WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER 16-28

INFORMATION SPILLOVERS, GAINS FROM TRADE, AND INTERVENTIONS IN FROZEN MARKETS

Braz Camargo
Sao Paulo School of Economics – FGV

Kyungmin Kim
University of Iowa

Benjamin Lester
Research Department
Federal Reserve Bank of Philadelphia

October 2016

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA
Ten Independence Mall, Philadelphia, PA 19106-1574 • www.philadelphiafed.org/research-and-data/

Information Spillovers, Gains From Trade, and Interventions in Frozen Markets

Braz Camargo[†]
Sao Paulo School of Economics - FGV

Kyungmin Kim
University of Iowa

Benjamin Lester[‡]
Federal Reserve Bank of Philadelphia

October 18, 2016

Abstract

We study government interventions in markets suffering from adverse selection. Importantly, asymmetric information prevents both the realization of gains from trade *and* the production of information that is valuable to other market participants. We find a fundamental tension in maximizing welfare: While some intervention is required to restore trading, too much intervention depletes trade of its informational content. We characterize the optimal policy that balances these two considerations and explore how it depends on features of the environment. Our model can be used to study a program introduced in 2009 to restore information production in the market for legacy assets.

[*]This paper previously circulated under the title "Subsidizing Price Discovery." We thank Philip Bond, Ken Burdett, Vincent Glode, and Pablo Kurlat for excellent discussions of this paper, along with Viral Acharya, George Alessandria, Roc Armenter, Gadi Barlevy, Mitchell Berlin, Hal Cole, Hanming Fang, Douglas Gale, Itay Goldstein, Boyan Jovanovic, Andrew Postlewaite, and Xianwen Shi for helpful comments. All errors are our own.

[†]Braz Camargo gratefully acknowledges financial support from CNPq.

1 Introduction

When markets fail, or "freeze," there are two types of welfare losses. The first, and most direct, is that *gains from trade* are left unrealized. But there is a second, indirect effect as well. Market transactions contain information that is often valuable to other agents in the economy. Hence, when trade is disrupted, so too is this important process of *information production.*

There are many channels through which the information contained in market transactions guides real economic decisions. For example, asset prices can contain information about a company or its investment opportunities, helping investors and managers to allocate resources more efficiently.[1] Alternatively, information produced about a certain type of asset can reduce information asymmetries in markets for similar assets, thereby helping other agents to realize gains from trade.[2] The information produced about a particular class of assets could also allow for a more accurate assessment of the balance sheet of a bank that owns these assets, which could be valuable to depositors who have to decide whether to withdraw their funds from such a bank, or regulators who need to decide whether to bail out such a bank if it faces financial distress.[3] In either case, more accurate asset prices could reduce the incidence of liquidating banks that would ultimately be solvent and bailing out banks that would ultimately find themselves insolvent.

Given the important role that markets play in both allocating resources and generating valuable information, a natural question is: when a market freezes, what role can and should policymakers play in "unfreezing" it? Answering this question has become particularly important in light of the crisis that occurred in 2007-2008, when the collapse of trade in several key financial markets had deleterious effects on the economy as a whole. However, nearly all of the literature that has emerged to study policy interventions in frozen markets has focused exclusively on the abil-

[1] See, e.g., Dow and Gorton (1997), Chen et al. (2007), Foucault and Gehrig (2008), Bakke and Whited (2010), and Foucault and Frésard (2012) for specific examples, and Bond et al. (2012) for a broad overview of the literature that studies the interaction between price informativeness and real investment decisions.

[2] For recent examples of papers that study information spillovers in financial markets, see Benveniste et al. (2003), who provide evidence from the IPO market, or Cespa and Foucault (2014), who document the effects of informational spillovers after a "flash crash." Also see Duffie et al. (2014) and Asriyan et al. (2015).

[3] Goldstein and Pauzner (2005) provide a model that describes how information about fundamentals can change the probability of a bank run. Hart and Zingales (2011), McDonald (2013), Flannery (2010), and Bond and Goldstein (2015) discuss several ways in which policymakers can utilize the information contained in current market prices.

ity of various government programs to prevent the first type of loss discussed above—unrealized gains from trade—while ignoring the effect of these programs on the amount of information being produced.

This paper studies the effects of government intervention in frozen markets on both gains from trade *and* information production. Within the context of a simple model, we identify a fundamental difference between restoring trade and promoting information production: the former requires only that buyers are willing to *participate* in exchange, while the latter requires that buyers have an incentive to *acquire information* before trading. Therefore, identifying the optimal policy requires understanding how an intervention affects both of these margins.

The relationship between the size of an intervention and a buyer's willingness to participate in exchange is fairly straightforward: as the government commits additional resources to support trade, buyers are more willing to participate. The relationship between the size of an intervention and information production, however, is more subtle. When markets are frozen, buyers will not participate if the intervention is too small, and hence no trade—and, subsequently, no information production—will occur. However, if the intervention subsidizes buyers too much, a moral hazard problem can emerge: buyers will choose to trade without first learning about the quality of the assets they are buying. If this occurs, gains from trade are realized, but there is no information contained in the transaction itself. Hence, when a market is frozen, small interventions will typically reduce both types of welfare losses. However, as the size of the intervention grows, eventually a tension arises—at some point, a policymaker can promote more trades only at the cost of reducing the informational content of those trades, and thus reducing the efficiency of decisions made by those who depend on that information.

Having described the main tradeoff that emerges from our analysis, we now describe our modeling exercise, and the many results that come out of it, in greater detail. We start, in Section 2, by constructing the simplest possible model to study the effects of government interventions in frozen markets on both gains from trade and information production. This model has four key ingredients.

First, a buyer and a seller have the opportunity to exchange an asset—which the buyer potentially values more than the seller—but there is *asymmetric information* about the quality of the seller's asset. In particular, we assume that the seller's asset is either of high or low quality, and that this is the seller's private information. This friction is not only a classic explanation for market failures in general, but also one of the most commonly cited reasons for the specific interruptions that occurred during the recent financial crisis (see, e.g., Gorton (2009)).

Second, we allow the buyer to *acquire information* about the quality of the asset before making the seller an offer. More specifically, the buyer can acquire a noisy signal about the quality of the asset at a cost, where this cost is stochastic and privately observed. As a result, the buyer's behavior—i.e., whether he trades and at what price—becomes a noisy signal itself about the quality of the asset. Moreover, this signal becomes more precise as the buyer's incentive to acquire information grows.

Third, we introduce a real economic decision for which the *information produced by the buyer is valuable.* In particular, we assume that there is a third agent, an "investor," who can allocate resources to a new project. However, there is uncertainty about the quality of this project—it may be of high or low quality—and the quality of the project is correlated with the quality of the seller's asset. We assume that the investor observes whether or not the buyer and seller trade, and at what price, and updates his beliefs before investing. Hence, as the amount of information contained in the trading outcome increases, so too does the efficiency of the investor's decision.

Finally, we introduce a simple *policy to "unfreeze" the market*, whereby the government provides partial insurance to the buyer against the risk of acquiring a low quality asset or a "lemon." In particular, we assume that if the buyer pays a high price and discovers that the asset is of low quality, then he suffers only a fraction γ of the loss, and the government shoulders the remainder. This is a natural policy to study for two reasons. First, it directly addresses the underlying friction in the market: the buyer is reluctant to trade with the seller because he is concerned about over-paying for a lemon. Therefore, providing him with a sufficient level of insurance can unambiguously restore trade.[4] Second, this form of intervention captures the essential features of several policies that

[4]The idea that insurance (or a warranty) provides a remedy for inefficiencies caused by adverse selection is well known in the literature; see, among others, Spence (1977) and Grossman (1981). The novelty of our analysis is to study the optimal insurance scheme from the viewpoint of a benevolent government. Unlike private agents, the government needs to take into consideration externalities to the broader economy; in our case, these externalities stem from the presence of information spillovers.

have actually been implemented in response to market interruptions. In fact, as we describe in detail later in the text, the program we study closely resembles the Public Private Investment Program for Legacy Assets, or PPIP, which was introduced in March of 2009 in order to "support market functioning and facilitate price discovery, mostly in the mortgage-backed securities market[.]"[5]

Within the context of this model, we study the properties of the optimal policy in Section 3. To start, we characterize the policy γ that maximizes gains from trade, ignoring the information spillovers to the investor's decision problem. More specifically, we assume that there is a cost to the public funds required to finance the government program, and identify the policy that balances these costs with the benefits of promoting trade between the buyer and seller. This exercise is similar, in spirit, to much of the existing work on optimal interventions in frozen markets. Then we do the complementary exercise: we characterize the policy that maximizes the benefits of information production, as captured by the payoffs from the investment project, ignoring all other effects of the intervention. Finally, we use these results to draw conclusions about the policy that maximizes overall welfare, paying careful attention to understanding how incorporating the effects of information spillovers alters the optimal policy.

We find that the presence of information spillovers does not, in general, always justify more or less aggressive intervention—this depends on several features of the economic environment. First, when the cost of public funds is small, we show that considerations for information production imply a more moderate policy: whereas the government might otherwise provide buyers with plenty of insurance to promote trade, incentivizing buyers to acquire information (i.e., reducing the moral hazard problem) requires offering a smaller subsidy. The opposite, however, is true when the cost of public funds is large. In this case, incorporating the effects of policy on the investor's decision prompts a more aggressive intervention. Second, absent considerations for information production, we show that policymakers will typically choose not to intervene if the adverse selection problem is very weak or very severe; in the former case trade likely occurs without intervention, while in the latter case the market is "too far gone" for intervention. When one introduces information spillovers, however, the policymaker is more likely to intervene even

[5]This quote is taken from the Quarterly Report of the U.S. Department of the Treasury, January 30, 2013.

in these extreme cases, i.e., the inaction region shrinks. Finally, we show that the policymaker is likely to intervene more aggressively when the information available to the buyer is more precise, and less so when it is difficult for the buyer to learn the true quality of the asset.

Having established these results within the context of a very simple model, we then systematically relax each of our strongest modeling assumptions in Section 4, proving that our main results still hold, and exploring additional insights that emerge from more sophisticated (and, perhaps, more realistic) environments. Three of these extensions are particularly noteworthy. First, we relax the assumption that trade occurs between a single buyer and a single seller, and instead consider an auction setting with multiple buyers. This extension allows prices to play a more meaningful role in conveying information than they play in our baseline model, and also reveals new insights about the interaction between information acquisition, the winner's curse, and the optimal policy. Second, we expand the set of policy instruments available to the government, endowing the policymaker with the ability to tax or subsidize buyers after reporting the quality of their asset (low *or* high). Interestingly, we find that the optimal policy may require the government to reward buyers when they acquire a high quality asset; this stands in contrast to the actual implementation of PPIP, which forced private investors to share profits when they acquired a high quality asset. Lastly, as we noted earlier, real investment decisions are not the only source of information spillovers. Hence, we study several alternative, potentially important decision problems that utilize the information produced by the buyer, and explore how the nature of these information spillovers can affect the size of the optimal intervention.

The Literature on Optimal Interventions in Frozen Markets. This paper primarily contributes to the young, but growing literature on optimal interventions in frozen markets. A non-exhaustive list includes Tirole (2012), Philippon and Skreta (2012), Chari et al. (2014), Camargo and Lester (2014), Guerrieri and Shimer (2014), Chiu and Koeppl (2011), Philippon and Schnabl (2013), House and Masatlioglu (2015), Diamond and Rajan (2012), Farhi and Tirole (2012), and Fuchs and Skrzypacz (2015).[6] As we noted above, the majority of this literature focuses on how govern-

[6]See Lester (2013) for a brief survey of this literature.

ment interventions can improve allocations, while ignoring the effects of these interventions on the process of information production.

To the best of our knowledge, the only other paper that explicitly studies the effects of government interventions on information production is Bond and Goldstein (2015). They highlight an interesting feedback effect that is absent from our analysis: the government decides how much to use market prices in formulating a policy, which affects the incentives of speculators to trade and hence changes the informational content of these prices. The focus of their analysis is very different from ours, though; most notably, they are not interested in inefficiencies due to adverse selection, and thus the interventions in their model play an entirely different role than in our environment.

2 The Model

In this section, we first describe the physical environment, highlighting the two channels that determine ex ante welfare: the direct gains that come from two agents trading, and the indirect gains generated by information spillovers. We then describe a natural form of government intervention, and derive the government's objective function.

2.1 Environment

There are three periods, indexed by $t = 0, 1, 2$, and three distinct, risk-neutral players: a buyer, a seller, and an investor. The buyer and the seller have the opportunity to trade in period 0, while the investor makes an investment decision in period 1. The investor cannot participate in period-0 trade, but observes the trading outcome before making his investment decision.

Period 0. The seller is endowed with one indivisible asset of quality $q_0 \in \{L, H\}$. If the asset is of high quality (H), then it yields $v > 0$ units of dividends at $t = 2$. If the asset is of low quality (L), then it yields no dividends at $t = 2$.

There are gains from trade between the buyer and the seller because of a difference in their time preferences, which, for example, can originate from a difference in their liquidity demands. Formally, we assume that the seller discounts period-2 consumption according to the discount factor $\beta = c/v$, for some $c < v$, while the buyer does not discount period-2 consumption. Hence, while a quality L asset is worthless to both the buyer and the seller, a quality H asset yields the buyer utility v and the seller utility $c < v$.

Although there are gains from trade between the buyer and the seller, there is also asymmetric information: the seller can observe the quality of her asset, but the buyer cannot. The buyer knows the ex ante probability that the asset is of quality H, which we denote by π_0, and this is common knowledge. The buyer also has the opportunity to inspect the asset at a cost k, where k is drawn from the interval $[0, \overline{k}]$, with $\overline{k} > v$, according to a cumulative distribution function $G(k)$ with density $g(k)$ that is bounded away from zero in $[0, \overline{k}]$.

If the buyer incurs the cost k, then he receives a private signal $s \in \{\ell, h\}$ about the quality of the asset. In order to deliver our results most clearly, we focus on a simple signal-generating process. In particular, the matrix below summarizes the probability of receiving signal s conditional on the true state being L or H

	H	L
h	1	$1-\rho$
ℓ	0	ρ

(1)

where $\rho \in (0, 1)$. Notice that, under this information structure, the buyer knows that the asset is of low quality if he receives signal ℓ, while there is residual uncertainty about the quality of the asset when he receives signal h.[7] We refer to the buyer as "informed" if he receives a signal and as "uninformed" if he chooses not to receive a signal.

We consider a simple, but commonly adopted, trading protocol: after observing the signal and updating his beliefs, the buyer makes a take-it-or-leave-it offer to the seller, who accepts or rejects.

Period 1. At $t = 1$, there is an investor who has the opportunity to "plant" new trees. The investment, however, is risky: the quality q_1 of the trees is either high (H) or low (L), and is unknown to the investor. If the investor chooses an investment level $i \geq 0$, the trees yield $Y(i)$ units

[7] Although the informational structure is rather stylized, it has a natural interpretation. One can imagine that there are certain "red flags" associated with low quality assets, corresponding to the signal ℓ in our environment. A buyer who studies a seller's asset will never uncover such a red flag if the asset is of high quality, while he may find one (with probability ρ) if the asset is of low quality. Many of our results are robust to other specifications, including the case in which the bad signal occurs with positive probability when the asset is of high quality.

of fruit at $t = 2$ if $q_1 = H$ and zero otherwise, where the function Y is continuously differentiable, strictly increasing, and strictly concave with $Y(0) = 0$. The cost of investing i units is $K(i)$ regardless of the quality of the trees, where the function K is continuously differentiable, strictly increasing, and convex with $K(0) = K'(0) = 0$. The investor does not discount between periods.

Importantly, the quality of the trees in period 1 is the same as the quality of the tree in period 0, so that the investor's prior belief that $q_1 = H$ is π_0.[8] Moreover, we assume that the investor observes whether trade occurs between the buyer and the seller at $t = 0$, along with the transaction price, before making his investment decision. Hence, trade in period 0 generates useful information for the investment decision in period 1. To see this, note that a buyer's offer in period 0 depends on the signal he receives. Since this signal is correlated with the quality of the seller's asset, and thus with the quality of trees in period 1, observing the outcome of the period 0 game between the buyer and seller can provide useful information for the investor in period 1.[9]

Period 2. At $t = 2$, all uncertainty is resolved: the asset that belonged to the seller in period 0 yields its dividends, as does the investment made in period 1. If the buyer offered p and the seller accepted, the seller's payoff is p and the buyer's payoff is $v - p$ if $q_0 = H$ and $-p$ otherwise. Alternatively, if the seller rejected the buyer's offer and retained her asset, the buyer's payoff is 0 and the seller's payoff is c if $q_0 = H$ and 0 otherwise. Finally, an investor who chose an investment level i receives a payoff $Y(i) - K(i)$ if $q_1 = H$ and $-K(i)$ otherwise.

2.2 Government Policy and Objective

A key source of inefficiency in the environment described above is the classic "lemons" problem: the buyer is reluctant to trade with the informed seller because he fears paying a positive price for a low quality asset. Not only does this hinder gains from trade from being realized at $t = 0$, but it

[8] For example, suppose the asset for sale at $t = 0$ is a mortgage-backed security, which will have a high payoff if demand for housing (and housing prices) increases over time, and a low payoff otherwise. Then, one could imagine that the investor at $t = 1$ is deciding how much to invest in a new real estate development, which will only be profitable if future demand for housing is high. Our analysis extends to the case in which q_1 and q_0 are positively correlated.

[9] In our baseline model, trade will only occur in period 0 at a price c, so that the investor learns only by observing whether or not trade occurred. As we discuss in Section 4.1, however, if a trading mechanism generates price dispersion, then the investor uses the transaction price to update his beliefs, too.

also inhibits information production, which is socially valuable at $t = 1$. A natural intervention, then, is for the government to offer the buyer insurance against the prospect of acquiring a low quality asset.

Policy. In order to implement such a policy, we consider the following intervention. If the buyer purchases the seller's asset at price p, the government offers the buyer the following choice after he learns the asset's quality at $t = 2$ (but before collecting dividends): if the buyer reports that the asset is of high quality, he retains the asset; if the buyer reports that the asset is of low quality, he surrenders the asset and receives a transfer $\tau = (1 - \gamma)p$, with $\gamma \in [0, 1]$. Thus, the government policy is tantamount to insurance: an "unlucky" buyer who pays price $p > 0$ and receives a low-quality asset loses only γp.

By offering buyers insurance against acquiring a lemon, this policy not only addresses the fundamental friction in the model, but also captures the key elements of interventions that have been implemented during financial crises in the past. Perhaps the best example of such an intervention is the Public-Private Investment Program for Legacy Assets, or PPIP, that was introduced in March of 2009 to rejuvenate the market for real estate loans and assets backed by these loans.[10] Under this program, when a buyer acquired an asset, he used his own equity to finance a fraction of the purchase price, the Treasury matched his cash outlay, and then the FDIC issued a *nonrecourse loan* for the remainder. If the buyer realized that he had purchased a lemon, he could simply default on the loan, surrender the asset, and lose only his initial equity investment; in the context of our model, γ denotes the fraction of the purchase price that the investor needed to finance with his own equity, while $\tau = (1 - \gamma)p$ denotes the implicit insurance offered by the government.[11]

As in most of the existing literature, we assume that transfers are costly; see, e.g., Tirole (2012).

[10]The idea of curing a frozen market by sharing in participants' potential losses was not exclusive to PPIP, though. For example, Swagel (2009) describes an FDIC proposal for foreclosure avoidance that included "a loss-sharing insurance plan, under which the federal government would make good on half of the loss suffered by a lender that modified a loan according to the IndyMac protocol but later saw the loan go into default and foreclosure." A similar philosophy underlies the "ring fence insurance schemes" he describes, whereby money from the Troubled Asset Relief Program was used to share losses on a large pool of assets owned by Citi.

[11]Under PPIP, in addition to offering investors insurance, the government also shared in the profits when an asset turned out to be worth more than the purchase price. We abstract from this feature here, for simplicity, and explore it further when we consider more sophisticated forms of government intervention in Section 4.

We capture this by assuming that there is a shadow cost $\lambda \geq 0$ per unit of public funds, so that the social cost of transfers of size τ is $(1 + \lambda)\tau$.

Objective. The government is benevolent and maximizes the total surplus. Let V_B be the buyer's expected payoff, V_S be the seller's expected payoff, and V_I be the investor's expected payoff. Moreover, let C denote the expected cost of the government policy. By influencing the buyer's incentives at the trading stage, government policy clearly affects V_B, V_S, and C directly. In addition, by changing the quantity and precision of the information produced in period 0, government policy also affects V_I indirectly. To summarize, the government's objective is to maximize

$$V_B + V_S - C + V_I.$$

In what follows, we will often decompose this objective. The first term $V_B + V_S + C$, corresponds to the objective often considered in the existing literature; we will refer to this as the "net gains from trade." The second piece, V_I, corresponds to the portion of total welfare that derives from information spillovers.

3 Trade, Investment, and Government Intervention

In this section, we first consider the period-0 trading problem, characterize optimal behavior, and identify the policy that maximizes net gains from trade. We then study the period-1 investment problem, analyze how the solution depends on the information generated by period-0 trade, and use this analysis to characterize the policy that maximizes the investor's expected payoff in period 1. Finally, we study the properties of the policy that maximizes welfare aggregated across both periods. Our analysis highlights a fundamental trade-off between maximizing gains from trade and maximizing information production.

3.1 Trade in Period 0

The seller's optimal behavior in period 0 is straightforward: she accepts an offer greater than her reservation value. When indifferent, we assume the seller also accepts an offer of c if the asset is of quality H but rejects an offer of 0 if the asset is of quality L. The first assumption is necessary for equilibrium existence. The second assumption simplifies the exposition without affecting its substance. For the buyer's behavior, we first characterize his optimal trading strategy given his information, and then derive his optimal information-acquisition strategy.

Optimal Offer Strategy. The buyer in period 0 will offer either c or 0, depending on his beliefs and the government's choice of γ. Hence, the expected payoff to an uninformed buyer is

$$V_B^u(\gamma) = \max\{\pi_0(v-c) - (1-\pi_0)\gamma c, 0\},$$

and the optimal strategy is to offer c if, and only if,

$$\gamma < \underline{\gamma} \equiv \min\left\{\frac{\pi_0(v-c)}{(1-\pi_0)c}, 1\right\}, \tag{2}$$

where we've assumed that the uninformed buyer offers c when indifferent, i.e., when $\gamma = \underline{\gamma}$.

