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ON THE INHERENT INSTABILITY
OF PRIVATE MONEY**

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Abstract

A primary concern in monetary economics is whether a purely private monetary regime is consistent with macroeconomic stability. I show that a competitive regime is inherently unstable due to the properties of endogenously determined limits on private money creation. Precisely, there is a continuum of equilibria characterized by a self-fulfilling collapse of the value of private money and a persistent decline in the demand for money. I associate these equilibrium allocations with self-fulfilling banking crises. It is possible to formulate a fiscal intervention that results in the global determinacy of equilibrium, with the property that the value of private money remains stable. Thus, the goal of monetary stability necessarily requires some form of government intervention.

Keywords: private money, self-fulfilling crises, macroeconomic stability

JEL classifications: E42, E44, G21

1. INTRODUCTION

Substantial changes in financial regulation, together with significant advances in information technology, have revived the debate on the role of private agents in the provision of payment services, especially because of the increased role of nonbank private firms in the supply of alternative payment instruments. For instance, financial innovations in the form of privately issued electronic money have increasingly gained the attention of economists and regulators alike. As a result, there has been renewed interest in the fundamental properties of a purely private monetary system. A primary concern is whether private agents are able to provide a stable monetary framework in the absence of government intervention.

Some economists have argued that many forms of government intervention in the monetary system can be a source of instability and that private markets are capable of providing a sound monetary framework. Others have argued that government control over the monetary system is necessary for achieving stability. In particular, there has been much emphasis on two polar views. Friedman (1959) has argued that the government should be the sole issuer of currency because private creation of government money substitutes will necessarily lead to excessive volatility in the supply of money and, consequently, an unstable monetary system. At the other extreme, we have the argument made by Hayek (1976) that private agents through private markets can effectively achieve desirable outcomes, even in the field of money and banking. According to this view, there is no reason to believe that any form of government intervention is necessary for the establishment of a stable monetary system.[1]

In this paper, I study the properties of a purely private monetary system and investigate whether it is possible to achieve a stable monetary framework under perfect competition. I show that a purely private monetary system is inherently unstable due to the properties of endogenously determined limits on private money creation. Precisely, there exist multiple equilibrium allocations characterized by a self-fulfilling collapse of the value of privately issued liabilities that circulate as a medium of exchange. In view of this intrinsic instability, I

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[1]See also King (1983) and Friedman and Schwartz (1986) for a critical examination of some proposals for monetary reform.

formulate a fiscal intervention that results in the global determinacy of equilibrium, with the unique equilibrium involving a stable value of private money (i.e., the ensuing equilibrium allocation is stationary). Thus, my results indicate that a competitive private monetary system requires a specific form of government intervention to ensure stability.

The important characteristics of the model are as follows. Buyers and sellers meet bilaterally in each period, and a seller is willing to produce for a buyer provided that the latter has something of value to offer in exchange. A buyer is able to access a production technology only when he is *not* searching for a trading partner, so he must rely on some store of value to trade with a seller. In this economy, the most productive investment projects are long term, so buyers cannot directly use the proceeds from investment projects as a means of payment. Neither can a buyer credibly use claims on these technologies to trade with a seller because the pair will never meet again. This characteristic of the physical environment, combined with a lack of commitment, also rules out the use of personal credit.

Although buyers and sellers trade bilaterally in dispersed locations, each one of them has an opportunity to visit a centralized location periodically. However, arrivals at and departures from the centralized location are imperfectly coordinated (in particular, a buyer does not overlap with any seller in the centralized location, and vice versa), so a buyer-seller pair cannot use the centralized location to settle debt. In this economy, a subset of agents, referred to as bankers, is permanently settled in the centralized location, so they can issue a transferable payment instrument that can be used to settle bilateral transactions because it can be redeemed in the centralized location. The problem is that agents do not observe the amount of collateral pledged as reserves to secure these privately issued claims. Combined with a lack of commitment, this gives nonbank agents a reason to distrust bankers.

The key economic decision in the model is the nonbank public's willingness to hold privately issued claims, referred to as notes, for transaction purposes. When an agent decides whether to obtain privately issued notes in exchange for something he is able to produce, he worries about whether the private agent who has issued them is willing to redeem them on demand. The willingness of the issuer to redeem his notes depends on the profitability of the note-issuing business. If the present value of the flow of income derived from the

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note-issuing business is sufficiently large, then the issuer is less inclined to renege on his promises, given that this decision will lead other agents not to trust him in future transactions (so he will no longer be able to issue notes that are widely accepted as a means of payment). Thus, to determine whether an issuer is willing to keep his promise, agents must form beliefs regarding the flow of income derived from the note-issuing business. I show that, under perfect competition, there exist multiple beliefs that are consistent with an equilibrium outcome, including a class of beliefs that is characterized by a self-fulfilling collapse of the value of private money. I associate this class of equilibrium allocations with self-fulfilling banking crises.

It is important to emphasize that the reason for the existence of multiple self-fulfilling equilibria involving a collapse of the value of private money is different from that giving rise to self-fulfilling inflationary equilibria in outside-money economies, such as those characterized in Wallace (1980), Woodford (1984), and Lagos and Wright (2003).[2] As previously described, in my analysis, the key element generating multiplicity of equilibrium is the distrust of banks (i.e., note-issuing agents), which requires traders to form beliefs regarding the continuation value of the note-issuing business to determine the acceptability of private notes as a means of payment. In other words, if we refer to the continuation value of the note-issuing business as the franchise value, then the key element giving rise to multiple equilibria is the nonbank public's beliefs regarding the evolution of the franchise value.

The existence of multiple equilibria due to endogenous limits on note issue is not invariant to the structure of the banking system. In particular, I show that a monopolistic banking system results in a unique equilibrium allocation, even though the decision to accept private notes in transactions requires agents to form beliefs regarding the evolution of the franchise value of the monopolist bank, as in the competitive banking system. This means that a monopolistic banking system is consistent with monetary stability.

