**WORKING PAPER NO. 15-20**
# ON THE WELFARE PROPERTIES OF FRACTIONAL RESERVE BANKING

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April 2015

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The author thanks Mitchell Berlin, Douglas Gale, Chao Gu, Todd Keister, Guido Menzio, Will Roberds, and David Skeie for excellent comments.

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**Abstract**

Monetary economists have long recognized a tension between the benefits of fractional reserve banking, such as the ability to undertake more profitable (long-term) investment opportunities, and the difficulties associated with it, such as the risk of insolvency for each bank and the associated losses to bank liability holders. I show that a specific banking arrangement (a joint-liability scheme) provides an effective mechanism for ensuring the *ex-post* transfer of reserves from liquid banks to illiquid banks, so it is possible to select a socially efficient reserve ratio in the banking system that preserves the safety of bank liabilities as a store of value and maximizes the rate of return paid to bank liability holders.

*Keywords*: fractional reserve banking, reserve management, risk sharing

*JEL classifications: E42, G21*

## 1. INTRODUCTION

One of the main characteristics of the modern banking system is the small amount of reserves in lawful money that banks hold relative to the amount of short-term liabilities (such as demand deposits) they issue. Economists refer to this practice as fractional reserve banking. The proponents of fractional reserve banking have argued that a fractional system allows banks to economize on noninterest-bearing reserves, permitting them to increase the return on their assets and, in the case of a competitive market for bank liabilities, pay a higher return to their liability holders. Implicit in this argument is the conjecture that a lower level of reserves necessarily translates into a higher return paid on a particular class of bank liabilities: those facilitating payments and settlement. This is usually viewed as a socially desirable outcome because one of the main functions of banks is to provide transaction services.

Fractional reserve banking is indeed a superior form of banking provided that each bank is able to borrow reserves from other banks if it suffers an unusual number of withdrawals. The fact that each bank holds only a fraction of its demandable liabilities in the form of highly liquid assets makes it prone to failure. A typical concern is whether fractional reserve banking renders the banking system insolvent in the event that interbank markets, for some reason, fail to perform the function of transferring reserves from more liquid banks to illiquid banks.[1] Thus, there is a clear tension between the benefits of fractional reserve banking, such as the ability to undertake more profitable (long-term) investment opportunities, and the difficulties associated with its implementation, such as the risk of insolvency for each bank.

The goal of this paper is to investigate whether it is possible to implement a fractional reserve system that allows member banks to provide maturity transformation and liquidity services in such a way that bank liabilities are widely accepted as a means of payment and

[1] For instance, Friedman (1959) argued in favor of a banking system with the property that each member bank holds in reserve the full value of its demandable liabilities. His main concern was precisely the stability of the banking system.

trade at par value. My main result is to show that a historically relevant form of private bank coalition (a joint-liability arrangement) allows the members of the banking system to engage in fractional reserve banking in such a way that bank liabilities are widely used as a medium of exchange and yield a higher rate of return. As opposed to markets, this organizational structure involves the monitoring and supervision of the activities of member banks.

The kind of bank coalition studied in this paper resembles the clearinghouse associations that developed in the U.S. in the 19th century, as described in Friedman and Schwartz (1963), Gorton (1984, 1985), Gorton and Mullineaux (1987), Selgin and White (1987), and Moen and Tallman (1992, 2000). These authors have provided evidence that the clearinghouse associations that were formed in some cities in the United States (e.g., New York, Boston, and Chicago) in the second half of the 19th century evolved into a coalition of banks that required each member bank to report its transactions, that imposed reserve requirements on each member bank, and that supervised the note-clearing process on a daily basis. This means that clearinghouse associations provided supervision and regulation services to member banks, allowing them to implement risk-sharing arrangements that would otherwise be impossible under a more decentralized organizational structure.

In my formal analysis, I construct a random-matching model in which privately issued liabilities circulate as a medium of exchange. Agents meet in pairs and use bank liabilities to trade. The redemption of bank liabilities happens periodically in a centralized location in which sellers who have sold goods to buyers take their bank liabilities to claim their face value. The key incentive problem within the banking system arises due to hidden action: It is necessary to provide banks with incentives to induce them to voluntarily report the creation of bank liabilities and hold the appropriate level of reserves. To deal with this incentive problem, a clearinghouse association (i.e., a recordkeeping and safekeeping device) requires member banks to report their transactions, imposes reserve requirements on each one of them, and supervises the clearing of bank liabilities at each date. Thus, the kind of monitoring provided by the clearinghouse allows each member bank to issue liabilities that effectively circulate as a medium of exchange.

I initially characterize equilibrium allocations in the absence of any interbank risk-sharing arrangement. In this case, a safe and sound banking system (i.e., one in which bank liability holders do not suffer losses due to bank failures) necessarily involves an institutional arrangement in which each banker is required to hold in reserve the full value of his demandable liabilities, as advocated by Friedman (1959). This strict collateral condition implies that each bank is fully solvent at any moment so that this form of banking certainly ensures the stability of the payment mechanism. However, I show that such a system costs something for the members of society. First, the banking system as a whole holds excess reserves at the end of each date, which is clearly inefficient because these resources could have been either consumed or invested in higher-return technologies. Second, the rate of return paid on bank liabilities is inefficiently low, which imposes a cost on those who hold these liabilities for transaction purposes.

Subsequently, I characterize the properties of a banking system in which each banker voluntarily chooses to become a member of a coalition that will issue liabilities that are effectively joint obligations of its members. Each banker continues to issue liabilities that identify him as a debtor, but the coalition publicly announces that, in the event an individual banker is unable to keep his promises, other members will honor any obligation of that member, according to their joint capacity. This joint-liability scheme is an effective arrangement that permits the appropriate *ex-post* transfer of reserves from liquid banks to illiquid banks, allowing them to reduce the share of funds invested in noninterest-bearing reserves and, consequently, increase the share of funds invested in interest-bearing assets. As a result, it is possible to eliminate excess reserves in the banking system and induce each banker to pay a socially effi cient return on bank liabilities without introducing the risk of individual bank failures.

The rest of the paper is structured as follows. Section 2 discusses the related literature. Section 3 presents the basic framework. Section 4 carefully describes the exchange mechanism. In Section 5, I characterize equilibrium allocations under a strict reserve requirement that imposes that banks must hold in reserve the full value of their demandable liabilities. In Section 6, I discuss the welfare implications of a joint-liability arrangement and fractional reserve banking. Section 7 concludes.

## 2. RELATED LITERATURE

My analysis is clearly related to the vast literature on inside money. Some prominent papers studying the properties of inside money include those by Champ, Smith, and Williamson (1996), Kahn and Roberds (1998, 1999), Cavalcanti and Wallace (1999a, 1999b), Williamson (1999), Azariadis, Bullard, and Smith (2001), Li (2001, 2006), Martin and Schreft (2006), Berentsen (2006), Mills (2007), Skeie (2008), He, Huang, and Wright (2008), Andolfatto and Nosal (2009), Huangfu and Sun (2011), Araujo and Minetti (2011), and Gu, Mattesini, Monnet, and Wright (2013), among others. In these papers, reserve management is not the focus of the analysis, so the welfare properties of alternative reserve policies are not studied.