Now consider the informed buyer. If he observes the signal ℓ, then the asset is surely of low quality, in which case he bids 0 and obtains a payoff of zero.[12] If he observes h, then he updates his belief to

$$\pi^h \equiv \frac{\pi_0}{\pi_0 + (1-\pi_0)(1-\rho)} > \pi_0.$$

Hence, the expected payoff for a buyer who receives signal h, as a function of γ, is

$$V_B^h(\gamma) = \max\{\pi^h(v-c) - (1-\pi^h)\gamma c, 0\},$$

and the payoff from offering c is strictly positive if, and only if,

$$\gamma < \overline{\gamma} = \min\left\{\frac{\pi^h(v-c)}{(1-\pi^h)c}, 1\right\} = \min\left\{\frac{\pi_0(v-c)}{(1-\pi_0)(1-\rho)c}, 1\right\}. \tag{3}$$

Notice that $\underline{\gamma} < \overline{\gamma}$ as long as $\underline{\gamma} < 1$ and that $\overline{\gamma} < 1$ if π_0 is small enough.

Since the buyer receives signal h with probability $\pi_0 + (1-\pi_0)(1-\rho)$, the (ex ante) expected

[12] An uninteresting multiplicity of optimal strategies arises if the government fully insures the buyer (i.e., $\gamma = 0$). In this case, we assume that the buyer still bids 0, which is the limit of his optimal strategy as γ decreases to 0.

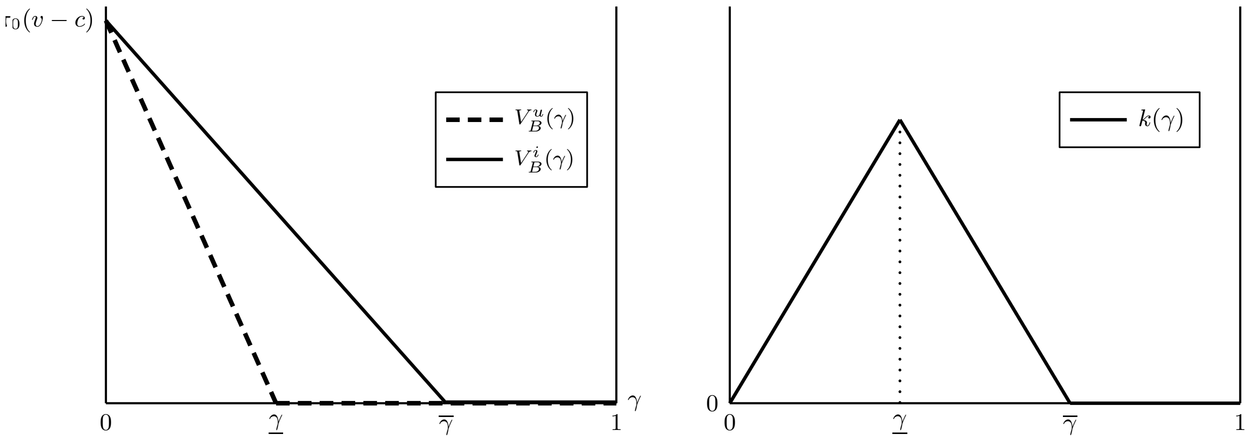

Figure 1: The left panel shows the expected payoffs of the informed buyer (solid, $V_B^i(\gamma)$) and the uninformed buyer (dashed, $V_B^u(\gamma)$), while the right panel shows the value of acquiring information $k(\gamma) \equiv V_B^i(\gamma) - V_B^u(\gamma)$.

payoff of the informed buyer is equal to

$$V_B^i(\gamma) \equiv [\pi_0 + (1-\pi_0)(1-\rho)]V_B^h(\gamma).$$

The left panel of Figure 1 plots $V_B^u(\gamma)$ and $V_B^i(\gamma)$. Note that $V_B^i(\gamma) \geq V_B^u(\gamma)$ for all $\gamma \in [0,1]$, so that the buyer always (at least weakly) prefers to be informed, and that both $V_B^u(\gamma)$ and $V_B^i(\gamma)$ are non-increasing in γ, so that the buyer always (at least weakly) prefers a more generous subsidy from the government.

Optimal Information Acquisition. The buyer's optimal information-acquisition strategy is a cutoff rule: inspect the asset if, and only if, $k \leq k(\gamma) \equiv V_B^i(\gamma) - V_B^u(\gamma)$. It is immediate to see that

$$k(\gamma) = \begin{cases} 0 & \text{if} \quad \gamma \geq \overline{\gamma} \\ \pi_0(v-c) - (1-\pi_0)(1-\rho)\gamma c & \text{if} \quad \gamma \in (\underline{\gamma}, \overline{\gamma}) \\ (1-\pi_0)\rho\gamma c & \text{if} \quad \gamma \in [0, \underline{\gamma}]. \end{cases}.$$

The right panel of Figure 1 depicts a typical shape of $k(\gamma)$. The most striking feature is that $k(\gamma)$ is single-peaked at $\underline{\gamma}$. To understand this result, first note that there is no value to inspection when $\gamma \geq \overline{\gamma}$: the lemons problem is sufficiently severe that the buyer would not be willing to offer

c even if he received the signal h. Naturally, then, the buyer will not acquire information at any positive cost when $\gamma \geq \overline{\gamma}$. When $\underline{\gamma} < \gamma < \overline{\gamma}$, however, the buyer will offer c if he receives the signal h, but will offer 0 if he remains uninformed. Hence, in this region, a marginal reduction in γ increases the expected payoff from being informed but has no effect on the expected payoff from being uninformed. As a result, the buyer's willingness to acquire information *increases* as the subsidy becomes more generous.

However, when γ falls below $\underline{\gamma}$, a moral hazard problem emerges: the insurance provided by the government is sufficiently generous that even uninformed buyers are willing to gamble and offer the seller a price c. From Figure 1, one can see that $V_B^u(\gamma)$ increases at a faster rate than $V_B^i(\gamma)$ as γ falls in this region. The reason is that uninformed buyers place a greater value on the insurance provided by the government, as they are more likely to purchase a low quality asset. Hence, in this region, a marginal reduction in γ causes $k(\gamma)$ to fall; that is, the buyer's willingness to acquire information *decreases* as the subsidy becomes more generous. In fact, this willingness disappears when the government fully insures the buyer. Lemma 1 summarizes.

Lemma 1. *The cutoff cost for information acquisition, $k(\gamma)$, is single-peaked, maximized at $\gamma = \underline{\gamma}$, and minimized at $\gamma = 0$.*

A consequence of Lemma 1 is that when the adverse selection problem is sufficiently severe, so that $\underline{\gamma} < 1$, the buyer has the strongest incentive to acquire information at an interior level of government subsidy. However, when the adverse selection is weak, so that $\underline{\gamma} = 1$, the buyer has the strongest incentive to acquire information when the government does not intervene.

Welfare Implications. Given the analysis above, it is straightforward to calculate the agents' expected payoffs and the government's expected cost as a function of the policy γ. The buyer's expected payoff is given by

$$V_B(\gamma) = G(k(\gamma)) \left[V_B^i(\gamma) - \mathbb{E}[k|k \leq k(\gamma)]\right] + [1 - G(k(\gamma))] \, V_B^u(\gamma).$$

Similarly, the seller's expected payoff is given by

$$V_S(\gamma) = \left\{\pi_0 + (1-\pi_0)\left[G(k(\gamma))(1-\rho) + [1-G(k(\gamma))]\mathbb{I}_{\{\gamma\leq\underline{\gamma}\}}\right]\right\}c,$$

where $\mathbb{I}_{\{\gamma\leq\underline{\gamma}\}}$ is the indicator function that is equal to 1 if $\gamma \leq \underline{\gamma}$ and 0 if $\gamma > \underline{\gamma}$. Is it easy to show that both V_B and V_S are decreasing in γ. Indeed, a decrease in the implicit subsidy offered by the government increases the buyer's loss in case he purchases a lemon. This, in turn, makes the buyer more cautious and, therefore, offer c less frequently, which hurts the seller.

Finally, the expected cost of this program to the government is equal to

$$C(\gamma) = (1+\lambda)(1-\pi_0)\left[G(k(\gamma))(1-\rho) + [1-G(k(\gamma))]\mathbb{I}_{\{\gamma<\underline{\gamma}\}}\right](1-\gamma)c.$$

Naturally, the government's expected cost increases as it promises more subsidy, for two reasons. First, ceteris paribus, a decrease in γ directly increases the expected transfer to the buyer conditional on acquiring a lemon. Second, an increase in the subsidy induces the buyer to become more aggressive and offer c more frequently.

Maximizing Net Gains from Trade. In the absence of (concerns for) information spillovers (i.e., $w = 0$), the optimal policy balances the benefits to the buyer and seller against the cost of the intervention. Let $\gamma_0^* = \text{argmax} V_B(\gamma) + V_S(\gamma) - C(\gamma)$ denote such a policy. The following proposition establishes a number of key properties of γ_0^*. We adopt the convention of assuming that when the government is indifferent between multiple values of γ, it chooses the maximum, i.e., the policy that implies the smallest subsidy.

Proposition 1. *The policy γ_0^* has the following properties:*

(1) The policy γ_0^* is increasing in λ, with $\gamma_0^* > 0$ for all $\lambda > 0$, $\lim_{\lambda\to 0}\gamma_0^* = 0$, and $\lim_{\lambda\to\infty}\gamma_0^* = 1$.

(2) For each $\lambda > 0$ and $c \in (0, v)$, there exist $0 < \underline{\pi} \leq \overline{\pi} < 1$ such that $\gamma_0^* = 1$ for all $\pi_0 \leq \underline{\pi}$ and $\pi_0 \geq \overline{\pi}$. Moreover, if λ is sufficiently small, then $\underline{\pi} < \overline{\pi}$ and $\gamma_0^* < \overline{\gamma}$ for all $\pi \in (\underline{\pi}, \overline{\pi})$.

(3) For each $\lambda > 0$ and $\pi_0 \in (0, 1)$, there exist $0 < \underline{c} \leq \overline{c} < v$ such that $\gamma_0^* = 1$ for all $c \leq \underline{c}$ and $c \geq \overline{c}$. Moreover, if λ is sufficiently small, then $\underline{c} < \overline{c}$ and $\gamma_0^* < \overline{\gamma}$ for all $c \in (\underline{c}, \overline{c})$.

The first result in Proposition 1 highlights that the optimal amount of insurance provided by the government is decreasing in the cost of public funds. At one extreme, if funding is costless, then full insurance maximizes period-0 welfare by ensuring that all gains from trade are realized. At the other extreme, if funding this type of program is too costly, then it is not worthwhile.

The second and third results highlight the relationship between the optimal intervention and the severity of the underlying lemons problem. Note that, in contrast to typical competitive models with asymmetric information—where trade occurs with probability 0 if π_0 is sufficiently small and probability 1 otherwise—the probability that trade occurs in our model is a continuous, increasing function of π_0 because of the information acquisition decision. As a result, the extent to which a market is "frozen" is a continuous variable, not a discrete one. The second result in Proposition 1 asserts that when the lemons problem is mild—i.e., when π_0 is sufficiently close to 1 or c is sufficiently close to zero—then the market is "not very frozen" and intervention is unnecessary. However, the third result in Proposition 1 states that when the lemons problem is more severe—i.e., when π_0 is sufficiently close to 0 or c is sufficiently close to v—then the market can be "very frozen" and the cost of restoring trade can be so large that it is not worthwhile. Therefore, our results suggest that interventions are only necessary when the problem of adverse selection is severe enough to disrupt trade, but not so severe that the market is "too far gone."

3.2 Investment in Period 1

We now derive the optimal investment decision in period 1 given beliefs about the quality of the trees. We analyze how these beliefs are influenced by the policy γ implemented in period 0, and then use this analysis to characterize the value of γ that maximizes the investor's expected payoff in period 1.

Optimal Investment Strategy. Let π_1 denote the investor's belief that the quality of the trees in period 1 is H after observing the period-0 trading outcome. Given these beliefs, the investor solves

$$\max_{i \geq 0} \pi_1 Y(i) - K(i).$$

Our assumptions on Y and K ensure that there is a unique, interior solution: the optimal investment level, which we denote by $I(\pi_1)$, is characterized by

$$\pi_1 Y'\left(I(\pi_1)\right) = K'\left(I(\pi_1)\right).$$

The assumptions on Y and K imply that $I(\pi_1)$ is strictly increasing in π_1, i.e., that the investor is more aggressive when he is more optimistic.

Let $\widehat{V}_I(\pi_1)$ denote the investor's interim expected payoff when his belief is π_1. Then

$$\widehat{V}_I(\pi_1) = \pi_1 Y(I(\pi_1)) - K(I(\pi_1)).$$

By the envelope theorem, $\widehat{V}_I'(\pi_1) = Y(I(\pi_1)) > 0$. Also, since both I and Y are strictly increasing functions, $\widehat{V}_I'(\pi_1)$ is strictly increasing in π_1. The following lemma summarizes these properties, which are useful below when we study the policy that maximizes the investor's expected payoff.

Lemma 2. $\widehat{V}_I(\pi_1)$ *is a strictly increasing and convex function of* π_1.

Policy and the Informational Content of Period-0 Trade. Let $\pi_1^T(\gamma)$ denote the investor's (posterior) belief that the quality of trees in period 1 is H after observing trade in period 0 when the government's choice of policy is γ. Similarly, let $\pi_1^N(\gamma)$ denote the investor's belief when trade does not occur in period 0.

We establish below that $\pi_1^N(\gamma) \leq \pi_0 \leq \pi_1^T(\gamma)$ for all $\gamma \in [0,1]$. Intuitively, since trade is more likely to occur in period 0 when the asset is of quality H than when it is of quality L, trade in period 0 is good news about the quality of the trees in period 1, while no trade is bad news. For our purpose, however, we need to understand *how much* information period-0 trade carries. This information is described by the (unconditional) distribution of the investor's beliefs, which depends on the policy γ; we denote this distribution $\Omega(\pi_1;\gamma)$.

Consider first the case when $\gamma \in (\underline{\gamma}, \overline{\gamma}]$. By Bayes' rule,

$$\pi_1^T(\gamma) = \frac{\pi_0 G(k(\gamma))}{\pi_0 G(k(\gamma)) + (1-\pi_0)G(k(\gamma))(1-\rho)} = \frac{\pi_0}{\pi_0 + (1-\pi_0)(1-\rho)} = \pi^h,$$

and

$$\pi_1^N(\gamma) = \frac{\pi_0[1-G(k(\gamma))]}{\pi_0[1-G(k(\gamma))]+(1-\pi_0)[1-G(k(\gamma))+G(k(\gamma))\rho]} \leq \pi_0.$$

There are a number of things to observe. First, note that $\pi_1^T(\gamma)$ is independent of γ: since trade only occurs when the buyer receives the signal h, observing trade is equivalent to observing h directly. Second, note that $\pi_1^N(\gamma)$ is strictly increasing in γ. To understand why, note that trade does not occur in period 0 either because the buyer is uninformed or because he received the bad signal, ℓ. In the former case, no additional information is revealed. In the latter case, the asset is surely of low quality. As γ increases over the range $(\underline{\gamma}, \overline{\gamma}]$, the probability of information acquisition decreases and it becomes less likely that trade did not occur because of a bad signal, leaving the investor less pessimistic. Finally, note that the unconditional probability of observing trade, $G(k(\gamma))[\pi_0 + (1-\pi_0)(1-\rho)]$, is strictly decreasing in γ. Taken together, these comparative statics results imply that $\Omega(\pi_1; \gamma)$ becomes less dispersed as γ increases. More precisely, an increase in γ increases $\Omega(\pi_1; \gamma)$ in the sense of second-order stochastic dominance.

Now consider the case when $\gamma \leq \underline{\gamma}$. In this region, the uninformed buyer also offers c. Therefore, trade does not occur in period 0 only when the buyer acquires information and receives signal ℓ. It then follows that

$$\pi_1^T(\gamma) = \frac{\pi_0}{\pi_0 + (1-\pi_0)[1-G(k(\gamma))(1-\rho)]} \geq \pi_0,$$

and $\pi_1^N(\gamma) = 0$. Thus, in contrast to the previous case, $\pi_1^N(\gamma)$ is independent of γ, while $\pi_1^T(\gamma)$ is strictly increasing in γ.[13] However, as in the previous case, the unconditional probability of observing trade in period 0, which is now $\pi_0 + (1-\pi_0)[1-G(k(\gamma))(1-\rho)]$, is strictly decreasing in γ. Taken together, these facts imply that $\Omega(\pi_1; \gamma)$ becomes more dispersed—i.e., decreases in the sense of second-order stochastic dominance—as γ increases. We summarize the properties of $\Omega(\pi_1; \gamma)$ in the following lemma.

Lemma 3. *For all $0 \leq \gamma < \gamma' \leq \underline{\gamma}$, $\Omega(\pi_1; \gamma)$ dominates $\Omega(\pi_1; \gamma')$ in the second-order stochastic*

[13] To understand why $\pi_1^T(\gamma)$ is increasing in γ, recall that $k(\gamma)$ is strictly increasing in γ when $\gamma \leq \underline{\gamma}$. Hence, the fraction of trades that can be attributed to informed buyers who received a good signal—as opposed to uninformed buyers who bid c with no additional information—is increasing as γ rises, which causes $\pi_1^T(\gamma)$ to increase as well.

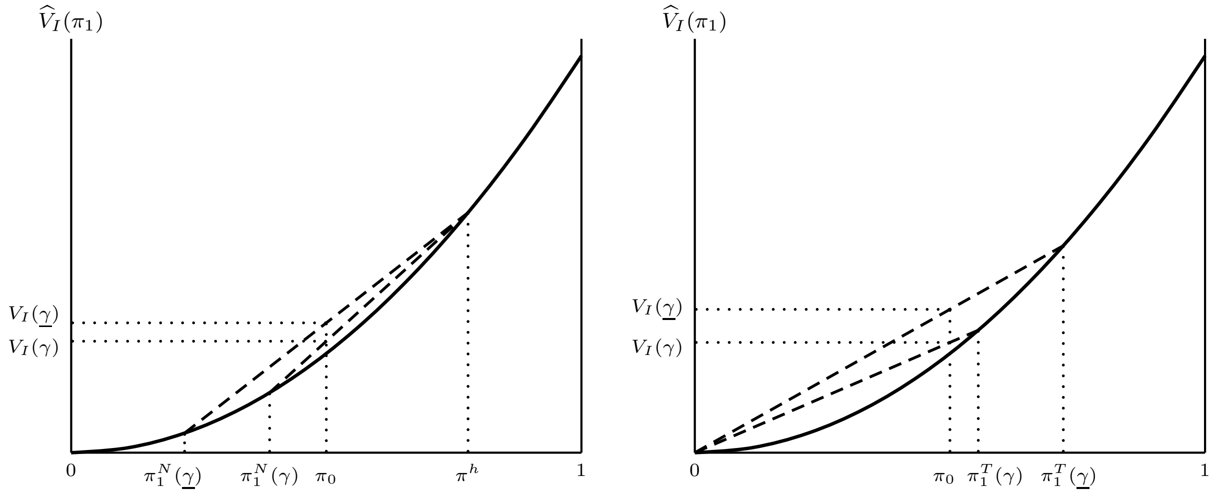

Figure 2: The left panel illustrates how the investor's expected payoff changes with γ in the interval $(\underline{\gamma}, \overline{\gamma}]$. The right panel illustrates the same when γ is in the interval $[0, \underline{\gamma}]$.

sense. For all $\underline{\gamma} < \gamma < \gamma' \leq \overline{\gamma}$, $\Omega(\pi_1; \gamma')$ dominates $\Omega(\pi_1; \gamma)$ in the second-order stochastic sense.

Maximizing Information Production. The investor's *ex ante* expected payoff as a function of the policy γ is

$$V_I(\gamma) = \mathbb{E}[\widehat{V}_I(\pi_1)],$$

where the expectation is taken with respect to $\Omega(\pi_1; \gamma)$. As we report in Proposition 2, since the investor's interim payoff $\widehat{V}_I(\pi_1)$ is strictly increasing and strictly convex in π_1, it follows immediately from Lemma 3 that $V_I(\gamma)$ is strictly increasing in γ when $\gamma \leq \underline{\gamma}$ and $V_I(\gamma)$ is strictly decreasing in γ when $\gamma > \underline{\gamma}$. The first fact is illustrated in the right panel of Figure 2, while the second fact is illustrated in the left panel of Figure 2.

Proposition 2. *The investor's expected payoff, $V_I(\gamma)$, is strictly increasing in γ when $\gamma \leq \underline{\gamma}$ and strictly decreasing in γ when $\gamma \in (\underline{\gamma}, \overline{\gamma}]$.*

Note that, in general, $V_I(\gamma)$ is discontinuous at $\gamma = \underline{\gamma}$; this is a consequence of the discrete change in the uninformed buyer's behavior in period 0 when $\gamma = \underline{\gamma}$. If $\lim_{\gamma \nearrow \underline{\gamma}} V_I(\gamma) \equiv V_I(\underline{\gamma}^-) \geq V_I(\underline{\gamma}^+) \equiv \lim_{\gamma \searrow \underline{\gamma}} V_I(\gamma)$, Proposition 2 implies that $\gamma_1^* = \text{argmax} V_I(\gamma)$ exists and is equal to $\underline{\gamma}$. On the other hand, if $V_I(\underline{\gamma}^-) < V_I(\underline{\gamma}^+)$, γ_1^* is not well-defined, as the government would like to set

$\gamma > \underline{\gamma}$ as close to $\underline{\gamma}$ as possible. While this issue of nonexistence is inconvenient from a theoretical point of view, it is less of a concern from a practical point of view: in the real world, policy choices typically lie on a finite set, in which case this issue essentially vanishes. Hence, we will ignore this detail going forward and treat the optimal policy in period 1 as $\gamma_1^* = \underline{\gamma}$.

3.3 Maximizing Total Welfare

The analysis above lays bare two very different reasons why policymakers may want to intervene in frozen markets: first, to promote the realization of gains from trade; and, second, to promote the production of valuable information. The former typically requires only that the intervention provides incentives for buyers to *participate* in trade. The latter, however, requires not only that buyers participate, but also that they have incentive to first *acquire information* about the asset for sale before attempting to buy it; otherwise, trade will not have any informational content.

