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Banking Panics and Output Dynamics

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Abstract

This paper develops a dynamic general equilibrium model with an essential role for an illiquid banking system to investigate output dynamics in the event of a banking crisis. In particular, it considers the ex-post efficient policy response to a banking crisis as part of the dynamic equilibrium analysis. It is shown that the trajectory of real output following a panic episode crucially depends on the cost of converting long-term assets into liquid funds. For small values of the liquidation cost, the recession associated with a banking panic is protracted as a result of the premature liquidation of a large fraction of productive banking assets to respond to a panic. For intermediate values, the recession is more severe but short-lived. For relatively large values, the contemporaneous decline in real output in the event of a panic is substantial but followed by a vigorous rebound in real activity above the long-run level.

Keywords: Banking panic, deposit contract, suspension of convertibility, time-consistent policies

JEL classification: E32, E42, G21

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1. INTRODUCTION

The relationship between banking crises and macroeconomic activity has gained renewed importance in the academic circles as a result of the recent global financial crisis. Many empirical studies have documented that banking crises are usually associated with significant decline in real activity across all sectors of the economy.[1] In addition, these studies have concluded that recessions associated with banking crises tend to be more severe and persistent, even though they have found considerable disparity in the behavior of real output across different episodes. An important conclusion in some of these studies is that the output dynamics following a banking panic seems to crucially depend on the way in which banking authorities have intervened to mitigate the adverse effects of a panic.[2]

The goal of this paper is to construct a dynamic general equilibrium model with an essential role for an illiquid banking system to investigate output dynamics in the event of a banking crisis. My contribution to the existing literature is to consider the ex-post efficient policy response to a banking crisis as part of the dynamic equilibrium analysis. Ennis and Keister (2009, 2010) have shown that a fragile banking system subject to a self-fulfilling panic can be the outcome of an optimal deposit contract when agents form their expectations based on the knowledge of the ex-post optimal policy response to a panic. In this paper, I consider the optimal deposit contract in a dynamic economy, given the expectation of an ex-post optimal policy intervention, and characterize output dynamics in the presence of a potentially fragile banking system.

The main advantage of adopting this approach is that it makes the theoretical analysis

[1] The classical reference is Friedman and Schwartz (1963). Prominent recent studies include Boyd, Kwak, and Smith (2005); Abiad, Balakrishnan, Brooks, Leigh, and Tytell (2009); Reinhart and Rogoff (2013); Jalil (2015); and Muir (forthcoming).

[2] In this paper, the terms "banking panic" and "banking crisis" are used interchangeably. In addition, I follow the definition provided in Calomiris and Gorton (1991) and refer to a panic or crisis as an event in which numerous depositors suddenly choose to exercise the option of converting their checkable deposits into currency from a significant number of banks in the banking system to such an extent that these banks suspend convertibility.

consistent with the documented panic episodes, given that in virtually all of these episodes government authorities resorted to suspensions of convertibility, deposit freezes, and banking holidays to end a systemic run on the banking system. As we will see, the output trajectory associated with a banking crisis crucially depends on the optimal liquidation strategy adopted as part of the equilibrium deposit contract.

In the analysis that follows, banks form their portfolio by issuing deposit-like claims to finance productive investments. A bank claim works as a transferable payment instrument and, for this reason, circulates as a medium of exchange in the economy. A key element of the analysis is that depositors may want to prematurely withdraw from the banking system before bank claims can circulate as a means of payment in decentralized markets. Thus, the occurrence of a banking panic will result in a contraction of the amount of liquid assets (i.e., bank claims) in the economy, affecting the real return on these assets and the agents' purchasing power in retail transactions. In addition, a banking panic affects the state of the banking portfolio in the post-panic period. As a result, the real return on liquid assets following a panic episode is altered, affecting output in the post-panic period.

In the event of a panic, the social planner, to be interpreted as a banking authority, will intervene to jointly decide the optimal rule for suspending the convertibility of deposits and the fraction of long-term assets that can be prematurely liquidated to respond to a banking panic. The planner's objective is to maximize the ex-post welfare of depositors by implementing an optimal liquidation strategy. As we will see, this optimal policy response to a banking panic will imply a specific pattern for the evolution of real output in a dynamic economy.

I show that the trajectory of real output following a panic episode crucially depends on the cost of converting long-term assets into liquid funds. For small values of the liquidation cost, the recession associated with a banking panic is protracted. Specifically, output remains below its socially efficient level in the post-panic period as a result of the premature liquidation of a large fraction of productive banking assets to respond to a panic episode. For intermediate values, the recession associated with a banking panic is more severe but short-lived, with output returning to its efficient level in the post-panic period. For relatively large values, the contemporaneous decline in real output in the event of a panic is substantial but followed by a vigorous rebound in real activity above the long-run level.

Thus, the theoretical analysis developed in this paper shows that an economy with an illiquid banking system can display different patterns for the evolution of output following a banking crisis as a result of a time-consistent policy response to a panic. Depending on how costly it is to prematurely convert long-term assets into liquid funds, the solution to the optimal liquidation problem can result in a quick recovery from a panic or even a post-recession boom. I believe that considering a time-consistent intervention provides a more realistic representation of the relationship between banking crises and output dynamics, making it consistent with the documented episodes.

The model has two main ingredients: (i) decentralized exchange with search frictions and (ii) dynamic portfolio analysis. The advantage of using a search-theoretic model to study consumer behavior is that it provides a more realistic representation of the effects of a banking panic in the presence of sequential service. Depositors who end up not being served in the event of a panic lose all their wealth and, consequently, cannot spend in decentralized markets, affecting the extensive margin of trade. Depositors who are served end up with less wealth available for spending in decentralized markets, affecting the intensive margin. Thus, there are fewer trade meetings, together with a reduction in the amount produced, amplifying the effects of premature liquidation due to a panic.

The dynamic analysis captures the persistence of the output loss associated with a banking panic by explicitly showing the effects of premature liquidation on the state of the banking portfolio in the post-panic period. As we will see, the evolution of capital as the determinant of the feasible set for the members of the banking system is a crucial mechanism to explain the persistence of the real effects of a banking panic.

The model has two key empirical implications. First, it implies that the value of bank claims declines during the panic-induced recession. Second, it predicts that the expected return on bank claims rises above the long-run level when the liquidation cost is relatively large. These empirical implications of the model seem to be consistent with the findings in Muir (forthcoming), who studies the behavior of expected returns across banking crises in

14 countries over 140 years. This author has shown that expected returns rise abnormally in a financial crisis as a result of the contemporaneous fall in asset prices associated with a systemic run on the banking system.

It is possible to argue that the model is consistent with these empirical findings. Because the banking authority liquidates only a small fraction of the productive capital in the banking system to respond to a panic when the liquidation cost is relatively large, it depresses the value of bank claims during the crisis but raises the expected return on bank claims going forward. As we will see, the expected inflow of new deposits in the post-panic period contributes to an increase in the value of the banking portfolio above its long-run value, so that the expected return on bank claims rises in a panic episode as observed in the data.

Finally, the framework developed in this paper is in line with the Friedman-Schwartz analysis of the real effects of banking panics; see Friedman and Schwartz (1953). These authors emphasize the decline in the money supply associated with a sharp contraction in bank deposits in the event of a banking panic as the main channel depressing real economic activity. Friedman and Schwartz have argued that the severity of the Great Depression was a direct consequence of the collapse of the banking system following several waves of widespread withdrawals from banks. My analysis builds an inside-money model that relies on fractional reserve banking to implement an efficient allocation. The possibility of a self-fulfilling banking panic as a result of an illiquid banking system is an important feature of the analysis. As we will see, the occurrence of a banking panic results in a significant decline in real economic activity that can be persistent. Consequently, my analysis takes the view that disturbances in the banking system induce a recession as the inside-money arrangement is severely disrupted in the event of a panic.

2. RELATED LITERATURE

The framework developed in this paper builds on two apparently distinct strands of the literature on money and banking. The first focuses on the study of panics as an equilibrium outcome under rational expectations. The seminal papers of Bryant (1980) and Diamond and Dybvig (1983) have initiated a vast literature on the real effects of panics. However, the vast majority of papers in this literature does not account for the fact that bank liabilities are widely used as a medium of exchange. The second strand focuses precisely on the role of money and other assets as a medium of exchange, following the influential contribution of Kiyotaki and Wright (1989). Following this tradition, Cavalcanti, Erosa, and Temzelides (1999) have modified the original Kiyotaki-Wright framework to study inside money creation (in the form of bank notes). However, the connection between the ability of banks to supply liquid assets and the possibility of panics has not been established.

More recently, some researchers have taken a monetary approach to banking, explicitly accounting for the fact that bank liabilities serve as a medium of exchange. A prominent paper taking this approach is that of Gu, Mattesini, Monnet, and Wright (2013), who study inside money creation in the form of bank deposits that serve as a means of payment. However, there is nothing in their analysis that resembles a banking panic. In this paper, I build on their basic framework and introduce some other elements based on Champ, Smith, and Williamson (1996) to create a socially useful role for a demand deposit contract. As should be expected, because these elements generate a socially beneficial role for the provision of liquidity insurance by the banking system, in addition to the supply of liquid assets, they also open the door to the possibility of self-fulfilling panics.

Very few papers in the literature have attempted to characterize the dynamic effects of a banking panic. A prominent analysis that identifies the effects of banking panics on capital accumulation and output is that of Ennis and Keister (2003). In a recently published paper, Gertler and Kiyotaki (2015) characterize the real effects of a banking panic in a dynamic framework with an endogenous liquidation price for banking assets. These studies assume that the convertibility of deposits cannot be suspended to prevent a bank run (or mitigate its real effects), so they do not attempt to characterize ex-post optimal policy responses to study the effects of a banking panic on the trajectory of output.

Finally, Martin, Skeie, and von Thadden (2014a, 2014b) and, more recently, Andolfatto, Berentsen, and Martin (2017) construct infinite-horizon models in which financial institutions borrow short-term and invest in long-term assets, making them subject to runs. To ensure tractability, these analyses do not emphasize history dependence in the same way as the present study does.

3. MODEL

Time $t = 0, 1, 2, \ldots$ is discrete, and the horizon is infinite. Each period is divided into three subperiods or stages. There exist two symmetric regions that are identical with respect to all fundamentals. There is no communication between these regions. In each region, there are three types of agents, referred to as buyers, sellers, and bankers, who are infinitely lived. There is a $[0, 1]$-continuum of each type in each region.

Agents in each region interact as follows. In the first stage, the group of buyers and the group of bankers get together in a centralized meeting. In the second stage, each buyer is randomly and bilaterally matched with a seller with probability $\lambda \in \left(\frac{1}{2}, 1\right)$. In the third stage, the group of sellers and the group of bankers get together in a centralized meeting. Thus, each type is able to interact with the other two types at each date, but not simultaneously.

At date 0, a fraction $\varepsilon \in [0, 1]$ of buyers in one region is randomly relocated to the other region and vice versa. I refer to a buyer who is relocated as a mover and to a buyer who is not relocated as a nonmover. A buyer finds out whether he is going to be permanently relocated at the end of the first stage, and the actual relocation occurs shortly after the idiosyncratic shock is realized. This shock is independently and identically distributed across agents. Unless otherwise explicitly stated, the relocation status of a buyer is privately observed until the moment he moves to the other location (when it becomes publicly observable). Note that no relocation occurs in subsequent periods $t \geq 1$.

There are two perfectly divisible commodities, referred to as good x and good y. A buyer is able to produce good x in the first subperiod. The available technology allows the buyer to produce either zero units or one unit. If good x is not properly stored in the subperiod it is produced, it will depreciate completely. Specifically, all buyers have access to an *indivisible* storage technology for good x, which can be costlessly liquidated at any moment. In particular, a buyer can store either one unit or nothing. A seller is able to produce good y in the second subperiod. Good y is perishable and cannot be stored, so it must be consumed in the subperiod it is produced.

A banker is unable to produce either good but has access to a *divisible* technology that uses x as input and that pays off at the beginning of the following date. Let $F(k)$ denote the payoff in terms of x when $k \in \mathbb{R}_+$ is the amount invested. Suppose the payoff function takes the form

$$F(k) = \begin{cases} (1+\rho)\,k \text{ if } 0 \leq k \leq \bar{\imath}, \\ (1+\rho)\,\bar{\imath} \text{ if } \bar{\imath} < k \leq 1, \end{cases}$$

with $\rho > 0$ and $\frac{1-\lambda}{1+\rho} \leq \bar{\imath} \leq 1 - \lambda$. If prematurely liquidated, the technology returns $\delta < 1$. Assume $\delta + \rho > 1$ and $0 < \varepsilon < 1 - \bar{\imath} < \lambda + \rho\bar{\imath}$. In addition, a banker has access to a perfectly *divisible* storage technology for x, which can be costlessly liquidated at any moment. Finally, a banker can also access a technology to costlessly create (and destroy) an indivisible, durable, and portable object, referred to as a bank claim, that perfectly identifies the banker as the issuer. An important characteristic of the environment is that a banker can access the productive technology only at the beginning of the period.

