $\frac{g'(C)}{g(C)} = \frac{g(c)^{1/\rho} \omega^{1/\rho} C^{-1/\rho}}{g(C)} = \frac{\omega^{1/\rho} C^{-1/\rho}}{(1-\omega)^{1/\rho} C^{1-1/\rho} + \omega^{1/\rho} C^{1-1/\rho}} \leq \frac{\omega^{1/\rho} C^{-1/\rho}}{\omega^{1/\rho} C^{1-1/\rho}} = \frac{1}{C}$. Combining the two inequalities gives us $\phi \leq \frac{\psi \Delta_i}{4C_i}$, which implies that the best response of lobby i is bounded by $\frac{\psi \Delta}{4\phi}$ for any level of the opponent's contributions. ■

B.0.2 Proof of Theorem 1

Proof. The key to the proof is the observation that setting agenda to j_m guarantees a candidate at least $\frac{1}{2}$ probability of winning regardless of the opponent's choice of the agenda, and that the probability of winning with $a_i = j_m$ is strictly greater than $\frac{1}{2}$ in any subgame where the opponent chooses $a_{-i} \neq j_m$.

To establish that observation, note that whenever $a_i = j_m$ and $a_{-i} \neq j_m$ then $\Delta_i(a_i, a_{-i}) > \Delta_{-i}(a_{-i}, a_i)$. From equation (3.1) in Lemma 1, this implies that

candidate i receives larger contributions (associated with lower marginal product of the contributions) and strictly greater probability of winning the election than their opponent. ■

B.0.3 Proof of Theorem 2

Proof. Since $\alpha > 1$, the extreme lobbies have the most intense preferences over the election outcomes, i.e., the largest Δ . Since the marginal cost of contributions ϕ is constant and the same across all lobbies, this implies that only the extreme lobbies may contribute in equilibrium. That is, equations (3.5) and (3.6) can hold with equality only for the extreme lobbies — the ones with largest Δ . ■

B.0.4 Proof of Theorem 3

Proof. The result follows simply from observing that in the no-polarization allocation, the first and the third term on the left-hand side of equation (4.3) are 0, while the second term is strictly positive. This means that the equilibrium condition is violated at no-polarization allocation. ■

B.0.5 Proof of Theorem 4

Proof. Any symmetric pure strategy subgame perfect equilibrium must satisfy the equilibrium equation (4.5). Note that the left-hand side of this equation is strictly decreasing in a_r , strictly positive when $a_r = \frac{L+R}{2}$, i.e., when there is no polarization, and becomes unboundedly negative as a_r approaches R . Thus, the equation has exactly one solution $a_r \in \left(\frac{L+R}{2}, R\right)$. ■

B.0.6 Proof of Theorem 5

Proof. (a) The proof follows immediately from (4.5) after dividing through by W_2 . Higher $\frac{W_1}{W_2}$ makes the first term more negative for any given value of $a_r > \frac{R+L}{2}$ so that the equilibrium value of a_r falls. (b) We can write the lobby's first period FOC in symmetric equilibrium as $\psi \frac{\epsilon_c^g(C)}{C} \Delta_R = \phi$, where $\epsilon_c^g = \frac{\partial g_i / \partial c_i}{g_i / c_i} = \frac{\omega^{1/\rho}}{(g/c)^{1-\rho}}$ and $\frac{\epsilon_c^g(C)}{C}$ is decreasing in C according to Lemma D.6. Thus, when a_r falls, Δ_R falls

and C also falls. The expression for the second period is analogous. ■

B.0.7 Proof of Theorem 6

Proof. (a) The proof follows immediately from (4.5) after writing $R, L = \bar{j} \pm \mu$ and increasing μ : \bar{j} is unchanged, but $R = \bar{j} + \mu$ rises. Thus, a_r rises, but less than μ . (b) Since a_r rises by less than μ and a_ℓ falls by less than μ , Δ_R increases. As in the argument in the proof of Theorem 5(b) thus implies that contributions increase. ■

B.0.8 Proof of Theorem 7

Proof. Since marginal contributions are positive (from Lemma D.1), the third term in equation (4.3) has the same sign as $\left(\frac{\partial \Delta_R}{\partial a_r} - \frac{\partial \Delta_L}{\partial a_r}\right)$, which is negative. The coefficient on λ in the first term is proportional to:

$$\left(-\Delta \frac{\partial^2 p_i}{\partial c_i^2}\right)^{-1} \left[\left(\frac{1}{2}s + \frac{\rho \phi'}{1 - \epsilon_{C'}^{g'}} \frac{C'_i}{\Delta_i}\right) \left(\frac{\partial \Delta_R}{\partial a_r} - \frac{\partial \Delta_L}{\partial a_r}\right) \pi(c) \frac{\partial p_i}{\partial c_i} + \frac{1}{2} \frac{\partial \pi}{\partial c} \frac{dV_{2R}}{da_r} \right], \quad (\text{B.1})$$

where we used equation (D.7) to substitute the following expression for the last term in the marginal contribution:

$$\frac{dV_{2R}(a_\ell, a_r)}{da_r} = p'_r(C'_\ell, C'_r) \cdot s \cdot \frac{d\Delta_R}{da_r} - \frac{\rho}{1 - \epsilon_{C'}^{g'}} \frac{C'_r}{\Delta_R} \phi' \frac{d\Delta_L}{da_r} \quad (\text{B.2})$$

Since the coefficient on λ is negative under the sufficient condition, it offsets the positive intercept $\beta \frac{W_2}{4} \bar{\kappa}$. Thus, a_r must be closer to $j_m = \frac{L+R}{2}$ in the new equilibrium. ■

C Extension: Informed voters

Theorem 1 in the main text contrasts with the analysis of collectivist policies (policies that affect everyone) in Baron (1994). Baron's core idea is that "informed voters" vote purely based on their policy preferences while the "uninformed voters"

are affected by campaign spending. Both Proposition 5 in [Baron \(1994\)](#) and our [Theorem 1](#) predict convergence of politicians' agendas, but for different reasons: [Baron \(1994\)](#) attributes the convergence to the influence of informed voters, and the point of convergence is the median voter's preferred policy. When we consider the same parametric case, but recognize the free-rider problem among lobbies and endogenize which lobbies contribute, we show that the policy convergence arises even in the absence of informed voters, and that the point of convergence is instead the midpoint between the extreme lobbies.

We now fully incorporate Baron's model into our framework, accounting for both centripetal forces (the one from the informed voters and the one from endogenous contributions), which may disagree regarding the center they pull towards. Unsurprisingly, we find convergence to a point in between the midpoint between the most extreme donors and the median (informed) voter.

To formally nest Baron's setup in our model, we change the contest function: Normalize the total number of voters to 1 and assume that they are divided into separate groups of informed and uninformed voters with the measure of uninformed voters given by ψ . The probability that candidate 1 wins the election, given spending levels and agendas, is then given by

$$p_1^B(C_1, C_2, a_1, a_2) = \psi \frac{C_1}{C_1 + C_2} + (1 - \psi) \begin{cases} \frac{a_1 + a_2}{2} & \text{if } a_1 < a_2, \\ \frac{1}{2} & \text{if } a_1 = a_2, \\ 1 - \frac{a_1 + a_2}{2} & \text{if } a_1 > a_2. \end{cases} \quad (\text{C.1})$$

Theorem C.1 *If $\alpha > 1$, informed voters are distributed uniformly on $[0, 1]$, and the candidates' sole objective is winning the election, then the pure strategy equilibrium has both candidates locating (choosing platforms) at agenda a^* located between the median informed voter's preferred policy $\frac{1}{2}$ and the midpoint between the two extreme potential donors, at $j_m = \frac{L+R}{2}$. Specifically, a^* solves*

$$\psi \frac{|U'_R(a^*)|}{|U'_L(a^*)| + |U'_R(a^*)|} + (1 - \psi)(1 - a^*) = \frac{1}{2}. \quad (\text{C.2})$$

Contributions are zero in equilibrium.

