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Makoto Nakajima
Federal Reserve Bank of Philadelphia Research Department

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Capital Income Taxation with Housing☆

Makoto Nakajima[1]

Federal Reserve Bank of Philadelphia

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Abstract

This paper quantitatively investigates capital income taxation in the general-equilibrium overlapping generations model with household heterogeneity and housing. Housing tax policy is found to affect how capital income should be taxed, due to substitution between housing and non-housing capital. Given the existing U.S. preferential tax treatment for owner-occupied housing, the optimal capital income tax rate is close to zero (1%), contrary to the high optimal capital income tax rate found with overlapping generations models without housing. A low capital income tax rate improves welfare by narrowing a tax wedge between housing and non-housing capital; the narrowed tax wedge indirectly nullifies the subsidies (taxes) for homeowners (renters) and corrects over-investment to housing. Naturally, when the preferential tax treatment for owner-occupied housing is eliminated, a high capital income tax rate improves welfare as in the model without housing.

Keywords: Housing, Optimal Taxation, Capital Income Taxation, Housing, Heterogeneous Agents, Life Cycle, Overlapping Generations, Incomplete Markets
JEL Classification: E21, H21, H24, R21

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[1]Research Department, Federal Reserve Bank of Philadelphia. Ten Independence Mall, Philadelphia, PA 19106-1574. E-mail: `makoto.nakajima@phil.frb.org`.

1. Introduction

Whether the government should tax capital income in the long run has been an important question and one that has been answered under a variety of assumptions. Chamley (1986) and Judd (1985) argue that the government should not tax capital income, using a model with an infinitely lived representative agent.[2] On the other hand, the optimal capital income tax rate is known to be different from zero in overlapping generations models. In particular, a recent study by Conesa et al. (2009) shows quantitatively that the optimal capital income tax rate is not only non-zero but also *very large*, using a calibrated overlapping generations model. What is missing in the discussion on optimal capital income taxation is housing, which consists of about one-third of the total capital of the U.S. economy and is the biggest single asset for the majority of U.S. households. Not only is housing large, but it is also different from non-housing capital and taxed very differently. The purpose of the paper is to revisit the optimality of capital income taxation, taking into account the unique characteristics of housing and housing tax policy.

How is housing different from non-housing capital? Notable differences are: (i) housing is held for the dual purpose of consumption and savings, (ii) housing can be either owned or rented, (iii) if owned, housing can be used as collateral for mortgage loans, and (iv) income from housing is taxed differently from non-housing capital income. In particular, in the U.S. there are two policies that favor housing, especially owner-occupied housing. First, imputed rents on owner-occupied housing are tax exempt. Second, the mortgage interest payment can be deducted from taxable income up to a certain limit. There are studies that investigate the implications of such housing tax policy, but mostly without a quantitative macroeconomic model. This paper is intended to bridge the gap between the literature on macroeconomic public finance, which typically ignores housing capital, and that on housing policy, where the quantitative general equilibrium model is rarely used.

In the U.S. and many other countries, owner-occupied housing enjoys various forms of implicit and explicit subsidies that non-housing capital does not enjoy. Rosen (1985) offers a good summary of the literature analyzing the effects of the government's policy toward housing. However, analysis of housing taxation in a realistically calibrated general equilibrium model started to appear only recently. The pioneer work is Gervais (2002). He analyzes such welfare gains from eliminating the preferential tax treatment for owner-occupied housing, using a calibrated overlapping generations model. Díaz and Luengo-Prado (2008) study the effect of the preferential tax treatment of owner-occupied housing on homeownership. This paper will not provide a positive theory of housing taxation. Instead, housing tax policy is taken as given, and optimal capital income taxation conditional on different housing policies is explored.

I employ the Ramsey approach to the optimal taxation problem in a limited sense. In this approach, the size of government expenditures in every period is exogenously given, a set of available distortionary tax instruments is assumed, and the optimal tax system within the set is explored. For the baseline experiment, I assume (i) the preferential tax treatment for owner-occupied housing that is present in the U.S., (ii) progressive labor income taxation, with the progressivity mimicking that of the U.S. federal income tax, and (iii) proportional capital income taxation. Under these assumptions, the optimal level of the capital income tax rate is investigated while maintaining

[2] It is further shown that the result holds true in less restrictive environments. Chari and Kehoe (1999) offer a good survey of the optimal taxation results within the Ramsey framework. Atkeson et al. (1999) show that the optimality of a zero capital income tax rate holds even if some assumptions are relaxed.

revenue neutrality. The assumption of the proportionality of the capital income tax is due to computational feasibility, but Conesa et al. (2009) find that the optimal tax system does not include progressive capital income tax in their model without housing.

There are three main findings. First, the optimal capital income tax rate is close to zero even in the life-cycle model, given the preferential tax treatment for owner-occupied housing. In the baseline experiment, the optimal capital income tax rate is found to be 1%. This is very different from 31%, which is obtained in the standard model without housing. The intuition is simple. When the imputed rents on owner-occupied housing are tax-exempt by assumption, lowering the capital income tax rate is equivalent to narrowing the tax wedge between housing and non-housing capital. There are two consequences. First, the narrowed tax wedge nullifies the subsidies to homeowners, who are typically higher earners, and taxes to renters, who are typically lower earners. Second, the narrowed tax wedge corrects the over-investment in housing capital. The numerical result shows that this simple intuition is actually very important in shaping the optimal capital income taxation. Second, when the preferential tax treatment for owner-occupied housing is eliminated, it becomes optimal to tax capital at a high rate again, as in the standard model without housing. In the baseline experiment, the optimal capital income tax rate is found to be 24%. When the tax wedge is eliminated by assumption, lowering the capital income tax rate no longer works to nullify the preferential tax treatment of owner-occupied housing. The two results above taken together suggest that housing tax/subsidy policy has a substantial effect on how capital income should be taxed. In other words, taxation of housing and non-housing capital should be considered as a package, because of the tight interaction between the two. Third, in either of the two cases discussed above, the welfare gain from moving from the baseline economy to the one with the optimal capital income tax rate is sizable: 1.2% of additional per-period consumption when the preferential tax treatment for owner-occupied housing is preserved, and 1.6% when the preferential tax treatment is eliminated. Consequently, implementing a high capital income tax rate, which is optimal in the model without housing, in the model with housing incurs a severe welfare loss.

To the best of my knowledge, Eerola and Maattanen (2013) are the only ones who study the optimal capital and housing taxation in a macroeconomic model. In particular, they investigate optimal housing taxation in the standard growth model with housing and non-housing capital. Using the standard Ramsey approach, they find that it is optimal to tax housing and non-housing capital at the same rate and close the tax wedge. It implies that, in the long-run, where it is optimal to have zero capital income tax as in Chamley-Judd, it is also optimal not to tax housing. Important differences from this paper are that my model features tenure decision between owning and renting, which is affected by preferential tax treatment of owner-occupied housing, market incompleteness that implies heterogeneity of households in various dimensions, and life cycle. The life-cycle aspect is especially important because Conesa et al. (2009) find that, in the model that features the life cycle, it is optimal to heavily tax non-housing capital. Relatedly, Erosa and Gervais (2002) and Garriga (2019) theoretically show that the optimal capital income tax rate is *not zero*. Moreover, I find that the tenure decision is crucial for the main results of the paper, as the capital income tax indirectly redistributes income between homeowners and renters.

Implications of market incompleteness to optimal capital income taxation have also been studied. Aiyagari (1995) argues that, in the presence of market incompleteness, the optimal capital income tax is not zero in the long run. In the economy with uninsured idiosyncratic shocks to earnings, agents have a precautionary savings motive, which pushes the aggregate savings above the efficient level in the complete markets model. A positive capital income tax can fix the over-accumulation of assets by countering the incentive to hold precautionary savings. Domeij and Heathcote (2004) build on the model used by Aiyagari (1995) and investigate optimal capital income taxation in the model, which features a realistic degree of the wealth inequality due to market incompleteness. They find that, taking into account the welfare loss during the transition, implementing a zero capital income tax generates a welfare loss. According to their baseline experiment, the optimal capital income tax rate is 39.7%. However, the long-run optimal capital income tax rate without consideration of the cost of transition is still zero. Fuster et al. (2008) study how the strength of altruism affects the welfare gain from various tax reforms.

The model developed in the current paper is built on the literature that develops general-equilibrium models with uninsured idiosyncratic shocks. The classic papers are Aiyagari (1994) and Huggett (1996). The pioneer papers that introduce housing or durable assets into the standard general-equilibrium model with uninsured idiosyncratic uncertainty are Gervais (2002), Fernández-Villaverde and Krueger (2011), Díaz and Luengo-Prado (2010), Nakajima (2005), and Chambers et al. (2009a).

The rest of the paper is organized as follows. Section 2 sets up the model and Section 3 describes how the model is calibrated and numerically solved. Some of the details of calibration are found in Appendices A.1 and A.2. Appendix A.3 gives further details of the computational methods. The properties of the baseline model economy with housing are studied in Section 4. In Section 5, the methodology for counterfactual experiments is explained. Appendix A.4 provides some details about the welfare criteria used here. Section 6 presents the main results of the paper. Section 7 investigates the role of housing in shaping the main results. Section 8 extends the baseline model in three ways. Additional robustness analyses are offered in Section 9. Section 10 concludes.

2. Model

The model is based on the general equilibrium overlapping generations model with uninsured idiosyncratic shocks to labor productivity and mortality, in particular Conesa et al. (2009). The novel feature of the model is that there are both housing and financial assets. The following four key characteristics of housing assets are explicitly incorporated into the model. First, housing assets play a dual role; housing generates services consumed by those who live in it and, at the same time, is a means for saving. Second, housing can be owned or rented. Third, homeowners can use their housing as collateral for mortgage loans. Using mortgage loans, agents can live in a house whose value is larger than the value of their total wealth. Fourth, there is a preferential tax treatment for owner-occupied housing through the tax-exemption of imputed rents and the mortgage interest payment deduction. Since the government can tax owner-occupied and rented housing and financial assets differently, the model can naturally be used to understand how the difference in taxes for housing, either owned or rented, and financial assets affects allocations, prices, and welfare.

2.1. Demographics

Time is discrete. In each period, the economy is populated by I overlapping generations of agents. In period t, a measure $(1+\gamma)^t$ of agents is born. γ is the population growth rate. Each generation is populated by a mass of agents, each of whom is measure zero. Agents are born at age 1 and could live up to age I. There is a probability of early death, represented by π_i. An age-i agent survives to age $i+1$ with probability π_i and dies at the end of age i. With probability $(1-\pi_i)$. I is the maximum possible age, which implies $\pi_I = 0$. Agents retire at age $1 < I_R < I$. Agents with age $i \le I_R$ are called *workers*, and those with age $i > I_R$ are called *retirees*. I_R is a parameter, implying that retirement is mandatory.

2.2. Preferences

An agent maximizes its expected lifetime utility. The utility function of an agent takes the standard time-separable form as follows:

$$\mathbb{E}\sum_{i=1}^{I}\beta^{i-1}u(c_i, d_i, m_i), \tag{1}$$

where c_i is the consumption of non-housing goods at age i, d_i is the consumption of housing services, and m_i is leisure. $\mathbb{E}$ is the expectation operator with respect to the information at the time of birth. β is the time discount factor. $u(.,.,.)$ is strictly increasing and strictly concave in all three arguments.

2.3. Endowment

Agents are endowed with one unit of time in each period and housing asset h_1 and financial asset a_1 at birth. I assume that $h_1 = 0$ and $a_1 = 0$. Agents can use their time either for work ℓ or for leisure m. Formally:

$$1 = \ell_i + m_i \tag{2}$$

for each age i.

Agents are heterogeneous in terms of labor productivity. Labor productivity has two components, $\overline{e}_i$ and e. $\overline{e}_i$ is a component associated with age or working experience of agents. Since agents are forced to retire at age I_R, $\overline{e}_i = 0$ for $i > I_R$. e is the stochastic component and independent of the age of agents. Each newborn draws the initial $e \in E = \{e_1, e_2, ..., e_{n_e}\}$ from $\{p_e^0\}$, where each of p_e^0 represents the probability assigned to each possible realization of e. The stochastic process for e is identical for all agents and independent across agents. In particular, $\log(e)$ is assumed to follow a finite-state first-order Markov process $(E, \{p_{ee'}\})$, where $p_{ee'}$ represents the Markov transition probability from e to e'. For an agent who supplies ℓ_i hours of work, the product $\ell_i\overline{e}_i e$ represents the individual labor supply of an age-i agent, measured in efficiency units.

2.4. Technology

There is a representative firm that has access to the following constant returns to scale technology:

$$Y_t = Z_t F(K_t, L_t), \tag{3}$$

where Y_t is output, Z_t is the level of total factor productivity, K_t is aggregate non-housing capital input, and L_t is aggregate labor input measured in efficiency units in period t, respectively. Since inputs are traded in competitive markets, the firm's profit will be zero in equilibrium. Non-housing capital depreciates at a constant rate δ_K. Housing capital is denoted by H_t and depreciates at a constant rate δ_H. There is a linear technology that converts between one unit of housing capital and one unit of non-housing capital costlessly. In sum, the aggregate resource constraint of the economy is the following:

$$C_t + G_t + K_{t+1} + H_{t+1} = (1-\delta_H)H_t + (1-\delta_K)K_t + Y_t, \tag{4}$$

where C_t is total private consumption, and G_t is public consumption. G_t is not valued by agents.

Housing capital H_t yields housing services D_t. Without loss of generality, the following linear production function is assumed:

$$H_t = D_t. \tag{5}$$

Because of the structure of the transformation technology, I can use H_t and D_t interchangeably.

2.5. Real Estate Sector

The real estate sector works as the intermediary for agents who rent housing.[3] In each period, a real estate firm borrows financial assets from saving agents and uses the assets to buy housing assets. The housing assets are rented out to renters, and the real estate firm receives the rent q_t, and uses it to pay back the cost of debt together with other costs. The following equation specifies the problem of a real estate firm renting out h_t in period t:

$$\max_{h_t} \left\{ (1-\delta_H)h_t + q_t h_t - (1+r_t)h_t - \tau_{P,t} h_t \right\}, \tag{6}$$

where $(1-\delta_H)h_t$ is the value of the house after depreciation, $q_t h_t$ is the rental income of the real estate firm, $(1+r_t)h_t$ is the financial cost associated with the housing assets, and $\tau_{P,t}$ is the property tax rate. Assuming free entry to the real estate sector, the equilibrium rent is determined by the zero profit condition and takes the following form:

$$q_t = r_t + \tau_{P,t} + \delta_H. \tag{7}$$

Basically, renters pay for the financial cost of the value of housing that they rent plus the property tax and the maintenance cost (depreciation) for the rented housing, through a real estate firm.

2.6. Market Structure

Without loss of generality, I assume that agents own *financial assets* instead of *non-housing capital*. One unit of financial assets is a claim to one unit of non-housing capital. In addition, financial assets capture *mortgage loans* as well. In particular, a positive amount of financial assets is a claim to the same amount of non-housing capital, while a negative amount of financial assets represents the amount of mortgage debt. The use of financial assets helps to ease the notation by combining non-housing capital and mortgage loans. In the same manner, I use the terms housing assets and housing capital interchangeably. Housing assets can be either owned or rented from the real estate sector.

Labor and financial assets are traded in competitive markets. By assumption, agents cannot trade state-contingent securities to insure away the shocks with respect to labor productivity or mortality. However, agents can save in the form of housing and financial assets and self-insure.

