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International Currency Dominance*

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Abstract

We present a micro-founded monetary model of the world economy to study international currency competition. Our model features both "unipolar" equilibria, with a single dominant international currency, and "multipolar" equilibria, in which multiple currencies circulate internationally. Governments can compete to internationalize their currencies by offering attractive interest rates on their sovereign debt. A large economy has a natural advantage in ensuring its currency becomes dominant, but if it lacks the fiscal capacity to absorb the global demand for liquid assets, the multipolar equilibrium emerges.

Keywords: Dominant Currency, International Monetary System, Interest-Rate Policy, Fiscal Capacity.

JEL classifications: E42, E58, G21.

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1 Introduction

Throughout history, international trade and finance have typically been dominated by a small number of global reserve currencies, despite the existence of dozens or hundreds of local currencies. In the colonial era, the Spanish dollar, the Dutch guilder, and the French franc circulated internationally until ultimately, the British pound sterling emerged as the globally dominant currency. The dollar emerged as a dominant currency following the advent of the Bretton Woods system, but there has been room for other reserve currencies as well, such as the pound, the Japanese yen, and later the Euro (Eichengreen, 2019).

While historically contingent events (such as the Bretton Woods agreement) have clearly played a role in shaping the international monetary system, macroeconomic fundamentals and government policies have mattered a great deal as well. The issuers of globally dominant currencies have typically been large economies with ample fiscal capacity, such as Britain in the Victorian era or the United States in the postwar period. Even the comparatively small Dutch economy of the 17th and 18th centuries had a large capacity to issue *private* debt via its highly developed financial markets.

In this paper, we focus on how macroeconomic fundamentals and government policies shape patterns of international currency circulation. In the long run, which countries' currencies will tend to circulate internationally? Are there fiscal or monetary policies that a government can implement to promote the internationalization of its currency? Is a system with a single dominant currency or a "multipolar" system with multiple international currencies more efficient?

Our approach employs New Monetarist tools (Lagos and Wright, 2005) to answer these classic questions about the international monetary system (Matsuyama, Kiyotaki, and Matsui, 1993; Wright and Trejos, 2001). The model focuses on money's role as an international *medium of exchange* – each country's government bonds may be used for international trade.[1] Our model features three types of international monetary regimes: an "autarky" equilibrium without an international currency, a "dominant currency" equilibrium in which only one country's bond circulates internationally, and a "multipolar" equilibrium in which multiple bonds coexist as international currencies.

The basic idea of the model is as follows. Our model's assumptions imply that agents have a "home bias" in currency choice: they have a higher valuation for domestic-currency bonds than foreigners do. Governments can compete to internationalize their currencies by offering an attractive interest rate on their debt, however. All else equal, a large country has an advantage in internationalizing its currency: foreign agents understand that they have a high likelihood of trading with domestic agents who value that currency highly. If there is no country with the

[1] Of course, the study of which commodities will serve as media of exchange more broadly dates back much further. Kiyotaki and Wright (1989) develop one of the first models to study the determination of the medium of exchange in equilibrium.

fiscal capacity to satisfy the *global* demand for liquid assets, though, the multipolar equilibrium will emerge, as agents seek out other currencies to meet their liquidity needs.

Model: The benchmark model has two countries, Home and Foreign, each being populated by two types of agents, buyers and sellers. It is set in discrete time, with each period having two subperiods. In the morning, buyers match anonymously at random with sellers in a decentralized market (DM). Sellers produce a *raw input* good that buyers can convert into an *intermediate input*. In the afternoon, a centralized market (CM) opens up in each country. Neoclassical firms combine intermediate inputs and labor to produce final output, which is consumed by domestic agents. Each country's government issues domestic nominal bonds, sets the nominal interest rate on those bonds, and levies taxes on domestic agents.

Our key assumptions lie in how we model the DM. There are two types of DM meetings: *domestic meetings* in which a buyer encounters a seller drawn from the same country with certainty and *international meetings* in which the two counterparties are drawn uniformly at random from the global population. Hence, as in Matsuyama, Kiyotaki, and Matsui (1993, MKM henceforth) there is a "home bias" in trade, and country size (in terms of population) matters: in international meetings, buyers are more likely to encounter sellers from the larger country (and vice-versa). As is typical in New Monetarist models, anonymity implies that agents require a medium of exchange to trade in the DM. We make one additional important assumption: in domestic meetings, the government *requires* the use of domestic-currency bonds. This assumption is meant to reflect legal restrictions, such as legal tender laws, that are commonly implemented in practice to promote the use of countries' domestic currencies.[2] By contrast, both types of bonds may be traded in international transactions.

The equilibrium pattern of international currency circulation obeys the familiar logic of international trade: a country's bonds tend to circulate internationally when foreign agents place a high enough valuation on those bonds vis-à-vis domestic agents. Our assumption that domestic-currency bonds are required for domestic transactions creates a wedge between agents' valuation of their own country's bonds and those issued by the other country, all else equal. The supply of bonds also matters for agents' bond valuations – the greater the supply of bonds held in a country, the lower the marginal value of additional bonds.

We first study how the international monetary regime depends on economic fundamentals, namely, the relative size of the two countries and the degree of global economic integration (captured in our model by the frequency of international transactions). We conjecture that larger size tends to favor the circulation of a country's currency. For example, when Home is large relative to Foreign, then Foreign agents are more willing to accept Home bonds. They know they are likely to have opportunities to trade with Home agents (who need those bonds

[2] This assumption is crucial to break the usual Kareken-Wallace (1981) indeterminacy result. Other papers in the literature take different approaches to indeterminacy: for example, Gomis-Porqueras, Kam, and Waller (2017) introduce the threat of counterfeiting in a two-country model to break indeterminacy.

for domestic transactions) in the future. On the other hand, Home agents are not likely to have future opportunities to trade with Foreign agents. Similarly, greater global integration tends to favor the circulation of *both* currencies.

We then examine the effects of government debt issuance policies on the international monetary regime. We conjecture that a country can internationalize its currency by issuing a large enough quantity of debt. When a country supplies enough debt, the marginal value of liquid assets for domestic agents will be low. Then, domestic agents will effectively be willing to export their country's bonds abroad. Indeed, we believe that this result can be used to derive a version of the classic "Triffin dilemma" in our model. A dominant currency equilibrium can emerge only if a country's capacity to supply bonds (i.e., its fiscal capacity) is large enough relative to global demand. Otherwise, there may emerge a "multipolar" equilibrium in which both countries' bonds circulate.

Extensions: Our benchmark model makes several assumptions to streamline the analysis. First, we assume that there are "domestic meetings" in which agents are simply *required* to use domestic-currency bonds, even though other means of exchange may be available. Second, we focus on a simple setting with two countries, which prevents us from analyzing *vehicle currencies* that circulate across foreign countries. Third, we make the strong assumption that there are no international financial markets in which agents can exchange one country's bonds for the other's, allowing differences in valuations to emerge across countries. We develop extensions of the model to relax each of these assumptions and show that none of them are essential for the main results.

The first extension studies an environment with imperfect currency recognizability (as is typical in the literature, see, e.g., Lester, Postlewaite, and Wright, 2012) rather than exogenous restrictions on currency usage. Specifically, we assume that (1) some sellers can recognize only domestic-currency bonds, and (2) buyers are disproportionately likely to run into sellers from their own country. The main results go through in this extension essentially unchanged: the model features the same equilibrium regimes, and countries can promote the use of their currencies by targeting a higher interest rate on their debt. The only difference is that the degree of imperfect recognizability puts limits on the difference between the two countries' interest rates: if almost all agents can recognize both currencies, then their interest rates must be close to one another to ensure that neither is driven out of circulation entirely.

The second extension considers an economy with one large country and a continuum of small countries. Again, there can be equilibria in which the large country's currency is dominant as well as equilibria in which all currencies circulate internationally. Importantly, in the dominant-currency equilibrium, small countries use the large country's currency to trade *with one another*. Hence, it is a dominant currency in the sense of Gopinath and Stein (2021): its share of global trade can far exceed the dominant country's share of global economic activity.

The final extension adds an international financial market to the model. As in the literature on over-the-counter markets (Duffie, Gârleanu, and Pedersen, 2005),[3] we assume that agents can sporadically access a foreign exchange (forex) market where Home and Foreign bonds can be directly exchanged. As long as bonds are not *perfectly* liquid in international markets (i.e., if agents cannot always access the forex market), there remain equilibria in which only one currency is used for international trade. However, the existence of a forex market does reduce the wedge in bond valuations and encourage the circulation of multiple currencies. We also use this extension to study how interest rate policy affects the exchange rate. When a country targets a higher interest rate, its real exchange rate *permanently* appreciates (in contrast to, for example, the New Keynesian literature, where interest rate hikes result in *temporary* appreciations).

Related literature: Our paper is most closely related to an extensive literature that studies how currencies compete internationally as exchange media. Methodologically, we contribute by developing a tractable two-country model with divisible currency and no restrictions on portfolio holdings, permitting us to build on the insights of an earlier generation of models (Matsuyama, Kiyotaki, and Matsui, 1993; Zhou, 1997; Wright and Trejos, 2001; Head and Shi, 2003; Camera and Winkler, 2003). We demonstrate that the model can be extended in several directions to study various issues in international finance. Zhang (2014) similarly presents a two-country model with divisible currencies but focuses on distinct issues (such as history-dependence rather than the role of fundamentals such as country size and fiscal capacity) and reaches different conclusions: in Zhang's model, an interest rate hike *discourages* the use of a country's currency. Madison (2024) introduces a two-country divisible-currency model that also has a different focus from ours: that model studies the interaction between fiscal policy and currency substitution in the domestic economy.

Our work is also related to a newer literature that studies the emergence of "dominant" currencies. Gopinath and Stein (2021) focus on frictions related to the *unit of account*: they argue that a dominant currency emerges due to a strategic complementarity between the denomination of trade invoicing and that of private debt. Coppola, Krishnamurthy, and Xu (2023) take the view that a dominant reserve currency tends to arise due to *payment* frictions in cross-border debt settlement: the currency that is most liquid in international markets will be the reserve asset. Farhi and Maggiori (2018) and Clayton et al. (2024) study competition among currencies as *stores of value* in a reputation game. Our model instead focuses on the competition between currencies as *exchange media* in the long run: we ask, which fundamental forces may lead a currency to become dominant as a liquid asset in the long run? We believe the medium of exchange role is important, as some of the earliest international currencies – the Venetian ducat and the Florentine florin – were quite literally those that were easiest to store and transport.

Finally, our paper is linked to a broader literature on the "exorbitant privilege" experienced in

[3]Several papers have introduced over-the-counter markets in New Monetarist economies, such as Geromichalos and Herrenbrueck (2016) and Herrenbrueck (2019).

recent years by the United States (Gourinchas and Rey, 2005, 2022). Gourinchas and Rey (2007) document how the low returns on US government debt sustain persistent trade deficits. Caballero, Farhi, and Gourinchas (2008) and Maggiori (2017) study the emergence of an exorbitant privilege in models with global financial imbalances. Jiang (2024) develops a model to explain the cyclical pattern of dollar convenience yields and returns on US debt, and Jiang and Richmond (2023) study a "global fiscal cycle" of competition across reserve currencies. In our model, the larger country has an exorbitant privilege in the sense that it can internationalize its currency even if it pays a relatively low interest rate. The country that issues the dominant currency also enjoys persistent trade surpluses financed by raising seigniorage revenues from foreigners.

Organization: The model and equilibrium conditions are introduced in Section 2. Section 3 analyzes the model's steady state and lays out the main results on international monetary regimes and comparative statics. Extensions to the model are developed in Section 4. Section 5 concludes. All proofs are in the Appendix.