Depending on the parameters, the unique equilibrium allocation under monopoly is exactly the same as the stationary allocation under perfect competition (with a constant value of private money), which is surprising given the existence of market power in the banking

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[2]See also Zhu (2003) and Jean, Rabinovich, and Wright (2010).

sector. To understand this result, it is important to keep in mind that even in a competitive banking system each banker obtains a strictly positive franchise value to induce the voluntary convertibility of private notes, which results in a lower rate of return on bank liabilities than what would be obtained under full commitment. The monopolist bank would be willing to pay a higher return on its liabilities than the level consistent with a competitive regime under limited commitment to raise the revenue from the sale of notes, maximizing its profits despite an increase in the cost of funds. Because a higher return on bank liabilities would lead the monopolist bank to strategically suspend convertibility, it is necessary to reduce the return to the competitive level (under limited commitment) to make it consistent with voluntary convertibility. Depending on the parameters, I also find that the return on bank liabilities under monopoly is lower than the return under perfect competition, so the stability of the banking system comes at a cost to the nonbank public.

In view of the difficulties associated with a competitive banking system, I formulate a fiscal intervention that results in the global determinacy of equilibrium under perfect competition. An important characteristic of the intervention is that the government is willing to provide an additional source of revenue to each banker if the private-sector demand for notes falls below a certain threshold level. When the demand for notes is strong, the fiscal rule imposes a tax on note-issuing agents. The announcement of a sustainable subsidy to note-issuing agents in case the demand for private notes falls below a certain level is sufficient to rule out self-fulfilling beliefs involving a persistent decline in the value and volume of private money because it essentially provides a lower bound for the franchise value.

The idea of using a fiscal instrument to obtain determinacy of equilibrium dates back at least to Benhabib, Schmitt-Grohe, and Uribe (2002), who study the properties of an outside-money economy where the monetary authority follows an interest-rate rule. In contrast to their analysis, in a private-money economy, the properties of an effective intervention must reflect the private incentives to create money. As previously mentioned, the mechanism leading to a self-fulfilling collapse of the value of private money under perfect competition involves beliefs regarding the franchise value, so a successful intervention must rule out trajectories with a persistently declining franchise value.

Finally, I demonstrate that, under the proposed intervention, the unique equilibrium allocation necessarily involves a stable value of private money. Thus, a purely private monetary regime under a competitive banking system is consistent with macroeconomic stability provided that the government is willing to intervene in the way described above to ensure the determinacy of equilibrium.

2. RELATED LITERATURE

The literature on inside money is vast.[3] To emphasize my contribution to this literature, it is helpful to explain how my analysis relates to a particular subset of papers. Cavalcanti, Erosa, and Temzelides (1999), using a standard random-matching model, have shown that a private monetary system can be stable in the sense that it is possible to show the existence of a stationary equilibrium. Their notion of stability is restricted to the existence of a stationary equilibrium with private money creation. My goal is precisely to characterize the complete set of equilibrium allocations (including nonstationary allocations), and I show that the analysis of nonstationary equilibria matters for the conclusions regarding the stability of private money.

Azariadis, Bullard, and Smith (2001) have characterized the dynamic properties of a purely private monetary system and a hybrid system in which privately and publicly issued notes coexist. These authors construct an overlapping generations model in which trade is imperfectly coordinated due to spatial separation. As a result, privately issued liabilities can circulate as a medium of exchange. In contrast to their analysis, my framework emphasizes the properties of endogenously determined limits on private money creation. This emphasis results in very different conclusions. In particular, I show that a competitive regime can result in very large aggregate fluctuations that, in most cases, drive the economy to autarky

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[3]Some prominent papers analyzing the issue include Champ, Smith, and Williamson (1996), Cavalcanti, Erosa, and Temzelides (1999), Cavalcanti and Wallace (1999a, 1999b), Kahn and Roberds (1999), Smith and Weber (1999), Williamson (1999), Azariadis, Bullard, and Smith (2001), Li (2001, 2006), Martin and Schreft (2006), Berentsen (2006), Mills (2007), He, Huang, and Wright (2008), Andolfatto and Nosal (2009), and Gu, Mattesini, Monnet, and Wright (2013a), among many others.

as a result of a self-fulfilling collapse of the banking system.

A recent paper by Gu, Mattesini, Monnet, and Wright (2013b) emphasizes the role of endogenous debt limits in determining the dynamics of pure credit economies. These authors construct a model of *bilateral* credit in which endogenous debt limits arise because of agents' inability to commit to their promises. In particular, they show that the set of equilibrium allocations can be very large, with some of them displaying interesting dynamics: Both deterministic and stochastic cycles, as well as chaos, are possible equilibrium outcomes. A key difference in my analysis is that privately issued claims circulate as a medium of exchange, which gives rise to a well-defined demand function for these claims. In addition, I take the analysis one step further and characterize a fiscal intervention that results in the global determinacy of equilibrium.

Finally, it is important to mention that my analysis is fundamentally different from that of Berentsen, Camera, and Waller (2007), who construct a model of money and banking based on the Lagos-Wright framework. The function of a banking system in their framework is to perform a reallocation of liquidity across heterogeneous consumers. There is no money creation by the banking system in their analysis. In addition, the banking system completely disappears when the inflation rate is sufficiently low. In my analysis, the members of the banking system create money and earn a positive profit despite perfect competition in the banking sector.

The rest of the paper is structured as follows. In Section 3, I present the basic framework. Section 4 provides a discussion of the main elements of the model and the exchange mechanism. In Section 5, I characterize equilibrium allocations under perfect competition. Section 6 investigates the properties of a monopolistic banking system. In Section 7, I return to the study of a perfectly competitive banking system and characterize a fiscal intervention that results in the global determinacy of equilibrium. Section 8 concludes.

