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Interchange Fees in Payment Networks: Implications for Prices, Profits, and Welfare*

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Abstract

In a two-sided model of the payment card market, we introduce a specific form of elastic demand (constant elasticity), merchant market power, ad valorem fees, and cash as an alternative. We derive the "credit card tax," consisting of an endogenously determined interchange fee and any rewards paid. We characterize how this tax influences prices, profits, and welfare. We also examine how these relationships vary under different assumptions about the elasticity of demand, merchant market power, and differentiation between cash and credit. Under the assumptions of our model, by endogenizing the credit card tax, we show that capping interchange fees benefits all consumers by lowering these taxes, even if rewards decrease.

Keywords: credit cards, two sided networks, merchant competition, interchange fees, regulation

JEL codes: L13, L40, G28, E42

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1 Introduction

This paper examines the level of a wholesale price-the interchange fee-typically set by a payment card network that influences the distribution of acceptance costs and benefits incurred or received by merchants and consumers. In the US, these fees typically range between 2% and 3% of the transaction value.[1] Interchange fees have been controversial for half a century, ever since payment card networks began to dominate purchases at the point-of-sale in the U.S. during the 1970s.

Merchants argue that interchange fees are too high, cutting into their profits and/or forcing them to pass on the cost to consumers.[2] In the United States, one policy response was the so-called Durbin Amendment, which sought to cap interchange fees paid to large debit card issuers at a level reasonable and proportional to the cost of processing debit transactions on behalf of merchants and consumers.[3] Also in the U.S., the proposed Credit Card Competition Act of 2023 aims to enhance competition and choice in the credit card network market, potentially lowering interchange fees.[4] The European Union implemented Regulation (EU) 2015/751, which caps interchange fees for both debit and credit card transactions.[5] Other countries, such as Australia and Canada, have introduced similar measures to regulate interchange fees and other perceived impediments to competition in retail payments.

Card networks and banks, on the other hand, argue that these regulations could reduce the benefits consumers receive, including credit card rewards. Rewards appear in myriad forms but they are tied to the transaction volume of the cardholder in order to incent greater usage. The average reward, according to Wang (2023) is about 1.45% of the value of a transaction.[6] This debate highlights the complex balance between reducing costs for merchants and maintaining benefits for consumers.

One of the complications in understanding the role and consequences of interchange fees is the consensus among economists that payments represent a classic example of a two-sided market that is intermediated by a platform-the payment card network. As such, pricing decisions that affect one side of the market (cardholders) affect decisions made by participants (merchants) on the other side of the market.

The purpose of this paper is to develop a more general model of a two-sided market with endogenous interchange fees, cardholder rewards, prices and the mix of payment instruments (card or cash) used. We can use this model to assess the impact of interchange fee cap regulation. Our model features a monopoly payment network and, to the best of our knowledge, is the first to integrate all of the following realistic elements of the credit card market (we discuss the literature in the next section): (i) elastic

[1] Interchange fees account for most of the merchant discount (on average about 2.25%, according to Wang (2023)) paid by merchants to their banks for processing credit card transactions.

[2] See https://thehill.com/business/4556451-visa-mastercard-to-lower-swipe-fees-in-merchant-antitrust-suit-settlement/

[3] See https://www.federalregister.gov/documents/2023/11/14/2023-24034/debit-card-interchange-fees-and-routing

[4] See https://www.congress.gov/bill/118th-congress/senate-bill/1838/text

[5] See https://www.legislation.gov.uk/eur/2015/751/body

[6] See https://www.americanbanker.com/opinion/death-by-a-thousand-caps-state-laws-could-kill-credit-card-rewards

product demand, (ii) merchants with market power, (iii) ad valorem fees and rewards, and (iv) cash as an alternative payment mode.

We parameterize the retail sector in three dimensions: the (constant) elasticity of demand (ε), the number of firms (n), and the conjectural variation (λ) retailers use when interpreting how their competitors will respond to their quantity decisions. The number of retailers and the conjectural variation combined determine the conduct parameter (γ) that describes the amount of competition in the final goods market. This setup permits us to understand more generally how consumer preferences, as captured by the elasticity of demand and the degree of differentiation between cash and credit, and merchant competition, endogenously determine retail prices, interchange fees and the consumer response to rewards that affect consumer demand, which, in turn, affects retail prices.

The model establishes an equilibrium where aggregate demand and merchant profits are influenced by what we term "credit card taxes," a key contribution of this paper. These taxes create a wedge between the price consumers pay-depending on whether they use cash or credit-and the price merchants receive for each unit of the final good. This difference represents the network revenue per dollar of transactions. The amount of credit card taxes is positively correlated with the interchange fee paid by merchants, while the tax credit card users pay is negatively correlated with the relative generosity of cardholder rewards. These taxes naturally emerge when we link the product market with network fees and rewards, an aspect largely overlooked in the existing literature.

One of our main findings is that an interchange fee cap benefits all consumers. Cash users benefit because a lower interchange fee is passed on to prices. The fact that credit card users also benefit is less obvious. As discussed above, one of the banks' main arguments against an interchange fee cap regulation is that such an intervention would lower the rewards for credit card users and hence hurt them. While rewards may indeed decrease, our analysis reveals that, due to competition between the two payment instruments, cash and credit, the reduction in rewards is smaller than the reduction of the interchange fee. The net effect is captured by the tax credit card users pay, which decreases post-regulation.

Schuh et al. (2010) attempt to quantify the size and distributional consequences of implicit monetary transfer to credit card users from non-card (or "cash") users that result from interchange fees and card rewards. They show that, on average, each cash-using household pays \$149 to card-using households, and each card-using household receives \$1,133 from cash users every year. Their calculations suggest this is a regressive transfer from low-income to high-income households. Using more recent data Felt et al. (2023) estimate that the lowest income cohorts in the U.S. (Canada) are paying 20 (60) basis points, respectively, more than the highest income cohorts in order to conduct their purchases.[7]

While we do not model heterogeneity in income, the resulting equilibrium in our model has a similar flavor: Cash users pay a higher tax than credit card users due to the presence of rewards when a trans-

[7] The calculations in both papers are derived using varying assumptions about the extent of pass through of card acceptance costs to consumers. Pass through is not formally modeled or estimated.

action is made with a credit card. However, the literature uses the term 'subsidy' loosely. Depending on the elasticity of demand, the equilibrium reward in our model can be so high that the tax for credit card users turns into a subsidy, in the sense that the price credit card users pay is lower than the price merchants charge. Hence, our analysis clarifies when credit card users are subsidized in absolute terms, as opposed to when they are subjected to a tax that is nevertheless lower than the tax of cash users.