In this section, we study the policy that maximizes total welfare, where both of these forces are active. We show that sometimes they reinforce each other—e.g., sometimes information spillovers give policymakers additional incentive to insure buyers and promote trade. However, in other cases, we show that the two forces can generate a tension for policymakers—e.g., sometimes the presence of information spillovers implies that promoting more trade in period 0 comes at the cost of less information, and thus less efficient decisions, in period 1.

Proposition 3. *Let* $\gamma^* = \text{argmax}_\gamma V_B(\gamma) + V_S(\gamma) - C(\gamma) + V_I(\gamma)$. *If* $0 < \gamma_0^* < \gamma_1^*$, *then* $\gamma^* > \gamma_0^*$. *On the other hand, if* $\overline{\gamma} > \gamma_0^* > \gamma_1^*$, *then* $\gamma^* < \gamma_0^*$.

Note that Proposition 3 implies that the presence of information spillovers does not, in general, always justify more or less aggressive intervention. Instead, this result highlights the fact that the optimal level of intervention depends on several features of the economic environment. We now discuss these features of the environment, and explain under what circumstances concerns about information production will lead to more or less government intervention.

The Cost of Public Funds. It follows from our results in Proposition 1 that $\gamma_0^* < \gamma_1^*$ when the cost of public funds, λ, is small, while $\gamma_0^* > \gamma_1^*$ when λ is sufficiently large. Intuitively, when the

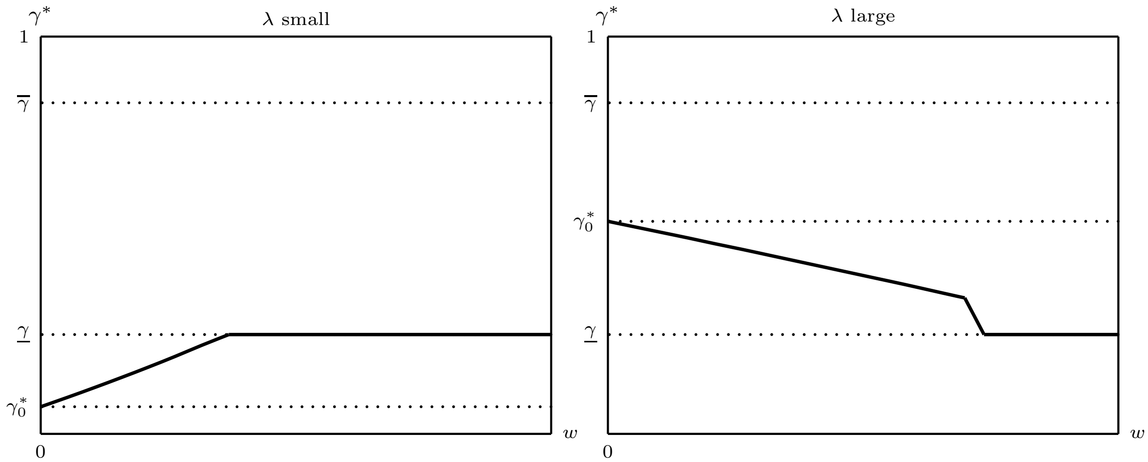

Figure 3: Both panels depict how the optimal policy γ^* varies as the information value of period-0 trade, measured by w, increases. The left panel is for the case in which the shadow cost of public funds (λ) is relatively small, while the right panel is for the opposite case in which the shadow cost is relatively large.

cost of public funds is small, the policymaker has a strong incentive to intervene and restore trade by providing significant levels of insurance to the buyer—that is, γ_0^* is relatively low. However, such an intervention undermines the informational content of market prices by encouraging only speculative (or uninformed) trade. As a result, when prices play an important role in guiding the period-1 investment decision, maximizing total welfare requires setting $\gamma^* > \gamma_0^*$. On the other hand, if λ is sufficiently large, then a policymaker focusing exclusively on period-0 gains from trade is reluctant to intervene and put public funds at risk—that is, γ_0^* is relatively high. In this case, the payoff from producing information provides an additional rationale for intervention, so that a policymaker maximizing total welfare will set $\gamma^* < \gamma_0^*$.

The Degree of Adverse Selection. We know from Proposition 1 that maximizing period-0 net gains from trade requires no intervention when adverse selection is either very mild or very severe. Given the results above, it follows that the presence of information spillovers can shrink the region for which no intervention is optimal. In other words, when adverse selection is relatively mild or severe, taking into account the effects of investment efficiency in period 1 will typically lead a policymaker to intervene more than he would otherwise. When adverse selection is more moderate,

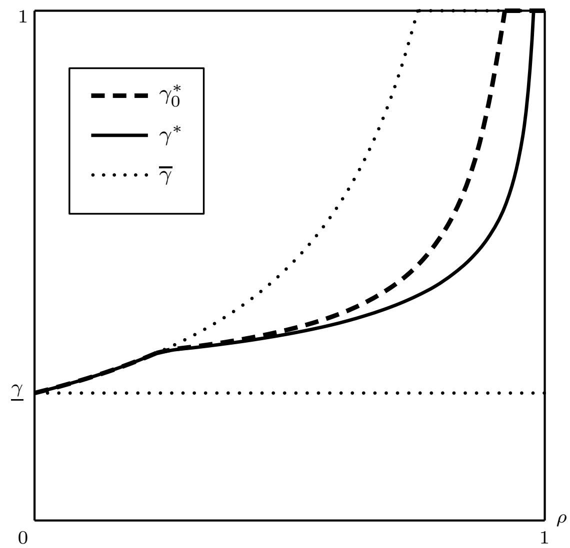

Figure 4: The overall optimal policy γ^* (solid) and the period-0 optimal policy γ_0^* (dashed) as functions of π_0 (left) and ρ (right).

on the other hand, the effects of information spillovers are less clear, and depend on the cost of public funds. In particular, as in the discussion above, when λ is small and policymakers would tend to be aggressive, information spillovers would be a force for more moderate intervention. When adverse selection is moderate and λ is large, on the other hand, information spillovers are again a rationale for even more intervention.

The Precision of the Signal. To conclude this section, we examine the relationship between the optimal intervention and the precision of the signal in period 0, ρ. Figure 4 plots γ_0^* and γ^* as a function of ρ when λ is relatively large.[14] First, note that no intervention is optimal in period 0 when ρ is small. Moreover, since the potential to generate information spillovers is limited when the signal is imprecise, no intervention also maximizes total welfare in this region of the parameter space. Then, as the signal becomes more precise, the presence of information spillovers provides

[14]One can derive comparative statics with respect to ρ analytically, though the analysis requires considering several different cases that depend on the combination of λ, π_0, and c. The numerical example we consider here illustrates the main economic insights that comes from this analysis, while avoiding the tedious, case-by-case algebra.

extra incentive to intervene, and hence $\gamma_0^* > \gamma^*$.[15] Finally, as the signal becomes very precise, the optimal policy in period 0 is again no intervention, as the availability of good information mitigates the initial lemons problem. However, incorporating period 1 payoffs shrinks the region where policymakers choose not to intervene; since the incidence of trade is a very valuable signal to the investor when ρ is close to 1, maximizing total welfare requires subsidizing trade in period 0, even when it has a very small effect on the payoffs of the buyer and seller.

4 Extensions and Robustness

In the previous section, we considered the simplest possible model in order to capture a fundamental tradeoff between providing incentives to promote trade and ensuring that these trades generate valuable information. In particular, we considered: (*i*) bilateral trade between a single buyer and a single seller; (*ii*) a uni-dimensional policy choice for the government; (*iii*) a highly stylized structure for the signal available to the buyer; and (*iv*) a very specific use of the information generated by trade, in the form of a simple investment decision. In this section, we systematically relax each of these assumptions, leaving all others in place. We show that the main insights we derive in our benchmark model survive. We also highlight several additional insights that emerge from these more complex (and, perhaps, more realistic) environments.

4.1 An Alternative Model of Period-0 Trade

In this section, we consider an alternative period-0 trading environment—a first-price auction with $N \geq 2$ buyers. We do this for several reasons. First, it is important to confirm that the basic insights generated in our benchmark model extend to settings with alternative trading protocols and multiple buyers. An auction is a natural choice for us, as it provides rigorous micro-foundations for price formation and, hence, has served as a workhorse model in the literature on information aggregation.[16] Second, though the bilateral bargaining problem we studied in our benchmark model

[15]For the same reasons discussed above, if we consider the case where λ is sufficiently small instead, then $\gamma_0^* < \gamma_1^*$ for intermediate values of π_0, and hence $\gamma^* > \gamma_0^*$.

[16]See, e.g., Wilson (1975), Milgrom (1979), Pesendorfer and Swinkels (1997), Pesendorfer and Swinkels (2000), Kremer (2002), Jackson (2003), and Lauermann and Wolinsky (2013) for information aggregation in large markets.

was highly tractable, our assumptions implied that the investor only learned from whether or not trade occurred, i.e., the *extensive* margin, but did not learn from the transaction price itself, i.e., the *intensive* margin. In the auction setting we consider, both of these margins are active, so that we can study how policy affects not only the information contained in trade, but the informational content of prices as well.[17] Lastly, it turns out that the auction setting is a fairly accurate description of the actual trading mechanism that was used to promote trade and information production in the aftermath of the 2007-2008 financial crisis, as part of the so-called Public Private Investment Program for Legacy Assets (or PPIP). Given the initial size and scope of this program, and the non-trivial possibility that a similar program will be utilized in future crises, we feel that developing a framework to understand the inherent trade-offs from this type of intervention—and to formalize the concept of an "optimal" policy—is an important contribution.[18]

We find that, in the context of an auction setting, the relationship between the level of insurance provided by the government and the incentive for buyers to acquire information is quite similar to the relationship we derived within the context of the bilateral bargaining game, so that the fundamental tension between ensuring trade and promoting information production is preserved in this market setting. However, our analysis also reveals new insights. In particular, since the winner's curse emerges in an environment with multiple bidders—and this discourages information acquisition—the policy that maximizes information production is more generous than the corresponding policy in the bilateral game; intuitively, the additional insurance offered in the auction setting counteracts the disincentives implied by the winner's curse.

Our work is more closely related to a recent literature on information aggregation in small markets, such as Ostrovsky (2012), Vives (2011), Rostek and Weretka (2012) and Rostek and Weretka (2015). This last paper is perhaps closest in spirit to ours, as they explore both information aggregation and welfare, albeit in a very different setting.

[17]Our analysis here is thus related to the literature that studies information acquisition in auctions; see, e.g., Matthews (1984), Persico (2000), Bergemann and Valimaki (2002), and Bergemann et al. (2009). Unlike our work, these papers are interested in neither the informational content of the winning bid, nor the effects of any form of intervention on information acquisition.

[18]As Timothy Geithner (then U.S. Secretary of the Treasury) described, PPIP would have initially provided financing for $500 billion of purchasing power for legacy loans and securities, with the potential to expand up to $1 trillion over time ("My Plan for Bad Banks Assets," The Wall Street Journal, March 23, 2009).

The Model. In order to minimize the departure from our benchmark model, we assume that in period 0 there is a single seller with an asset of quality $q_0 \in \{L, H\}$, which is the seller's private information, and $N \geq 2$ ex ante homogeneous buyers. The preferences and inspection technology are the same as before. In particular, each buyer $i \in \{1, \dots, N\}$ can acquire a signal $s_i \in \{\ell, h\}$ at cost k_i, where each s_i is an i.i.d. draw from the distribution described in (1) and each k_i is an i.i.d. draw from a cumulative distribution function $G(k)$, which we assume has the same properties as in Section 3. Each buyer's inspection decision is unobservable to other buyers.

After buyers have the opportunity to acquire a signal, each one can submit a bid. The seller then either accepts the highest bid, in which case trade occurs at that bid price, or rejects them all. The investor's problem in $t = 1$ is unchanged and, as in the benchmark model, all uncertainty is resolved in $t = 2$.

To facilitate comparison with our earlier results, we consider the same policy as before, in which a buyer who has acquired a low quality asset can surrender the asset in exchange for a transfer $\tau = (1 - \gamma)p$ from the government, with $\gamma \in [0, 1]$. In order to focus on the most relevant cases, we also make use of the following two assumptions:

$$\pi_0(v - c) - (1 - \pi_0)c < 0; \tag{4}$$

$$\pi^h(v - c) - (1 - \pi^h)c > 0. \tag{5}$$

As before, π^h is the posterior belief of a buyer who observes the signal h. The first assumption implies that the initial adverse selection problem is sufficiently severe that buyers are not willing to place a "serious" bid $b \geq c$ without inspecting the asset. If we let $\underline{\gamma}$ be given by (2), then (4) implies that $\underline{\gamma} < 1$. The second assumption implies that inspection is sufficiently informative to generate the potential for trade; that is, a buyer who receives the good signal is willing to bid $b \geq c$. If we let $\overline{\gamma}$ be given by (3), then (5) implies that $\overline{\gamma} = 1$.

Strategies and Equilibrium The optimal strategies of the seller in period 0 and the investor in period 1 are the same as in the benchmark model, and hence we take their behavior as given in what follows. We also maintain our assumption that, when indifferent between accepting and rejecting an offer, the seller rejects if her asset is of quality L but accepts if her asset is of quality H.

Now consider the strategy of a buyer in period 0, which has two components. First, a buyer must decide whether to inspect the asset. Again, the optimal inspection strategy is a cutoff rule: buyer i inspects the asset if, and only if, he draws $k_i \leq k$ for some cutoff cost k. Second, after receiving a signal $s \in \{\ell, h\}$, or after deciding to remain uninformed ($s = u$), a buyer formulates an optimal bidding strategy as a function of his signal $s \in \{\ell, h, u\}$. Anticipating that our equilibrium will involve mixed strategies, we let the cumulative distribution function $F_s(b)$ represent the mixed bidding strategy of a buyer with signal s, i.e., $F_s(b)$ is the probability that a buyer with signal s bids b or less. We focus on symmetric equilibria of the period-0 game.

Trade in Period 0. Notice that the buyers' equilibrium inspection and bidding strategies must be jointly characterized: the value of inspection for buyers depends on their bidding behavior, which in turn depends on the inspection decisions of other buyers. This makes the analysis significantly more complicated than that of the single-buyer case. In the interest of space, we sketch the intuition behind the equilibrium construction here, and relegate the formal arguments to the Appendix.[19]

Loosely speaking, the equilibrium construction proceeds in two steps. First, we take as given the probability η that each buyer acquires the signal. We then solve for the equilibrium bidding strategy of buyers for each signal $s \in \{\ell, h, u\}$, taking as given both η and the policy γ, and use the payoffs from these bidding strategies to derive the cutoff cost for inspection, $k(\eta, \gamma)$. Then, in the second step, we identify the equilibrium probability of inspection given the policy γ, $\eta(\gamma)$, as a solution to the fixed point problem $\eta = G(k(\eta, \gamma))$. Given $\eta(\gamma)$, we construct the equilibrium cutoff cost for inspection, which is $k(\gamma) = k(\eta(\gamma), \gamma)$.

In the Appendix, we show that for each γ there exists a unique symmetric equilibrium of the period-0 game and provide a complete characterization of this equilibrium. The structure of this equilibrium depends on whether γ is above or below a threshold $\widetilde{\gamma} \in (0, \underline{\gamma})$. In particular, when $\gamma \geq \widetilde{\gamma}$, so that the government is providing relatively little insurance, uninformed buyers

[19]Our equilibrium construction resembles the equilibrium construction in Cao and Shi (2001), who consider (costly) information acquisition in a common value auction with a similar information structure. The focus of their analysis is different, though. Unlike us, they study how information acquisition and bidding behavior depend on the number of bidders in the auction, and use their results to explain some facts about the market for bank loans in the U.S.

always bid 0, while informed buyers bid according to a continuous distribution $F_h(b)$ with support $[c, \bar{b}_h]$, where $\bar{b}_h > c$. On the other hand, when $\gamma < \widetilde{\gamma}$, so that the government is providing a relatively high level of insurance, uninformed buyers also place serious bids. More specifically, when $\gamma < \widetilde{\gamma}$, uninformed buyers draw bids from a distribution $F_u(b)$ that contains a mass point at zero, and is otherwise continuous and strictly increasing in $[c, \bar{b}_u]$, with $\bar{b}_u > c$. Meanwhile, buyers who received the signal h continue to bid more aggressively than uninformed buyers: they bid according to a continuous distribution $F_h(b)$ with support $[\underline{b}_h, \bar{b}_h]$, where $\bar{b}_u = \underline{b}_h < \bar{b}_h$.

In Lemma 4, below, we use the characterization of the equilibrium of the period-0 game to examine how the cutoff cost for inspection responds to changes in policy. As in the benchmark model, we show that $k(\gamma)$ is single-peaked, minimized when the government provides full insurance, and maximized at an interior level of subsidy when adverse selection is severe. Indeed, the only difference is that $k(\gamma)$ is maximized at a value of $\gamma = \widetilde{\gamma} \in (0, \underline{\gamma})$ in the auction setting, whereas $k(\gamma)$ was maximized at $\gamma = \underline{\gamma}$ in the bilateral bargaining game.

Lemma 4. *There exists a unique $\tilde{\gamma} \in (0, \underline{\gamma})$ such that $k(\gamma)$ is strictly increasing in γ when $\gamma \leq \tilde{\gamma}$ and strictly decreasing otherwise. Moreover, $k(\gamma)$ converges to zero as γ decreases to zero.*

Intuitively, the source of the non-monotonicity described in Lemma 4 is similar to what we described after Lemma 1. When γ is relatively large, a marginal reduction in γ provides additional insurance to informed buyers, who bid $b \geq c$ when they receive the signal h, and hence increases the expected payoff to a buyer from acquiring information. Since uninformed buyers do not bid $b \geq c$ in this region of the parameter space, their expected payoff is unaffected by the reduction in γ. Hence, the incentive to become informed increases as γ decreases in this region. On the other hand, when γ is relatively small, the moral hazard problem emerges: buyers have incentive to bid $b \geq c$ even when they are uninformed. In this region, as γ decreases, uninformed buyers bid more aggressively and the payoff to informed buyers falls. As a result, buyers become less willing to acquire the costly signal as γ decreases in this region.

The threshold $\widetilde{\gamma}$ is the value of γ that makes uninformed buyers exactly indifferent between bidding $b \geq c$ and not bidding. In the bilateral bargaining model, the corresponding value of γ was

$\underline{\gamma}$. When $N \geq 2$, an uniformed buyer requires more insurance to be indifferent, as this compensates him for the adverse effects of the winner's curse. Hence, $\widetilde{\gamma} < \underline{\gamma}$.

Maximizing Net Gains from Trade. We close this subsection by reporting some key properties of the policy that maximizes the net gains from trade in period 0, which we again denote γ_0^*. As in the benchmark model, we establish that it is optimal to fully insure buyers—which guarantees that all gains from trade are realized—when the cost of public funds converges to zero. On the other hand, the size of the optimal government intervention converges to zero as these costs become large. The proof is in the Appendix.

Proposition 4. *The policy γ_0^* is such that* $\lim_{\lambda \to 0} \gamma_0^* = 0$ *and* $\lim_{\lambda \to \infty} \gamma_0^* = 1$.

Investment in Period 1. We now turn our attention to the relationship between the policy γ, the informational content of trade and prices in period 0, and the payoffs of the investor in period 1. Let $\phi(p;\gamma)$ denote the investor's posterior belief that the asset is of quality H after observing a winning bid p when the government policy is γ, which can be constructed using the equilibrium strategies of buyers. The following result reports basic properties of $\phi(p;\gamma)$.

Lemma 5. *The posterior belief $\phi(p;\gamma)$ satisfies the following properties:* (*i*) $\phi(0;\gamma) < \phi(c;\gamma)$*;* (*ii*) $\phi(p;\gamma)$ *is strictly increasing in p when $p \in [c, \overline{b}_h]$; and* (*iii*) $\phi(\overline{b}_h;\gamma) = \pi^h$.

The first two facts in Lemma 5 are intuitive. Indeed, since buyers who observe ℓ only bid zero, while buyers who observe u or h sometimes bid $b \geq c$, observing trade at some price $p \geq c$ is more indicative that the asset is of high quality than observing no trade. Moreover, as p increases, so too does the conditional probability that the losing buyers received signal h or u (as opposed to ℓ) but bid $b \leq p$. However, for any $p < \overline{b}_h$, bids less than p are more likely when the asset is of low quality. Hence, $\phi(p;\gamma) < \pi^h$ for all $p \in \left[c, \overline{b}_h\right)$ because of what the investor infers about the losing bids. It is only when $p = \overline{b}_h$ that observing the winning bid is equivalent to observing the signal h, for in this case bids less than $\overline{b}_h$ have the same probability regardless of the asset's type.

Maximizing Information Production. We now establish that, as in the benchmark model, the policy that maximizes information production is interior, which confirms the intuition that promoting price discovery requires *some intervention* in order to encourage buyers to trade, but *not so much* intervention that the typical buyer is trading without first inspecting the asset.

Proposition 5. *The investor's expected payoff achieves a maximum at some* $\gamma_1^* \in (0, \widetilde{\gamma})$.

To understand the result in Proposition 5, suppose first that $\gamma > \widetilde{\gamma}$. In this region, an increase in γ causes less information acquisition, as $k(\gamma)$ declines, and has no effect on the bidding behavior of uninformed buyers. Hence, one can show that the distribution of posterior beliefs increases in the sense of second-order stochastic dominance as γ increases, which implies that the investor's expected payoff decreases in γ over this interval; much like the case of a single buyer, government policy affects only the extensive margin in this region of the parameter space.

This is not the case, however, when $\gamma < \widetilde{\gamma}$. In this region, an increase in γ causes more information acquisition—that is, $k(\gamma)$ increases—but there are also effects on the intensive margin. In particular, as γ increases, $\bar{b}_u$ falls and fewer uninformed buyers bid $b \geq c$. As a result, uninformed buyers become less distinguishable from buyers who received the signal ℓ, and thus the informational content of prices falls. Indeed, one can show that this latter effect dominates at values of γ sufficiently close to $\widetilde{\gamma}$, so that $V_I(\gamma)$ achieves a maximum at $\gamma < \widetilde{\gamma}$. Intuitively, when γ is close to $\widetilde{\gamma}$, the effect of a change in γ on the extensive margin is of second-order importance, and so the intensive margin dominates. Moreover, $V_I(\gamma)$ achieves a minimum at $\gamma = 0$, for in this case there is no information acquisition, and so the distribution of posterior beliefs is concentrated at the prior belief. Thus, γ_1^* must lie in $(0, \widetilde{\gamma})$.