2 Model

This section lays out a two-country model of the international monetary system. The government of each country issues bonds that can circulate as exchange media both domestically and internationally. Each government imposes legal restrictions that *require* the use of domestic bonds in some transactions, rendering a country's agents the natural holders of domestic bonds. We begin by filling in the details of the model and then characterize the model's equilibrium, which will set the stage to determine the conditions under which each currency will circulate internationally.

Environment: The global economy consists of two countries: Home (H) and Foreign (F). Each country $j \in \{H, F\}$ is populated by a mass n_j of two types of agents, buyers and sellers. Time is discrete and continues forever, $t \in \{0, 1, 2, \dots\}$. Each period is divided into two subperiods, the first ("morning") with a decentralized market (DM) and the second ("afternoon") with a centralized market (CM) in each country. In the DM, buyers are matched randomly with (domestic or foreign) sellers according to a process specified later. Each country has a government that levies taxes in the CM and issues nominal bonds. There is no aggregate uncertainty.

Production takes place in two stages. In the DM, sellers can produce a *raw input* good z at a linear utility cost, which buyers can convert into an *intermediate input* good x using a one-to-one linear technology, $x = z$. In each country's CM, there are competitive firms (owned by domestic agents) that combine intermediate inputs and labor to produce a perishable final output good y according to a separable production function

$$y = f(x) + \ell,$$

where $f : \mathbb{R}_+ \to \mathbb{R}$ is increasing, continuously differentiable, and concave and satisfies Inada conditions. Agents who supply labor in the CM incur a linear disutility cost.

Agents derive utility $u(c)$ from consumption of c units of final output, and they discount payoffs using a common subjective discount factor $\beta \in (0,1)$. The function $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}$ is assumed to be increasing, concave, and twice continuously differentiable. An agent's lifetime utility can then be written as

$$U = \sum_{t=0}^{\infty} \beta^t (-z_t + u(c_t) - \ell_t),$$

where z_t denotes production of raw inputs in the DM and ℓ_t is labor supplied in the afternoon.

Assets and markets: There are two financial assets: Home and Foreign nominal bonds. A bond issued by government $j \in \{H, F\}$ pays a nominal interest rate $1 + i_t^j$ from t to $t+1$.

In the afternoon of each period, a centralized market opens up in each country. There are Walrasian markets for labor, intermediate inputs, final output, and bonds. The two CMs are physically separate, so goods and assets may trade at different prices in each. We take final output as the numeraire in each market. The wage in CM $j \in \{H, F\}$ in period t is denoted w_t^j, the price of intermediate inputs is ψ_t^j, and the price of a country-i bond in country j's CM is ϕ_{it}^j. Henceforth, we will denote the *real* return on country-j bonds (for domestic agents) by

$$1 + r_t^j \equiv \frac{\phi_{j,t+1}^j}{\phi_{jt}^j}(1 + i_t^j),$$

and we let

$$\varepsilon_{it}^j \equiv \frac{\phi_{it}^j}{\phi_{it}^i}$$

denote the price of country-i bonds in country j relative to country i. So, for instance, if $\varepsilon_F^H < 1$, then F-bonds trade at a discount in H relative to F.

In the DM, buyers and sellers meet randomly in pairs. With probability δ, a buyer (seller) meets a seller (buyer) from the same country in a *domestic transaction*. With probability λ, a buyer (seller) meets a seller (buyer) drawn at random from the entire world in an *international transaction*. Thus, the probability of meeting a Home seller in an international transaction is

$$\xi \equiv \frac{n_H}{n_H + n_F}.$$

In what follows, λ can be interpreted as a measure of global economic integration, whereas ξ is a measure of the relative size of Home, as in MKM.

Buyers and sellers are anonymous, and their trading histories are private information, so a medium of exchange is essential for trade to take place in the DM. Buyers make take-it-or-leave-it offers to sellers (i.e., they have all the bargaining power in DM meetings). Furthermore, we assume that governments impose *legal restrictions* on the assets that can be used in domestic transactions. For now, we simply assume that *only* domestic bonds can be used in domestic transactions. One interpretation of this assumption is that the domestic government requires that domestic currency be accepted as legal tender in some transactions. It implies that country-j buyers will be the natural holders of country-j bonds, since they can use those bonds for both domestic and international transactions.

Governments: Governments pay their debts by levying lump-sum taxes τ_t^j and issuing a quantity of nominal bonds B_t^j in the domestic CM. Each government's policy consists of a path of bond issuances $\{B_0^j, B_1^j, \dots\}$ and a sequence of nominal interest rates $\{i_0^j, i_1^j, \dots\}$ on domestic-currency debt. We denote the growth rate of the stock of country-j debt by $\mu_t^j \equiv B_t^j / B_{t-1}^j - 1$.

Government j's budget constraint (in terms of country-j final output) can be written as

$$\frac{1}{1 + r_t^j} b_t^j = b_{t-1}^j - \tau_t^j, \tag{1}$$

where $b_t^j \equiv \phi_t^j B_t^j$ denotes the real quantity of country-j bonds outstanding in period t. The real rate on country-j debt is determined in equilibrium. Taxes $\{\tau_0^j, \tau_1^j, \dots\}$ adjust so that (1) holds in each period.

2.1 Equilibrium

In this section, we solve firms' and agents' optimization problems and characterize equilibrium.

Firms: The problem faced by firms in country j is standard: they purchase a quantity x_t^j of intermediate inputs and hire labor ℓ_t^j to maximize profits, taking prices ψ_t^j, w_t^j as given:

$$\max_{x_t^j, \ell_t^j} \; f(x_t^j) + \ell_t^j - \psi_t^j x_t^j - w_t^j \ell_t^j.$$

This optimization problem yields the usual first-order conditions, which state that the marginal product of intermediate inputs should be set equal to their price,

$$\psi_t^j = f'(x_t^j), \tag{2}$$

and that the marginal product of labor should be set equal to the wage,

$$w_t^j = 1. \tag{3}$$

Bellman equations: Next, we turn to the problem faced by a country-j buyer in the CM. A country-j buyer enters the period-t CM with an endowment of intermediate inputs x_t^j and a portfolio of bonds (b_{jt}^j, b_{it}^j). The buyer chooses consumption c_t^j, labor supply ℓ_t^j, and a portfolio of bonds $(b^j_{j,t+1}, b^j_{i,t+1})$, taking prices as given, subject to the budget constraint

$$c^j_t + \frac{1}{1+r^j_t} b^j_{j,t+1} + \frac{1}{1+r^i_t} \varepsilon^j_{it} b^j_{i,t+1} \leq \psi^j_t x^j_t + w^j_t \ell^j_t + b^j_{jt} + \varepsilon^j_{it} b^j_{it} - \tau^j_t + \Pi^j_t, \tag{4}$$

where τ^j_t denotes taxes levied in the CM and Π^j_t denotes profits received from firms.

Denote the buyer's CM value function by $W^j_t(x, b^j_j, b^j_i)$, and let $V^j_{t+1}(b^{j\prime}_j, b^{j\prime}_i)$ denote the buyer's *expected* value in the subsequent DM when choosing bond holdings $(b^{j\prime}_j, b^{j\prime}_i)$. The value function satisfies the Bellman equation

$$W^j_t(x, b^j_j, b^j_i) = \max_{c,\ell,b^{j\prime}_j,b^{j\prime}_i} \; u(c) - \ell + \beta V^j_{t+1}(b^{j\prime}_j, b^{j\prime}_i) \;\; \text{s.t.} \;\; (4). \tag{5}$$

The first-order condition for labor supply ℓ equates the marginal utility of consumption in country j times the wage with the marginal disutility of labor:

$$w^j_t u'(c^j_t) = 1. \tag{6}$$

Under the assumption that agents' utility functions are quasi-linear in labor, this result implies that their value functions are linear in the value of their assets:

Proposition 1 *There exists a constant* $\hat{W}^j_t$ *for each* t *such that*

$$W^j_t(x, b^j_j, b^j_i) = \hat{W}^j_t + \psi^j_t x + b^j_j + \varepsilon^j_{it} b^j_i. \tag{7}$$

DM bargaining: We can now solve for bargaining outcomes in the DM. We begin by analyzing domestic transactions in country j. Proposition 1 implies that a seller from country j is willing to sell one unit of raw inputs per unit of domestic bonds received (in real terms). The buyer has an equal valuation of bonds but values a unit of raw inputs at ψ^j_t. Thus, the buyer uses all available domestic bonds to purchase inputs from the seller if $\psi^j_t > 1$, is indifferent to the quantity purchased if $\psi^j_t = 1$, and does not buy inputs from the seller if $\psi^j_t < 1$. Denoting the buyer's value function in a domestic meeting by $V^{j,D}_t(b^j_j, b^j_i)$, we have

$$V^{j,D}_t(b^j_j, b^j_i) = (\psi^j_t - 1)^+ b^j_j + \hat{W}^j_t + b^j_j + \varepsilon^j_{it} b^j_i. \tag{8}$$

Similarly, if a country-j buyer meets a country-j seller in an international meeting, both the buyer and the seller value bonds from country $i \neq j$ at ε^j_{it}. The buyer uses all available funds to buy raw inputs if $\psi^j_t > 1$, so the buyer's value $V^{j,j}_t(b^j_j, b^j_i)$ in such a meeting satisfies

$$V^{j,j}_t(b^j_j, b^j_i) = (\psi^j_t - 1)^+ (b^j_j + \varepsilon^j_{it} b^j_i) + \hat{W}^j_t + b^j_j + \varepsilon^j_{it} b^j_i. \tag{9}$$

Finally, we must consider what happens if a country-j buyer encounters a country-i seller (where $i \neq j$) in an international meeting. The seller values one real unit of country-j bonds at ε^i_{jt} (and is willing to produce an equal quantity of raw inputs in exchange), whereas the buyer's valuation of country-j bonds is equal to 1. If the buyer spends a unit of country-j bonds in a meeting with a country-i seller, then, she receives raw inputs that she values at $\psi^j_t \varepsilon^i_{jt}$. The buyer spends all of her country-j bonds if $\psi^j_t \varepsilon^i_{jt} > 1$. Using similar reasoning, we can conclude that the buyer spends country-i bonds if $\psi^j_t > \varepsilon^j_{it}$. Then, the value of a country-j buyer in a meeting with a country-i seller is

$$V^{j,i}_t(b^j_j, b^j_i) = (\varepsilon^i_{jt}\psi^j_t - 1)^+ b^j_j + (\psi^j_t - \varepsilon^j_{it})^+ b^j_i + \hat{W}^j_t + b^j_j + \varepsilon^j_{it} b^j_i. \tag{10}$$

Collecting these results, the buyer's expected value in the DM is then

$$V^j_t(b^j_j, b^j_i) = \delta V^{j,D}_t(b^j_j, b^j_i) + \lambda\big(\xi V^{j,j}_t(b^j_j, b^j_i) + (1-\xi) V^{j,i}_t(b^j_j, b^j_i)\big). \tag{11}$$

Capital flows and intermediate goods production: It remains to specify capital flows between the two countries and the quantities of intermediate goods produced in each. As argued above, country-j bonds will flow to country i if $\psi^j_t \varepsilon^i_{jt} \geq 1$, and they will flow back from i to j if $\psi^i_t \geq \varepsilon^i_{jt}$. Let b^j_{it} denote the quantity of country-i bonds held by agents in country j. Then, the quantity of Home bonds held in Foreign, b^F_{Ht}, satisfies

$$b^F_{H,t+1} = (1 + r^H_t)\Big(\big(1 - \mathbf{1}\{\psi^F_t \geq \varepsilon^F_{Ht}\}\lambda\xi\big) b^F_{Ht} + \mathbf{1}\{\psi^H_t \varepsilon^F_{Ht} \geq 1\}\lambda(1-\xi) b^H_{Ht}\Big), \tag{12}$$

and the quantity x^F_t of intermediate inputs produced in Foreign satisfies

$$x^F_t = \delta b^F_{Ft} + \lambda(1-\xi)(\varepsilon^F_{Ht} b^F_{Ht} + b^F_{Ft}) + \lambda\xi\big(\mathbf{1}\{\psi^F_t \geq \varepsilon^F_{Ht}\} b^F_{Ht} + \mathbf{1}\{\psi^F_t \varepsilon^H_{Ft} \geq 1\}\varepsilon^H_{Ft} b^F_{Ft}\big). \tag{13}$$