Proof. In equilibrium, a marginal deviation by either candidate generates dis-

continuous changes in informed voter shares, which are exactly balanced by the unequal contributions. An infinitesimal move towards the median voter increases the informed voter share from $\frac{1}{2}$ to either a^* or $1 - a^*$, whichever is larger. The location of a^* is such that this is exactly offset by the disadvantage in relative contributions from such a deviation. Lobbies' first order conditions (3.1) for arbitrary $a_i \neq a_{-i}$ form the system

$$\begin{aligned}\psi \frac{c_{-i}}{(c_i + c_{-i})^2} (U_i(a_i) - U_i(a_{-i})) &= \phi, \\ \psi \frac{c_i}{(c_i + c_{-i})^2} (U_{-i}(a_{-i}) - U_{-i}(a_i)) &= \phi,\end{aligned}$$

which combines to

$$\frac{c_i}{c_{-i}} = \frac{U_i(a_i) - U_i(a_{-i})}{U_{-i}(a_{-i}) - U_{-i}(a_i)}.$$

Taking the limit as a_i gets arbitrarily close to a_{-i} (and dividing both the numerator and the denominator by $a_i - a_{-i}$), we get

$$\lim_{a_i \rightarrow a_{-i}} \frac{c_i}{c_{-i}} = \frac{U'_i(a_{-i})}{-U'_{-i}(a_{-i})}, \quad (\text{C.3})$$

where $i \in \{l, r\}$. We define a^* as the agenda that exactly balances the loss in popular support among informed voters with the gain from the larger share of contributions:

$$\psi \frac{|U'_L(a^*)|}{|U'_L(a^*)| + |U'_R(a^*)|} + (1 - \psi)a^* = \frac{1}{2}.$$

Note that the above equation, derived by considering a deviation to the left of a^* , is equivalent to the one from considering a deviation to the right:

$$\psi \frac{|U'_R(a^*)|}{|U'_L(a^*)| + |U'_R(a^*)|} + (1 - \psi)(1 - a^*) = \frac{1}{2}. \quad (\text{C.4})$$

This is an equilibrium because moving marginally from a^* leads to exactly offsetting discontinuous changes in the two components of the probability of an election win. Moving further away from a^* then lowers a candidate's probability of winning: Moving towards the median voter yields constant marginal gain from informed voters but increasing marginal loss from relative contributions. On the other hand, moving towards the midpoint j_m yields a decreasing marginal benefit

from the (relative) contributions, while generating constant marginal cost from the informed voters. ■

Example 1 Consider an example with $\alpha = 2$ so that $|U'_j(a)| = 2|a - j|$. Then the equilibrium exists and is given by:

$$a^* = \frac{\psi R + (1 - \psi - 0.5)(R - L)}{\psi + (1 - \psi)(R - L)} = \begin{cases} \frac{R}{1+R-L} & \text{if } \psi = \frac{1}{2}, \\ \frac{1}{2} & \text{if } j_m = \frac{1}{2}. \end{cases}$$

If we also set $\psi = 0.5$ and $L = 0 < 0.5 = R$ we thus have that $a^* = \frac{1}{3}$, so that $j_m = \frac{1}{4} < a^* < \frac{1}{2}$, with $\frac{1}{2}$ being the median voter.

D Auxiliary Appendix: Not for Publication

D.1 Supplementary Data Appendix: Robustness

This appendix establishes robustness of empirical results documented in Section 4.1.

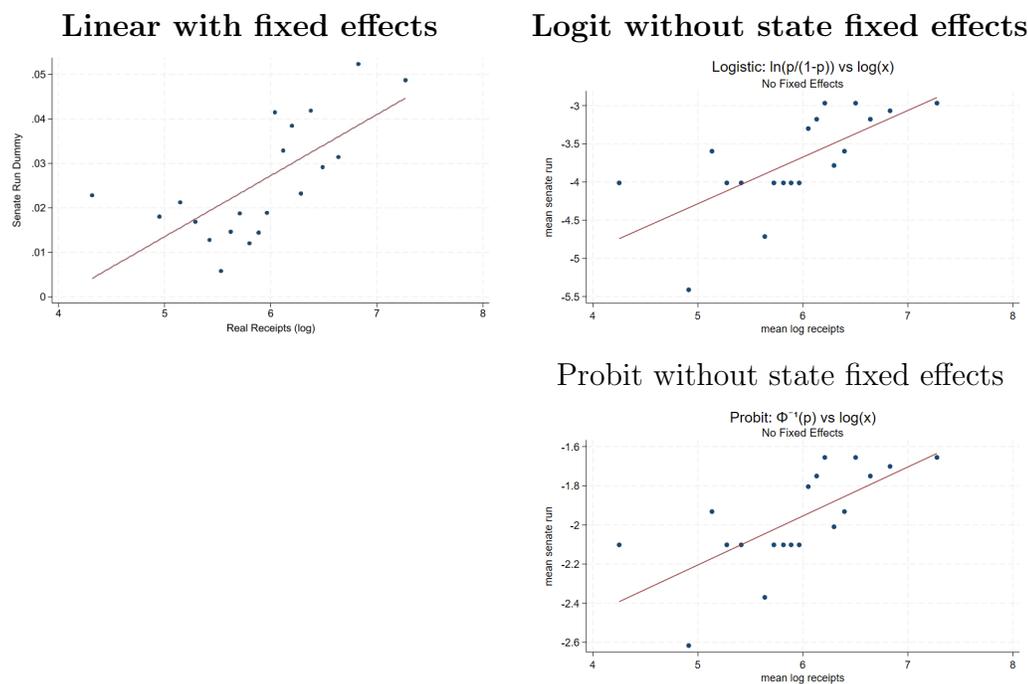


Figure D.1: Campaign donations and Senate Run Probability: Binned scatter plots

	(1) No controls	(2) Some controls	(3) Benchmark	(4) Partisanship controls
Real Receipts (log)	0.831*** (0.283)	0.808*** (0.274)	0.803*** (0.273)	0.812*** (0.278)
# districts in state		-0.089 (0.075)	-0.096 (0.073)	-0.118* (0.063)
Republican dummy			0.680** (0.276)	0.637* (0.337)
District partisanship				0.379* (0.214)
District partisanship \times party				-0.478 (0.376)
Senate Run Mean	0.03	0.03	0.03	0.03
Senate Run SD	0.16	0.16	0.16	0.16
Real Receipts Mean	456.2	456.2	456.2	456.5
Real Receipts SD	387.5	387.5	387.5	387.8
Additional Controls	none	none	none	none
Fixed Effects	by state & year			