3. MODEL

Time is discrete, and the horizon is infinite. Each period is divided into two subperiods. There are two physical commodities: good 1 and good 2. Good 1 can be produced only in the first subperiod, and good 2 can be produced only in the second subperiod. If not immediately consumed, good 1 perishes completely. Good 1 can also be used as input in a production process. In particular, there exists a perfectly divisible investment technology that returns $\beta^{-1} > 1$ units of good 1 at date $t+1$ for each unit of good 1 invested at date t. Good 2 cannot be stored and completely depreciates if not immediately consumed.

There are three types of infinitely lived agents, referred to as buyers, sellers, and bankers, with a $[0,1]$ continuum of each type. Each seller wants to consume good 1 but is unable to produce it. Each buyer is able to produce this good using a divisible technology that delivers one unit of the good for each unit of effort he exerts. Each buyer wants to consume good 2, but only a seller is able to produce such a good. In particular, each seller is endowed with a divisible technology that requires one unit of effort to produce each unit of good 2. Finally, each banker wants to consume good 1 but cannot produce either good.

There exists a centralized location where interactions happen as follows. Buyers and sellers visit the centralized location periodically, whereas bankers are permanently settled in this location. Specifically, each buyer and each seller visit the centralized location only in the first subperiod. As in Freeman (1996a), an important characteristic of the environment is that the group of buyers and the group of sellers do *not* overlap in the centralized location.[4] In particular, all buyers arrive at the centralized location first and depart from the centralized location *before* all the sellers arrive. In the second subperiod, each buyer is randomly matched with a seller in such a way that each buyer finds a seller. Following the literature, I refer to the second market as the decentralized market. See Figure 1 for a sequence of events within a period.

Let me now explicitly describe preferences. Let $y_t \in \mathbb{R}_+$ denote a buyer's production of good 1, and let $q_t \in \mathbb{R}_+$ denote his consumption of good 2. His preferences are represented

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[4]See also Freeman (1996b).

by

$$\sum_{t=0}^{\infty} \beta^t \left[-y_t + u\left(q_t\right) \right],$$

where $\beta \in (0,1)$. The function $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing, and strictly concave, with $u'(0) = \infty$ and $u(0) = 0$. Let $x_t^s \in \mathbb{R}_+$ denote a seller's consumption of good 1, and let $n_t \in \mathbb{R}_+$ denote his effort level to produce good 2. His preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c\left(n_t\right) \right],$$

where $c : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, increasing, and convex, with $c(0) = 0$. Let $x_t^b \in \mathbb{R}_+$ denote a banker's consumption of good 1. Each banker has preferences represented by

$$\sum_{t=0}^{\infty} \beta^t x_t^b.$$

Note that, for simplicity, all types of agents have the same discount factor over periods.

Buyers and sellers are anonymous, and their trading histories are privately observable. The trading history of each banker is publicly observable. Finally, the amount invested by any individual in the productive technology is privately observable (i.e., other people do not know how much an individual has invested in the productive technology at each date).

4. EXCHANGE MECHANISM

To understand why trade is difficult in this economy, consider what happens in the second subperiod. A buyer wants to purchase good 2 from the seller with whom he is currently matched but is unable to offer something of value in exchange because the proceeds from investment in the productive technology are unavailable for use in the decentralized market. In addition, because a buyer and a seller will not meet bilaterally again in a large economy, claims on the proceeds from investment in the productive technology cannot be credibly used as a means of payment. Recall that an important characteristic of the environment is that buyers and sellers do not overlap in the centralized location.[5] Each banker has an

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[5] In this respect, my model departs from the standard Lagos-Wright framework in a fundamental way. See Lagos and Wright (2005).

opportunity to trade sequentially with buyers and sellers, respectively, in the centralized location. As a result, bankers are able to play an essential intermediation role in this economy. Precisely, a banker is able to provide transaction services by issuing a transferable payment instrument in the form of perfectly divisible notes.

This payment instrument is extremely useful for a buyer because it allows him to purchase good 2 from a seller in the decentralized market. Figure 2 shows how privately issued notes circulate in the economy. Each buyer has an opportunity to acquire notes while visiting the centralized location in the first subperiod. Specifically, a buyer is able to achieve his desired note holdings by producing and selling good 1 in the market in exchange for notes. In the second subperiod, the buyer is randomly matched with a seller. To obtain some amount of good 2 from his trading partner, he transfers his notes to the seller. In the following period, the seller is able to exchange notes for good 1 while visiting the centralized location because the issuer is supposed to retire them at some expected value. This means that a privately issued note is a convertible payment instrument.

Agents are willing to trade a privately issued payment instrument provided that they believe the issuer will be willing to retire it at some expected value at the following date. In this case, each seller is willing to accept privately issued notes as a means of payment, so each buyer is willing to use them as a temporary store of value.

What makes it difficult for a buyer or a seller to trust a banker's promise? Another important characteristic of the environment is that agents do not observe the amount of collateral (if any) each banker holds in reserve to secure his circulating liabilities. In this respect, the availability of public knowledge of the banker's trading history, together with the possibility of endogenously punishing any banker who reneges on his promises, is crucial for the circulation of private notes. In the decentralized market, a seller does not trust a buyer's IOU because he knows the latter cannot be (endogenously) punished in case of default. But the same seller is willing to accept a banker's note as a means of payment because he knows a banker can be (endogenously) punished if he fails to fulfill the promise of retiring his notes at the expected face value.

The existence of a centralized location where note holders can exchange privately issued

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claims for goods implies that a banker's notes will be periodically presented for redemption. The banker's willingness to retire notes today depends on the exchange value of notes in future periods. If future monetary conditions are more favorable for him, then the continuation value of his note-issuing business is higher, so he will be less inclined to renege on his promises. As a result, his ability to raise funds today through the issuance of notes increases because note holders know that he will have more to lose if he reneges on his promises. This means that the creation of private notes at any given date crucially depends on beliefs about the value associated with the note-issuing business in the future. And this is the key to understanding my results.