Let me now provide the details of the interaction between buyers and bankers in period 0. As in Wallace (1988, 1990), suppose that, after initially meeting with the group of bankers in the first stage, all buyers remain isolated from each other so that no trade can occur among them. However, each buyer has the ability to contact the group of bankers once, after learning his type (i.e., his relocation status). Specifically, assume that buyers' types are revealed in a fixed order determined by the index i so that buyer i discovers her relocation status before buyer i' if and only if $i < i'$. As we will see, this feature of the environment implies that the banking system pays depositors as they arrive to withdraw and cannot condition current payments to depositors on future information.

Finally, let me describe agents' preferences. A buyer is a consumer of y, whereas a banker and a seller are consumers of x. Let $x_t \in \{0, 1\}$ denote a buyer's production of x at date t, and let $y_t \in \mathbb{R}_+$ denote consumption of y at date t. A buyer's preferences are represented by

$$-\gamma x_t + u\left(y_t\right),$$

where $\gamma \in \mathbb{R}_+$ and $u : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, increasing, and strictly concave, with $u(0) = 0$ and $u'(0) = \infty$. As previously mentioned, the production technology of x allows a buyer to produce either zero units or one unit at each date. But keep in mind that good x is perfectly divisible.

Let $y_t \in \mathbb{R}_+$ denote a seller's production of y at date t, and let $x_t \in \mathbb{R}_+$ denote consumption of x at date t. A seller's preferences are represented by

$$v\left(x_t\right) - w\left(y_t\right),$$

where $v : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, strictly increasing, and concave, with $v(0) = 0$, and $w : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, strictly increasing, and convex, with $w(0) = 0$. Let $y^* \in \mathbb{R}_+$ denote the quantity satisfying $u'(y^*) = w'(y^*)$. Assume $w(y^*) \geq v\left(1 + \frac{\rho \bar{\iota}}{\lambda}\right)$. Let $\beta \in (0,1)$ denote the common discount factor for buyers and sellers. Assume $\beta(1+\rho) > 1$.

A banker derives instantaneous utility x_t in period t if his consumption of x is given by $x_t \in \mathbb{R}_+$. Let $\hat{\beta} \in (0,1)$ denote the banker's discount factor. Assume $\hat{\beta}(1+\rho) \leq 1$.

4. PRELIMINARIES

To see why a banking arrangement is essential in this economy, it is easier to start with the second stage. In this stage, a buyer is randomly matched with a seller with probability λ. A buyer wants y but is unable to produce x for a seller at that time. The pair can trade if the buyer has x in storage. As we have seen, any nonbank agent can convert x into an indivisible unit of storage and vice versa. Is this trading arrangement socially desirable? By adopting this trading strategy, agents hold, at any point in time, an inefficiently large amount of inventories for transaction purposes. These inventories could be either consumed or productively invested.

A superior arrangement can be obtained if a group of bankers is willing to provide a medium of exchange that serves as an alternative to storage. Note that a banker is able to interact with the group of buyers in the first stage and with the group of sellers in the third stage. In the first stage, a buyer can produce one unit and "deposit" it with a banker. In exchange for the buyer's deposit, the banker issues a bank claim certifying the amount originally deposited plus any promised interest payment and entitles the bearer to receive this amount on demand in the third stage. If a seller is willing to accept a privately issued claim in exchange for output, then he is able to redeem this claim in the third stage, so we can think of this stage as the settlement stage.

If a banker is willing to issue a bank claim that promises to pay a higher return than storage, then it is a dominant strategy for a buyer to deposit with a banker. The only problem with this arrangement is that, at date 0, a depositor may need to withdraw funds if he finds out he is a mover. Otherwise, he would have taken into account the inability to withdraw funds on demand when making the deposit decision. Because of a lack of communication across regions, it is impossible to transfer a claim on the banking system in one region to the banking system in the other region. Consequently, a mover needs to hold wealth in the form of storage prior to relocation.

Recall that a banker can access the productive technology only at the beginning of the period (before the realization of the idiosyncratic relocation shock). To be able to offer valuable transaction services to depositors, the members of the banking system need to receive deposits at the beginning of the period to make their portfolio decision. At that time, a depositor does not know whether he is going to be permanently relocated to the other region. Thus, the one-shot relocation shock gives rise to a legitimate demand for withdrawals at date 0, with the withdrawal option providing insurance against the relocation risk. At any subsequent date, the withdrawal option is not socially valuable, so the deposit contract will simply not allow depositors to prematurely withdraw.

A mover who is able to withdraw funds prior to relocation is willing to redeposit these funds in the other region as long as he believes that the banking system there has the ability to pay a higher expected return on deposits than storage. As we shall see, this *expected* flow of resources across regions due to random relocations does not disrupt the investment plans of banks. Although a nonmover does not need to withdraw, we will see that a nonmover is willing to withdraw if he believes that other nonmovers are also withdrawing and the banking portfolio is illiquid, given that depositors are sequentially served when withdrawing from the banking system. In this case, the previously described payment mechanism will be severely disrupted.

5. SYMMETRIC INFORMATION

As a useful benchmark, it is helpful to start the analysis by assuming that a depositor's relocation status at date 0 is publicly observable. The members of the banking system offer a demand deposit contract specifying that, in exchange for one unit of x, a depositor receives an *indivisible* bank claim, which is a transferable instrument that entitles the bearer to receive $\phi_t \in \mathbb{R}_+$ units of x in the settlement stage (third stage). Throughout the paper, I assume that there is perfect monitoring of the activities of bankers and that a deposit contract can be perfectly enforced.

A depositor can potentially withdraw from the banking system after learning his relocation status at date 0. Given the indivisibility of the storage technology available to buyers (recall that they can store either one unit or nothing), we can assume, without loss of generality, that the feasible payments from the banking system to the depositors when withdrawing early lie in the set $\{0, 1\}$. Although the banking system could feasibly offer a payment amount that is strictly less than one, the depositor acts as if the payment amount was zero, given that he cannot store anything less than one unit. As we will see, this assumption regarding the storage technology available to buyers is crucial for the tractability of the distributions of asset holdings across agents.

When there is symmetric information, the members of the banking system are able to perfectly distinguish depositors who have a legitimate motive for exercising the withdrawal option (movers) from depositors who are not going to be relocated and do not need to withdraw (nonmovers). In this case, the banking system can condition the withdrawal option on the depositor's relocation status, so only movers are able to withdraw prior to relocation. As a result, there cannot be a banking panic under this type of contract.

5.1. Distributions

To characterize an equilibrium allocation, it is helpful to start by describing the distributions of asset holdings across different types of agents. These distributions can be summarized as follows. Let $m_t^1 \in [0,1]$ denote the measure of buyers holding one unit of assets (either storage or bank deposits) prior to the formation of bilateral matches, let $m_t^2 \in [0,1]$ denote the measure of sellers holding one unit of assets shortly after bilateral matches are dissolved, and let $m_t^3 \in [0,1]$ denote the volume of redemptions in the settlement stage. In what follows, I will demonstrate that all buyers voluntarily choose to deposit with the banking system and that a depositor is willing to hold at most one unit of bank deposits at any given moment.

If each buyer chooses to hold wealth in the form of bank deposits, then an equilibrium is consistent with the following invariant distributions:

$$m_t^1 = 1 \tag{1}$$

and

$$m_t^2 = m_t^3 = \lambda \tag{2}$$

for all dates $t \geq 0$. These distributions imply that each buyer enters the second stage holding a bank claim and that a measure λ of sellers enters the settlement stage holding a bank claim and chooses to redeem these claims. As we shall see, no buyer will choose to use storage for transaction purposes in equilibrium (a mover stores one unit during relocation but chooses to redeposit it in the banking system upon arrival in the new region).

5.2. Buyers

Given these distributions, I now describe the Bellman equation for a buyer. Let $V_t \in \mathbb{R}$ denote the expected utility of a buyer prior to the formation of bilateral matches at date t. The Bellman equation is given by

$$V_t = \lambda \left[u\left(y_t\right) + \beta\left(-\gamma + V_{t+1}\right)\right] + (1-\lambda)\,\beta V_{t+1}. \tag{3}$$

Here $y_t \in \mathbb{R}_+$ denotes the quantity traded in a bilateral meeting.

With probability λ, a buyer will be matched with a seller and will be able to consume, entering the following period without assets. Then, he will be able to rebalance his portfolio by producing one unit and depositing it in the banking system. With probability $1-\lambda$, a buyer will not find a trading partner, entering the following period with the same asset holdings. If each buyer is willing to trade with a seller and is willing to produce to rebalance his portfolio, then the conjecture $m_t^1 = 1$ for all $t \geq 0$ is consistent with individual behavior.

5.3. Sellers

Let $W_t \in \mathbb{R}$ denote the expected utility of a seller. The Bellman equation for a seller is given by

$$W_t = \lambda \left[-w\left(y_t\right) + v\left(\phi_t\right) + \beta W_{t+1}\right] + \left(1-\lambda\right)\beta W_{t+1}. \tag{4}$$

Recall that a bank claim entitles the bearer to receive ϕ_t units of x in the settlement stage. In the previous equation, I have conjectured that a seller will redeem a bank claim in the settlement stage instead of holding on to it to claim redemption in a subsequent period. As we shall see, this conjecture will be confirmed in equilibrium. If each seller accepts to produce y_t units in exchange for a bank claim, then the conjecture $m_t^2 = \lambda$ for all $t \geq 0$ is consistent with individual behavior.

5.4. Bankers

When a banker issues a bank claim to a buyer, the latter will be able to spend it at the current date with probability λ, so a seller will claim the face value with the same probability. With probability $\left(1-\lambda\right)\lambda$, a seller will claim the face value at the following date. With probability $\left(1-\lambda\right)^2\lambda$, a seller will claim the face value two dates after issuance and so on. Because an individual banker faces idiosyncratic risk when issuing a bank claim (i.e., uncertainty regarding the date at which the claim will be presented for redemption), the members of the banking system have an incentive to engage in a risk-sharing scheme.

An effective arrangement can be constructed as follows. Suppose that all bankers agree that an individual banker who has an opportunity to issue a bank claim is supposed to save a fraction $z_t \in [0,1]$ of the deposit amount. All bankers then decide how to invest all savings subject to the constraint that all claims presented for redemption in the settlement stage must be retired at the promised value ϕ_t. In other words, a banker is supposed to make a contribution z_t every time he has an opportunity to issue a bank claim in exchange for a disbursement ϕ_t on his behalf every time someone wants to retire a claim issued by him.

Let me now describe the investment decisions of the members of the banking system. Let $k_t \in \mathbb{R}_+$ denote per-capita investment in the productive technology, and let $s_t \in \mathbb{R}_+$ denote per-capita investment in storage. At date 0, the resource constraint for the members of the banking system is given by

$$s_0 + k_0 = z_0. \tag{5}$$

In addition, we must have $s_0 \geq \varepsilon$ so that the banking system can meet the expected withdrawal demand of movers. In any subsequent period $t \geq 1$, we have

$$k_t + s_t = F\left(k_{t-1}\right) + \lambda z_t + s_{t-1} - \lambda \phi_{t-1} \tag{6}$$

and

$$\lambda \phi_t \leq s_t. \tag{7}$$

At any date $t \geq 1$, a fraction λ of bankers is able to issue a bank claim, so the per-capita inflow of funds is given by λz_t. The per-capita disbursement due to redemptions is $\lambda \phi_t$. Constraint (7) reflects the fact that the productive technology pays off only at the beginning of the following period, so at least part of the amount invested in storage has to be liquidated to meet expected redemptions in the settlement stage. I have implicitly assumed that bankers do not want to prematurely liquidate the productive technology. As we will see, this is consistent with equilibrium behavior under symmetric information.

Let $J_t \in \mathbb{R}$ denote the expected utility of a banker. At date 0, we have

$$J_0 = 1 - z_0 + \hat{\beta} J_1, \tag{8}$$

given that each banker has an opportunity to issue a bank claim. At any subsequent date $t \geq 1$, we have

$$J_t = \lambda \left(1 - z_t + \hat{\beta} J_{t+1}\right) + (1 - \lambda)\, \hat{\beta} J_{t+1}. \tag{9}$$

A banker is able to consume $1 - z_t$ every time he has an opportunity to issue a bank claim. Because $\hat{\beta}\,(1 + \rho) \leq 1$, a banker is willing to immediately consume any retained earnings. Note that the expected utility of a banker does not depend on the amount of bank claims he has previously issued because of the implementation of a risk-sharing scheme.