As for the housing assets, agents can either own or rent housing assets but the choice is exclusive. When renting, an agent pays the unit cost of housing, which is the rental cost q_t to a real estate firm. When owning, an agent has to pay a property tax, an owner-occupied housing tax (which is zero in the baseline), and a depreciation. The interpretation of the depreciation is the maintenance cost. There is a minimum size constraint of housing assets. Moreover, the minimum size is different depending on the tenure: $\underline{h}^r$ for rental properties and $\underline{h}^o$ for owner-occupied housings. This is a

[3] The setup of the real estate sector is the same as in Gervais (2002) and Nakajima (2005). Chambers et al. (2009b) construct a model in which homeowners can become landlords and supply rental properties to renters.

parsimonious way to capture the lumpiness of housing, and this assumption was originally used by Gervais (2002).

The assumption of different minimum sizes deserves some discussion. Think of a model without the minimum size restrictions. Because of the preferential tax treatment for owner-occupied housing, agents in the model choose to own housing rather than renting as long as it is feasible. On the other hand, the homeownership rate in the U.S. is historically around 64%. This implies that there are some additional costs of owning, which makes about one-third of households in the U.S. rent rather than own. Assuming minimum size restrictions is one parsimonious way to achieve the relatively low homeownership rate. It is important to point out, however, that the main results of the paper are robust to other assumptions, such as higher moving costs pertaining to ownership and additional costs of owning.

When owning, an agent can use the value of housing assets as collateral. In particular, an agent can borrow up to $(1-\lambda)$ of the value of housing assets that the agent owns. Collateralized borrowing is called a mortgage loan. Mortgage loans in the model capture both primary mortgage loans and other types of loans that are secured by the value of housing. There is no unsecured loan. If interpreted as the standard primary mortgage loan, λh is the down payment to own housing of value h. If interpreted as a secondary mortgage loan, a home equity loan, or a home equity line of credit, $(1-\lambda)h$ is the maximum value of mortgages an agent can take out from the housing asset of value h.

Housing services cannot be traded. Since marginal utility from housing services is strictly positive, an agent consumes all the housing services generated by the housing asset owned or rented.

2.7. Government Policy

The government is engaged in the following three activities: (i) collecting various forms of taxes to finance the public expenditure each period G_t, (ii) collecting estate taxes and distributing them to all surviving agents in a lump sum, and (iii) running the pay-as-you-go social security program.

The government must spend G_t in period t. $\{G_t\}_{t=0}^{\infty}$ is exogenously given. It is the standard setup in the optimal taxation problem. For simplicity, I assume that the government must balance the budget each period. In other words, the government must collect taxes whose total amount is G_t in every period t. There are five types of taxes: (i) proportional capital income tax with the tax rate $\tau_{K,t}$; (ii) labor income tax represented by the tax function $T_t(.)$, which captures the progressivity of the U.S. tax code; (iii) property tax with the tax rate $\tau_{P,t}$; (iv) proportional tax for the imputed rents of owner-occupied housing, with the tax rate $\tau_{H,t}$; and (v) proportional subsidy (negative tax) for mortgage interest payment with the tax rate $\tau_{M,t}$. This captures the mortgage interest payment deduction.

Since time of death is stochastic, and there is no private annuity market, there are accidental bequests.[4] The government imposes a 100% estate tax rate on accidental bequests and distributes all the proceeds equally to all the surviving agents using a lump-sum transfer, in each period. t_t denotes the lump-sum transfer for each agent in period t.

Finally, the government runs a simple social security program. The government collects payroll taxes from labor income at the rate $\tau_{S,t}$. All the proceeds are equally distributed to all the retired agents in each period. The social security benefit is denoted by $b_{i,t}$, where $b_{i,t} = 0$ for $i \leq I_R$, and

[4] In Section 8.2, I introduce a warm-glow bequest motive, which generates intended bequests.

$b_{i,t} = \overline{b}_t$ for all $i > I_R$. Notice that, since the amount of benefit is the same for all agents regardless of the amount contributed, this particular social security program has a strong redistribution effect, even more than the U.S. social security program.

2.8. Agents' Problem

The agents' problem is formulated recursively. I use a prime to denote a variable in the next period. An agent is characterized by the set of individual state variables (i, e, x), where i is age, e is the stochastic component of individual productivity, and x is total wealth. The use of total wealth x instead of a pair of housing and financial assets (h, a) as a state variable reduces the size of the state space and thus greatly simplifies the problem. But the transformation is valid only as long as there is no fixed cost of changing housing or financial asset holdings, and thus it is not necessary to keep track of the portfolio allocation determined in the previous period. The recursive problem for an agent with individual state (i, e, x) and in time t is below:

$$V_t(i, e, x) = \max \{V_t^o(i, e, x), V_t^r(i, e, x)\} \tag{8}$$

$$V_t^o(i, e, x) = \max_{c \geq 0, h^o \geq \underline{h}^o, a \geq -(1-\lambda)h^o, x' \geq 0, \ell \in [0,1]} \left\{ u(c, h^o, 1 - \ell) + \beta \pi_i \sum_{e'} p_{ee'} V_{t+1}(i + 1, e', x') \right\} \tag{9}$$

subject to

$$x + t_t = h^o + a \tag{10}$$

$$(1 + \tilde{r}_t)a + (1 - \delta_H - \tau_{P,t} - r_t \tau_{H,t})h^o + w_t e \overline{e}_i \ell (1 - \tau_{S,t}) - T_t(w_t e \overline{e}_i \ell) + b_{i,t} = c + x' \tag{11}$$

$$\tilde{r}_t = \begin{cases} r_t(1 - \tau_{K,t}) & \text{if } a \geq 0 \\ r_t(1 - \tau_{M,t}) & \text{if } a < 0 \end{cases} \tag{12}$$

$$V_t^r(i, e, x) = \max_{c \geq 0, h^r \geq \underline{h}^r, \ell \in [0,1]} \left\{ u(c, h^r, 1 - \ell) + \beta \pi_i \sum_{e'} p_{ee'} V_{t+1}(i + 1, e', x') \right\} \tag{13}$$

subject to

$$(1 + r_t(1 - \tau_{K,t}))(x + t_t) + w_t e \overline{e}_i \ell (1 - \tau_{S,t}) - T_t(w_t e \overline{e}_i \ell) + b_{i,t} = c + x' + q_t h^r. \tag{14}$$

Equation (8) represents the tenure decision. $V_t^o(i, e, x)$ and $V_t^r(i, e, x)$ are the values conditional on owning and renting, respectively. The two Bellman equations that follow define the values conditional on the tenure choice.

The Bellman equation (9) is the problem of a homeowner. A homeowner chooses consumption c, financial assets a (which captures savings by a positive value and mortgage loans by a negative value), owned housing assets h^o, wealth carried over to the next period x', and hours worked ℓ to maximize the sum of the current utility and the expected discounted value in the next period, subject to the constraints listed above and explained below.

The first constraint (10) is the asset allocation constraint. The sum of the total wealth x and the lump-sum transfer t_t is allocated to housing assets h^o and financial assets a. Notice that the agent can borrow up to $(1 - \lambda)h^o$ using mortgage loans collateralized by the value of owned housing assets h^o. In the case in which an agent is using mortgage loans, the size of housing h will be larger than total wealth. House size h is subject to the minimum size restriction $h \geq \underline{h}^o$.

The second constraint (11) is the budget constraint. The first term on the left-hand side is the principal and after-tax interest income of financial assets. More explanation of the after-tax interest income is found below. The second term represents the value of owned housing assets after paying the property tax, the owner-occupied housing tax and the maintenance cost. The housing tax is represented as the proportion of the interest rate ($r_t \tau_{H,t}$), which makes it easier to compare the cost of renting and owning. The third term is labor income net of the social security tax (with the rate of $\tau_{S,t}$). $w_t e \overline{e}_i \ell$ is the before-tax labor income. The fourth term is the labor income tax, which is characterized by the tax function $T_t(.)$. The last term on the left-hand side is the social security benefit $b_{i,t}$. As $b_{i,t} = 0$ for $i \leq I_R$, the social security benefit is zero for working agents. The right-hand side consists of non-housing consumption c and total wealth carried over to the next period x'.

Equation (12) defines the after-tax interest rate. When the agent is saving ($a \geq 0$), the saving yields the before-tax return of r_t but is subject to the proportional capital income tax at the rate of $\tau_{K,t}$. When the agent is borrowing ($a < 0$), the agent pays the interest rate for the amount of the mortgage loans, but there is a tax deduction whose amount is defined as the proportion $\tau_{M,t}$ of mortgage interest payments.

The Bellman equation (13) is the problem of a renter. A renter chooses h^r, which is bounded from below by $\underline{h}^r$. A renter does not make an asset allocation decision because all the wealth is invested into financial assets by definition of a renter. (14) is the budget constraint for a renter. There is no term for the owner-occupied housing asset and there is a cost of rental properties $q_t h^r$ on the right-hand side. The financial asset a for a homeowner corresponds to $(x + t_t)$ for the renter, since renters have only financial assets. The after-tax interest rate $\tilde{r}_t$ is always the interest rate net of the capital income tax rate $\tau_{K,t}$ because renters cannot borrow using mortgage loans.

The solution to the dynamic programming problem above yields optimal decision rules $c = g_{c,t}(i, e, x)$, $h^o = g_{o,t}(i, e, x)$, $h^r = g_{r,t}(i, e, x)$, $a = g_{a,t}(i, e, x)$, $\ell = g_{\ell,t}(i, e, x)$, and $x' = g_{x,t}(i, e, x)$. The tenure decision is included in $h^o = g_{o,t}(i, e, x)$ and $h^r = g_{r,t}(i, e, x)$. In particular, if an agent is an owner, $h^r = g_{r,t}(i, e, x) = 0$. The opposite holds if an agent is a renter.

2.9. Equilibrium

I define the recursive competitive equilibrium and the stationary recursive competitive equilibrium of the economy. In the latter, prices are constant over time. The population size is growing at the constant rate γ, but the age composition of the population is time invariant.

Let $M = \{1, 2, ..., I\} \times E \times X$, where $x \in X \subset \mathbb{R}^+$. X is assumed to be compact. The upper bound is set such that it is never binding and thus the solution to the problem with the bound is the same as the one without. M is the space of individual states. Let $m \in M$ be an element of M. Let $\mathbb{M}$ be the Borel *σ-algebra* generated by M, and let μ the probability measure defined over $\mathbb{M}$. I will use a probability space $(M, \mathbb{M}, \mu)$ to represent a type distribution of agents.

Definition 1 (Recursive competitive equilibrium). *Given sequences of government expenditures $\{G_t\}_{t=0}^{\infty}$, social security tax rates $\{\tau_{S,t}\}_{t=0}^{\infty}$, total factor productivity $\{Z_t\}_{t=0}^{\infty}$, and initial conditions K_0, H_0, μ_0, a recursive competitive equilibrium is a sequence of value functions $\{V_t(i, e, x)\}_{t=0}^{\infty}$, optimal decision rules, $\{g_{c,t}(i, e, x)\}_{t=0}^{\infty}$, $\{g_{o,t}(i, e, x)\}_{t=0}^{\infty}$, $\{g_{r,t}(i, e, x)\}_{t=0}^{\infty}$, $\{g_{a,t}(i, e, x)\}_{t=0}^{\infty}$, $\{g_{\ell,t}(i, e, x)\}_{t=0}^{\infty}$, $\{g_{x,t}(i, e, x)\}_{t=0}^{\infty}$, measures $\{\mu_t\}_{t=0}^{\infty}$, aggregate stock of housing and non-housing capital and aggre-*

gate labor supply $\{K_t\}_{t=0}^{\infty}$, $\{H_t\}_{t=0}^{\infty}$, $\{L_t\}_{t=0}^{\infty}$, *prices* $\{r_t\}_{t=0}^{\infty}$, $\{w_t\}_{t=0}^{\infty}$, $\{q_t\}_{t=0}^{\infty}$, *transfers* $\{t_t\}_{t=0}^{\infty}$, *tax policies* $\{\tau_{K,t}, T_t(.), \tau_{P,t}, \tau_{H,t}, \tau_{M,t}\}_{t=0}^{\infty}$, *social security benefits* $\{b_{i,t}\}_{t=0}^{\infty}$, *such that:*

1. $\{V_t(i,e,x)\}_{t=0}^{\infty}$ *is a solution to the agent's problem defined above.* $\{g_{c,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{o,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{r,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{a,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{\ell,t}(i,e,x)\}_{t=0}^{\infty}$, *and* $\{g_{x,t}(i,e,x)\}_{t=0}^{\infty}$, *are the associated optimal decision rules.*
2. *The representative firm maximizes its profit. Equivalently,* r_t *and* w_t *satisfy the following marginal conditions for all* t*:*

$$r_t = Z_t F_K(K_t, L_t) - \delta_K \tag{15}$$
$$w_t = Z_t F_L(K_t, L_t) \tag{16}$$

3. *The real estate sector is competitive. Consequently, rent is determined as follows:*

$$q_t = r_t + \tau_{P,t} + \delta_H \tag{17}$$

4. *The following market clearing conditions are satisfied for all* t*:*

$$K_t = \int_M g_{a,t}(i,e,x) - g_{r,t}(i,e,x)\ d\mu_t \tag{18}$$
$$H_t = \int_M g_{o,t}(i,e,x) + g_{r,t}(i,e,x)\ d\mu_t \tag{19}$$
$$L_t = \int_M \bar{e}_i e g_{\ell,t}(i,e,x)\ d\mu_t \tag{20}$$

5. $\{\mu_t\}_{t=0}^{\infty}$ *is consistent with the transition function* $Q_t(m, \mathbb{M})$, *which is consistent with the optimal decision rules and the laws of motion for* i *and* e*. Specifically, the following law of motion is satisfied:*

$$\mu_{t+1}(\mathbb{M}) = \int_M Q(m, \mathbb{M})\ d\mu_t \tag{21}$$

6. *The following government budget balance condition is satisfied:*

$$G_t = \int_M T_t(w_t \bar{e}_i e g_{\ell,t}(i,e,x)) + \max(g_{a,t}(i,e,x),0) r_t \tau_{K,t} + \min(g_{a,t}(i,e,x),0) r_t \tau_{M,t} + g_{o,t}(i,e,x) r_t \tau_{H,t} + (g_{o,t}(i,e,x) + g_{r,t}(i,e,x)) \tau_{P,t}\ d\mu_t$$

7. *The total amount of accidental bequests is equal to the total amount of lump-sum transfers. In particular, the following budget balance condition is satisfied:*

$$(1+\gamma) \int_M t_{t+1}\ d\mu_{t+1} = \int_M (1-\pi_i) g_{x,t}(i,e,x)\ d\mu_t \tag{22}$$

8. *The social security program budget balances. In particular, the following budget balance condition is satisfied:*

$$\int_M w_t \bar{e}_i e g_{\ell,t}(i,e,x) \tau_{S,t}\ d\mu_t = \int_M b_{i,t}\ d\mu_t \tag{23}$$

Definition 2 (Stationary recursive competitive equilibrium). *A stationary recursive competitive equilibrium is a recursive competitive equilibrium where tax policies, total factor productivity, value functions, optimal decision rules, prices, transfers, and social security benefits are time-invariant. Government expenditures and aggregate variables are growing at the constant rate γ and thus are time-invariant if normalized by the population size.*

Notice that the market clearing condition for non-housing capital stock includes $-g_{r,t}(i,e,x)$. This is because real estate firms borrow exactly the same amount of housing assets as they rent. The market clearing condition for the housing capital stock includes owner-occupied housing assets and the amount of housing assets rented. The five terms in the integrand in the government budget constraint denote labor income taxes, capital income taxes, mortgage interest payment deduction, owner-occupied housing taxes, and property taxes, respectively.

Since I focus on the stationary equilibrium, I drop the time subscripts hereafter.

3. Calibration and Computation

I will first describe how the baseline model economy with both housing and financial assets is calibrated. In the last section, I will discuss how the version of the model economy with only financial assets is calibrated and compare the two economies. The section closes with brief description of the computational methods used to solve the model.