Discussion. Propositions 4 and 5 imply that the main insights derived from our benchmark model extend to this alternative model of period-0 trade, with multiple buyers and a different trading protocol. Namely, as in our benchmark model, $\gamma_1^* > \gamma_0^*$ when the cost of public funds is small and there is a tension between promoting trade and information production. Alternatively, when the cost of public funds is large, $\gamma_1^* < \gamma_0^*$ and considerations for information production provide policymakers with added incentive to intervene.

4.2 More General Policy Options

A natural concern is whether the tension we highlight between generating trade and promoting information production is a consequence of the limited form of intervention we allow the policymaker to utilize. In this section, we enrich the set of tools available to the policymaker. We show that offering more instruments does *not* resolve the fundamental tension derived in Section 3; if anything, the nature of this tension actually *sharpens*. In addition to confirming some of our previous results, the analysis below also generates new insights that may be relevant for future interventions aimed at promoting information production.

Description of Policy. We maintain our assumption that the buyer observes the quality of the asset and then reports to the policymaker whether it is of high or low quality before the dividends are realized in period 2. As in the analysis above, if the buyer acquires an asset at price p and reports quality L, he forfeits the asset and receives a transfer of $(1-\gamma_L)p$ from the government, with $\gamma_L \in [0,1]$. However, if the same buyer reports quality H, he retains the asset and must pay the government a fraction of the profits, $(1-\gamma_H)(v-p)$, with $\gamma_H \geq 0$.[20] Notice that our baseline model corresponds to the case where $\gamma_H = 1$. Incentive compatibility for the buyer after acquiring a low- or high-quality asset, respectively, are

$$(1-\gamma_L)p \;\geq\; -(v-p)(1-\gamma_H) \tag{6}$$

$$v-(1-\gamma_H)(v-p) \;\geq\; (1-\gamma_L)p. \tag{7}$$

Introducing a second dimension to the policymaker's toolbox, in the form of γ_H, is important for at least two reasons. First, as we establish below, allowing for a multi-dimensional policy reveals important new insights about the optimal design of interventions. Second, this formulation more accurately captures the Public-Private Investment Program for Legacy Assets that was ac-

[20]The assumptions that $\gamma_L \leq 1$ and $\gamma_H \geq 0$ are ex-post participation constraints. If $\gamma_L > 1$ ($\gamma_H < 0$), the buyer would have an incentive to keep the asset for himself and make no report to the government in case he purchased a low (high) quality asset. Moreover, it is clear that it is not optimal for the government to set $\gamma_L < 0$, otherwise the buyer would have an incentive to make an arbitrarily large offer p and report L. Notice, however, that we *do* allow $\gamma_H > 1$, in which case the buyer receives a transfer *from* the government if he reports H. Also notice that $\gamma_L \in [0,1]$ and $\gamma_H \geq 0$ imply that (7) automatically holds.

tually used during the recent financial crisis. In particular, under this program, the government not only insured investors against losses, but also took a portion of the gains when the investment proved profitable.

We assume that government interventions are costly, regardless of whether the government offers a transfer to, or receives a transfer from, the agents in the economy. Formally, the social cost of a transfer $\tau \in \mathbb{R}$ is $(1+\lambda_+)\tau$ if $\tau > 0$ and $(1+\lambda_-)\tau$ if $\tau < 0$, where $\lambda_- \leq 0 < \lambda_+$. This assumption captures in a reduced-form way that there are distortions in both *raising* funds to pay for this intervention and *spending* tax revenue. That is, it costs more than \$1 for the government to inject \$1 into the market, while \$1 in the hands of the government is worth less than \$1 in the hands of the private agents in the economy. The assumption that $\lambda_- \leq 0$ implies that the government has no incentive to intervene in the market without any form of market failure.

To highlight our main insights most clearly, we will focus on the case where λ_+ is sufficiently small that $\gamma_0^* \leq \underline{\gamma}$ in our baseline model i.e., when $\gamma_H = 1$; recall that this is the case where considerations for information spillovers provide incentive for less government intervention. The discussion below can be modified in a straightforward fashion to cover the opposite case where λ_+ is sufficiently large.

The Buyer's Behavior. Fix the policy (γ_L, γ_H), and let $\gamma \equiv \gamma_L/\gamma_H$. Since both γ_L and γ_H are nonnegative, the buyer either offers 0 or c. Following the same steps as in Section 3, and using the same notation, one can show that the expected payoffs of the uninformed buyer and the informed buyer are

$$V_B^u(\gamma_L, \gamma_H) = \max\{\pi_0\gamma_H(v_H - c) - (1-\pi_0)\gamma_L c, 0\},$$

and

$$V_B^i(\gamma_L, \gamma_H) = \max\{\pi_0\gamma_H(v_H - c) - (1-\pi_0)(1-\rho)\gamma_L c, 0\},$$

respectively. Let $k(\gamma_L, \gamma_H)$ denote the cutoff cost such that the buyer inspects the asset if, and only if, $k \leq k(\gamma_L, \gamma_H)$. Since $k(\gamma_L, \gamma_H) = V_B^i(\gamma_L, \gamma_H) - V_B^u(\gamma_L, \gamma_H)$, it follows that

$$k(\gamma_L, \gamma_H) = \begin{cases} 0, & \text{if} \quad \gamma > \overline{\gamma}, \\ \pi_0 \gamma_H (v - c) - (1 - \pi_0)(1 - \rho)\gamma_L c, & \text{if} \quad \gamma \in (\underline{\gamma}, \overline{\gamma}), \\ (1 - \pi_0)\rho \gamma_L c, & \text{if} \quad \gamma \in [0, \underline{\gamma}], \end{cases}$$

where $\underline{\gamma}$ and $\overline{\gamma}$ are identical to those in Section 3.

Figure 5 describes how the cutoff cost varies according to γ_L and γ_H. If the ratio γ is smaller than $\underline{\gamma}$, then $k(\gamma_L, \gamma_H)$ is independent of γ_H and the level set is a horizontal line. In this parameter region, both the uninformed buyer and the buyer who observes the signal h offer c, and thus a high quality asset always trades. Therefore, γ_H, which is relevant only when the asset is of high quality, does not influence the buyer's incentive regarding information acquisition. If the ratio γ exceeds $\underline{\gamma}$, then the level curve is strictly increasing. In this parameter region, the buyer obtains a positive expected payoff only when he acquires information and observes $s = h$. Therefore, the two policy variables become substitutes for each other: if γ_L increases (less subsidy in state L), then γ_H must increase (less transfer in state H) to preserve the buyer's incentive to acquire information.

Optimal Policies. Using the buyer's optimal response to different policy choices, (γ_L, γ_H), we now establish several important results. First, the policy maximizing net payoffs in period 0 is unchanged from our benchmark model: ignoring the effect of information spillovers, the government optimally sets γ_L equal to the value γ_0^* derived in Section 3 and γ_H equal to 1. Second, the ratio γ that maximizes information production is also unchanged from our benchmark model: focusing exclusively on period-1 payoffs, the government optimally chooses γ_H and γ_L so that $\gamma = \underline{\gamma}$ and the uninformed buyer is indifferent between making a serious offer and not.[21] However, to maximize information production, in fact the government sets $\gamma_H > 1$—that is, it *rewards* buyers for acquiring a high quality asset—and this policy induces more information acquisition than in the benchmark model with a uni-dimensional policy. As such, the difference between the

[21]For the same reasons as in the benchmark model, for a fixed value of γ_H, $k(\gamma_L, \gamma_H)$ increases in γ_L when $\gamma < \underline{\gamma}$ and decreases in γ_L otherwise.

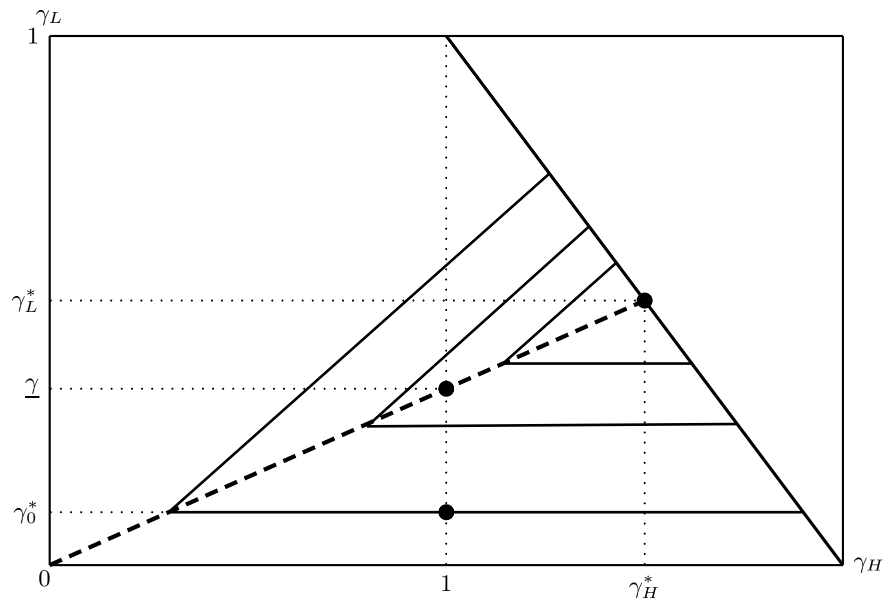

Figure 5: The three kinked lines represent the level sets $\{(\gamma_L, \gamma_H) : k(\gamma_L, \gamma_H) = k\}$ for three different values of k. The diagonal solid line depicts the incentive compatibility constraint for a low quality asset, (6).

amount of information acquisition that maximizes period-0 and period-1 payoffs *increases* when policymakers have access to a richer set of instruments.

Proposition 6. *If λ_+ is sufficiently small, then the policy maximizing period-0 payoffs coincides with the optimal period-0 policy when γ_H is constrained to be equal to 1. In addition, the policy that maximizes period-1 payoffs is $\gamma_H^* = v/[v - c(1 - \underline{\gamma})] > 1$ and $\gamma_L^* = \gamma_H^* \underline{\gamma}$.*

In Figure 5, we plot the optimal policies for period-0 and period-1 payoffs in both the benchmark model and in the extended model with a more general policy. To understand why the policy maximizing period-0 payoffs is unchanged, notice that the buyer's behavior is constant over any horizontal line below $\gamma \leq \underline{\gamma}$: on any such line, the probability that the buyer acquires information is independent of γ_H, and thus gains from trade are constant in γ_H, too. Since government interventions are always costly, the optimal policy is the one that minimizes any sort of government intervention, which reduces to setting $\gamma_H = 1$ and $\gamma_L = \gamma_0^*$.

Now consider the policy that maximizes information production, and thus payoffs in period 1. As in the benchmark model, the policy that maximizes information production is the policy that maximizes the buyer's incentive to acquire information, i.e., the policy that maximizes $k(\gamma_L, \gamma_H)$.[22] As one can see from Figure 5, (γ_L^*, γ_H^*) is the policy that maximizes $k(\gamma_L, \gamma_H)$ within the set of feasible, incentive compatible policy pairings.

To understand why $\gamma_H^* > 1$, consider a policy (γ_L, γ_H) with $\gamma = \underline{\gamma}$. An increase in γ_H, keeping the ratio γ constant, causes $V_B^i(\gamma_L, \gamma_H)$ to increase while keeping $V_B^u(\gamma_L, \gamma_H)$ constant and equal to zero. Hence, the policymaker maximizes information acquisition by setting γ_H to the maximum possible value, subject to satisfying the incentive constraint (6). Intuitively, since the government wants to promote *both* participation and information acquisition, it wants to increase the buyer's payoffs from trading in a way that disproportionately rewards outcomes that are more likely to occur for a buyer who inspects the asset.[23]

Importantly, this last result suggests that the tension between maximizing gains from trade and maximizing information production becomes *more pronounced* when the government possesses additional policy instruments, as the optimal policy for information production that is valuable in period 1 moves further away from the policy that maximizes social gains from trade in period 0.

4.3 Alternative Information Structures

In the benchmark model, the information structure in period 0 had the following features: (*i*) there is a cost to acquire information; (*ii*) there are only two signals; (*iii*) the precision of the signals is asymmetric, i.e., $\mathbb{P}(h|H) \neq \mathbb{P}(\ell|L)$; and (*iv*) the precision of the signals is exogenously determined. In this subsection, we demonstrate that, while these assumptions make it easier to derive a number of results in our benchmark model, our main qualitative results are robust to other specifications. Specifically, we consider two alternative information structures for period-0 asset trade: the first jointly relaxes (*i*) and (*ii*), while the second jointly relaxes (*iii*) and (*iv*).

[22] To understand why this is true, note that, as in the benchmark model, the distribution of the investor's posterior beliefs decreases in the sense of second-order stochastic dominance as $k(\gamma_L, \gamma_H)$ increases.

[23] This result is consistent with a fundamental principle in information economics—namely, that the optimal way to mitigate moral hazard is to reward the agent when the realized outcome is the one that is most likely to occur when the agent takes a desirable action. See Hölmstrom (1979) and Milgrom (1981).

Costless Information Acquisition with Many Signals. Suppose the buyer in period 0 receives a signal s about the quality of the asset at no cost. Let $F_q(s)$ denote the cumulative distribution function from which the signal s is drawn when the asset is of quality $q_0 = q \in \{L, H\}$. Assume that $F_q(s)$ has everywhere-positive density $f_q(s)$ for $q \in \{L, H\}$, and that the likelihood ratio function $l(s) \equiv f_H(s)/f_L(s)$ is strictly increasing, with $l(0) > 0$ and $l(1) < \infty$.

Given a policy $\gamma \in [0, 1]$, the buyer offers c if, and only if,

$$\pi_0 f_H(s)(v - c) - (1 - \pi_0) f_L(s) \gamma c \geq 0,$$

which is equivalent to

$$l(s) \geq \frac{(1 - \pi_0)\gamma c}{\pi_0 (v - c)}. \tag{8}$$

Let γ^1 and γ^2 be such that

$$\gamma^1 = \min \left\{ \frac{\pi_0 (v - c)}{1 - \pi_0} l(0), 1 \right\},$$

and

$$\gamma^2 = \min \left\{ \frac{\pi_0 (v - c)}{1 - \pi_0} l(1), 1 \right\}.$$

Since $l(s)$ is strictly increasing, the buyer's optimal offer strategy is a cutoff rule: offer c if, and only if, $s \geq s(\gamma)$, where

$$s(\gamma) = \begin{cases} 0 & \text{if} \quad \gamma < \gamma^1, \\ l^{-1}\left(\dfrac{(1 - \pi_0)\gamma c}{\pi_0 (v - c)} \right) & \text{if} \quad \gamma \in [\gamma^1, \gamma^2] \\ 1 & \text{if} \quad \gamma > \gamma^2. \end{cases} .$$

Thus, if γ is sufficiently small, the buyer offers c regardless of his signal. On the other hand, if γ is sufficiently large, even the most optimistic buyer is unwilling to offer c. Finally, note that $s(\gamma)$ is strictly increasing in γ on (γ^1, γ^2): as the government provides more insurance, the buyer is willing to purchase the asset for a wider range of signals. Clearly, then, expected gains from trade decrease in γ.

As a result, the policy that maximizes net gains from trade in period 0, which we again denote by γ_0^*, optimally trades off between realizing these gains from trade and the social cost of government funds. It is possible to show that γ_0^* has the same properties as in the benchmark model. In particular, γ_0^* is increasing in the shadow cost of public funds λ, converges to γ^1 as λ decreases to zero, and converges to γ^2 as λ increases to infinity.

Unlike in our benchmark model, the policy that maximizes information production, which we denote by γ_1^*, depends on the precise shapes of the distributions F_H and F_L, as well as the investor's production technology, as captured by Y and K. Technically, this is because the (unconditional) distributions of the investor's posterior beliefs as functions of γ cannot be ranked in terms of second-order stochastic dominance. Nevertheless, it is clear that γ_1^* never takes an extremal value: if $\gamma \leq \gamma^1$, then trade always takes place in period 0, while if $\gamma \geq \gamma^2$, then the asset never trades in period 0. In both cases, the investor learns nothing from period-0 trade: the distribution of his posterior beliefs is degenerate at the prior π_0. Given the strict convexity of the investor's indirect utility function, both policy choices are suboptimal from the period-1 perspective. Hence, in general, $\gamma_1^* \in (\gamma^1, \gamma^2)$.

The discussion above implies that the conclusions drawn in Proposition 3 are largely preserved under this alternative information structure. In particular, since γ_0^* is close to γ^1 when λ is small, while γ_1^* is independent of λ and bounded away from 0, it is clear that $\gamma_0^* < \gamma_1^*$ and thus $\gamma^* > \gamma_0^*$ when the shadow cost of public funds is small: the concern for information production induces the government to be more conservative. In the opposite case, when λ is sufficiently large, $\gamma_0^* > \gamma_1^*$ and thus $\gamma^* < \gamma_0^*$: the concern for information production provides an incentive for the government to intervene more aggressively than it would otherwise.

Symmetric Signals with Endogenous Precision. Now suppose, instead, that there are two signals, h and ℓ, and the information structure is symmetric: $\mathbb{P}(h|H) = \mathbb{P}(\ell|L) = \rho$, where $\rho \geq 1/2$. Moreover, suppose ρ is a choice variable: the buyer can acquire a signal with precision ρ at cost $g(\rho)$, where $g(\rho)$ is differentiable, strictly increasing, strictly convex, and such that $g'(1/2) = 0$.

As in the benchmark model, the buyer acquires information if, and only if, he conditions his offer on the realized signal, i.e., if he offers c if, and only if, he receives signal h. Then, a buyer who acquires information solves

$$V_B^i(\gamma) = \max_{\rho} \pi_0\rho(v-c) - (1-\pi_0)(1-\rho)\gamma c - g(\rho),$$

which has a unique optimal solution $\rho(\gamma)$. It is straightforward to establish that $\rho(\gamma)$ is increasing in γ, so that more insurance reduces the buyer's incentive to acquire information.

To complete the characterization of the buyer's optimal strategy, note that the expected payoff of an uninformed buyer is given by

$$V_B^u(\gamma) = \max\{\pi_0(v-c) - (1-\pi_0)\gamma, 0\}.$$

Now let γ^3 and γ^4 be the values of γ such that $V_B^u(\gamma^3) = V_B^i(\gamma^3)$ and $V_B^i(\gamma^4) = 0$.[24] If $\gamma \leq \gamma^3$, then the buyer does not acquire information and offers c. Alternatively, if $\gamma \in (\gamma^3, \gamma^4]$, then the buyer acquires information, chooses the signal precision $\rho(\gamma)$, and offers c if, and only if, he receives signal h. Finally, if $\gamma > \gamma^4$, then the buyer does not acquire information and offers 0.

Given the buyer's behavior, the policy maximizing net gains from trade in period 0 looks a lot like the optimal policy in the benchmark model: when λ is very small, the government sets $\gamma_0^* = \gamma^3$ to ensure that all buyers trade; when λ takes an intermediate value, γ_0^* falls into the interval (γ^3, γ^4) and increases in λ; and when λ is sufficiently large, γ_0^* is equal to γ^4 and no trade occurs.

Information production, on the other hand, is maximized when $\gamma = \gamma^4 = \gamma_1^*$. To see why, note that the distribution of the investor's posterior beliefs is degenerate if either $\gamma \leq \gamma^3$ or $\gamma > \gamma^4$, since the buyer simply does not acquire information in this range. Over the interval $(\gamma^3, \gamma^4]$, the investor's posterior beliefs are equal to

$$\pi_1^T(\gamma) = \frac{\pi_0\rho(\gamma)}{\pi_0\rho(\gamma) + (1-\pi_0)(1-\rho(\gamma))},$$

if trade occurs in period 0, and

$$\pi_1^N(\gamma) = \frac{\pi_0(1-\rho(\gamma))}{\pi_0(1-\rho(\gamma)) + (1-\pi_0)\rho(\gamma)},$$

[24]It is possible to show that there is a unique γ such that $V_B^u(\gamma) = V_B^i(\gamma)$.

if the asset does not trade in period 0. Since $\rho(\gamma)$ is strictly increasing, $\pi_1^T(\gamma)$ is strictly increasing in γ, while $\pi_1^N(\gamma)$ is strictly decreasing. This implies that the distribution of the investor's beliefs strictly decreases in γ in the sense of second-order stochastic dominance, which in turn implies that the investor's expected payoff is strictly increasing in γ over the interval $[\gamma^3, \gamma^4]$.

Given these properties of γ_0^* and γ_1^*, again the basic conclusions from Proposition 3 are preserved. Our main insights are thus robust to alternative information structures.

4.4 Alternative Period-1 Decision Problems

In our benchmark model, we introduced information spillovers by assuming that there was an investor who had to make a real investment decision and that trade generated information about the investment's uncertain return. In reality, many agents in the economy can benefit when trade is more informative. In this section, we introduce another plausible source of information spillovers, and show that the implications for the optimal government policy are essentially identical. This should convince the reader that the specific decision problem we considered in our benchmark model is not crucial for our results, per se. However, it is also important to note that more information production is not always better; to close this section, we introduce a different decision problem that highlights the fact that information spillovers can be positive *or negative*. Naturally, the sign of this externality has substantial effects on the optimal policy.