5.5. Terms of Trade and Output

Let me now determine the terms of trade in the first and second stages. Start with the second stage. In a bilateral meeting, the terms of trade are determined by Nash bargaining. For simplicity, I assume the buyer makes a take-it-or-leave-it offer to the seller. A buyer is willing to trade provided $u\,(y_t) - \beta\gamma \geq 0$, and a seller is willing to trade provided $-w\,(y_t) + v\,(\phi_t) \geq 0$. Because the seller's participation constraint is binding when the buyer has all the bargaining power, the amount produced is given by

$$y_t = w^{-1}\,(v\,(\phi_t))\,. \tag{10}$$

It remains to verify whether a buyer is willing to produce to acquire a bank claim in stage 1, given the term of trade in stage 2. The buyer's participation constraint is given by

$$U\,(\phi_t) \geq \frac{\gamma\,(1 - \beta + \beta\lambda)}{\lambda}, \tag{11}$$

where the function $U : \mathbb{R}_+ \to \mathbb{R}_+$ is defined by

$$U\,(\phi_t) \equiv u\left(w^{-1}\,(v\,(\phi_t))\right).$$

Note that $U\,(\phi_t)$ is increasing and strictly concave in ϕ_t, with $U\,(0) = 0$. Because a buyer has the ability to store goods, it follows that

$$\phi_t \geq 1, \tag{12}$$

which implies that the rate of return on bank deposits must be positive in equilibrium. In other words, bank deposits must command a higher purchasing power than storage to induce a buyer to become a depositor.

The banker's participation constraint is given by $z_t \leq 1$. Throughout the analysis, I assume that the terms of trade in the deposit market are such that a banker earns zero profits in equilibrium, so we must have

$$z_t = 1 \tag{13}$$

for all $t \geq 0$. In addition, the investment plan implemented by the members of the banking system must maximize the expected utility of depositors.

Finally, we need to specify production of x in stage 1. Total output of x is

$$x_0 = 1 \tag{14}$$

at date 0 and satisfies the law of motion

$$x_t = \lambda + F(k_{t-1}) \tag{15}$$

at any subsequent date $t \geq 1$. As previously mentioned, a fraction λ of buyers enters the period without purchasing power and produces one unit to rebalance their portfolio.

5.6. Equilibrium

Given these descriptions of individual behavior and feasibility conditions, it is now possible to provide a formal definition of equilibrium under symmetric information.

Definition 1 *An equilibrium consists of value functions $\{V_t, W_t, J_t\}_{t=0}^{\infty}$, an investment plan $\{k_t, s_t, z_t\}_{t=0}^{\infty}$, a sequence describing the value of bank deposits $\{\phi_t\}_{t=0}^{\infty}$, a sequence specifying sectorial outputs $\{x_t, y_t\}_{t=0}^{\infty}$, and distributions $\{m_t^1, m_t^2, m_t^3\}_{t=0}^{\infty}$ such that **(i)** the distributions $\{m_t^1, m_t^2, m_t^3\}_{t=0}^{\infty}$ satisfy (1)-(2); **(ii)** the value functions $\{V_t, W_t, J_t\}_{t=0}^{\infty}$ satisfy the Bellman equations (3)-(4) and (8)-(9); **(iii)** the investment plan $\{k_t, s_t, z_t\}_{t=0}^{\infty}$ satisfies*

(5)-(6) and (13) and is consistent with the maximization of the expected utility of depositors; ***(iv)*** *the sequence of values* $\{\phi_t\}_{t=0}^{\infty}$ *satisfies (7) and (11)-(12); and* ***(v)*** *the quantities* $\{x_t, y_t\}_{t=0}^{\infty}$ *satisfy (10) and (14)-(15).*

The first step towards the characterization of an equilibrium allocation is to derive an investment plan consistent with the maximization of the expected utility of depositors. To derive an optimal investment plan, it is useful to make the following assumption.

Assumption 1 Assume $U'\left(\frac{1-\bar{\iota}}{\lambda}\right) < \beta\left(1+\rho\right)U'\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)$.

This condition is likely to hold when the rate of return on the productive technology is sufficiently large, which is consistent with previously made assumptions. The following lemma describes the optimal investment plan. All proofs are provided in the appendix.

Lemma 2 *Consider the following portfolio choice:* $k_0 = \bar{\iota}$ *and* $s_0 = 1 - \bar{\iota}$ *at date* 0*;* $k_t = \bar{\iota}$ *and* $s_t = \lambda + \rho\bar{\iota}$ *at any subsequent date* $t \geq 1$*. In addition, suppose* $z_t = 1$ *for all* $t \geq 0$*. This investment plan is the unique solution consistent with the maximization of the expected utility of depositors.*

An important property of the optimal investment plan refers to the state of the banking system at the time withdrawal requests can be made. Because the per-capita liquidation value of banking assets satisfies

$$s_0 + \delta k_0 = 1 - \left(1-\delta\right)\bar{\iota} < 1,$$

it is impossible to meet the demand for withdrawals if, for some reason, all depositors choose to exercise the withdrawal option. Thus, we can say that the banking system is illiquid and potentially subject to a self-fulfilling panic. When the agent's relocation status is publicly observable, the fact that the optimal investment plan implies an illiquid banking system has no consequence for the equilibrium allocation. Because the members of the banking system can perfectly differentiate movers from nonmovers, it is possible to deny a withdrawal order made by a nonmover to preserve the investment plan, so the fact that the banking system is illiquid has no consequence for the equilibrium allocation.

Note that movers, who temporarily hold storage during relocation, are willing to redeposit their balances upon arrival in the new region, so the previously described investment plan is not disrupted. To formally show existence, I need to make an additional assumption to guarantee that the buyer's participation constraint is satisfied.

Assumption 2 Assume $\lambda\left[U\left(1+\frac{\rho\bar{\iota}+(1-\lambda)}{\lambda}\right)-U\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right]\leq\gamma\leq\frac{\lambda U(1)}{1-\beta+\beta\lambda}$.

This assumption also implies that a depositor is willing to hold at most one unit of bank deposits at any moment. Now I can formally establish existence.

Proposition 3 *There exists an equilibrium with* $\phi_0=\frac{1-\bar{\iota}}{\lambda}$ *and* $\phi_t=1+\frac{\rho\bar{\iota}}{\lambda}$ *for all* $t\geq 1$. *The ensuing equilibrium allocation is Pareto optimal.*

In this equilibrium, a buyer produces one unit in period 0 and consumes $w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right)$ if he has a trading opportunity. A seller who finds a buyer in period 0 produces $w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right)$ and consumes $\frac{1-\bar{\iota}}{\lambda}$. In subsequent periods, a buyer consumes $w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right)$ when he has a trading opportunity and produces one unit when he needs to rebalance his portfolio, and a seller produces $w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right)$ and consumes $1+\frac{\rho\bar{\iota}}{\lambda}$ when he has a trading opportunity.

An important property of the equilibrium allocation is that the banking system is able to accumulate the socially efficient amount of capital, which allows it to provide perfect insurance against the relocation risk and to offer a payment instrument with a higher purchasing power than storage. This socially beneficial role of a banking system has been demonstrated by assuming that a depositor's relocation status is publicly observable. As we will see, this assumption is not innocuous.

6. ASYMMETRIC INFORMATION

Suppose the relocation status of a buyer is privately observable, as initially described. As a result, the members of the banking system cannot distinguish a mover from a nonmover at the time withdrawal requests can be made. In this section, I characterize the equilibrium allocation under a fixed banking contract that does not allow for the suspension of the convertibility of deposits. I believe this is a useful intermediate step to understand the mechanics of a banking panic and its real effects. In addition, it makes my results comparable to those of Gertler and Kiyotaki (2015), who consider a fixed banking contract of the same type. In the following section, I consider the optimal banking contract as part of the equilibrium definition.

In the absence of suspension of convertibility, it is possible to have a banking panic if all nonmovers decide to prematurely withdraw. This means that, at the initial date, the members of the banking system have to make their portfolio decision contemplating the possibility of a banking panic.

I allow agents to coordinate their actions based on the realization of a sunspot variable, as in Cooper and Ross (1998), Peck and Shell (2003), Ennis and Keister (2006), and Allen and Gale (2007). There is a publicly observable random variable $S \in \{n, r\}$ with no effects on fundamentals but potentially with an effect on behavior due to expectations. Suppose $\Pr(S = r) = \pi \in (0, 1)$. The realization of S occurs *shortly after* the relocation status of each buyer is privately revealed at date 0.

As we will see, in equilibrium, all buyers voluntarily choose to hold wealth in the form of deposits. After investment decisions have been made at date 0, a random fraction ε of depositors is going to be permanently relocated and so chooses to exercise the withdrawal option. Nonmovers choose whether to withdraw depending on the realization of the sunspot variable *and* the state of the banking system. Specifically, nonmovers optimally choose to withdraw when the banking system is illiquid and $S = r$ is realized and choose not to withdraw otherwise. Thus, the realization $S = r$ does not trigger a bank run if the banking portfolio is liquid, so the choice of the banking portfolio is crucial for the occurrence of a panic in equilibrium.

Recall that buyers' types are revealed in a fixed order determined by the index i so that depositors contact the banking system sequentially before relocation occurs. As in Ennis and Keister (2010), the payments made from the banking system in period 0 can be summarized by a function $\sigma : [0, 1] \rightarrow \{0, 1\}$, referred to as a *banking policy*. The value $\sigma(j)$ is the payment given to the jth depositor to withdraw in period 0. The arrival point of a depositor j depends not only on her index i but also on the actions of depositors with lower indexes. As previously mentioned, we can restrict attention to payment amounts in the subset $\{0,1\}$ as a result of the indivisibility of the storage technology available to depositors.

In this section, I consider the following banking policy: $\sigma(j)=1$ if $j\in[0,s_0+\delta k_0]$ and $\sigma(j)=0$ otherwise. Because $\delta<1$ and $s_0+k_0=1$, we have $s_0+\delta k_0<1$. In this case, the banking system pays one unit to any depositor withdrawing in period 0 as long as it has funds. In what follows, I define the equilibrium allocation for the whole economy, given this *fixed* banking policy. In the subsequent section, I will consider the optimal banking policy as part of the equilibrium.

6.1. Distributions

As in the previous section, it is helpful to start by describing the distributions of asset holdings across different types of agents at different moments within the period. Let $m_t^1(S)\in[0,1]$ denote the measure of buyers holding one unit of assets prior to the formation of bilateral matches, let $m_t^2(S)\in[0,1]$ denote the measure of sellers holding one unit of assets shortly after bilateral matches are dissolved, and let $m_t^3(S)\in[0,1]$ denote the volume of redemptions in the settlement stage. Note that these distributions depend on the aggregate state S realized at date 0.

If each buyer chooses to hold wealth in the form of bank deposits, then an equilibrium allocation is consistent with the following distributions:

$$m_0^1(S)=\left[1-\hat{I}(S)\right](s_0+\delta k_0)+\hat{I}(S), \tag{16}$$

$$m_0^2(S)=\lambda m_0^1(S), \tag{17}$$

$$m_0^3(S)=m_0^2(S)\,\hat{I}(S) \tag{18}$$

for each $S\in\{n,r\}$, with $\hat{I}(S)$ representing an indicator function defined by

$$\hat{I}(S)=\begin{cases}0 \text{ if } S=r,\\ 1 \text{ otherwise.}\end{cases} \tag{19}$$

The per-capita liquidation value of the assets of the banking system at the time withdrawal requests can be made is given by $s_0 + \delta k_0$. Because the feasible choices of s_0 and k_0 always imply $s_0 + \delta k_0 < 1$, the banking portfolio is illiquid in period 0.

In the absence of a panic, the nonbank public is able to trade using bank deposits as a means of payment, so the volume of redemptions in the settlement stage is given by λ. In the event of a panic, the banking system is liquidated, so the nonbank public temporarily reverts to storage to settle bilateral transactions. In this case, a seller is able to consume one unit shortly after trading with a buyer, so nothing happens in the settlement stage.

Following the initial date, the distributions are given by

$$m_t^1(n) = m_t^1(r) = 1, \tag{20}$$

$$m_t^2(n) = m_t^2(r) = \lambda, \tag{21}$$

$$m_t^3(n) = m_t^3(r) = \lambda \tag{22}$$

for all $t \geq 1$. Because there is no shock after date 0, the distributions of asset holdings are invariant, given that the banking system does not allow depositors to withdraw.

6.2. Bankers

As previously described, the members of the banking system engage in a risk-sharing scheme when issuing bank claims to the public. An investment plan consists of a vector (k_0, s_0, z_0) and a sequence

$$\{k_t(S), s_t(S), z_t(S)\}_{t=1}^{\infty}$$

satisfying the following feasibility conditions. At date 0, we must have

$$k_0 + s_0 = z_0, \tag{23}$$

given that no one is a depositor at the beginning of period 0. At date 1, we must have

$$k_1(S) + s_1(S) = F(k_0)\hat{I}(S) + \left[m_0^3(S) + 1 - \hat{I}(S)\right] z_1(S) \tag{24}$$

for each $S \in \{n, r\}$. Note that the feasible set for the members of the banking system at date 1 depends on whether a panic occurred at date 0. I have implicitly assumed that, in the absence of a panic, no one stores goods across periods, which is shown to be consistent with optimal individual behavior. In any subsequent period $t \geq 2$, we must have

$$k_t(S) + s_t(S) = F(k_{t-1}(S)) + \lambda z_t(S) \tag{25}$$

for each $S \in \{n, r\}$. In addition, the per-capita amount s_0 invested in storage at date 0 must be sufficiently large to meet the expected withdrawal orders of movers: $s_0 \geq \varepsilon$.