One prominent paper that explicitly accounts for reserve management is that of Cavalcanti, Erosa, and Temzelides (1999).[2] In this paper, the authors characterize an equilibrium allocation corresponding to a banking system for which regulation is weaker than 100% reserve requirements. In contrast to their work, my analysis focuses on the welfare properties of interbank arrangements as a means of enhancing reserve management. Also, my framework allows me to fully characterize the effects on prices and quantities (in their model, prices are exogenous).

My results can also be viewed as a response to the narrow banking proposal, as described in Wallace (1996). That author uses the Diamond-Dybvig model (see Diamond and Dybvig, 1983) to show that a banking system that issues liabilities fully backed by safe short-term assets is socially undesirable. In his concluding remarks, Wallace points out that, in reality, bank liabilities serve as a means of payment (something not captured in the Diamond-Dybvig framework) and raises some concerns about how this property would influence the conclusions. My analysis emphasizes precisely the role that bank liabilities play in facilitating transactions.

[2] See also Cavalcanti, Erosa, and Temzelides (2005).

A recent paper that also studies the benefits of fractional reserve banking is that of Chari and Phelan (2014). These authors find that, under some circumstances, the adoption of 100% reserve requirements is socially desirable because of the existence of a social cost (in terms of resources devoted to the banking system) associated with the private creation of government currency substitutes. In my analysis, the usefulness of a fractional reserve system relies on the possibility of constructing an incentive-feasible interbank arrangement that allows banks to provide maturity transformation and supply a widely accepted payment instrument that strictly dominates government-supplied (noninterest-bearing) fiat currency.

Finally, it is important to mention that my results have a similar flavor to those obtained by Berentsen, Camera, and Waller (2007) in that the introduction of an interbank arrangement of the kind described above allows society to better allocate resources. In my analysis, the welfare gains come entirely from better management of banking reserves, which can only be achieved through the implementation of an incentive-feasible risk-sharing scheme among the members of the banking sector.

## 3. MODEL

Time $t = 0, 1, 2, ...$ is discrete, and the horizon is infinite. Each period is divided into three subperiods or stages. There are two physical commodities, referred to as good $x$ and good $y$, which are perfectly divisible. There are three types of agents, indexed by $i = 1, 2, 3$, who are infinitely lived. There is a $[0, 1]$ continuum of each type.

Types 2 and 3 want to consume good $x$, whereas type 1 wants to consume good $y$. If good $x$ is not properly stored in the subperiod it is produced, it will depreciate completely. Good $y$ is perishable and cannot be stored, so it must be consumed in the subperiod it is produced. Type 1 is able to produce good $x$ only in the first subperiod. Type 2 is able to produce good $y$ only in the second subperiod. Type 3 is unable to produce either good but has access to the technology to perfectly store good $x$ at any moment. In the first subperiod, each type 3 also has access to a (divisible) investment technology that requires good $x$ as input and yields a fixed return $\rho > 1$ (in terms of good $x$) only at the beginning of the following date. Finally, each type 3 has access to a technology that allows him to create, at zero cost, an indivisible and durable object, referred to as a note, that perfectly identifies him. This means that notes issued by each type 3 are perfectly distinguishable from those issued by other agents so that counterfeiting will not be a problem.

I now explicitly describe preferences. Let $x_t \in \{0, 1\}$ denote type 1's production of good $x$ at date $t$, and let $y_t \in \mathbb{R}_+$ denote his consumption of good $y$ at date $t$. Type 1's preferences are represented by

$$u(y_t) - \gamma x_t,$$

where $\gamma \in \mathbb{R}_+$ and $u : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable, increasing, and strictly concave, with $u(0) = 0$ and $u'(0) = \infty$. The production technology of good $x$ allows type 1 to produce either zero or one unit of good $x$ at each date. But keep in mind that good $x$ is perfectly divisible.

Let $y_t \in \mathbb{R}_+$ denote type 2's production of good $y$ at date $t$, and let $x_t \in \mathbb{R}_+$ denote his consumption of good $x$ at date $t$. Type 2's preferences are represented by

$$v(x_t) - \omega y_t,$$

where $\omega \in \mathbb{R}_+$, and $v : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable, increasing, and concave, with $v(0) = 0$. Type 3 derives utility $x_t$ if his consumption of good $x$ at date $t$ is $x_t \in \mathbb{R}_+$. Finally, let $\beta \in (0, 1)$ denote the common discount factor over periods. Assume $\rho \leq \beta^{-1}$.

In each subperiod, there is a distinct round of interactions. In the first subperiod, each type 1 is randomly matched with a type 3. In the second subperiod, each type 1 is randomly matched with a type 2 with probability $\lambda \in (0, 1)$. In the third subperiod, all type 2s and all type 3s meet in a centralized location. I assume that, after meeting with a type 1 bilaterally in the first subperiod, all type 3s immediately move to the centralized location.

All type 2s arrive at the centralized location only in the third subperiod.

## 4. EXCHANGE MECHANISM

To describe the exchange process, it is convenient to refer to type 1 as a buyer, to type 2 as a seller, and to type 3 as a banker. To better understand these labels, it is easier to start with the second stage. In this stage, each buyer is randomly matched with a seller with probability $\lambda$. Because the buyer wants good $y$ but is unable to produce good $x$ for the seller at that time, the pair will be able to trade only if a medium of exchange is made available.[3] As will become clear, a banker will be able to provide such a medium of exchange in the form of tradable liabilities redeemable on demand, referred to as bank notes. Thus, the objects a buyer and a seller trade are good $y$ and notes.

A buyer will be able to acquire a note in the first stage when he is randomly matched with a banker. In this stage, each buyer has access to the technology to produce good $x$, so the objects a buyer and a banker trade are good $x$ and notes. Finally, in the third stage, the group of sellers and the group of bankers interact in a centralized location. In this stage, a seller has an opportunity to redeem any note (i.e., to convert a privately issued obligation into good $x$) that he has received from a buyer (if any) in the previous stage, so we can think of this stage as the settlement stage. Thus, two objects can be traded: good $x$ and notes. Note that no production takes place during the settlement stage.

[Insert Figure 1]

This payment arrangement works perfectly well provided that each banker is willing to set aside (i.e., invest in storage) the appropriate amount of good $x$ to have enough resources, referred to as reserves, to retire a note in case it is presented for redemption in the settlement stage (an event that happens with a positive probability). What makes the implementation of such an arrangement difficult is that not all trades in the economy are perfectly observable.

Each banker is able to observe the actions of other bankers in the centralized location. The bilateral trades in the first and second stages are privately observable, i.e., only the pair of agents participating in the meeting knows the amounts traded. As a result, each banker may have an incentive to issue notes without fully securing them with storage.

In addition, a banker may want to opportunistically access previously accumulated re-

[3] With a continuum of agents and random matching, the probability that a buyer finds the same seller again is zero. In addition, there is no technology allowing the pair to record quantities traded.

serves. This means that the possibility of a banker having many notes outstanding following a history of successful trade meetings and few redemptions creates a problem. In particular, a banker who has issued notes that remain in circulation (those issued to buyers who have not had an opportunity to trade with a seller) and who has held reserves to secure these notes may want to opportunistically consume these reserves in case they become very large. The short-term payoff of defection for a banker will be enormous in some cases, making him more likely to renege on his promises.