Table D.1: Campaign donations and Senate Run Probability: Logit

	(1) No controls	(2) Some controls	(3) Benchmark	(4) Partisanship controls
Real Receipts (log)	0.374*** (0.118)	0.361*** (0.113)	0.355*** (0.114)	0.368*** (0.117)
# districts in state		-0.031 (0.028)	-0.035 (0.027)	-0.044* (0.023)
Republican dummy			0.298** (0.119)	0.275* (0.152)
District partisanship				0.165* (0.085)
District partisanship \times party				-0.200 (0.169)
Senate Run Mean	0.03	0.03	0.03	0.03
Senate Run SD	0.16	0.16	0.16	0.16
Real Receipts Mean	456.2	456.2	456.2	456.5
Real Receipts SD	387.5	387.5	387.5	387.8
Additional Controls	none	none	none	none
Fixed Effects	by state & year			

Table D.2: Campaign donations and Senate Run Probability: Probit

Table D.3: Campaign donations and Senate Run Probability: Linear Probability Model, by presidential election years

	No Election	Pres Election	No Election	Pres Election
Real Receipts (log)	0.009* (0.005)	0.015*** (0.005)	0.013** (0.005)	0.015** (0.006)
# districts in state	-0.004** (0.002)	0.000 (0.001)	-0.003** (0.001)	-0.000 (0.001)
Republican dummy	0.005 (0.007)	0.020** (0.008)	0.005 (0.007)	0.021** (0.008)
R-Squared	0.05	0.07	0.06	0.07
Inside [0,1]	0.77	0.81	0.72	0.79
Senate Run Mean	0.02	0.03	0.02	0.03
Senate Run SD	0.15	0.17	0.15	0.17
Real Receipts Mean	449.9	463.8	449.9	463.8
Real Receipts SD	351.9	413.9	351.9	413.9
Additional Controls	none	none	none	none
Fixed Effects	by state	by state	by state & year	by state & year
States	49	49	49	49
Observations	2086	2414	2086	2414

Table D.4: Campaign donations and Senate Run Probability: Effects of lagged donations

	LPM	Probit	Logit
Real Receipts (log)	0.018** (0.008)	0.366** (0.159)	0.788** (0.349)
Lagged Receipts	-0.006 (0.006)	-0.116 (0.107)	-0.186 (0.246)
Double Lagged Receipts	-0.004 (0.007)	-0.032 (0.105)	-0.115 (0.245)
# districts in state	-0.003*** (0.001)	-0.089*** (0.031)	-0.228*** (0.078)
Republican dummy	0.013** (0.006)	0.304** (0.144)	0.695** (0.335)
Senate Run Mean	0.02	0.03	0.03
Senate Run SD	0.15	0.16	0.16
Real Receipts Mean	455.2	451.2	451.2
Real Receipts SD	397.0	397.4	397.4
Additional Controls	none	none	none
Fixed Effects	by state	by state	by state
Observations	3026	2696	2696

D.2 Auxiliary Appendix: Derivations

D.2.1 Contributions as a function of agendas

The left lobby's FOC (in the second period) w.r.t. C'_ℓ is simply given by:

$$\frac{\partial p'_\ell}{\partial C'_\ell} \underbrace{(U_L(a_\ell) - U_L(\tilde{a}_r))}_{\equiv \Delta(a_\ell, \tilde{a}_r)} = \frac{\phi'}{s}, \quad (\text{D.1})$$

while the FOC in the first period reflects a static tradeoff as well as a dynamic tradeoff whose conditional probability is λ :

$$\begin{aligned} & \frac{\partial p_\ell(c_\ell, c_r)}{\partial c_\ell} \left[U_L(a_\ell) + \beta\lambda(\pi(c_\ell)V_{2L}(a_\ell, \bar{a}_r) + (1 - \pi(c_\ell))V_{2L}(\bar{a}_\ell, \bar{a}_r)) \right. \\ & \quad \left. - U_L(a_r) - \beta\lambda(\pi(c_r)V_{2L}(\bar{a}_\ell, a_r) + (1 - \pi(c_r))V_{2L}(\bar{a}_\ell, \bar{a}_r)) \right] \\ & + p_\ell(c_\ell, c_r)\beta\lambda \frac{\partial \pi(c_\ell)}{\partial c_\ell} (V_{2L}(a_\ell, \bar{a}_r) - V_{2L}(\bar{a}_\ell, \bar{a}_r)) = \phi \end{aligned} \quad (\text{D.2})$$

While the dynamic tradeoff is important for setting incentives for candidates, an equilibrium that is symmetric across districts (so that $a_i = \bar{a}_i, i = \ell, r$) and within districts (so that $c_\ell = c_r$), the FOC collapses to a static tradeoff:

$$\frac{\partial p_\ell(c_\ell, c_r)}{\partial c_\ell} \underbrace{[U_L(\bar{a}_\ell) - U_L(\bar{a}_r)]}_{\equiv \Delta_L(\bar{a}_\ell, \bar{a}_r)} = \phi \quad (\text{D.3})$$

Candidates, however, understand that their agenda choice affects their lobby's contributions both via the current and the expected future tradeoffs via equation (D.2).

As candidates change their agendas, contributions adjust such that lobbies' FOC hold, thereby implicitly defining marginal contributions. In general, period

1 contributions are characterized by the following equation:

$$\begin{aligned}
& \left(\frac{\partial^2 p_\ell}{\partial c_\ell^2} \frac{\partial c_\ell}{\partial a_\ell} + \frac{\partial^2 p_\ell}{\partial c_\ell \partial c_r} \frac{\partial c_r}{\partial a_\ell} \right) \cdot \left[U_L(a_\ell) + \beta \lambda (\pi(c_\ell) V_{2L}(a_\ell, \bar{a}_r) + (1 - \pi(c_\ell)) V_{2L}(\bar{a}_\ell, \bar{a}_r)) \right. \\
& \quad \left. - U_L(a_r) - \beta \lambda (\pi(c_r) V_{2L}(\bar{a}_\ell, a_r) + (1 - \pi(c_r)) V_{2L}(\bar{a}_\ell, \bar{a}_r)) \right] \\
& + \frac{\partial p_\ell(c_\ell, c_r)}{\partial c_\ell} \cdot \left[\frac{\partial U_L(a_\ell)}{\partial a_\ell} + \beta \lambda \frac{\partial \pi(c_\ell)}{\partial c_\ell} \frac{\partial c_\ell}{\partial a_\ell} (V_{2L}(a_\ell, \bar{a}_r) - V_{2L}(\bar{a}_\ell, \bar{a}_r)) \right. \\
& \quad \left. + \beta \lambda \pi(c_\ell) \frac{\partial V_{2L}(a_\ell, \bar{a}_r)}{\partial a_\ell} - \beta \lambda \frac{\partial \pi(c_r)}{\partial c_r} \frac{\partial c_r}{\partial a_\ell} (V_{2L}(\bar{a}_\ell, a_r) - V_{2L}(\bar{a}_\ell, \bar{a}_r)) \right] \\
& + \left(\frac{\partial p_\ell}{\partial c_\ell} \frac{\partial c_\ell}{\partial a_\ell} + \frac{\partial p_\ell}{\partial c_r} \frac{\partial c_r}{\partial a_\ell} \right) \cdot \beta \lambda \frac{\partial \pi(c_\ell)}{\partial c_\ell} \cdot (V_{2L}(a_\ell, \bar{a}_r) - V_{2L}(\bar{a}_\ell, \bar{a}_r)) \\
& + p_\ell(c_\ell, c_r) \cdot \beta \lambda \left[\frac{\partial^2 \pi}{\partial c_\ell^2} \frac{\partial c_\ell}{\partial a_\ell} \cdot (V_{2L}(a_\ell, \bar{a}_r) - V_{2L}(\bar{a}_\ell, \bar{a}_r)) + \frac{\partial \pi(c_\ell)}{\partial c_\ell} \cdot \frac{\partial V_{2L}(a_\ell, \bar{a}_r)}{\partial a_\ell} \right] = 0
\end{aligned} \tag{D.4}$$

In general, marginal contributions are shaped by how sensitive the winning probability function is to changing contributions (first two lines) and by how much static and dynamic payoffs change.