5. COMPETITIVE BANKING SYSTEM

As previously mentioned, an important decision in the model is the amount of notes each banker is willing to supply and the nonbank public is willing to hold. This means that, in a purely private monetary system, the money supply is endogenously determined. In what follows, I restrict attention to symmetric equilibria in which all private notes trade at the same price, thus paying the same return. This means that, in equilibrium, the notes issued by any pair of bankers are perfect substitutes, which is consistent with individual rationality only if agents believe both bankers will be willing to redeem them at the expected face value.

Let $\phi_t \in \mathbb{R}_+$ denote the value of a privately issued note in the centralized location at date t. This means that a banker who decides to issue a note is able to receive ϕ_t units of good 1 in exchange. A banker who issues a note at date t is expected to retire it at date $t+1$ at the current market value. This means that a banker who has issued a note at date t is supposed to retire it at date $t+1$ by exchanging it for ϕ_{t+1} units of good 1. Thus, the real rate of return on notes is given by ϕ_{t+1}/ϕ_t. The requirement that a banker is supposed to retire a note in the following period makes it equivalent to a debt instrument with a market-determined real return ϕ_{t+1}/ϕ_t.

Throughout the paper, I use the same equilibrium concept as in Lagos and Wright (2005), which is a blend of traditional Arrow-Debreu components and axiomatic bargaining ele-

ments, with one important change. In my framework, private claims are issued in a Walrasian market by agents who cannot commit to redeem them at the expected face value at the following date. Thus, I have to find a way to ensure that those who have issued claims do not prefer to renege on their promises. To deal with this issue in a tractable way, I replace the traditional Arrow-Debreu equilibrium (in the centralized market) with the equilibrium concept introduced by Alvarez and Jermann (2000), who adapted the traditional Arrow-Debreu equilibrium to an environment characterized by a lack of commitment. This seems to be a natural choice of equilibrium concept to study private money in the Lagos-Wright framework. Thus, my equilibrium concept is the same as in the original Lagos-Wright model adapted to an environment where private claims are issued in a Walrasian market and used as a means of payment in a decentralized market.

Let me start by describing the decision problem of a typical buyer. Let $W(m,t)$ denote the value function for a buyer holding $m \in \mathbb{R}_+$ privately issued notes at the beginning of the first subperiod, and let $V(m,t)$ denote the value function for a buyer holding $m \in \mathbb{R}_+$ notes at the beginning of the second subperiod. The Bellman equation for a buyer in the first subperiod is given by

$$W(m,t) = \max_{(y,\hat{m})\in\mathbb{R}_+^2} [-y + V(\hat{m},t)]$$

subject to the budget constraint

$$\phi_t \hat{m} = y + \phi_t m.$$

Note that $\hat{m} \in \mathbb{R}_+$ represents the buyer's note holdings after trading in the centralized location, and $y \in \mathbb{R}_+$ represents his production of good 1. In the case of an interior solution for y, the value $W(m,t)$ can be written as $W(m,t) = \phi_t m + W(0,t)$, where the intercept is given by

$$W(0,t) = \max_{\hat{m}\in\mathbb{R}_+} [-\phi_t \hat{m} + V(\hat{m},t)]. \tag{1}$$

In the decentralized market, the buyer makes a take-it-or-leave-it offer to the seller.[6] The buyer chooses the amount of good 2, represented by $q \in \mathbb{R}_+$, that the seller is supposed to

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[6]In what follows, nothing hinges on this particular choice of the bargaining protocol.

produce and the amount of notes, represented by $d \in \mathbb{R}_+$, that he is supposed to transfer to the seller. Formally, the terms of trade (q, d) are determined by the solution to the following problem:

$$\max_{(q,d) \in \mathbb{R}_+^2} \left[u(q) - \beta \phi_{t+1} d \right]$$

subject to the seller's participation constraint

$$-c(q) + \beta \phi_{t+1} d \geq 0$$

and the buyer's liquidity constraint

$$d \leq m, \tag{2}$$

with $m \in \mathbb{R}_+$ representing the buyer's note holdings at the beginning of the decentralized market. Let $q^* \in \mathbb{R}_+$ denote the quantity satisfying $u'(q^*) = c'(q^*)$ (i.e., q^* is the surplus-maximizing quantity). The solution to the buyer's problem is as follows:

$$q(m, t) = \begin{cases} c^{-1}(\beta \phi_{t+1} m) & \text{if } m < (\beta \phi_{t+1})^{-1} c(q^*), \\ q^* & \text{if } m \geq (\beta \phi_{t+1})^{-1} c(q^*), \end{cases}$$

$$d(m, t) = \begin{cases} m & \text{if } m < (\beta \phi_{t+1})^{-1} c(q^*), \\ (\beta \phi_{t+1})^{-1} c(q^*) & \text{if } m \geq (\beta \phi_{t+1})^{-1} c(q^*). \end{cases}$$

The Bellman equation for a buyer holding $m \in \mathbb{R}_+$ notes at the beginning of the decentralized market is given by

$$V(m, t) = u(q(m, t)) + \beta W(m - d(m, t), t + 1). \tag{3}$$

Using the fact that $W(m, t)$ is an affine function in its first argument, I can rewrite (3) as follows:

$$V(m, t) = u(q(m, t)) + \beta \phi_{t+1} [m - d(m, t)] + \beta W(0, t + 1).$$

Thus, the first-order condition for the optimal choice of note holdings is given by

$$-1 + \frac{\partial V}{\partial m}(m, t) \leq 0,$$

with equality if $m > 0$. If $\phi_{t+1}/\phi_t < \beta^{-1}$, then the optimal choice of note holdings satisfies the following condition:

$$\frac{u'\left(q\left(m,t\right)\right)}{c'\left(q\left(m,t\right)\right)} = \frac{\phi_t}{\beta\phi_{t+1}}, \tag{4}$$

with $q\left(m,t\right) = c^{-1}\left(\beta\phi_{t+1}m\right)$. Thus, (4) gives the individual demand for notes as a function of the rate of return on notes. Note that all buyers choose to hold the same quantity of notes regardless of their beginning-of-period wealth, so condition (4) also gives the aggregate demand for privately issued money as a function of its expected rate of return.