In view of these difficulties, let me explain how each banker will be able to issue liabilities that effectively circulate as a medium of exchange. At the beginning of date zero, all bankers agree to establish a clearinghouse association that will work merely as a recordkeeping and safekeeping institution. It will accept deposits from member banks and will coordinate the clearing of privately issued liabilities. Each banker can be a member of the clearinghouse at no cost but has to follow its rules. The clearinghouse requires each banker to report any meeting in the first stage in which a note has been issued. For each note issued, the banker is required to store a fraction of the face value of the note (in terms of good $x$), to be interpreted as reserves backing the issuance of his note. Thus, each banker is required to "deposit" reserves with the clearinghouse every time he announces the creation of a note so that he cannot opportunistically access his reserves in future periods. Thus, the clearinghouse provides recordkeeping and safekeeping services to all member banks.

Recall that, shortly after meeting with buyers bilaterally in the first stage, all bankers meet in the centralized location so that they have an opportunity to report the creation of notes and deposit the appropriate amount of reserves with the clearinghouse. Each one of them can also invest in the productive technology. Note that sellers arrive at the centralized location only in the third subperiod, so they do not observe the amounts deposited by each banker. See Figure 2 for a representation of the payment mechanism.

[Insert Figure 2]

Any banker who fails to report the issuance of a note will have his membership permanently revoked. His deviation will be publicly observable to all members of the clearinghouse only when an unreported note is presented for redemption in the settlement stage, which may take several periods to happen.

Finally, I assume that each agent can carry, at most, one indivisible unit of money at any moment.[4] This means that individual note holdings are restricted to the set $\{0, 1\}$. On the other hand, there is *no* restriction on the number of notes each banker is allowed to issue at any moment except for that imposed by the matching technology and agents' willingness to trade. This means that the number of notes issued by a banker belongs to the set $\{0, 1, 2, ...\}$.

## 5. STRICT RESERVE REQUIREMENT

In this section, I characterize stationary equilibrium allocations assuming that the members of the clearinghouse do not engage in any sort of risk-sharing scheme. Thus, to guarantee the solvency of each member bank, the clearinghouse will require each banker to keep in reserve the full face value of any note he has issued. In other words, each banker will have to adopt a 100% reserve policy to retain his membership.

### 5.1. Equilibrium

Throughout the paper, I restrict attention to equilibria for which there exists an invariant distribution of note holdings across buyers, an invariant distribution of note holdings across sellers, and an invariant volume of note creation and note redemption by the members of the banking sector. These invariant distributions can be summarized as follows. Let $m^1 \in [0, 1]$ denote the invariant measure of buyers holding a note at the end of the first stage, let $m^2 \in [0, 1]$ denote the invariant measure of sellers holding a note at the end of the second stage, and let $m^3 \in [0, 1]$ denote the invariant volume of outstanding notes that are retired in the third stage. I only consider equilibria for which $m^1 = 1$ and $m^2 = m^3 = \alpha\lambda$, where $\alpha \in [0, 1]$ denotes the probability that the seller will accept a privately issued note in

[4] In this respect, the model developed in this paper relates to the second generation of search-theoretic models of monetary exchange, following the ideas in Shi (1995) and Trejos and Wright (1995).

exchange for his output. Thus, if each banker truthfully reports the creation of notes, there will be no uncertainty with respect to the total volume of redemptions in the settlement stage.

Let me start by describing the Bellman equations for each buyer. Let $V^0$ denote the beginning-of-period expected discounted utility of a buyer not holding a note, and let $V^1$ denote the beginning-of-period expected discounted utility of a buyer holding a note. The Bellman equations for a buyer are given by

$$V^0 = -\gamma + \alpha\lambda \left[u(y) + \beta V^0\right] + (1 - \alpha\lambda)\beta V^1, \tag{1}$$

$$V^1 = \alpha\lambda \left[u(y) + \beta V^0\right] + (1 - \alpha\lambda)\beta V^1. \tag{2}$$

Here, $y \in \mathbb{R}_+$ denotes the quantity of good $y$ that he will be able to purchase from the seller with whom he is matched in exchange for a note.

If the buyer starts the period without a note, then he will be able to obtain one from the banker with whom he is currently matched, in which case he will produce one unit.[5] A newly issued note costs one unit of good $x$ and is a promise to pay $\phi \in \mathbb{R}_+$ units of good $x$ on demand to the note holder. Then, with probability $\lambda$, the buyer will be matched with a seller in the second stage, in which case the buyer will be able to consume $y \in \mathbb{R}_+$ with probability $\alpha$ (and will enter the following period without a note). With probability $1-\alpha\lambda$, the buyer will not trade in the second stage and will hold on to his note.

Each buyer is able to save in the form of liabilities issued by bankers (a store of value) until he has an opportunity to consume. This means that the buyer's wealth is completely determined by the equilibrium value of bank liabilities.

Let $W^0$ denote the expected discounted utility of a seller who does not find a trading partner in the second stage, and let $W^1$ denote the expected discounted utility of a seller who finds a trading partner. In a stationary equilibrium, the Bellman equations for a seller are given by

$$W^0 = \beta \left[\lambda W^1 + (1 - \lambda) W^0\right], \tag{3}$$

[5] Thus, each buyer enters the second stage holding one unit of money. In this respect, my model bears a resemblance to those of Lagos and Wright (2005) and Rocheteau and Wright (2005).

$$W^1 = \max_{\alpha \in [0,1]} \alpha \left[ -\omega y + v(\phi) \right] + \beta \left[ \lambda W^1 + (1-\lambda) W^0 \right]. \tag{4}$$

I have implicitly assumed that the seller believes the banker is willing to deposit with the clearinghouse the face value of each note previously issued, so by accepting a banker's note in trade, the seller expects to receive the face value $\phi$ with probability one. Below I provide the conditions that make this belief consistent with an equilibrium outcome.

Now, consider the Bellman equations for each banker. Let $J^0$ denote the expected discounted utility of a banker who is currently matched with a buyer not holding a note in the first stage, and let $J^1$ denote the expected discounted utility of a banker who is currently matched with a buyer holding a note. For each note issued in the first stage, the banker will be required to set aside the amount $\phi \in [0,1]$ in order to meet his future obligation. In a stationary equilibrium, the Bellman equations for a banker are given by

$$J^0 = 1 - \phi + \beta \left[ \alpha\lambda J^0 + (1-\alpha\lambda) J^1 \right], \tag{5}$$

$$J^1 = \beta \left[ \alpha\lambda J^0 + (1-\alpha\lambda) J^1 \right]. \tag{6}$$

Note that a banker who has issued a note (and who has truthfully reported it to the clearinghouse) is able to immediately consume the amount $1-\phi$. The consumption decision is trivial: The banker will save exactly the required amount because $\beta\rho \leq 1$.

The expected discounted utility of each banker does not depend on the number of notes he has issued. On the equilibrium path, each banker is willing to deposit with the clearinghouse the amount $\phi$ for each note issued so that he can immediately consume $1-\phi$ every time he is able to issue a note. Because the clearinghouse will ensure the solvency of each individual member, the number of notes outstanding for each banker will not influence his probability of failure. In particular, the probability of failure will be zero because the clearinghouse either requires each member to deposit the full face value of each note or ensures the *ex post* transfer of reserves from liquid banks to illiquid banks if fractional reserve banking is feasible (see the next section).