Evaluating at a symmetric within-district equilibrium, the cross-derivative drops out of the first term (Lemma D.5). Symmetry across districts furthermore cancels the changes in the dynamic tradeoffs due to differences in the level of the value function:

$$\begin{aligned}
& \left(\frac{\partial^2 p_\ell}{\partial c_\ell^2} \frac{\partial c_\ell}{\partial a_\ell} \right) \Delta_L(\bar{a}_\ell, \bar{a}_r) + \frac{\partial p_\ell(c_\ell, c_r)}{\partial c_\ell} \cdot \frac{\partial \Delta_L}{\partial a_\ell} \Big|_{a_\ell = \bar{a}_\ell} \\
& + \beta \lambda \frac{\partial V_{2L}(\bar{a}_\ell, \bar{a}_r)}{\partial a_\ell} \cdot \left[\frac{\partial p_\ell(c_\ell, c_r)}{\partial c_\ell} \cdot \pi(c_\ell) + p_\ell(c_\ell, c_r) \cdot \frac{\partial \pi(c_\ell)}{\partial c_\ell} \right] = 0
\end{aligned} \tag{D.5}$$

This tells us how the candidates' own contributions change as a function of their agenda.

Repeating the process by differentiating the lobby's FOC with respect to the opposing candidate's agenda gives us the following expression in a symmetric equi-

librium within and across districts:

$$\begin{aligned} & \left(\frac{\partial^2 p_\ell}{\partial c_\ell^2} \frac{\partial c_\ell}{\partial a_r} \right) \cdot \Delta_L + \frac{\partial p_\ell(c_\ell, c_r)}{\partial c_\ell} \cdot \frac{\partial \Delta_L}{\partial a_r} \Big|_{a_r = (\bar{a}_r)} \\ & - \beta \lambda \frac{\partial p_\ell(c_\ell, c_r)}{\partial c_\ell} \pi(c_r) \frac{\partial V_{2L}(\bar{a}_\ell, \bar{a}_r)}{\partial a_r} = 0 \end{aligned} \quad (\text{D.6})$$

This expression has one term less than (D.5) because there is no marginal effect on the payoff if the lobby's own candidate is promoted, because this involves the right candidate from other districts.

To evaluate these expressions, we need to compute the marginal continuation values based on the value function in (4.2). We can use an envelope condition to simplify the terms in the value function that involve the lobby's choice variable::

$$\begin{aligned} \frac{dV_{2L}(a_\ell, a_r)}{da_\ell} &= p'_\ell(C'_\ell, C'_r) \cdot s \cdot \frac{dU_L(a_\ell)}{da_\ell} + s(U_L(a_\ell) - U_L(a_r)) \frac{\partial p'_\ell(C'_\ell, C'_r)}{\partial C'_r} \frac{\partial C'_r}{\partial a_\ell} \\ &= p'_\ell(C'_\ell, C'_r) \cdot s \cdot \frac{d\Delta_L(a_\ell)}{da_\ell} - \phi' \frac{\partial C'_r}{\partial a_\ell}, \end{aligned} \quad (\text{D.7})$$

where we used the definition of Δ as well as the fact that $\frac{\partial p_\ell}{\partial c_r} = -\frac{\partial p_r}{\partial c_r}$ and the lobby's FOC in equilibrium to simplify.

Here, the first term is always positive. The second term is negative, but zero near the no-polarization case for $\rho > 1$.

Analogously:

$$\begin{aligned} \frac{dV_{2L}(a_\ell, a_r)}{da_r} &= (1 - p'_\ell(C'_\ell, C'_r)) \cdot s \cdot \frac{dU_L(a_r)}{da_r} + s \cdot (U_L(a_\ell) - U_L(a_r)) \frac{\partial p'_\ell(C'_\ell, C'_r)}{\partial C'_r} \frac{\partial C'_r}{\partial a_r} \\ &= -(1 - p'_\ell(C'_\ell, C'_r)) \cdot s \cdot \frac{d\Delta_L}{da_r} - s \cdot \Delta_L(a_\ell, a_r) \frac{\partial p'_\ell(C'_\ell, C'_r)}{\partial C'_\ell} \frac{\partial C'_r}{\partial a_r} \end{aligned} \quad (\text{D.8})$$

The first term is negative because the more extreme agenda of the non-aligned candidate lowers the continuation value for the lobby. The second term is negative because the opposing lobby's contribution increases and thus lowers the winning probability.

Now differentiate the FOC for period 2 w.r.t a_i :

$$0 = \left(\frac{\partial^2 p'_\ell}{(\partial C'_\ell)^2} \frac{dC'_\ell}{da_\ell} + \frac{\partial^2 p'_\ell}{\partial C'_\ell \partial C'_r} \frac{dC'_r}{da_\ell} \right) \Delta(a_\ell, \tilde{a}_r) + \frac{\partial p'_\ell}{\partial C'_\ell} \frac{d\Delta_\ell}{da_\ell} \quad (\text{D.9})$$

$$0 = \left(\frac{\partial^2 p'_\ell}{(\partial C'_\ell)^2} \frac{dC'_\ell}{da_r} + \frac{\partial^2 p'_\ell}{\partial C'_\ell \partial C'_r} \frac{dC'_r}{da_r} \right) \Delta(a_\ell, \tilde{a}_r) + \frac{\partial p'_\ell}{\partial C'_\ell} \frac{d\Delta_\ell}{da_r} \quad (\text{D.10})$$

Using Lemma D.5 implies that the cross-derivative drops out. Simplifying and collecting the results, we have the following Lemma:

Lemma D.1 *In a symmetric equilibrium within and across districts, the marginal contributions are given by:*

$$\frac{\partial c'_r}{\partial a_r} = \left(-\Delta \frac{\partial^2 p'_i}{(\partial c'_i)^2} \right)^{-1} \frac{\partial p'_i}{\partial c'_i} \frac{\partial \Delta_R}{\partial a_r} \quad (\text{D.11})$$

$$\frac{\partial c'_\ell}{\partial a_r} = \left(-\Delta \frac{\partial^2 p'_i}{(\partial c'_i)^2} \right)^{-1} \frac{\partial p'_i}{\partial c'_i} \frac{\partial \Delta_L}{\partial a_r} \quad (\text{D.12})$$

$$\begin{aligned} \frac{\partial c_r}{\partial a_r} &= \left(-\Delta \frac{\partial^2 p_i}{\partial c_i^2} \right)^{-1} \left(\frac{\partial p_i}{\partial c_i} \frac{\partial \Delta_R}{\partial c_r} + \left(\frac{1}{2} s \frac{\partial \Delta_R}{\partial a_r} - s \left(-\frac{\partial^2 p'_i}{\partial c_i'^2} \right)^{-1} \left(\frac{\partial p'_i}{\partial c'_i} \right)^2 \frac{\partial \Delta_L}{\partial a_r} \right) \beta \lambda \right. \\ &\quad \left. \times \left(\pi(c) \frac{\partial p_i}{\partial c_i} + \frac{1}{2} \frac{\partial \pi}{\partial c} \right) \right) \quad (\text{D.13}) \end{aligned}$$