The sequence $\{\phi_t(S)\}_{t=0}^{\infty}$ representing the value of liquid assets must satisfy

$$[\lambda \phi_0(S) - s_0] \hat{I}(S) = 0$$

at date 0 and $\lambda \phi_t(S) = s_t(S)$ at any subsequent date. When there is no panic, the value of liquid assets is the same as the face value of bank deposits. When there is a panic, the value of liquid assets is 1 (i.e., the technological rate of return associated with storage). Thus, the state-dependent value of liquid assets at date 0 is given by

$$\phi_0(S) = \begin{cases} 1 \text{ if } S = r, \\ \frac{s_0}{\lambda} \text{ otherwise.} \end{cases} \tag{26}$$

At any subsequent date $t \geq 1$, we have

$$\phi_t(S) = \frac{s_t(S)}{\lambda}. \tag{27}$$

In the following section, I will consider suspension of convertibility as part of an optimal arrangement. As we will see, the value of liquid assets will depend on the optimal point for suspending the convertibility of deposits.

Let $J_0 \in \mathbb{R}$ denote the expected utility of a banker at date 0, and let $J_t(S) \in \mathbb{R}$ denote the expected utility at a subsequent date t. At date 0, the value J_0 satisfies

$$J_0 = 1 - z_0 + \hat{\beta}[\pi J_1(r) + (1 - \pi) J_1(n)]. \tag{28}$$

At date 1, the value function is given by

$$J_1(S) = \left[m_0^3(S) + 1 - \hat{I}(S)\right][1 - z_1(S)] + \hat{\beta} J_2(S). \tag{29}$$

At any subsequent date $t \geq 2$, we have

$$J_t(S) = \lambda [1 - z_t(S)] + \hat{\beta} J_{t+1}(S). \tag{30}$$

If a panic did not occur at date 0, then a banker is able to issue a bank claim with probability λ at date 1. If a panic occurred at date 0, then each banker is able to issue a bank claim because no one is a depositor at the beginning of period 1. So far, I have conjectured that a depositor is willing to deposit in the banking system contemplating the possibility of a banking panic. Thus, it is necessary to verify whether this conjecture is consistent with individual behavior.

6.3. Buyers

At each date, a buyer has an opportunity to produce x and deposit it in the banking system. A depositor will hold a bank claim until he has an opportunity to spend it. In a bilateral meeting, the buyer's surplus is given by $u(y_t(S)) - \beta\gamma \geq 0$ and the seller's surplus is given by $-w(y_t(S)) + v(\phi_t(S)) \geq 0$. Given that the buyer makes a take-it-or-leave-it offer to the seller, we must have

$$y_t(S) = w^{-1}(v(\phi_t(S))) \tag{31}$$

for each $S \in \{n, r\}$. As in the previous section, it is convenient to work with the indirect utility function $U(\phi) \equiv u\left(w^{-1}(v(\phi))\right)$.

Let $V_0 \in \mathbb{R}$ denote the postdeposit expected utility of a buyer at date 0, and let $V_t(S) \in \mathbb{R}$ denote the expected utility at any subsequent date. At date 0, the value V_0 must satisfy

$$\begin{aligned} V_0 &= \pi\{-p\beta\gamma + (1-p)\lambda[U(\phi_0(r)) - \beta\gamma] + \beta V_1(r)\} \\ &\quad + (1-\pi)\{\lambda[U(\phi_0(n)) - \beta\gamma] + \beta V_1(n)\}. \end{aligned} \tag{32}$$

Here $p \in [0,1]$ represents the probability of loss in the event of a panic, which must satisfy

$$p = 1 - s_0 - \delta k_0.$$

Given that the banking portfolio is illiquid, a panic occurs when $S = r$ so that the banking system in each region is liquidated. Because depositors are sequentially served, an individual depositor is able to withdraw one unit with probability $s_0+\delta k_0<1$, given that all depositors are submitting a withdrawal order.

In the event of a panic, only a fraction $s_0+\delta k_0<1$ of buyers enters the second stage holding one unit of x in storage, so the number of trade meetings is given by $\lambda\left(s_0+\delta k_0\right)<\lambda$. Thus, a banking panic affects both the quantity traded in each bilateral meeting (intensive margin) and the total number of trade meetings (extensive margin).

At a subsequent date $t\geq 1$, the values $V_t\left(S\right)$ must satisfy

$$V_t\left(S\right)=\lambda\left[U\left(\phi_t\left(S\right)\right)-\beta\gamma\right]+\beta V_{t+1}\left(S\right), \tag{33}$$

given that a panic will not occur in other periods.

So far, I have implicitly assumed that each buyer is willing to deposit in the banking system, even though a panic can occur with probability π. A buyer is willing to deposit in the banking system if the following participation constraint is satisfied:

$$\pi\left(1-p\right)\lambda U\left(1\right)+\left(1-\pi\right)\lambda U\left(\phi_0\left(n\right)\right)\geq\lambda U\left(1\right)+\pi p\left(1-\lambda\right)\beta\gamma. \tag{34}$$

Note that a bank claim commands a higher purchasing power than storage when a panic does not occur, but a buyer who chooses to store goods is not subject to loss if a panic occurs. Thus, a buyer is willing to hold bank claims provided that the expected rate of return on deposits is sufficiently large to compensate him for the possibility of suffering a loss in the event of a panic.

6.4. Sellers

Let $W_0\in\mathbb{R}$ denote the expected utility of a seller at date 0, and let $W_t\left(S\right)\in\mathbb{R}$ denote the expected utility at a subsequent date t. The value W_0 satisfies

$$W_0=\lambda m_0^1\left(S\right)\left[-w\left(y_0\left(S\right)\right)+v\left(\phi_0\left(S\right)\right)\right]+\beta\left[\pi W_1\left(r\right)+\left(1-\pi\right)W_1\left(n\right)\right]. \tag{35}$$

At any subsequent date $t\geq 1$, the sequence of value functions satisfies

$$W_t\left(S\right)=\lambda\left[-w\left(y_t\left(S\right)\right)+v\left(\phi_t\left(S\right)\right)\right]+\beta W_{t+1}\left(S\right). \tag{36}$$

A seller is willing to produce for a buyer in exchange for a unit of assets provided that the value of the asset is sufficiently large to compensate him for the disutility of production. At date 0, the occurrence of a panic affects the probability with which a seller finds a buyer with purchasing power in the decentralized market.

6.5. Participation Constraints

In addition to the previously described conditions, it must be the case that a buyer is willing to produce to rebalance his portfolio, given the terms of trade in the decentralized market. At date 0, the following participation constraints must hold:

$$\pi(1-p)\lambda U(1) + (1-\pi)\lambda U(\phi_0(n)) \geq (1-\beta+\lambda\beta)\gamma + \pi p(1-\lambda)\beta\gamma \tag{37}$$

Note that condition (34) implies that (37) is necessarily satisfied under Assumption 2.

The banker's participation constraint is $z_0 \leq 1$ at date 0 and $z_t(S) \leq 1$ at a subsequent date t, given $S \in \{n, r\}$. Because the terms of trade in the deposit market are such that the banker earns zero profits, we must have

$$z_0 = 1 \text{ and } z_t(S) = 1 \text{ at any } t \geq 1. \tag{38}$$

In addition, the investment plan implemented by the members of the banking system maximizes the expected utility of depositors.

6.6. Postdeposit Coordination Game

Consider now the postdeposit coordination game in period 0. All depositors play this game after learning their relocation status. Formally, let $\tau_i : \{n, r\} \times \{m, s\} \to \{0, 1\}$ denote depositor i's withdrawing plan, and let τ denote the strategy profile of all agents. In what follows, $\tau_i = 0$ represents withdrawing in period 0, and $\tau_i = 1$ represents not withdrawing. Additionally, m means the depositor is a mover, and s means he is a nonmover. Because depositors are isolated, they do not observe other agents' actions. Although these actions take place sequentially, depositors can be thought of as choosing their strategies simultaneously.

It is clear that a mover always chooses to withdraw from the banking system prior to relocation so that $\tau_i(S, m) = 0$ for any $S \in \{n, r\}$. A nonmover decides whether to withdraw based on his beliefs regarding the actions of other depositors. It is a best response for a nonmover to withdraw if the banking system is illiquid and he believes all other nonmovers are withdrawing. It is a best response for a nonmover not to withdraw if he believes all other nonmovers are not withdrawing. Thus, widespread withdrawals are a pure-strategy Nash equilibrium of the coordination game when the banking system is illiquid. In addition, there exists another pure-strategy Nash equilibrium with the property that movers withdraw and nonmovers do not withdraw.

Given an illiquid banking portfolio, $\tau_i(n, s) = 1$ is the nonmover's best response if she believes that other nonmovers will not withdraw upon seeing the realization $S = n$, and $\tau_i(r, s) = 0$ is the nonmover's best response if she believes that other nonmovers will withdraw upon seeing the realization $S = r$.

6.7. Equilibrium

To provide a complete description of equilibrium, it remains to specify total output of x. Let $x_0 \in \mathbb{R}_+$ denote sectorial output at date 0, and let $x_t(S) \in \mathbb{R}_+$ represent sectorial output in any subsequent period $t \geq 1$. We have

$$x_0 = 1, \tag{39}$$

$$x_1(S) = [\lambda + F(k_0)]\, \hat{I}(S) + [\lambda(1-p) + p]\left[1 - \hat{I}(S)\right], \tag{40}$$

$$x_t(S) = \lambda + F(k_{t-1}(S)) \text{ for any } t \geq 2. \tag{41}$$

If a panic occurred in period 0, there is no productive investment coming to fruition in period 1, so total output in the post-panic period is determined by the measure of depositors who lost their wealth in the process of liquidation of the banking system and by the measure of depositors who were served and had a consumption opportunity.

Given these descriptions of individual behavior and feasibility conditions, it is now possible to provide a formal definition of equilibrium under asymmetric information.

Definition 4 *An equilibrium is a set of values consisting of a vector* (V_0, W_0, J_0) *and a sequence*

$$\{V_t(S), W_t(S), J_t(S)\}_{t=1}^{\infty},$$

an investment plan consisting of a vector (k_0, s_0, z_0) *and a sequence*

$$\{k_t(S), s_t(S), z_t(S)\}_{t=1}^{\infty},$$

a sequence $\{\phi_t(S)\}_{t=0}^{\infty}$ *describing the value of liquid assets, a set of quantities* $(x_0, y_0(n), y_0(r))$ *and* $\{x_t(S), y_t(S)\}_{t=1}^{\infty}$ *specifying sectorial outputs, distributions*

$$\{m_t^1(S), m_t^2(S), m_t^3(S)\}_{t=0}^{\infty},$$

and a strategy profile τ *such that **(i)** the distributions satisfy (16)-(22); **(ii)** the value functions satisfy (28)-(30), (32)-(33), and (35)-(36); **(iii)** the investment plan satisfies (23)-(25) and (38) and is consistent with the maximization of the expected utility of depositors; **(iv)** the values* $\{\phi_t(S)\}_{t=0}^{\infty}$ *satisfy (26)-(27) and (34); **(v)** the sectorial outputs satisfy (31) and (39)-(41); and **(vi)** the strategy profile* τ *is an equilibrium of the postdeposit coordination game.*

In what follows, I describe the equilibrium allocation and demonstrate the conditions for existence. In equilibrium, the distributions of asset holdings are given by

$$m_0^1(n) = 1 > 1 - (1-\delta)\bar{\iota} = m_0^1(r),$$

$$m_0^2(n) = \lambda > \lambda - \lambda(1-\delta)\bar{\iota} = m_0^2(r),$$

$$m_0^3(n) = \lambda > 0 = m_0^3(r).$$

The optimal portfolio choice in period 0 continues to be given by $k_0 = \bar{\iota}$ and $s_0 = 1 - \bar{\iota}$. In period 1, we have $k_1(n) = k_1(r) = \bar{\iota}$ and

$$s_1(n) = \rho\bar{\iota} + \lambda > 1 - \bar{\iota} = s_1(r).$$

At any subsequent date $t \geq 2$, the banking portfolio is $k_t(n) = k_t(r) = \bar{\iota}$ and $s_t(n) = s_t(r) = \lambda + \rho\bar{\iota}$. These choices imply the values

$$\phi_0(n) = \frac{1-\bar{\iota}}{\lambda} > 1 = \phi_0(r), \tag{42}$$

$$\phi_1(n) = 1 + \frac{\rho\bar{\iota}}{\lambda} > \frac{1-\bar{\iota}}{\lambda} = \phi_1(r), \tag{43}$$

$$\phi_t(n) = \phi_t(r) = 1 + \frac{\rho\bar{\iota}}{\lambda} \tag{44}$$

in a subsequent period $t \geq 2$. The following lemma establishes the optimality of the previously described banking portfolio.