A banker who meets a buyer holding a note can offer his own note in exchange for the buyer's note. In this case, the banker can claim the face value of someone else's note only if he reports the acquisition of such a note to the clearinghouse, in which case the clearinghouse will require him to hold reserves due to the issuance of his own note. Thus, such a trade will bring no extra benefit to the banker unless the buyer gives him some extra amount of good $x$ together with his note. But, in this case, the buyer will clearly be better off by holding on to his (previously acquired) note. Thus, a swap of notes happens if and only if both agents are indifferent. For simplicity, I assume that both choose not to swap notes.[6]

Throughout the paper, I am interested in the subset of equilibrium allocations for which $\alpha = 1$ (i.e., a seller accepts privately issued notes with probability one). When $\alpha = 1$, the number of trades in the second stage is maximized. Because the acceptability of money is endogenously determined, it is natural to focus on equilibrium allocations that maximize the number of transactions in the economy. For simplicity, I will simply impose $\alpha = 1$ from now on. As we shall see, this assumption will be consistent with individual behavior.

The terms of trade in the first and second stages are determined as follows. Start with the second stage. I assume the buyer makes a take-it-or-leave-it offer to the seller, so he will be able to capture all surplus from trade. In a bilateral meeting, the buyer's surplus from trade is given by

$$u(y) + \beta V^0 - \beta V^1 = u(y) - \beta\gamma,$$

and the seller's surplus from trade is given by

$$-\omega y + v(\phi).$$

The buyer is willing to make any offer such that

$$u(y) - \beta\gamma \geq 0,$$

and the seller accepts the buyer's offer if and only if

$$-\omega y + v(\phi) \geq 0.$$

[6] Even if both agents swapped notes, the total volume of reserves would remain unchanged because the redemption of a note by another banker simply means a transfer of reserves within the banking system, which does not affect the total stock of notes available to the nonbank public.

This participation constraint will always bind if the buyer has all the bargaining power, so the quantity of good $y$ produced in exchange for a note will be given by

$$y = \omega^{-1} v(\phi). \tag{7}$$

Consider now the terms of trade in the first stage. In a trade meeting, the buyer's participation constraint is given by

$$-\gamma + \lambda u\left(\omega^{-1} v(\phi)\right) + \beta(1-\lambda)\left(V^1 - V^0\right) \geq 0.$$

Using (1) and (2), I can rewrite this participation constraint as follows:

$$u\left(\omega^{-1} v(\phi)\right) \geq \frac{\gamma\left[1-\beta(1-\lambda)\right]}{\lambda}. \tag{8}$$

The banker's participation constraint is given by

$$J^0 \geq \beta\left[\lambda J^0 + (1-\lambda) J^1\right],$$

which simply requires

$$\phi \leq 1. \tag{9}$$

This means that an equilibrium value $\phi$ must satisfy both (8) and (9).

Each banker has the option of not reporting his newly issued note to the clearinghouse. The punishment for failing to report any newly issued note (and setting aside the required amount of reserves) is the immediate termination of membership when a deviation is detected. Thus, each banker truthfully reports the creation of a note in the first stage if and only if

$$1 - \phi + \beta\left[\lambda J^0 + (1-\lambda) J^1\right] \geq J^d, \tag{10}$$

where $J^d$ denotes the value associated with his best deviation. The left-hand side gives the banker's expected discounted utility when he chooses to truthfully report the creation of a note. The right-hand side gives his expected discounted utility if he adopts his best deviation strategy. This means that each banker is willing to deposit with the clearinghouse the full face value of each note he has issued provided that the equilibrium value of notes is such that his expected discounted utility is at least the same as that which he would obtain by adopting his best deviation strategy. His best deviation strategy may involve issuing some notes without holding the appropriate amount of reserves (i.e., engaging in fractional reserve banking).

Formally, a deviation strategy specifies the dates at which the banker chooses to truthfully report the creation of notes (depositing the appropriate amount with the clearinghouse) and the dates at which the banker chooses not to report the creation of notes, given a required (constant) deposit amount. Throughout the paper, I restrict attention to pure strategies.

My first step is to show that the value of deviation is bounded below by 1 and is bounded above by $(1-\beta+\lambda\beta)^{-1}\left[1-(1-\lambda)^2\beta\right]$, regardless of the required deposit amount $\phi$. To verify that $J^d \geq 1$, note that a banker who decides to deviate at any given date is able to immediately consume one unit of good $x$. His decision to not deposit reserves with the clearinghouse will certainly affect his continuation value. But in any case, his continuation value is at least zero. Thus, I have just shown that $J^d \geq 1$. To show that $J^d$ has an upper bound, consider the hypothetical case in which a banker who has deviated at some date $t$ is able to deviate at each subsequent date without increasing his probability of failure (for instance, because each note holder will freely dispose of his notes). In this case, the maximum expected discounted utility he can obtain is given by

$$\bar{J} = 1 + (1-\lambda)\,\beta J',$$

where the value $J'$ satisfies

$$J' = \lambda + (1-\lambda)\,\beta J'.$$

When he initially deviates at some date $t$, he is able to immediately consume one unit of good $x$. He will be able to continue trading only with probability $1-\lambda$, which is precisely the probability that the buyer who has acquired his note does not find a trading partner in the second stage. If his deviation is not detected at date $t$, he will be able to issue a new note at date $t+1$ with probability $\lambda$. After date $t$, his probability of failure will not increase (even though more than one note has been issued without the corresponding amount of reserves) because I have assumed that whoever acquires his notes after date $t$ will freely dispose of them so that his probability of survival continues to be given by $1-\lambda$ at the end of each date. It is straightforward to show that

$$\bar{J}=\frac{1-(1-\lambda)^2\beta}{1-(1-\lambda)\beta}.$$

Thus, the value associated with his best deviation $J^d$ is indeed bounded:

$$1\leq J^d\leq\frac{1-(1-\lambda)^2\beta}{1-(1-\lambda)\beta}. \tag{11}$$

I will now demonstrate an important property of the value associated with a best deviation strategy.

**Lemma 1** *Let $J^d$ be the value associated with a best deviation strategy when the required deposit amount is given by $\phi\in[0,1]$, and let $\hat{J}^d$ be the value associated with a best deviation strategy when the required deposit amount is given by $\hat{\phi}\geq\phi$. Then, it follows that $\hat{J}^d\leq J^d$.*

**Proof.** To prove this claim, consider a deviation strategy that delivers the maximum payoff $\hat{J}^d$ when the required deposit amount is $\hat{\phi}$. Formally, a deviation strategy specifies the dates at which the banker chooses to truthfully report the creation of notes (depositing the appropriate amount with the clearinghouse) and the dates at which the banker chooses not to report the creation of notes. Suppose now that we hold this strategy constant; that is, suppose the banker chooses the same dates to report the creation of notes and the same dates to not report the creation of notes.