Our analysis predicts how merchant market power influences equilibrium taxes and merchant profits. As the product market becomes more competitive, the equilibrium tax for credit card users decreases. For cash users, the equilibrium tax decreases if only if the product demand elasticity is low. Aggregate merchant profits exhibit an inverse U-shape relative to merchant market power. Interestingly, there is a region where increased competition among merchants leads to higher equilibrium profits. This occurs because the network, which competes with cash, reduces the tax for credit card users as product market competition intensifies, thereby encouraging higher consumption levels.

We also derive predictions regarding how credit and cash substitutability affects equilibrium fees, rewards and taxes. When the two payment modes become closer substitutes, the tax credit card users pay increases, mainly due to lower rewards. The tax cash users pay also decreases, but only when product demand is relatively elastic. Otherwise, the tax and interchange fee increase.

The remainder of this paper is organized as follows. Section 2 provides a brief overview of the literature. Section 3 lays out the fundamental structure of the model, while Section 4 presents the equilibrium analysis. Section 5 examines the impact of an interchange fee cap regulation. Section 6 concludes.

2 Literature review

Over the last three decades, an extensive theoretical literature on two-sided markets has emerged, and many of those papers examine the implications for payment systems; see Rochet and Tirole (2006), Rysman (2009) and Jullien, Pavan, and Rysman (2021) for reviews of this literature. Our purpose here is simply to sketch the main findings related to our work and highlight our contributions.

Our modeling assumptions are most closely aligned with those in Wang (2010) and Shy and Wang (2011), both of which employ a constant elasticity demand framework and incorporate ad valorem interchange fees and rewards. Wang (2010) further assumes a perfectly competitive merchant sector and a fixed distribution of cash and credit card users. Among other contributions, he analyzes the effects of an interchange fee cap and concludes that such regulation reduces retail prices, increasing consumption and thereby enhancing consumer welfare. We extend this result by demonstrating that consumer benefits persist even in a more realistic setting characterized by merchant market power and competition between payment instruments.

Shy and Wang (2011) compare 'proportional' and 'fixed' transaction fees in a model where merchants possess market power, but cash is not available as an alternative payment method. Our primary innovation relative to Shy and Wang (2011) is the explicit inclusion of cash as an alternative payment mode. In our framework, each consumer chooses their preferred payment method, allowing the network to influence both the intensive and extensive margins of payment behavior. In the absence of cash, an indeterminacy arises: the interchange fee and reward cannot be uniquely determined.[8] Consequently, without cash, meaningful comparative statics cannot be derived to analyze how product market competition and other key parameters affect the interchange fee and rewards separately. In particular, the impact of interchange fee regulation cannot be assessed. Any cap imposed on the interchange fee becomes ineffective, as the network can adjust rewards to offset the cap, thereby maintaining the effective tax-and thus prices and welfare-at their pre-regulation equilibrium levels.

Baxter (1983) developed the first formal model of interchange fees in a payment scheme. In that paper, Baxter makes three key assumptions: i) issuers and acquirers make no profit (perfect competition), ii) merchants do not use card acceptance strategically, i.e., to attract consumers from rival merchants who do not accept a card and iii) there is no merchant heterogeneity in the benefit of accepting cards. Schmalensee (2002) develops a model that explores the double marginalization problem that emerges when networks set an interchange fee that is paid to card issuers and the acquiring bank for the merchant separately sets a merchant discount for processing transactions.

In Rochet and Tirole (2002), the consumers receive a different benefit from transacting using cards rather than paying cash. There is a single payment network that sets an interchange fee, but it is not ad valorem. The retail market is a Hotelling model, but consumers face the choice of purchasing a fixed quantity of their preferred good. Rochet and Tirole (2011) develop a similar model, but here there is competition between networks modelled using the Hotelling framework. Wright (2004) uses a model similar to Rochet and Tirole (2002) and relaxes all three assumptions of Baxter (1983). Bedre-Defolie and Calvano (2013) show that networks oversubsidize card usage and overtax merchants.

Wang and Wright (2017, 2018) assume Bertrand competition among sellers in a market with many different goods that vary widely in their costs and values. The authors show that ad valorem fees and taxes represent an efficient form of price discrimination relative to uniform fees that disadvantage low-cost, low-value goods. Wang (2023) develops a structural empirical approach to a two-sided market of payments. He finds that interchange fee caps increase welfare by reducing rewards, retail prices, and credit card use. Overall consumer welfare increases because low-income consumers, who rely less on credit, pay lower prices, but high-income consumers get hurt because their rewards decrease. In our model, while there is no income heterogeneity, we predict that the reduction in rewards will not completely offset the interchange fee reduction due to a cap.

Edelman and Wright (2015) shows that an intermediary always chooses to impose price coherence

[8] Wang (2010) resolves this indeterminacy by assuming that the network (or issuers collectively) faces a convex cost function for processing card payments, which uniquely determines the interchange fee and rewards, even though merchants accepting cards do not serve cash-paying customers.

if it has the ability to do so. In the context of payments this is the equivalent of the “no surcharge” rule that precludes merchants from charging customers a different price based on how they choose to pay. An important qualification is that the fees examined in that paper are not ad valorem.

The prevailing methodology in most of the above papers includes one or more of the following assumptions: (1) consumer demand is inelastic, (2) fees are specific rather than ad valorem, (3) Bertrand competition is assumed in the product market, (4) cash transactions are not allowed, and (5) the benefits and costs experienced by both merchants and consumers due to card usage are independent of each other and of the product price, implying that interchange fees and rewards do not affect retail prices. While these simplifications facilitate analysis, they diverge significantly from real-world observations.

Our study aims to bridge this gap by offering a model that encompasses many realistic features of the credit card market.