In the derivation of the demand function previously described, I have implicitly assumed that each seller voluntarily accepts privately issued notes in exchange for his output in the second subperiod. A seller's decision to accept private notes as a means of payment depends on his beliefs about the issuer's willingness to redeem them at the expected face value. A seller is willing to accept privately issued notes as a means of payment provided that the amount of notes issued by each banker does not exceed an upper bound $\bar{B}_t \in \mathbb{R}_+$ at each date $t \geq 0$. If this upper bound is exceeded at some date t, then each seller refuses to accept privately issued notes as a means of payment in the decentralized market. This means that a seller's acceptance rule depends on the current and all future bounds on note issue. As a result, the sequence of individual limits on note issue $\left\{\bar{B}_t\right\}_{t=0}^{\infty}$ is an equilibrium object because it essentially determines the acceptability of privately issued notes in bilateral trades. As we shall see, it is possible to construct a sequence $\left\{\bar{B}_t\right\}_{t=0}^{\infty}$ such that the seller's acceptance rule is individually rational, which is an equilibrium requirement.

Now I describe the decision problem of a typical banker. Let $J\left(n,s,t\right)$ denote the value function for a banker who issued $n \in \mathbb{R}_+$ notes and invested $s \in \mathbb{R}_+$ in the productive technology at the previous date. The banker's decision problem can be formulated as follows:

$$J\left(n,s,t\right) = \max_{(x,\hat{n},\hat{s})\in\mathbb{R}_+^3}\left[x + \beta J\left(\hat{n},\hat{s},t+1\right)\right], \tag{5}$$

subject to the budget constraint

$$\hat{s} + x + \phi_t n = \beta^{-1}s + \phi_t\hat{n}$$

and the upper bound on the number of notes that can be issued at each date

$$\hat{n} \leq \bar{B}_t.$$

Here $\hat{s} \in \mathbb{R}_+$ denotes the amount of good 1 the banker decides to invest in the productive technology at the current date, $x \in \mathbb{R}_+$ denotes his current consumption, and $\hat{n} \in \mathbb{R}_+$ denotes the number of notes he decides to issue at the current date. Note that the constraint $\hat{n} \leq \bar{B}_t$ is equivalent to the solvency constraint in Alvarez and Jermann (2000). Following their equilibrium concept, each banker takes the sequences $\{\phi_t\}_{t=0}^{\infty}$ and $\{\bar{B}_t\}_{t=0}^{\infty}$ as given when making his individual decisions.

If $\phi_{t+1}/\phi_t < \beta^{-1}$, then each banker finds it optimal to issue as many notes as possible (i.e., he chooses $\hat{n} = \bar{B}_t$). Because the rate of return paid on his liabilities (his cost of funds) is lower than the rate of return on the productive technology, the banker makes a positive profit by issuing notes and investing the proceeds in the productive technology. Also, note that because the return on the productive technology equals his rate of time preference, the banker is indifferent between immediately consuming and reinvesting his earnings. Therefore, an optimal investment decision is given by $\hat{s} = \phi_t \bar{B}_t$, which can be interpreted as the decision to voluntarily hold in reserve all proceeds from the sale of notes in the current period. In this case, the banker's optimal consumption is given by

$$x_t = \phi_{t-1}\bar{B}_{t-1}\left(\beta^{-1} - \frac{\phi_t}{\phi_{t-1}}\right).$$

Thus, the banker's lifetime discounted utility is given by

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_{\tau-1}\bar{B}_{\tau-1}\left(\beta^{-1} - \frac{\phi_\tau}{\phi_{\tau-1}}\right).$$

This means that his lifetime utility at any point in time depends on the sequence of individual limits on note issue and the value of private notes today and in all future periods.

5.1. Equilibrium

Let $\pi_t \equiv \phi_{t+1}/\phi_t$ denote the real rate of return on money. For any rate of return $\pi_t < \beta^{-1}$, the liquidity constraint (2) is binding. Let $q_t \in \mathbb{R}_+$ denote the amount of good 2 traded in

each bilateral meeting at date t. Thus, we can rewrite (4) as follows:

$$\frac{u'(q_t)}{c'(q_t)} = \frac{1}{\pi_t \beta}. \tag{6}$$

This condition determines production in each bilateral meeting as a function of the expected return on notes. Thus, I can use (6) to implicitly define $q_t = q(\pi_t)$, with $q'(\pi_t) > 0$ for any $\pi_t > 0$. Thus, a higher rate of return on notes results in a larger amount produced and traded in each bilateral meeting. Real money balances, as a function of the rate of return π_t, are given by

$$a(\pi_t) \equiv \frac{c(q(\pi_t))}{\beta \pi_t}. \tag{7}$$

Note that the demand for notes can be either increasing or decreasing in the rate of return π_t, depending on the specification of preferences.

In equilibrium, the market-clearing condition

$$\phi_t \bar{B}_t = a(\pi_t) \tag{8}$$

must hold at each date t. This condition guarantees that, given an expected return π_t, each banker is willing to supply the amount of notes specified in (7). Then, a particular sequence of expected returns $\{\pi_t\}_{t=0}^{\infty}$ is consistent with convertibility if and only if

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} a(\pi_{\tau-1})(\beta^{-1} - \pi_{\tau-1}) \geq a(\pi_{t-1})(\beta^{-1} - \pi_{t-1}) + a(\pi_t) \tag{9}$$

holds at each date $t \geq 1$. As in Kehoe and Levine (1993) and Alvarez and Jermann (2000), these constraints allow each banker to issue as many notes as possible without inducing him to opportunistically renege on his promises. The left-hand side gives the banker's beginning-of-period lifetime utility. The right-hand side gives the short-term payoff the banker gets if he decides *not* to hold in reserve the proceeds from the sale of notes at date t. In this case, he can increase his current consumption by the amount $a(\pi_t)$, but he will inevitably suspend convertibility at date $t+1$, resulting in the autarkic payoff from date $t+1$ onward.