Lemma 5 *Consider the following portfolio choice: $k_0 = \bar{\iota}$ and $s_0 = 1 - \bar{\iota}$; $k_1(n) = k_1(r) = \bar{\iota}$, $s_1(n) = \rho\bar{\iota} + \lambda$, and $s_1(r) = 1 - \bar{\iota}$; $k_t(n) = k_t(r) = \bar{\iota}$ and $s_t(n) = s_t(r) = \lambda + \rho\bar{\iota}$ at all dates $t \geq 2$. In addition, suppose $z_0 = 1$ and $z_t(n) = z_t(r) = 1$ at any $t \geq 1$. This investment plan maximizes the expected utility of depositors when agents expect a banking panic to occur at date* 0 *with probability π provided*

$$\pi \leq \hat{\pi} \equiv \frac{\beta(1+\rho)U'\left(1+\frac{\rho\bar{\iota}}{\lambda}\right) - U'\left(\frac{1-\bar{\iota}}{\lambda}\right)}{(1-\delta)\{\beta\gamma + \lambda[U(1) - \beta\gamma]\} + \beta(1+\rho)U'\left(1+\frac{\rho\bar{\iota}}{\lambda}\right) - U'\left(\frac{1-\bar{\iota}}{\lambda}\right)}. \tag{45}$$

Provided that the probability of a panic is sufficiently small, the optimal portfolio choice involves undertaking all productive projects in the economy at any date regardless of the realization of the aggregate state S. Because this portfolio choice results in an illiquid banking system, a banking panic occurs when the sunspot signal r is realized, given that agents believe that nonmovers will prematurely withdraw funds from the banking system under the fixed banking policy.

Given the previously described investment plan, sectorial output is given by

$$x_0 = 1$$

and

$$y_0(n) = w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right) > w^{-1}(v(1)) = y_0(r)$$

in period 0. In the following period, we have

$$x_1(n) = \lambda + (1+\rho)\bar{\iota} > (1-\lambda)(1-\delta)\bar{\iota} + \lambda = x_1(r)$$

and

$$y_1(n) = w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right) > w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right) = y_1(r).$$

In all subsequent periods $t \geq 2$, sectorial output is given by

$$x_t(n) = x_t(r) = \lambda + (1+\rho)\,\bar{\iota}$$

and

$$y_t(n) = y_t(r) = w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right).$$

A formal statement of existence is provided in the following proposition.

Proposition 6 *There exists an equilibrium with the property that a banking panic occurs in period* 0 *if the sunspot signal* r *is realized provided* $\pi \leq \pi^*$ *for some* $\pi^* > 0$*.*

The effects of a banking panic are persistent. Note that production and consumption decline substantially in the event of a panic, remaining below their efficient levels in the aftermath of the panic. The efficient level of decentralized-market trading activity is reached only two dates after the onset of the banking panic. Thus, in the absence of suspension of convertibility, the occurrence of a banking panic results in a protracted recession.

The banking panic disrupts the investment plan of the members of the banking system, who make all capital expenditure decisions in the economy. Because $1-(1-\delta)\,\bar{\iota} < 1$, not all depositors will be served in the event of a panic. Thus, a banking panic substantially reduces the purchasing power of buyers in the decentralized market, affecting both the intensive margin and the extensive margin (as a result of sequential service). In addition, it wipes out all capital goods coming to fruition in period 1 so that the feasible set for the members of the banking system in the post-panic period is the same as in the initial period.

The advantage of using a search-theoretic model to study consumer behavior is that it provides a more realistic representation of the effects of a banking panic in the presence of sequential service. Depositors who end up not being served in the event of a panic lose all their wealth and, consequently, cannot spend in the decentralized market, affecting the extensive margin of trade. Depositors who are served end up with a smaller wealth available for spending in the decentralized market, affecting the intensive margin. Thus, there are fewer trade meetings, together with a reduction in the amount produced, amplifying the effects of premature liquidation due to a panic.

The dynamic analysis captures the persistence of the output loss associated with a banking panic by explicitly showing the effects of premature liquidation on the state of the banking portfolio in the post-panic period. As we will see, the evolution of capital as the determinant of the feasible set for the members of the banking system is a crucial mechanism to explain the persistence of the real effects of a banking panic. To advance in this direction, we need to explore the properties of an equilibrium in which agents expect a banking authority to optimally intervene to respond to a panic episode.

7. OPTIMAL SUSPENSION OF CONVERTIBILITY

In this section, I consider the endogenous choice of the banking policy to maximize the expected utility of depositors, taking the agents' strategy profile in the postdeposit game as given. Following the terminology in Ennis and Keister (2009, 2010), I consider an equilibrium without commitment. These authors have demonstrated that the effectiveness of a suspension-of-convertibility policy in removing any incentive to join a run relies heavily on the assumption that the banking authority responsible for implementing such a suspension can fully commit to its ex-ante policies. In the absence of commitment, they have shown that suspension of convertibility does not always eliminate bank runs. In this section, I follow their approach by focusing on time-consistent suspension policies. I also follow the majority of papers in the literature and refer to the entity solving the banking problem as *the planner*. Later on, I provide some interpretations.

Without loss of generality, we can restrict attention to banking policies of the form:

$$\sigma(j) = 1 \text{ if } j \in [0, \hat{\varepsilon}]$$

and

$$\sigma(j) = 0 \text{ otherwise.}$$

Thus, the value $\hat{\varepsilon} \in [0,1]$ represents the freeze point, that is, the fraction of depositors that the planner chooses to serve before suspending convertibility. In addition, let $\hat{k} \in \mathbb{R}_+$ denote the amount of productive investments the planner chooses to prematurely liquidate in response to a panic. Feasibility requires

$$\hat{\varepsilon} \leq s_0 + \delta k_0$$

and

$$\hat{k} \leq k_0.$$

If the banking system freezes deposits before all resources have been depleted, then the value of a bank claim in period 0 will not necessarily drop to zero. In other words, the fact that $\sigma(j) = 0$ for $j > \hat{\varepsilon}$ does not mean that depositors who have their withdrawal request denied in the event of a panic will necessarily get no consumption in period 0. Note also that the planner can liquidate a fraction of productive capital to serve these depositors in an attempt to smooth their consumption.

7.1. Distributions

As in the previous section, we can summarize the distributions of liquid assets by considering asset holdings across different types at the end of each trading stage. The measure of buyers holding one unit of liquid assets prior to the formation of bilateral matches in period 0 is now given by

$$m_0^1(S) = \left[1 - I\left(S, \hat{\varepsilon}, \hat{k}, s_0\right)\right] \left[\hat{\varepsilon} + (1-\varepsilon)(1-\hat{\varepsilon})\right] + I\left(S, \hat{\varepsilon}, \hat{k}, s_0\right) \tag{46}$$

for each aggregate state $S \in \{n, r\}$. The indicator function $I\left(S, \hat{\varepsilon}, \hat{k}, s_0\right)$ is defined as

$$I\left(S, \hat{\varepsilon}, \hat{k}, s_0\right) = \begin{cases} 0 \text{ if } S = r \text{ and } \frac{s_0 + \delta\hat{k} - \hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)} < 1 \\ 1 \text{ otherwise.} \end{cases}$$

In addition, we have

$$m_0^2(S) = \lambda m_0^1(S) \tag{47}$$

and

$$m_0^3(S) = m_0^2(S)\, I\left(S, \hat{\varepsilon}, \hat{k}, s_0\right). \tag{48}$$

In the event of a panic, all depositors attempt to withdraw from the banking system in each region. A fraction $\hat{\varepsilon}$ of depositors is served before convertibility is suspended. A fraction $\varepsilon(1-\hat{\varepsilon})$ consists of movers who have not been served so that these agents lose their wealth in the event of a panic. As we will see, the value of liquid assets in the event of a panic will differ across asset holders. The distributions in subsequent periods are described by (20)-(22).

7.2. Bankers

The feasible set for the group of bankers continues to be described by (23)-(25). The value of bank deposits in the event of a panic now depends on the planner's liquidation strategy, which involves determining the optimal freeze point, given by $\hat{\varepsilon}$, and the amount of productive capital to be prematurely liquidated, given by $\hat{k}$.

The value of bank deposits in the event of a panic is given by

$$\phi\left(\hat{\varepsilon},\hat{k},s_0\right) \equiv \frac{s_0+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)}. \tag{49}$$

The numerator provides the available resources in the banking system after the implementation of the planner's liquidation strategy. The denominator in (49) reflects the fact that movers who have their withdrawal request denied end up losing their wealth so that $(1-\hat{\varepsilon})(1-\varepsilon)$ provides the measure of remaining depositors after the panic.

As we can see, the planner's banking policy, summarized by the pair $\left(\hat{\varepsilon},\hat{k}\right)$, influences the contemporaneous value of bank deposits in the event of a panic. A higher freeze point $\hat{\varepsilon}$ lowers the numerator in (49) as the value of the liquid portion of the banking portfolio declines, but raises the denominator as the number of remaining depositors shrinks. It is clear that if the planner chooses to prematurely liquidate a larger fraction of productive capital, the contemporaneous value of bank deposits goes up. However, this choice can be costly from a social perspective, given that the planner gives up the full return $1+\rho$ in the following period for the immediate amount $\delta<1$.

In period 1, the value of deposits following a panic episode is given by

$$\phi_+\left(\hat{\varepsilon},\hat{k},s_0\right) \equiv \frac{(1+\rho)\left(1-s_0-\hat{k}\right)+1-(1-\lambda)(1-\varepsilon)(1-\hat{\varepsilon})-k_1(r)}{\lambda}. \tag{50}$$

In (50), the term

$$(1+\rho)\left(1-s_0-\hat{k}\right)$$

describes the proceeds from investment coming to fruition in period 1, given that a panic occurred in the previous period. The term

$$1-(1-\lambda)(1-\varepsilon)(1-\hat{\varepsilon})$$

describes the inflow of new deposits, given that a measure $(1-\lambda)(1-\varepsilon)(1-\hat{\varepsilon})$ of agents enters period 1 as deposit holders.

It is clear from (50) that an increase in $\hat{k}$ significantly reduces the value of bank deposits in the post-panic period, given that $\beta(1+\rho)>1$. A variation in $\hat{\varepsilon}$ also has an unambiguous effect on the value of bank deposits in the post-panic period. In particular, an increase in the freeze point $\hat{\varepsilon}$ reduces the number of frozen accounts in the event of a panic, resulting in a larger inflow of new deposits in the post-panic period.

The banker's Bellman equations are given by

$$J_0 = 1 - z_0 + \hat{\beta}\left[\pi J_1(r) + (1-\pi) J_1(n)\right], \tag{51}$$

$$\begin{aligned} J_1(r) &= \left[1 - I\left(r, \hat{\varepsilon}, \hat{k}, s_0\right)\right]\left[1-(1-\lambda)(1-\varepsilon)(1-\hat{\varepsilon})\right]\left[1 - z_1(r)\right] \\ &\quad + I\left(r, \hat{\varepsilon}, \hat{k}, s_0\right)\lambda\left[1 - z_1(r)\right] + \hat{\beta} J_2(r), \end{aligned} \tag{52}$$

$$J_1(n) = \lambda\left[1 - z_1(n)\right] + \hat{\beta} J_2(n). \tag{53}$$

In period 1, all buyers enter the decentralized market as deposit holders, given that uncertainty has been completely resolved in period 0. The occurrence of a panic in period 0 influences the inflow of new deposits in the banking system in the post-panic period, as shown in equation (52).

7.3. Buyers

The buyer's value function in period 0 is given by

$$\begin{aligned} V_0 &= \pi\left[1-I\left(r,\hat{\varepsilon},\hat{k},s_0\right)\right]\left\{\begin{array}{c}(1-\hat{\varepsilon})\left\{-\varepsilon\gamma+(1-\varepsilon)\left[U\left(\phi\left(\hat{\varepsilon},\hat{k}\right)\right)-\beta\gamma\right]\right\}\\ +\hat{\varepsilon}\lambda\left[U(1)-\beta\gamma\right]+\beta V_1(r)\end{array}\right\} \\ &+\pi I\left(r,\hat{\varepsilon},\hat{k},s_0\right)\left\{\lambda\left[U\left(\phi_0(r)\right)-\beta\gamma\right]+\beta V_1(r)\right\} \\ &+(1-\pi)\left\{\lambda\left[U\left(\phi_0(n)\right)-\beta\gamma\right]+\beta V_1(n)\right\}. \end{aligned} \tag{54}$$

Depending on the planner's liquidation strategy, a banking panic, defined as an event in which nonmovers attempt to withdraw from the banking system, does not occur, in which case $\phi_0(r)=\phi_0(n)$. The state-contingent value of bank deposits in period 0 is given by

$$\phi_0(S)=\begin{cases}\phi\left(\hat{\varepsilon},\hat{k},s_0\right) & \text{if } S=r \text{ and } \phi\left(\hat{\varepsilon},\hat{k},s_0\right)<1\\ \frac{s_0}{\lambda} & \text{otherwise.}\end{cases} \tag{55}$$

As we have seen, a depositor who is not served in the event of a panic completely loses his wealth only if he is a mover. Otherwise, he remains a deposit holder and can trade in the decentralized market, even though the real value of his bank claim is less than one. Table 1 summarizes the value of asset holdings in the event of a panic.