Bank Bailouts. Suppose that we replace the investor's problem in period 1 with a decision problem for the government. In particular, at $t = 1$, assume that there is an agent (the "bank") that owes another agent (the "creditor") an amount d to be paid at $t = 2$. The bank owns an asset of quality $q_1 \in \{L, H\}$ that pays a random dividend δ at $t = 2$. The quality q_1 is unknown to the government. If $\delta < d$, the bank must default and a deadweight loss D ensues. Otherwise, the bank repays the loan and retains the remainder, $\delta - d$. The dividend δ is drawn from a cumulative distribution function $G_q(\delta)$ when $q_1 = q \in \{L, H\}$. We assume that $G_L(\delta)$ and $G_H(\delta)$ are twice differentiable and strictly convex, with a common support $[0, \overline{\delta}]$, where $\overline{\delta} > d$. Moreover, $G_H(\delta) < G_L(\delta)$ for all

$\delta \in (0, \overline{\delta})$. Finally, for simplicity, we assume that $q_1 = q_0$.[25]

Now, suppose the government can inject an amount $e \geq 0$ of "equity" into the bank at $t = 1$, which can be used to repay the creditor at $t = 2$. Thus, when $q_1 = q$, by providing e units of equity, the government can reduce the probability of default from $G_q(d)$ to $G_q(d - e)$. However, as in the benchmark model, there is a cost of public funds: dispensing e units of equity has a social cost of θe, with $\theta > 0$. If we let π_1 be the government's posterior belief that $q_1 = H$, then the government's objective function in period 1 is

$$\begin{aligned} \widehat{V}_1(e, \pi_1) \;&=\; (1 - \pi_1) \left\{ -G_L(d - e)D + \int_{d-e}^{\overline{\delta}} (\delta + e - d) dG_L(\delta) \right\} \\ &\quad + \pi_1 \left\{ -G_H(d - e)D + \int_{d-e}^{\overline{\delta}} (\delta + e - d) dG_H(\delta) \right\} - (1 + \theta)e, \end{aligned}$$

which, after integration by parts, we can rewrite as

$$\widehat{V}_1(e, \pi_1) = \overline{\delta} - d - \left[(1 - \pi_1)G_L(d - e) + \pi_1 G_H(d - e)\right] D - \theta e.$$

The strict convexity of $G_L(\delta)$ and $G_H(\delta)$ implies that for each π_1 there is a unique $e^* = e^*(\pi_1)$ that maximizes $\widehat{V}_1(e, \pi_1)$. In an abuse of notation, let $\widehat{V}_1(\pi_1) = \widehat{V}_1(e^*(\pi_1), \pi_1)$ be the government's indirect utility function in period 1. The envelope theorem implies that

$$\widehat{V}_1'(\pi_1) = G_L(d - e^*(\pi_1)) - G_H(d - e^*(\pi_1)) > 0.$$

Now observe, by the implicit function theorem, that[26]

$$\frac{de^*(\pi_1)}{d\pi_1} = \frac{g_H(d - e^*(\pi_1)) - g_L(d - e^*(\pi_1))}{\pi_1 g_H'(d - e^*(\pi_1)) + (1 - \pi_1) g_L'(d - e^*(\pi_1))}.$$

It follows from this that $\widehat{V}_1''(\pi_1) > 0$. So, $\widehat{V}_1(\pi_1)$ is strictly increasing and strictly convex function of π_1. Hence, as in the benchmark model, the *ex ante* expected welfare in period 1, $V_1(\gamma) = \mathbb{E}[\widehat{V}_1(\pi_1)]$, is strictly increasing in γ when $\gamma < \underline{\gamma}$ and strictly decreasing in γ when $\gamma \in (\underline{\gamma}, \overline{\gamma}]$, and

[25] As in our benchmark model, one could easily relax the assumption that the qualities of the assets in $t = 0$ and $t = 1$ are *perfectly* correlated, so long as the correlation is sufficiently strong.

[26] A sufficient condition for $e^* > 0$ regardless of π_1 is that $\min\{g_L(d), g_H(d)\} > \theta$.

all of the results in Section 3 go through unchanged.

Spillovers to Other Traders. Now suppose that, in period 1, a new buyer and seller meet with the opportunity to trade. These agents have identical preferences to those in the benchmark model—the buyer places a value v on a high quality asset, the seller places a value $c \in (0, v)$ on a high quality asset, and a low quality asset is worth 0 to both. Moreover, assume that the quality of the seller's asset in period 1, q_1, is the same as the quality of the seller's asset in period 0, q_0. After observing the outcome of trade in period 0 and updating his beliefs, the buyer in period 1 makes a take-it-or-leave-it offer. In order to isolate the effects of information spillovers from trade in period 0, we assume that there is no government intervention in period 1.

In this case, the buyer offers c if $\pi_1 \geq c/v$ and 0 otherwise, so that the policy maximizing period-1 social surplus is the policy that maximizes the probability that $\pi_1 \geq c/v$. Maximizing this probability is different than maximizing the dispersion in posterior beliefs, though. In particular, when π_0 is small, information production in period 0 can be good for period-1 social payoffs: observing trade in $t = 0$ can generate posterior beliefs higher than c/v, so that gains from trade are realized in $t = 1$. However, if $\pi_0 \geq c/v$, then information production in period 0 can potentially *reduce* social welfare in period 1: observing no trade in $t = 0$ can generate a posterior belief lower than π_0, which can discourage buyers from trading even though such trades are socially beneficial.

In this latter case—when additional information is harmful—the optimal policy for period-1 welfare is to suppress information acquisition by setting $\gamma_1^* = 0$.[27] Therefore, when information production has negative effects on social welfare, the implications for policymakers can be quite different. For example, when information production increases welfare in period 1, the optimal policy can involve less subsidy than the policy that maximizes net gains from trade. However, when information production decreases welfare in period 1, the optimal policy always involves more subsidy than the policy that maximizes net gains from trade.

[27] Indeed, it is possible to show that welfare in period 1 is strictly decreasing in γ when $\pi_0 \geq c/v$.

5 Conclusion

One of the most important questions to emerge from the financial crisis of 2007-2008 was whether the government could (and should) intervene in a frozen market. As economists begin to grapple with this question, it is important to correctly identify the various costs and benefits of intervention. To date, most of the existing literature has identified the benefit of intervention as restoring gains from trade, while the costs of intervention have typically been associated with either a direct cost of taxpayer dollars or an indirect cost of encouraging risky behavior in the future (i.e., moral hazard).

This paper identifies and studies a margin that has been mostly ignored: information spillovers. Information produced in financial markets can have widespread effects on economic activity. We show that, in a market suffering from adverse selection, policymakers face an important trade-off between restoring gains from trade and maximizing information production; namely, while some amount of intervention may be required in order to incentivize buyers to participate in the market, too much intervention can erode the informational content of transaction prices. Hence, policymakers face a delicate balance, which must be calibrated based on, e.g., the cost of public funds and the severity of the lemons problem.

This paper is among the first to study the effects of government interventions in frozen markets on price discovery, and many important questions remain. Some of these questions apply to interventions in frozen markets *in general*. For example, it would be interesting to study how dynamic considerations would change sellers' incentives to trade their assets and buyers' incentives to produce information at a given time. If sellers have more than one asset, they may choose not to sell a portion of their assets to avoid marking other assets to market prices, as in Milbradt (2012) and Bond and Leitner (2015). Similarly, if buyers expect information to be revealed and prices to rise, they may choose to delay trading, as in Camargo and Lester (2014).

Still other questions apply to *specific* forms of government intervention in frozen markets. For example, in our analysis of the auction setting with $N \geq 2$ buyers—which captures many of the features of the PPIP—there are several important considerations that we did not study here. One obvious question is whether there is an optimal number of buyers that the government should let participate in each auction. Moreover, since the PPIP intervention reduced the amount of equity buyers needed to purchase assets, it also relaxed their budget constraints. This encourages participation, but can also discourage information acquisition by enhancing the winner's curse.[28] We explore both of these issues in some detail in the working paper version, Camargo et al. (2013), but leave a more comprehensive analysis for future work.

Finally, markets fail for reasons other than adverse selection, such as debt overhang ((Myers, 1977)), "cash-in-the-market" pricing ((Allen and Gale, 1994)), or financial contagion ((Allen and Gale, 2000)). When trade is disrupted for these reasons, the absence of information production may still be costly, but the most natural form of intervention may differ from the policies we consider. Hence, our analysis is best applied to markets where asymmetric information is a first-order problem, i.e., where the assets for sale are idiosyncratic and their properties are difficult for potential buyers to determine.[29] Therefore, a natural question remains: how do interventions affect information production when markets fail for reasons other than adverse selection? We leave this, too, for future work.

[28]Budget constraints relax the winner's curse since in their presence an individual can win a common value auction despite not being the most optimistic bidder.

[29]Commonly cited examples of assets that are vulnerable to adverse selection include derivatives like mortgage-backed securities and credit-default swaps, but asymmetric information is also relevant in many markets for real assets, like companies, capital, or real estate.

References

Franklin Allen and Douglas Gale. Limited market participation and volatility of asset prices. *The American Economic Review*, pages 933–955, 1994.

Franklin Allen and Douglas Gale. Financial contagion. *The Journal of Political Economy*, 108(1): 1–33, 2000.

Vladimir Asriyan, William Fuchs, and Brett Green. Information spillovers in asset markets with correlated values. *working paper*, 2015.

Tor-Erik Bakke and Toni M Whited. Which firms follow the market? an analysis of corporate investment decisions. *Review of Financial Studies*, 23(5):1941–1980, 2010.

Lawrence M Benveniste, Alexander Ljungqvist, William J Wilhelm, and Xiaoyun Yu. Evidence of information spillovers in the production of investment banking services. *The Journal of Finance*, 58(2):577–608, 2003.

Dirk Bergemann and Juuso Valimaki. Information acquisition and efficient mechanism design. *Econometrica*, 70(3):1007–1033, 2002.

Dirk Bergemann, Xianwen Shi, and Juuso Välimäki. Information acquisition in interdependent value auctions. *Journal of the European Economic Association*, 7(1):61–89, 2009.

Philip Bond and Itay Goldstein. Government intervention and information aggregation by prices. *forthcoming, Journal of Finance*, 2015.

Philip Bond and Yaron Leitner. Market run-ups, market freezes, and leverage. 2015.

Philip Bond, Alex Edmans, and Itay Goldstein. The real effects of financial markets. *Annu. Rev. Financ. Econ.*, 4(1):339–360, 2012.

Braz Camargo and Benjamin Lester. Trading dynamics in decentralized markets with adverse selection. *Journal of Economic Theory*, 153:534–568, 2014.

Braz Camargo, Kyungmin Kim, and Benjamin Lester. Subsidizing price discovery. *Federal Reserve Bank of Philadelphia, Working Paper No. 13-20*, 2013.

Melanie Cao and Shouyong Shi. Screening, bidding, and the loan market tightness. *European Finance Review*, 5(1-2):21–61, 2001.

Giovanni Cespa and Thierry Foucault. Illiquidity contagion and liquidity crashes. *Review of Financial Studies*, 27(6):1615–1660, 2014.

VV Chari, Ali Shourideh, and Ariel Zetlin-Jones. Reputation and persistence of adverse selection in secondary loan markets. *American Economic Review*, 104(12):4027–70, 2014.

Qi Chen, Itay Goldstein, and Wei Jiang. Price informativeness and investment sensitivity to stock price. *Review of Financial Studies*, 20(3):619–650, 2007.

Jonathan Chiu and Thorsten V. Koeppl. Market making of last resort: Optimal intervention when financial markets are frozen. *Working Paper, Bank of Canada and Queen's University*, 2011.

Douglas W Diamond and Raghuram G Rajan. Illiquid banks, financial stability, and interest rate policy. *Journal of Political Economy*, 120(3):552–591, 2012.

James Dow and Gary Gorton. Stock market efficiency and economic efficiency: Is there a connection? *The Journal of Finance*, 52(3), 1997.

Darrell Duffie, Piotr Dworczak, and Haoxiang Zhu. Benchmarks in search markets. *NBER working paper series*, No. 20620, 2014.

Emmanuel Farhi and Jean Tirole. Collective moral hazard, maturity mismatch and systemic bailouts. *The American Economic Review*, 102:60–93, 2012.

Mark Flannery. Stabilizing large financial institutions with contingent capital certificates. *CAREFIN Research Paper*, (04), 2010.

Thierry Foucault and Laurent Frésard. Cross-listing, investment sensitivity to stock price, and the learning hypothesis. *Review of Financial Studies*, 25(11):3305–3350, 2012.

Thierry Foucault and Thomas Gehrig. Stock price informativeness,cross-listings, and investment decisions. *Journal of Financial Economics*, 88(1):146–168, 2008.

William Fuchs and Andrzej Skrzypacz. Government interventions in a dynamic market with adverse selection. *Journal of Economic Theory*, 158:371–406, 2015.

Itay Goldstein and Ady Pauzner. Demand–deposit contracts and the probability of bank runs. *Journal of Finance*, 60(3):1293–1327, 2005.

Gary Gorton. Information, liquidity, and the (ongoing) panic of 2007. *American Economic Review*, 99(2):567–72, 2009.

Sanford J Grossman. The informational role of warranties and private disclosure about product quality. *Journal of Law and Economics*, pages 461–483, 1981.

Veronica Guerrieri and Robert Shimer. Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. *The American Economic Review*, 104(7):1875–1908, 2014.

Oliver Hart and Luigi Zingales. A new capital regulation for large financial institutions. *American Law and Economics Review*, 13(2):453–490, 2011.

Bengt Hölmstrom. Moral hazard and observability. *The Bell journal of economics*, pages 74–91, 1979.

Christopher L House and Yusufcan Masatlioglu. Managing markets for toxic assets. *Journal of Monetary Economics*, 70:84–99, 2015.

Matthew O. Jackson. Efficiency and information aggregation in auctions with costly information. *Review of Economic Design*, 8(2):121, 2003.

Ilan Kremer. Information aggregation in common value auctions. *Econometrica*, 70(4):1675–1682, 2002.

Stephan Lauermann and Asher Wolinsky. Search with adverse selection. *mimeo*, 2013.

Benjamin Lester. Breaking the ice: government interventions in frozen markets. *Business Review*, (Q4):19–25, 2013.

Steven Matthews. Information acquisition in discriminatory auctions. *Bayesian models in economic theory, ed. by M. Boyer, and R. Kihlstrom*, 49:1477–1500, 1984.

Robert L McDonald. Contingent capital with a dual price trigger. *Journal of Financial Stability*, 9 (2):230–241, 2013.

Konstantin Milbradt. Level 3 assets: Booking profits and concealing losses. *Review of Financial Studies*, 25(1):55–95, 2012.

Paul R Milgrom. A convergence theorem for competitive bidding with differential information. *Econometrica*, 47(3):679–688, 1979.

Paul R Milgrom. Rational expectations, information acquisition, and competitive bidding. *Econometrica*, pages 921–943, 1981.

Stewart C Myers. Determinants of corporate borrowing. *Journal of Financial Economics*, 5:147–175, 1977.

Michael Ostrovsky. Information aggregation in dynamic markets with strategic traders. *Econometrica*, 80(6):2595–2647, 2012.

Nicola Persico. Information acquisition in auctions. *Econometrica*, 68(1):135–148, 2000.

Wolfgang Pesendorfer and Jeroen M Swinkels. The loser's curse and information aggregation in common value auctions. *Econometrica*, 65(6):1247–1281, 1997.

Wolfgang Pesendorfer and Jeroen M Swinkels. Efficiency and information aggregation in auctions. *American Economic Review*, 90(3):499–525, 2000.

Thomas Philippon and Philipp Schnabl. Efficient recapitalization. *The Journal of Finance*, 68(1): 1–42, 2013.

Thomas Philippon and Vasiliki Skreta. Optimal interventions in markets with adverse selection. *The American Economic Review*, 102(1):1–28, 2012.

Marzena Rostek and Marek Weretka. Price inference in small markets. *Econometrica*, 80(2): 687–711, 2012.

Marzena Rostek and Marek Weretka. Information and strategic behavior. *Journal of Economic Theory*, 158:536–557, 2015.

Michael Spence. Consumer misperceptions, product failure and producer liability. *The Review of Economic Studies*, pages 561–572, 1977.

Phillip Swagel. The financial crisis: an inside view. *Brookings Papers on Economic Activity*, 2009 (1):1–63, 2009.

Jean Tirole. Overcoming adverse selection: How public intervention can restore market functioning. *The American Economic Review*, 102(1):29–59, 2012.

Xavier Vives. Strategic supply function competition with private information. *Econometrica*, 79 (6):1919–1966, 2011.

Robert Wilson. A bidding model of perfect competition. *The Review of Economic Studies*, 44(3): 511–518, 1975.

Appendix

Proofs of Results in Section 3

Proof of Proposition 1

Let $W_0(\gamma) = V_B(\gamma) + V_S(\gamma) - C(\gamma)$ be welfare in period 0. We first prove that $\text{argmax}_\gamma W_0(\gamma)$ is always compact and non-empty, so that $\gamma_0^* = \max\{\gamma : \gamma \in \text{argmax}_\gamma W_0(\gamma)\}$ is always well-defined, and establish some useful properties of $W_0(\gamma)$. We then establish the properties of γ_0^* as a function of λ, π_0, and c.

i) First observe that

$$\begin{aligned}
V_B(\gamma) &= (1 - G(k(\gamma)))V_B^u(\gamma) + G(k(\gamma))\{V_B^i(\gamma) - \mathbb{E}[k|k \le k(\gamma)]\} \\
&= V_B^u(\gamma) + G(k(\gamma))\{V_B^i(\gamma) - V_B^u(\gamma) - \mathbb{E}[k|k \le k(\gamma)]\} \\
&= V_B^u(\gamma) + G(k(\gamma))\{k(\gamma) - \mathbb{E}[k|k \le k(\gamma)]\}.
\end{aligned}$$

Since

$$\mathbb{E}[k|k \le k(\gamma)] = \frac{1}{G(k(\gamma))} \int_0^{k(\gamma)} k dG(k) = k(\gamma) - \frac{1}{G(k(\gamma))} \int_0^{k(\gamma)} G(k)dk,$$

we then have that

$$V_B(\gamma) = \max\{\pi_0(v - c) - (1 - \pi_0)\gamma c, 0\} + \int_0^{k(\gamma)} G(k)dk.$$

Now observe that if $\gamma > \underline{\gamma}$, so that trade occurs only when the buyer inspects the asset, then

$$V_S(\gamma) - C(\gamma) = \pi_0 c + (1 - \pi_0)G(k(\gamma))(1 - \rho)[1 - (1 + \lambda)(1 - \gamma)]c,$$

and that if $\gamma \le \underline{\gamma}$, so that the uninformed buyer also offers c, then

$$V_S(\gamma) - C(\gamma) = \pi_0 c + (1 - \pi_0)[G(k(\gamma))(1 - \rho) + 1 - G(k(\gamma))][1 - (1 + \lambda)(1 - \gamma)]c.$$

Hence, $\gamma > \underline{\gamma}$ implies that

$$W_0(\gamma) = \pi_0 c + (1 - \pi_0)G(k(\gamma))(1 - \rho)[(1 + \lambda)\gamma - \lambda]c + \int_0^{k(\gamma)} G(k)dk,$$

while $\gamma \le \underline{\gamma}$ implies that

$$W_0(\gamma) = \pi_0 v - (1 - \pi_0)\gamma c + (1 - \pi_0)(1 - \rho G(k(\gamma)))[(1 + \lambda)\gamma - \lambda]c + \int_0^{k(\gamma)} G(k)dk.$$

Clearly, $W_0(\gamma)$ is differentiable in γ for all $\gamma \neq \underline{\gamma}$. Moreover, if $\underline{\gamma}^+ \equiv \lim_{\varepsilon \searrow 0}(\underline{\gamma} + \varepsilon)$, then

$$W_0(\underline{\gamma}) = W_0(\underline{\gamma}^+) + (1 - \pi_0)(1 - G(k(\gamma)))[(1 + \lambda)\underline{\gamma} - \lambda]c. \tag{9}$$

We break the analysis in two cases: $(1 + \lambda)\underline{\gamma} - \lambda \leq 0$ and $(1 + \lambda)\underline{\gamma} - \lambda > 0$.

Suppose first that $(1 + \lambda)\underline{\gamma} - \lambda \leq 0$; notice that $\lambda > 0$ and $\underline{\gamma} < 1$ in this case. Given that $k'(\gamma) = (1 - \pi_0)\rho c$ if $\gamma < \underline{\gamma}$,

$$\frac{\partial W_0(\gamma)}{\partial \gamma} = (1 - \pi_0)c \left\{\lambda(1 - \rho G(k(\gamma))) - (1 - \pi_0)\rho^2 g(k(\gamma))[(1 + \lambda)\gamma - \lambda]c\right\} \tag{10}$$

for all $\gamma < \underline{\gamma}$. Hence, $W_0(\gamma)$ is strictly increasing in γ for all $\gamma \in [0, \underline{\gamma}]$. Now observe that $W_0(\underline{\gamma}) \leq W_0(\underline{\gamma}^+)$. Moreover, since $k'(\gamma) = -(1 - \pi_0)(1 - \rho)c$ if $\gamma > \underline{\gamma}$,

$$\frac{\partial W_0(\gamma)}{\partial \gamma} = (1 - \pi_0)(1 - \rho)c \{\lambda G(k(\gamma)) - (1 - \pi_0)g(k(\gamma))(1 - \rho)[(1 + \lambda)\gamma - \lambda]c\} \tag{11}$$

for all $\gamma > \underline{\gamma}$. Thus, $W_0(\gamma)$ is also strictly increasing in a neighborhood of $\underline{\gamma}$. Therefore, there exists $\varepsilon < 1 - \underline{\gamma}$ such that $W_0(\underline{\gamma} + \varepsilon) > W_0(\gamma)$ for all $\gamma < \underline{\gamma} + \varepsilon$. Given that $W_0(\gamma)$ is continuous in $[\underline{\gamma} + \varepsilon, 1]$, we can then conclude that $\mathrm{argmax}_\gamma \mathrm{W}_0(\gamma)$ is compact and non-empty.