3.1. Demographics

One period is set as one year. Age 1 in the model corresponds to the actual age of 22. I is set at 79, meaning that the maximum actual age is 100. I_R is set at 43, implying that the agents start life in retirement at the actual age of 65. The annual population growth rate, γ, is set at 1.2%. This growth rate corresponds to the average annual population growth rate of the U.S. over the last 50 years. The survival probabilities $\{\pi_i\}_{i=1}^{I}$ are taken from the life table in Social Security Administration (2007).[5] Figure A.1 in Appendix A.1 shows the conditional survival probabilities used.

3.2. Preferences

For the baseline calibration, I use the following non-separable functional form:

$$u(c,d,m) = \frac{((c^{\psi}d^{1-\psi})^{\eta}m^{1-\eta})^{1-\sigma}}{1-\sigma}. \tag{24}$$

A Cobb-Douglas aggregator between (composite-)consumption goods and leisure is standard in the literature and is used by Conesa et al. (2009) as well. A Cobb-Douglas aggregator between non-housing consumption goods and housing services is a special form of a CES (constant elasticity of substitution) aggregator with unit elasticity. The assumption of unit elasticity between housing and non-housing goods is also used by Fernández-Villaverde and Krueger (2011). They refer to empirical studies estimating the elasticity and claim that the unit elasticity is in the middle of various estimates.

[5]Table 4.C6 of Social Security Administration (2007). The survival probability of males conditional on age is used.

ψ is calibrated later to match the relative size of the housing and non-housing capital stock in equilibrium. η is pinned down such that average hours worked are 0.33 of the disposable time for workers in equilibrium. σ is pinned down such that the coefficient of relative risk aversion associated with the composite goods of housing services and non-housing consumption goods is 2.0. This is a commonly used value in the literature.[6] The other parameter for preference, β, will be calibrated such that the aggregate amount of wealth in the model matches the U.S. counterpart.

For sensitivity analysis, I will use the following separable utility function as well:

$$u(c, d, m) = \frac{(c^{\psi} d^{1-\psi})^{1-\sigma}}{1-\sigma} + \eta \frac{m^{1-\rho}}{1-\rho}. \tag{25}$$

Leisure m is separable from consumption of aggregated goods, and consumption of non-housing goods c and housing services d is non-separable and aggregated with a Cobb-Douglas aggregator. σ, which represents the coefficient of relative risk aversion, is set at 2.0. ρ is set at 3, which corresponds to the Frisch elasticity of 0.5. This value is consistent with various estimates using micro data. Conesa et al. (2009) also use $\rho = 3.0$.

3.3. Endowment

The average life-cycle profile of earnings $\{\bar{e}_i\}_{i=1}^{I}$ is taken from Hansen (1993). Since Hansen (1993) estimates labor productivity for groups consisting of five ages (for example, ages 20-24, 25-29,...), his estimates are smoothed out using a quadratic function. Figure A.2 in Appendix A.1 shows the life-cycle profile of the average labor productivity used in the model. Since retirement is mandatory at the model age of I_R, $\bar{e}_i = 0$ for $i > I_R$.

As for the stochastic component of agents' earnings, I use the data on the cross-sectional variances of log of the hourly wage of the heads of households in the Panel Study on Income Dynamics (PSID). According to the PSID data, the cross-sectional variance of log of the hourly wage of heads of household of age 22 is 0.197, and the same statistic for heads of household of age 64 is 0.674, and the cross-sectional variance is almost linearly increasing. Appendix A.2 includes details about the empirical procedure. I basically follow the methodology of Storesletten et al. (2004) but derive the cross-sectional variances of *hourly wages* of the heads of households over the life-cycle, instead of those of the *total earnings* of households.

In the model, I assume that the initial distribution of $\log e$ is the normal $N(0, \sigma_e^2)$ and $\log e$ follows the following AR(1) process:

$$\log e' = \rho_e \log e + \epsilon \tag{26}$$

with $\epsilon \sim N(0, \sigma_\epsilon^2)$. There are three parameters, ρ_e, σ_e and σ_ϵ, that characterize the stochastic process. These three parameters are pinned down to capture the properties of the PSID data described above. First, σ_e^2 is set at 0.197 so that the cross-sectional variance of $\log e$ for agents of age 1 (corresponding to the actual age of 22) in the model is equal to the cross-sectional variance of log of the hourly wage of age-22 households. Second, in the data, cross-sectional variance almost linearly increases. It means that the persistence parameter ρ_e must be close to unity for the stochastic process of the model to replicate the property. Therefore, ρ_e is set at 0.99. Finally, σ_ϵ

[6] Specifically, σ is set to satisfy $1 - CRRA = \eta(1-\sigma)$, where $CRRA$ is the coefficient of relative risk aversion and is set at 2.0.

is pinned down such that the stochastic process used in the model implies that the cross-sectional variance of $\log e$ for age-43 agents (corresponding to the actual age of 64) is 0.674. This procedure leads to $\sigma_\epsilon^2 = 0.02058$.

Finally, the AR(1) process is approximated using a finite-state first-order Markov process. I use $n_e = 9$ as the number of states. For a highly persistent process, it is difficult for the discretized stochastic process to replicate the original process with a small number of n_e. The AR(1) process obtained above is converted into the Markov process using the method proposed by Tauchen (1986). In the standard Tauchen (1986) method, abscissas are distributed with equal space between $-\nu\sigma_e$ and $\nu\sigma_e$, where the scale parameter ν is set at 2 and σ_e is the unconditional standard deviation of e. Instead of the standard method with $\nu = 2$, I calibrate ν so that the discretized stochastic process generates the same variance as the original process for age-43 agents (corresponding to the actual age of 64). This procedure yields $\nu = 1.5$. The initial distribution of $\log e$ is approximated by assigning the probabilities to each of the grids obtained by applying the Tauchen (1986) method, similar to the way used in Tauchen (1986) for Markov process.

3.4. Technology

The production function is of the standard Cobb-Douglas type:

$$Y = ZK^\theta L^{1-\theta} \tag{27}$$

with $\theta = 0.247$ computed using the National Income and Product Accounts (NIPA). The value of θ is lower than the value usually used in the literature. This is because, in the current model, a part of the widely defined capital income associated with housing capital is removed from the definition of capital income for this economy with two kinds of capital.[7] I also calibrate the model with only non-housing capital and financial assets. I recalibrate θ such that there is no distinction between housing and non-housing capital and obtain $\theta = 0.326$, which is consistent with the commonly used value for one-asset models. The depreciation rate for non-housing capital is $\delta_K = 0.109$. The depreciation rate for housing capital is $\delta_H = 0.017$. Both are computed using the data on depreciation in NIPA. Since there is no shock to total factor productivity, Z is normalized to one.

3.5. Housing Market

There are three parameters pertaining to the housing market: the down payment requirement ratio λ, and the minimum sizes of owned and rented properties, $\underline{h}^o$ and $\underline{h}^r$. I set $\lambda = 0.20$. This is consistent with the typical down payment ratio of primary mortgage loans (20%) or a loan-to-value (LTV) ratio of 80%. As for the minimum size restrictions, I set $\underline{h}^r = 0$. I calibrate $\underline{h}^o$ such that the model generates the homeownership rate in the recent U.S. economy. The homeownership rate is historically around 64% in the U.S. This number is chosen as the calibration target. Notice that, without the strictly positive minimum restriction $\underline{h}^o$, the homeownership rate in the model will be substantially higher than the observed rate because of the preferential tax treatment of homeownership.

[7] Díaz and Luengo-Prado (2010) follow the same calibration strategy and obtain a similarly low θ of 0.26.

3.6. Government Policy

Following Domeij and Heathcote (2004), who use proportional taxes for capital and labor income, I use $\tau_K = 40\%$ for the baseline capital income tax rate.[8] As for housing taxes, since the imputed rents of owner-occupied housing in the U.S. are not taxed, I set $\tau_H = 0\%$ for the baseline rate. The baseline rate for the mortgage interest payment deduction is set at 23%. This number is the average marginal subsidy associated with mortgage interest payments, computed by Feenberg and Poterba (2004).

In order for the baseline model economy to capture key features of the current U.S. tax system, it is crucial to capture the progressivity of the federal income tax rate. I use the results of Gouveia and Strauss (1994), who estimate the progressive tax schedule of the U.S. federal income tax between 1979 and 1989, using the following functional form:

$$T(y) = \tau_0(y - (y^{-\tau_1} + \tau_2)^{-1/\tau_1}), \tag{28}$$

where y is taxable income and $T(y)$ is the corresponding tax bill. Gouveia and Strauss (1994) obtain $\tau_0 = 0.258$, $\tau_1 = 0.768$, and $\tau_2 = 0.031$. There are two issues when using their results in the current model. First, since the tax schedule (28) is estimated for incomes in 1990 U.S. dollars and is not unit-independent, normalization is necessary. I follow Erosa and Koreshkova (2007) and normalize τ_2, using the following formula, and obtain $\tilde{\tau}_2$, which is used in the model:

$$\tilde{\tau}_2 = \tau_2 \left(\frac{\overline{y}^{\text{model}}}{\overline{y}^{\text{US1990}}} \right), \tag{29}$$

where $\overline{y}^{\text{model}}$ is the average income in the model, and $\overline{y}^{\text{US1990}}$ is the average U.S. household income in 1990, which is about USD 50,000. The second issue is that I use the progressive tax function only for labor income, since I assume a proportional capital income tax rate and will investigate the welfare consequences of changing the constant capital income tax rate. I will use the average labor income in the model as $\overline{y}^{\text{model}}$, and leave other parameters intact.

Finally, considering that half of the social security contribution is paid by the employer and not subject to income tax, the tax function used in the current model is characterized as follows:

$$T(y) = \tau_0 \left(y\left(1 - \frac{\tau_S}{2}\right) - \left(\left(y\left(1 - \frac{\tau_S}{2}\right)\right)^{-\tau_1} + \tilde{\tau}_2 \right)^{-1/\tau_1} \right). \tag{30}$$

In order to investigate the importance of the progressivity of the labor income tax, I also investigate the model economy with proportional labor income tax as a part of the sensitivity analysis.

In the U.S., there is no federal tax for owner-occupied housing, but different local governments impose residential property taxes with different rates. For example, according to the government of the District of Columbia, if the tax rates applied in the largest city in each state are compared, the median effective tax rate in 2004 is 1.54%. The National Association of Home Builders (NAHB) reports that, according to self-reported property tax rates in the 2000 Census, the national average property tax rate in 2000 was 1.127%. Based on the evidence, τ_P is set at 1.1%. As sensitivity

[8] The tax rates are the averages between 1990 and 1996 of the effective tax rates computed by Mendoza et al. (1994). McGrattan (1994) and Joines (1981) obtain similar effective tax rates for the U.S.

analysis, the case where $\tau_P = 0$ is also studied later. $\tau_P = 0$ pertains to the idea that the property taxes levied by local governments are benefit taxes whose proceeds are used by local governments to provide goods and services necessary for those who pay the taxes.

The social security tax rate τ_S is set at 7.4%. According to Social Security Administration (2007), the average labor income in 2003 is USD 32,808, while the average annual benefit of retired workers is USD 11,065.[9] The replacement ratio, defined as the ratio between the two, is 33.7%. The 7.4% social security tax rate in the model is determined such that, when the government is balancing the budget in each period, the model replicates the replacement ratio.[10]

Since all the tax policies are set exogenously, the size of the government expenditure is obtained ex-post in the stationary equilibrium of the model economy with the baseline specification. In the baseline model with the tax rates described above, the total amount of government expenditures relative to output, including social security expenditures, turns out to be 21.5%, which is close to the average size of expenditures of the U.S. federal government.

3.7. Endogenously Calibrated Parameters

As I mentioned above, three parameters regarding the preference, the time discount factor β, the parameter that determines the relative value of the utility from housing services, ψ, the parameter that determines the relative value of leisure, η; and the minimum size of housing owned, $\underline{h}^o$, are calibrated *endogenously*. More specifically, the four parameters are calibrated such that four closely related targets are simultaneously satisfied in the stationary equilibrium of the baseline model economy. The four targets are the total value of housing capital stock and that of non-housing capital stock, the average hours spent working, and the homeownership rate. According to the NIPA, the average value for the period 2002-2006 of private housing capital relative to output ($\frac{H}{Y}$) is 1.29, while the same statistic for non-housing capital ($\frac{K}{Y}$) for the same period is 1.47. In total, the average value of total private capital stock over output is 2.76 in the U.S. As for the time spent on work, on average, workers spent one-third of their disposable time for work. Therefore, I use $\overline{\ell} = 0.33$ as the target. The target homeownership rate is 64%.

To pin down the four parameters, it is necessary to compute the equilibrium of the model repeatedly with a different set of parameter values, until the four statistics generated by the model are close to the corresponding targets. Even though there is no guarantee that all the targets can be satisfied, because of the non-linear nature of the problem, the calibration process turned out to be successful, and it is found that $\beta = 0.9774$ $\psi = 0.8874$, $\eta = 0.3612$, and $\underline{h}^o = 0.5305$ jointly satisfy the four targets: $\frac{H}{Y} = 1.29$, $\frac{K}{Y} = 1.47$, $\overline{\ell} = 0.33$, and the homeownership rate of 0.64.

3.8. Model Economy with One Asset

One of the key exercises in the paper is to compare the optimal capital income tax rate in the model economy with both housing and financial assets (two-asset model) and in the economy without housing (one-asset model). This one-asset model is constructed by treating housing assets as part of financial assets. Table 1 compares the two model economies.

In the economy without housing, the parameter controlling the capital share of income, θ, is

[9] This number is computed by multiplying the monthly benefit of retired workers of USD 922.1 by 12.

[10] Government budget balance implies $\tau_S m_W \overline{e} = \overline{b} m_R$ where m_W and m_R are measures of workers and retirees, respectively, and $\overline{e}$ and $\overline{b}$ represent average labor income and benefits, respectively. Plugging in $\frac{\overline{b}}{\overline{e}} = 0.337$ and $\frac{m_R}{m_W} = 0.221$ yields $\tau_S = 0.074$.

Table 1: Comparison of the Model Economies

Economy	Two-asset model	One-asset model
Aggregate statistics		
(H+K)/Y	2.7600	2.7600
H/Y	1.2900	–
K/Y	1.4700	2.7600
Parameters		
β	0.9774	0.9863
ψ	0.8874	–
η	0.3612	0.3637
$\underline{h}^o$	0.5306	–
θ	0.2470	0.3260
δ_H	0.0170	–
δ_K	0.1090	0.0660

higher because the capital income includes what is generated by housing capital. According to NIPA, θ for the one-asset model turns out to be 0.326, which is close to the value usually used in the models with one type of capital. The depreciation rate for capital δ_K is also adjusted, taking into account that capital in the one-asset model also includes housing capital, which depreciates more slowly than non-housing capital. Naturally, the depreciation rate is lower. According to NIPA, the annual depreciation rate associated with capital in the one-asset model is 6.6%.

Notice that the parameters β and η are re-calibrated for the one-asset model such that the model satisfies the capital output ratio of 2.76 and the average fraction of time spent working at 0.33. ψ and $\underline{h}^o$ are not used in a meaningful way in the one-asset model.