Table 1: Value of Assets In a Panic

Number of depositors	$\hat{\varepsilon}\varepsilon$	$(1-\hat{\varepsilon})\varepsilon$	$(1-\varepsilon)\hat{\varepsilon}$	$(1-\varepsilon)(1-\hat{\varepsilon})$
Value of assets	1	0	1	$\phi\left(\hat{\varepsilon},\hat{k},s_0\right)$

In period 1, we have

$$V_1(S)=\lambda\left[U\left(\phi_1(S)\right)-\beta\gamma\right]+\beta V_2(S), \tag{56}$$

where the value of bank deposits satisfies

$$\phi_1(S)=\begin{cases}\phi_+\left(\hat{\varepsilon},\hat{k},s_0\right) & \text{if } S=r \text{ and } \phi\left(\hat{\varepsilon},\hat{k},s_0\right)<1\\ \frac{s_1}{\lambda} & \text{otherwise.}\end{cases} \tag{57}$$

As we have seen, the value of bank deposits in the post-panic period is given by $\phi_+\left(\hat{\varepsilon}, \hat{k}, s_0\right)$, defined in (50). In other periods and states, the value function is described by (33).

Finally, the buyer is willing to produce and deposit in the banking system provided

$$-\gamma + V_0 \geq -\gamma + \lambda\left[U(1) - \beta\gamma\right] + \beta\left[\pi V_1(r) + (1-\pi)V_1(n)\right]. \tag{58}$$

To be consistent with the previously described distributions of asset holdings, buyers must be willing to deposit in the banking system, even though a banking panic can occur in equilibrium.

7.4. Sellers

As in the previous section, the output in a bilateral meeting as a function of the state-contingent value of bank claims is given by

$$y_t(S) = w^{-1}\left(v\left(\phi_t(S)\right)\right) \tag{59}$$

for each $S \in \{n, r\}$ at all dates $t \geq 0$. In the event of a panic, there are some meetings in which the buyer gives x directly in exchange for y. In these meetings, the seller gets one unit of x in exchange for

$$y^s \equiv w^{-1}\left(v(1)\right)$$

units of y.

The seller's Bellman equation in period 0 is given by

$$\begin{aligned} W_0 &= \pi\left[1 - I\left(r, \hat{\varepsilon}, \hat{k}, s_0\right)\right]\lambda\left\{\begin{array}{c} \hat{\varepsilon}\left[-w\left(y^s\right) + v(1)\right] \\ +(1-\varepsilon)(1-\hat{\varepsilon})\left[-w\left(y_0(r)\right) + v\left(\phi_0(r)\right)\right] \end{array}\right\} \\ &\quad +\pi I\left(r, \hat{\varepsilon}, \hat{k}, s_0\right)\lambda\left[-w\left(y_0(r)\right) + v\left(\phi_0(r)\right)\right] \\ &\quad +(1-\pi)\lambda\left[-w\left(y_0(n)\right) + v\left(\phi_0(n)\right)\right] + \beta\left[\pi W_1(r) + (1-\pi)W_1(n)\right]. \end{aligned} \tag{60}$$

As previously mentioned, the banking panic affects both the intensive and the extensive margin. Later on, I show the aggregate implications for decentralized-market output. In other periods and states, the Bellman equation is given by (36).

7.5. Postdeposit Coordination Game

When choosing his strategy in the coordination game, a depositor takes into account the planner's optimal liquidation strategy. A nonmover chooses $\tau_i(S,s) = 0$ if $S = r$ and $\phi\left(\hat{\varepsilon}, \hat{k}, s_0\right) < 1$ and chooses $\tau_i(S,s) = 1$ otherwise. If the banking portfolio is illiquid in view of the planner's optimal liquidation strategy, then a nonmover believes that other nonmovers will withdraw upon seeing the realization $S = r$.

7.6. Equilibrium

The output of x is given by (39) in period 0 and by (41) in any period $t \geq 2$. In period 1, we have

$$\begin{aligned} x_1(S) &= [\lambda + (1+\rho)(1-s_0)]\, I\left(S, \hat{\varepsilon}, \hat{k}, s_0\right) \\ &+ \left\{ \begin{array}{c} 1-(1-\lambda)\left[(1-\varepsilon)(1-\hat{\varepsilon})+\hat{\varepsilon}\right] \\ +(1+\rho)\left(1-s_0-\hat{k}\right) \end{array} \right\} \left[1 - I\left(S, \hat{\varepsilon}, \hat{k}, s_0\right)\right]. \end{aligned} \tag{61}$$

Before we formally define equilibrium, it is helpful to provide an additional definition. Let $V\left(\hat{\varepsilon}, \hat{k}, \tau\right)$ denote the indirect expected utility of depositors as a function of the banking policy $\left(\hat{\varepsilon}, \hat{k}\right)$ and the strategy profile τ.

Definition 7 *An equilibrium is a set of values consisting of a vector* (V_0, W_0, J_0) *and a sequence*

$$\{V_t(S), W_t(S), J_t(S)\}_{t=1}^{\infty},$$

an investment plan consisting of a vector (k_0, s_0, z_0) *and a sequence*

$$\{k_t(S), s_t(S), z_t(S)\}_{t=1}^{\infty},$$

a sequence $\{\phi_t(S)\}_{t=0}^{\infty}$ *describing the value of liquid assets, a set of quantities* $(x_0, y_0(n), y_0(r))$ *and* $\{x_t(S), y_t(S)\}_{t=1}^{\infty}$ *specifying sectorial outputs, distributions*

$$\left\{m_t^1(S), m_t^2(S), m_t^3(S)\right\}_{t=0}^{\infty},$$

a strategy profile τ*, and a banking policy* $\left(\hat{\varepsilon},\hat{k}\right)$ *such that* ***(i)*** *the distributions satisfy (20)-(22) and (46)-(48);* ***(ii)*** *the value functions satisfy (30), (33), (36), (51)-(54), (56), and (60);* ***(iii)*** *the investment plan satisfies (23)-(25) and (38) and is consistent with the maximization of the expected utility of depositors;* ***(iv)*** *the values* $\{\phi_t(S)\}_{t=0}^{\infty}$ *satisfy (27), (55), and (57);* ***(v)*** *the sectorial outputs satisfy (39), (41), (59), and (61);* ***(vi)*** *the strategy profile* τ *is an equilibrium of the postdeposit coordination game; and* ***(vii)*** *the pair* $\left(\hat{\varepsilon},\hat{k}\right)$ *satisfies*

$$V\left(\hat{\varepsilon},\hat{k},\tau\right)\geq V\left(\hat{\varepsilon}',\hat{k}',\tau\right)$$

for all feasible values $\left(\hat{\varepsilon}',\hat{k}'\right)$*.*

Given this equilibrium definition, the next step is to solve the optimal banking problem so that we can characterize an equilibrium allocation. The planner chooses a pair $\left(\hat{\varepsilon},\hat{k}\right)$ to maximize the expected utility of depositors

$$\begin{aligned}&\hat{\varepsilon}\lambda\left[U(1)-\beta\gamma\right]+(1-\hat{\varepsilon})\left\{-\varepsilon\beta\gamma+(1-\varepsilon)\lambda\left[U\left(\phi\left(\hat{\varepsilon},\hat{k},1-\bar{\iota}\right)\right)-\beta\gamma\right]\right\}\\&+\beta\lambda\left[U\left(\phi_{+}\left(\hat{\varepsilon},\hat{k},1-\bar{\iota}\right)\right)-\beta\gamma\right]\end{aligned}$$

subject to

$$\varepsilon\leq\hat{\varepsilon}\leq1-\bar{\iota}+\delta\hat{k},$$

$$0\leq\hat{k}\leq\bar{\iota},$$

$$U\left(\phi\left(\hat{\varepsilon},\hat{k},1-\bar{\iota}\right)\right)-\beta\gamma\geq\beta\lambda\left[U\left(\phi_{+}\left(\hat{\varepsilon},\hat{k},1-\bar{\iota}\right)\right)-\beta\gamma\right],\qquad(62)$$

$$\phi\left(\hat{\varepsilon},\hat{k},1-\bar{\iota}\right)=\frac{1-\bar{\iota}+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)},$$

and

$$\phi_{+}\left(\hat{\varepsilon},\hat{k},1-\bar{\iota}\right)=\frac{(1+\rho)\left(\bar{\iota}-\hat{k}\right)+1-(1-\lambda)(1-\varepsilon)(1-\hat{\varepsilon})-\bar{\iota}}{\lambda}.$$

The first and second constraints are feasibility conditions. The freeze point cannot exceed the available liquid funds in the banking system. In addition, the fraction of productive investments the planner chooses to prematurely liquidate to respond to a banking panic cannot exceed the amount previously invested in the productive technology. The third restriction is the participation constraint for a depositor who had his deposit account frozen and currently has a trading opportunity in the decentralized market. Condition (62) needs to be satisfied because such a depositor can wait until date 1 to spend his bank claim.

The trade-offs in the previously described optimization problem are as follows. When the planner chooses a larger value for the freeze point, it increases the proportion of movers with purchasing power entering the decentralized market. Selecting a larger value for the freeze point has two opposite effects on the value of deposits in the event of a panic, given by $\phi\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)$. It reduces the available funds in the banking system after liquidation to pay bank claims presented for redemption (the numerator of $\phi\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)$). However, a larger value for the freeze point reduces the number of remaining depositors in the event of a panic (the denominator of $\phi\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)$).

In addition, a larger value for the freeze point increases the inflow of funds into the banking system in period 1, allowing the planner to raise the purchasing power of deposits in the post-panic period. Because any nonmover with a frozen bank account who does not have a trading opportunity in period 0 will remain a deposit holder in the following period, a larger value for the freeze point implies that fewer agents will enter period 1 as deposit holders, resulting in a larger inflow of *new* deposits into the banking system.

The decision to prematurely liquidate productive investments provides the planner with more funds to deal with the banking panic but lowers total output at the following date, given that a smaller amount of capital will come to fruition in period 1 when some premature liquidation occurs in period 0. As a result, the decision to prematurely liquidate productive investments increases the value of deposits in the event of a panic but reduces the purchasing power of deposits in the post-panic period.

If the solution to the optimization problem implies $\frac{1-\bar{\iota}+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)} \geq 1$, then the nonmovers are better off if they do not attempt to withdraw. In this case, a banking panic does not materialize and the ensuing allocation is the Pareto optimal allocation described in Proposition 3. In other words, a time-consistent suspension-of-convertibility policy successfully eliminates panics.

If the solution implies $\frac{1-\bar{\iota}+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)} < 1$, then a nonmover will choose to withdraw if she believes that other nonmovers will do the same. In this case, a banking panic occurs if the signal r is realized. In the event of a panic, a fraction $\hat{\varepsilon}$ of depositors is served so that each one of them holds one unit in storage before entering the decentralized retail market. A fraction $(1-\hat{\varepsilon})\,\varepsilon$ of depositors consists of movers who are not served, who arrive at the new region without purchasing power. They need to wait until the following period to rebalance their portfolio. Finally, a fraction $(1-\hat{\varepsilon})\,(1-\varepsilon)$ of depositors consists of nonmovers who were not served. These agents enter the decentralized market holding a claim worth $\phi\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right) < 1$, so a deposit holder who has a trading opportunity purchases a smaller amount in period 0.

A nonmover who has been served in the event of a panic can potentially choose to redeposit funds in the banking system after relocated agents from the other region arrive. If $\frac{1-\bar{\iota}+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)} < 1$, these nonmovers will optimally choose not to redeposit in the banking system in period 0, given their knowledge of the optimal freeze point. Similarly, the movers with purchasing power choose not to redeposit in the banking system upon arrival in the new region when $\frac{1-\bar{\iota}+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)} < 1$.

The solution to the previously described optimization problem crucially depends on the liquidation cost $1-\delta$. If the liquidation cost is relatively small, then it is likely that the solution involves the premature liquidation of a substantial fraction of productive investments in the event of a panic. As previously described, the benefits of premature liquidation are twofold. First, premature liquidation allows the planner to serve more depositors in the event of a panic in an attempt to maximize the number of movers who are able to withdraw prior to relocation. Second, premature liquidation increases the value of deposits for those with a frozen bank account who currently have a trading opportunity in the decentralized market.

If the liquidation cost is sufficiently large, then it is likely that the planner will optimally choose to prematurely liquidate only a small fraction of productive investments. An interesting property of the optimal banking policy is that, when the solution involves a small amount of premature liquidation, the panic-induced contraction is followed by a vigorous rebound in real activity above the long-run level.

Proposition 8 *If the optimum $\hat{k}$ is sufficiently small, then* $\phi_+\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right) > 1 + \frac{\rho\bar{\iota}}{\lambda}$.