Let $J'$ denote the value associated with the aforementioned strategy when the required deposit amount is now given by $\phi$. If we decrease the value of the required deposit amount for the banker from $\hat{\phi}$ to $\phi$, then the banker's consumption when he chooses to report the creation of a note is greater than or equal to the amount he gets when $\hat{\phi}$ is the required deposit amount. His consumption when he chooses not to report the creation of a note is exactly the same as the amount he gets when $\hat{\phi}$ is the required deposit amount. Thus, it follows that $J'\geq\hat{J}^d$. Because $J^d\geq J'$ by definition, we conclude $J^d\geq\hat{J}^d$. ■

It remains to verify whether a seller's decision to accept privately issued notes with probability one is consistent with individual rationality. A seller's decision to accept a note issued by a banker in exchange for his output is based on the available information he has about the issuer, which is provided by the clearinghouse. In particular, the clearinghouse provides a record of compliance with its rules for each member bank (i.e., agents observe the membership status of each banker). Each seller knows that the clearinghouse requires member banks to deposit reserves to secure outstanding notes and that it expels members issuing notes without depositing the appropriate amount of reserves (when the deviation is detected). Thus, the decision to become a member is viewed as a signal of "financial rectitude," which will influence a seller's decision to accept notes issued by a member bank.

It is individually rational for each seller to choose $\alpha = 1$ (i.e., to accept privately issued notes with probability one) provided that (10) and (7) are satisfied. Recall that the seller observes the face value $\phi$ associated with a note and a banker's membership status. When the value $\phi$ is such that (10) is satisfied, each seller knows that member banks are willing to deposit with the clearinghouse the required amount of reserves for each note issued. Thus, he is willing to accept a note issued by a member bank with probability one when the terms of trade are given by (7).

Finally, note that the participation constraints (8) and (9) impose both a minimum and a maximum value of notes consistent with equilibrium:

$$v^{-1}\left(\omega u^{-1}\left(\frac{\gamma\left[1-\beta\left(1-\lambda\right)\right]}{\lambda}\right)\right) \leq \phi \leq 1. \tag{12}$$

The minimum value arises owing to the buyer's participation constraint, whereas the maximum value arises because of the banker's participation constraint.

Finally, suppose that each buyer starts date zero without a note so that each banker has an opportunity to issue a note to the buyer with whom he is initially matched. Given these requirements, it is now straightforward to formally define a stationary equilibrium.

**Definition 2** *A stationary monetary equilibrium for the economy previously described is an array* $\left\{J^0, J^1, J^d, V^0, V^1, W^0, W^1, \phi, y, m^1, m^2, m^3\right\}$ *satisfying* $m^1 = 1$, $m^2 = m^3 = \lambda$, *(1)-(7), (10), and (12). In addition,* $J^d$ *is the value associated with a banker's best deviation strategy when each banker is required to deposit with the clearinghouse the amount* $\phi$*, contingent on the creation of a note.*

In a stationary equilibrium, the measure of bankers who issue a note in the first stage is given by $\lambda$ at each date $t \geq 1$, so the total volume of reserves increases by the amount $\lambda\phi$ when buyers rebalance their portfolios. In the third stage, a fraction $\lambda$ of all outstanding notes is retired, so the total volume of reserves decreases by $\lambda\phi$. This means that, in a stationary equilibrium, the total volume of reserves at the end of the period is exactly the same as the volume at the beginning of the period.

### 5.2. Existence

Within the set of stationary equilibria, I am interested in a class of equilibrium allocations with the following properties: (i) each seller accepts notes with probability one and (ii) the banker's truth-telling constraint holds with equality. The requirement that (10) holds with equality allows me to obtain the highest equilibrium value of bank liabilities consistent with truthful reporting. The idea is to focus on equilibrium allocations for which the nonbank public is able to receive the highest rate of return on their money holdings.

To show the existence of an equilibrium, I will make an additional parametric assumption.

**Assumption 1** Assume $1 - \frac{(1-\beta)\left[1-(1-\lambda)^2\beta\right]}{(1-\beta+\beta\lambda)^2} \geq v^{-1}\left(\omega u^{-1}\left(\frac{\gamma[1-\beta(1-\lambda)]}{\lambda}\right)\right)$.

This assumption ensures that the upper bound on the value associated with a best deviation strategy is not too large so that the buyer's participation constraint is satisfied even if the equilibrium gross return on notes is less than one. This assumption is not restrictive provided that, given parametric values for $\beta$, $\lambda$, and $\gamma$ and functional forms for $u(\cdot)$ and $v(\cdot)$, one is willing to make the disutility parameter $\omega$, which can also be interpreted as a seller's level of productivity, sufficiently small. Under Assumption 1, I can establish the existence of a stationary equilibrium with the aforementioned properties.

**Proposition 3** *There exists a stationary monetary equilibrium with a binding truth-telling constraint. In this equilibrium, the value of notes satisfies*

$$\bar{\phi} = \frac{1 - \beta(1-\lambda) - (1-\beta)\bar{J}^d}{1-\beta(1-\lambda)}. \tag{13}$$

*Also, the end-of-period excess reserves are given by* $(1-\lambda)\bar{\phi}$ *at each date.*

**Proof.** To show the existence of a value of notes $\bar{\phi}$ and a value $\bar{J}^d$ associated with a best deviation strategy such that the banker's truth-telling constraint holds with equality and $\bar{J}^d$ is the maximum value associated with a best deviation strategy when $\bar{\phi}$ is the required deposit amount, I construct candidates $\{\phi_s\}_{s=0}^{\infty}$ and $\left\{J_s^d\right\}_{s=0}^{\infty}$ as follows. Define

$$J_0^d = \frac{1-(1-\lambda)^2\beta}{1-(1-\lambda)\beta}.$$

Thus, the first element of $\left\{J_s^d\right\}_{s=0}^{\infty}$ is the upper bound $\bar{J}$. If (10) holds with equality, then the value of notes is given by

$$\phi_0 = \frac{1-\beta(1-\lambda)-(1-\beta)J_0^d}{1-\beta(1-\lambda)}.$$

Given this choice for the value of notes, there exists a value associated with a best deviation strategy, $J_1^d$. It follows that $J_1^d \leq J_0^d$ because $J_0^d$ equals the upper bound. Given $J_1^d$, I can define $\phi_1$ as follows:

$$\phi_1 = \frac{1-\beta(1-\lambda)-(1-\beta)J_1^d}{1-\beta(1-\lambda)}.$$

Given this choice for the value of notes, there exists a value associated with a best deviation strategy, $J_2^d$. From Lemma 1, we have $J_2^d \leq J_1^d$ because $\phi_1 \geq \phi_0$. Following the same steps as those previously described, I can define an increasing sequence $\{\phi_s\}_{s=0}^{\infty}$ and a decreasing sequence $\left\{J_s^d\right\}_{s=0}^{\infty}$. Because $\left\{J_s^d\right\}_{s=0}^{\infty}$ is bounded, it converges to a unique limit $\bar{J}^d \geq 1$. Because $\{\phi_s\}_{s=0}^{\infty}$ is bounded, it converges to a unique limit $\bar{\phi} \leq 1$. Assumption 1 guarantees that $\bar{\phi} > v^{-1}\left(\omega u^{-1}\left(\frac{\gamma[1-\beta(1-\lambda)]}{\lambda}\right)\right) > 0$ (i.e., the upper bound $\bar{J}$ is not too large).