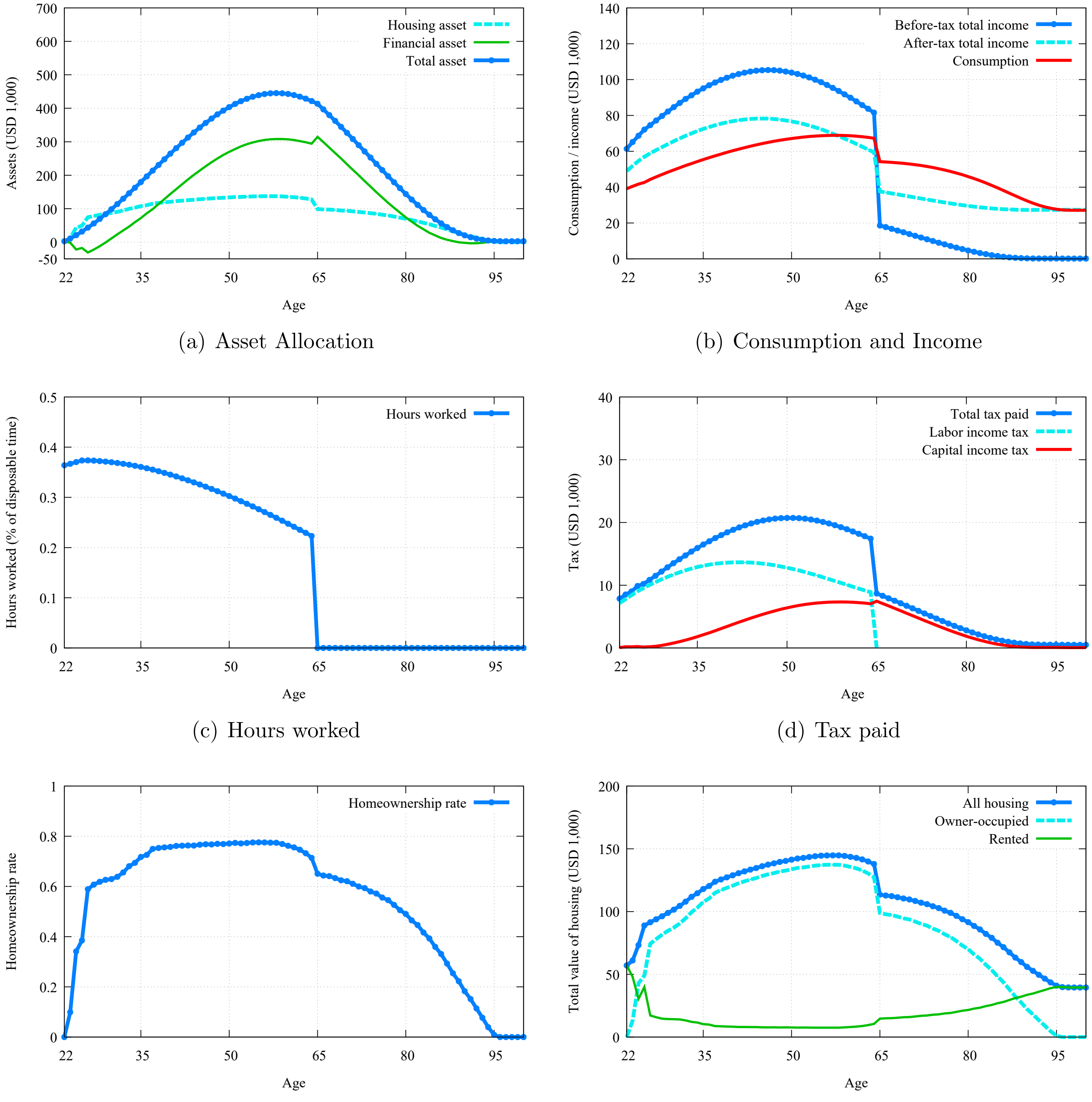
3.9. Computation

Since the model cannot be solved analytically, numerical methods are used to compute the stationary equilibrium of the model. The solution method is a standard one for overlapping generations models.[11] In solving the problem of an individual agent, the optimal decision rules are approximated using piecewise linear functions, and the optimal decision rules are obtained backwards, starting from the last period of life.

A challenge for the current model is that there are two types of assets. When the set of individual state variables includes two endogenous continuous state variables, the model is very difficult to solve with a reasonable level of accuracy. This is especially so if there is a tenure choice as well as labor-leisure decision. However, it is feasible to solve the current model because there is only one continuous state variable, which is the total wealth x. The set of individual state variables of agents does not include h and a separately but does include only x, because the allocation between h and a does not affect the agents' optimal decision.

In obtaining the aggregate statistics, I implement a simulation with $1,000,000$ agents in each generation. Appendix A.3 includes further details of the computation.

[11]For more details on the computational methods employed here, see Ríos-Rull (1999).



(a) Asset Allocation

(b) Consumption and Income

(c) Hours worked

(d) Tax paid

(e) Homeownership Rate

(f) Housing Assets: Owned and Rented

Figure 1: Average Life-Cycle Profiles in the Baseline Model Economy

4. Properties of the Baseline Model Economy

Figure 1 exhibits the life-cycle profiles in the baseline model economy. Figure 1(a) shows the average life-cycle profile of housing and financial asset holdings, as well as the total wealth in the baseline model economy. The most striking feature of the figure is that the portfolio allocation between housing and financial assets varies greatly with age. At the beginning of their working lives, agents save to prepare for the down payment on their first house. They rent while doing so. Then agents borrow using mortgage loans and accumulate housing assets. Around age 30, average agents finish repaying mortgage loans and start accumulating savings in the form of financial assets, after accumulating sufficient housing assets to support a desirable amount of housing service consumption. After retirement, agents reduce financial asset holdings more quickly than housing assets, because agents need housing for consumption of housing services. Toward the end of the life-cycle, agents reduce holdings of both types of assets. The hump shape of durable goods, whose main component is housing, is well documented by Fernández-Villaverde and Krueger (2011). In terms of the ratio of housing assets over total wealth, the ratio is much higher for young agents because they use leverage when they own housing assets whose value is larger than the value of their total wealth. The ratio keeps going down as agents accumulate financial assets relative to housing assets.

Figure 1(b) shows the average life-cycle profile of before- and after-tax income and consumption in the baseline model economy. The after-tax total income in the figure includes the social security benefit and excludes tax payments and social security contributions. The profile of the after-tax total income is flatter than that of the before-tax total income, not only because of the intergenerational transfer through the social security program, but also because workers are taxed more heavily than retirees. The life-cycle profile of consumption is even flatter than after-tax income, but still remains hump shaped. The hump shape of non-housing consumption of U.S. consumers is carefully documented by Gourinchas and Parker (2002) and Fernández-Villaverde and Krueger (2011).

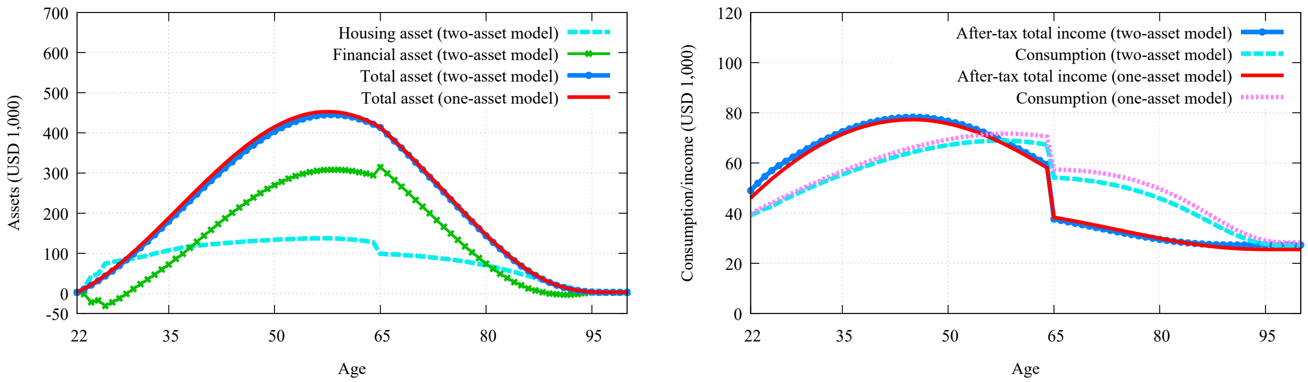
Figure 1(c) shows the average life-cycle profile of hours worked. Hours worked increase in the early 20s as agents try to accumulate assets to purchase a house. After the mid-20s, the profile is mostly decreasing over the life-cycle, because of the income effect for the earlier stage of working life and because of the substitution effect for the latter stage of working life. After the retirement age of 64, there are no hours for work.

Figure 1(d) shows the average life-cycle profile of tax payments. The majority of taxes paid by workers is labor income tax. Workers close to retirement age pay approximately the same amount of labor and capital income tax. Retirees pay only capital income and property tax.

Figure 1(e) shows the life-cycle of the homeownership rate in the baseline model economy. The overall average homeownership rate is calibrated to be 64% in the model. It exhibits a hump shape, as in the U.S. data. The ratio is low for young agents, peaks around age 55, and goes down after retirement age.

Figure 1(f) shows the average life-cycle profile of all housing assets, including owner-occupied as well as rented housing. The profile for owner-occupied housing is the same as the one shown in Figure 1(a). For very young and very old agents, more housing assets are rented rather than owned. Therefore, the average life-cycle profile of total housing assets is located higher than the profile of owner-occupied housing for the young and retirees. Consumption of non-housing goods, which is captured by the total housing holdings in Figure 1(f), is hump shaped, as in the U.S. data.

Figures 2(a) and 2(b) compare average life-cycle profiles of assets and consumption between the two-asset baseline model and the one-asset model. The one-asset model is calibrated to the same set of targets as the two-asset model as long as it is feasible. The striking feature is that the life-cycle profiles of total wealth and consumption are very close to each other, although there is an interesting life-cycle profile for the asset portfolio between the housing and financial assets in the two-asset model.



(a) Asset Allocation

(b) Consumption and Income

Figure 2: Comparison of Two-Asset and One-Asset Model

5. Design of Experiments

5.1. Design of Alternative Tax Systems

A tax system is defined as $\mathcal{T} = (\tau_K, \tau_H, \tau_M, \tau_P, \tau_0, \tau_1, \tau_2)$ for the two-asset model and $\mathcal{T} = (\tau_K, \tau_0, \tau_1, \tau_2)$ for the one-asset model. A revenue-neutral tax system is a tax system that generates the total tax revenue equal to the government expenditure G, which is obtained in the stationary equilibrium of the baseline economy. Suppose a housing tax system (τ_H, τ_M, τ_P) (only for the two-asset model), and two parameters of the labor income tax system (τ_1, τ_2) are given. Then, for a capital income tax rate τ_K, a revenue-neutral tax system is given by a τ_0 that guarantees revenue neutrality. In other words, a revenue-neutral tax system is characterized by the capital income tax rate τ_K. The main experiment of the paper is to find the optimal revenue-neutral tax system $\mathcal{T}^*$, which maximizes the social welfare (defined in the next section).

Let me make four remarks about the design of the experiments. First, since all the experiments are implemented in a revenue-neutral manner, the total tax revenue is the same across all alternative tax systems, at 21.5% of output in the two-asset model. Alternative assumption is to assume that $\frac{G}{Y}$ is constant across experiments, but I follow Conesa et al. (2009) and assume that the G is kept the same. I also find that the main results are not affected by fixing $\frac{G}{Y}$ instead of G. Second, τ_0 is associated with the average level of the labor income tax. Roughly speaking, changing τ_0 while leaving τ_1 and τ_2 unchanged is equivalent to shifting the average tax rate without affecting the degree of progressivity. Third, in the case that proportional labor income tax is used instead of the progressive labor income tax, as a part of the robustness exercises, the proportional labor income tax rate τ_L is adjusted to ensure revenue neutrality. Fourth, Conesa et al. (2009) explore the optimal combination of (τ_K, τ_0, τ_1) while using τ_2 to ensure revenue neutrality, using a model without housing. I do not jointly search for the optimal labor income tax schedule like they do, mainly because the current model is substantially harder to solve than theirs, which makes searching jointly for the optimal capital income tax rate and the optimal labor income schedule infeasible. However, it is likely that the intuition of the main findings of the paper remain valid for cases when the optimal labor income tax schedule is searched jointly with the capital income tax rate. For example, using proportional labor income tax instead of the progressive one does not change the main result of the paper, as shown in Section 9.

For the two-asset model, it is necessary to pre-set the housing tax system (τ_H, τ_M, τ_P), which is not present in the standard one-asset environment. I investigate three cases. In the first case, I maintain the preferential tax treatment for owner-occupied housing. Specifically, I keep $\tau_H = 0$, $\tau_M = 0.23$ and $\tau_P = 0.011$. In the second case, I require that the existing preferential tax treatment for owner-occupied housing be eliminated. In particular, I impose that the housing and financial assets be taxed at the same rate, i.e., $\tau_K = \tau_H = \tau_M$ and change τ_K $(= \tau_H = \tau_M)$ to various rates. It will be shown in Section 6.2 that the choice of τ_M does not play a significant role for the result; what matters is the choice of τ_H. τ_P is kept at the baseline rate of 1.1%. Finally, I will allow both τ_K and τ_H to be chosen independently without a restriction and investigate the optimal combination of the two. I keep τ_P and τ_M at the baseline rates of 0.011 and 0.23, respectively. It turns out that the choice of τ_M does not play an important role for the results, either.

5.2. Welfare Measures

In comparing the social welfare in economies with different tax systems, I use the ex-ante expected utility of newborns in the stationary equilibrium. This social welfare is used by Conesa et al. (2009), which makes the comparison straightforward. This also corresponds to the long-run social welfare in the model with infinitely lived agents. Technically, the social welfare is computed by integrating the value of the newborns into the stationary equilibrium with respect to the initial shock to individual labor productivity. The welfare criterion is useful in taking into account both the efficiency effect due to tax reforms and the redistribution or insurance aspect of tax reforms. The consideration of the latter is crucially important in experiments where markets are incomplete, and therefore, agents are ex-post heterogeneous.

Like Conesa et al. (2009), I do not use the utilitarian welfare function of the living agents in the initial stationary equilibrium with transition path taken into account, as the measure of social welfare, for the following reasons.[12] First, it is extremely challenging to find the optimal tax system taking the transition into account, since the model is hard to solve even for a stationary equilibrium. Second, the social welfare employed here and the associated definition of the optimality correspond to the long-run optimal taxation in the growth model, which is typically used in the literature. The employed social welfare makes the comparison with the literature straightforward.

In measuring the magnitude of the welfare gain or loss, I use the percentage changes in the consumption of non-housing goods. This is a standard measure for welfare analysis in the literature. Using this measure, the welfare gain by moving from one tax system to another is defined as the percentage increment ϵ to the consumption of non-housing goods in every period and under every contingency in the economy with the original tax system, which equates average welfare in the economy with the original tax system to that of the economy with the alternative tax system. A positive ϵ implies that agents are better off by being born into the economy with the alternative tax system, in the expected ex-ante sense. Notice that, in the current model, there are three sources of utility, namely, consumption of non-housing goods, consumption of housing services, and leisure, but percentage ϵ is added only to the consumption of non-housing goods in computing the welfare gain. In other words, welfare changes associated with changes in the consumption of housing services as well as leisure are converted and merged into the welfare changes in the consumption of non-housing

[12]The separate appendix of Conesa et al. (2009) shows that their main result – that the optimal capital income tax rate is substantially higher than zero – is strengthened when the transition to the new steady state is taken into account.

goods in computing the welfare gain.

Moreover, for an analytical purpose, I decompose ϵ as follows:

$$\epsilon = \epsilon_a + \epsilon_d, \tag{31}$$

where ϵ_a measures the welfare gain associated with changes in aggregate consumption. In particular, ϵ_a measure the welfare gain by uniformly increasing non-housing and housing consumption and leisure of all agents in each period and state by the growth rates of aggregate consumption. I call ϵ_a the *aggregate* effect. For an economy without heterogeneity and life-cycle, the aggregate effect coincides with the total welfare effect. On the other hand, ϵ_d represents the welfare gain associated with the redistribution of consumption across age and node. I call the effect the *redistribution* effect. Their formal definitions are provided in Appendix A.4.