The quantity of Foreign bonds held in Home, b^H_{Ft}, satisfies

$$b^H_{F,t+1} = (1 + r^F_t)\Big(\big(1 - \mathbf{1}\{\psi^H_t \geq \varepsilon^H_{Ft}\}\lambda(1-\xi)\big) b^H_{Ft} + \mathbf{1}\{\psi^F_t \varepsilon^H_{Ft} \geq 1\}\lambda\xi b^F_{Ft}\Big), \tag{14}$$

and the quantity x^H_t of intermediate inputs produced in Home satisfies

$$x^H_t = \delta b^H_{Ht} + \lambda\xi(\varepsilon^H_{Ft} b^H_{Ft} + b^H_{Ht}) + \lambda(1-\xi)\big(\mathbf{1}\{\psi^H_t \geq \varepsilon^H_{Ft}\} b^H_{Ft} + \mathbf{1}\{\psi^H_t \varepsilon^F_{Ht} \geq 1\}\varepsilon^F_{Ht} b^H_{Ht}\big). \tag{15}$$

World equilibrium: We can now provide a formal definition of equilibrium for the world economy. A world equilibrium is a sequence of prices $\{\psi^j_t, r^j_t, w^j_t, \varepsilon^j_{j't}\}_{j,j'\neq j}$, quantities $\{x^j_t, y^j_t\}_j$, and bond portfolios $\{b^j_{j't}\}_{j,j'}$ such that (2)-(3) hold at all dates, agents choose their CM allocation optimally in each country, taking prices as given, equations (12)-(15) hold at all dates, and bond holdings satisfy $b_t^H = b_{Ht}^H + b_{Ht}^F$ and $b_t^F = b_{Ft}^F + b_{Ft}^H$ at all dates.

3 Steady-state analysis

In this section, we search for a steady-state equilibrium in which quantities and prices $\{i^j, \mu^j, \psi^j, x^j, r^j, \varepsilon_{j'}^j\}$ are constant over time. The Fisher equation yields the real rate for each country's debt in terms of the nominal rate and the growth rate of the stock of bonds:

$$1 + r^j = \frac{1 + i^j}{1 + \mu^j}. \tag{16}$$

In a steady-state equilibrium, one can view each government's policy as targeting a particular real rate r^j on domestic-currency debt.

Agents' labor-leisure optimization (6) implies that output is equal to $y_t^j = y^*$ in each country, where

$$u'(y^*) = 1.$$

We also have

$$\psi^j = f'(x^j),$$

where x^j is the quantity of intermediate inputs produced by country-j buyers.

The key equations in the model are the Euler equations determining the rate of return on each asset. In a steady state, the return on any asset is $\frac{1}{\beta}$ minus a *liquidity premium* that depends on the gains from trade that a buyer expects to receive when carrying that asset into the DM.

To begin, consider Home bonds. When a Home buyer spends one unit of Home bonds in a (domestic or international) meeting with a Home seller, they realize gains from trade $\psi^H - 1$. By contrast, when a Home buyer meets a Foreign seller in an international meeting, they may not trade because they have different valuations of Home bonds. As shown in the previous section, trade takes place only if the Foreign seller's valuation, ε_H^F, times the buyer's marginal value of liquidity, ψ^H, exceeds 1, the buyer's valuation of the bond. The gains from trade, correspondingly, are $(\psi^H \varepsilon_H^F - 1)^+$. Similarly, a Foreign buyer carrying Home bonds will always trade (if allowed) when meeting a Foreign seller, but trade takes place with Home sellers only if $\frac{\psi^F}{\varepsilon_H^F} \geq 1$. Table 1 gives the gains from trade from carrying one real unit of Home bonds into the DM, for each type of buyer and each type of meeting.

After finding the gains from trade in each type of meeting, we can derive the two Euler equations for Home bonds. For a Home buyer, we have

$$\frac{1}{\beta(1 + r^H)} - 1 \geq (\delta + \lambda\xi)(\psi^H - 1) + \lambda(1 - \xi)(\psi^H \varepsilon_H^F - 1)^+, \tag{17}$$

	Domestic	International, H seller	International, F seller
H buyer	$\psi^H - 1$	$\psi^H - 1$	$(\psi^H \varepsilon_H^F - 1)^+$
F buyer	0	$\big(\psi^F \frac{1}{\varepsilon_H^F} - 1\big)^+$	$\psi^F - 1$

Table 1: Gains from trade in the DM from exchanging Home bonds, as a function of the buyer's identity and the type of meeting.

with equality if $b_H^H > 0$, and for a Foreign buyer, we have

$$\frac{1}{\beta(1+r^H)} - 1 \geq \lambda\big(\xi(\psi^F \frac{1}{\varepsilon_H^F} - 1)^+ + (1-\xi)(\psi^F - 1)\big), \tag{18}$$

with equality if $b_H^F > 0$. Suppose that Home bond holdings are strictly positive for Home and Foreign buyers. Given the marginal value of liquidity in each country, ψ^H and ψ^F, these two equations pin down Foreign agents' relative valuation ε_H^F via the equation

$$(\delta + \lambda\xi)(\psi^H - 1) + \lambda(1-\xi)(\psi^H \varepsilon_H^F - 1)^+ = \lambda\big(\xi(\psi^F \frac{1}{\varepsilon_H^F} - 1)^+ + (1-\xi)(\psi^F - 1)\big), \tag{19}$$

as well as the real rate of return r^H on Home bonds. According to (19), Foreign buyers' relative valuation of Home bonds must adjust so that the liquidity benefit enjoyed by Foreign buyers is equal to that enjoyed by Home buyers – otherwise, Foreign buyers would not be willing to hold Home bonds that return r^H.

Equations (17)-(42) have three unknowns: the marginal value of liquidity ψ^j in each country and Foreign buyers' relative valuation of Home bonds, ε_H^F. As ε_H^F varies, these equations trace out a downward-sloping locus of marginal values of liquidity (ψ^H, ψ^F) consistent with equilibrium in the market for Home bonds. A higher marginal value of liquidity ψ^F in Foreign drives up the relative valuation ε_H^F. In turn, an increase in ε_H^F improves the returns that Home buyers obtain when trading Home bonds internationally. Therefore, Home buyers are still willing to hold Home bonds even if their marginal valuation of liquid assets ψ^H falls. In equilibrium, ψ^H must fall to equalize the return on domestic bonds to Home buyers with the liquidity premium $\frac{1}{\beta(1+r^H)} - 1$.

Exactly analogous calculations yield the Euler equations for Foreign bonds. For a Home buyer, we have

$$\frac{1}{\beta(1+r^F)} - 1 \geq \lambda\big(\xi(\psi^H - 1) + (1-\xi)(\psi^H \frac{1}{\varepsilon_F^H} - 1)^+\big), \tag{20}$$

with equality if $b_F^H > 0$, and for a Foreign buyer, we have

$$\frac{1}{\beta(1+r^F)} - 1 \geq \lambda\xi(\psi^F \varepsilon_F^H - 1)^+ + (\delta + \lambda(1-\xi))(\psi^F - 1), \tag{21}$$

with equality if $b_F^F > 0$. Just as with Home bonds, when Home buyers' valuation ε_F^H is high enough, then Foreign bonds will circulate internationally. Equations (43)-(44) trace out a distinct downwards-sloping locus (ψ^H, ψ^F) consistent with equilibrium in the market for Foreign bonds.

Given the real interest rates r^j targeted by each government, the Euler equations pin down the marginal value of liquidity in each country as well as the relative bond valuations $\varepsilon_H^F, \varepsilon_F^H$. Therefore, real interest rate policies are sufficient to characterize patterns of international currency circulation in this economy.

Figure 1 illustrates the determination of equilibrium in (ψ^H, ψ^F) space. The blue and red lines represent the loci of points consistent with equilibrium in the markets for Home and Foreign bonds, respectively. Analytically, we can demonstrate the existence of a unique steady-state equilibrium given interest rate policies.

Proposition 2 *Given interest rate policies (r^H, r^F), there exists a unique steady-state equilibrium.*

The pattern of international currency circulation depends on where these two curves intersect. The figure illustrates that there are four possible regimes:

1. **Regime A (for "autarky")**: Neither currency circulates internationally ($\psi^H \varepsilon_H^F < 1$ and $\psi^F \varepsilon_F^H < 1$).
2. **Regime H**: Only H bonds circulate internationally ($\psi^H \varepsilon_H^F \geq 1$ and $\psi^F \varepsilon_F^H < 1$).
3. **Regime F**: Only F bonds circulate internationally ($\psi^H \varepsilon_H^F < 1$ and $\psi^F \varepsilon_F^H \geq 1$).
4. **Regime C (for "coexistence")**: Two international currencies coexist ($\psi^H \varepsilon_H^F \geq 1$ and $\psi^F \varepsilon_F^H \geq 1$).

When ψ^H is low enough relative to ψ^F (i.e., when Foreign agents value liquidity more than Home agents), the economy tends to be in Regime H, where Home bonds are the "dominant" currency. The intuition mirrors that in models of international trade: in this case, Foreign agents value liquid assets much more than Home agents, so their relative valuation ε_H^F of Home bonds is high. Home agents can then obtain favorable terms of trade in international meetings, incentivizing them to "export" their bonds to Foreign. Similarly, the economy tends to be in Regime F when ψ^H is high relative to ψ^F.

The economy tends to be in Regime C when the marginal value of liquidity is high in *both* countries; i.e., when the demand for liquid assets is large relative to the global supply. When ψ^H and ψ^F are both high, buyers can sell intermediate inputs at high prices. Hence, the loss from trading a bond internationally (due to the fact that domestic agents tend to have higher valuations of their own countries' bonds) is outweighed by the immediate gains from trade between the buyer and the seller. By contrast, the economy can be in Regime A when ψ^H and ψ^F are both close to each other and low – then, gains from trade are too small for buyers to trade their domestic-currency bonds to low-valuation sellers. The following proposition formalizes this result.

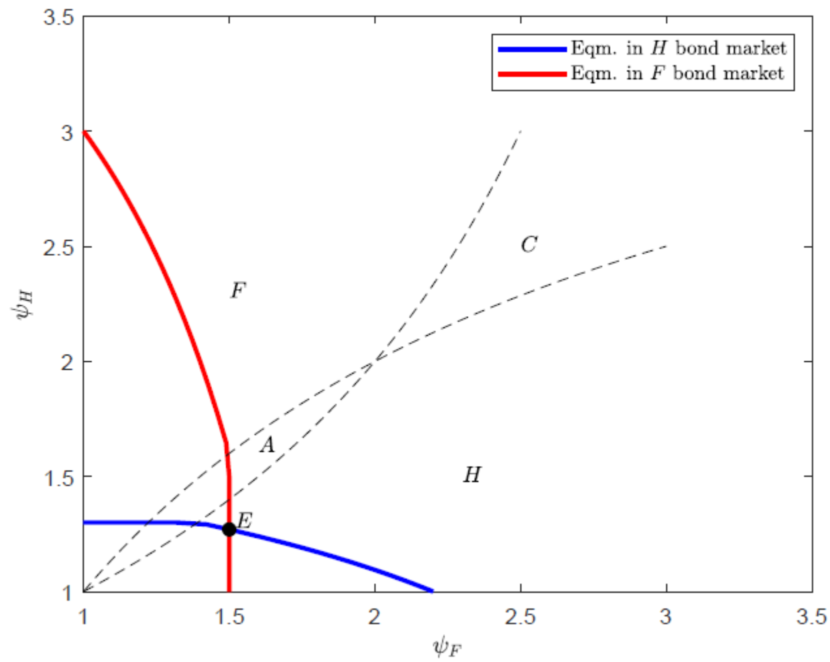

Figure 1: An illustration of determination of equilibrium. The blue (red) line represents values of (ψ^H, ψ^F) consistent with equilibrium in the Home (Foreign) bond market. The equilibrium is marked by point E. The figure is plotted with $\delta = 0.15$, $\lambda = 0.1$, $\xi = 0.5$, $\beta = 0.95$, $r^H = -0.01$, and $r^F = -0.05$.