3 Structure of the market

We consider an industry consisting of n firms (merchants), $j = 1, ..., n$, producing a single homogeneous product. The output of firm j is denoted by x_j and the industry output by $X = \sum_{j=1}^{n} x_j$. All the merchants have the same cost structure $C(x) = cx$, where $c > 0$ is a constant marginal cost. The consumer price is given by an inverse demand function $P(X)$, with derivative $P_X(X) < 0$ and elasticity $\varepsilon \equiv \frac{P}{XP_X} < 0$. Consumers, when purchasing goods, can use either cash or a credit card.

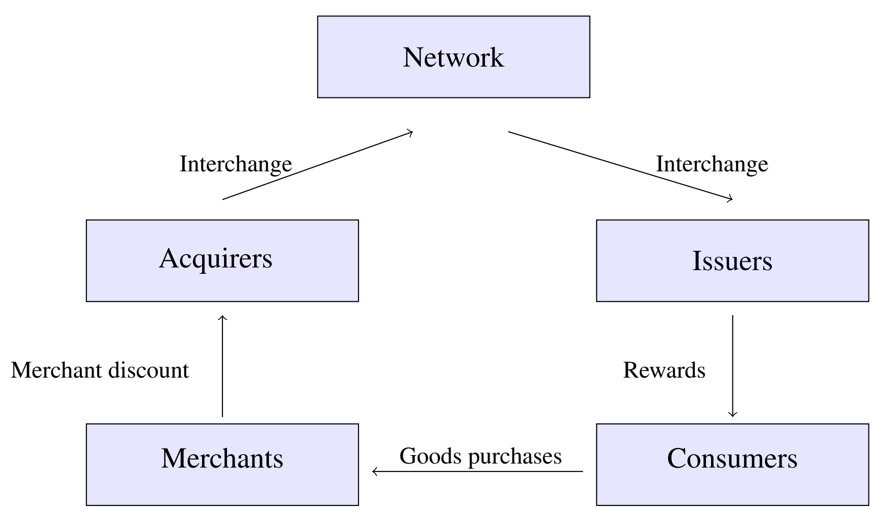

Figure 1: Payment flows in the Network

For credit card users, there exists a monopoly payment network denoted by $\mathcal{N}$, with $N_A < n$ acquiring banks and $N_I < n$ issuing banks. Banks are homogeneous and compete à la Bertrand for merchants and cardholders. In Figure 1, we present the payment flows in the network. Each acquiring bank α

chooses its merchant discount m^α to attract merchants, and each issuing bank ι chooses the reward $R^\iota \in [\underline{r}, 1]$ to attract users/consumers and to influence the value of consumer transactions. If $R^\iota < 0$, then the reward becomes a fee. The network chooses the interchange fee, $i \in [0, 1]$, which becomes each acquirer's marginal cost. Part of i, denoted by r, goes to the issuing banks to fund the rewards, and the rest is kept by the network. For simplicity, all other costs to process a transaction are assumed to be zero.

The reward for the credit card is a percentage of the value of the transaction that a consumer who uses the credit card receives as cashback from the issuing bank. The acquiring bank charges a merchant discount fee, which is a percentage of the value of the transaction that is paid by the merchant to the acquiring bank when a consumer uses the credit card. We assume that the network cannot price discriminate across banks, i.e., all acquirers pay the same interchange fee to the network, and all issuers receive the same fraction of the interchange fee from the network.

Consumers exhibit horizontal preferences between cash and credit, as discussed further in Section 4.4. Network fees and rewards affect the value of transactions within the payment network and can incentivize cash users to switch. We assume that merchants do not impose surcharges, meaning they cannot differentiate prices based on the payment method.[9] Furthermore, we consider a mature card market in which the extensive margin of card adoption is largely saturated (e.g., Wang (2010)). As a result, all merchants are assumed to accept both cash and credit.

We analyze a five-stage game with simultaneous and independent moves in each stage. In stage 1, the network sets the interchange fee, i, and chooses how much of the interchange fee, r, will be given to each issuing bank. In stage 2, each acquiring bank sets the merchant discount m^α, and each issuing bank sets the reward R^ι. In stage 3, each merchant chooses its level of output. In stage 4, each consumer chooses whether to use credit or cash. In stage 5, consumers make purchases. We will look for a subgame-perfect Nash equilibrium in pure strategies.

4 Equilibrium analysis

4.1 Acquiring and issuing banks

Acquiring banks compete in merchant fees m^α. The network interchange fee i is each acquiring bank's marginal cost. Given that acquiring banks are homogeneous and compete for merchants à la Bertrand, each acquiring bank sets the same merchant discount $m^\alpha = i$, for all α. Acquiring banks earn zero profits in equilibrium.

Issuing banks compete in rewards R^ι. Each issuing bank receives from the network part of the interchange fee $r < i$. This is the maximum amount, per dollar of transactions, that each issuing bank

[9] Foster et al. (2024) find that surcharging remains relatively uncommon.

can give to users as a reward. (If $r < 0$ then issuing banks pay the network a fee.) Given that issuing banks are homogeneous and compete à la Bertrand for users, each issuing bank sets the same reward $R^{\iota} = r$, for all ι. Issuing banks earn zero profits in equilibrium.

Therefore, in the unique equilibrium, for credit card transactions each merchant pays a merchant discount i and each consumer receives a reward r.

4.2 Consumers

Let μ be the endogenous fraction, of a unit mass, of consumers who use a credit card, and the remaining fraction, $1-\mu$, uses cash. We will describe how μ is determined in Section 4.4. Consumers who use cash pay a price P, while consumers who use the credit card pay $P \cdot (1-r)$.[10]

There are two goods, x and a numeraire good y whose price is normalized to one. Each consumer's preferences over the two goods are captured by the following quasi-linear utility $U = \frac{kx(1/x)^{1/k}}{k-1} + y$, with $k > 1$. Each consumer maximizes her utility subject to a budget constraint. This yields the standard constant elasticity individual demand (with $\varepsilon = -k$). The demand of a consumer who uses cash is $x_{ch} = \frac{1}{P^k}$ and the demand of a consumer who uses credit is $x_{cc} = \frac{1}{(P \cdot (1-r))^k}$. The aggregate demand is $X = \mu x_{cc} + (1-\mu)x_{ch}$ and the aggregate inverse demand function is

$$P(X, r, \mu) = \left(\frac{1}{X}\right)^{1/k} \left(\frac{\mu}{(1-r)^k} + 1 - \mu\right)^{1/k}. \tag{4.1}$$

4.3 Merchant competition

All merchants accept cash and credit. We assume that for each merchant j, a fraction μ of its sales are paid with a credit card, while a fraction $1-\mu$ are paid with cash. Each merchant takes the rewards and the interchange fee as given and chooses its output x_j to maximize (average) profits given by

$$\begin{aligned} \pi_j &= (\mu \cdot (1-i)P(X, r, \mu) + (1-\mu)P(X, r, \mu))x_j - cx_j \\ &= (1-\mu i)P(X, r, \mu)x_j - cx_j, \end{aligned}$$

where $P(X, r, \mu)$ is given by (4.1).