6. Optimal Capital Income Taxation

6.1. With Preferential Treatment for Owner-Occupied Housing

The right columns of Table 2 summarize the effect of implementing the optimal capital income tax rate in the baseline two-asset model economy. For comparison, the result from the one-asset model economy is also shown (left columns). In the one-asset model where the difference between housing and non-housing capital is ignored, the optimal capital income tax rate is found to be 31% (see the second column of Table 2). The rate is not only non-zero, but far from zero, as Conesa et al. (2009) find. The optimal capital income tax rate found here (31%) is close to the optimal rate of Conesa et al. (2009), which is 36%. As Conesa et al. (2009) argue, it is optimal to tax capital income heavily in a model where there is a life-cycle saving motive and thus the saving decisions of agents are not strongly elastic against changes in the after-tax interest rate. They argue that the inelasticity of saving relative to labor supply makes a high capital income tax rate optimal in the economy with life-cycle, while it is optimal *not* to tax capital in an economy without life-cycle.

When the capital income tax rate is lowered from the baseline level of 40% to the optimal level of 31% in the one-asset model, the average labor income tax rate has to be increased to guarantee the revenue neutrality. Naturally, capital stock increases, by 2.3%, while labor supply declines by 1.5%. Aggregate output and aggregate consumption decline, by 0.5% and 1.3%, respectively. The total welfare gain is equivalent to a mere 0.1% increase in consumption equivalence. Although the aggregate effect is negative, reflecting the decline in aggregate consumption, the positive redistribution effect more than offsets the negative aggregate effect. The overall size of the welfare gain by moving from the baseline economy to the one with the optimal tax rate is small, because the optimal capital income tax rate (31%) is close to the baseline value (40%).

Now, turn to the fifth column of Table 2. It is found that the optimal capital income tax rate in the two-asset economy when the preferential tax treatment of owner-occupied housing is preserved (i.e., $\tau_H = 0\%$ and $\tau_M = 23\%$) to be 1%. The optimal capital income tax rate is close to zero and is remarkably different from the case in which there is no distinction between housing and non-housing assets (one-asset economy), where the optimal capital tax rate is 31%. τ_0, which roughly represents the average labor income tax level, must be increased from 26% to 36% to keep revenue neutrality. Output in the new steady state with the capital income tax rate of 1% declines by 0.64%, since a decline in the labor supply (−4.0%) dominates an increase in the non-housing capital stock (+11.2%).

Table 2: Optimal Capital Taxation: With Preferential Tax Treatment for Owner-Occupied Housing

Economy	One-asset model			Two-asset model		
	Baseline	Optimal	$\tau_K = 0.01$[1]	Baseline	Optimal	$\tau_K = 0.31$[2]
Tax rates						
τ_H	–	–	–	0.000	0.000	0.000
τ_M	–	–	–	0.230	0.230	0.230
τ_K	0.400	0.310	0.010	0.400	0.010	0.310
τ_0^3	0.258	0.299	0.429	0.258	0.360	0.285
% change from the baseline[4]						
Output(=Y)	0.589	−0.45	−3.23	0.404	−0.64	−0.05
Total capital/Y	2.760	+2.30	+8.32	2.760	+0.09	+0.35
Housing capital/Y	–	–	–	1.290	−12.56	−2.58
Non-housing capital/Y	2.760	+2.30	+8.32	1.470	+11.18	+2.92
Average hours worked	0.330	−1.52	−6.63	0.330	−4.19	−1.01
Labor supply	0.360	−1.54	−6.89	0.356	−4.04	−0.99
Consumption	0.372	−1.34	−6.74	0.253	−3.22	−0.74
Homeownership rate[5]	–	–	–	0.640	0.361	0.619
% change in welfare[6]						
Overall effect (ϵ)	–	+0.09	−0.78	–	+1.20	+0.44
Aggregate effect (ϵ_a)	–	−0.13	−0.58	–	−2.16	−0.53
Redistribution effect (ϵ_d)	–	+0.22	−0.20	–	+3.36	+0.97

[1] Optimal level for the two-asset model.
[2] Optimal level for the one-asset model.
[3] Adjusted to guarantee revenue neutrality.
[4] Level is shown for the baseline economy.
[5] Level is shown for all economies.
[6] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.

The overall welfare gain is large at 1.2% in consumption equivalence. In terms of the composition, the aggregate effect is negative (−2.2%), as a result of a drop in aggregate consumption and housing capital stock, but the large positive redistribution effect (+3.4%) more than offsets the negative aggregate effect. Also notice that if the capital income tax rate found to be optimal using the one-asset model (31%) is implemented (see the last column of Table 2), there is still a welfare gain (0.4%) but the gain is smaller than would have been achieved by implementing a 1% capital income tax. On the other hand, if the optimal capital income tax rate of 1% is implemented in the one-asset model, there is a large welfare loss (−0.8%).

Figure 3 compares the welfare effect of changing the capital income tax rate between 0% to 60%, for both one-asset and two-asset models. For the two-asset model, two cases are shown, with (dotted red line) and without (dashed cyan line) preserving preferential tax treatment of housing. The case without will be discussed in the next section. A dot for each line represents the optimal point. Since the baseline tax rate is 40%, the welfare effect for the one-asset model (solid blue line)

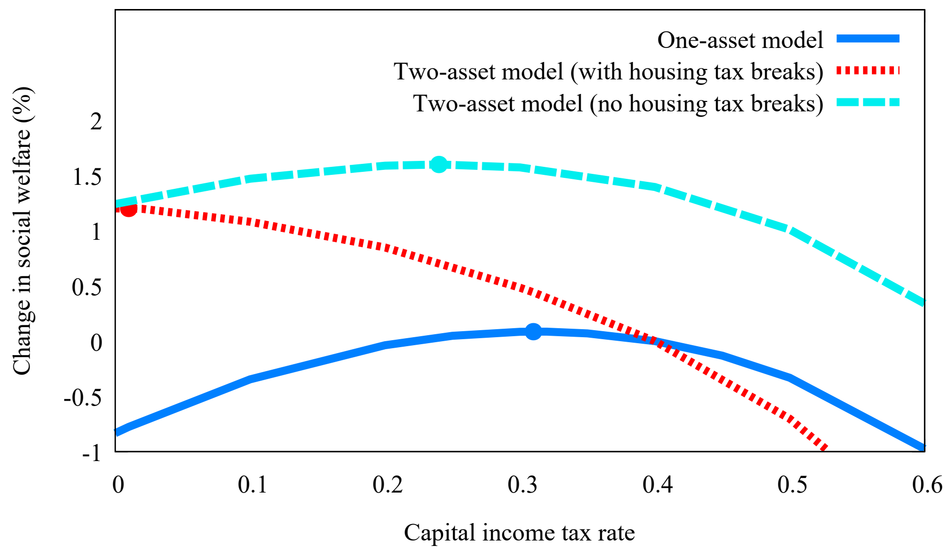
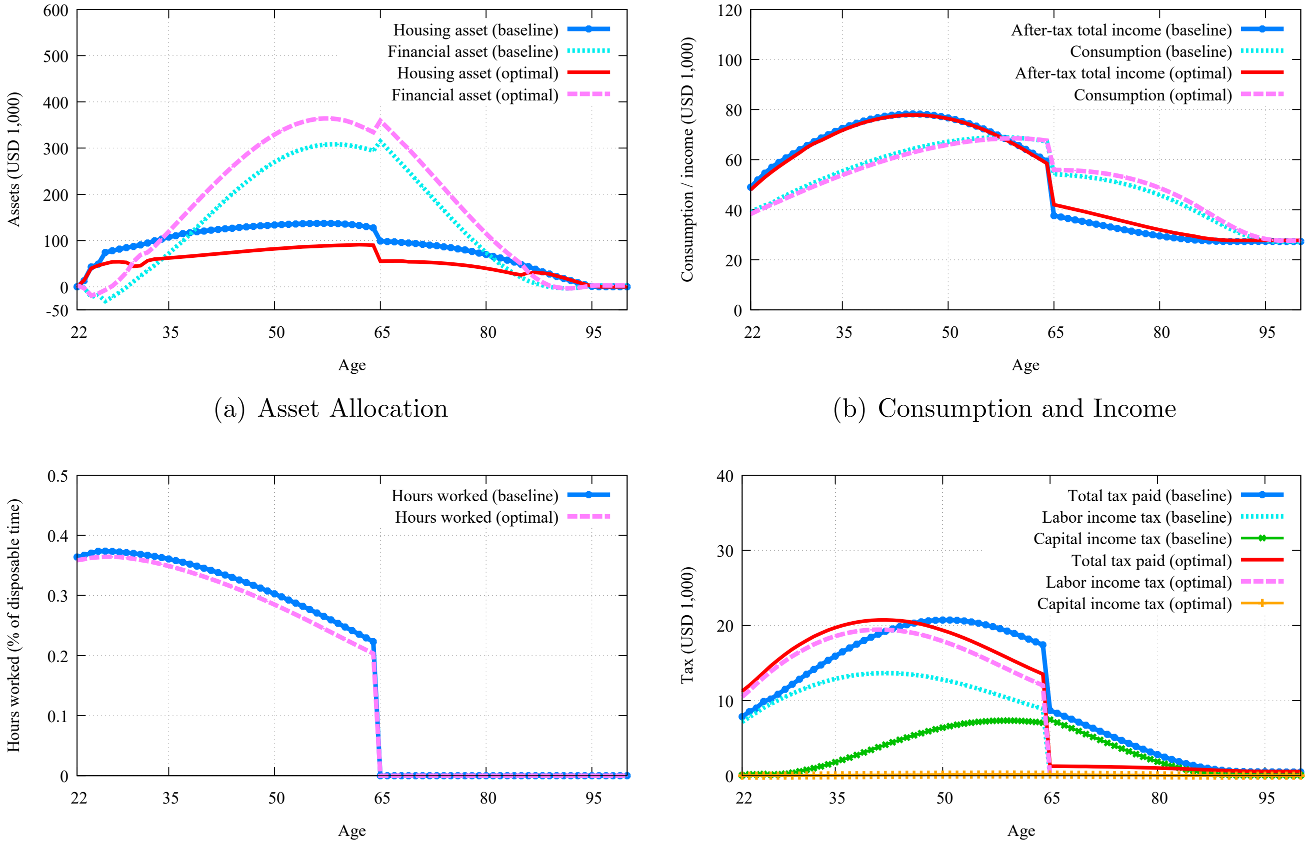


Figure 3: Comparison of Welfare Effects of Changing Capital Income Tax

and that for the two-asset model with housing tax breaks preserved associated with a 40% capital income tax rate is zero. From there, while social welfare quickly starts to decline as the capital income tax rate is lowered from the baseline rate of 40% in the one-asset model, social welfare is almost monotonically increasing in the two-asset model.

What generates the difference in the optimal capital income tax rate between the one-asset model and the two-asset model? There are three key intuitions. First, when the capital income tax rate is increased, agents can evade the higher tax by shifting their portfolio to housing. Although this intuition is more straightforward if there are closer substitutes, such as other financial assets that are not subject to the increased tax, the same logic applies to housing, which is an imperfect substitute for financial assets. When the capital income tax rate is lowered in the two-asset model, this evading behavior is weakened. Second, lowering the capital income tax rate while keeping the exemption for imputed rents of owner-occupied housing means nullifying the tax wedge between the two types of capital and thus correcting the over-accumulation of housing capital. When there is no tax for owner-occupied housing but capital income is heavily taxed, the tax wedge between the two discourages investment in non-housing capital compared with investment in housing capital. Third, a higher capital income tax rate accompanied by the preferential tax treatment for owner-occupied housing is a subsidy for homeowners at the expense of renters, who tend to hold less wealth and earn lower income. To see this more clearly, let me compare the cost of renting and owning. The unit cost of renting a house is the rent, $q = r + \tau_P + \delta_H$. What is the cost of owning? For an agent who does not take a mortgage, it is $\tilde{q} = \tau_P + \delta_H + r\tau_H + r(1 - \tau_K)$. The first three terms are straightforward; a homeowner has to pay property tax, maintenance costs (depreciation), and housing tax, if there is one. The last term $r(1 - \tau_K)$ represents the opportunity cost of owning a house instead of investing in financial assets. The last term has $(1 - \tau_K)$ because the financial asset return is subject to the capital income tax. If q and $\tilde{q}$ are compared, it is easy to see that owning is less costly if $\tau_K > \tau_H$. It is trivially satisfied in the baseline model (and in the U.S. economy), where $\tau_H = 0$. In other words, renters pay more than homeowners to enjoy the same house by $\tau_K - \tau_H$. There are renters in the baseline model in spite of the benefit of being a homeowner



(a) Asset Allocation

(b) Consumption and Income

(c) Hours Worked

(d) Taxes

Figure 4: Average Life-Cycle Profiles of Two-Asset Model with the Optimal Capital Tax Rate (with Preferential Tax Treatment for Owner-Occupied Housing)

precisely because of the lumpiness of housing; agents cannot own a house that is smaller than a positive lower bound $\underline{h}^o$. In the baseline model, renters suffer by either paying extra to enjoy the same amount of housing services consumption or living in a larger house than they would if tax breaks did not exist, in order to enjoy the tax benefits for homeowners (although they would like to remain renters if there is no tax break for homeowners). Since renters typically have lower income and lower wealth, the preferential tax treatment for homeowners works as a tax for lower-income and low-wealth agents who choose to rent. The tax wedge between τ_K and τ_H, which represents the additional tax for renters, declines (disappears) when the capital income tax rate is brought down (to zero), in case the preferential tax treatment for owner-occupied housings is preserved. You can see in Table 2 that the homeownership rate drops from the baseline level of 64% to 36% when the capital income tax rate is lowered to 1% and owning relative to renting becomes less attractive. How important are the three channels mentioned above? To answer the question, I investigate optimal capital income taxation in various alternative models in Section 7.

Figure 4 compares the average life-cycle profiles of model economies with the baseline tax rates and the optimal tax rates together with the preferential tax treatment for owner-occupied housing. Figure 4(a) shows that there is a substantial portfolio reallocation from housing to financial assets.

Table 3: Optimal Capital Taxation: Without Preferential Tax Treatment for Owner-Occupied Housing

Economy	Baseline	Optimal			
Preferential tax treatment for housing	Yes	Yes	No	Only τ_M	Only τ_H
Tax rates					
τ_H	0.000	0.000	$=\tau_K$	$=\tau_K$	0.000
τ_M	0.230	0.230	$=\tau_K$	0.230	$=\tau_K$
τ_K	0.400	0.010	0.240	0.240	0.010
τ_0^1	0.258	0.360	0.281	0.281	0.359
% change from the baseline[2]					
Output(=Y)	0.404	−0.64	+1.44	+1.44	−0.52
Total capital/Y	2.760	+0.09	−4.58	−4.58	−0.17
Housing capital/Y	1.290	−12.56	−18.56	−18.56	−13.46
Non-housing capital/Y	1.470	+11.18	+7.69	+7.69	+11.49
Average hours worked	0.330	−4.19	−1.22	−1.22	−4.15
Labor supply	0.356	−4.04	−0.99	−0.99	−4.01
Consumption	0.253	−3.22	+0.71	+0.71	−3.10
Homeownership rate[3]	0.640	0.361	0.254	0.189	0.324
% change in welfare[4]					
Overall effect (ϵ)	–	+1.20	+1.60	+1.60	+1.24
Aggregate effect (ϵ_a)	–	−2.16	−2.08	−2.08	−2.23
Redistribution effect (ϵ_d)	–	+3.36	+3.68	+3.68	+3.47

[1] Adjusted to guarantee revenue neutrality.

[2] Level is shown for the baseline economy.