$$\begin{aligned} &= \left(-\Delta \frac{\partial^2 p_i}{\partial c_i^2} \right)^{-1} \left(\frac{\partial p_i}{\partial c_i} \frac{\partial \Delta_R}{\partial c_r} + \left(\frac{1}{2} s \frac{\partial \Delta_R}{\partial a_r} - \frac{\rho \phi'}{1 - \epsilon_{C'}^g} \frac{C'_i}{\Delta_i} \frac{\partial \Delta_L}{\partial a_r} \right) \beta \lambda \right. \\ &\quad \left. \times \left(\pi(c) \frac{\partial p_i}{\partial c_i} + \frac{1}{2} \frac{\partial \pi}{\partial c} \right) \right) \quad (\text{D.14}) \end{aligned}$$

$$\begin{aligned} \frac{\partial c_\ell}{\partial a_r} &= \left(-\Delta \frac{\partial^2 p_i}{\partial c_i^2} \right)^{-1} \left(\frac{\partial p_i}{\partial c_i} \frac{\partial \Delta_L}{\partial c_r} + \left(\frac{1}{2} s \frac{\partial \Delta_L}{\partial a_r} - s \left(-\frac{\partial^2 p'_i}{\partial c_i'^2} \right)^{-1} \left(\frac{\partial p'_i}{\partial c'_i} \right)^2 \frac{\partial \Delta_R}{\partial a_r} \right) \beta \lambda \pi(c) \frac{\partial p_i}{\partial c_i} \right) \\ &= \left(-\Delta \frac{\partial^2 p_i}{\partial c_i^2} \right)^{-1} \left(\frac{\partial p_i}{\partial c_i} \frac{\partial \Delta_L}{\partial c_r} + \left(\frac{1}{2} s \frac{\partial \Delta_L}{\partial a_r} - \frac{\rho \phi'}{1 - \epsilon_{C'}^g} \frac{C'_i}{\Delta_i} \frac{\partial \Delta_R}{\partial a_r} \right) \beta \lambda \pi(c) \frac{\partial p_i}{\partial c_i} \right) \quad (\text{D.15}) \end{aligned}$$

Note that in the above, $\frac{\partial \Delta_R}{\partial a_r} = R - a_r > 0$ and $\frac{\partial \Delta_L}{\partial a_r} = a_r - L > 0$. Moreover, we can replace $\frac{C'_i}{\Delta_i} = \frac{1}{4} \psi \frac{s}{\phi'} \epsilon_{C'}^g$.

Proof. See above for the derivation of the marginal contributions. The result on $\frac{C'_i}{\Delta_i}$ follows from Lemma D.6. ■

Lemma D.2 When polarization is small enough, $\frac{dC_r}{da_r} > \frac{dC_\ell}{da_\ell}$ if $\lambda > 0$.

Proof. Using Lemma D.1, we have:

$$\frac{\partial c_r}{\partial a_r} = \left(-\Delta \frac{\partial^2 p_i}{\partial c_i^2} \right)^{-1} \left(\frac{\partial p_i}{\partial c_i} \frac{\partial \Delta_r}{\partial c_r} + \frac{1}{2} s \left(\frac{\partial \Delta_r}{\partial a_r} - \frac{\epsilon_{c'}^{g'}}{1 - \epsilon_{c'}^{g'}} \frac{1}{2} \rho \psi \frac{\partial \Delta_\ell}{\partial a_r} \right) \right) \beta \lambda \left(\pi(c) \frac{\partial p_i}{\partial c_i} + \frac{1}{2} \frac{\partial \pi}{\partial c} \right) \quad (\text{D.16})$$

If $a_r \searrow j_m = \frac{L+R}{2}$, then $C_r \searrow 0$. From Lemma D.6, it follows that the term loading on $\frac{\partial \Delta_\ell}{\partial a_r}$ is approximately zero, leaving us with

$$\frac{\partial c_r}{\partial a_r} \approx \left(-\Delta \frac{\partial^2 p_i}{\partial c_i^2} \right)^{-1} \left(\frac{\partial p_i}{\partial c_i} + \frac{1}{2} s \beta \lambda \left(\pi(c) \frac{\partial p_i}{\partial c_i} + \frac{1}{2} \frac{\partial \pi}{\partial c} \right) \right) \frac{\partial \Delta_r}{\partial a_r} \quad (\text{D.17})$$

The same reasoning leads us to this analogous expression:

$$\frac{\partial c_\ell}{\partial a_r} \approx \left(-\Delta \frac{\partial^2 p_i}{\partial c_i^2} \right)^{-1} \left(1 + \frac{1}{2} s \beta \lambda \pi(c) \right) \frac{\partial p_i}{\partial c_i} \frac{\partial \Delta_\ell}{\partial a_r} \quad (\text{D.18})$$

For small amounts of polarization, the ratio of marginal contributions thus behaves as:

$$\frac{\partial c_r / \partial a_r}{\partial c_\ell / \partial a_r} \approx \frac{1 + \frac{1}{2} s \beta \lambda \left(\pi(c) + \frac{1}{2} \frac{\partial \pi}{\partial c} \frac{\partial p_i}{\partial c_i} \right) \frac{\partial \Delta_r / \partial a_r}{\partial \Delta_\ell / \partial a_r}}{1 + \frac{1}{2} s \beta \lambda \pi(c)} \quad (\text{D.19})$$

By Lemma D.3, the last term is approximately 1. In contrast, by Assumption 2, the first term is bounded away from 1. Thus, $\frac{\partial c_r / \partial a_r}{\partial c_\ell / \partial a_r} > 1$ for small amounts of polarization. ■

Lemma D.3 $\frac{d\Delta_\ell / da_\ell}{d\Delta_r / da_r} < 1 \Leftrightarrow a_\ell < j_m$, where $j_m = \frac{L+R}{2}$. Similarly, $\frac{d\Delta_r / da_r}{d\Delta_\ell / da_\ell} < 1 \Leftrightarrow a_r > j_m$. The ratio of derivatives is continuous in a_r and converge to unity if $a_\ell = j_m$ or $a_r = j_m$.

Proof. Consider $(i, j) = (\ell, r)$. The other case is analogous. We get that

$$\frac{d\Delta_\ell / da_\ell}{d\Delta_r / da_r} = \left(\frac{a_\ell - j}{\bar{j} - a_\ell} \right)^{\alpha-1} < 1 \Leftrightarrow a_\ell - \underline{j} < \bar{j} - a_\ell \Leftrightarrow a_\ell < 0.5(\bar{j} + \underline{j}) = j_m$$

■

D.2.2 Results on functional forms

Lemma D.4 *Under symmetry, we have that $\frac{\partial p_i}{\partial c_{-i}} = -\frac{\partial p_i}{\partial c_i}$.*

Proof. Since $p_i + p_{-i} = 1$ we have that $\frac{\partial p_i}{\partial c_{-i}} = \frac{\partial(1-p_{-i})}{\partial c_{-i}} = -\frac{\partial p_{-i}}{\partial c_{-i}}$. The desired result then follows from symmetry. ■

Lemma D.5 *Under the functional form assumptions for the winning probability, in a symmetric equilibrium within districts, $\frac{\partial^2 p_i}{\partial c_i \partial c_i'} = 0$.*

Proof.

$$p_i(C_i, C_{-i}) = (1 - \psi) \frac{1}{2} + \psi \frac{g(\bar{C}, C_i)}{g(\bar{C}, C_i) + g(\bar{C}, C_{-i})}, \quad (\text{D.20})$$

where $g(\bar{C}, C_i) = \left((1 - \omega)^{1/\rho} \bar{C}^{1-1/\rho} + \omega^{1/\rho} C_i^{1-1/\rho} \right)^{\frac{\rho}{\rho-1}}$, $\bar{C} > 0, \rho > 1, \omega \in (0, 1), \psi \in (0, 1]$.