Because the planner optimally chooses to preserve productive capital coming to fruition in period 1 when the liquidation cost is large, it follows that, for a sufficiently large liquidation cost, the occurrence of a banking panic leads to a sharp contemporaneous decline in output that is followed by a vigorous expansion in real activity above the long-run level.

The next step is to numerically solve the banking optimization problem to illustrate some important properties of the equilibrium allocation. Before doing so, it is helpful to define aggregate output in the decentralized market. In the event of a panic, the decentralized-market output is given by

$$\lambda\left[\hat{\varepsilon}y^s + (1-\varepsilon)(1-\hat{\varepsilon})\, w^{-1}\left(v\left(\phi\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)\right)\right)\right].$$

In the post-panic period, the decentralized-market output is given by

$$\lambda w^{-1}\left(v\left(\phi_+\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)\right)\right).$$

If the solution to the banking problem implies that a banking panic is an outcome of the postdeposit game, then we must have

$$\lambda\left[\hat{\varepsilon}y^s + (1-\varepsilon)(1-\hat{\varepsilon})\, w^{-1}\left(v\left(\phi\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)\right)\right)\right] < \lambda w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right)$$

so that the contemporaneous decentralized-market output necessarily declines in the event of a panic. In the post-panic period, the decentralized-market output can be smaller or large than the long-run level, depending on the liquidation cost. As we have seen, the steady-state level of output is $\lambda w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right)$. If the liquidation cost is large, then the result in the previous proposition is likely to hold so that

$$\lambda w^{-1}\left(v\left(\phi_+\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)\right)\right) > \lambda w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right).$$

Thus, the post-panic output rises above its long-run level when the planner chooses to liquidate a small fraction of productive capital.

In what follows, consider the functions $u(y) = (1-\sigma)^{-1} y^{1-\sigma}$ and $v(x) = (1-\eta)^{-1} x^{1-\eta}$, with $0 < \sigma < 1$ and $0 < \eta < 1$. For simplicity, assume $w(y) = y$. In addition, suppose

$\beta = .96$, $\hat{\beta} = .8$, $\bar{\iota} = .3$, $\varepsilon = .25$, $\sigma = .5$, $\gamma = .5$, $\eta = .5$, and $\rho = .5$. Table 2 provides the solution to the optimal banking problem for different values of the liquidation cost $1 - \delta$. As should be expected, the optimal freeze point and the amount of capital prematurely liquidated in the event of a panic are both decreasing in the liquidation cost.

Table 2: *Ex-Post* Optimal Intervention

$1-\delta$	welfare	$\hat{k}$	$\hat{\varepsilon}$	$\phi\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)$	$\phi_{+}\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)$
.02	203.78	.29	.97	.70	1.04
.04	203.15	.24	.88	.71	1.11
.06	202.66	.18	.80	.72	1.17
.08	202.30	.13	.70	.73	1.25

Because it is relatively costly to obtain additional funds by liquidating productive investments when $1-\delta$ is large, the planner will allow a large intertemporal variation in the value of deposits in the event of a panic. Interestingly, as the liquidation cost rises, the value of frozen deposits remains roughly constant. The large intertemporal disparity in the value of deposits comes from the sharp increase in the value of deposits in the post-panic period. Note that there is a maximum intertemporal dispersion consistent with a solution to the optimization problem, given that the participation constraint (62) must be satisfied.

Let me now investigate the behavior of aggregate output. In particular, I focus on decentralized-market output. Figure 1 plots the deviation of output from the socially efficient level. In Figure 1, I show the evolution of output when the liquidation cost is relatively small ($1 - \delta = 0.02$). In this case, the decline in output associated with a banking panic is followed by a recovery period characterized by a suboptimal level of real activity, so we can say that the recession associated with a systemic run on the banking system is protracted. The optimal liquidation strategy involves considerable premature liquidation of productive investments to mitigate the adverse effects of a banking panic. Note that the planner allows almost all depositors to withdraw in the event of a panic. To mitigate the effects of the panic on the small fraction of remaining depositors, the planner liquidates a substantial fraction of productive capital in the banking system to preserve the value of frozen deposits. The premature liquidation of capital implies that the value of deposits in the post-panic period remains below its socially efficient level, resulting in a protracted recession.

[Figure 1]

In Figure 2, the liquidation cost is set at 4 percent. It is clear that the contemporaneous decline in output is larger than that depicted in Figure 1. However, it is reasonable to say that the recovery from a panic episode occurs in the subsequent period. Although the level of output remains slightly below the socially efficient level in the post-panic period, from a practical perspective, we can confidently say that real activity quickly recovers from a panic-induced recession in this case. Note that, in the event of a panic, the planner freezes deposits significantly earlier than in the previous case. As should be expected, the planner liquidates a smaller fraction of productive investments to respond to a panic episode. Note that the proportional decline in the amount of liquidation is larger than that of the freeze point. As a result, the value of deposits in the post-panic period approaches (from below) its steady-state level so that output quickly recovers from the panic-induced recession. Thus, we can conclude that, for intermediate values of the liquidation cost, the recession associated with a banking panic is short-lived.

[Figure 2]

Figures 3 and 4 plot the deviation in output for larger values of the liquidation cost. As we can see, the occurrence of a panic causes a severe contemporaneous decline in output. In both cases, the decline in real activity is followed by a surge in output, given the planner's decision to preserve a larger fraction of productive capital. Note that the freeze point is significantly smaller, given the planner's attempt to maintain the value of frozen deposits roughly constant as the liquidation cost rises. We can see that both $\hat{\varepsilon}$ and $\hat{k}$ decline as $1-\delta$ rises. Because $\partial\phi/\partial\hat{k} > 0$ and $\partial\phi/\partial\hat{\varepsilon} < 0$, it is necessary to lower the freeze point to avoid liquidating a large fraction of productive capital. Consequently, the decision to preserve productive capital in the banking portfolio yields a vigorous rebound in real activity in the aftermath of the banking crisis. Note that the positive contribution to the value of deposits in the post-panic period stemming from a smaller amount of premature liquidation dominates the negative contribution associated with an earlier freeze point, given that $\partial\phi_+/\partial\hat{k} = \lambda^{-1}(1+\rho) > \lambda^{-1}(1-\lambda)(1-\varepsilon) = \partial\phi_+/\partial\hat{\varepsilon}$.

[Figure 3]

[Figure 4]

The previous numerical example illustrates two key empirical implications. First, the model implies that the value of bank claims declines during the panic-induced recession. Second, the model predicts that the expected return on bank claims rises above the long-run level when the liquidation cost is relatively large. As previously mentioned, these empirical implications of the model seem to be consistent with the findings in Muir (forthcoming), who has shown that expected returns rise abnormally in a financial crisis as a result of the contemporaneous fall in asset prices associated with a systemic run on the banking system.

It is possible to argue that the model is consistent with these empirical findings. As we have seen, the banking authority liquidates only a small fraction of the productive capital in the banking system to respond to a panic when the liquidation cost is relatively large. This response depresses the value of bank claims during the crisis but raises the expected return on bank claims going forward, given that the value of the banking portfolio does not fall by a large amount in the post-panic period. In addition, the expected inflow of new deposits in the post-panic period contributes to an increase in the value of the banking portfolio. As a result, the dispersion between the contemporaneous value of bank claims, given by $\phi\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)$, and its post-panic value, given by $\phi_+\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)$, grows as the liquidation cost rises (see Table 2). Because $\phi_+\left(\hat{\varepsilon}, \hat{k}, 1-\bar{\iota}\right)$ goes up as the liquidation cost rises, the expected return on bank claims can rise abnormally (i.e., above its efficient long-run level) in a panic episode.

It remains to provide an interpretation for the solution to the planner's problem in the previous analysis. The most common interpretation in the banking literature is to view the planner as a benevolent banking authority with legal power to suspend the convertibility of deposits and liquidate banking assets in the event of a banking crisis, such as the Federal Deposit Insurance Corporation (FDIC) in the United States. Gorton and Metrick (2012) have interpreted the 2007-08 financial crisis as a bank run on the so-called shadow banking system. Indeed, some of the policy responses adopted in response to the crisis resembled those described in this paper. In this broader interpretation of the financial system, the banking authority that I refer to in my analysis would have to include other government agencies to reflect the broader scope of the shadow banking system.

In reality, the liquidation cost is endogenously determined and can be influenced by government intervention, such as the purchase of privately issued assets by a central bank. In my analysis, I have treated the liquidation cost as fixed and invariant to the type of intervention implemented by the banking authority. It is an interesting extension of the framework to allow for an endogenous liquidation value to study the effects of asset purchases by a central bank.

8. CONCLUSIONS

This paper has developed a macroeconomic model of bank liquidity provision to study output dynamics following a banking crisis. I have argued that one of the advantages of using a search-theoretic model to study consumer behavior is that it provides a more realistic representation of the effects of a banking panic in the presence of sequential service. As we have seen, depositors who end up not being served in the event of a panic lose all their wealth and, consequently, cannot spend in decentralized markets, affecting the extensive margin of trade. Depositors who are served have their wealth substantially reduced, affecting the intensive margin. Thus, there are fewer trade meetings, together with a reduction in the amount produced, amplifying the effects of premature liquidation in the event of a panic.

The dynamic analysis captures the persistence of the output loss associated with a banking panic by explicitly tracing the effects of premature liquidation on the state of the banking portfolio following a panic episode. It has been shown that the evolution of capital as the determinant of the feasible set for the members of the banking system is a crucial mechanism to explain the persistence of the real effects of a banking panic.

As we have seen, the equilibrium trajectory of real output in the event of a panic can follow distinct patterns, depending on the liquidation cost that a banking authority faces when jointly deciding the optimal rule for suspending the convertibility of deposits and the fraction of long-term assets that can be prematurely liquidated to respond to a banking crisis. As we have seen, a protracted recession is associated with a banking crisis when the liquidation cost is relatively low. For intermediate values of the liquidation cost, the contemporaneous contraction in output is more severe but the recession associated with a banking panic is short-lived, given that the economy fully recovers in the post-panic period. When the liquidation cost is sufficiently large, the contemporaneous decline in real output in the event of a panic is substantial but followed by a vigorous rebound in real activity above the long-run efficient level.

REFERENCES

[1] A. Abiad, R. Balakrishnan, P. Brooks, D. Leigh, I. Tytell. "What's the damage? Medium-term output dynamics after banking crises," IMF working paper 245 (2009).

[2] F. Allen, D. Gale. *Understanding Financial Crises*, Oxford University Press, Oxford, New York, 2007.

[3] D. Andolfatto, A. Berentsen, F. Martin. "Money, banking, and financial markets," unpublished manuscript, 2017.

[4] J. Boyd, S. Kwak, B, Smith. "The real output losses associated with modern banking crises," *Journal of Money, Credit and Banking* 37 (2005), pp. 977-999.

[5] J. Bryant. "A model of reserves, bank runs, and deposit insurance," *Journal of Banking and Finance* 4 (1980), pp. 335-44.

[6] C. Calomiris, G. Gorton. “The origins of banking panics: Models, facts, and bank regulation.” In Hubbard, R. Glenn, ed., *Financial Markets and Financial Crises*. Chicago: Chicago University Press, 1991, pp. 109-173.

[7] R. Cavalcanti, A. Erosa, T. Temzelides. “Private money and reserve management in a random-matching model,” *Journal of Political Economy* 107 (1999), pp. 929-945.

[8] B. Champ, B. Smith, S. Williamson. “Currency elasticity and banking panics: Theory and evidence,” *Canadian Journal of Economics* 29 (1996), pp. 828-64.

[9] R. Cooper, T. Ross. “Bank runs: Liquidity costs and investment distortions,” *Journal of Monetary Economics* 41 (1998), pp. 27-38.

[10] D. Diamond, P. Dybvig. “Bank runs, deposit insurance, and liquidity,” *Journal of Political Economy* 91 (1983), pp. 401-19.

[11] H. Ennis, T. Keister. “Economic growth, liquidity, and bank runs,” *Journal of Economic Theory* 109 (2003), pp. 220-45

[12] H. Ennis, T. Keister. “Bank runs and investment decisions revisited,” *Journal of Monetary Economics* 53 (2006), pp. 217-32.

[13] H. Ennis, T. Keister. “Bank runs and institutions: The perils of interventions,” *American Economic Review* 99 (2009), pp. 1588-1607.

[14] H. Ennis, T. Keister. “Banking panics and policy responses,” *Journal of Monetary Economics* 57 (2010), pp. 404-419.

[15] M. Friedman, A. Schwartz. *A Monetary History of the United States, 1867-1960*, Princeton University Press, Princeton, 1963.

[16] M. Gertler, N. Kiyotaki. “Banking, liquidity, and bank runs in an infinite horizon economy,” *American Economic Review* 105 (2015), pp. 2011-43

[17] G. Gorton, A. Metrick. “Securitized banking and the run on repo,” *Journal of Financial Economics*, 104 (2012), pp. 425-451.