Suppose now that $(1 + \lambda)\underline{\gamma} - \lambda > 0$. Since $\lim_{\gamma \nearrow \underline{\gamma}} W_0(\gamma) = W_0(\underline{\gamma}) > W_0(\underline{\gamma}^+)$, $W_0(\gamma)$ is upper semicontinuous at $\gamma = \underline{\gamma}$. Given that $W_0(\gamma)$ is continuous at any $\gamma \neq \underline{\gamma}$, it is an upper semicontinuous function then. So, $\mathrm{argmax}_\gamma \mathrm{W}_0(\gamma)$ is compact and non-empty as well.

ii) Fix $\pi_0 \in (0, 1)$ and $c \in (0, v)$. Suppose first that $\lambda = 0$. In this case, the private and social costs of any bid by the buyer coincide. Hence,

$$W_0(\gamma) = \pi_0 \left\{\left[G(k(\gamma)) + (1 - G(k(\gamma)))\mathbb{I}_{\{\gamma \leq \underline{\gamma}\}}\right] v + (1 - G(k(\gamma)))\mathbb{I}_{\{\gamma > \underline{\gamma}\}} c\right\} - \int_0^{k(\gamma)} k dG(k),$$

and so $W_0(0) = \pi_0 v$, while $W_0(\gamma) < \pi_0 v$ whenever $\gamma > 0$. Therefore, $\gamma_0^* = 0$. Suppose now that $\lambda > 0$. It follows easily from (10) that $\partial W_0(0^+)/\partial \gamma > 0$, so that $\gamma_0^* > 0$. Now observe from (10) and (11) and the fact that $g(k)$ is bounded away from zero that for all $\gamma > 0$, $\partial W_0(\gamma)/\partial \gamma < 0$ if λ is sufficiently small. Since (9) implies that $W_0(\underline{\gamma}) > W_0(\underline{\gamma}^+)$ if λ is small enough, we then have that $\lim_{\lambda \to 0} \gamma_0^* = 0$. Next, observe from (10) and (11) that

$$\frac{\partial^2 W_0(\gamma)}{\partial \lambda \partial \gamma} = (1 - \pi_0)c[1 - \rho G(k(\gamma)) + (1 - \pi_0)\rho^2 g(k(\gamma))(1 - \gamma)] > 0$$

for all $\gamma < \underline{\gamma}$, and that

$$\frac{\partial^2 W_0(\gamma)}{\partial \lambda \partial \gamma} = (1 - \pi_0)(1 - \rho)c[G(k(\gamma)) + (1 - \pi_0)g(k(\gamma))(1 - \rho)(1 - \gamma)c] > 0$$

for all $\gamma > \underline{\gamma}$. A standard monotone comparative statics argument then shows that γ_0^* is increasing in λ. To finish, notice from (10) and (11) that for all $\gamma \in (0, \underline{\gamma}) \cup (\underline{\gamma}, \overline{\gamma})$, $\partial W_0(\gamma)/\partial\gamma > 0$ if λ is sufficiently large. Whether $\underline{\gamma} < 1$ or $\underline{\gamma} = 1$, we can conclude that $\lim_{\lambda\to\infty} \gamma_0^* = \overline{\gamma}$; in the first case, $(1+\lambda)\underline{\gamma} - \lambda \leq 0$ if λ is large enough, which in turn implies that it is suboptimal to set $\gamma \leq \underline{\gamma}$.

iii) Fix $\lambda > 0$ and $c \in (0, v)$. First, let π_0 be small enough that $(1+\lambda)\overline{\gamma} - \lambda < 0$. It follows from (11) that $W_0(\gamma)$ is strictly increasing in γ when $\gamma \in (\underline{\gamma}, \overline{\gamma}]$. Since $(1+\lambda)\underline{\gamma} - \lambda < 0$ also implies $\gamma_0^* > \underline{\gamma}$ by (10), we can conclude that $\gamma_0^* = 1$. Now let π_0 be sufficiently close to one that $\underline{\gamma} = \overline{\gamma} = 1$. Given that $\rho^2 g(k(\gamma))[(1+\lambda)\gamma - \lambda]c$ is bounded, (10) implies that $W_0(\gamma)$ is strictly increasing in γ when $\gamma \in [0, \underline{\gamma}]$, so that $\gamma_0^* = 1$ as well. To finish, suppose that π_0 is such that $\overline{\gamma} < 1$, in which case $k(\overline{\gamma}) = \overline{0}$. Then, using once again the fact that $g(k)$ is bounded away from zero, (11) implies that $\lim_{\gamma\nearrow\overline{\gamma}} \partial W_0(\gamma)/\partial\gamma < 0$ as long as λ is small enough that $(1+\lambda)\overline{\gamma} - \lambda > 0$; recall that $G(0) = 0$.

iv) Fix $\lambda > 0$ and $\pi_0 \in (0, 1)$. First, suppose that c is close enough to v that $(1+\lambda)\overline{\gamma} - \lambda < 0$; notice that $\overline{\gamma} \to 0$ as $c \to v$. The same argument as in the previous step shows that $\gamma_0^* = 1$ in this case. Now assume that c is close enough to 0 that $\underline{\gamma} = \overline{\gamma} = 1$; notice that $\underline{\gamma} \to 1$ as $c \to 0$. The same argument as in the previous step shows that $\gamma_0^* = 1$ in this case. To finish, suppose that c is such that $\overline{\gamma} < 1$. The same argument as in the previous step shows that $\gamma_0^* < \overline{\gamma}$ if λ is sufficiently small. This concludes the proof.

Proof of Proposition 3

Let $W(\gamma) = W_0(\gamma) + V_I(\gamma)$. Suppose first that $0 < \gamma_0^* < \gamma_1^*$. Since $W_0(\gamma) \leq W_0(\gamma_0^*)$ for all $\gamma \in [0, 1]$ by the definition of γ_0^* and $V_I(\gamma) < V_I(\gamma_0^*)$ for all $\gamma < \gamma_0^*$ by Proposition 2, we have that $W(\gamma) < W(\gamma_0^*)$ for all $\gamma < \gamma_0^*$. Moreover, given that $\gamma_0^* \in (0, \underline{\gamma})$ is a local maximum of W_0 and W_0 is differentiable at γ_0^*, so that $W_0'(\gamma_0^*) = 0$, we also have that $W(\gamma)$ is strictly increasing in γ in a neighborhood of γ_0^*. Thus, $\gamma^* > \gamma_0^*$. A similar argument shows that $\gamma^* < \gamma_0^*$ if $\overline{\gamma} > \gamma_0^* > \gamma_1^*$.

Equilibria of the Period-0 Game in the Auction Setting

Here, we characterize the symmetric equilibria of the period-0 game in the auction setting. As in the benchmark model, a buyer who observes the signal ℓ learns that the asset is of low quality and bids 0.[30] Therefore, in what follows, we take the behavior of these buyers as given, and concentrate on the behavior of buyers with $s \in \{u, h\}$. Note that bidding $b \in (0, c)$ is suboptimal for a buyer with $s \in \{u, h\}$ when $\gamma > 0$, as such an offer is only accepted when the asset is of low quality, in which case the buyer surely suffers a loss of γb.

We first consider the full subsidy case, i.e., the case when $\gamma = 0$, and then consider the partial subsidy case. For each $s \in \{u, h\}$, denote the minimum and maximum of the support of F_s by $\underline{b}_s$ and $\overline{b}_s$, respectively. In addition, let π^s be the (posterior) belief of a buyer with signal s, and let

[30]When $\gamma > 0$, it is weakly dominant for a buyer with $s = \ell$ to bid 0. When $\gamma = 0$, such a buyer is indifferent between any offer he makes, and we assume that he also bids 0. The result that there is no information acquisition in equilibrium when $\gamma = 0$ does not depend on the assumption that a buyer with $s = \ell$ bids 0.

$V_s(b)$ be the expected payoff to such a buyer if he bids b. Finally, let V_s be the equilibrium payoff of a buyer with signal s.

Full Subsidy The result below shows that there is a unique equilibrium when $\gamma = 0$, and that $k = 0$ in this case. Thus, since $G(0) = 0$, there is no information acquisition when the government fully insures the buyers against the possibility of purchasing a lemon.

Proposition 7. *Suppose that $\gamma = 0$. There is a unique equilibrium, and $k = 0$ in this equilibrium.*

Proof. Let $\xi_H(b)$ be the probability that a buyer who bids $b \geq c$ wins the auction when $q_0 = H$. Then $V_s(b) = \pi^s \xi_H(b)(v - b)$ for all $b \geq c$. We claim that $V_h = 0$, so that in equilibrium no buyer has an incentive to inspect the asset. Suppose, by contradiction, that $V_h > 0$. Then $\underline{b}_h \geq c$ and $\overline{b}_h < v$. Given that $V_u \geq V_u(\underline{b}_h) = (\pi_0/\pi^h)V_h(\underline{b}_h) = (\pi_0/\pi^h)V_h > 0$, we then have that $\underline{b}_u \geq c$ and $\overline{b}_u < v$ as well. Therefore, $\underline{b} = \min\{\underline{b}_u, \underline{b}_h\} \in [c, v)$ and $\overline{b} = \max\{\overline{b}_u, \overline{b}_h\} < v$. A standard argument now shows that $\xi_H(b)$ has no mass points in $[\underline{b}, \overline{b}]$. This, however, implies that $V_u(\underline{b}) = V_h(\underline{b}) = 0$, so that either $V_u = 0$ or $V_h = 0$, a contradiction. To finish, note that if $\xi_H(b) > 0$ for some $b \in [c, v]$, then $V_h > 0$. Thus, $\xi_H(b) = 0$ for all $b \in [c, v)$, so that a buyer with signal $s \in \{u, h\}$ bids v with probability one. ■

Partial Subsidy We divide the analysis of the partial subsidy case in three parts. We first show that if $\gamma > 0$, then there exists no equilibrium of the period-0 game in which the probability η that a buyer inspects the asset is zero. Then, we determine the bidding behavior of buyers taking the probability $\eta \in (0, 1)$ of inspection as given; the assumption that $G(v - c) < 1$ implies that $\eta < 1$ in equilibrium. Finally, we endogenize η.

Lemma 6. *Suppose that $\gamma > 0$. There is no equilibrium with $\eta = 0$.*

Proof. We show that if $\eta = 0$, then buyers can profitably deviate by inspecting the asset if the cost of doing so is small enough. Suppose that $\eta = 0$, so that no buyer inspects the asset. There are two cases to consider: $\gamma \geq \underline{\gamma}$ and $\gamma < \underline{\gamma}$.

Let $\gamma \geq \underline{\gamma}$. In this case, $V_u = 0$. However, the payoff to buyer i if he inspects the asset and bids $c + \varepsilon$, with $\varepsilon > 0$, if $s = h$ is

$$[\pi_0 + (1 - \rho)(1 - \pi_0)][\pi^h(v - c - \varepsilon) - (1 - \pi^h)\gamma(c + \varepsilon)] - k_i,$$

which is positive if both k_i and ε are small enough; recall that $\pi^h(v - c) - (1 - \pi^h)c > 0$ by assumption. Hence, $\eta = 0$ cannot be part of an equilibrium.

Now let $\gamma < \underline{\gamma}$. We claim that $V_u = 0$ as well. Indeed, a standard argument shows that since no buyer inspects the asset, F_u has no mass points in $[c, v)$. Now observe that if $V_u > 0$, then $\underline{b}_u \geq c$. However, $V_u(\underline{b}_u) = 0$, as a buyer who bids $\underline{b}_u$ loses the auction with probability one, contradiction. Thus, $V_u = 0$. Next, we claim that $\overline{b}_u \in (c, v)$. It is clear that $\overline{b}_u < v$. Besides, if $\overline{b}_u = 0$, then a buyer with $s = u$ can profitably deviate by bidding c. Hence, $\overline{b}_u > c$. Therefore, $V_u(\overline{b}_u) = \pi_0(v - \overline{b}_u) - (1 - \pi_0)\gamma\overline{b}_u = 0$. To finish, notice that the payoff to buyer i if he inspects the asset and bids $\overline{b}_u + \varepsilon$, with $\varepsilon > 0$, if $s = h$ is

$$[\pi_0 + (1 - \rho)(1 - \pi_0)][\pi^h(v - \overline{b}_u - \varepsilon) - (1 - \pi^h)\gamma(c + \varepsilon)] - k_i.$$

Since $\pi^h(v-\bar{b}_u)-(1-\pi^h)\gamma\bar{b}_u>0$, the above payoff is positive if both k_i and ε are small enough. Thus, once again, $\eta=0$ cannot be part of an equilibrium. This concludes the proof. ■

Suppose that each buyer inspects the asset with probability $\eta\in(0,1)$ and for each $q\in\{L,H\}$, let $Q_q(b)$ be the ex ante probability that a buyer bids b or less when $q_0=q$; notice that $Q_H(b)=(1-\eta)F_u(b)+\eta F_h(b)$, while $Q_L(b)=(1-\eta)F_u(b)+\eta[\rho+(1-\rho)F_h(b)]$. Now, for each $b\geq c$ and $q\in\{L,H\}$, let $\xi_q(b)$ be given by

$$\xi_q(b)=\sum_{s=0}^{N-1}\frac{1}{s+1}\binom{N-1}{s}Q_q(b^-)^{N-1-s}[Q_q(b)-Q_q(b^-)]^s,$$

where $Q_q(b^-)=\lim_{\varepsilon\searrow 0}Q_q(b-\varepsilon)$. By construction, $\xi_q(b)$ is the probability that a buyer who bids $b\geq c$ wins the auction when $q_0=j$. Notice that $\xi_q(b)$ is nondecreasing in b. Indeed, if $b_1>b_2$, then $F_s(b_1^-)\geq F_s(b_2)$ for each $s\in\{u,h\}$ implies that $\xi_q(b_1)\geq Q_q(b_1^-)^{N-1}\geq Q_q(b_2)^{N-1}\geq\xi_q(b_2)$. Now let $\xi_q(b^+)=\lim_{\varepsilon\searrow 0}\xi_q(b+\varepsilon)$. The result below establishes some useful properties of $\xi_q(b)$.

Lemma 7. *The following two facts hold:*

$$\xi_q(b)=\frac{1}{N}\sum_{s=0}^{N-1}Q_q(b)^sQ_j(b^-)^{N-1-s}; \tag{12}$$

$$\frac{\xi_q(b)}{\xi_q(b^+)}=\frac{1}{N}\sum_{s=0}^{N-1}\left(\frac{Q_q(b^-)}{Q_q(b)}\right)^s. \tag{13}$$

Proof. Notice that

$$\sum_{s=0}^{N-1}\frac{1}{s+1}\binom{N-1}{s}a^sb^{N-1-s}=\frac{(a+b)^N-b^N}{Na}$$

for all $a,b>0$.[31] Hence,

$$\xi_q(b)=\frac{Q_q(b)^N-Q_q(b^-)^N}{N[Q_q(b)-Q_q(b^-)]}$$

and, since $\xi_q(b^+)=Q_q(b)^{N-1}$,

$$\frac{\xi_q(b)}{\xi_q(b^+)}=\sum_{s=0}^{N-1}\frac{1}{s+1}\binom{N-1}{s}\left(\frac{Q_q(b^-)}{Q_q(b)}\right)^{N-1-s}\left[1-\frac{Q_q(b^-)}{Q_q(b)}\right]^s=\frac{1-(Q_q(b^-)/Q_q(b))^N}{N[1-(Q_q(b^-)/Q_q(b))]}.$$

Facts (12) and (13) now follow since $c^N-d^N=(c-d)\sum_{s=0}^{N-1}c^sd^{N-1-s}$ for all $c,d>0$. ■

The next result establishes some basic properties of the equilibrium bidding strategies of buyers when they inspect the asset with probability $\eta\in(0,1)$.

[31] For a proof of this fact, let $A(y)=\sum_{s=0}^{N-1}\frac{1}{1+s}\binom{N-1}{s}(ya)^sb^{N-1-s}$ and $B(y)=yA(y)$. The desired result holds since $A(1)=\int_0^1 B'(y)dy$ and $B'(y)=(b+ya)^N$ by the binomial formula.

Lemma 8. *Suppose that $\gamma > 0$ and take as given the probability $\eta \in (0,1)$ of inspection. The following holds in equilibrium: (i) $\bar{b}_u \leq \underline{b}_h$; (ii) $\bar{b}_h > c$; (iii) $F_s(b)$ is continuous and strictly increasing in b when $b \in [\max\{c, \underline{b}_s\}, \bar{b}_s]$ for each $s \in \{u,h\}$; and (iv) $\underline{b}_u = 0$.*

Proof. (*i*) The result is obvious if $\bar{b}_u = 0$. Suppose then that $\bar{b}_u \geq c$. Since the expected payoff to a type $s \in \{u,h\}$ buyer who bids b is

$$V_s(b) = \pi^s \xi_H(b)(v-b) - (1-\pi^s)\xi_L(b)\gamma b,$$

we have that

$$V_h(b) = \frac{1}{\pi_0 + (1-\pi_0)(1-\rho)} V_u(b) + \frac{(1-\pi_0)\rho}{\pi_0 + (1-\pi_0)(1-\rho)} \xi_L(b)\gamma b; \tag{14}$$

recall that $\pi^h = \pi_0/[\pi_0 + (1-\rho)(1-\pi_0)]$. The second term in the right-hand side of (14) is strictly increasing in b. In addition, by the optimality of $\bar{b}_u$ for a buyer with $s = u$, we have that $V_u(\bar{b}_u) \geq V_u(b)$ for all $b \in [c, \bar{b}_u]$. It then follows that $V_h(\bar{b}_u) > V_h(b)$ for all $b \in [c, \bar{b}_u]$, which implies that $\underline{b}_h \geq \bar{b}_u$.

(*ii*) Suppose that $F_h(0) = 1$. By (*i*), this implies that $F_u(0) = 1$ as well. Hence, the payoff to a buyer with $s = h$ who bids $b = c$ is equal to $\pi^h(v-c) - (1-\pi^h)\gamma c$, which is greater than zero by assumption. Thus, bidding $b = 0$ is suboptimal for a buyer with $s = h$, a contradiction.

(*iii*) We begin by establishing that there are no atoms on the relevant region of the support. First, notice that $\pi^s(v-b) - (1-\pi^s)\gamma b > 0$ if b is a mass point of F_s. Indeed, $Q_H(b^-) < Q_L(b^-)$ when $b \in [\underline{b}_u, \bar{b}_h]$ is a mass point of either F_u or F_h—from (*i*), if $b \in [\underline{b}_u, \bar{b}_u]$, then $F_h(b) < 1$. Hence, (12) implies that $\xi_H(b) < \xi_L(b)$ if $b \in [\underline{b}_u, \bar{b}_h]$ is a mass point of either F_u or F_h. The desired result follows from the fact that

$$V_s(b) = [\xi_H(b) - \xi_L(b)]\pi^s(v-b) + \xi_L(b)[\pi^s(v-b) - (1-\pi^s)\gamma b].$$

Now let $\mu(b) = \xi_H(b)/\xi_L(b)$. We claim that $\mu(b^+) \geq \mu(b)$ for all $b \in [\underline{b}_u, \bar{b}_h]$. Indeed, by (13),

$$\mu(b^+) \geq \mu(b) \Leftrightarrow \frac{\xi_L(b)}{\xi_L(b^+)} \geq \frac{\xi_H(b)}{\xi_H(b^+)} \Leftrightarrow \frac{Q_L(b) - Q_L(b^-)}{Q_L(b)} \leq \frac{Q_H(b) - Q_H(b^-)}{Q_H(b)}.$$

The desired result follows from the fact that $Q_L(b) \geq Q_H(b)$ and

$$\begin{aligned} Q_L(b) - Q_L(b^-) &= (1-\lambda)(F_u(b) - F_u(b^-)) + \lambda(1-\rho)(F_h(b) - F_h(b^-)) \\ &\leq (1-\lambda)(F_u(b) - F_u(b^-)) + \lambda(F_h(b) - F_h(b^-)) = Q_H(b) - Q_H(b^-). \end{aligned}$$

Suppose then that b is a mass point of F_s. This implies that

$$V_s(b^+) = \pi^s \xi_h(b^+)(v-b)\left\{1 - \frac{(1-\pi^s)\gamma b}{\pi^s(v-b)\mu(b^+)}\right\}$$
$$\geq \pi^s \xi_h(b^+)(v-b)\left\{1 - \frac{(1-\pi^s)\gamma b}{\pi^s(v-b)\mu(b)}\right\} > \pi^s \xi_h(b)(v-b)\left\{1 - \frac{(1-\pi^s)\gamma b}{\pi^s(v-b)\mu(b)}\right\},$$

where the strict inequality follows from the fact that $\xi_H(b^+) > \xi_H(b)$ and $\pi^s(v-b) > (1-\pi^s)\gamma b$. Thus, bidding b is suboptimal for a buyer with signal s, a contradiction.

Now we establish that there are no gaps. Suppose that F_u is constant in some interval $[b_1, b_2] \subseteq (\max\{c, \underline{b}_u\}, \overline{b}_u]$; if $b_1 = \max\{c, \underline{b}_u\}$, then b_1 is a mass point of F_u. In this case, an uninformed buyer strictly prefers bidding b_1 to b_2, for both bids imply the same and positive probability of winning, while the first bid implies a smaller payment. Thus, $F_u(b)$ is strictly increasing in b when $b \in [\max\{c, \underline{b}_u\}, \overline{b}_u]$. A similar argument applies to F_h.

(*iv*) Suppose that $\underline{b}_u > 0$ and consider a buyer with $s = u$ who bids $\underline{b}_u$. By (*i*) and (*iii*), the buyer wins if, and only if, all other buyers observe the signal ℓ, which is only possible if the asset is of low quality. So, the expected payoff to the buyer is strictly negative, which cannot be the case. ■

We now make use of Lemma 8 to characterize the equilibrium bidding strategies of buyers for each $\gamma > 0$ and $\eta \in (0,1)$. Let $\underline{\eta}(\gamma) \geq 0$ be the smallest value of η such that

$$\left[1 + \frac{\eta\rho}{1-\eta}\right]^{N-1} \geq \frac{\pi_0(v-c)}{(1-\pi_0)\gamma c}$$

and let $\overline{\eta}(\gamma) \in (\underline{\eta}(\gamma), 1)$ be the only value of η that satisfies

$$\left[1 + \frac{\eta\rho}{1-\eta}\right]^{N-1} = \frac{\pi^h(v-c)}{(1-\pi^h)\gamma c}.$$

Lemma 9. *There is a unique symmetric equilibrium for each $\gamma > 0$ and $\eta \in (0,1)$. If $\eta < \underline{\eta}(\gamma)$, then $\underline{b}_h = \overline{b}_u > c$. On the other hand, if $\underline{\eta}(\gamma) \leq \eta \leq \overline{\eta}(\gamma)$, then $\overline{b}_u = 0$ and $\underline{b}_h = c$. Finally, if $\eta > \overline{\eta}(\gamma)$, then $\underline{b}_h = \overline{b}_u = 0$.*

Proof. We know from (*iii*) in Lemma 8 that if $b \geq c$, then

$$V_s(b) = \pi^s\left[(1-\eta)F_u(b) + \eta F_h(b)\right]^{N-1}(v-b)$$
$$-(1-\pi^s)\left\{(1-\eta)F_u(b) + \eta\left[\rho + (1-\rho)F_h(b)\right]\right\}^{N-1}\gamma b, \quad (15)$$

We also know from (*i*), (*ii*), and (*iv*) in Lemma 8 that the following three mutually exclusive cases are also exhaustive: $\overline{b}_u > c$, $\overline{b}_u = 0$ and $\underline{b}_h > c$, and $\underline{b}_h = 0$.