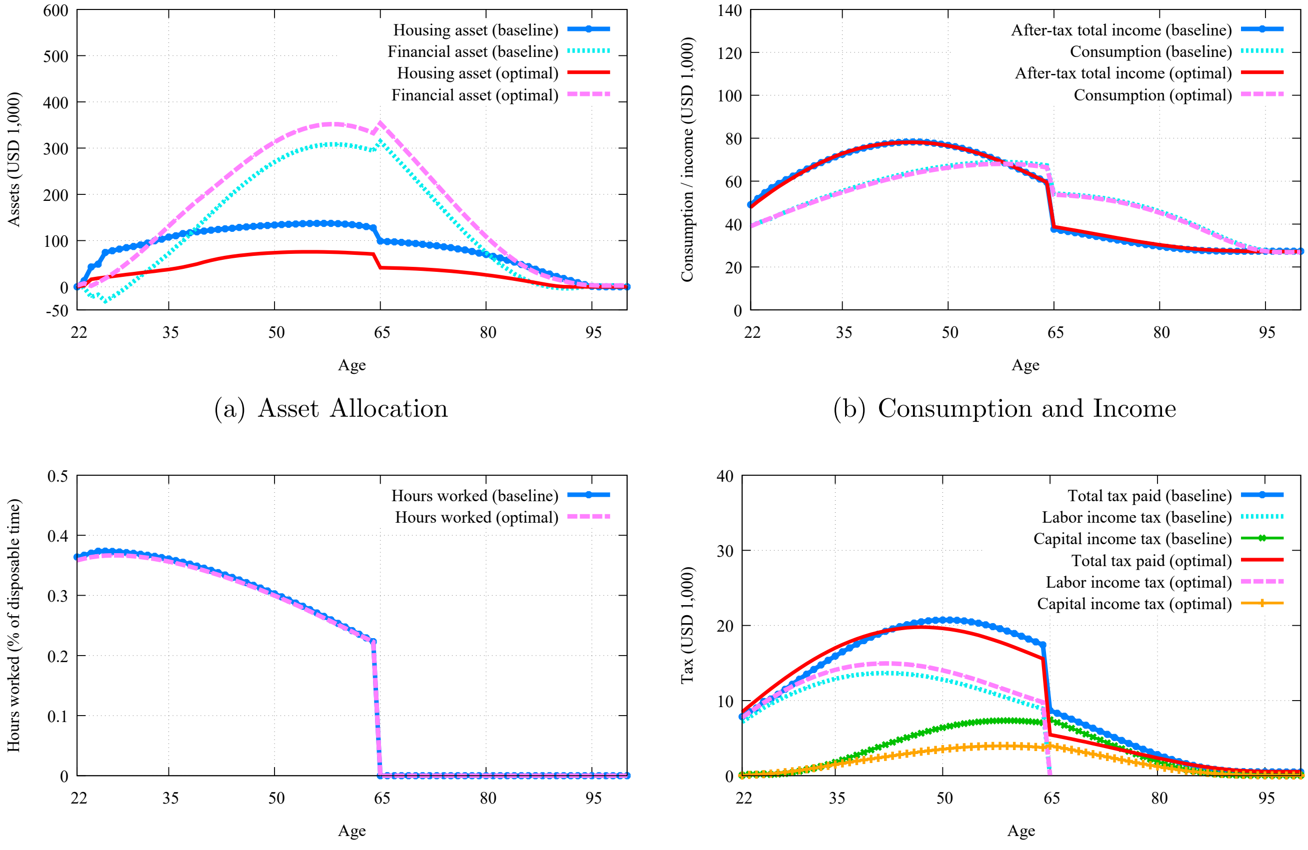
[3] Level is shown for all economies.

[4] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.

Figures 4(b) and 4(c) show that life-cycle profiles of consumption and hours shift down slightly. Consistently, Figure 4(d) shows that there is a noticeable change in the life-cycle profile of tax payments; the young, who are more likely to be borrowing-constrained, and the middle-aged, who are the most productive, pay more taxes in the alternative tax system as labor income tax becomes more important as a source of government income. Finally, notice that the welfare gain that stems from the narrowed tax wedge between housing and non-housing capital income is enjoyed at the expense of a higher average labor income tax. But lowering the capital income tax rate is not the only way to narrow the wedge, if the tax rate applied to imputed rents of owner-occupied housing is not fixed at zero as in the baseline. If the gap is narrowed without applying a high labor income tax rate, there might be an even larger welfare gain. That is shown to be the case in the next section.

6.2. Without Preferential Tax Treatment for Owner-Occupied Housing

In this section I will show that the optimal capital income tax rate crucially depends on the tax system associated with housing. The previous section showed that the two-asset model developed in this paper has a very different implication regarding how to tax capital, compared with the



(a) Asset Allocation

(b) Consumption and Income

(c) Hours Worked

(d) Taxes

Figure 5: Average Life-Cycle Profiles of Two-Asset Model with the Optimal Capital Tax Rate (without Preferential Tax Treatment for Owner-Occupied Housing)

standard one-asset model that doesn't distinguish housing and non-housing. In this section, I will show that how income from non-housing capital should be taxed depends on how housing is taxed. Table 3 summarizes the main results. The first and the second columns are duplicated from Table 2 for comparison; they show the properties of the baseline model, and the properties of the economy under the optimal capital income tax rate, conditional on the current preferential tax treatment for owner-occupied housing being preserved. The remaining three columns exhibit properties of the model economy with the optimal capital income tax rate, under three alternative housing tax systems.

The third column of Table 3 contains the most important results. I restrict tax rates for imputed rents of owner-occupied housing (τ_H) as well as the mortgage interest payment deduction rate (τ_M) to be equal to the capital income tax rate (τ_K) and find the optimal τ_K under this restriction. We can interpret the experiment as finding the optimal capital income tax rate when the preferential tax treatment for housing is eliminated. Remember that $r(\tau_K - \tau_H)$ represents the additional unit cost that renters have to pay to live in the same house as homeowners and this becomes zero under the current housing tax system. As you can see, the optimal capital tax rate is 24%, which is substantially higher than the optimal rate obtained under the restriction that the preferential tax treatment for owner-occupied housing is preserved. In terms of the welfare effect, the welfare gain of implementing this tax system is even larger, at 1.6% of flow consumption, than the one obtained in the previous section (1.2%). In Figure 3, the welfare effect of changing the capital income tax rate conditional on eliminating the preferential tax treatment for owner-occupied housings is also drawn (dashed cyan line). At the capital income tax rate of zero, the effect on social welfare is almost identical with and without the exemption of imputed rents; the only difference is the deduction rate of the mortgage interest payment (τ_M). But the welfare effect as the capital income tax rate is increased is strikingly different. The difference is generated because the tax wedge between housing and capital income changes differently as the capital income tax rate is increased. In the case investigated in the previous section, a higher capital income tax implies a higher tax wedge, or more favorable tax treatment for owner-occupied housing, but the tax wedge does not change in the case studied in this section. Also notice that eliminating the tax wedge while keeping the capital income tax rate at the baseline level (40%) alone generates a large (1.4%) welfare gain. Compared with the size of the welfare gain, achieving the optimal level of the capital income tax rate (additional 0.2%) is small. It is also suggested by the fact that the dashed line in Figure 3 is substantially above the dotted line.

Why is the size of the welfare gain even higher? To answer this question, see Figure 5, which compares the baseline model economy and the economy with the optimal capital income tax rate, and without preferential tax treatment for owner-occupied housing. First, Figure 5(a) shows that the optimal tax system induces agents to shift their portfolio from housing to financial assets. The effect on the asset portfolio is similar to the case in which housing's preferential tax treatment is preserved (see Figure 4(a)). Figure 5(b) shows that the average life-cycle profile of consumption does not change significantly by implementing the optimal tax system, as in the case studied in the previous section. On the other hand, 5(c) shows that the average life-cycle profile of hours declined in the case of preserving preferential tax treatment for owner-occupied housing, while hours do not change noticeably in the current case. This is because the labor income tax does not need to be raised in the current case ($\tau_0 = 0.28$) compared with the case studied in the last section ($\tau_0 = 0.36$), This can be seen in how life-cycle profile of tax payment is different (Figures 4(d) and 5(d)). In the case studied in the previous section, the optimal tax system shifts the burden of taxes to younger and more productive agents. On the other hand, in the current case, the average life-cycle profile of total taxes changes less by implementing the optimal tax system. This is where the additional welfare gain is coming from. In the case in which the preferential tax treatment for owner-occupied housing is preserved by assumption, the capital income tax rate is lowered, and the average labor income tax rate must be raised, to narrow the tax wedge between housing and financial assets. In the case in which the preferential tax treatment is eliminated, since the tax wedge is already zero by assumption, there is no need to increase the average labor income tax rate to fill the tax wedge. In other words, when the preferential tax treatment for owner-occupied housing is eliminated, there is no trade-off between narrowing the tax wedge between housing and financial assets and avoiding a severe labor supply distortion for productive agents.

The last two columns of Table 3 basically show that the mortgage interest payment deduction is not crucial in determining how capital income should be taxed. More specifically, the fourth column shows the case in which the housing tax rate (τ_H) is restricted to be equal to the capital income tax rate (τ_K), but the mortgage interest payment deduction rate (τ_M) is left at the baseline rate of 23%. The result is that the optimal tax system is virtually identical to the one just presented; the level of τ_M does not matter for the optimal level of τ_K. The last column is associated with the case

Table 4: Optimal Capital Taxation: Role of Housing

Economy	One-asset model		Two-asset model			
	Opt. τ_K	Welfare[1]	Opt. τ_K^2	Welfare[1]	Opt. τ_K^3	Welfare[1]
1. Baseline	0.31	+0.09	0.01	+1.20	0.24	+1.60
2. No renting	–	–	0.15	+0.36	0.22	+0.51
3. Lower down payment	–	–	0.00	+1.16	0.24	+1.58
4. No renting and no debt	–	–	0.16	+0.35	0.23	+0.49
5. Additional ownership cost	–	–	0.17	+2.88	0.26	+2.96
6. Additional rental cost	–	–	0.00	+0.91	0.25	+1.40

[1] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.
[2] The preferential tax treatment for owner-occupied housing is preserved.
[3] The preferential tax treatment for owner-occupied housing is eliminated.

in which the tax exemption for owner-occupied housing is preserved ($\tau_H = 0$) while τ_M is restricted to be equal to τ_K. Again, the optimal tax system is very similar to the one in which both τ_H and τ_M are fixed at their baseline values. In sum, the role of the mortgage interest payment deduction is minor compared with the importance of taxation of imputed rents of owner-occupied housing.

6.3. Optimal Combination of Housing and Capital Taxation

In the last two sections, I explore optimal capital income taxation, given the housing tax systems. What does the optimal taxation system look like if there is no restriction and the housing tax policy can be chosen as well? Based on the finding in the previous section that the mortgage interest payment deduction rate (τ_M) does not play an important role in shaping the optimal taxation system, I fix τ_M at the baseline rate of 23%, and explore the optimal combination of the capital income tax rate (τ_K) and the tax rate for imputed rents of owner-occupied housing (τ_H). As in the previous cases, I use the average labor income tax rate (τ_0) to guarantee revenue neutrality. Somewhat surprisingly, the optimal combination of (τ_K, τ_H) turns out to be the one found under the restriction of $\tau_K = \tau_H$, i.e., the case shown in the fourth column of Table 3. The optimal combination is found to be $(\tau_K, \tau_H) = (0.24, 0.24)$. The optimal tax system in which τ_K and τ_H can be chosen independently is consistent with the following two intuitions. First, as in Conesa et al. (2009), it is optimal to tax capital (and housing) at a high tax rate in the overlapping generations model. Second, it is optimal not to create a tax wedge between two assets.

7. Role of Housing

In this section, I explore the role of various features of housing by studying the variants of the baseline model. All models are recalibrated following the same calibration strategy as in the baseline model. Table 4 summarizes the results.

7.1. Role of Tenure Decision

The first row of Table 4 replicates the baseline results, for comparison. The second row shows the case in which there is no tenure decision in the two-asset model. In particular, the option of renting is eliminated and the lower bound of owned housing assets $\underline{h}^o$ is set at zero; agents cannot rent housing but can own housing of any size, as long as the down payment constraint is satisfied. Interestingly, although the optimal capital income tax rate is still lower (15% compared to 22%) when the preferential tax treatment for owner-occupied housing is preserved, the difference is smaller than in the baseline, and the welfare gains are also noticeably smaller. For example, when housing and financial assets are taxed equally, the welfare gain associated with the optimal tax rate is 0.51% of flow consumption, instead of the baseline welfare gain of 1.60%. We can interpret this as suggesting that a large part of the welfare gain in the two-asset baseline model, and the major reasoning behind the baseline results, is associated with the redistribution effect between homeowners and renters. In the baseline experiment, reducing the tax wedge between housing and non-housing capital implies a reduction in subsidies to homeowners, who are on average higher earners, at the expense of renters, who are on average lower earners. In the current setup without tenure decision, the benefit of lowering the capital income tax rate is smaller because there is no gain associated with nullifying implicit subsidies to homeowners. However, there is still a welfare gain from correcting over-investment of housing capital.

7.2. Role of Mortgage Market

In order to study the importance of the mortgage market, in the third row of Table 4, the down payment ratio is set at 10% ($\lambda = 0.1$) instead of 20%. A reduction in the down payment requirement is one of the major changes that has happened in the U.S. mortgage markets since the 1990s. According to Chambers et al. (2009a), the average down payment ratio declined from 21.6% in 1995 to 16.3% in 2003 for mortgages offered by the Federal Housing Administration (FHA) and 29.8% to 24.1% for other loans. The question concerning the experiment is the effect of the lowered down payment requirement on optimal capital income taxation. It turns out that the results with a lower down payment are close to those of the baseline model economy, suggesting that the down payment ratio does not have a substantial effect on the optimal level of the capital income tax.

In the fourth row, I modify the model without rental market further by shutting down the mortgage market. The optimal capital income tax rates with and without tax benefits for ownership are 16% and 23%, respectively, close to the optimal rates in the first experiment without rental market (15% and 22%, respectively). This implies that the mortgage market is not crucially important in shaping the main results.

7.3. Additional Costs of Housing

In the last two rows, I introduce additional costs of either ownership (5th row) or renting (6th row). Additional costs of ownership can be interpreted as higher moving costs associated with owned housing, although it is not precise because households can adjust housing tenure and size costlessly each period. For the current experiment, I assume that homeowners have to pay an additional 1% cost of home value. Interestingly, the optimal capital income tax rate is significantly higher (17%) compared with the baseline experiment (1%) even when the tax benefit of ownership is preserved. The reason is that the additional cost of ownership works as additional tax for ownership. Therefore, it is not necessary to lower that capital income tax rate close to zero to eliminate the tax wedge between housing and financial assets. Indeed, when the capital income tax rate is lowered from 40% to 17%, homeownership rate declines from 62.8% (calibrated level) to 0.6%. Even with the relatively small decrease in the capital income tax rate, the welfare gains are significantly larger, at 2.9% of flow consumption. This is because incentivising households away from homeownership saves homeowners from this additional cost of ownership. When the tax benefits of homeownership are eliminated, the optimal tax capital income tax rate is close to the baseline result (26%), but the welfare gain is again significantly larger (3.0% instead of 1.6%).

In the last row of Table 4, I assume an additional cost of renting rather than owning. This is interpreted as the costs associated with moral hazard (renters do not maintain the house as well as owners). In this case, the optimal capital income tax rates are similar to the baseline results. However, interestingly, the homeownership rate does not go down as much in this experiment. When the preferential tax treatment of owner-occupied housing is preserved, the homeownership rate declines from 64% to 36% in the baseline experiment (see Table 2) when the capital income tax rate is lowered to 1%, while the homeownership rate declines from 64% to 52% in the model with additional rental costs. In this experiment, inducing too many households to rent is also costly. Interestingly, however, the size of the welfare gains are only slightly lower compared with the baseline experiments.