Proposition 3 *Home currency circulates internationally in equilibrium if and only if*

$$\psi^H \leq \frac{\delta + \lambda(1-\xi)(\psi^F - 1)}{\delta - \lambda\xi(\psi^F - 1)}. \tag{22}$$

Foreign currency circulates internationally in equilibrium if and only if

$$\psi^F \leq \frac{\delta + \lambda\xi(\psi^H - 1)}{\delta - \lambda(1-\xi)(\psi^H - 1)}. \tag{23}$$

It remains to determine the quantity of bonds of each type that will be held in each country. We let b_i^j denote the quantity of country-i bonds held in country j in steady state. Whether a country-j bond circulates internationally is determined by the buyer's valuation of intermediate inputs (captured by ψ^j) and whether sellers in the other country value their bond (captured by ε_j^i). Home agents' holdings of Foreign bonds satisfy

$$b_F^H = (1+r^F)\Big(\big(1 - \lambda(1-\xi)\mathbf{1}\{\psi^H \geq \varepsilon_F^H\}\big)b_F^H + \lambda\xi\mathbf{1}\{\psi^F\varepsilon_F^H \geq 1\}b_F^F\Big), \tag{24}$$

whereas Foreign holdings of Home bonds satisfy

$$b_H^F = (1+r^H)\Big(\big(1 - \lambda\xi\mathbf{1}\{\psi^F \geq \varepsilon_H^F\}\big)b_H^F + \lambda(1-\xi)\mathbf{1}\{\psi^H\varepsilon_H^F \geq 1\}b_H^H\Big). \tag{25}$$

The final pair of equations to construct a steady-state equilibrium is obtained by combining the firm's first-order condition for intermediate inputs and the equations describing total production of intermediate inputs in each country. Precisely, equations (2) and (13)-(15) allow us to write the production of intermediate inputs in the Home country as

$$(f')^{-1}(\psi^H) = \delta b_H^H + \lambda\xi(\varepsilon_F^H b_F^H + b_H^H) + \lambda(1-\xi)\big(\mathbf{1}\{\psi^H \geq \varepsilon_F^H\}b_F^H + \mathbf{1}\{\psi^H\varepsilon_H^F \geq 1\}\varepsilon_H^F b_H^H\big) \tag{26}$$

and the production of intermediate inputs in the Foreign country as

$$(f')^{-1}(\psi^F) = \delta b_F^F + \lambda(1-\xi)(\varepsilon_H^F b_H^F + b_F^F) + \lambda\xi\big(\mathbf{1}\{\psi^F \geq \varepsilon_H^F\}b_H^F + \mathbf{1}\{\psi^F\varepsilon_F^H \geq 1\}\varepsilon_F^H b_F^F\big). \tag{27}$$

Equations (17)-(42), (43)-(44), (24)-(25), and (26)-(27) fully determine a steady-state equilibrium: they are eight equations in the eight unknowns $(\psi^j, b_i^j, \varepsilon_i^j)$.[4]

We can use this simple model to answer two questions. First, under what conditions on *fundamentals* does each country's currency circulate internationally? Second, what *policies* can promote the internationalization of a currency? We address each in turn.

[4] Agents' holdings b_j^j of domestic bonds are pinned down by $b^j = b_j^j + b_j^i$.

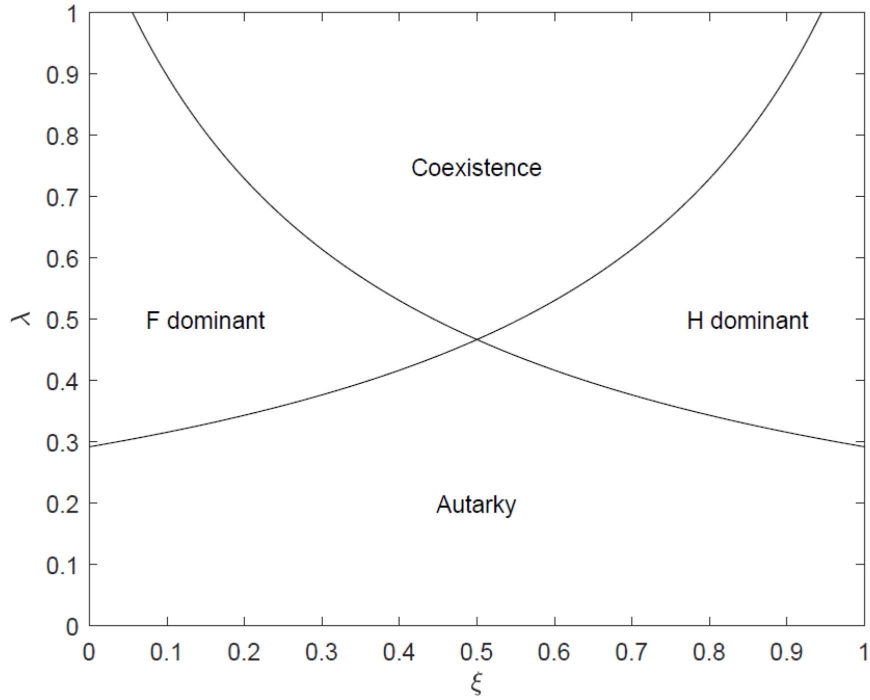

Figure 2: An illustration of the international monetary regime that corresponds to each parameter combination (λ, ξ).

3.1 The determinants of currency internationalization

We begin by holding government policies fixed and asking how patterns of international currency exchange depend on (1) the degree of global economic integration (captured by the parameter λ) and (2) the relative sizes of the two countries (captured by ξ).

Our main result is that greater global integration tends to promote the circulation of both currencies, whereas an increase in the size of one country tends to promote the circulation of its currency at the expense of the other's. The following conjecture provides a formal statement.

Proposition 4 *Fix a baseline steady-state equilibrium with government policies* (r^H, r^F) *and parameter values* (λ, ξ)*. Then, holding government policies fixed,*

1. *If H circulates internationally in the baseline equilibrium, it circulates internationally in a steady-state equilibrium when parameter values are* (λ', ξ) *with* $\lambda' \geq \lambda$ *or* (λ, ξ') *with* $\xi' \geq \xi$*.*

2. *If F circulates internationally in the baseline equilibrium, it circulates internationally in a steady-state equilibrium when parameter values are* (λ', ξ) *with* $\lambda' \geq \lambda$ *or* (λ, ξ') *with* $\xi' \leq \xi$*.*

Figure 2 illustrates the region of the parameter space in which each regime emerges.

The key intuition underlying this result is that in this model, a country's domestic agents are the highest-valuation owners of its government's bonds. Consider Home bonds. Home agents are the natural holders of their country's bonds due to the legal restrictions requiring the use of their own currency in domestic transactions. Foreign agents, by contrast, can use those bonds only in international transactions and forgo the benefits of liquidity in domestic transactions. Hence, they value Home bonds at a discount, $\varepsilon_H^F < 1$.

Due to these forgone liquidity benefits, it is costly to hold Home bonds outside of Home for extended periods of time. Foreign agents spend those bonds in transactions with Home sellers whenever possible. When the global economy is more integrated (higher λ), or when Home is larger (higher ξ), it is less costly for Foreign agents to hold Home bonds: under these conditions, the expected delay until a bond returns back to Home is shorter. This decreases the gap between Home and Foreign agents' valuations, reducing the discount applied by Foreign agents and increasing ε_H^F. Once ε_H^F is large enough, the Home currency will circulate internationally. An analogous argument applies to Foreign-currency bonds.

To see this result more concretely, consider an initial equilibrium in which the Home currency circulates internationally with $\varepsilon_H^F < 1$. In this case, Foreign agents' Euler equation can be written as

$$\varepsilon_H^F = \beta(1+r^H) \times \left(\varepsilon_H^F + \underbrace{\lambda\xi(\psi^F - \varepsilon_H^F)}_{\text{Meet w/}H} + \underbrace{\lambda(1-\xi)(\psi^F - 1)\varepsilon_H^F}_{\text{Meet w/}F} \right).$$

This Euler equation can be rearranged to obtain Foreign buyers' valuation:

$$\varepsilon_H^F = \underbrace{\frac{1}{1-\beta(1-\lambda)(1+r^H)}}_{\text{Discount factor}} \times \Big(\underbrace{\lambda\xi\psi^F}_{\text{Meet w/}H} + \underbrace{\lambda(1-\xi)\varepsilon_H^F\psi^F}_{\text{Meet w/}F} \Big).$$

With probability $\lambda\xi$, a Foreign buyer meets a Home seller and spends the bond, which the seller values at one unit. Thus, the buyer gains a unit of intermediate inputs valued at ψ^F. In a meeting with a Foreign seller (which occurs with probability $\lambda(1-\xi)$), the buyer receives goods worth $\psi^F\varepsilon_H^F$, since the seller values the bond at only ε_H^F. This terminal payoff is discounted at $\beta(1-\lambda)(1+r^H)$, since the probability of a meeting is λ.

In partial equilibrium, with the rate of return r^H and the value of liquidity ψ^F fixed, an increase in ξ increases the expected terminal payoff: conditional on an international meeting, the buyer is more likely to encounter a Home seller who values the bond more. An increase in λ increases the probability of getting to enjoy the bond's liquidity benefits and thus also increases foreign buyers' valuation ε_H^F.

3.2 The effects of policy

Next, we consider the effects of government policies on the patterns of international currency circulation. We focus on the effect of a government targeting a higher real interest rate on its debt, holding fixed the real rate paid by the other government's debt. (That is, we do not consider strategic interactions in which one government responds to a change in the other's debt issuance.)

In the context of a steady-state equilibrium, a policy for government j consists simply of a real interest rate target r^j.

Simply put, a government can make its domestic currency more attractive by increasing its real interest rate target. By doing so, the government decreases the liquidity premium $\frac{1}{\beta(1+r^j)} - 1$ on domestic-currency debt (i.e., the opportunity cost incurred by bond holders). An increase in the domestic-currency real rate has two effects.

First, an increase in country j's real rate target r^j incentivizes country-j buyers to hold a greater quantity of domestic-currency bonds. By holding more liquid assets, these buyers can purchase a greater quantity of raw inputs, driving down the marginal value of liquidity ψ^j.

Second, an increase in r^j may crowd out the other country's debt. A decrease in ψ^j reduces country-j buyers' relative valuation of foreign-currency debt, $\varepsilon^j_{j'}$. In turn, this worsens the terms of trade that foreign buyers will get by trading foreign-currency bonds internationally, reducing the liquidity value of foreign-currency debt. Both domestic and foreign buyers then choose to hold smaller quantities of foreign-currency bonds.

Once a government's real rate target is high enough, its debt is guaranteed to circulate internationally. As country j's real rate policy approaches the Friedman rule, $r^j \to \frac{1}{\beta} - 1$, in fact, the other country's debt is eventually driven out of circulation entirely: even foreign buyers are willing to forgo the opportunity to trade in domestic meetings, since the opportunity cost of holding country j's debt is much smaller than that of holding their own country's debt. The following proposition summarizes these arguments.