Then

$$p_i = \frac{1}{2}(1 - \psi) + \psi \frac{g_i}{g_i + g_{-i}}$$

where

$$g_i = \left((1 - \omega)^{1/\rho} \bar{C}^{1-1/\rho} + \omega^{1/\rho} C_i^{1-1/\rho} \right)^{\frac{\rho}{\rho-1}}.$$

It follows that

$$\frac{\partial g_i}{\partial C_i} = \left(\frac{\omega g_i}{C_i} \right)^{1/\rho}$$

and that

$$\begin{aligned} \frac{\partial p_i}{\partial C_i} &= \psi \left(\frac{1}{g_i + g_{-i}} - \frac{g_i}{(g_i + g_{-i})^2} \right) \frac{\partial g_i}{\partial C_i} \\ &= \psi \frac{g_{-i}}{(g_i + g_{-i})^2} \left(\frac{\omega g_i}{C_i} \right)^{1/\rho} \end{aligned} \quad (\text{D.21})$$

and

$$\begin{aligned} \frac{\partial p_i}{\partial C_{-i}} &= -\psi \frac{g_i}{(g_i + g_{-i})^2} \frac{\partial g_{-i}}{\partial C_{-i}} \\ &= -\psi \frac{g_i}{(g_i + g_{-i})^2} \left(\frac{\omega g_{-i}}{C_{-i}} \right)^{1/\rho} \end{aligned} \quad (\text{D.22})$$

Note that at $C_i = C_{-i}$ we have that $\frac{\partial p_i}{\partial C_{-i}} = -\frac{\partial p_i}{\partial C_i}$.

The second derivative is

$$\frac{\partial^2 p_i}{\partial C_i^2} = -\frac{1}{\rho} \frac{\partial p_i}{\partial C_i} \frac{1}{C_i} + \frac{1}{\rho} \frac{\partial p_i}{\partial C_i} \frac{1}{g_i} \left(\frac{\omega g_i}{C_i} \right)^{1/\rho} - \frac{2}{g_i + g_{-i}} \frac{\partial p_i}{\partial C_i} \left(\frac{\omega g_i}{C_i} \right)^{1/\rho} \quad (\text{D.23})$$

At the symmetric equilibrium $C_i = C_{-i}$ and $g_i = g_{-i}$ so that

$$\frac{\partial^2 p_i}{\partial C_i^2} \Big|_{C_i=C_{-i}} = -\frac{1}{\rho} \frac{1}{C_i} \frac{\partial p_i}{\partial C_i} \left(1 - \frac{\partial g_i / \partial C_i}{g_i / C_i} (1 - \rho) \right) \equiv -\frac{1}{\rho} \frac{1}{C_i} \frac{\partial p_i}{\partial C_i} (1 + (\rho - 1) \epsilon_g^c) < 0 \quad (\text{D.24})$$

The second derivative with respect to the other candidate's contributions is :

$$\begin{aligned} \frac{\partial^2 p_i}{\partial C_{-i}^2} &= -(-2) \psi \frac{g_i}{(g_i + g_{-i})^3} \left(\frac{\omega g_i}{C_i} \right)^{2/\rho} - \psi \frac{g_i}{(g_i + g_{-i})} \frac{\partial^2 g_{-i}}{\partial C_{-i}^2} \\ &= \frac{\partial p_i}{\partial C_{-i}} \left(\frac{\omega g_i}{C_i} \right)^{1/\rho} \left(-2 \frac{\partial g_{-i} / \partial C_{-i}}{g_i + g_{-i}} - \frac{1}{\rho} \frac{1}{C_i} \left(1 - \frac{\partial g_{-i} / \partial C_{-i}}{g_{-i} / C_{-i}} \right) \right) \end{aligned} \quad (\text{D.25})$$

Using symmetry:

$$\begin{aligned} \frac{\partial^2 p_i}{\partial C_{-i}^2} &= -\frac{1}{\rho} \frac{\partial p_i}{\partial C_i} \left(-\rho \frac{1}{C_i} \frac{\partial g_i / \partial C_i}{g_i / C_i} - \frac{1}{C_i} \left(1 - \frac{\partial g_{-i} / \partial C_{-i}}{g_{-i} / C_{-i}} \right) \right) \\ &= \frac{1}{\rho} \frac{1}{C_i} \frac{\partial p_i}{\partial C_i} (1 + (\rho - 1) \epsilon_g^c) = -\frac{\partial^2 p_i}{\partial C_i^2} \end{aligned} \quad (\text{D.26})$$

The second cross derivative is

$$\begin{aligned} \frac{\partial^2 p_i}{\partial C_i \partial C_{-i}} &= \frac{\partial}{\partial C_{-i}} \frac{\partial p_i}{\partial C_i} \\ &= \frac{\partial p_i}{\partial C_i} \left(\frac{1}{g_{-i}} - \frac{2}{g_i + g_{-i}} \right) \left(\frac{\omega g_{-i}}{C_{-i}} \right)^{1/\rho} \end{aligned} \quad (\text{D.27})$$

This equals zero when evaluated at $C_i = C_{-i}$.

$$\frac{\partial^2 p_i}{\partial C_i \partial C_{-i}} \Big|_{C_i=C_{-i}} = 0. \quad (\text{D.28})$$

■

Lemma D.6 Define ϵ_c^g as the elasticity of g_t w.r.t. contributions c_t : $\epsilon_c^g \equiv \frac{\partial g_t / \partial c_t}{g_t / c_t}$.

1. ϵ_c^g is strictly increasing in c ,
2. ϵ_c^g / c is strictly decreasing in c
3. $0 \leq \epsilon_c^g \leq \epsilon_g^{\max}$, where $\epsilon_g^{\max} \equiv \frac{\omega^{1/\rho} c_{\max}^{1-1/\rho}}{(1-\omega)^{1/\rho} \bar{c}^{1-1/\rho} + \omega^{1/\rho} c_{\max}^{1-1/\rho}} \equiv \epsilon_{\max} < 1$.
4. As $C \searrow 0$, $\epsilon_c^g \searrow 0$
5. $\frac{C'_i}{\Delta_i} = \frac{\psi}{4} \frac{s}{\phi'} \epsilon_{c'}^{g'}$.