In selecting its output each merchant j conjectures that other merchants' responses will be such that $\frac{dX}{dx_j} = \lambda$, the conjectural variation λ being taken as a fixed constant throughout. The case $\lambda = 1$ corresponds to the Cournot conjecture. When $\lambda = 0$, conjectures are 'competitive' and we obtain the Bertrand outcome. When $\lambda = n$, each firm believes that all other active firms will behave exactly as it

[10] We use $\cdot$ to distinguish between multiplication, i.e., $P \cdot (1-r)$, and a function $P(X)$, when the two are not immediately distinguishable from the context.

does: tacit collusion among incumbent firms then being perfect (in the sense that aggregate profits are maximized conditional on the number of firms). It will be assumed throughout that $\lambda \in [0, n]$.[11]

The first order condition of the representative merchant is (omitting arguments)

$$\frac{\partial \pi_j}{\partial x_j} = (1 - \mu i) \left(P_X \frac{dX}{dx_j} x_j + P \right) - c = 0. \tag{4.2}$$

Restricting attention to symmetric equilibria, this becomes

$$\begin{aligned} (1 - \mu i) \left(P_X X \gamma + P \right) = c \quad &\Rightarrow \quad P \cdot \left(1 + \frac{\gamma}{\varepsilon} \right) = \frac{1}{(1 - \mu i)} \\ &\Rightarrow \quad P = \frac{c}{\left(1 - \frac{\gamma}{k} \right)} \frac{1}{(1 - \mu i)}, \end{aligned} \tag{4.3}$$

where $\gamma \equiv \frac{\lambda}{n} \in [0, 1]$.[12] As γ decreases, the merchant market becomes more competitive and price approaches marginal cost c (adjusted by the $1 - \mu i$). From the merchant's perspective, a lower interchange fee has the same effect as a reduction in marginal cost, and therefore the same effect on price.

Since in our case the elasticity of the slope of inverse demand, $E \equiv -\frac{P_{XX} X}{P_X}$, is $\frac{1+k}{k} < 2$, the second order condition, which is $2 - \gamma E > 0$, is satisfied. The stability condition requires that $1 + \gamma \cdot (1 - E) > 0$ (see, for example, Seade (1980) and Delipalla and Keen (1992)), which is also satisfied. From the first-order condition (4.3), a non-negative price-cost margin implies that the elasticity must satisfy $\gamma - k < 0$, which also holds.

From (4.3), it follows that the (average) price merchants receive, P^m, is $(1 - \mu i)P$. It is instructive at this juncture to introduce the expressions for the tax consumers pay due to a credit card. The tax credit card users pay is $z_{cc} \equiv \frac{1-r}{1-\mu i}$, making the price they pay $P^{cc} = z_{cc} P^m$. The tax cash users pay is $z_{ch} \equiv \frac{1}{1-\mu i}$, making the price they pay $P^{ch} = z_{ch} P^m$. A higher interchange fee increases the tax for all consumers, while a higher reward lowers it for the consumers who use a credit card.

The price merchants receive in the presence of a credit card is the same as the price without a credit card (i.e., using (4.3), P^m is not a function of z_{cc} or z_{ch}),

$$P^m = \frac{ck}{k - \gamma} \geq c. \tag{4.4}$$

Thus, in total, consumers pay the entire burden of a tax or receive the entire benefit of a subsidy. This is a well-known feature of the constant elasticity demand. Under other assumptions about demand, it is possible that some of the incidence of the credit card tax would fall on merchants.

[11] See Seade (1980), Bresnahan (1981) and Delipalla and Keen (1992) for similar modeling frameworks.

[12] Note that γ is similar to the conduct parameter θ in Weyl and Fabinger (2013) and Miklos-Thal and Shaffer (2021). Following most of the literature, we assume that the conduct parameter is not a function of aggregate output. For cases where the conduct parameter is a function of aggregate output, see Miklós-Thal and Shaffer (2021).

Efficiency dictates that $P = c$. We can have $P > c$ either because $\gamma > 0$, or $i > 0$, or both. The first source of inefficiency arises when competition in the merchant market is imperfect. The second source of inefficiency is due to the credit card tax levied by the payment network. Since $i > 0$ (otherwise network profit cannot be positive), there is a double-marginalization: the first mark-up is from the merchants when they have market power and the second mark-up is from the payment network (that has market power).

4.4 Endogenizing credit card and cash user shares

We assume that credit and cash are 'differentiated'. We model differentiation using the circular model of Salop (1979).[13] In particular, on a unit circumference circle, the network is located at 0, cash is located at $\frac{1}{2}$ and users/consumers are uniformly distributed on the circumference with density one. We assume that each consumer receives a gross benefit $V > 0$, pays a price P^{cc} when making purchases with a credit card, a price P^{ch} from making purchases with cash and incurs a linear per-unit of distance to a credit card transportation cost $t > 0$. The parameter t captures the degree of differentiation between credit and cash. For tractability, we assume that V is independent of the volume of transactions. Thus, the consumer located at $x \in \left[0, \frac{1}{2}\right]$, if she uses the credit card obtains a net utility $V - P^{cc} - tx$ and if she uses cash obtains a net utility $V - P^{ch} - t\left(\frac{1}{2} - x\right)$. The mass of consumers who use a credit card is given by

$$\mu = \frac{1}{2} + \frac{P^{ch} - P^{cc}}{t} = \frac{1}{2} + \frac{c \cdot (z_{ch} - z_{cc})}{t \cdot \left(1 - \frac{\gamma}{k}\right)}. \tag{4.5}$$

First, note that the second term of μ in (4.5) is zero when r is zero. This means rewards are the key to changing payment shares. Second, the importance of the transportation cost t is relative to the cost of merchandise. The less competitive is the retail market, i.e., higher γ, the more sensitive is the payment market share for a given transportation cost and that is because prices are higher and rewards are affected by prices.