As previously mentioned, a seller's decision rule specifies that he refuses to accept privately issued notes in the decentralized market if the amount of notes issued by a banker

exceeds the upper bound $\bar{B}_t = a\left(\pi_t\right)/\phi_t$ at some date. Thus, I can interpret the banker's decision to renege on his promises as the dissolution of his note-issuing business, given that nobody will be willing to produce to acquire his notes in future periods if he refuses to convert his notes into goods today.

According to the Alvarez-Jermann equilibrium concept, the convertibility constraints (9) must hold with equality at each date, so I can rewrite (9) as follows:

$$-a\left(\pi_t\right) + \beta J_{t+1} = 0, \tag{10}$$

where $J_t = \sum_{\tau=t}^{\infty} \beta^{\tau-t} a\left(\pi_{\tau-1}\right)\left(\beta^{-1} - \pi_{\tau-1}\right)$ denotes the banker's lifetime utility at the beginning of date t.[7] I can also rewrite (5) as follows:

$$J_t = a\left(\pi_{t-1}\right)\left(\beta^{-1} - \pi_{t-1}\right) + \beta J_{t+1}. \tag{11}$$

Note that the term $a\left(\pi_{t-1}\right)\left(\beta^{-1} - \pi_{t-1}\right)$ gives the banker's current profit at date t. Specifically, at date $t-1$, the banker received the amount $a\left(\pi_{t-1}\right)$ in exchange for his notes and invested this amount in the productive technology, obtaining the revenue $\beta^{-1} a\left(\pi_{t-1}\right)$ at date t. Because each claim issued at date $t-1$ yields a real return π_{t-1} at date t, his current disbursement is given by $\pi_{t-1} a\left(\pi_{t-1}\right)$. Thus, his profit will be given by the difference between the revenue $\beta^{-1} a\left(\pi_{t-1}\right)$ and the disbursement $\pi_{t-1} a\left(\pi_{t-1}\right)$. As I have shown, he will immediately consume any profit he makes.

Combining (10) with (11), I obtain the following equilibrium law of motion for the rate of return on notes:

$$a\left(\pi_t\right) = \pi_{t-1} a\left(\pi_{t-1}\right). \tag{12}$$

A formal definition of a perfect-foresight equilibrium is now provided.

Definition 1 *A perfect-foresight equilibrium is a sequence $\{\pi_t\}_{t=0}^{\infty}$ satisfying $0 \leq \pi_t \leq \beta^{-1}$ and (12) at each date t.*

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[7] As suggested by a referee, Bertrand competition among bankers would provide a rationale for a binding convertibility constraint as an equilibrium requirement in the Alvarez-Jermann formulation.

Note that (10) indicates that the supply of notes today depends on the continuation value of the note-issuing business. If future monetary conditions are more favorable for each banker, then the value of his note-issuing business is higher, making it more costly for him to renege on his promises. In this case, the supply of notes today is higher. If future monetary conditions are less favorable for each banker, then he will be more inclined to renege on his promises. In this case, the supply of notes today is lower.

For the rest of the paper, I focus on the case in which the demand for private notes is strictly increasing in the expected return on notes. As we shall see, this is precisely the case in which we observe self-fulfilling collapses of the monetary mechanism, the main topic of this paper.

Assumption 1 Assume $c(q) = q$ and $u(q) = (1 - \sigma)^{-1} q^{1-\sigma}$, with $0 < \sigma < 1$.

Given this assumption, I can formally establish the existence and uniqueness of a stationary equilibrium.

Proposition 2 $\pi_t = 1$ for all $t \geq 0$ is the unique interior stationary equilibrium.

Proof. Note that $\pi_{t-1} = 0$ when $\pi_t = 0$ because the demand function $a(\pi)$ goes to zero as π converges to zero from above. When $\pi_t = \beta^{-1}$, it follows from (12) that $\pi_{t-1} < \beta^{-1}$. Also, note that $\pi = 1$ is a fixed point.

Using the *Implicit Function Theorem*, we have

$$\frac{d\pi_{t-1}}{d\pi_t} = \frac{a'(\pi_t)}{\pi_{t-1}a'(\pi_{t-1}) + a(\pi_{t-1})} > 0$$

for any $\pi_t \in [0, \beta^{-1}]$. This implies that the law of motion for π_t is strictly increasing, i.e., equation (12) implicitly defines a strictly increasing mapping $\pi_t = f(\pi_{t-1})$. From the *Inverse Function Theorem*, it follows that

$$\frac{d\pi_t}{d\pi_{t-1}} = \frac{\pi_{t-1}a'(\pi_{t-1}) + a(\pi_{t-1})}{a'(\pi_t)} > 0$$

for any $\pi_{t-1} \in [0, f^{-1}(\beta^{-1})]$. In particular, we have

$$\frac{d\pi_t}{d\pi_{t-1}}\bigg|_{\pi_{t-1}=\pi_t=1} = \frac{a'(1) + a(1)}{a'(1)} > 1,$$

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which means that the mapping $\pi_t = f(\pi_{t-1})$ crosses the 45-degree line from below at the point $(\pi_{t-1}, \pi_t) = (1, 1)$. Thus, the unique interior stationary solution is given by $\pi_t = 1$ for all $t \geq 0$. In this case, the amount of good 2 produced by the seller in each bilateral meeting is given by the quantity $\hat{q}$ satisfying

$$\frac{u'(\hat{q})}{c'(\hat{q})} = \frac{1}{\beta}.$$

∎

In this equilibrium, the exchange value of notes remains constant over time. Agents do not expect monetary conditions to change over time, so the amount of notes issued at each date, as well as their expected return, remains constant over time. In particular, agents expect the gross return on notes to be one in all future periods and know that, as long as the amount of funds raised from the sale of notes equals $a(1) = \beta^{\frac{1}{\sigma} - 1}$ at each date, each banker will be willing to maintain the convertibility of bank liabilities. As a result, no banker will ever renege on his promises along the equilibrium path.