[18] C. Gu, F. Mattesini, C. Monnet, R. Wright. "Banking: A new monetarist approach," *Review of Economic Studies* 80 (2013), pp. 636-60.

[19] A. Jalil. "A new history of banking panics in the United States, 1825-1929: Construction and applications," *American Economic Journal: Macroeconomics* 7 (2015), pp. 295-330.

[20] N. Kiyotaki, R. Wright. "On money as a medium of exchange," *Journal of Political Economy* 97 (1989), pp. 927-54.

[21] A. Martin, D. Skeie, E. von Thadden. "The fragility of short-term secured funding markets," *Journal of Economic Theory* 149 (2014a), pp. 15-42.

[22] A. Martin, D. Skeie, E. von Thadden. "Repo runs," *Review of Financial Studies* 27 (2014b), pp. 957-89.

[23] T. Muir. "Financial Crises and Risk Premia," *Quarterly Journal of Economics*, forthcoming.

[24] J. Peck, K. Shell. "Equilibrium bank runs," *Journal of Political Economy* 111 (2003), pp. 103-23.

[25] C. Reinhart, K. Rogoff. "Banking crises: An equal opportunity menace," *Journal of Banking and Finance* 37 (2013), pp. 4557-73.

[26] N. Wallace. "Another attempt to explain an illiquid banking system: The Diamond and Dybvig model with sequential service taken seriously," *Federal Reserve Bank of Minneapolis Quarterly Review* 12 (1988), pp. 3-16.

[27] N. Wallace. "A banking model in which partial suspension is best," *Federal Reserve Bank of Minneapolis Quarterly Review* 14 (1990), pp. 11-23.

APPENDIX

A.1. Proof of Lemma 2

To establish the optimality of the proposed portfolio when agents do not expect the occurrence of a panic, consider the following variational argument. At date 0, the marginal change in the expected utility of a depositor is given by

$$-U'\left(\frac{1-k_0}{\lambda}\right)+\beta\left(1+\rho\right)U'\left(1+\frac{\left(1+\rho\right)k_0-\bar{\iota}}{\lambda}\right).$$

Because

$$U'\left(\frac{1-\bar{\iota}}{\lambda}\right)<\beta\left(1+\rho\right)U'\left(1+\frac{\rho\bar{\iota}}{\lambda}\right),$$

it follows that

$$-U'\left(\frac{1-k_0}{\lambda}\right)+\beta\left(1+\rho\right)U'\left(1+\frac{\left(1+\rho\right)k_0-\bar{\iota}}{\lambda}\right)>0$$

for any $k_0<\bar{\iota}$. Because the productive technology pays off nothing for anything invested above the threshold $\bar{\iota}$, we must have $k_0=\bar{\iota}$ at the optimum.

In any subsequent date $t\geq 1$, the marginal change in the expected utility of a depositor is given by

$$-U'\left(1+\frac{\left(1+\rho\right)\bar{\iota}-k_0}{\lambda}\right)+\beta\left(1+\rho\right)U'\left(1+\frac{\left(1+\rho\right)k_0-\bar{\iota}}{\lambda}\right)$$

for any $k_0<\bar{\iota}$. Note that

$$U'\left(1+\frac{\left(1+\rho\right)k_0-\bar{\iota}}{\lambda}\right)>U'\left(1+\frac{\left(1+\rho\right)\bar{\iota}-k_0}{\lambda}\right)$$

for any $k_0<\bar{\iota}$. Because $\beta\left(1+\rho\right)>1$, it follows that

$$-U'\left(1+\frac{\left(1+\rho\right)\bar{\iota}-k_0}{\lambda}\right)+\beta\left(1+\rho\right)U'\left(1+\frac{\left(1+\rho\right)k_0-\bar{\iota}}{\lambda}\right)>0$$

for any $k_0<\bar{\iota}$. Given that the productive technology pays off nothing for anything invested above $\bar{\iota}$, we must have $k_t=\bar{\iota}$ at the optimum for any $t\geq 1$. **Q.E.D.**

A.2. Proof of Proposition 3

Because the members of the banking system maximize the expected utility of depositors, it must be the case that condition (7) holds as equality in equilibrium. Given the investment plan described in Lemma 2, it follows that $\phi_0 = \frac{1-\bar{\iota}}{\lambda}$ and $\phi_t = 1 + \frac{\rho\bar{\iota}}{\lambda}$ for all $t \geq 1$.

To demonstrate that the equilibrium allocation is Pareto optimal, note that it is impossible to make a banker better off without making a depositor worse off. It remains to verify whether it is possible to achieve a higher level of expected utility for a depositor without making other agents worse off. There is one relevant feasible deviation that I need to check to conclude that the allocation is indeed Pareto optimal. Suppose that a buyer who enters the period as a deposit holder decides to produce one unit of x and transfer it to a banker with the expectation that the banker can raise the purchasing power of *existing* deposits (i.e., no additional deposit claim is issued). Note that it is infeasible to increase the level of investment in the productive technology, given that the economywide productive capacity is fully utilized. Thus, these additional resources are necessarily invested in storage. In this case, it is feasible to implement the value

$$1 + \frac{\rho\bar{\iota}}{\lambda} + \frac{1-\lambda}{\lambda} = 1 + \frac{\rho\bar{\iota} + (1-\lambda)}{\lambda}.$$

Note that each banker remains indifferent and that the original investment plan is not altered in other periods. Now I need to verify whether a buyer who enters the period as a deposit holder is willing to produce in order to increase the purchasing power of deposits in this way. A depositor is willing to produce provided that

$$-\gamma + \lambda U\left(1 + \frac{\rho\bar{\iota} + (1-\lambda)}{\lambda}\right) > \lambda U\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right).$$

Rearranging this expression, we obtain the following condition:

$$\gamma < \lambda\left[U\left(1 + \frac{\rho\bar{\iota} + (1-\lambda)}{\lambda}\right) - U\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right)\right].$$

If $\gamma \geq \lambda\left[U\left(1 + \frac{\rho\bar{\iota}+(1-\lambda)}{\lambda}\right) - U\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right)\right]$, then a deposit holder is better off if he does not produce an extra unit of the good to raise the purchasing power of existing deposits. As a result, there is no feasible deviation that can increase the expected utility of a depositor without making other agents worse off, which means that the aforementioned equilibrium allocation is Pareto optimal. **Q.E.D.**

A.3. Proof of Lemma 5

To show the optimality of the proposed portfolio choice when agents expect the occurrence of a banking panic with probability π, consider the following variational argument at date 0. The marginal change in the expected utility of a depositor is

$$-\pi(1-\delta)\beta\gamma - \pi(1-\delta)\lambda\left[U(1)-\beta\gamma\right] +$$

$$+(1-\pi)\left[-U'\left(\frac{1-k_0}{\lambda}\right)+\beta(1+\rho)U'\left(1+\frac{(1+\rho)k_0-\bar{\iota}}{\lambda}\right)\right]$$

for any $k_0 \in (0, \bar{\iota})$. If the probability π associated with the realization r satisfies (45), then the previously described marginal change is strictly positive, indicating a corner solution (i.e., $k_0 = \bar{\iota}$) to the decision problem when agents contemplate the possibility of a banking panic. **Q.E.D.**

A.4. Proof of Proposition 6

Given the investment plan described in Lemma 5, the equilibrium value of liquid assets is described by (42)-(44). It remains to verify whether a buyer is willing to deposit in the banking system knowing that a banking panic occurs with probability π at date 0. The buyer is willing to deposit provided

$$(1-\pi)\lambda\left[U\left(\frac{1-\bar{\iota}}{\lambda}\right)-U(1)\right] \geq \pi(1-\delta)\bar{\iota}\left\{\beta\gamma+\lambda\left[U(1)-\beta\gamma\right]\right\}.$$

This conditions holds if and only if

$$\pi \leq \bar{\pi} \equiv \frac{\lambda\left[U\left(\frac{1-\bar{\iota}}{\lambda}\right)-U(1)\right]}{(1-\delta)\bar{\iota}\left\{\beta\gamma+\lambda\left[U(1)-\beta\gamma\right]\right\}+\lambda\left[U\left(\frac{1-\bar{\iota}}{\lambda}\right)-U(1)\right]}.$$

Given that the proposed investment plan maximizes the expected utility of depositors only if the probability π associated with the realization r satisfies (45), existence requires

$$\pi \leq \pi^* = \min\left\{\bar{\pi}, \hat{\pi}\right\}.$$

In this case, we obtain an equilibrium with the property that a banking panic occurs if the sunspot signal r is realized. **Q.E.D.**

A.5. Proof of Proposition 7

Note that

$$\phi_+\left(\hat{\varepsilon}, 0, 1-\bar{\iota}\right) = \frac{1-(1-\lambda)(1-\varepsilon)(1-\hat{\varepsilon})}{\lambda} + \frac{\rho\bar{\iota}}{\lambda}.$$

Because $1-(1-\lambda)(1-\varepsilon)(1-\hat{\varepsilon}) > \lambda$, it follows that

$$\phi_+\left(\hat{\varepsilon}, 0, 1-\bar{\iota}\right) > 1 + \frac{\rho\bar{\iota}}{\lambda}.$$

Then, there is $\bar{k} > 0$ such that

$$\phi_+\left(\hat{\varepsilon}, \bar{k}, 1-\bar{\iota}\right) = 0$$

and

$$\phi_+\left(\hat{\varepsilon}, k, 1-\bar{\iota}\right) > 1 + \frac{\rho\bar{\iota}}{\lambda}$$

for all $k \in \left(0, \bar{k}\right)$. **Q.E.D.**

Figure 1. Output Deviation ($1 - \delta = .02$)

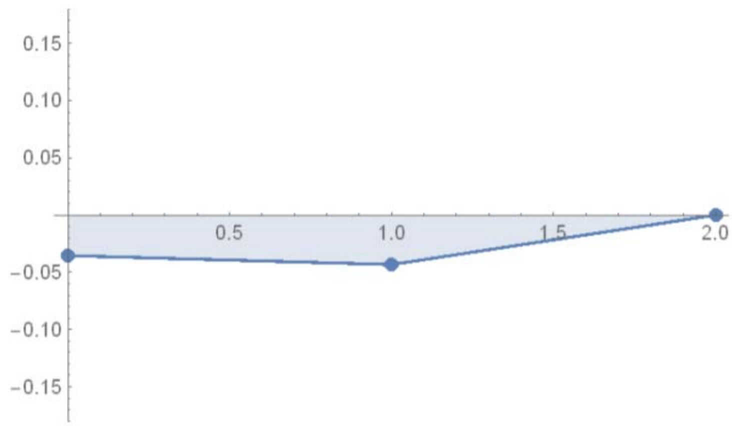

Figure 2. Output Deviation ($1 - \delta = .04$)

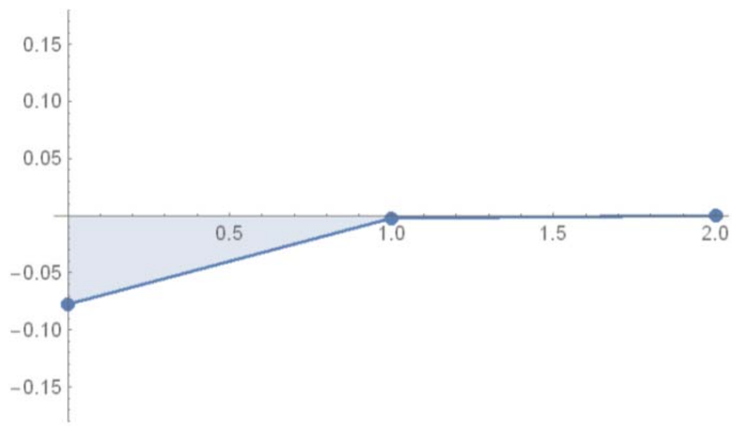

Figure 3. Output Deviation ($1 - \delta = .06$)

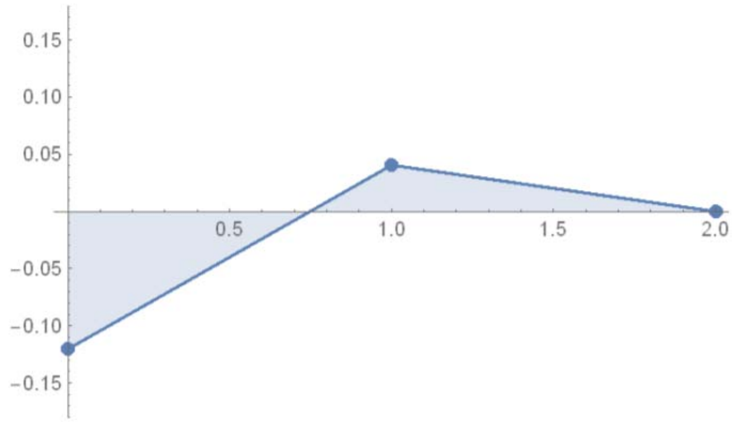

Figure 4. Output Deviation ($1 - \delta = .08$)