Proposition 5 *Consider a country $j \in \{H, F\}$ and fix a real rate target r^k for country $k \neq j$. Then:*

1. *An increase in r^j decreases the steady-state equilibrium value of ψ^j, $\frac{d\psi^j}{dr^j} < 0$;*

2. *If currency k circulates internationally given policies (r^j, r^k), then $\frac{d\psi^k}{dr^j}\big|_{r^j,r^k} > 0$;*

3. *For large enough r^j, currency j is the dominant global currency;*

4. *Currency k is driven out of circulation if*

$$\frac{1}{\beta(1+r^j)} - 1 < \frac{\lambda\xi_j}{\delta + \lambda\xi_k}\left(\frac{1}{\beta(1+r^k)} - 1\right),$$

   where ξ_j denotes country j's share of the global population.

Taking stock of our results, each country can attempt to internationalize its currency (or even to make its currency the world's dominant currency) by targeting a higher real interest rate on its domestic debt. All else equal, the larger country has an advantage: it does not have to target as high a real rate to internationalize its currency.

3.3 Limited fiscal capacity

The internationalization of a country's currency has clear welfare benefits for its citizens. When a country's currency circulates internationally, that country can run a persistent trade deficit, increasing domestic output.

If both governments were free to compete à la Bertrand and target any real rate on their debt (in a one-shot game), both would find it optimal to run the Friedman rule. The liquidity premium on debt would be competed away, and neither country would be able to run a trade deficit.

However, if countries are limited in their fiscal capacity, it may not be feasible to do so. Suppose that country j attempts to internationalize its currency by setting a high real rate close to $\frac{1}{\beta} - 1$, driving the other country's currency out of circulation entirely. Then country j would supply liquid assets elastically to satisfy world demand at a high interest rate. It would therefore be forced to issue large quantities of debt that would need to be partially backed by taxes. If the government's ability to tax its domestic citizens is limited, then it would eventually be unable to pay the interest expenses on its debt.

Suppose that country j cannot raise more than $\overline{\tau}^j$ per capita each period in lump-sum taxes from domestic citizens. Let $b_j(r^j, r^k)$ denote total steady-state bond issuance by country j when it sets real rate r^j and the other country sets real rate r^k. Naturally, $b_j(\cdot)$ is an increasing function of r^j and a (weakly) decreasing function of r^k, since the two countries' bonds are partial substitutes. Furthermore, there is a discontinuity at the point where r^j becomes large enough that currency j circulates internationally: then, the government must supply enough bonds to satisfy the demand of both domestic and foreign agents. The government budget constraint (1) then implies that r^j must be low enough that

$$r^j b_j(r^j, r^k) \leq \xi^j \overline{\tau}^j, \tag{28}$$

where ξ^j is country j's share of the global population.

Limited fiscal capacity implies that the smaller an economy is relative to the rest of the world, the more difficult it will be for that country to satisfy global liquid asset demand. In fact, a country that is too small lacks the debt capacity to issue a globally dominant currency.

Proposition 6 *For ξ small (large) enough, (28) does not hold in any equilibrium in which H (F) is the dominant currency.*

This result relates to the "Triffin Dilemma": a supplier of the world's reserve currency must have a great deal of fiscal capacity or else face mounting and unsustainable debts. Therefore, only a large enough economy can play this role, and if its growth does not keep pace with the global economy's, its hegemonic status will eventually fade.

We further conjecture that if the other country sets a high enough interest rate, then it is not feasible for country j to internationalize its currency. In this case, the country with a larger fiscal capacity can attempt to prevent the other country from internationalizing its currency by targeting a high real rate on its debt. Currency competition will not lead to a situation in which both countries run the Friedman rule. Instead, one country will target an interest rate just high enough to guarantee dominant currency status for its debt. The country with the dominant currency will be able to run persistent trade surpluses financed by seigniorage revenues.

4 Extensions

Our benchmark model studies the determinants of currency internationalization under several specific assumptions. First, we assume a country's domestic agents are natural holders of its debt by supposing that its government exogenously requires the use of domestic currency in specific transactions. Second, we study a simplified two-country environment, precluding our model from addressing the properties of "vehicle currencies" that serve as a common medium of exchange among many foreign countries. Third, we assume that differences in asset valuation arise across countries because there are no international markets in which Home and Foreign agents can directly trade assets. In this section, we relax each of these assumptions in turn and show that our main results continue to hold qualitatively.

4.1 An extension with imperfect recognizability

In this section, we drop the assumption that the government requires the use of Home currency in domestic transactions, so there are no longer "domestic meetings" as in the benchmark model. Instead, we assume:

1. *Home-biased matching*: Buyers are more likely to meet sellers from their own country. With probability δ, a buyer automatically meets a domestic seller, whereas with probability λ, a buyer meets a seller drawn from the world population uniformly at random.

2. *Imperfect recognizability*: Not all sellers are capable of recognizing the other country's currency. Specifically, a fraction σ of sellers can recognize both currencies, whereas the remaining fraction $1 - \sigma$ recognize only their domestic currency.

Due to the fact that (1) country-j buyers meet sellers from their own country more often than other sellers, and (2) some country-j sellers accept only currency j, domestic agents will tend to value a country's currency more than agents in other countries. This is precisely what the assumption of "domestic meetings" accomplished in the benchmark model.

We demonstrate that the results in this modification of the model are similar by deriving the steady-state Euler equations and showing that, just as in the benchmark model, the same four equilibrium regimes arise. Furthermore, the comparative statics with respect to policies are unchanged as well.

Consider first Home currency. A Home buyer meets a domestic seller who is certain to accept Home bonds with probability $\delta+\lambda\xi$. With probability $\lambda(1-\xi)$, the buyer meets a Foreign seller instead, who can recognize Home currency with probability σ. The buyer's Euler equation for Home currency reads

$$\frac{1}{\beta(1+r^H)}-1=\delta(\psi^H-1)+\lambda(\xi(\psi^H-1)+(1-\xi)\sigma(\psi^H\varepsilon_H^F-1)^+). \tag{29}$$

A Foreign buyer meets a domestic seller with probability $\delta+\lambda(1-\xi)$, and that seller is willing to accept Home currency with probability σ. With probability $\lambda\xi$, the Foreign buyer meets a Home seller, so the buyer's Euler equation for Home bonds is

$$\frac{1}{\beta(1+r^H)}-1=\delta\sigma(\psi^F-1)+\lambda\big(\xi(\frac{\psi^F}{\varepsilon_H^F}-1)^+ +(1-\xi)\sigma(\psi^F-1)\big). \tag{30}$$

The only difference from our previous Euler equations is in the matching probabilities. As before, these equations also define a region of (ψ^H,ψ^F) space in which Home currency circulates internationally. In particular, Home currency circulates whenever the marginal value of liquidity in Home is sufficiently small relative to that in Foreign,

$$\psi^H\le\frac{\delta+\sigma(\delta+\lambda(1-\xi))(\psi^F-1)}{\delta-\lambda\xi(\psi^F-1)}. \tag{31}$$

Similarly, the Euler equations for Foreign bonds are

$$\frac{1}{\beta(1+r^F)}-1=\delta\sigma(\psi^H-1)+\lambda\big(\xi\sigma(\psi^H-1)+(1-\xi)(\frac{\psi^H}{\varepsilon_F^H}-1)^+\big), \tag{32}$$

for Home buyers, and

$$\frac{1}{\beta(1+r^F)}-1=\delta(\psi^F-1)+\lambda(\xi\sigma(\psi^F\varepsilon_F^H-1)^+ +(1-\xi)(\psi^F-1)), \tag{33}$$

for Foreign buyers. Foreign currency circulates internationally if

$$\psi^F\le\frac{\delta+\sigma(\delta+\lambda\xi)(\psi^H-1)}{\delta-\lambda(1-\xi)(\psi^H-1)}. \tag{34}$$

We can use (31) and (34) to delineate the four regimes of international currency circulation: H, F, A, and C. An autarky regime exists in (ψ^F,ψ^H)-space if and only if

$$\delta>\sqrt{\big((\delta+\lambda\xi)\sigma+\lambda(1-\xi)\big)\big((\delta+\lambda(1-\xi))\sigma+\lambda\xi\big)}.$$

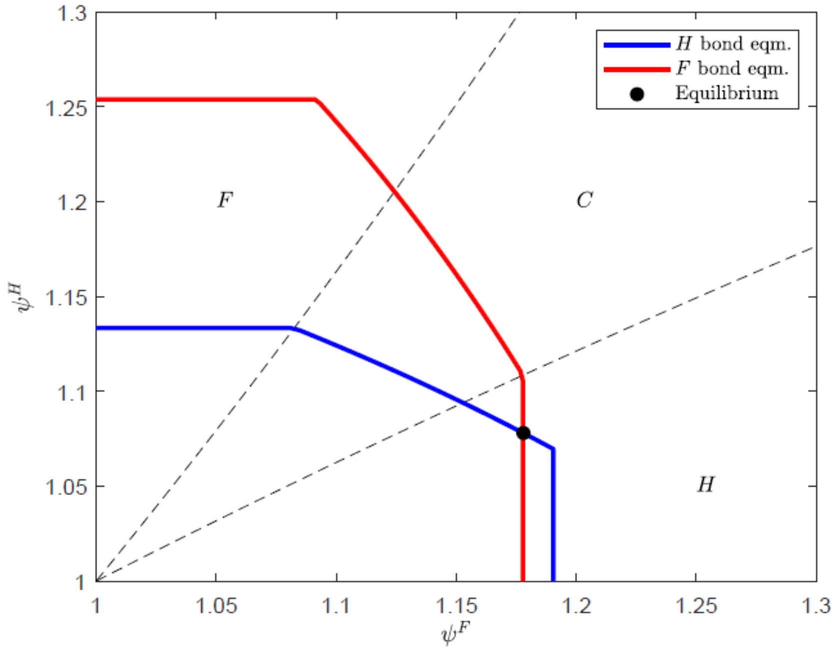

Figure 3: The determination of equilibrium in the extension with imperfect currency recognizability. The blue line represents equilibrium in the market for H bonds, whereas the red line represents equilibrium in the market for F bonds. Parameter values are $\rho = 0.05$, $r_H = 0.02$, $r_F = 0.01$, $\xi = 0.5$, $\sigma = 0.7$, and $\delta = \lambda = 0.15$.

As long as currencies are imperfectly recognizable ($\sigma < 1$), this inequality holds if there is sufficient home bias in matching (δ large enough or λ small enough). Otherwise, only regimes H, F, and C can exist.

The equilibrium can be derived from real interest rates (r^H, r^F) as in the benchmark model. Again, (29)-(30) trace out a locus of points (ψ^H, ψ^F) consistent with equilibrium in the market for Home bonds, whereas (32)-(33) trace out the equilibrium in the Foreign bond market. (See Figure 3.)

There should be a unique intersection of these two curves as long as

$$\sigma \leq \frac{\frac{1}{\beta(1+r^H)} - 1}{\frac{1}{\beta(1+r^F)} - 1} \leq \frac{1}{\sigma}.$$

That is, the liquidity premium on one currency cannot be much larger than the premium on the other. This inequality shows why imperfect recognizability is essential: if $\sigma = 1$, the two currencies are perfect substitutes. The currency with a larger liquidity premium gets driven out of the market entirely. Imperfect recognizability limits the degree of competition between the currencies and permits both to survive in equilibrium. By contrast, in the benchmark model, the difference between the liquidity premia on the two countries' debt was limited by the frequency of domestic meetings in which the government requires the use of local currency.

We can also use the previous analysis to see why home-biased matching is important ($\delta > 0$). If $\delta = 0$, then (31) and (34) show that both currencies circulate internationally for any (ψ^H, ψ^F). That is, when matching patterns are the same across countries, the economy is always in the coexistence regime.

Comparative statics are exactly as in the benchmark model. If F circulates internationally, then an increase in r^H will raise ψ^H but lower ψ^F. On the other hand, if F does not circulate internationally, an increase in r^H increases ψ^H without affecting ψ^F. Figure 3 illustrates this result.