Note that each banker is able to consume

$$a(1)\left(\beta^{-1} - 1\right) = \beta^{\frac{1}{\sigma}}(1 - \beta)$$

at each date, which is precisely the flow of income derived from the note-issuing business in a stationary equilibrium. In equilibrium, each individual banker is willing to maintain the convertibility of bank liabilities to retain this constant flow of income. Although the market for privately issued notes is perfectly competitive, each banker earns a positive profit from the note-issuing business. Because agents lack commitment and private portfolios are not publicly observable, only a strictly positive franchise value is consistent with an equilibrium outcome. Perfect competition ensures that this franchise value is the *minimum* stationary value consistent with the voluntary convertibility of privately issued notes.

Thus, it is possible to construct an equilibrium with the property that the value of privately issued notes is stable over time so that the equilibrium allocation is stationary. However, it is also possible to construct other equilibria in which the exchange value of notes is not constant over time, with agents fully anticipating that privately issued notes

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will persistently depreciate over time. These equilibria exist because other beliefs regarding the value of the note-issuing business will also be consistent with an equilibrium outcome (i.e., a self-fulfilling prophecy). In these equilibria, the amount of goods produced and traded in the decentralized market will vary over time, and the dynamics will be completely driven by expectations about future monetary conditions.

5.2. Self-Fulfilling Collapses

In this subsection, I characterize equilibria for which people expect monetary conditions to constantly deteriorate over time. I interpret this kind of equilibrium as a self-fulfilling collapse of the banking system characterized by a persistent decline in the amount of notes in circulation driven by expectations that monetary conditions will persistently deteriorate over time. As we shall see, this kind of equilibrium will have an adverse impact on trading activity. In particular, the quantities produced and traded in the decentralized market will monotonically decline over time.

As I have shown in the proof of Proposition 2, it follows that

$$\frac{d\pi_t}{d\pi_{t-1}} = \frac{\pi_{t-1}a'(\pi_{t-1}) + a(\pi_{t-1})}{a'(\pi_t)} > 0,$$

which means that (12) implicitly defines a strictly increasing mapping $\pi_t = f(\pi_{t-1})$. In particular, we have

$$\left.\frac{d\pi_t}{d\pi_{t-1}}\right|_{\pi_{t-1}=\pi_t=1} = \frac{a'(1) + a(1)}{a'(1)} > 1,$$

which means that the mapping $\pi_t = f(\pi_{t-1})$ crosses the 45-degree line from below at the unique interior steady state $(\pi_{t-1}, \pi_t) = (1, 1)$. Note also that, for any initial condition $\pi_0 > 1$, there will be no equilibrium because the condition $\pi_t \leq \beta^{-1}$ will necessarily be violated at some finite date t. See Figure 3.

For any initial condition $\pi_0 \in (0, 1)$, there exists a unique equilibrium trajectory that is monotonically decreasing. Along this equilibrium path, real money balances decrease monotonically over time and converge to zero, so the equilibrium allocation approaches autarky as $t \to \infty$. The decline in the desired amount of real balances follows from the

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optimization problem of each buyer when the value of privately issued notes persistently depreciates over time (i.e., the anticipated decline in the purchasing power of private money leads agents to reduce their real money balances over time). As a result, buyers and sellers will be able to trade smaller amounts of goods in the decentralized market. I summarize these findings in the following proposition.

Proposition 3 *For each initial condition $\pi_0 \in (0,1)$, there exists a unique equilibrium trajectory $\{\pi_t\}_{t=0}^{\infty}$ with the property that the rate of return on notes is strictly decreasing and converges to zero.*

I interpret this kind of equilibrium as a self-fulfilling collapse of the banking system. As I have shown, the determination of equilibrium quantities and prices completely depends on agents' beliefs regarding the value of the note-issuing business. Because agents believe that the exchange value of private notes will persistently decrease over time (so agents expect the value of the note-issuing business to decline), the amount of funds devoted to each banker is lower at the current date, so the number of notes in circulation today is lower. In fact, the number of notes in circulation monotonically decreases over time, resulting in a decreasing amount of goods produced and traded in the decentralized market. From a buyer's standpoint, his demand for private notes declines over time because he expects the purchasing power of these claims to continue to depreciate along the equilibrium trajectory, allowing him to purchase ever smaller amounts of goods from his trading partners.

It is important to emphasize that a declining sequence of expected returns on notes affects each banker's ability to raise funds through the issuance of notes because it reduces the continuation value of the note-issuing business, which can be defined as the bank's franchise value. As we have seen, a larger franchise value today increases the banker's ability to raise funds through the issuance of notes because note holders know that the banker will have more to lose if he reneges on his promises. A declining sequence of expected returns on notes implies that the franchise value will monotonically decrease along the equilibrium path, so agents know that each banker's incentive to maintain the convertibility of notes is declining over time. Accordingly, the nonbank agents reduce their demand for private

money as its purchasing power persistently depreciates.

To verify that the banker's franchise value declines along any nonstationary equilibrium trajectory, we can rewrite the dynamic system in terms of the banker's continuation value J_t. Thus, (12) can be rewritten as

$$J_t h\left(J_t\right) = J_{t+1},$$

where $h\left(J\right) = a^{-1}\left(\beta J\right)$. Because $h\left(J\right)$ is strictly increasing, a nonstationary equilibrium necessarily involves a monotonically decreasing sequence for the continuation value J_t.