The amount of good $y$ produced and traded in each bilateral meeting in the second stage is given by

$$y = \omega^{-1}v\left(\bar{\phi}\right).$$

Using (1) and (2), we obtain the values $V^0$ and $V^1$:

$$V^0 = \frac{\lambda\left[u\left(\omega^{-1}v\left(\bar{\phi}\right)\right)-\gamma\beta\right]}{1-\beta} - \gamma,$$

$$V^1 = \frac{\lambda\left[u\left(\omega^{-1}v\left(\bar{\phi}\right)\right)-\gamma\beta\right]}{1-\beta}.$$

Because of the assumption that the buyer has all the bargaining power when trading with a seller, it follows that $W^0 = W^1 = 0$. The banker's expected discounted utilities are given by $J^0 = \bar{J}^d$ and $J^1 = \bar{J}^d - 1 + \bar{\phi}$.

Finally, I need to show that the end-of-period excess reserves are $(1 - \lambda)\,\bar{\phi}$. First, note that, at the end of each date, all sellers who have acquired a note are able to convert it into $\bar{\phi}$ unit of good $x$. Note also that there is no reason for them to delay the redemption of a note. Because $m^1 = 1$ in a stationary equilibrium and note holdings are constrained to the set $\{0, 1\}$, the total volume of reserves at the end of the first stage must be $\bar{\phi}$. Because $m^2 = \lambda$, the total volume of reserves decreases by the amount $\lambda\bar{\phi}$ at the end of the third stage. This means that the end-of-period volume of excess reserves is $(1 - \lambda)\,\bar{\phi}$. ■

In a stationary equilibrium, each banker consumes $1 - \bar{\phi}$ unit of good $x$ when he has an opportunity to issue a note, each buyer consumes $\omega^{-1}v\left(\bar{\phi}\right)$ when he has an opportunity to trade with a seller and produces one unit when he acquires a note, and each seller produces $\omega^{-1}v\left(\bar{\phi}\right)$ and consumes $\bar{\phi}$ when he has an opportunity to trade with a buyer.

A safe banking system of the kind described in this section costs something for nonbanks. As I have shown, the equilibrium value $\bar{\phi}$ is determined in such a way that each banker obtains a flow of income derived from the note-issuing business, which is sufficient to induce him to deposit reserves to fully secure his demandable liabilities. Because bankers have to be induced to hold the appropriate level of reserves, it means that there exists an endogenous minimum value associated with the note-issuing business consistent with an equilibrium without bank failures. This endogenous franchise value is necessary for the implementation of a banking system with the property that bankers fully secure their demandable liabilities with safe, short-term assets so that bank failures and losses to note holders do not occur in equilibrium.

Note that no banker invests in the productive technology if each banker is required to fully collateralize with storage his demandable liabilities. The requirement of depositing the full face value of each note with the clearinghouse in the form of noninterest-bearing reserves is imposed to guarantee the solvency of each individual banker. When each individual banker follows this policy, the banking system as a whole ends up holding excess reserves, so the banking sector provides liquidity services without accomplishing maturity transformation.

In the next section, I consider an institutional arrangement that allows the members of the banking system to invest at least some of their funds in the productive technology and that simultaneously guarantees the solvency of each individual banker. Thus, such an arrangement allows banks to simultaneously provide maturity transformation and useful liquidity services.

## 6. FRACTIONAL RESERVE SYSTEM

The goal of this section is to characterize an incentive-feasible arrangement within the banking sector that preserves the safety of bank liabilities as a store of value but permits the members of society to achieve a better allocation of resources by taking advantage of more profitable investment opportunities. Suppose now that, at the beginning of date zero, the members of the clearinghouse association agree to issue notes that are effectively joint obligations of its members. Each banker continues to issue notes that identify him as a debtor, but the clearinghouse publicly announces that, in the event that an individual banker is unable to redeem his own notes, other members will honor any obligation of such a member, according to their joint capacity. Under this arrangement, each banker is entitled to use other bankers' reserves to meet his own obligation in case he is called for redemption provided that he is willing to pledge his own reserves to redeem the notes issued by other bankers.

The clearinghouse is responsible for supervising the required deposit amounts by the members of the coalition. When dealing with each individual member, the clearinghouse needs to induce him to truthfully report the issuance of notes and voluntarily deposit the appropriate amount of resources, which is the same as saying that it needs to ensure that the truth-telling constraint (10) is satisfied. The clearinghouse will also determine the amounts to be invested in storage and the productive technology, which will form the portfolio of the coalition.

The main difference from the previous case is that now the relevant measure to determine the solvency of each banker who is called for redemption is the ratio of the value of *all* reserves of the coalition to the value of *all* notes that are presented for redemption. Because the members of the coalition know that not all outstanding notes will be presented for redemption at each date, it is possible to invest at least some fraction of the funds in the productive technology to obtain a higher rate of return provided that each member is willing to deposit the appropriate amount of resources with the clearinghouse, which means that he is willing to engage in a joint-liability arrangement.

### 6.1. Equilibrium

As in the previous section, each buyer starts date zero without any note. Thus, at date zero, each banker has an opportunity to issue a note to the buyer with whom he is initially matched. Let $s \in \mathbb{R}_+$ denote the *constant* required deposit amount for each member bank, contingent on the creation of a note. In the previous section, this amount was also equal to the face value of notes. As we shall see, it may be different from the the equilibrium value of notes when the coalition is able to invest some of the available funds in the productive technology.

Let $i^p_{t+1} \in \mathbb{R}_+$ denote the per capita amount invested in the productive technology at each date, and let $i^s_{t+1} \in \mathbb{R}_+$ denote the per capita amount invested in storage at each date $t \geq 0$, where per capita means per member bank. These investment decisions are made in the first stage when bankers get together in the centralized location. Under a joint-liability arrangement, the per capita resource constraint for the clearinghouse at any date $t \geq 1$ is given by

$$i^p_{t+1} + i^s_{t+1} = \rho i^p_t + \lambda s + i^s_t - \lambda \phi^c, \tag{14}$$

where

$$i^s_{t+1} \geq \lambda \phi^c. \tag{15}$$

Here, $\phi^c \in \mathbb{R}_+$ denotes the equilibrium value of notes when bankers choose to implement a joint-liability arrangement. In equilibrium, a fraction $\lambda$ of bankers will report the creation of notes and will deposit $s$ units of good $x$ with the clearinghouse so that the per capita deposit amount is given by $\lambda s$. In equilibrium, the aggregate volume of redemptions is given by $\lambda\phi^c$, so the per capita disbursement to note holders is also given by $\lambda\phi^c$. Constraint (15) reflects the fact that the productive technology pays off only at the beginning of the following date so that part of the investment in storage will have to be liquidated to pay note holders.

At date $t = 0$, the per capita resource constraint for the clearinghouse is given by

$$i_1^p + i_1^s = s, \tag{16}$$

where

$$i_1^s \geq \lambda\phi^c. \tag{17}$$

At date $t = 0$, the *per capita* deposit amount is given by $s$ because, as in the previous section, each banker is able to issue a note in the first stage.