4.5 Network's decisions

Using (4.1) and (4.3) the equilibrium aggregate output, for any given interchange fee and reward, is

$$\begin{aligned} X &= \mu x_{cc} + (1-\mu)x_{ch} = \frac{(k-\gamma)^k}{(ck)^k}\left(\frac{\mu \cdot (1-\mu i)^k}{(1-r)^k} + (1-\mu)(1-\mu i)^k\right) \\ &\quad \left(\text{using } z_{cc} \equiv \frac{1-r}{1-\mu i} \text{ and } z_{ch} \equiv \frac{1}{1-\mu i}\right) \\ &= \frac{(k-\gamma)^k}{(ck)^k}\left(\frac{\mu}{z_{cc}^k} + \frac{1-\mu}{z_{ch}^k}\right), \end{aligned} \tag{4.6}$$

[13] See Jeon and Rey (2022) for a similar assumption regarding differentiation between two platforms.

where μ is given by (4.5).

The network profit consists of the interchange fee minus the rewards to consumers times the total value of transactions in the market from consumers who make purchases with a credit card, $(i-r)P\mu x_{cc}$. We assume that transaction processing costs are zero.[14] Using (4.3), the network profit can be expressed as follows

$$\pi_{\mathcal{N}}(i,r) = \frac{c\mu \cdot (i-r)}{\left(1-\frac{\gamma}{k}\right)(1-\mu i)} x_{cc}(i,r). \tag{4.7}$$

It will be more revealing if, using the credit card taxes $z_{cc} \equiv \frac{1-r}{1-\mu i}$ and $z_{ch} \equiv \frac{1}{1-\mu i}$, we express (4.7) as an explicit function of these taxes

$$\pi_{\mathcal{N}}(z_{cc}, z_{ch}) = \frac{c \cdot ((\mu z_{cc} + (1-\mu) z_{ch}) - 1)}{\left(1-\frac{\gamma}{k}\right)} x_{cc}(z_{cc}). \tag{4.8}$$

The network chooses z_{cc} and z_{ch} to maximize (4.8). For example, network can increase z_{ch} by increasing the interchange fee i and it can keep z_{cc} fixed by simultaneously increasing the reward r. The weighted average tax affects network profits and since $z_{ch} > 1$ (because $i > 0$) the network can subsidize credit card users by choosing $z_{cc} < 1$ and still make positive profits $(\mu z_{cc} + (1-\mu) z_{ch} > 1)$. We will come back to this later in this section.

Using (4.8) and (4.5), let's understand the trade-offs the network faces when it changes the taxes for cash and credit card users. As a benchmark, observe that when there are no credit card users, $\mu = 0$, then $z_{ch} = 1$ and the network profit is zero. First, we consider the trade-off with respect to the tax for cash users. When z_{ch} increases profits increase, holding μ fixed. However, from (4.5), the share of credit card users, μ, also increases, which reduces profits per transaction assuming $r > 0$ ($z_{ch} > z_{cc}$), which is confirmed in equilibrium. Since credit card users 'cost' the network relatively more than cash users, due to the rewards, any increase in the share of credit at the expense of cash lowers profits per transaction, all else equal. Finally, because $z_{ch} \equiv \frac{1}{1-\mu i}$, a higher μ reinforces the increase in z_{ch}. Second, we consider the trade-off with respect to the tax for credit card users. When the network increases z_{cc}, profits increase while holding μ and individual consumption x_{cc} fixed. Additionally, μ decreases, leading to further profit increases per transaction as discussed above. However, individual consumption using a credit card, x_{cc}, decreases, illustrating the usual tradeoff between margin and volume. Finally, a lower μ mitigates the increase in z_{ch}, since $z_{cc} \equiv \frac{1-r}{1-\mu i}$.

The network profit function, after we substitute (4.5) into (4.8), is given by

$$\pi_{\mathcal{N}}(z_{cc}, z_{ch}) = \frac{k\left(\frac{k-\gamma}{ck}\right)^k c \cdot \left(\left(\left(\frac{z_{cc}}{2} + \frac{z_{ch}}{2} - 1\right)t - c(z_{cc} - z_{ch})^2\right)k - \frac{t\gamma \cdot (z_{cc} + z_{ch} - 2)}{2}\right) z_{cc}^{-k}}{t \cdot (k-\gamma)^2}. \tag{4.9}$$

[14] Adding a constant marginal processing cost would not affect our results qualitatively.

The profit-maximizing taxes, which are the solutions to the system of first-order conditions, are given by

$$z^*_{cc} = \frac{(16c-t)k+\gamma t}{16c(k-1)} \text{ and } z^*_{ch} = \frac{(16c+3t)k^2-(3\gamma+4)tk+4\gamma t}{16(k-1)ck}. \tag{4.10}$$

In Appendix A.1, we show that the network profit function is quasi-concave in z_{cc} and z_{ch} if and only if $t < \frac{16ck}{k-\gamma}$. This condition ensures that the first-order conditions (with equality) are sufficient for a maximum and also guarantees $z^*_{cc} > 0$. If this condition is not met, the solution is at a corner and becomes uninteresting.

Cash users always pay a higher tax in equilibrium, $z^*_{ch} - z^*_{cc} = \frac{t\cdot(k-\gamma)}{4ck} > 0$, which implies that, $\mu = \frac{3}{4}$, i.e., 75% of consumers use a credit card and 25% use cash.[15] It also implies that consumers receive rewards in equilibrium, $r^* > 0$.

Using (4.10), (4.5), and $z_{cc} \equiv \frac{1-r}{1-\mu i}$ and $z_{ch} \equiv \frac{1}{1-\mu i}$ we can derive the unique equilibrium interchange fee and reward

$$i^* = \frac{12k^2t-((12\gamma+16)t-64c)k+16\gamma t}{(48c+9t)k^2-(9\gamma+12)tk+12\gamma t} \text{ and } r^* = \frac{4(k-\gamma)(k-1)t}{(16c+3t)k^2-(3\gamma+4)tk+4\gamma t}. \tag{4.11}$$

Without cash as an alternative payment mode (e.g., Shy and Wang (2011)), i and r cannot be uniquely determined. In this scenario, only the tax for credit card users, $z \equiv \frac{1-r}{1-i}$, is pinned down. Consequently, meaningful comparative statics cannot be performed.