7.4. Discussion of Other Features

The main intuition of this paper is that capital income tax can be used to nullify the existing preferential tax treatment of owner-occupied housing.[13] There is a non-trivial welfare gain from it because there is no reason to subsidize homeownership in the model. This is a common assumption in the literature or the policy discussion. For example, Gervais (2002) obtains welfare gains from eliminating mortgage interest deduction because there is no inherent reason to subsidize homeownership in the model. Rosen (1985) argues that it is difficult to justify the U.S. housing policy from an efficiency or a redistribution point of view and concludes that "paternalism and political considerations seem to be the source of this policy." Naturally, if paternalism, geographical consideration, or political consideration is incorporated into the model, the optimal taxation could be different from what is obtained here. For an example of paternalism, support for housing could be justified if consumers' preference exhibits hyperbolic discounting, and housing is useful as a commitment device to avoid over-consumption. See Laibson (1997) for this line of argument. For literature review of research incorporating geographical consideration in public finance, see Glaeser (2013). I leave it for future research.

There are other elements that are abstracted from the model for simplicity, but potentially important. Sommer and Sullivan (2018) argue that eliminating mortgage interest deduction could induce house price changes, and create redistribution between homeowners and renters, which is not considered in this paper. Similarly, capital gains from housing, which has been an important driver for U.S. wealth inequality dynamics (Kuhn et al. (forthcoming)), is not considered. Boerma (2019) argues that housing consumption is complementary to leisure, and thus subsidies to housing can be understood as subsidies to leisure, disincentivising work. Also, it is easy to imagine that any policy that weakens preferential tax treatment to homeowners is politically difficult, as homeowners are the majority of U.S. households. Analysis that explicitly considers transition from the current state to the new state can capture such consideration. These considerations are also left for future research.

Table 5: Optimal Capital Taxation: Extended Models

Economy	One-asset model		Two-asset model			
	Opt. τ_K	Welfare[1]	Opt. τ_K^2	Welfare[1]	Opt. τ_K^3	Welfare[1]
1. Baseline	0.31	+0.09	0.01	+1.20	0.24	+1.60
2. High inequality	0.38	+0.00	0.00	+0.90	0.36	+1.81
3. Slow wealth decumulation	0.38	+0.01	0.03	+1.35	0.34	+2.09
4. Government debt	0.33	+0.05	0.04	+1.03	0.26	+1.48

[1] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.
[2] The preferential tax treatment for owner-occupied housing is preserved.
[3] The preferential tax treatment for owner-occupied housing is eliminated.

Table 6: Wealth and Income Inequality

Gini index	Earnings	Income	Total Assets	Housing Assets	Financial Assets
U.S. Data[1]	0.610	0.537	0.806	0.674	1.095
Baseline, one-asset	0.538	0.504	0.623	–	0.623
Baseline, two-asset	0.530	0.506	0.628	0.480	0.806
Extended, one-asset	0.631	0.600	0.792	–	0.792
Extended, two-asset	0.624	0.600	0.793	0.568	1.038

[1] U.S. data are from the Survey of Consumer Finances (SCF), 2004.

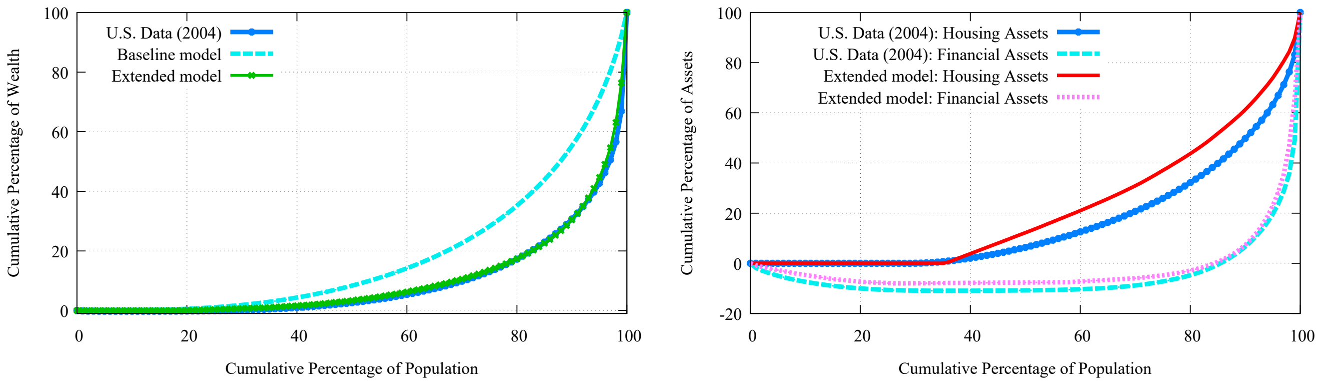
8. Extensions

In the calibration of the baseline model, I stay close to the calibration of Conesa et al. (2009) to facilitate comparison. However, the model does not capture salient properties of the U.S. data in some dimensions. In this section, I address three issues. First, in Section 8.1, I modify the model so that the model replicates the observed extreme concentration of income and wealth. Second, in Section 8.2, I modify the model so that it replicates the slow decumulation of wealth after retirement. Finally, in Section 8.3, I introduce government debt. Table 5 summarizes the results from these experiments.

8.1. Income and Wealth Inequality

There is a substantial inequality in income and wealth in the U.S., and the degree of inequality has been rising in the recent decades. The first row of Table 6 shows Gini indexes of income and wealth among U.S. households in 2004, computed using the Survey of Consumer Finances (SCF). Gini index of earnings, income, and wealth is 0.61, 0.54, and 0.81, respectively. Moreover, as studied by Díaz and Luengo-Prado (2010), inequality of housing (Gini of 0.67) is relatively equally distributed compared with non-housing assets (Gini of 1.10), due partially to the role of housing as

[13] I thank Axelle Ferriere, for giving inspiration for this section.



(a) Total Assets (b) Housing and Financial Assets

Figure 6: Lorenz Curves: U.S. and Model Economies

durable consumption goods.

Compared with the U.S. data, the baseline models (shown in the second and the third rows) exhibit lower inequality in earnings, income, and wealth. Are the main results presented in the previous section sensitive to the degree of inequality? In order to investigate the issue, I extend the model so that the model replicates the high concentration of income and wealth. In particular, following Castañeda et al. (2003), I add an additional "super-productive" state which is extremely high, which is intended to capture the top 1% of income distribution. Once a household becomes super-productive, the household stays with 80% probability, which makes the expected duration of such state 5 years. Any household has the same probability of becoming super-productive, and the probability of becoming super-productive is calibrated such that 1% of households are in the state. The productivity level of super-productive state is calibrated to match approximately the wealth Gini (0.793 in the extended two-asset model compared to 0.806 in the data). The bottom two rows of Table 6 confirm that the two extended models generally replicate the observed inequality of earnings, income, and total assets in the U.S. data. Moreover, the extended two-asset model also replicates the feature of the data that financial (non-housing) assets are significantly more unevenly distributed (Gini of 1.10 in the data and 1.04 in the model) than housing assets (Gini of 0.67 in the data and 0.57 in the model). Figure 6 compares Lorenz curves for total assets (Figure 6(a)) as well as housing and financial assets (Figure 6(b)). Figure 6(a) shows that, compared with the baseline model, the extended two-asset model replicates the Lorenz curve of total assets in the data very well.[14] In Figure 6(b), the extended two-asset model replicates the empirical Lorenz curves for housing and financial assets well, although housing assets are less unequally distributed in the model, possibly because the model does not have the illiquid nature of housing nor regional house price dispersion.

Comparing the first and the second rows of Table 5, the main results obtained in the previous section are robust to extending the model by matching income and wealth inequality. The optimal level of capital income tax rate is very high (38%) in the one-asset model, while it is 0% in the two-asset model when the preferential tax treatment of owner-occupied housing is preserved. If

[14]The Lorenz curve for total assets is similar in the extended one-asset model.

such preferential tax treatment is eliminated, the optimal capital income tax rate in the model with housing becomes 36%, close to the level in the one-asset model, suggesting that closing the tax wedge between housing and financial assets is crucial.

8.2. Slow Wealth Decumulation in Retirement

Another dimension in which the baseline model does not do well in replicating the U.S. data is that retirees decumulate wealth slowly in the data compared with the prediction of the standard life-cycle model.[15] This could be due to a bequest motive, a precautionary saving motive for medical expense risk, or the illiquid nature of housing. For simplicity, I extend the baseline model by introducing a warm-glow bequest motive, and calibrate the parameter controlling the strength of bequest motive such that average wealth at age 85 relative to that of 65 is about 2/3.[16]

Comparing the first and the third rows of Table 5, it is easy to see that extending the model by introducing utility from bequest does not change the main results presented in the previous section. The optimal rate of capital income taxation is 38% in the one-asset model, while it is 3% in the model with housing, when preferential tax treatment of owner-occupied housing is preserved. When it is eliminated, the gain from closing the tax wedge between housing and financial assets by lowering the capital income tax rate disappears, and the optimal capital income tax rate becomes close to that in the one-asset model (34%).

8.3. Government Debt

As studied by Aiyagari and McGrattan (1998), government debt provides liquidity that can be used to smooth consumption in an incomplete-market model. Does the existence of government debt change the way capital income is taxed? I will not study the optimal provision of government debt as in Aiyagari and McGrattan (1998). Instead, I assume that government issues debt whose quantity is 67% of GDP, and investigate the optimal level of capital income taxation.[17] Again, the last row of Table 5 confirms that the main results of the paper are robust to introducing government debt. Even in the model with government debt, the optimal capital income tax rate in the one-asset model is high at 33%, while it is low (4%) in the model with housing, with preferential tax treatment of owner-occupied housing. The optimal tax rate becomes high (26%) when the tax benefits of homeownership are eliminated.

9. Robustness Analysis

Table 7 summarizes the results when the main experiments are implemented under alternative assumptions. The first two columns show the optimal capital income tax rates and the associated welfare gains in the one-asset model under alternative assumptions. The last four columns are associated with the two-asset model. The third and fourth columns show the optimal tax system with the preferential tax treatment for owner-occupied housing being preserved, and the last two columns are for the case in which the housing tax breaks are eliminated.

[15]See De Nardi et al. (2016) for a survey of the literature.

[16]The target is documented by Nakajima and Telyukova (forthcoming). Following De Nardi et al. (2010), utility from bequest is parameterized as $v(x) = \chi_0 \frac{(\chi_1 + x)^{1-\sigma}}{1-\sigma}$. χ_1 is obtained by converting the value obtained by De Nardi et al. (2010) into the unit in my model. See Nakajima and Telyukova (2018) for more details.

[17]67% of GDP is the benchmark quantity of government debt in Aiyagari and McGrattan (1998).

Table 7: Optimal Capital Taxation: Robustness

Economy	One-asset model		Two-asset model			
	Opt. τ_K	Welfare[1]	Opt. τ_K^2	Welfare[1]	Opt. τ_K^3	Welfare[1]
1. Baseline	0.31	+0.09	0.01	+1.20	0.24	+1.60
2. Separable utility	0.20	+0.35	0.00	+1.17	0.15	+1.35
3. Inelastic labor	0.00	+1.90	0.00	+2.60	0.00	+2.67
4. Consumption tax	0.35	+0.02	0.03	+0.98	0.29	+1.65
5. No property tax	0.31	+0.09	0.00	+1.35	0.32	+2.43
6. Proportional tax	0.44	+0.02	0.04	+0.46	0.45	+1.71
7. Lower income ineq.	0.37	+0.02	0.19	+0.36	0.34	+1.37

[1] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.
[2] The preferential tax treatment for owner-occupied housing is preserved.
[3] The preferential tax treatment for owner-occupied housing is eliminated.

9.1. Alternative Assumption on Preference

The first row (labeled as *baseline*) replicates the main results of the baseline model, for comparison. The second row (labeled as *separable utility*) presents the results of the model with the separable utility function instead of the non-separable one. The functional form is shown in Section 3.2. The alternative model is calibrated to match the same set of targets as in the baseline model. Although the optimal tax rates are lower than in the baseline, the main results of the baseline model are valid with the separable utility function; the optimal capital income tax rate in the two-asset model when the preferential tax treatment for owner-occupied housing is preserved is lower (actually it is zero) than in the one-asset model where there is no distinction between housing and non-housing assets (20%). On the other hand, the optimal capital income tax rate is substantially higher (15%) when the tax breaks for owner-occupied housing are eliminated.

The third row (labeled as *inelastic labor*) is an interesting case. It is assumed that labor is inelastically supplied. Under this assumption, labor income tax is non-distortionary, although there are distributional consequences. Therefore, there is no efficiency loss from labor income taxation. Not surprisingly, under the assumption of inelastic labor supply, the optimal capital income tax rate is zero for all cases; it is optimal to raise tax revenues solely from non-distortionary labor income taxation rather than distortionary capital income taxation.

9.2. Alternative Tax Systems

The fourth row (labeled as *consumption tax*) of Table 7 shows the results of the model with a consumption tax. In particular, the consumption tax captures the sales tax that is present in most U.S. states; there is a proportional tax for all non-housing goods but there is no tax for rents. The consumption tax rate is set at 5%, which is the estimate of Mendoza et al. (1994) and is used by Conesa et al. (2009) as well. As you can see in the table, the main result of the baseline model is not significantly affected by introducing the consumption tax. The optimal capital income tax rate in the one-asset model is 35%, which is exactly the same as in Conesa et al. (2009). On the other hand, the optimal capital income tax rate is 3% with the housing tax breaks, and 29% without the tax breaks.

The fifth row (labeled as *no property tax*) is the case in which the property tax, whose baseline value is 1.1%, is eliminated. Since property tax plays no role in the one-asset model, the results are the same for the one-asset model as for the baseline case. For the two-asset model, as in the baseline experiments, it is optimal not to tax capital if the preferential tax treatment for owner-occupied housing is preserved, while it is optimal to tax capital at a high rate when the housing tax breaks are eliminated.

The sixth row (labeled as *proportional tax*) is the case in which proportional labor income tax is used instead of the baseline progressive labor income tax. The main results of the paper still survive; the optimal capital income tax rate with and without tax breaks for housing is 4% and 45%, respectively, while the optimal rate is 44% under the one-asset model.

9.3. Lower Income Inequality

Finally, in the seventh row (labeled as *lower income inequality*), I investigate the link between the degree of income inequality and the socially desirable capital income tax rate. In particular, I halve the standard deviation of the individual productivity shocks and redo the experiments. In the case of the one-asset model, the optimal capital income tax rate is higher (37%) than in the baseline experiment (31%). The optimal capital income tax rates are higher in the experiments with the two-asset model as well. In particular, the optimal rate is now 19% even when the housing tax breaks are preserved. When the dispersion of productivity and income is smaller, the gain from weakening the redistribution effect from owners to renters is small, which reduces the welfare gain from lowering the capital income tax rate in the economy with a lower income dispersion. However, the key main result that the capital income tax rate should be lower when the preferential tax treatment for owner-occupied housing exists is still valid. The effect of lowering income variances is more sizable than the result in Section 8.1 because adding super-productive earners does not affect the households at the margin between owning and renting as significantly as lowering income variances for all households.

10. Conclusion

This paper quantitatively investigates optimal capital income taxation in the general-equilibrium overlapping generations model in which characteristics of housing and the U.S. preferential tax treatment of owner-occupied housing are carefully captured. There are three main findings. First, given the current preferential tax treatment for owner-occupied housing, the optimal capital income tax rate is close to zero. In the baseline experiment, the optimal rate is found to be 1%. It is very different from the optimal rate in the standard model in which housing and non-housing capital are not distinguished (31%). The key intuition is that lowering the capital income tax rate works to narrow the tax wedge between housing and non-housing and thus indirectly nullifies the preferential tax treatment for homeowners at the expense of renters. Second, if the preferential tax treatment for owner-occupied housing is eliminated, it is optimal to tax housing and non-housing at a higher rate. In the baseline model, the optimal rate is 24%. When the tax breaks are eliminated, lowering the capital income tax no longer generates the welfare gain of narrowing the tax wedge of the two kinds of capital. The general message of the experiments is that optimal capital income taxation should be analyzed as a package together with other taxes; housing tax policy affects the answer to how the government should tax capital income. Finally, the welfare gain of implementing the optimal tax system is large, above 1% of flow consumption in both cases. It implies that implementing the optimal capital income tax rate obtained from the model without housing incurs a large welfare loss.