Proof.

$$\epsilon_c^g = \frac{\partial g_t / \partial c_t}{g_t / c_t} = \frac{(\omega g / c)^{1/\rho}}{g / c} = \frac{\omega^{1/\rho}}{(g / c)^{1-\rho}} \quad (\text{D.29})$$

$$= \frac{\omega^{1/\rho} c^{1-1/\rho}}{(1-\omega)^{1/\rho} \bar{c}^{1-1/\rho} + \omega^{1/\rho} c^{1-1/\rho}} \in [0, 1) \quad (\text{D.30})$$

Since $\rho > 1$, the $\epsilon_c^g \searrow 0$ as $c \searrow 0$, this proves part 1, part 4, and the first inequality in part 3.

We can also use this $[0, 1)$ bound on ϵ_c^g from (D.30) together with the result that $c < \frac{\Delta_{\max}}{4\phi}$ from (D.33) to construct a sharper bound of ϵ_c^g as:

$$\epsilon_c^g = \frac{\omega^{1/\rho} c^{1-1/\rho}}{(1-\omega)^{1/\rho} \bar{c}^{1-1/\rho} + \omega^{1/\rho} c^{1-1/\rho}} < \frac{\omega^{1/\rho} c_{\max}^{1-1/\rho}}{(1-\omega)^{1/\rho} \bar{c}^{1-1/\rho} + \omega^{1/\rho} c_{\max}^{1-1/\rho}} \equiv \epsilon_{\max} \quad (\text{D.31})$$

This proves the second inequality in part 3.

To prove part 2, notice that

$$\frac{\epsilon_c^g}{c} = \frac{\omega^{1/\rho} c^{-1/\rho}}{(1-\omega)^{1/\rho} \bar{c}^{1-1/\rho} + \omega^{1/\rho} c^{1-1/\rho}}, \quad (\text{D.32})$$

which is strictly decreasing for $\rho > 1$.

We can characterize $\frac{C'_i}{\Delta_i}$ from the lobbies FOC $\frac{\partial p'_i}{\partial C'_i} \Delta_i = \phi'/s$:

$$\begin{aligned} \phi' &= \frac{\partial p'_i}{\partial c'_i} s \Delta_i \\ \Rightarrow \frac{C'_i}{\Delta_i} &= \frac{\partial p'_i / \partial C'_i}{\phi'} s C' = \frac{1}{4} \frac{c'_i}{g'_i} \frac{\partial g'_i}{\partial C'_i} \frac{s}{\phi'} \equiv \frac{s}{4\phi'} \epsilon_{c'}^{g'} \end{aligned} \quad (\text{D.33})$$

This proves part 5. ■

D.2.3 Existence proof

Theorem D.1 *If $1 < \rho < \frac{2}{\psi} \frac{1 - \epsilon_{\max}}{\epsilon_{\max}}$, a symmetric pure-strategy equilibrium with*

strictly positive polarization exists, where $\epsilon_{\max} = \frac{\omega^{1/\rho} \left(s \frac{(R-L)^2}{4\phi'} \right)^{1-1/\rho}}{(1-\omega)^{1/\rho} \bar{c}^{1-1/\rho} + \omega^{1/\rho} \left(s \frac{(R-L)^2}{4\phi'} \right)^{1-1/\rho}}$

Intuitively, this condition guarantees that the marginal contribution is dominated by the corresponding marginal utility to the lobby: In Lemma D.1, this means that all the loading of every term in $\frac{\partial c_r}{\partial a_r}$ on $\frac{\partial \Delta_r}{\partial a_r}$ is higher than the loading on the cross-derivative $\frac{\partial \Delta_\ell}{\partial a_r}$.

Proof. Multiplying the candidate's FOC by $\frac{\partial^2 p_i}{\partial c_i^2}$:

$$\begin{aligned} &\frac{\partial^2 p_i}{\partial c_i^2} \frac{dC_r}{da_r} \left(\frac{dp_i}{dC_i} (W_1 + \frac{1}{2} \beta \pi(C) \bar{W}_2) + \frac{1}{4} \beta \bar{W}_2 \frac{d\pi}{dC} \right) + \frac{1}{2} \beta \pi \lambda \frac{dp'_i}{dC'_i} \bar{W}_2 \frac{\partial^2 p_i}{\partial c_i^2} \left(\frac{\partial^2 p'_i}{(\partial c'_i)^2} \right)^{-1} \frac{\partial^2 p'_i}{(\partial c'_i)^2} \frac{dC'_r}{da_r} \\ &= \frac{\partial^2 p_i}{\partial c_i^2} \frac{dC_\ell}{da_r} \left(\frac{dp_i}{dC_i} (W_1 + \frac{1}{2} \beta \pi(C) \bar{W}_2) \right) + \frac{1}{2} \beta \pi \lambda \frac{dp'_i}{dC'_i} \bar{W}_2 \frac{\partial^2 p_i}{\partial c_i^2} \left(\frac{\partial^2 p'_i}{(\partial c'_i)^2} \right)^{-1} \frac{\partial^2 p'_i}{(\partial c'_i)^2} \frac{dC'_\ell}{da_r} \end{aligned} \quad (\text{D.34})$$

The ratio of second derivatives in the first and the second period is given by:

$$\frac{\partial^2 p_i}{\partial c_i^2} \left(\frac{\partial^2 p'_i}{(\partial c'_i)^2} \right)^{-1} = \frac{C'_i}{C_i} \frac{\partial p_i / \partial C_i}{\partial p'_i / \partial C'_i} \frac{1 + (\rho - 1) \epsilon_g^c}{1 + (\rho - 1) \epsilon_g^{c'}} = \frac{C'_i}{C_i} \frac{\phi}{\phi'/s} \frac{1 + (\rho - 1) \epsilon_g^c}{1 + (\rho - 1) \epsilon_g^{c'}}$$

where the second equality uses that $\frac{\partial p_i / \partial c_i}{\partial p'_i / \partial c'_i} = \frac{\phi}{\phi'/s}$ from the lobby's FOCs in symmetric equilibrium (and along the sticky branch with $\Delta'_i = \Delta_i$). If $\phi = \frac{\phi'}{s}$, the ratio of marginal winning probabilities and of the second derivatives equals 1.

Plugging in the ratio of second derivatives and multiplying by Δ_i gives:

$$\begin{aligned}
& \Delta_i \frac{\partial^2 p_i}{\partial c_i^2} \frac{dC_r}{da_r} \left(\frac{dp_i}{dC_i} (W_1 + \frac{1}{2} \beta \pi(C) \bar{W}_2) + \frac{1}{4} \beta \bar{W}_2 \frac{d\pi}{dC} \right) + \frac{1}{2} \beta \pi \lambda \bar{W}_2 \frac{C'_i}{C_i} \frac{1 + (\rho - 1) \epsilon_g^c}{1 + (\rho - 1) \epsilon_{g'}^c} \frac{\phi}{\phi'/s} \Delta_i \frac{\partial^2 p'_i}{(\partial c'_i)^2} \frac{\partial p'_i}{\partial c'_i} \frac{dC'_r}{da_r} \\
& = \Delta_i \frac{\partial^2 p_i}{\partial c_i^2} \frac{dC_\ell}{da_r} \left(\frac{dp_i}{dC_i} (W_1 + \frac{1}{2} \beta \pi(C) \bar{W}_2) \right) + \frac{1}{2} \beta \pi \lambda \bar{W}_2 \frac{C'_i}{C_i} \frac{1 + (\rho - 1) \epsilon_g^c}{1 + (\rho - 1) \epsilon_{g'}^c} \frac{\phi}{\phi'/s} \Delta_i \frac{\partial^2 p'_i}{(\partial c'_i)^2} \frac{\partial p'_i}{\partial c'_i} \frac{dC'_\ell}{da_r}
\end{aligned} \tag{D.35}$$