4.2 An extension with multiple small countries

We next consider an extension of the benchmark model with many countries. There is one large country ("Home") containing a fraction ξ of the world's population as well as a continuum of small "Foreign" countries with equal-sized populations totaling the remaining fraction $1-\xi$. We show that it is possible for the Home currency to emerge as a dominant currency, so that it is the primary medium of exchange used even in meetings between agents from different small Foreign countries. There are also "coexistence" regimes in which all Foreign currencies circulate internationally.

In this setting, we look for a symmetric equilibrium in which all Foreign countries set the same interest rate r^F and their currencies are valued equally. Note that now there are three relative valuations to keep track of: Home agents' valuation of (any) Foreign currency ε_F^H, Foreign country j agents' valuation of Home currency ε_H^F, and Foreign country j agents' valuation of some other country k's currency. The relative valuations ε_k^j will all be equal to one another. We denote this relative valuation by ε_X^F.

The Euler equations for Home bonds are exactly the same as in the benchmark model. For Foreign currency, there are three Euler equations to consider. The first is the Euler equation for a Home agent holding the currency of some country j:

$$\frac{1}{\beta(1+r^F)} - 1 \geq \lambda\big(\xi(\psi^H - 1) + (1-\xi)(\psi^H \frac{\varepsilon_X^F}{\varepsilon_F^H} - 1)^+\big). \tag{35}$$

The main difference from the two-country case is that now, a Home agent runs into a Foreign agent from some random country with probability $1-\xi$. Since each country is small, this agent is not from the country that issued the bond (with positive probability). The Foreign agent's valuation of the bond relative to the Home agent's is $\frac{\varepsilon_X^F}{\varepsilon_F^H}$.

Second, there is an Euler equation for Foreign agents holding their own bonds:

$$\frac{1}{\beta(1+r^F)} - 1 \geq \delta(\psi^F - 1) + \lambda\big(\xi(\psi^F \varepsilon_F^H - 1)^+ + (1-\xi)(\psi^F \varepsilon_X^F - 1)^+\big). \tag{36}$$

With probability $\lambda\xi$, the agent runs into a Home seller that can recognize Foreign currency, whose relative valuation is ε_F^H. With probability $\lambda(1-\xi)$, the agent runs into a Foreign seller from a different country who accepts this currency, whose valuation is ε_X^F. Note that the Euler equation for Foreign agents holding their own bonds implies

$$\frac{1}{\beta(1+r^F)} - 1 \geq \delta(\psi^F - 1). \tag{37}$$

Finally, there is an Euler equation for Foreign agents holding *another* country's currency:

$$\frac{1}{\beta(1+r^F)} - 1 \geq \lambda\big(\xi(\psi^F \frac{\varepsilon_F^H}{\varepsilon_X^F} - 1)^+ + (1-\xi)(\psi^F - 1)\big). \tag{38}$$

We will focus on equilibria in which, at the very least, agents choose to hold their own country's currency, so that (36) holds with equality. If a currency is valued at all outside its own country, then (35) and (38) hold as well. We will also show, however, that there are conditions under which agents outside country j do not value its currency.

If currency j is valued outside country j, then it is simple to show that $\frac{\varepsilon_F^H}{\varepsilon_X^F} = \frac{\psi^H}{\psi^F}$ – that is, Home agents' valuation of currency j relative to Foreign agents' is equal to the ratio of the marginal value of liquidity in Home to that in Foreign. Then (35) and (38) collapse to

$$\frac{1}{\beta(1+r^F)} - 1 = \lambda\big(\xi(\psi^H - 1) + (1-\xi)(\psi^F - 1)\big). \tag{39}$$

This is a downwards-sloping locus of points (ψ^H, ψ^F) consistent with equilibrium in the market for country-j bonds if those bonds circulate internationally.

We can now determine whether Foreign currency circulates internationally in equilibrium. Combining (39) and (37), it is possible to show that Foreign currency circulates if and only if

$$\frac{\psi^H - 1}{\psi^F - 1} \geq \frac{\delta - \lambda(1-\xi)}{\lambda\xi}. \tag{40}$$

Foreign currencies tend not to circulate internationally when the Home country is large or when the economy is highly globally integrated (high λ). If (40) does not hold, country j's currency *is not valued at all* outside of country j.

The equilibrium equations in the market for Home bonds, and the conditions for Home currency to circulate internationally, are exactly as in the benchmark model. The comparative statics with respect to interest rates are also identical. The different regimes and the determination of equilibrium are illustrated in Figure 4.

When there are multiple countries, the Home country's currency can become a dominant currency when its liquidity premium is sufficiently low. In this case, it will be the *only* currency

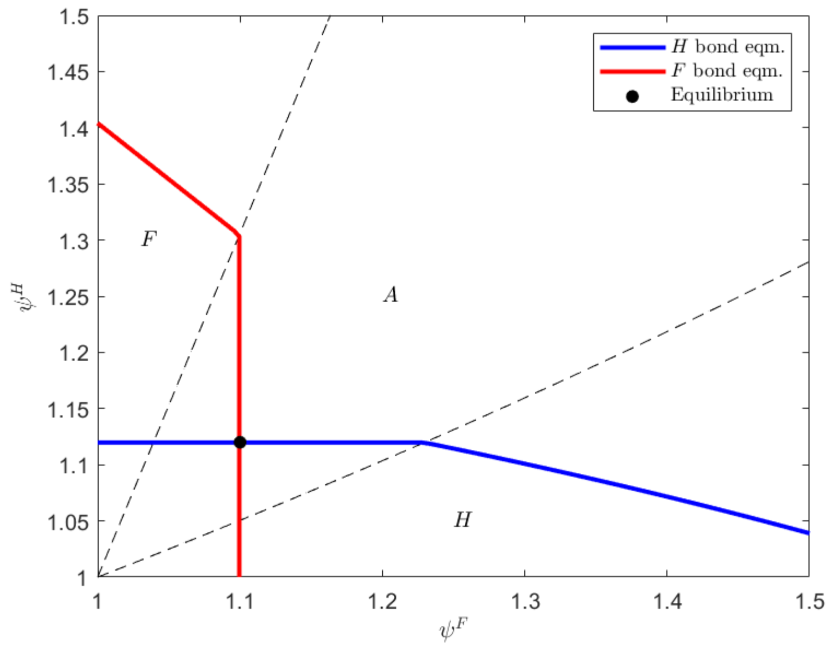

Figure 4: The determination of equilibrium in the extension with multiple small foreign countries. The blue line represents equilibrium in the market for H bonds, whereas the red line represents equilibrium in the market for F bonds. Parameter values are $\rho = 0.05$, $r_H = 0.02$, $r_F = 0.03$, $\xi = 0.5$, and $\delta = \lambda = 0.15$.

that circulates internationally – even when two small countries trade, they exchange Home bonds. Home's government will be able to sustain this equilibrium only if its fiscal capacity is large enough to sustain the demand for globally tradable liquid assets. Otherwise, Home will be forced to cut its real rate target, possibly leading to a regime in which all currencies are traded internationally. (Of course, if there were more than one large country, a "multipolar" equilibrium could also emerge, in which several small countries' currencies circulate.)

4.3 Adding international financial markets

The final modification of the model we consider is an extension with an international market in which agents can trade financial assets. In the benchmark model, the only way to trade assets across borders was in occasional meetings between buyers and sellers. We made the simplifying assumption that there was no market, centralized or otherwise, where the Home and Foreign currencies could be exchanged.

Of course, in reality, there are large over-the-counter international currency markets. We extend the model to accommodate such markets as follows. There is a centralized international market in which Home and Foreign bonds can be traded for one another at a rate of e_t Home bonds per Foreign bond. All agents take this nominal exchange rate as given. To capture the fact that international currency markets are not perfectly liquid, we assume that agents cannot access the international currency market at all times. Instead, with probability $\nu \in (0,1)$ each period, a buyer receives an opportunity to go to the foreign exchange (forex) market.

The existence of a forex market reduces barriers to international currency circulation. In the benchmark model, a currency may not circulate internationally because that country's domestic agents value it far more than foreigners do. The introduction of a forex market reduces the wedge between domestic and foreign valuations of a country's bonds, since it creates additional opportunities for bonds to be swapped back to their natural owners.

Let

$$q_t \equiv \frac{\phi^H_{Ht}}{\phi^F_{Ft}} e_t$$

denote the *real* exchange rate in the forex market. With this notation, the (steady-state) Euler equations dictating agents' willingness to hold Home bonds are

$$\frac{1}{\beta(1+r^H)} - 1 \geq (\delta + \lambda\xi)(\psi^H - 1) + \lambda(1-\xi)(\psi^H \varepsilon^F_H - 1)^+ + \nu(\frac{\varepsilon^F_H}{q} - 1)^+, \tag{41}$$

for Home agents, and

$$\frac{1}{\beta(1+r^H)} - 1 \geq \lambda\big(\xi(\psi^F \frac{1}{\varepsilon^F_H} - 1)^+ + (1-\xi)(\psi^F - 1)\big) + \nu(\frac{1}{q\varepsilon^F_H} - 1)^+, \tag{42}$$

for Foreign agents. The only differences from the Euler equations in the benchmark model are the terms representing the payoffs agents obtain when trading in the forex market. A Home agent who chooses to trade Home bonds in the forex market receives $\frac{1}{q}$ Foreign bonds valued at ε^F_H, hence the last term in Home buyers' Euler equation. Similar logic can be used to derive the Euler equation for Foreign buyers.

Analogously, the Euler equations in the market for Foreign bonds are

$$\frac{1}{\beta(1+r^F)} - 1 \geq \lambda\big(\xi(\psi^H - 1) + (1-\xi)(\psi^H \frac{1}{\varepsilon^H_F} - 1)^+\big) + \nu(\frac{q}{\varepsilon^H_F} - 1)^+, \tag{43}$$

for Home buyers, and

$$\frac{1}{\beta(1+r^F)} - 1 \geq \lambda\xi(\psi^F \varepsilon^H_F - 1)^+ + (\delta + \lambda(1-\xi))(\psi^F - 1) + \nu(q\varepsilon^H_F - 1)^+, \tag{44}$$

for Foreign buyers.

For the forex market to clear, the real exchange rate q must satisfy

$$\varepsilon^H_F \leq q \leq {\varepsilon^F_H}^{-1}. \tag{45}$$

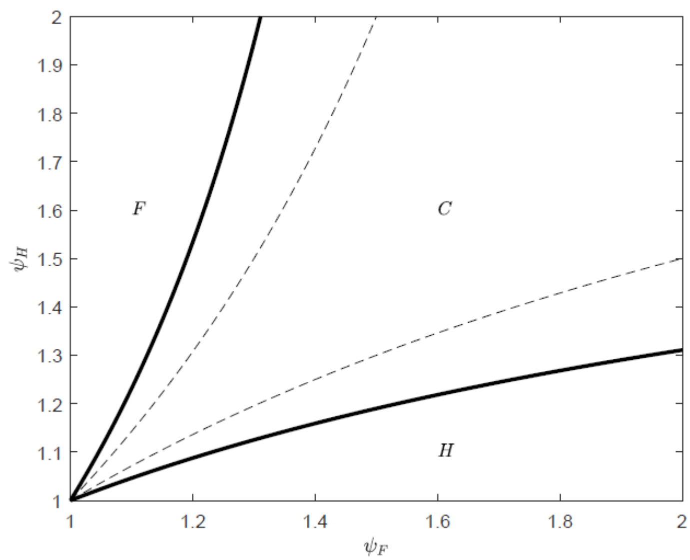

Figure 5: A plot of the equilibrium monetary regimes in the extension of the model with a forex market. The dashed black lines represent the boundaries of the H-dominant and the F-dominant regions in the benchmark model *without* a forex market, whereas the solid black lines represent the boundaries in the model extension. The parameter values are $\delta = 0.3$, $\lambda = 0.4$, $\xi = 0.5$, and $\nu = 0.07$.