In a stationary equilibrium, we have $i_t^p = i_{t+1}^p = i^p$ and $i_t^s = i_{t+1}^s = i^s$ for all $t \geq 0$. Because the productive technology strictly dominates storage as a store of value, we have that both (15) and (17) hold with equality at an optimum. This means that, in a stationary equilibrium, the clearinghouse's per capita resource constraints are given by

$$i^p + \lambda\phi^c = \rho i^p + \lambda s, \tag{18}$$

$$i^s = \lambda\phi^c, \tag{19}$$

$$i^p + \lambda\phi^c = s. \tag{20}$$

Finally, it is necessary to include the buyer's participation constraint

$$\phi^c \geq v^{-1}\left(\omega u^{-1}\left(\frac{\gamma\left[1-\beta\left(1-\lambda\right)\right]}{\lambda}\right)\right) \tag{21}$$

and the banker's participation constraint

$$1 - s + \beta\left[\lambda J^0 + \left(1-\lambda\right)J^1\right] \geq J^d. \tag{22}$$

Now, it is straightforward to define a stationary equilibrium in the presence of a joint-liability arrangement.

**Definition 4** *A stationary monetary equilibrium under a joint-liability arrangement is an array* $\{J^0, J^1, J^d, V^0, V^1, W^0, W^1, \phi^c, y, s, i^p, i^s, m^1, m^2, m^3\}$ *satisfying* $m^1 = 1$, $m^2 = m^3 = \lambda$, *(1)-(7), and (18)-(22). In addition,* $J^d$ *is the value associated with a banker's best deviation strategy when each banker is required to deposit with the clearinghouse the amount* $s$*, contingent on the creation of a note.*

It should be clear that any equilibrium allocation under 100% reserve requirements is also feasible under a joint-liability arrangement. Formally, given an equilibrium allocation under 100% reserve requirements, it is possible to construct an equilibrium allocation under a joint-liability arrangement by setting $s = \phi$ and $i^p = 0$, where $\phi$ is the equilibrium value of notes under 100% reserve requirements. Thus, the set of equilibrium allocations under a joint-liability arrangement is at least the same as the set of equilibrium allocations under 100% reserve requirements. Next, I will show that it is larger in a nontrivial way.

### 6.2. Welfare

Following Cavalcanti and Wallace (1999a), I want to find a stationary monetary equilibrium allocation that maximizes the expected discounted utility of each buyer subject to delivering at least the (beginning-of-period) expected discounted utility $\lambda W^1 + (1-\lambda) W^0 \geq 0$ to each seller and at least the (beginning-of-period) expected discounted utility $\lambda J^0 + (1-\lambda) J^1 \geq \bar{J}^d - (1-\lambda)\left(1-\bar{\phi}\right)$ to each banker, where $\bar{J}^d$ and $\bar{\phi}$ are the values defined in Proposition 3. Note that the required expected utility levels for each seller and each banker are exactly the same as those that each one of them receives under 100% reserve requirements. Thus, I want to find an investment policy that allows me to maximize the expected discounted utility of each buyer, keeping each seller and each banker indifferent.

**Proposition 5** *There exists a stationary monetary equilibrium under a joint-liability arrangement for which* $s = \bar{\phi}$*, the banker's truth-telling constraint binds, and the value of notes is given by*

$$\phi^c = \phi^c_* \equiv \frac{\bar{\phi}}{\lambda}\left(1 - \frac{1-\lambda}{\rho}\right) > \bar{\phi}.$$

*This equilibrium achieves the highest value of notes consistent with a joint-liability arrangement and satisfies the required utility levels previously described. Thus, it is a solution to the optimum problem.*

**Proof.** We have seen that $s = \bar{\phi}$ and $\bar{J}^d$ satisfy the banker's truth-telling constraint with equality, and $\bar{J}^d$ is the value associated with a best deviation strategy when the required deposit amount is $s = \bar{\phi}$ (Proposition 3). If there exists another pair $\left(s, J^d\right)$, with either $s \neq \bar{\phi}$ or $J^d \neq \bar{J}^d$, satisfying the truth-telling constraint with equality, it must be the case that the banker's participation constraint is either violated or holds as a strict inequality. If it holds as a strict inequality, it must be the case that $s \leq \bar{\phi}$ (Lemma 1). Thus, maximizing the expected discounted utility of each buyer subject to the required expected utility levels previously described is equivalent to finding the maximum equilibrium value of notes consistent with a stationary equilibrium under a joint-liability arrangement when $s = \bar{\phi}$ and the banker's truth-telling constraint binds.

Using (20) and $s = \bar{\phi}$, we have

$$i^p = \bar{\phi} - \lambda \phi^c. \tag{23}$$

Using (18) and $s = \bar{\phi}$, we have

$$i^p = \frac{\lambda \left(\phi^c - \bar{\phi}\right)}{\rho - 1}. \tag{24}$$

The solution to equations (23) and (24) is given by

$$\phi^c = \frac{\bar{\phi}}{\lambda} \left(1 - \frac{1 - \lambda}{\rho}\right)$$

and

$$i^p = \frac{\bar{\phi}\left(1 - \lambda\right)}{\rho}.$$

Thus, these values are consistent with a stationary monetary equilibrium in which $s = \bar{\phi}$ and the banker's truth-telling constraint binds. ■

At date zero, each banker deposits $s = \bar{\phi}$ with the clearinghouse so that the *per capita* amount of resources available to the coalition is given by $\bar{\phi}$. Because the *per capita* amount invested in the productive technology is given by

$$\frac{\bar{\phi}\,(1-\lambda)}{\rho}, \tag{25}$$

the *per capita* amount invested in storage is given by

$$\bar{\phi} - \frac{\bar{\phi}\,(1-\lambda)}{\rho} = \bar{\phi}\left(1 - \frac{1-\lambda}{\rho}\right).$$

Because only a fraction $\lambda$ of outstanding notes will be presented for redemption in the settlement stage, the payment to each note holder is given by $\phi_*^c$.

After date zero, only a fraction $\lambda$ of bankers will have an opportunity to issue a note in the first stage. The *per capita* contribution is then given by $\lambda\bar{\phi}$. Because the *per capita* investment amount at each date is given by (25), the clearinghouse starts each period $t \geq 1$ with the *per capita* amount $\bar{\phi}\,(1-\lambda)$. Thus, the *per capita* amount of resources available to the coalition is given by $\bar{\phi}$, which is exactly the same as the amount the coalition had at date zero. Thus, the coalition is able to pay $\phi_*^c$ to each note holder at each subsequent date as well. Note that, at the end of each period, there is nothing invested in storage so that there are no excess reserves in the system.

The bank coalition invests in storage only the amount required to cover expected redemptions and invests the remaining resources in the productive technology. The members of the clearinghouse know that not all notes in circulation will be presented for redemption at the end of each date, so by creating a mechanism for pooling reserves to avoid individual insolvency, it is possible to reduce the amount of non-interest-bearing reserves in the system and, consequently, increase the amount of interest-bearing assets held by the coalition. As a result, it is possible to raise the equilibrium return on bank notes from $\bar{\phi}$ to $\phi_*^c$. This efficient management of reserves is feasible because of the implementation of a joint-liability arrangement.

Let me now establish an important result regarding the welfare properties of a joint-liability arrangement. Under some additional conditions, the previously described solution to the optimum problem is a Pareto optimal allocation. Thus, there exists a stationary monetary equilibrium under a joint-liability arrangement that is socially efficient.