Case 1: $\overline{b}_u > c$.

For each $b \in [c, \bar{b}_u]$, $F_u(b)$ is derived from the facts that $F_h(b) = 0$ and $V_u(b) = 0$; notice that $F_u(0) = F_u(c)$. In addition, combining $F_u(\bar{b}_u) = 1$ with $V_u(\bar{b}_u) = 0$, we obtain

$$\bar{b}_u = \frac{\pi_0(1-\eta)^{N-1}v}{\pi_0(1-\eta)^{N-1} + (1-\pi_0)(1-\eta+\eta\rho)^{N-1}\gamma}.$$

We see immediately that $\underline{b}_h = \bar{b}_u$ when $\underline{b}_u > c$. Hence, V_h is determined by considering a type h buyer who bids $\bar{b}_u$. From (14) in the proof of Lemma 8 and $F_h(\bar{b}_u) = 0$, we find that

$$\begin{aligned} V_h &= \frac{(1-\pi)\rho}{\pi + (1-\pi)(1-\rho)}(1-\eta+\eta\rho)^{N-1}\gamma\bar{b}_u \\ &= \frac{\rho v}{\pi_0 + (1-\pi_0)(1-\rho)}\left\{\frac{1}{(1-\pi_0)(1-\eta+\eta\rho)^{N-1}\gamma} + \frac{1}{\pi_0(1-\eta)^{N-1}}\right\}^{-1}, \end{aligned} \tag{16}$$

where the second equality follows from substituting $\bar{b}_u$ in the first expression for V_h and arranging the terms. Finally, for each $b \in [\underline{b}_h, \bar{b}_h]$, $F_h(b)$ is derived from (15) and the fact that $V_h(b) = V_h$, while $\bar{b}_h$ is determined from $F_h(\bar{b}_h) = 1$.

A necessary and sufficient condition for the equilibrium described in the above paragraph to exist is that $F_u(c) \in (0, 1)$. From $V_u(c) = 0$, we obtain

$$\left[1 + \frac{\eta\rho}{(1-\eta)F_u(c)}\right]^{N-1} = \frac{\pi_0(v-c)}{(1-\pi_0)\gamma c}.$$

So, $F_u(c) > 0$ if, and only if, $\gamma < \underline{\gamma}$, and $F_u(c) < 1$ if, and only if, $\eta < \underline{\eta}(\gamma)$; note that $\underline{\eta}(\gamma) > 0$ if, and only if, $\gamma < \underline{\gamma}$. It is clear from the explicit construction above that the equilibrium is unique.

Suppose now that $\eta < \underline{\eta}(\gamma)$, so that $\gamma < \underline{\gamma}$ a fortiori. Then $\bar{b}_u = 0$ implies that the payoff to a type u buyer from bidding $b = c$ is at least

$$\pi_0(1-\eta)^{N-1}(v-c) - (1-\pi_0)(1-\eta+\eta\rho)^{N-1}\gamma c,$$

which is positive given that $\eta < \underline{\eta}(\gamma)$. Thus, $\bar{b}_u > c$.

Case 2: $\bar{b}_u = 0$ and $\underline{b}_h > c$.

First note that $\underline{b}_h = c$; if $\underline{b}_h > c$, then a buyer with $s = h$ strictly prefers bidding c to $\underline{b}_h$, as this decreases his payment without changing his probability of winning. Second, V_h is determined by considering a type h buyer who bids $\underline{b}_h = c$. From (15), it follows that

$$V_h = \pi^h(1-\eta)^{N-1}(v-c) - (1-\pi^h)(1-\eta+\eta\rho)^{N-1}\gamma c. \tag{17}$$

Finally, for each $b \in [c, \bar{b}_h]$, $F_h(b)$ is derived from (15) and the fact that $V_h(b) = V_h$, while $\bar{b}_h$ is derived from $F_h(\bar{b}_h) = 1$.

The equilibrium under consideration exists if, and only if, a buyer with $s = h$ has no incentive to bid 0 and a buyer with $s = u$ has no incentive to bid more than c. The first condition is that

$V_h \geq 0$, which is equivalent to $\eta \leq \overline{\eta}(\gamma)$. The second condition is that $V_u(b) \leq 0$ for all $b \geq c$. Since $\pi_0 < \pi^h$ implies that a buyer with $s = u$ strictly prefers b to $b' > b$ whenever a buyer with $s = h$ is indifferent between b and b', a necessary and sufficient condition for $V_u(b) \leq 0$ for all $b \geq c$ is that $V_u(c) \leq 0$, which is equivalent to $\eta \geq \underline{\eta}(\gamma)$. It is clear from the explicit construction above that the equilibrium is unique.

Case 3: $\underline{b}_h = 0$.

Note that if a buyer with $s = h$ is indifferent between bidding $b = 0$ and bidding $b \in [c, \overline{b}_h]$, then $F_h(b)$ must be such that

$$V_h(b) = \pi^h \left[1 - \eta + \eta F_h(b)\right]^{N-1}(v - b) - \pi^h \left[1 - \eta + \eta\left(\rho + (1-\rho)F_h(b)\right)\right]^{N-1}\gamma b = 0.$$

A necessary and sufficient condition for this equilibrium to exist is that $F_h(c) > 0$. Straightforward algebra shows that $F_h(c) > 0$ is equivalent to $\eta > \overline{\eta}(\gamma)$.

Suppose now that $\eta > \overline{\eta}(\gamma)$. Then $\underline{b}_h > 0$ implies that

$$V_h(c) \leq \pi^h(1-\eta)^{N-1}(v - c) - (1 - \pi^h)(1 - \eta + \eta\rho)^{N-1}\gamma b < 0,$$

a contradiction. Thus, $\underline{b}_h = 0$. ■

We complete the description of the equilibria of the period-0 game by endogenizing the inspection decision of buyers. For each $\gamma > 0$ and $\eta \in (0, 1)$, let $V_B^i(\eta, \gamma)$ be the ex ante payoff to a buyer if he inspects the asset and $V_B^u(\eta, \gamma)$ be the ex ante payoff to a buyer if he does not inspect the asset; both payoffs can be computed from Lemma 9. The cutoff cost for inspection given γ and η is $k(\eta, \gamma) = \max\{V_B^i(\eta, \gamma) - V_B^u(\eta, \gamma), 0\}$, and the equilibrium probability of inspection given γ is a fixed point of the map $\eta \mapsto G(k(\eta, \gamma))$. The next result shows that for each $\gamma > 0$ this map has a unique fixed point $\eta = \eta(\gamma)$, with $\eta(\gamma) \in (0, \overline{\eta}(\gamma))$. Thus, in equilibrium, the probability of inspection is never high enough that buyers with $s = h$ bid 0 with positive probability.

Lemma 10. *For each $\gamma \in (0, 1]$ there is a unique $\eta = \eta(\gamma) \in (0, 1)$ such that $\eta = G(k(\eta, \gamma))$. Moreover, $\eta \in (0, \overline{\eta}(\gamma))$.*

Proof. Fix $\gamma \in (0, 1]$. We know from the proof of Lemma 9 that $V_B^u(\eta, \gamma) \equiv 0$ and $V_B^i(\eta, \gamma) = [\pi_0 + (1 - \rho)(1 - \pi_0)]V_h$, where: (*i*) V_h is given by (16) if $\eta \in (0, \underline{\eta}(\gamma))$; (*ii*) V_h is given by (17) if $\eta \in [\underline{\eta}(\gamma), \overline{\eta}(\gamma)]$; and (*iii*) $V_h = 0$ otherwise. From this it follows that $k(\eta, \gamma)$ is continuous and strictly decreasing in η if $\eta \in (0, \underline{\eta}(\gamma)) \cup (\underline{\eta}(\gamma), \overline{\eta}(\gamma))$, and $k(\eta, \gamma) = 0$ if $\eta > \overline{\eta}(\gamma)$. Straightforward algebra also shows that $k(\overline{\eta}(\gamma), \gamma) = 0$ and $\lim_{\eta \nearrow \underline{\eta}(\gamma)} k(\eta, \gamma) = k(\underline{\eta}(\gamma), \gamma)$. Therefore, $k(\eta, \gamma) = V_B^i(\eta, \gamma)$ is continuous and nondecreasing in η, so that $G(k(\eta, \gamma))$ is also continuous and nondecreasing in η. Given that $\lim_{\eta \searrow 0} G(k(\eta, \gamma)) > 0$, we can then conclude that the map $\eta \mapsto G(k(\eta, \gamma))$ has a unique fixed point $\eta = \eta(\gamma)$. Clearly, $\eta \in (0, \overline{\eta}(\gamma))$. ■

It follows from the above analysis that for each $\gamma > 0$ there is a unique symmetric equilibrium, and either $\underline{b}_h = \overline{b}_u > c$ or $\overline{b}_u = \underline{b}_h = c$ in equilibrium. The proposition below summarizes.

Proposition 8. *For each $\gamma > 0$ there is a unique symmetric equilibrium of the period-0 game. The equilibrium cutoff cost k for information acquisition is such that either $G(k) \in (0, \underline{\eta}(\gamma))$ or*

$G(k) \in [\underline{\eta}(\gamma), \overline{\eta}(\gamma))$. In the second case, $\overline{b}_u = 0$ and $\underline{b}_h = c$, so that only the buyers with $s = h$ bid seriously. In the first case, $\underline{b}_h = \overline{b}_u > c$, so that the uninformed buyers can also bid seriously.

Proofs of Remaining Results in Section 4

Proof of Lemma 4

Notice that $V_B^i(G(k), \gamma) \leq \lim_{k\to 0} V_B^i(G(k), \gamma) = V_B^i(0^+, \gamma)$ for all $k \geq 0$ by Lemma 10. Moreover, $V_B^i(0^+, \gamma)$ converges to zero as γ decreases to 0 by Lemma 9 and the fact that $\underline{\eta}(\gamma) > 0$ if γ is small enough. Hence, $k(\gamma)$ converges to 0 as γ decreases to 0.

We now show that there is a unique $\widetilde{\gamma} \in (0, 1)$ such that $k(\gamma)$ is increasing in γ when $\gamma \leq \widetilde{\gamma}$ and decreasing otherwise. Notice first that (16) in the proof of Lemma 9 implies that $V_B^i(\eta, \gamma)$ is increasing in γ if $\eta \leq \underline{\eta}(\gamma)$, while (17) in the proof of Lemma 9 implies that $V_B^i(\eta, \gamma)$ is decreasing in γ if $\eta > \underline{\eta}(\gamma)$. Hence, $k(\gamma)$ is increasing in γ if $\eta(\gamma) \leq \underline{\eta}(\gamma)$ and is decreasing in γ otherwise. A straightforward argument shows that $k(\gamma)$ is also continuous in γ. Now let $\underline{k}(\gamma)$ be the smallest value of k such that $G(k) \geq \underline{\eta}(\gamma)$. Notice that $\underline{k}(\gamma)$ is continuous and decreasing in γ when $\gamma \leq \underline{\gamma}$. Moreover, $\underline{k}(0) = 1$ and $\underline{k}(\overline{\gamma}) = 0$ if $\gamma \geq \underline{\gamma}$. Given that $k(\gamma) > 0$ for all $\gamma > 0$ and $k(\gamma) < 1$ if $\overline{\gamma}$ is small enough, we then have that there is $\gamma \in (0, 1)$ with $k(\gamma) = \underline{k}(\gamma)$. Let $\widetilde{\gamma}$ be the greatest value of γ for which $k(\gamma) = \underline{k}(\gamma)$. Then $\eta(\widetilde{\gamma}) = \underline{\eta}(\widetilde{\gamma})$, as $\underline{k}(\widetilde{\gamma}) = k(\widetilde{\gamma}) > 0$. By the properties of $k(\gamma)$, $\gamma < \widetilde{\gamma}$ implies that $k(\gamma) < k(\widetilde{\gamma})$. To finish, notice that since $\underline{k}(\gamma) > \underline{k}(\widetilde{\gamma})$ for all $\gamma < \widetilde{\gamma}$, we can then conclude that there is no $\gamma < \widetilde{\gamma}$ such that $k(\gamma) = \underline{k}(\gamma)$. This implies the desired result.

Proof of Proposition 4

We first show that $\lim_{\lambda\to\infty} \gamma_0^* = 1$. Let $W_0(\gamma)$ be welfare in period 0 when the policy is γ. Since $\Pi_L(\gamma) = \eta(\gamma)(1-\rho) + (1-\eta(\gamma))[1 - F_u(0)]$ is the probability that a buyer bids c or more when the asset is of low quality and the policy is γ, it follows that

$$W_0(\gamma) \leq \pi_0 v - (1-\pi_0)\left\{1 - [1-\Pi_L(\gamma)]^N\right\}\lambda(1-\gamma)c.$$

A consequence of the equilibrium characterization of the period-0 game in the auction setting is that for all $\gamma' < 1$, there is $\alpha > 0$ such that $\Pi_L(\gamma) \geq \alpha$ for all $\gamma \leq \gamma'$. Hence, for all $\gamma' < 1$, there is $\lambda' > 0$ such that $W_0(\gamma) < 0$ for all $\gamma \leq \gamma'$ if $\lambda \geq \lambda'$. Given that $W_0(1) \geq 0$ regardless of λ, we can then conclude that $\lim_{\lambda\to\infty} \gamma_0^* = 1$.

We now show that $\lim_{\lambda\to 0} \gamma_0^* = 0$. First note that if $\lambda = 0$, so that the private and social costs of any winning bid coincide, then

$$W_0(\gamma) = \pi_0\left\{\left[1-(1-\Pi_H(\gamma))^N\right]v + (1-\Pi_H(\gamma))^N c\right\} - N\int_0^{k(\gamma)} k dG(k),$$

where $\Pi_H(\gamma)$ is the probability that a buyer bids c or more when the asset is of quality H and the policy is γ. Since $\Pi_H(\gamma) < \Pi_H(0) = 1$ for all $\gamma > 0$ by the equilibrium characterization of the period-0 game, it follows that $\gamma_0^* = 0$ when $\lambda = 0$. Now observe that (unlike the benchmark model) $W_0(\gamma)$ is jointly continuous in λ and γ. Indeed, it follows from Lemma 9 that for each $b \geq 0$, the distributions $F_u(b)$ and $F_h(b)$ are continuous functions of γ. Moreover, $\eta(\gamma)$ is a continuous function of γ. Thus, the distributions of winnings bids conditional on the asset's quality are continuous functions of γ in the weak topology of probability measures. Consequently, the theorem of the maximum implies that the correspondence $\lambda \rightrightarrows \text{argmax}_\gamma W_0(\gamma)$ is upper hemicontinuous. Given that $\gamma = 0$ is the unique optimal choice of policy when $\lambda = 0$, it must be that $\lim_{\lambda \to 0} \gamma_0^* = 0$.

Proof of Lemma 5

Consider first the case where $\eta \in (0, \underline{\eta}(\gamma))$, so that $\bar{b}_u > c$. Suppose that $p = 0$. In this case, each buyer either observed $s = \ell$ or $s = u$. Therefore,

$$\phi(0;\gamma) = \frac{\pi}{\pi_0 + (1-\pi_0)\left[1 + \dfrac{\eta(\gamma)\rho}{(1-\eta(\gamma))F_u(0)}\right]^N}.$$

Now suppose that $p \in [c, \bar{b}_u]$. In this case, the winner must be uninformed and any buyer who inspected the asset must have observed $s = \ell$, so that

$$\phi(p;\gamma) = \frac{\pi_0}{\pi_0 + (1-\pi_0)\left[1 + \dfrac{\eta(\gamma)\rho}{(1-\eta(\gamma))F_u(p)}\right]^{N-1}}.$$

Since $F_u(c) = F_u(0) > 0$ and $F_u(p)$ is strictly increasing in p when $p \in [c, \bar{b}_u]$, it is easy to see that $\phi(0;\gamma) < \phi(c;\gamma)$ and that $\phi(p;\gamma)$ is strictly increasing in p when $p \in [c, \bar{b}_u]$. Finally, if $p \in [\underline{b}_h, \bar{b}_h]$, then the winner must have observed $s = h$. In this case,

$$\phi(p;\gamma) = \frac{\pi_0}{\pi_0 + (1-\pi_0)(1-\rho)\left\{1 + \dfrac{\eta(\gamma)\rho[1-F_h(p)]}{1-\eta(\gamma)+\eta(\gamma)F_h(p)}\right\}^{N-1}}.$$

It is easy to see that $\phi(\underline{b}_h;\gamma) > \phi(\bar{b}_u;\gamma)$. Moreover, given that $F_h(p)$ is strictly increasing in p when $p \in [\underline{b}_h, \bar{b}_h]$, we have that $\phi(p;\gamma)$ is strictly increasing in p when $p \in [\underline{b}_h, \bar{b}_h]$. To finish, note that $F_h(\bar{b}_h) = 1$ implies that $\phi(\bar{b}_h;\gamma) = \pi^h$.

Consider now the case where $\eta \in [\underline{\eta}(\gamma), \bar{\eta}(\gamma)]$. Then, since now $F_u(0) = 1$, we have that

$$\phi(0;\gamma) = \frac{\pi_0}{\pi_0 + (1-\pi_0)\left[1 + \dfrac{\eta(\gamma)\rho}{1-\eta(\gamma)}\right]^N}$$

and

$$\phi(p;\gamma) = \frac{\pi_0}{\pi_0 + (1-\pi_0)(1-\rho)\left\{1 + \dfrac{\eta(\gamma)\rho[1-F_h(p)]}{1-\eta(\gamma)+\eta(\gamma)F_h(p)}\right\}^{N-1}}$$

for all $p \in [c, \bar{b}_h]$. We see immediately that $\phi(0;\gamma) < \phi(c;\gamma)$, that $\phi(p;\gamma)$ is strictly increasing in p when $p \in [c, \bar{b}_h]$, and that $\phi(\bar{b}_h;\gamma) = \pi^h$.

Proof of Proposition 5

We first derive the unconditional distribution of posterior beliefs $\Omega(\pi_1;\gamma)$ for each $\gamma \in (0,1]$. Let $\Omega_j(\pi_1;\gamma)$ be the probability that the investor's posterior belief is π_1 or less when the policy is γ and $q_0 = j \in \{L, H\}$. Moreover, let $\phi(p) = \phi(p;\gamma)$ and $\phi^{-1}(\pi_1) = \phi^{-1}(\pi_1;\gamma)$ be the inverse of $\phi(p)$, which is well-defined by Lemma 5. If $\gamma < \tilde{\gamma}$, then

$$\Omega_H(\pi_1;\gamma) = \begin{cases} 0 & \text{if } \pi_1 \in [0, \phi(0)) \\ [(1-\eta(\gamma))F_u(0)]^N & \text{if } \pi_1 \in [\phi(0), \phi(c)) \\ [(1-\eta(\gamma))F_u(\phi^{-1}(\pi_1))]^N & \text{if } \pi_1 \in \left[\phi(c), \phi(\bar{b}_u)\right) \\ [(1-\eta(\gamma))]^N & \text{if } \pi_1 \in \left[\phi(\bar{b}_u), \phi(\underline{b}_h)\right) \\ [1-\eta(\gamma)+\eta(\gamma)F_h(\phi^{-1}(\pi_1))]^N & \text{if } \pi_1 \in \left[\phi(\underline{b}_h), \phi(\bar{b}_h)\right] \end{cases},$$

and if $\gamma \geq \tilde{\gamma}$, then

$$\Omega_H(\pi_1;\gamma) = \begin{cases} 0 & \text{if } \pi_1 \in [0, \phi(0)) \\ [(1-\eta(\gamma))]^N & \text{if } \pi_1 \in [\phi(0), \phi(c)) \\ [1-\eta(\gamma)+\eta(\gamma)F_h(\phi^{-1}(\pi_1))]^N & \text{if } \pi_1 \in \left[\phi(c), \phi(\bar{b}_h)\right] \end{cases}. \quad (18)$$

Similarly, $\gamma < \tilde{\gamma}$ implies that

$$\Omega_L(\pi_1;\gamma) = \begin{cases} 0 & \text{if } \pi_1 \in [0, \phi(0)) \\ [(1-\eta(\gamma))F_u(0)+\eta(\gamma)\rho]^N & \text{if } \pi_1 \in [\phi(0), \phi(c)) \\ [(1-\eta(\gamma))F_u(\phi^{-1}(\pi_1))+\eta(\gamma)\rho]^N & \text{if } \pi_1 \in \left[\phi(c), \phi(\bar{b}_u)\right) \\ (1-\eta(\gamma)+\eta(\gamma)\rho)^N & \text{if } \pi_1 \in \left[\phi(\bar{b}_u), \phi(\underline{b}_h)\right) \\ \{1-\eta(\gamma)+\eta(\gamma)[\rho+(1-\rho)F_h(\phi^{-1}(\pi_1))]\}^N & \text{if } \pi_1 \in \left[\phi(\underline{b}_h), \phi(\bar{b}_h)\right] \end{cases},$$

while $\gamma \geq \tilde{\gamma}$ implies that

$$\Omega_L(\pi_1;\gamma) = \begin{cases} 0 & \text{if } \pi_1 \in [0, \phi(0)) \\ (1-\eta(\gamma)+\eta(\gamma)\rho)^N & \text{if } \pi_1 \in [\phi(0), \phi(c)) \\ \{1-\eta(\gamma)+\eta(\gamma)[\rho+(1-\rho)F_h(\phi^{-1}(\pi_1))]\}^N & \text{if } \pi_1 \in \left[\phi(c), \phi(\bar{b}_h)\right] \end{cases}. \quad (19)$$

The unconditional distribution of posterior beliefs is

$$\Omega(\pi_1;\gamma) = \pi_0\Omega_H(\pi_1;\gamma) + (1-\pi_0)\Omega_L(\pi_1;\gamma).$$

In the rest of the proof we use the properties of $\Omega(\pi_1;\gamma)$ to: (*i*) first show that $V_I(\gamma)$ is strictly decreasing in γ if $\gamma > \widetilde{\gamma}$; (*ii*) then show that $V_I(\widetilde{\gamma}-\varepsilon) > V_I(\widetilde{\gamma})$ for ε positive but sufficiently small; and (*iii*) finally show that $V_I(\widetilde{\gamma}) > \lim_{\gamma\to 0} V_I(\gamma) = 0$.