The equilibrium network profits, after substituting (4.10) into (4.9), are given by

$$\pi_{\mathcal{N}}(z^*_{cc}, z^*_{ch}) = \frac{((16c-t)k+\gamma t)\left(\frac{16(k-\gamma)}{ck}\right)^k \left(\frac{(16c-t)k+\gamma t}{c(k-1)}\right)^{-k}}{16(k-1)(k-\gamma)}. \tag{4.12}$$

The equilibrium total merchant profit is

$$\Pi^m(z^*_{cc}, z^*_{ch}) = (P^m - c)X = \frac{c\gamma}{k-\gamma}\frac{(k-\gamma)^k}{(ck)^k}\left(\frac{3}{4(z^*_{cc})^k} + \frac{1}{4(z^*_{ch})^k}\right)$$
$$= \frac{c\gamma(ck)^{-k}(k-\gamma)^{k-1}\left(3\cdot 16^k\left(\frac{(16c-t)k+\gamma t}{k-1}\right)^{-k} + \left(\frac{(16c+3t)k^2-(3\gamma+4)tk+4\gamma t}{16(k-1)ck}\right)^{-k}\right)}{4}. \tag{4.13}$$

[15] The independence of shares from the parameters is a characteristic of the constant elasticity demand. In a more general demand scenario, where the elasticity varies with aggregate output, μ would depend on the key parameters in equilibrium.

4.6 Comparative statics

We examine how demand elasticity, merchant market power and degree of differentiation between cash and credit affect the equilibrium variables, with particular emphasis on the taxes credit card and cash users are paying.

We summarize the effect of demand elasticity in the following Proposition.

Proposition 1 *The equilibrium taxes as a function of the demand elasticity are given as follows:*

a) *Low demand elasticity: $k < \gamma + \frac{16c}{t}$. Both groups of consumers pay a tax. Credit card users receive a reward $r^* \geq 0$. Cash users pay a higher tax than credit card users, $z^*_{ch} > z^*_{cc} > 1$.*

   – *Credit card users pay a higher price than the price merchants charge: $P^{cc} = z^*_{cc}P^m = \frac{(16c-t)k^2+\gamma kt}{16(k-1)(k-\gamma)} > P^m = \frac{ck}{k-\gamma}$.*

b) *High demand elasticity: $k > \gamma + \frac{16c}{t}$. Cash users pay a tax, while credit card users are subsidized by receiving a relatively high reward, $z^*_{ch} > 1 > z^*_{cc}$.*

   – *Credit card users pay a lower price than the price merchants charge: $P^{cc} = z^*_{cc}P^m = \frac{(16c-t)k^2+\gamma kt}{16(k-1)(k-\gamma)} < P^m = \frac{ck}{k-\gamma}$.*

c) *As demand elasticity increases (higher k):*

   – *the tax credit card users pay, z^*_{cc}, is decreasing,*

   – *the difference between the taxes, $z^*_{ch} - z^*_{cc}$, is increasing,*

   – *if $k < \frac{4}{3}$, the tax credit cash users pay, z^*_{ch}, is decreasing,*

   – *if $k > \frac{4}{3}$, the tax credit cash users pay, z^*_{ch}, is decreasing if and only if $t < \frac{16k^2c}{3\gamma k^2-8\gamma k+k^2+4\gamma}$.*

It is often suggested by some academics and practitioners that cash users are 'subsidizing' credit card users if the latter are receiving rewards (see, for example, Schuh et al. (2010) and Felt et al. (2023)). Here we wish to use the term differently. We have already defined the credit card taxes that card and cash users pay, and we established that cash users pay a higher tax than do card users if rewards are positive. By subsidy, we mean that rewards are so high relative to interchange fees that card users are paying less than P; in other words, the mark-up is less than 1. Proposition 1 reveals that when demand elasticity is relatively high, credit card users are indeed subsidized, paying a lower price than the price merchants charge. Conversely, when demand elasticity is low, credit imposes a tax on both groups of consumers, although the tax is lower for credit card users due to the rewards.

We summarize the effect of merchant market power in the Proposition below.

Proposition 2 *As product market competition intensifies (lower γ):*

a) the equilibrium interchange fee, $i^* > 0$, decreases if and only if product demand elasticity is low, $k < \frac{4}{3}$,

b) the equilibrium reward, $r^* > 0$, increases,

c) the equilibrium tax of credit card users, z^*_{cc}, decreases,

d) the equilibrium tax of cash users, z^*_{ch}, decreases if and only if product demand elasticity is low, $k < \frac{4}{3}$,

e) equilibrium network profits, $\pi_{\mathcal{N}}(z^*_{cc}, z^*_{ch})$, increase and

f) aggregate merchant equilibrium profits, $\Pi^m(z^*_{cc}, z^*_{ch})$, are inverse U-shaped.

The effect of merchant competition arises from the influence of γ on μ. It can be understood as follows. From (4.5), as γ decreases, μ also decreases for a fixed $z_{ch} - z_{cc} > 0$. Intuitively, in a more competitive merchant market, prices are lower. Since taxes are ad valorem, a tax differential in favor of credit provides a smaller advantage to credit relative to cash. Therefore, the network has an incentive to increase $z_{ch} - z_{cc} > 0$ to restore its market share. This is achieved by decreasing z_{cc} and increasing z_{ch} or decreasing z_{ch} but at a lower rate.

Aggregate merchant profits can increase as competition in the product market intensifies. This occurs because the network, which competes with cash, lowers the tax on credit card users, who constitute the majority of consumers, thereby encouraging increased consumption. In other words, there is a range of values for the market conduct parameter in which the double marginalization problem is first order in economic significance. This result contrasts with the findings of Shy and Wang (2011), who show that aggregate merchant profits monotonically decrease as the market becomes more competitive.

We summarize the effect of differentiation between cash and credit in the Proposition below (the proof is straightforward and is omitted).