**Proposition 6** *Suppose $\rho = \beta^{-1}$. Then, the solution to the optimum problem described in the previous proposition is Pareto optimal provided*

$$\gamma \geq \lambda \left[ u\left(\omega^{-1} v\left(\phi_c^* + \frac{1-\lambda}{\lambda}\right)\right) - u\left(\omega^{-1} v\left(\phi_c^*\right)\right)\right].$$

**Proof.** It is clear that it is not possible to make either a seller or a banker better off without making a buyer worse off. It remains to verify whether it is possible to achieve a higher level of expected utility for a buyer without making other agents worse off. There is one relevant feasible deviation that I need to check to conclude that the allocation is indeed Pareto optimal.

Suppose that a buyer who holds a note decides to produce a unit of good $x$ and transfer it to a banker with the expectation that the banker can raise the purchasing power of *existing* notes (i.e., no additional unit of money is issued). Because $\rho = \beta^{-1}$, there is no intertemporal gain for a buyer from a higher level of investment in the productive technology. Thus, suppose that these additional resources are invested in storage and that the banker is willing to pay them out as an excess return to anyone claiming redemption in the settlement stage. In this case, it is feasible to implement the following return on deposits:

$$\phi_c^* + \frac{1-\lambda}{\lambda}.$$

Note that each banker remains indifferent and that the original investment plan is not altered in other periods. Now, I need to verify whether a buyer holding a note is willing to produce in order to increase the purchasing power of notes in this way. A note holder is willing to produce provided that

$$-\gamma + \lambda u\left(\omega^{-1} v\left(\phi_c^* + \frac{1-\lambda}{\lambda}\right)\right) > \lambda u\left(\omega^{-1} v\left(\phi_c^*\right)\right).$$

Rearranging this expression, we obtain the following condition:

$$\gamma < \lambda \left[ u\left(\omega^{-1} v\left(\phi_c^* + \frac{1-\lambda}{\lambda}\right)\right) - u\left(\omega^{-1} v\left(\phi_c^*\right)\right)\right].$$

If $\gamma \geq \lambda \left[ u\left(\omega^{-1} v\left(\phi_c^* + \frac{1-\lambda}{\lambda}\right)\right) - u\left(\omega^{-1} v\left(\phi_c^*\right)\right)\right]$, then a buyer is better off if he does not produce a unit of good $x$ to raise the purchasing power of existing notes. As a result, there is no feasible deviation that can increase the expected utility of buyers without making other agents worse off, which means that the aforementioned equilibrium allocation is indeed Pareto optimal. ■

Note that the assumed indivisibility of money holdings, combined with the unity upper bound, is not restrictive in the sense that it is possible to implement an efficient allocation as an equilibrium outcome even though money holdings are restricted to the set $\{0, 1\}$.

Finally, I can interpret the solution to the optimum problem as the outcome of a regulatory framework that imposes a collateral requirement on the portfolio of the members of the banking system. In contrast to the previous section, this optimum requirement allows the members of the banking system to invest in storage only the amount required to cover expected redemptions at each date, so a fraction of the aggregate collateral can be held in the form of interest-bearing assets, given that the banking system has provided a mechanism for transferring reserves within its members to ensure that note holders receive the full face value of notes.

## 7. CONCLUSION

This paper has emphasized the welfare properties of a historically relevant banking arrangement: a joint-liability scheme. As opposed to markets, this organizational form involves the monitoring and supervision of the activities of member banks. I have shown that it offers an effective response to a well-known tension between the benefits of fractional reserve banking and the risks associated with it. In particular, I have demonstrated that it is possible to allow member banks to take advantage of profitable (long-term) investment opportunities and, at the same time, provide socially optimal liquidity services in the form of notes that trade at par value.

The banking arrangement described in this paper ensures the appropriate *ex-post* transfer of reserves from liquid banks to illiquid banks so that it is possible to raise the rate of return paid on bank liabilities, benefiting those who hold them for transaction purposes. By providing each member of the banking system with a minimum requisite value associated with the note-issuing business (i.e., a franchise value), which is required to induce each one of them to hold the appropriate level of reserves to meet the expected demand for note redemptions at each date, it is possible to derive a socially efficient reserve ratio in the banking system that maximizes the return paid to bank liability holders.

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Figure 1: Sequence of Events within a Period

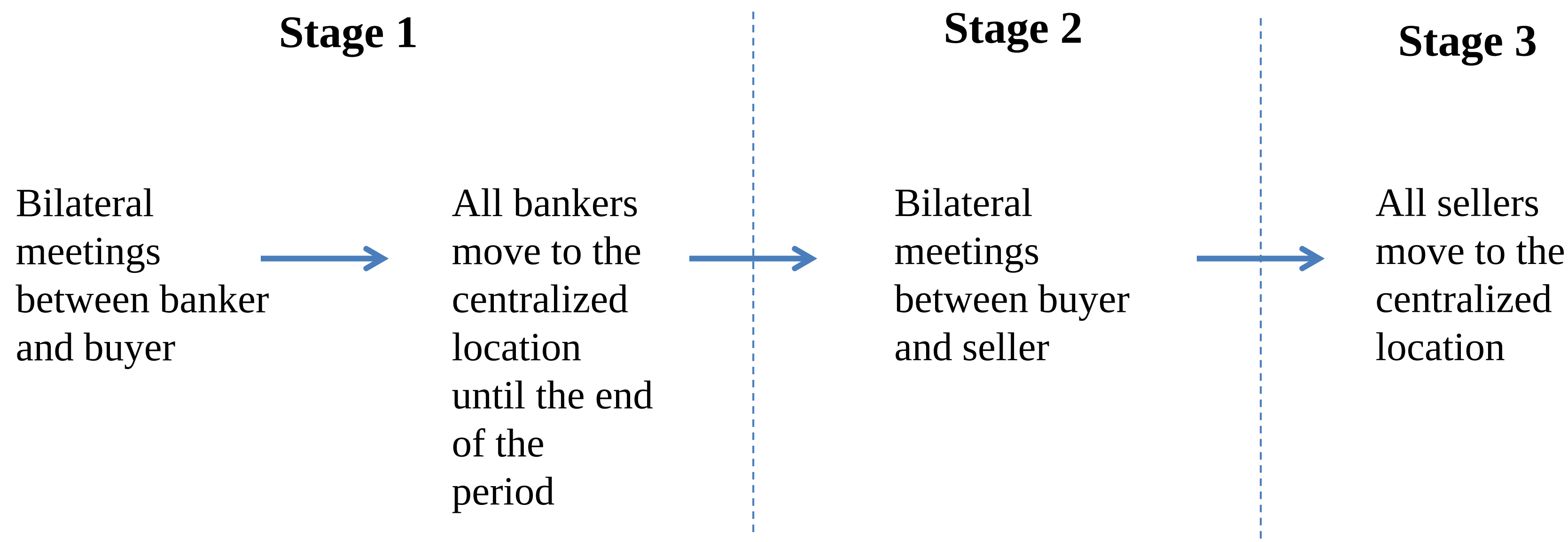

Figure 2: Creation and Redemption of Bank Notes