Step 1. $V_I(\gamma)$ is strictly decreasing in γ if $\gamma > \widetilde{\gamma}$.

Suppose that $\gamma > \tilde{\gamma}$. We begin by establishing some useful properties of $\Omega(\pi_1;\gamma)$. A straightforward consequence of Lemma 5 is that if $\pi_1 \in [\phi(c), \pi^h]$, then $\phi(p) \leq \pi_1$ if, and only if,

$$a = \left[\frac{(1-\pi_1)\pi_0}{\pi_1(1-\pi_0)(1-\rho)}\right]^{\frac{1}{N-1}} \leq 1 + \frac{\eta(\gamma)\rho\,[1-F_h(p)]}{1-\eta(\gamma)+\eta(\gamma)F_h(p)};$$

note that $a \geq 1$ since $\pi_1 \leq \pi^h$. Hence,

$$F_h(\phi^{-1}(\pi_1)) = 1 - \frac{1}{\eta(\gamma)}\frac{a-1}{a-1+\rho},$$

and so $\eta(\gamma)[1-F_h(\phi^{-1}(\pi_1))]$ is independent of γ for all $\pi_1 \in [\phi(c), \pi^h]$. Therefore, (18) and (19) imply that $\Omega(\pi_1;\gamma)$ is independent of γ when $\pi_1 \in [\phi(c), \pi^h]$. Another consequence of (18) and (19) is that $\Omega(\pi_1;\gamma) = \pi_0(1-\eta(\gamma))^N + (1-\pi_0)(1-\eta(\gamma)+\eta(\gamma)\rho)^N$ for all $\pi_1 \in [\phi(0), \phi(c))$. Thus, since $\eta(\gamma)$ is strictly decreasing in γ when $\gamma \geq \widetilde{\gamma}$, $\Omega(\pi_1;\gamma)$ is strictly increasing in γ when $\pi_1 \in [\phi(0), \phi(c))$.

We now compute the derivative of $V_I(\gamma) = \mathbb{E}[\widehat{V}_I(\pi_1)]$ with respect to γ. First note that integration by parts and $\Omega(\phi(c);\gamma) = \Omega(\phi(0);\gamma)$ imply that

$$\begin{aligned}\mathbb{E}[\widehat{V}_I(\pi_1)] &= \Omega(\phi(0);\gamma)\widehat{V}_I(\phi(0)) + \int_{\phi(c)}^{\pi^h} \widehat{V}_I(\pi_1)d\Omega(\pi_1;\gamma) \\ &= \widehat{V}_I(\pi^h) + \Omega(\phi(0);\gamma)\left[\widehat{V}_I(\phi(0)) - \widehat{V}_I(\phi(c))\right] - \int_{\phi(c)}^{\pi^h} \widehat{V}_I'(\pi_1)\Omega(\pi_1;\gamma)d\pi_1.\end{aligned}$$

Given that

$$\frac{d}{d\gamma}\int_{\phi(c)}^{\pi^h} \widehat{V}_I'(\pi_1)\Omega(\pi_1;\gamma)d\pi_1 = -\Omega(\phi(c);\gamma)\widehat{V}_I'(\phi(c))\frac{d\phi(c)}{d\gamma}$$

by the fundamental theorem of calculus and the fact that $\Omega(\pi_1;\gamma)$ is independent of γ when $\pi_1 \in [\phi(c), \pi^h]$, we then have that

$$\frac{d\mathbb{E}[\widehat{V}_I(\pi_1)]}{d\gamma} = \frac{d\Omega(\phi(0);\gamma)}{d\gamma}\left[\widehat{V}_I(\phi(0)) - \widehat{V}_I(\phi(c))\right] + \Omega(\phi(0);\gamma)\widehat{V}_I'(\phi(0))\frac{d\phi(0)}{d\gamma}.$$

Since $\widehat{V}_I(\phi(c)) > \widehat{V}_I(\phi(0)) + \widehat{V}_I'(\phi(0))(\phi(c)-\phi(0))$ by the strictly convexity of $\widehat{V}_I(\phi)$ and

$d\Omega(\phi(0),\gamma)/d\gamma > 0$, the equation for $d\mathbb{E}[\widehat{V}_I(\pi_1)]/d\gamma$ derived above implies that

$$\frac{d\mathbb{E}[\widehat{V}_I(\pi_1)]}{d\gamma} < \widehat{V}_I'(\phi(0)) \left\{ -\frac{d\Omega(\phi(0);\gamma)}{d\gamma}\phi(c) + \frac{d}{d\gamma}\left[\Omega(\phi(0);\gamma)\phi(0)\right] \right\}.$$

We claim that the right-hand side of the above equation is zero. Indeed,

$$\frac{d\Omega(\phi(0);\gamma)}{d\gamma} = -N\left[\pi_0(1-\eta(\gamma))^{N-1} + (1-\pi_0)(1-\rho)(1-\eta(\gamma)+\eta(\gamma)\rho)^{N-1}\right]\frac{d\eta(\gamma)}{d\gamma},$$

and so Lemma 5 implies that

$$-\frac{d\Omega(\phi(0);\gamma)}{d\gamma}\phi(c) = N\pi_0(1-\eta(\gamma))^{N-1}\frac{d\eta(\gamma)}{d\gamma}.$$

The desired result follows from the fact that Lemma 5 also implies that $\Omega(\phi(0);\gamma)\phi(0) = \pi_0(1-\eta(\gamma))^N$. We can then conclude that $V_I(\gamma)$ is strictly decreasing in γ when $\gamma \in (\widetilde{\gamma}, 1]$.

Step 2. $V_I(\widetilde{\gamma}-\varepsilon) > V_I(\widetilde{\gamma})$ for ε positive but sufficiently small.

Suppose that $\gamma < \widetilde{\gamma}$. By the same argument as in Step 1, $\Omega(\pi_1;\gamma)$ is independent of γ when $\pi_1 \in [\underline{b}_h, \pi^h]$. In addition, as in Step 1, if $\pi_1 \in [\phi(c), \phi(\bar{b}_h)]$, then

$$F_u(\phi^{-1}(\pi_1)) = \frac{\eta(\gamma)}{(1-\eta(\gamma))(\hat{a}-1)},$$

where $\hat{a} > 1$ is the only variable that depends on π_1. Hence, when $\pi_1 \in [\phi(c), \phi(\bar{b}_u)]$, (18) and (19) imply that $\Omega(\pi_1;\gamma) = \Psi(\pi_1, \eta(\gamma))$, where $\Psi(\pi_1, \eta)$ is strictly increasing in η. In particular, since $\eta(\gamma)$ is strictly increasing in γ when $\gamma < \widetilde{\gamma}$, $\Omega(\pi_1;\gamma)$ is strictly increasing in γ when $\pi_1 \in [\phi(c), \phi(\bar{b}_u)]$. Now observe from the proof of Lemma 9 that

$$(1-\eta(\gamma))F_u(0) = \frac{\eta(\gamma)\rho}{(\underline{\gamma}/\gamma)^{1/(N-1)} - 1}.$$

Hence, (18) and (19), together with the fact that $\eta(\gamma)$ is strictly increasing in γ when $\gamma < \widetilde{\gamma}$, also imply that $\Omega(\phi(0),\gamma)$ is strictly increasing in γ.

We now compute $V_I'(\gamma)$. Integration by parts implies that

$$\begin{aligned}\mathbb{E}[\widehat{V}_I(\pi_1)] \;=\; & \widehat{V}_I(\pi^h) + \Omega(\phi(0);\gamma)\left[\widehat{V}_I(\phi(0)) - \widehat{V}_I(\phi(c))\right] + \Omega(\phi(\bar{b}_u);\gamma)\left[\widehat{V}_I(\phi(\bar{b}_u)) - \widehat{V}_I(\phi(\underline{b}_h))\right] \\ & - \int_{\phi(c)}^{\phi(\bar{b}_u)} \widehat{V}_I'(\pi_1)\Omega(\pi_1;\gamma)d\pi_1 - \int_{\phi(\underline{b}_h)}^{\pi^h} \widehat{V}_I'(\pi_1)\Omega(\pi_1;\gamma)d\pi_1,\end{aligned}$$

where we used the fact that $\Omega(\phi(c);\gamma) = \Omega(\phi(0);\gamma)$ and $\Omega(\phi(\underline{b}_h);\gamma) = \Omega(\phi(\bar{b}_u);\gamma)$. By the fundamental theorem of calculus and the fact that $\Omega(\pi_1;\gamma)$ is independent of γ when $\pi_1 \in [\phi(c), \pi^h]$, we then have

$$\frac{d\mathbb{E}[\widehat{V}_I(\pi_1)]}{d\gamma} = \frac{d\Omega(\phi(0);\gamma)}{d\gamma}\left[\widehat{V}_I(\phi(0)) - \widehat{V}_I(\phi(c))\right] + \Omega(\phi(0);\gamma)\widehat{V}_I'(\phi(0))\frac{d\phi(0)}{d\gamma} + \frac{d\Omega(\phi(\bar{b}_u);\gamma)}{d\gamma}\left[\widehat{V}_I(\phi(\bar{b}_u)) - \widehat{V}_I(\phi(\underline{b}_h))\right] - \int_{\phi(c)}^{\phi(\bar{b}_u)} \widehat{V}_I'(\pi_1)\frac{d\Omega(\pi_1;\gamma)}{d\gamma}d\pi_1.$$

Since $\Omega(\phi(0);\gamma)$ is strictly increasing in γ and $\widehat{V}_I(\pi_1)$ is strictly convex in π_1,

$$\frac{d\mathbb{E}[\widehat{V}_I(\pi_1)]}{d\gamma} < \widehat{V}_I'(\phi(0))\left\{-\frac{d\Omega(\phi(0);\gamma)}{d\gamma}\phi(c) + \frac{d}{d\gamma}\left[\Omega(\phi(0);\gamma)\phi(0)\right]\right\} + \frac{d\eta(\gamma)}{d\gamma}\left\{\frac{\partial\Psi(\pi_1;\eta(\gamma))}{\partial\eta}\left[\widehat{V}_I(\phi(\bar{b}_u)) - \widehat{V}_I(\phi(\underline{b}_h))\right] - \int_{\phi(c)}^{\phi(\bar{b}_u)} \widehat{V}_I'(\pi_1)\frac{\partial\Psi(\pi_1;\eta(\gamma))}{\partial\eta}d\gamma d\pi_1\right\}.$$

Now observe that $\Omega(\phi(0);\gamma)\phi(0) = \pi_0[(1-\eta(\gamma))F_u(0)]^N$. Moreover, straightforward algebra shows that

$$\frac{d\Omega(\phi(0);\gamma)}{d\gamma}\phi(c) = \frac{d}{d\gamma}[\Omega(\phi(0);\gamma)\phi(0)] + \phi(c)(1-\pi_0)N[(1-\eta(\gamma))F_u(0)+\eta(\gamma)\rho]^{N-1}\rho\frac{d\eta(\gamma)}{d\gamma}.$$

Given that $d\eta(\widetilde{\gamma})/d\gamma = 0$, we can then conclude that $V_I'(\widetilde{\gamma}) < 0$. Hence, there exists $\varepsilon > 0$ such that $V_I(\gamma)$ is strictly decreasing in γ when $\gamma \in (\widetilde{\gamma} - \varepsilon, \widetilde{\gamma})$.

Step 3. $V_I(\widetilde{\gamma}) > \lim_{\gamma\to 0} V_I(\gamma)$.

By Jensen's inequality, $V_I(\widetilde{\gamma}) > \widehat{V}_I(\pi_0)$. The result follows from the fact that $\lim_{\gamma\to 0} k(\gamma) = 0$, and so $\Omega(\pi_1;\gamma)$ converges to the degenerate distribution that assigns probability one to π_0 as γ decreases to zero.

Proof of Proposition 6

We first consider the optimal period-0 policy. Let $V_B(\gamma_L,\gamma_H)$, $V_S(\gamma_L,\gamma_H)$, and $C(\gamma_L,\gamma_H)$ be, respectively, the buyer's expected payoff, the seller's expected payoff, and the government's expected cost when the policy is (γ_L,γ_H). Welfare in period 0 is $W_0(\gamma_L,\gamma_H) = V_B(\gamma_L,\gamma_H) + V_S(\gamma_L,\gamma_H) - C(\gamma_L,\gamma_H)$. Now let $k = k(\gamma_L,\gamma_H)$ be the buyer's cutoff cost for inspection and $\gamma \equiv \gamma_L/\gamma_H$. The same argument as in the proof of Proposition 1 shows that

$$V_B(\gamma_L,\gamma_H) = \max\{\pi_0\gamma_H(v-c) - (1-\pi_0)\gamma_L c, 0\} + \int_0^k G(s)ds.$$

Moreover, it is easy to see that

$$V_S(\gamma_L,\gamma_H) = \left\{\pi_0 + (1-\pi_0)\left[G(k)(1-\rho) + [1-G(k)]\mathbb{I}_{\{\gamma\le\underline{\gamma}\}}\right]\right\}c;$$

as in the benchmark model, an uninformed buyer bids seriously if, and only if, $\gamma \leq \underline{\gamma}$, and an informed buyer bids seriously if, and only if, he observes the signal h. Finally, observe that

$$C(\gamma_L, \gamma_H) \;=\; (1-\pi_0)\left[G(k)(1-\rho) + [1-G(k)]\mathbb{I}_{\{\gamma \leq \underline{\gamma}\}}\right] g((1-\gamma_L)c) \\ +\pi_0 \left[G(k) + [1-G(k)]\mathbb{I}_{\{\gamma \leq \underline{\gamma}\}}\right] g((\gamma_H - 1)(v-c)),$$

where $g(\tau) = (1+\lambda_+)\tau$ if $\tau \geq 0$ and $g(\tau) = (1+\lambda_-)$ if $\tau < 0$.

Suppose first that $\gamma > \underline{\gamma}$. It follows from the above expressions for $V_B(\gamma_L, \gamma_H)$, $V_S(\gamma_L, \gamma_H)$, and $C(\gamma_L, \gamma_H)$ that

$$W_0(\gamma_L, \gamma_H) \;=\; \int_0^k G(s)ds + \pi_0 c \\ +G(k)\left\{(1-\pi_0)(1-\rho)[1-(1+\lambda_+)(1-\gamma_L)]c - \pi_0 g((\gamma_H - 1)(v-c))\right\}.$$

Notice that if $\gamma_H < 1$, then

$$\frac{\partial W_0(\gamma_L, \gamma_H)}{\partial \gamma_H} = -\lambda_- G(k)\pi_0(v-c).$$

Hence, it is optimal to set $\gamma_H \geq 1$ if $\lambda_- < 0$, while any $\gamma_H \in [0,1]$ yields the same payoff to the government if $\lambda_- = 0$. In particular, there is no loss of generality in assuming that $\gamma_H \geq 1$. Now observe that if $\gamma_H \geq 1$, then

$$(1-\pi_0)(1-\rho)[1-(1+\lambda_+)(1-\gamma_L)]c - \pi_0 g((\gamma_H - 1)(v-c)) \\ = \pi_0(1+\lambda_+)(v-c) - (1-\pi_0)(1-\rho)\lambda c - (1+\lambda_+)k.$$

Thus, when $\gamma_H \geq 1$, $W_0 = W_0(\gamma_L, \gamma_H)$ depends on (γ_L, γ_H) only through the cutoff cost k, i.e., $W_0 = W_0(k)$, where

$$W_0(k) = \pi_0 c + G(k)\left[\pi_0(1+\lambda_+)(v-c) - (1-\pi_0)(1-\rho)\lambda_+ c\right] + \int_0^k G(s)ds - (1+\lambda_+)G(k)k.$$

To finish, observe that since $\int_0^k G(s)ds \leq G(k)k$, we have that

$$W_0(k) \leq \pi_0 c + G(k)\{\pi_0(1+\lambda_+)(v-c) - (1-\pi_0)(1-\rho)\lambda_+ c\}.$$

In particular, when λ_+ is sufficiently small, and so the term in brackets is necessarily positive,

$$W_0(k) \leq \pi_0 c + G(k^*)\{\pi_0(1+\lambda_+)(v-c) - (1-\pi_0)(1-\rho)\lambda_+ c\}, \tag{20}$$

where $k^* = k(\gamma_L^*, \gamma_H^*) = \gamma_H^* \rho \pi_0(v-c)$ is the highest cutoff cost for inspection possible; this last fact is useful below.

Suppose now that $\gamma \leq \underline{\gamma}$. In this case, the above expressions for $V_B(\gamma_L, \gamma_H)$, $V_S(\gamma_L, \gamma_H)$, and $C(\gamma_L, \gamma_H)$ imply that

$$W_0(\gamma_L, \gamma_H) = \pi_0 \gamma_H (v-c) - (1-\pi_0)\gamma_L c + \int_0^k G(s)ds + \pi_0 c$$
$$+(1-\pi_0)[(1+\lambda_+)\gamma_L - \lambda_+]c - \pi_0 g((\gamma_H - 1)(v-c)) - \rho G(k)(1-\pi_0)[(1+\lambda_+)\gamma_L - \lambda_+]c.$$

Since the cutoff cost k does not depend on γ_H, it follows that

$$\frac{\partial W_0(\gamma_L, \gamma_H)}{\partial \gamma_H} = \pi_0(v-c) - \pi_0(1+\lambda_+)(v-c) < 0$$

if $\gamma_H > 1$. Thus, conditional on setting $\gamma \leq \underline{\gamma}$, it is optimal for the government to either set $\gamma_H \leq 1$ or set $\gamma_H > 1$ and $\gamma = \underline{\gamma}$. We show that the latter option is always suboptimal. For this, let $\gamma_H > 1$ and $\gamma_L = \underline{\gamma}\gamma_H$. Moreover, let $\widehat{W}_0(\gamma_H) = W_0(\underline{\gamma}\gamma_H, \gamma_H)$ and $\widehat{k} = (1-\pi_0)\rho\underline{\gamma}\gamma_H c$. Then,

$$\begin{aligned}\widehat{W}_0(\gamma_H) &= \int_0^{\widehat{k}} G(s)ds + \pi_0 c + (1-\pi_0)[(1+\lambda_+)\underline{\gamma}\gamma_H - \lambda_+]c \\ &\quad -(1-\pi_0)\rho G(\widehat{k})[(1+\lambda_+)\underline{\gamma}\gamma_H - \lambda_+]c - \pi_0(1+\lambda_+)(\gamma_H - 1)(v-c),\end{aligned}$$

and so, since $\pi_0(v-c) = (1-\pi_0)\underline{\gamma}c$,

$$\frac{\partial \widehat{W}_0(\gamma_H)}{\partial \gamma_H} = -\lambda_+ G(\widehat{k})(1-\pi_0)\rho\underline{\gamma}c - [(1-\pi_0)\rho c]^2 \underline{\gamma} g(\widehat{k})[(1+\lambda_+)\underline{\gamma}\gamma_H - \lambda_+].$$

Now observe that if $(1+\lambda_+)\underline{\gamma}\gamma_H - \lambda_+ = 1 - (1+\lambda_+)(1-\underline{\gamma}\gamma_H) < 0$, then

$$\begin{aligned}\widehat{W}_0(\gamma_H) &< \int_0^{\widehat{k}} G(s)ds + \pi_0 c \\ &\quad +G(\widehat{k})\{(1-\pi_0)(1-\rho)[1-(1+\lambda_+)\underline{\gamma}\gamma_H]c - \pi_0(1+\lambda_+)(\gamma_H - 1)(v-c)\},\end{aligned}$$

in which case the government can increase its payoff by reducing γ_H marginally (so increasing γ slightly above $\underline{\gamma}$). Finally, observe that if $\gamma_H < 1$, then

$$\frac{\partial W_0(\gamma_H)}{\partial \gamma_H} = -\lambda_- \pi_0(v-c)$$

and so it is optimal for the government to set $\gamma_H \geq 1$.

To finish the analysis of the period-0 optimal policy, observe that

$$W_0(0,1) = \pi_0 v - (1-\pi_0)\lambda_+ c.$$

Thus, by (20) and the fact that $G(k^*) < 1$, $W_0(0,1) > W_0(k)$ for all $k \leq k^*$ if λ_+ is small enough; notice that $\gamma_H^* < v/(v-c)$ implies that $k^* < \rho\pi_0 v$. Hence, when λ_+ is sufficiently small, it is optimal for the government to set $\gamma \leq \underline{\gamma}$; the payoff the government obtains when it sets $\gamma > \underline{\gamma}$ is bounded above by the payoff the government obtains when it sets $\gamma_H = 1$ and fully insures the buyer in case he purchases a low quality asset. Given that it is optimal for the government to set $\gamma_H = 1$ when $\gamma \leq \underline{\gamma}$, we can then conclude that if λ_+ is sufficiently small, then the optimal policy in period-0 coincides with the optimal policy in the benchmark model—if it is optimal for the government to set $\gamma \leq \underline{\gamma}$ in the general policy case, then it surely is optimal for the government to set $\gamma_0^* \leq \underline{\gamma}$ in the uni-dimensional policy case.

We now consider the optimal period-1 policy. Let $\Omega(\pi_1; \gamma_L, \gamma_H)$ be the distribution of posterior beliefs for the investor given the policy pair (γ_L, γ_H). As in the benchmark model, $\Omega(\pi_1; \gamma_L, \gamma_H)$ depends on the policy (γ_L, γ_H) only through the cutoff cost k. Thus, the same argument as in Section 3 shows that: (*i*) if the ratio $\gamma = \gamma_L/\gamma_H$ is greater than $\underline{\gamma}$, then $\Omega(\pi_1; \gamma_L, \gamma_H)$ increases in the second order stochastic sense as k decreases; and (*ii*) if the ratio $\gamma = \gamma_L/\gamma_H$ is smaller than $\underline{\gamma}$, then $\Omega(\pi_1; \gamma_L, \gamma_H)$ increases in the second-order stochastic sense as k decreases. Therefore, the government maximizes the investor's welfare by maximizing k. This concludes the proof.