The main findings suggest that regardless of what kind of housing policy is assumed, the optimal capital income tax rate is associated with nullifying the preferential tax treatment for owner-occupied housing. Therefore, if there is some non-economic reason that supports homeownership, it becomes optimal to keep the capital income tax rate high, as in Conesa et al. (2009). However, as discussed in Section 7.4, one should be aware that the reason why capital income should be heavily taxed is different between the models with and without housing. Explicitly incorporating the benefits of the preferential tax treatment for owner-occupied housing in the analysis is left for future work.

Finally, as I mentioned in the introduction, different countries have different policies toward housing taxation. According to the European Central Bank (2003), the U.S., the U.K., Germany, and France do not tax the imputed rents of owner-occupied housing, but Denmark, the Netherlands, and Sweden do tax imputed rents. Why is there such difference across countries? Understanding the cross-country differences of housing taxation is also important and left for future research.

Appendix

A.1. Calibration Details

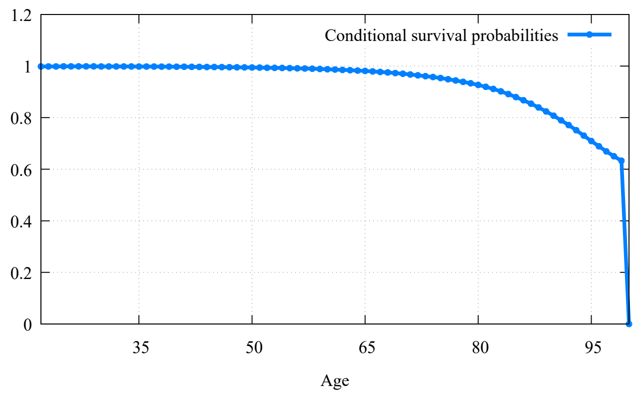


Figure A.1: Conditional Survival Probabilities

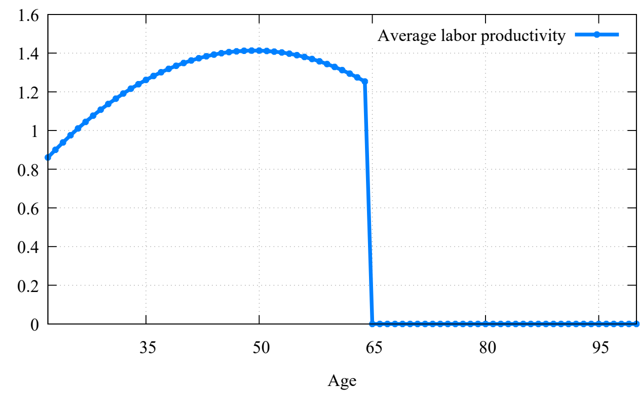


Figure A.2: Average Life-Cycle Profile of Labor Productivity

A.2. Computing Cross-Sectional Variances of Hourly Wage from PSID

I use the Panel Study on Income Dynamics (PSID), waves 1967-1996.[18] Since each wave of PSID covers income and hours worked in the previous year, the data set covers the years 1966-1995. Following Storesletten et al. (2004), I construct each *year* as an overlapping panel of three years. For example, *year* 1968 consists of actual years of 1967, 1968, and 1969. This overlapping panel structure helps maintain a broad cross-section and a stable age distribution, but still enables identification of time series parameters.

Following Storesletten et al. (2004), I use the households with these characteristics: (i) the head is between 22 and 64 years of age, (ii) the household has a positive core weight, (iii) labor income of the head is not top-coded, (iv) hourly wage of the head (computed by dividing the annual labor income of the head by the total hours worked by the head in the same year) is above half of the minimum wage of the respective year, (v) hours worked by the head are between 520 and 5096, and (vi) all the conditions are satisfied in two consecutive years. The nominal hourly wage is deflated using the consumer price index (CPI) for respective years.

I compute the cross-sectional variances of the logarithm of real hourly wage of the heads of households, for each age between 22 and 64. The variances are net of cohort effect; i.e., the variance for each age represents the one for households whose heads are born in the same year. This is accomplished by a cohort and age dummy-variable regression developed by Deaton and Paxson (1994) and also used in Storesletten et al. (2004).

Figure A.3 shows the age effect on the cross-sectional variances of logarithm of the real hourly wage of heads of households. The age effect is normalized by adding the cohort effect of households whose heads are age 42 in 1994 (the last year in the sample). The cross-sectional variance takes the value of 0.197 and 0.674 for age 22 and 64, respectively, and almost linearly increases between age 22 and 64.

[18] I do not use waves after 1996, when PSID is no longer annual, but biennial.

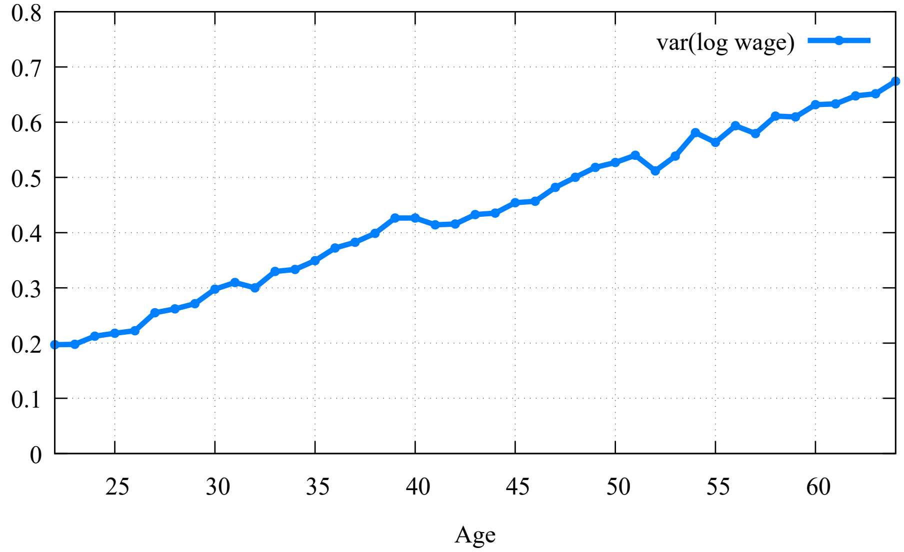


Figure A.3: Cross-Sectional Variances of Log-Hourly Wages

A.3. Computation Details

This appendix gives details about the solution algorithm of the stationary recursive competitive equilibrium. I focus on the baseline model with housing and a progressive labor income tax. The solution algorithm of other model economies is basically the same, with minor modifications:

1. Fix the capital income tax rate τ_K as well as the housing tax system τ_H, τ_M, and τ_P. The baseline values are $\tau_K = 0.40$, $\tau_H = 0$, $\tau_M = 0.23$, and $\tau_P = 0.011$. For the case of the baseline model, a parameter of the progressive labor income tax rate, τ_0, is fixed at the estimated value of $\tau_0 = 0.258$. In other experiments, τ_0 is adjusted together with prices such that the total tax revenue is the same as the one obtained in the baseline model (revenue neutrality).
2. Guess prices r, w, amount of lump-sum transfer t, amount of social security benefit $\bar{b}$, and tax rate τ_0 (it is fixed for the baseline model). The equilibrium price of rental properties is easily computed using $q = r + \tau_P + \delta_H$.
3. Given $(r, w, t, \bar{b}, \tau_0)$, solve the problem of agents. Specifically, follow the steps below and find the optimal decision rules for consumption, $c = g_c(i, e, x)$, housing conditional on owning, $h^o = g_o(i, e, x)$, housing conditional on renting, $h^r = g_r(i, e, x)$, financial asset holdings, $a = g_a(i, e, x)$, total assets for the next period, $x' = g_x(i, e, x)$, and hours worked, $\ell = g_\ell(i, e, x)$.
   (a) Find the optimal decisions in the last period of life I. It is easier because age-I is the last period of life and thus the maximand contains only the current utility for age I.
   (b) Compute the value for age I, using the obtained optimal decisions.
   (c) Given the value function for age I obtained in the last step, go back one step and find the optimal decision rules for age $I-1$. The value function in the next period is interpolated using a spline approximation.
   (d) Keep going back up to age 1.
4. Having obtained the optimal decision rules $c = g_c(i, e, x)$, $h^o = g_o(i, e, x)$, $h^r = g_r(i, e, x)$, $a = g_a(i, e, x)$, $x' = g_x(i, e, x)$, and $\ell = g_\ell(i, e, x)$, run a simulation with N agents (I use $N = 1,000,000$ agents). Specifically, follow the steps below:

(a) For each of N agents, draw the initial e from $\{p_e^0\}$, using a random number generator. Initial x is set at zero. Initial i is 1.

(b) For each of N agents, compute the optimal decisions $(c, h^o, h^r, a, x', \ell)$ using the optimal decision rules. Notice that in order to compute the optimal tenure choice, values conditional on owning and renting must be computed and compared for each agent. Optimal decision rules are interpolated with piecewise linear functions.

(c) Once optimal decisions are obtained, update the individual state variables. e is updated using the first-order Markov process together with another draw from a random number generator. x is updated using the optimal decision rule $x' = g_x(i, e, x)$.

(d) Keep updating the individual state variables up to age I.

5. Using the simulation results, compute the aggregate variables. When aggregating individual variables, normalize the measure of a single newborn as 1. Because of population growth and mortality risk, the measure of a single age-2 agent is $\frac{\pi_1}{1+\gamma}$. Similarly, the measure of a single age-i agent is $\frac{\prod_{j=0}^{i-1} \pi_j}{(1+\gamma)^{i-1}}$ (set $\pi_0 = 1$), and so on.

6. Use the aggregate variables to construct new guesses $(\hat{r}, \hat{w}, \hat{t}, \hat{b}, \hat{\tau}_0)$.

   (a) The new prices, $\hat{r}$ and $\hat{w}$, can be constructed using the profit-maximizing conditions for the firm, and the aggregate capital stock and labor supply that are obtained by aggregating individual agents' decision.

   (b) The new amount of transfer $\hat{t}$ can be constructed by computing the total amount of accidental bequests (total amount of assets, taking into account interests and depreciation, held by the agents that are not surviving), and dividing the total accidental bequests by the number of living agents in the next period.

   (c) The new social security benefit $\hat{b}$ can be constructed by computing the total social security contribution and dividing the total by the number of retirees.

   (d) In case τ_0 is used to guarantee revenue neutrality, the new $\hat{\tau}_0$ can be obtained from the government's balanced budget constraint. $\hat{\tau}_0$ is chosen such that the balanced budget is achieved. In the case of the baseline model, tax rates are all fixed; τ_0 is always set at 0.258 and there is no need for updating τ_0.

7. Compare the old and the new guess for prices. If the distance of the two is smaller than a predetermined tolerance level, an equilibrium was found. Otherwise, update the guess and go back to step 3.

8. Calibrating the model requires repeatedly solving the stationary equilibrium with a different set of parameter values. If, in a stationary equilibrium with a set of parameters, all the calibration targets are satisfied up to a predetermined tolerance level, the calibration is done. Otherwise, change the parameters and solve the stationary equilibrium again.

A.4. Definition of the Welfare Measures

This appendix defines the welfare criteria for comparing two economies $j = 0, 1$. Economy 0 is the baseline economy and economy 1 is the counterfactual one. For example, economy 1 can be a stationary equilibrium with a tax system featuring the optimal capital income tax rate. The optimal combination of consumption of non-housing goods, housing services, and leisure of an age-i agent in economy j, conditional on the initial $e = e_0$ and the history of realization of labor productivity shocks $\tilde{e}$, is denoted by $(c_i^j(e_0, \tilde{e}), d_i^j(e_0, \tilde{e}), m_i^j(e_0, \tilde{e}))$. Moreover, let $\tilde{s}$ denote the history of realizations of mortality shocks. In particular, $\tilde{s}_i = 1$ when the agent is alive in age-i, and $\tilde{s}_i = 0$ when the agent is dead in age-i.

The ex-ante expected welfare of a newborn in economy j in the stationary equilibrium can be represented as follows:

$$\omega^j = \sum_{e_0} p_e^0 \sum_{\tilde{e}} \sum_{\tilde{s}} \ \tilde{p}_{\tilde{e}|e_0} \ \tilde{p}_{\tilde{s}} \ \sum_{i=1}^{I} \mathbb{1}_{\tilde{s}_i=1} \beta^{i-1} u(c_i^j(e_0,\tilde{e}), d_i^j(e_0,\tilde{e}), m_i^j(e_0,\tilde{e})), \tag{A.1}$$

where p_e^0 is the probability with which e_0 is drawn, $\tilde{p}_{\tilde{e}|e_0}$ is the probability of a history $\tilde{e}$ conditional on e_0, $\tilde{p}_{\tilde{s}}$ is the unconditional probability of a history $\tilde{s}$, $\mathbb{1}$ is the indicator function, which takes the value of 1 if the statement attached to it is true, and 0 otherwise. In particular, $\mathbb{1}_{\tilde{s}_i=1}$ means that the agent is alive in age-i.

The welfare gain by moving from the economy 0 to the economy 1, measured by the uniform percentage increase in non-housing consumption goods, ϵ, can be defined implicitly as follows:

$$\omega^1 = \sum_{e_0} p_e^0 \sum_{\tilde{e}} \sum_{\tilde{s}} \ \tilde{p}_{\tilde{e}|e_0} \ \tilde{p}_{\tilde{s}} \ \sum_{i=1}^{I} \mathbb{1}_{\tilde{s}_i=1} \beta^{i-1} u(c_i^0(e_0,\tilde{e})(1+\epsilon), d_i^0(e_0,\tilde{e}), m_i^0(e_0,\tilde{e})). \tag{A.2}$$

Notice that the social welfare in economy 1 (left-hand side) is equated to the social welfare in economy 0 where the consumption of non-housing goods in each age and node is increased by the proportion ϵ (right-hand side).

Now suppose the average consumption of non-housing goods, housing services, and leisure increased by the proportion g^c, g^d and g^m, respectively, by moving from economy 0 to economy 1. The welfare gain measured by a uniform percentage increase in non-housing consumption goods, associated with the average increase in consumption of non-housing goods, housing services and leisure is labeled the *aggregate effect*, ϵ_a. This is defined implicitly as follows:

$$\begin{aligned}
&\sum_{e_0} p_e^0 \sum_{\tilde{e}} \sum_{\tilde{s}} \tilde{p}_{\tilde{e}|e_0} \ \tilde{p}_{\tilde{s}} \ \sum_{i=1}^{I} \mathbb{1}_{\tilde{s}_i=1} \beta^{i-1} u(c_i^0(e_0,\tilde{e})(1+\epsilon_a), d_i^0(e_0,\tilde{e}), m_i^0(e_0,\tilde{e})) \\
&\quad = \sum_{e_0} p_e^0 \sum_{\tilde{e}} \sum_{\tilde{s}} \tilde{p}_{\tilde{e}|e_0} \ \tilde{p}_{\tilde{s}} \ \sum_{i=1}^{I} \mathbb{1}_{\tilde{s}_i=1} \beta^{i-1} u(c_i^0(e_0,\tilde{e})(1+g^c), d_i^0(e_0,\tilde{e})(1+g^d), m_i^0(e_0,\tilde{e})(1+g^m)).
\end{aligned} \tag{A.3}$$

The *redistribution effect*, ϵ_d, which is the welfare gain measured by uniform percentage increase in non-housing consumption goods, associated with changes in distribution of consumption of non-housing goods, housing services and leisure, is defined as the residual, as follows:

$$\epsilon_d = \epsilon - \epsilon_a. \tag{A.4}$$

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