Proposition 3 *As cash and credit become more substitutable (lower t):*

a) the equilibrium tax cash users pay, z^*_{ch}, is decreasing if and only if product demand elasticity is high, $k > \frac{4}{3}$,

b) the equilibrium interchange fee i^* is decreasing if and only if product demand elasticity is high, $k > \frac{4}{3}$,

c) the equilibrium tax credit card users pay, z^*_{cc}, is increasing,

d) the equilibrium reward, r^*, is decreasing and

e) the difference between the taxes, $z^*_{ch} - z^*_{cc}$, is decreasing.

The finding that the tax for credit card users increases as cash and credit become more substitutable might seem counterintuitive. One might expect that, as the network faces stronger competition from cash, it would increase the rewards offered to credit card users, thereby lowering the tax for these consumers. The intuition is as follows. From (4.5), and given that $z_{ch} > z_{cc}$, a lower t increases the share of credit card users, holding the taxes fixed. As we have already discussed, this leads to lower network profits per transaction (because credit card users cost the network relatively more than cash users). The network lowers the rewards and the difference between the two taxes to mitigate this negative effect.

5 Regulation

We use the model to shed light on the effect of an interchange fee cap regulation. Specifically, we assume that a regulator imposes a cap which marginally lowers the interchange fee from its private equilibrium value. This is equivalent to reducing the tax, z_{ch}, paid by cash users. We differentiate the network's profit function, (4.9), with respect to the two taxes to obtain

$$\frac{\partial \pi^2_{\mathcal{N}}(z_{cc}, z_{ch})}{\partial z_{cc} \partial z_{ch}} = -\frac{2 z_{cc}^{-(1+k)} c k^2 \left(\left((z_{cc} - z_{ch})c + \frac{t}{4} \right) k - c z_{cc} - \frac{\gamma t}{4} \right) \left(\frac{k-\gamma}{ck} \right)^k}{t(k-\gamma)^2}. \tag{5.1}$$

When we evaluate (5.1) at the equilibrium, (4.10), we obtain

$$\frac{\partial \pi^2_{\mathcal{N}}(z^*_{cc}, z^*_{ch})}{\partial z_{cc} \partial z_{ch}} = \frac{2 \left(\frac{16(k-\gamma)}{ck} \right)^k \left(\frac{(16c-t)k+\gamma t)}{c(k-1)} \right)^{-k} k^2 c^2}{t(k-\gamma)^2} > 0.$$

A positive value of the above derivative indicates that when z_{ch} decreases, due to a regulation, the marginal profitability of z_{cc} also decreases. Consequently, the network will adjust the reward r to affect the tax for credit card users z_{cc}. However, the likely reduction in r will not be big enough to offset the reduction in the interchange fee i, so z_{cc} will definitely decrease. Since the price merchants receive is not affected by the taxes, the prices consumers pay decrease. Merchant profits also increase because consumption increases. We summarize this in the following Proposition.

Proposition 4 *A regulation that marginally lowers the interchange fee from its equilibrium value will reduce the taxes for both cash and credit card payers, z_{cc} and z_{ch} respectively, even if the rewards to credit card users decrease. Both credit card and cash users will benefit. Merchant profits will also increase.*

The above result is local, starting from the unregulated equilibrium. We can examine an interchange fee cap starting from any pair of taxes. It follows that (5.1) is positive if and only if $t < \tilde{t} \equiv \frac{4c(kz_{ch}-(k-1)z_{cc})}{k-\gamma}$. This condition requires that the degree of differentiation between cash and credit is not very high. We summarize in the Proposition below.

Proposition 5 *Assume $t < \tilde{t}$. A regulation that lowers the interchange fee will reduce the taxes for both cash and credit card payers, z_{cc} and z_{ch} respectively, even if the rewards to credit card users decrease. Both credit card and cash users will benefit. Merchant profits will also increase.*

In other words, as long as cards and cash are sufficiently close substitutes, an interchange fee cap that represents more than a marginal change relative to the private equilibrium can enhance welfare for both card and cash users, and increase profits for merchants, at the expense of network profits. This holds true even if credit card rewards decrease. It's important to note that these results are based on the assumption of constant elasticity of demand. While additional effects may arise under different assumptions about the nature of demand, the identified outcomes will persist.

6 Conclusion

The model developed in this paper features a two-sided market with a monopoly payment network, interchange fees in the form of an ad valorem markup, consumers who can choose to pay with either credit or cash, merchants with market power, and elastic demand. Credit card taxes emerge from the interchange fees and rewards, creating a wedge between the prices consumers pay and the prices merchants receive, all endogenously determined. The model establishes an equilibrium where aggregate demand and merchant profits are influenced by these credit card taxes.

Merchants argue that the high fees for processing credit card transactions erode their profits and force them to raise prices, negatively affecting consumers. Those concerns have prompted governments worldwide to introduce regulations aimed at curbing these fees. Networks and banks, on the other hand, contend that lowering interchange fees would compel them to reduce the rewards offered to credit card users, thereby disadvantaging their customers.

Under the assumptions of our model, we demonstrate that capping interchange fees can benefit all consumers, and merchants, by lowering the credit card tax, despite a decrease in credit card rewards. Simply put, with endogenous prices, and assuming constant elasticity of demand, the reduction in rewards is less significant compared to the decrease in interchange fees, resulting in a net reduction in the taxes paid by credit card users. This occurs because the network competes with cash as an alternative payment method.

As in other models, we show that credit card users pay a lower tax than cash users due to the rewards offered to those who make purchases with cards. Under certain circumstances, rewards can be so high that the tax associated with using a credit card turns into an actual subsidy-cardholders pay less, on net, than merchants charge.

Finally, as competition in the product market intensifies, the equilibrium tax paid by credit card users decreases. The tax paid by cash users decreases only if the demand elasticity is relatively high. Aggregate merchant profits can increase with intensified product market competition due to the significant double marginalization problem within a certain range of the market conduct parameter. More intense product market competition prompts the network, which competes with cash, to lower the tax for credit card users. This, in turn, significantly increases their consumption, offsetting the negative impact of competition on merchant profits. In models that do not include cash as an alternative payment mode, merchant profits strictly decline as competition increases.

The model generates testable implications about the effect of product market competition, the degree of differentiation between cash and credit and the elasticity of demand on the taxes for credit and cash users.

We have made significant progress in developing a more realistic model of a payment network. However, there remain several simplifying assumptions that we aim to relax in ongoing research. For example, in our model, we assume the presence of a single monopoly payment network. In reality, multiple payment card networks typically exist.