Using these results and plugging in for the marginal contributions from Lemma D.1 lets us re-write the candidate's FOC as:

$$\begin{aligned}
& \frac{\partial \Delta_r}{\partial a_r} \left(\left(\frac{\partial p_i}{\partial c_i} + \frac{1}{2} s \beta \lambda \left(\pi \frac{\partial p_i}{\partial c_i} + \frac{1}{2} \frac{\partial \pi}{\partial c} \right) \right) \left(\frac{\partial p_i}{\partial c_i} (W_1 + \beta \pi(c) \bar{W}_2) + \frac{1}{4} \bar{W}_2 \frac{\partial \pi}{\partial c_i} \right) + \frac{1}{2} \beta \lambda \bar{W}_2 \frac{1 + (\rho - 1) \epsilon_g^c}{1 + (\rho - 1) \epsilon_{g'}^c} \frac{\phi}{\phi'/s} \frac{C'_i}{C_i} + (W_1 + \beta \pi(c) \bar{W}_2) \left(\frac{\partial p_i}{\partial c_i} \right)^2 \beta \lambda \pi \frac{\rho \phi'}{1 + \epsilon_{g'}^c} \frac{C'_i}{\Delta_i} \right) \\
& = \frac{\partial \Delta_\ell}{\partial a_r} \left(\left(\frac{\rho \phi'}{1 - \epsilon_{g'}^c} \frac{C'_i}{\Delta_i} \right) \beta \lambda \left(\pi(c) \frac{\partial p_i}{\partial c_i} + \frac{1}{2} \frac{\partial \pi}{\partial c} \right) \left(\frac{\partial p_i}{\partial c_i} (W_1 + \beta \pi(c) \bar{W}_2) + \frac{1}{4} \bar{W}_2 \frac{\partial \pi}{\partial c_i} \right) + \frac{\partial p_i}{\partial c_i} \left(1 + \frac{1}{2} s \beta \lambda \pi(c) \right) \frac{\partial p_i}{\partial c_i} (W_1 + \beta \pi(c) \bar{W}_2) \right) \\
& \quad + \frac{\partial \Delta_\ell}{\partial a_r} \frac{1}{2} \beta \lambda \bar{W}_2 \frac{1 + (\rho - 1) \epsilon_g^c}{1 + (\rho - 1) \epsilon_{g'}^c} \frac{\phi}{\phi'/s} \frac{C'_i}{C_i}
\end{aligned} \tag{D.36}$$

Expressing this equation as $\frac{\partial \Delta_r}{\partial a_r} A = \frac{\partial \Delta_\ell}{\partial a_r} B$ and using the functional forms, we get that $a_r = \frac{Ar+B\ell}{A+B}$. Since associating candidate r with lobby r requires that $a_r \geq a_\ell$ so that a natural conjecture is that $\frac{A}{A+B} > \frac{1}{2}$ if $A > B > 0$.

$$\delta_a = \frac{A - B}{2(A + B)} (r - \ell) \tag{D.37}$$

Since $0 \leq \epsilon_g^c < 1$, all terms in A and B are strictly positive. Hence, $A, B > 0$. If $\lambda = 0$, we also immediately have the desired result that $A > B > 0$ because $\pi' > 0$.

We now derive conditions under which this result holds also with $\lambda > 0$.

We start with the difference $A - B$:

$$\begin{aligned}
A - B &= \left(\frac{\partial p_i}{\partial c_i} + \frac{1}{2} s \beta \lambda \left(\pi \frac{\partial p_i}{\partial c_i} + \frac{1}{2} \frac{\partial \pi}{\partial c} \right) \right) \left(\frac{\partial p_i}{\partial c_i} (W_1 + \beta \pi(c) \bar{W}_2) + \frac{1}{4} \bar{W}_2 \frac{\partial \pi}{\partial c_i} \right) \\
&\quad + (W_1 + \beta \pi(c) \bar{W}_2) \left(\frac{\partial p_i}{\partial c_i} \right)^2 \beta \lambda \pi \frac{\rho \phi'}{1 - \epsilon_{c'}^{g'}} \frac{C'_i}{\Delta_i} \\
&\quad - \left(\frac{\rho \phi'}{1 - \epsilon_{c'}^{g'}} \frac{C'_i}{\Delta_i} \right) \beta \lambda \left(\pi(c) \frac{\partial p_i}{\partial c_i} + \frac{1}{2} \frac{\partial \pi}{\partial c} \right) \left(\frac{\partial p_i}{\partial c_i} (W_1 + \beta \pi(c) \bar{W}_2) + \frac{1}{4} \bar{W}_2 \frac{\partial \pi}{\partial c_i} \right) \\
&\quad - \frac{\partial p_i}{\partial c_i} \left(1 + \frac{1}{2} s \beta \lambda \pi(c) \right) \frac{\partial p_i}{\partial c_i} (W_1 + \beta \pi(c) \bar{W}_2) \tag{D.38}
\end{aligned}$$

The following result from Lemma D.6 (Part 5) – which holds in symmetric equilibrium across districts – helps to simplify:

$$\frac{C'_i}{\Delta_i} = \frac{\psi s}{4 \phi'} \epsilon_{c'}^{g'} \tag{D.39}$$

Plugging in from (D.39) and (D.33) and re-arranging yields:

$$\begin{aligned}
A - B &= \frac{\partial p_i}{\partial c_i} \frac{1}{4} \bar{W}_2 \frac{\partial \pi}{\partial c_i} + \frac{1}{2} \left(1 - \frac{\psi}{2} \frac{\rho \epsilon_{c'}^{g'}}{1 - \epsilon_{c'}^{g'}} \right) s \beta \lambda \left(\pi(c) \frac{\partial p_i}{\partial c_i} + \frac{1}{2} \frac{\partial \pi}{\partial c} \right) \frac{1}{4} \bar{W}_2 \frac{\partial \pi}{\partial c_i} \\
&\quad + \frac{1}{2} \left(1 - \frac{\psi}{2} \frac{\rho \epsilon_{c'}^{g'}}{1 - \epsilon_{c'}^{g'}} \right) s \beta \lambda \frac{1}{2} \frac{\partial \pi}{\partial c} \left(\frac{\partial p_i}{\partial c_i} (W_1 + \beta \pi(c) \bar{W}_2) \right). \tag{D.40}
\end{aligned}$$

A sufficient condition for this expression to be positive is for ρ to satisfy $\frac{2}{\rho} > \psi \frac{\epsilon_{c'}^{g'}}{1 - \epsilon_{c'}^{g'}}$. Since the RHS of this inequality is strictly increasing in ϵ_g^c , we can weaken this condition by plugging in the maximum value for ϵ_g^c from Lemma D.6, which is a function of parameters only:

$$\frac{2}{\rho} > \psi \frac{\epsilon_{\max}}{1 - \epsilon_{\max}}, \quad \epsilon_{\max} \equiv \frac{\omega^{1/\rho} \left(s \frac{(r-\ell)^2}{4\phi'} \right)^{1-1/\rho}}{(1 - \omega)^{1/\rho} \bar{c}^{1-1/\rho} + \omega^{1/\rho} \left(s \frac{(r-\ell)^2}{4\phi'} \right)^{1-1/\rho}}.$$

■