If the exchange rate is too high, Foreign agents will not be willing to trade their Home bonds in the forex market, since they are receiving too low a price. On the other hand, if the exchange rate is too low, Home agents are not willing to trade their Foreign bonds.

As before, there are four possible regimes of international currency circulation: a "dominant currency" equilibrium for each country, an autarky equilibrium, and a "coexistence" equilibrium in which both bonds circulate internationally. The existence of the forex market simply provides an additional incentive for agents to accept the other country's currency, since they will be able to trade it back for their own currency later on.

Figure (5) illustrates this point in a numerical example with $\delta < \lambda$ (so that there is no autarky equilibrium). In the extension with a forex market, the coexistence region of (ψ_F, ψ_H)-space is larger than in the benchmark model. However, for sufficiently low ψ_H (ψ_F), an H-dominant (F-dominant) equilibrium continues to exist. Hence, the inclusion of a forex market does not overturn the main results: as long as there is some illiquidity in the forex market, there will still be dispersion in bond valuations that leads to dominant-currency equilibria.

The existence of a forex market also permits us to analyze how interest rate policies affect the real exchange rate.

Proposition 7 *Fix policies r^H, r^F that yield a dominant-currency equilibrium. The real exchange rate q is (locally) increasing in r^F and decreasing in $r^H F$.*

While typical New Keynesian models often find that interest rate hikes tend to cause a country's real exchange rate to appreciate *temporarily*, in our model, a permanent increase in the real rate on a country's bonds (i.e., a decrease in the liquidity premium) *permanently* increases the exchange rate. The intuition is straightforward. When Home bonds pay a higher interest rate, Foreign agents' relative valuation of those bonds increases as well, so they are willing to exchange a greater quantity of goods for Home bonds. In an equilibrium where H-currency is dominant, the real exchange rate reflects foreign agents' valuation ($q = {\varepsilon_H^F}^{-1}$), so it appreciates.

5 Conclusion

We develop a micro-founded model to study which assets will be used as international media of exchange. Governments can compete to internationalize their currencies by increasing the real interest rate paid by their bonds. Larger countries have a natural advantage at establishing themselves as dominant currency issuers in this competition. This advantage may dissipate, though, as the global economy becomes more integrated or as the large country's share of global trade decreases.

Methodologically, our model contributes by offering a tractable environment to study several issues in international finance. For instance, we extend our model in several directions to demonstrate the robustness of our conclusions. Furthermore, the model could also be extended to study optimal policies and game-theoretic interactions between competing currency issuers. We leave these issues to future research.

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A Proofs

Proof of Proposition 1. Consider the Bellman equation (5). It can be written in Lagrangian form as

$$W_t^j(x, b_j^j, b_i^j) = \max_{c,\ell,b_j^{j\prime},b_i^{j\prime}} u(c) - \ell + \beta V_{t+1}^j(b_j^{j\prime}, b_i^{j\prime}) - \lambda_t^j \Bigg(c + \frac{1}{1+r_t^j} b_j^{j\prime} + \frac{1}{1+r_t^i} \varepsilon_{it}^j b_i^{j\prime} - \psi_t^j x - w_t^j \ell - b_j^j - \varepsilon_{it}^j b_i^j - \Pi_t^j + \tau_t^j \Bigg)$$

The first-order conditions are

$$\begin{aligned}
(c) &: u'(c) = \lambda_t^j; \\
(\ell) &: 1 = w_t^j \lambda_t^j; \\
(b_j^{j\prime}) &: \beta(1+r_t^j) \frac{\partial}{\partial b_j^{j\prime}} V_{t+1}^j(b_j^{j\prime}, b_i^{j\prime}) = \lambda_t^j; \\
(b_i^{j\prime}) &: \frac{\beta(1+r_t^i)}{\varepsilon_{it}^j} \frac{\partial}{\partial b_i^{j\prime}} V_{t+1}^j(b_j^{j\prime}, b_i^{j\prime}) = \lambda_t^j.
\end{aligned}$$

Note that the first-order conditions imply $w_t^j u'(c) = 1$, so consumption for any country-j buyer is independent of bond holdings. Hence, $c = c_t^j = u'^{-1}\big(\frac{1}{w_t^j}\big)$, and λ_t^j is identical across all country-j buyers. Then, the first-order conditions for bond holdings imply that all country-j buyers hold the same portfolio of bonds. The budget constraint then yields

$$\begin{aligned}
\ell &= \frac{1}{w_t^j} \Bigg(c_t^j + \frac{1}{1+r_t^j} b_{j,t+1}^j + \frac{1}{1+r_t^i} \varepsilon_{it}^j b_{i,t+1}^j - \psi_t^j x - b_j^j - \varepsilon_{it}^j b_i^j - \Pi_t^j + \tau_t^j \Bigg) \\
&= \frac{1}{w_t^j} \big(K_t^j - \psi_t^j x - b_j^j - \varepsilon_{it}^j b_i^j \big),
\end{aligned}$$

where K_t^j is constant across country-j buyers. Hence, the value function can be written as

$$W_t^j(x, b_j^j, b_i^j) = u(c_t^j) + \frac{1}{w_t^j} \big(\psi_t^j x + b_j^j + \varepsilon_{it}^j b_i^j - K_t^j \big) + \beta V_{t+1}^j(b_{j,t+1}^j, b_{i,t+1}^j). \tag{46}$$

We obtain the desired result after imposing the equilibrium condition $w_t^j = 1$, setting

$$\hat{W}_t^j = u(c_t^j) - K_t^j + \beta V_{t+1}^j(b_{j,t+1}^j, b_{i,t+1}^j), \tag{47}$$

where

$$K_t^j \equiv c_t^j + \frac{1}{1+r_t^j} b_{j,t+1}^j + \frac{1}{1+r_t^i} \varepsilon_{it}^j b_{i,t+1}^j - \Pi_t^j + \tau_t^j.$$

■

Proof of Proposition 2. Henceforth, let

$$l^j \equiv \frac{1}{\beta(1+r^j)} - 1 \tag{48}$$

denote the *liquidity premium* on currency j.

Define functions $\psi_H(\varepsilon_H^F), \psi^F(\varepsilon_H^F)$ via the conditions for equilibrium in the H-bond market:

$$l^H = (\delta + \lambda\xi)\big(\psi_H(\varepsilon_H^F) - 1\big) + \lambda(1-\xi)\big(\psi_H(\varepsilon_H^F)\varepsilon_H^F - 1\big)^+, \tag{49}$$

$$l^H = \lambda\bigg(\xi(\frac{\psi_F(\varepsilon_H^F)}{\varepsilon_H^F} - 1)^+ + (1-\xi)(\psi_F(\varepsilon_H^F) - 1)\bigg). \tag{50}$$

Recalling that $\psi_H \geq 1$, the maximum level of ε_H^F consistent with (49) is

$$\overline{\varepsilon}_H^F \equiv 1 + \frac{l^H}{\lambda(1-\xi)} \geq 1.$$

Similarly, since $\psi_F \geq 1$, the minimum level of ε_H^F consistent with (50) is

$$\underline{\varepsilon}_H^F \equiv \left(1 + \frac{l^H}{\lambda\xi}\right)^{-1} \leq 1.$$

For any $\varepsilon_H^F \in [\underline{\varepsilon}_H^F, \overline{\varepsilon}_H^F]$, a unique solution $(\psi_H(\varepsilon_H^F), \psi_F(\varepsilon_H^F))$ to (49) - (50) exists.

Define

$$\psi_{H,\max} \equiv 1 + \frac{l^H}{\delta + \lambda\xi}, \quad \overline{\psi}_F \equiv 1 + \frac{l^H}{\lambda(1-\xi)}.$$

Then we have

$$\psi_H(\varepsilon_H^F) = \begin{cases} \psi_{H,\max} & \varepsilon_H^F \in \left[\underline{\varepsilon}_H^F, \frac{1}{\psi_{H,\max}}\right] \\ \frac{l^H+\delta+\lambda}{\delta+\lambda(\xi+(1-\xi)\varepsilon_H^F)} & \varepsilon_H^F \in \left(\frac{1}{\psi_{H,\max}}, \overline{\varepsilon}_H^F\right] \end{cases}, \tag{51}$$

$$\psi_F(\varepsilon_H^F) = \frac{l^H + \lambda}{\lambda(\frac{\xi}{\varepsilon_H^F} + 1 - \xi)} \quad \forall \ \ \varepsilon_H^F \in \left[\underline{\varepsilon}_H^F, \overline{\varepsilon}_H^F\right]. \tag{52}$$

Clearly, ψ_H is decreasing in ε_H^F and ψ_F is increasing in ε_H^F. Therefore, the locus of points (ψ_H, ψ_F) that solve this system of equations for *some* value of ε_H^F implicitly defines a function $\hat{\psi}_H(\psi_F)$ on the interval $\psi_F \in [1, \overline{\psi}_F]$.

Claim 8 *The function $\hat{\psi}_H(\psi_F)$ is decreasing and concave and satisfies $\hat{\psi}_H(1) = \psi_{H,\max}$, $\hat{\psi}_H(\overline{\psi}_F) = 1$, and $\hat{\psi}'(\overline{\psi}_F) > -1$.*

Proof. Solving (51)-(52),

$$\hat{\psi}_H(\psi_F) = \begin{cases} \psi_{H,\max} & \psi_F \in \left[1, \frac{l^H+\lambda}{\lambda(\xi\psi_{H,\max}+1-\xi)}\right] \\ (l^H+\delta+\lambda)\frac{l^H+\lambda\xi-\lambda(1-\xi)(\psi_F-1)}{\lambda\xi(l^H+\lambda)+\delta(l^H+\lambda\xi-\lambda(1-\xi)(\psi_F-1))} & \text{otherwise} \end{cases}$$

Then, in the region where $\hat{\psi}_H(\cdot)$ is not constant,

$$\hat{\psi}'_H(\psi_F) = -\frac{\lambda^2\xi(1-\xi)(l^H+\delta+\lambda)(l^H+\lambda)}{\big(\lambda\xi(l^H+\lambda)+\delta(l^H+\lambda\xi-\lambda(1-\xi)(\psi_F-1))\big)^2}. \tag{53}$$

Note that the term inside parentheses in the denominator is positive, since even when $\psi_F = \overline{\psi}_F$, we have

$$\delta\lambda(1-\xi)(\overline{\psi}_F - 1) = \delta l^H < \lambda\xi(l^H+\lambda) + \delta(l^H+\lambda\xi).$$

Therefore, the derivative in (53) is negative, as desired. Note, furthermore, that the function is concave: the derivative is decreasing in ψ_F, since the denominator is a decreasing function of ψ_F, the numerator is constant, and the sign of the derivative is negative.

Finally, note that the derivative (53) evaluated at $\overline{\psi}_F$ is

$$\hat{\psi}'(\overline{\psi}_F) = -\frac{\lambda^2\xi(1-\xi)(l^H+\delta+\lambda)(l^H+\lambda)}{(\lambda\xi(l^H+\delta+\lambda))^2} = (1-\xi)\frac{l^H+\lambda}{l^H+\delta+\lambda} > -1.$$

■

Using exactly analogous arguments, we can prove that there exists a decreasing and concave function $\hat{\psi}_F(\psi_H)$ on an interval $[1, \overline{\psi}_H]$ (where $\overline{\psi}_H = 1 + \frac{\ell_F}{\lambda\xi}$) representing the locus of points consistent with equilibrium in the market for F-bonds. The function $\hat{\psi}_F$ ranges from $\psi_{F,\max}$ to 1, and $\hat{\psi}'_F(\overline{\psi}_H) = 1$. As above,

$$\overline{\psi}_H \equiv 1 + \frac{l^F}{\lambda\xi}, \quad \psi_{F,\max} \equiv 1 + \frac{l^F}{\delta+\lambda(1-\xi)}.$$

Claim 9 *There exists a unique equilibrium under policies* (l^H, l^F) *if and only if*

$$\left(1 + \frac{\delta}{\lambda(1-\xi)}\right)^{-1} \le \frac{l^H}{l^F} \le 1 + \frac{\delta}{\lambda\xi}. \tag{54}$$

Otherwise, an equilibrium does not exist.