Another assumption in this paper is that, while product demand is elastic, it is assumed to have constant elasticity. This assumption has two main implications worth noting. First, none of the incidence of the credit card tax is borne by merchants; while the volume may change, retailer margins remain unaffected. Under more general assumptions about demand, merchants might bear some portion of the credit card tax, complicating any welfare analysis of potential regulations. Second, the constant elasticity assumption rules out the possibility that the elasticity may change with aggregate output, the level of which is affected by the credit card tax.

Preliminary results from our ongoing research suggest that new and interesting effects arise from relaxing either of these assumptions, although often at the expense of analytic solutions.

Finally, we assumed Bertrand competition among issuing and acquiring banks. At the expense of substantial complication, introducing market power at this layer of the network would yield additional effects and insights.

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A Appendix: Proofs

A.1 Second order condition

The second derivative of the network profit, (4.9), with respect to z_{cc} is given by

$$\frac{\partial^2 \pi_{\mathcal{N}}}{\partial z_{cc}^2} = -\frac{A}{t(k-\gamma)^2},$$

where $A \equiv c z_{cc}^{-2-k} \left(\frac{k-\gamma}{ck}\right)^k k^2 (c(k-1)(k-2)z_{cc}^2 - 2((cz_{ch} + t/4)k - \gamma(t/4))(k-1)z_{cc} + (1+k)((cz_{ch}^2 - (1/2)tz_{ch} + t)k + t\gamma(z_{ch}-2)/2))$. When we evaluate the second derivative at the solutions to the first order conditions, (4.10), it is negative if and only if $t < \frac{16ck}{k-\gamma}$. The second derivative of the network profit with respect to z_{ch} is given by,

$$\frac{\partial^2 \pi_{\mathcal{N}}}{\partial z_{ch}^2} = -\frac{\left(\frac{k-\gamma}{ck}\right)^k z_{cc}^{-k} 2k^2 c^2}{t(k-\gamma)^2} < 0.$$

The determinant of the Hessian matrix of $\pi_{\mathcal{N}}(z_{cc}, z_{ch})$ is given by

$$\frac{\left(k(z_{cc}-z_{ch})^2(k-1)c^2 + \frac{3\left(\left(z_{cc}-\frac{z_{ch}}{3}-\frac{2}{3}\right)k - z_{cc}+\frac{z_{ch}}{3}-\frac{2}{3}\right)t(k-\gamma)c}{2} + \frac{t^2(k-\gamma)^2}{8}\right) 2k^4 \left(\frac{k-\gamma}{ck}\right)^{2k} c^2 z_{cc}^{-2(1+k)}}{t^2(k-\gamma)^4},$$

which when evaluated at the solutions to the first order conditions, (4.10), is positive if and only if $t < \frac{16ck}{k-\gamma}$.

A.2 Proof of Proposition 1

Note that $z^*_{cc} < 1$ if and only if $k > \gamma + \frac{16c}{t}$, or $t > \frac{16c}{k-\gamma}$. The effect of demand elasticity on the taxes is: $\frac{\partial z^*_{cc}}{\partial k} < 0 \Leftrightarrow t < \frac{16c}{1-\gamma}$, which is always satisfied since $\frac{16c}{1-\gamma} > \frac{16ck}{k-\gamma}$, $\frac{\partial z^*_{ch}}{\partial k} < 0 \Leftrightarrow t < \frac{16k^2c}{3\gamma k^2-8\gamma k+k^2+4\gamma}$ and $\frac{\partial(z^*_{ch}-z^*_{cc})}{\partial k} > 0$. Moreover, $\frac{16k^2c}{3\gamma k^2-8\gamma k+k^2+4\gamma} < \frac{16c}{1-\gamma}$ and $\frac{16k^2c}{3\gamma k^2-8\gamma k+k^2+4\gamma} > \frac{16ck}{k-\gamma} \Leftrightarrow k < \frac{4}{3}$. Therefore, the tax credit card users pay is decreasing, and the difference between the taxes is increasing, as demand becomes more elastic. The effect of elasticity on the tax cash users pay is ambiguous. When $k < \frac{4}{3}$, z_{ch} is decreasing as k increases, while when $k > \frac{4}{3}$ it is decreasing if and only if $t < \frac{16k^2c}{3\gamma k^2-8\gamma k+k^2+4\gamma}$.

The equilibrium price credit card users pay is $P^{cc} = z^*_{cc}P^m = \frac{(16c-t)k^2+\gamma kt}{16(k-1)(k-\gamma)} \lesseqgtr P^m$ and the equilibrium price cash card users pay is $P^{ch} = z^*_{ch}P^m = \frac{(16c+3t)k^2-(3\gamma+4)tk+4\gamma t}{16(k-1)(k-\gamma)} > P^m$. It can be verified that both prices are increasing in γ.

A.3 Proof of Proposition 2

The effect of merchant market power on the taxes is: $\frac{\partial z^*_{cc}}{\partial \gamma} > 0$, $\frac{\partial z^*_{ch}}{\partial \gamma} > 0 \Leftrightarrow k < \frac{4}{3}$ and $\frac{\partial(z^*_{ch}-z^*_{cc})}{\partial \gamma} < 0$. The effect of the degree of differentiation between cash and credit on the taxes is: $\frac{\partial z^*_{cc}}{\partial t} < 0$, $\frac{\partial z^*_{ch}}{\partial t} > 0 \Leftrightarrow k > \frac{4}{3}$ and $\frac{\partial(z^*_{ch}-z^*_{cc})}{\partial t} > 0$.

From (4.11), it can be verified that $\frac{\partial i^*}{\partial \gamma} > 0 \Leftrightarrow k < \frac{4}{3}$ and $\frac{\partial r^*_1}{\partial \gamma} < 0$.

It can be verified, using (4.12), that $\frac{\partial \pi_{\mathcal{N}}}{\partial \gamma} < 0$, if $t < \frac{16ck}{k-\gamma}$.

Using (4.13), while $\frac{\partial \Pi^m}{\partial \gamma} > 0$ for low γ, because $\Pi^m = 0$ at $\gamma = 0$, we have shown numerically that for a wide range of permissible parameter values Π^m is inverse U-shaped in γ.