Proof. Consider the functions $\hat{\psi}_H(\psi_F)$, $\hat{\psi}_F(\psi_H)$ plotted in (ψ_F, ψ_H) space (with ψ_F on the x-axis). The curve $\hat{\psi}_H(\psi_F)$ intersects the y-axis at $\psi_{H,\max}$ and the x-axis at $\overline{\psi}_F$. Likewise, the curve $\hat{\psi}_F(\psi_H)$ intersects the y-axis at $\overline{\psi}_H$ and the x-axis at $\psi_{F,max}$.

If (54) holds, then

$$\frac{l^H}{\delta+\lambda\xi} \leq \frac{l^F}{\lambda\xi} \quad \Leftrightarrow \quad \psi_{H,\max} \leq \overline{\psi}.$$

Similarly, we can show that $\psi_{F,\max} \leq \overline{\psi}_F$. Hence, $\hat{\psi}_H(\cdot)$ hits the y-axis below $\hat{\psi}_F(\cdot)$ but hits the x-axis to the right of $\hat{\psi}_F(\cdot)$. By the intermediate value theorem, the two curves must intersect. Moreover, the two curves intersect only once: Proposition 8 implies that the slope of $\hat{\psi}_H$ is greater than -1 everywhere, whereas the slope of $\hat{\psi}_F$ is less than -1 everywhere.

On the other hand, if (54) does not hold, then these same properties demonstrate that either

1. $\psi_{H,\max} > \overline{\psi}_H$ but $\psi_{F,\max} < \overline{\psi}_F$; or
2. $\psi_{H,\max} < \overline{\psi}_H$ but $\psi_{F,\max} > \overline{\psi}_F$.

In either case, it is easy to check that one of the two curves lies entirely inside the other, so the two curves cannot intersect. ■

This concludes the proof of Proposition 2. ■

Proof of Proposition 3. We demonstrate the result for H bonds only. The corresponding inequality for F bonds can be derived analogously.

The Euler equation (42) implies that

$$\varepsilon_H^F = \frac{\lambda\xi\psi^F}{l^H + \lambda\xi - \lambda(1-\xi)(\psi^F - 1)}. \tag{55}$$

Furthermore, from Home agents' Euler equation (17) for Home bonds,

$$l^H \geq (\delta + \lambda\xi)(\psi^H - 1), \tag{56}$$

which holds with equality whenever H currency does not circulate internationally. Home currency does not circulate whenever $\psi^H \varepsilon_H^F \leq 1$. Given the inequality (56), it suffices to show

$$\begin{aligned} \lambda\xi\psi^H\psi^F &\leq (\delta+\lambda\xi)(\psi^H - 1) + \lambda\xi - \lambda(1-\xi)(\psi^F - 1) \\ \Leftrightarrow \psi^H &\geq \frac{\delta + \lambda(1-\xi)(\psi^F - 1)}{\delta - \lambda\xi(\psi^F - 1)}. \end{aligned}$$

Conversely, suppose this inequality does not hold, and that H currency does not circulate regardless. We have

$$\begin{aligned} \psi^H \varepsilon_H^F &= \frac{\lambda\xi\psi^H\psi^F}{l^H + \lambda\xi - \lambda(1-\xi)(\psi^F - 1)} \\ &= \frac{\lambda\xi\psi^H\psi^F}{(\delta+\lambda\xi)(\psi^H - 1) + \lambda\xi - \lambda(1-\xi)(\psi^F - 1)}. \end{aligned}$$

We know the right-hand side is greater than 1 (contradicting the assumption that H does not circulate), since

$$\lambda\xi\psi^H\psi^F > (\delta+\lambda\xi)(\psi^H-1)+\lambda\xi-\lambda(1-\xi)(\psi^F-1)$$

by assumption. Therefore, whenever

$$\psi^H \leq \frac{\delta+\lambda(1-\xi)(\psi^F-1)}{\delta-\lambda\xi(\psi^F-1)},$$

H circulates internationally, proving the result. ■

Proof of Proposition 4. We show that given initial equilibrium values of (ψ^H, ψ^F), if H (resp. F) circulates at the initial parameter configuration (δ, λ, ξ), then it continues to circulate under the new configuration (δ, λ', ξ') for $\lambda' \geq \lambda$ or $\xi' \geq \xi$ ($\xi' \leq \xi$).

H circulates if (22) holds. Notice that the right-hand side is increasing in λ as long as $\psi^F \geq 1$: the numerator increases and the denominator decreases. The right-hand side is also increasing as a function of ξ:

$$\begin{aligned}\frac{\partial}{\partial\xi}\frac{\delta+\lambda(1-\xi)(\psi^F-1)}{\delta-\lambda\xi(\psi^F-1)} &= -\frac{\lambda(\psi^F-1)}{\delta-\lambda\xi(\psi^F-1)}+\frac{\lambda(\psi^F-1)(\delta+\lambda(1-\xi)(\psi^F-1))}{(\delta-\lambda\xi(\psi^F-1))^2}\\ &= \frac{\lambda(\psi^F-1)}{\delta-\lambda(\psi^F-1)}\Big(-1+\frac{\delta+\lambda(1-\xi)(\psi^F-1)}{\delta-\lambda\xi(\psi^F-1)}\\ &> 0\end{aligned}$$

This proves the result for H. The result for F is proven in an exactly analogous way. ■

Proof of Proposition 5. We prove the comparative statics results for r^H. The proofs for r^F follow analogously. We use the same notation as in the proof of Proposition 2.

Recall that the equilibrium (ψ^H, ψ^F) is the intersection between the curves $\hat{\psi}^H(\psi^F)$ and $\hat{\psi}^F(\psi_H)$. The location of $\hat{\psi}^H(\psi^F)$ depends only on l^H. This locus is horizontal in the region where H does not circulate internationally and downward-sloping in the region where H circulates. Similarly, the location of $\hat{\psi}^F(\psi^H)$ depends on l^F, it is downward-sloping in the region where F circulates, and it is vertical in the region where F does not circulate. The proof of Proposition 2 showed that $\hat{\psi}^F(\psi^H)$ is steeper than the slope of $\hat{\psi}^H(\psi^F)$ everywhere.

An increase in r^H shifts the locus $\hat{\psi}^H(\psi^F)$ inwards. In the region where $\hat{\psi}^F(\psi^H)$ is vertical (i.e., when F does not circulate), this results in a decrease in ψ^H with no change in ψ^F. On the other hand, if F circulates, the inward shift results in a decrease in ψ^H and an *increase* in ψ^F. This proves points (1) and (2) of the proposition.

To demonstrate part (3), first consider $r^H = \frac{1}{\beta}$. Clearly, if $r^F < \frac{1}{\beta}$, r^H must be the dominant global currency: the only way the Euler equation (17) can hold is if $\psi^H = 1$, so if $\psi^F > 1$, then the equilibrium must lie in region H. The locus of points where H circulates internationally is continuous in r^H, so it must also be that H is the dominant currency for all r^H sufficiently close to $\frac{1}{\beta}$.

Finally, we prove point (4). The proof of Proposition 2 demonstrated that there does not exist an equilibrium where both H and F are valued if

$$l^H \geq \frac{\delta + \lambda\xi}{\lambda\xi} l^F.$$

In this case, the locus of points in (ψ^F, ψ^H)-space describing equilibrium in the market for H-bonds lies entirely outside the locus of points consistent with equilibrium in the market for F-bonds. Hence, there exists an equilibrium in which H-bonds are not valued and even Home agents hold only F-bonds. ■

Proof of Proposition 6. We demonstrate the result for country H and a fixed foreign policy r^F.

In any equilibrium where H is the dominant currency, (44) implies that

$$\psi^F = 1 + \frac{l^F}{\delta + \lambda(1-\xi)}, \tag{57}$$

where l^F is defined in (48). Then, for H to be the dominant currency, Proposition 3 implies that we must have

$$\psi^H \leq \overline{\psi}^H(l^F) \equiv \frac{\delta + \frac{\lambda(1-\xi)}{\delta+\lambda(1-\xi)} l^F}{\delta - \frac{\lambda\xi}{\delta+\lambda(1-\xi)} l^F}. \tag{58}$$

Since country-H buyers' demand for liquid assets must be entirely satisfied by country-H bonds,

$$b_H^H \geq f'^{-1}(\overline{\psi}^H(l^F)), \tag{59}$$

and the H interest rate must be bounded below by some $\underline{r}^H$ as $\xi \to 0$. But then the capital flow equation (14) implies

$$b_F^F \geq \frac{\lambda(1-\xi)}{1-\lambda\xi} b_H^H \to \frac{\lambda}{1-\lambda} b_H^H \quad \text{as} \quad \xi \to 1. \tag{60}$$

Inequality (59) implies that b_H^H stays bounded away from zero, which implies that Foreign holdings of Home bonds b_H^F are bounded away from zero as well. But the debt capacity inequality (28) implies that as $\xi \to 0$, Home cannot supply a non-zero quantity of debt to the rest of the world, proving that there does not exist an equilibrium in which H is the dominant currency. ■

B Model extensions

This section establishes results for the model extensions in greater detail.

B.1 Adding international financial markets

We begin by characterizing the conditions under which an equilibrium with H as the dominant currency exists. Consider an equilibrium in which H buyers are exactly indifferent between holding their H-bonds or trading them to F sellers in international meetings. In such an equilibrium, $\varepsilon_H^F = \frac{1}{\psi^H}$. Furthermore, for the international bond market to clear, F agents must be willing to continue holding their H-bonds whenever they access the market. Hence,

$$q = \frac{1}{\varepsilon_H^F} = \psi^H. \tag{61}$$

Then (17) implies

$$\frac{1}{\beta(1+r^H)-1} = (\delta + \lambda\xi)(\psi^H - 1). \tag{62}$$

The Euler equation for Foreign agents holding Home bonds is

$$\frac{1}{\beta(1+r^H)} - 1 = \lambda\big(\xi(\psi^F \frac{1}{\varepsilon_H^F} - 1) + (1-\xi)(\psi^F - 1)\big), \tag{63}$$

so both Euler equations are identical to those in the benchmark model. We conclude that a necessary condition is that (22) must hold.

Next, we turn to the market for F bonds. Since H is dominant, F bonds do not circulate. Then (44) yields

$$\frac{1}{\beta(1+r^F)} - 1 = (\delta + \lambda(1-\xi))(\psi^F - 1), \tag{64}$$

and (43) implies

$$\varepsilon_F^H = \frac{(\lambda(1-\xi)+\nu)\psi^H}{(\delta+\lambda(1-\xi))(\psi^F-1) + \lambda(1-\xi) + \nu - \lambda\xi(\psi^H-1)}. \tag{65}$$

If, in fact, F does not circulate in equilibrium, we must have $\psi^F\varepsilon_F^H \leq 1$. Equation (65) can be rearranged to show that this inequality is equivalent to

$$\psi^F \geq \frac{\delta - \nu + \lambda\xi(\psi^H-1)}{\delta - \nu\psi^H - \lambda(1-\xi)(\psi^H-1)}. \tag{66}$$

Proof of Proposition 7. The proposition follows immediately from the fact that, as demonstrated above, $q = \psi^H$ in an H-dominant equilibrium. Then, since Proposition 5 implies that an increase in r^H decreases ψ^H, an increase in r^H also leads to an appreciation of the H-currency (i.e., a decrease in q). ■