The existence of equilibria with undesirable properties for initial conditions arbitrarily close to $\pi_0 = 1$ implies that a private monetary system is necessarily unstable. These equilibria arise because there is no condition to pin down the initial value of notes. As a result, multiple beliefs regarding the exchange value of notes in future periods are consistent with an equilibrium outcome. In this respect, inside money shares some of the same properties as outside fiat money, namely, indeterminacy of equilibrium. See Wallace (1980), Woodford (1984), and Lagos and Wright (2003) for a description of the properties of outside money.[8]

Despite this similarity, it is important to keep in mind that the mechanism leading to the existence of multiple self-fulfilling equilibria involving a collapse of the value of private money is different from that giving rise to self-fulfilling inflationary equilibria in outside-money economies. As previously described, the key element generating multiplicity of equilibrium is the distrust of banks, which requires buyers and sellers to form beliefs regarding the evolution of the franchise value to determine the acceptability of private notes as a means of payment.

6. MONOPOLISTIC BANKING SYSTEM

In the previous section, I have characterized the properties of a perfectly competitive banking system. I have shown that an important characteristic of the ensuing private monetary system is the existence of multiple equilibrium allocations with undesirable properties.

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[8] See also Gu, Mattesini, Monnet, and Wright (2013b) for a study of the dynamic properties of pure credit economies.

A natural question to ask is whether a concentrated banking system is consistent with the determinacy of equilibrium. In this section, I provide at least a partial answer to this question by showing that a monopolistic banking system results in a unique equilibrium allocation. Under certain conditions, this unique equilibrium allocation is exactly the same as the stationary allocation obtained under perfect competition (with the same equilibrium return on notes). However, there exists a region of the parameter space in which a stable banking system under monopoly results in a lower equilibrium return on notes, making the nonbank public worse off.

Suppose now that there is only one banker in the economy and assume that all other characteristics of the environment remain the same. The monopolist banker takes the demand function $a(\pi_t)$ as given when making his individual choices. As in a typical monopolist problem, the banker is able to directly choose the return on notes by selecting the amount of notes he wants to supply at each date. The monopolist's problem can be formulated as follows:

$$\max_{\{x_t, s_t, \pi_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t x_t$$

subject to the budget constraint

$$s_t + x_t + \pi_{t-1} a(\pi_{t-1}) = \beta^{-1} s_{t-1} + a(\pi_t),$$

the individual rationality constraint

$$-a(\pi_t) + \sum_{\tau=t+1}^{\infty} \beta^{t-\tau} x_\tau \geq 0, \tag{13}$$

and the feasibility conditions $x_t \geq 0$, $s_t \geq 0$, and $0 \leq \pi_t \leq \beta^{-1}$ for all $t \geq 0$. Note that, under a monopolistic banking system, the individual rationality constraint does not need to hold with equality, as in a competitive equilibrium (a consequence of the Alvarez-Jermann equilibrium concept).

The monopolist's decision problem previously described can be viewed as the decision problem faced by a typical monopolist who is required to make intertemporal decisions regarding the supply of a good or service. What is perhaps nonstandard is the requirement

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that the choice of any allocation must satisfy an individual rationality constraint as a consequence of a lack of commitment. As we shall see, the presence of this additional constraint results in interesting new results regarding the properties of a solution to the monopolist's problem.

To solve this problem, ignore the individual rationality constraint for a moment. In this case, the first-order necessary condition for π_t is given by

$$a'(\pi_t)(1 - \beta\pi_t) = \beta a(\pi_t). \tag{14}$$

I am also temporarily ignoring the feasibility condition $0 \leq \pi_t \leq \beta^{-1}$. Note that the first-order condition (14) implies a constant value for the return on notes. This means that, under certain regularity conditions, there exists a unique value π^m consistent with utility maximization under a monopolistic banking system. Note also that one possible choice for the amount invested in the productive technology is $s_t = a(\pi_t)$ at each date $t \geq 0$, which implies that the feasibility condition $x_t \geq 0$ is satisfied at each date $t \geq 0$.

The following result establishes the existence of a solution to the monopolist's problem under Assumption 1.

Proposition 4 *There exists a solution to the monopolist's problem. In addition, any solution necessarily involves a constant value for the return on notes, i.e., we have $\pi_t = \pi^m$ for all $t \geq 0$. If $\beta \geq 1 - \sigma$, then $\pi^m = \beta^{-1}(1 - \sigma)$. If $\beta < 1 - \sigma$, then $\pi^m = 1$.*

Proof. We can rewrite the monopolist's maximization problem as

$$\max_{\{s_t, \pi_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[a(\pi_t) - \pi_{t-1} a(\pi_{t-1}) + \beta^{-1} s_{t-1} - s_t \right],$$

with $a(\pi_t) - \pi_{t-1} a(\pi_{t-1}) + \beta^{-1} s_{t-1} - s_t \geq 0$ and $0 \leq \pi_t \leq \beta^{-1}$ at each date $t \geq 0$. Define $g(\pi) \equiv (1 - \beta\pi) a(\pi)$ for any $\pi \in [0, \beta^{-1}]$. Under Assumption 1, it follows that

$$g(\pi) = (1 - \beta\pi)(\beta\pi)^{\frac{1}{\sigma}-1}.$$

Then, there exists a unique value π^* satisfying $g'(\pi^*) = 0$. Note that $g''(\pi^*) < 0$, which means that the first-order condition $g'(\pi^*) = 0$ is both necessary and sufficient for a maxi-

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mum. Note also that condition (14) becomes

$$(1 - \beta\pi^*)\left(\frac{1}{\sigma} - 1\right) = \beta\pi^*,$$

so $\pi^* = \beta^{-1}(1 - \sigma)$. Because $\sigma < 1$, we have $\pi^* < \beta^{-1}$, satisfying feasibility.

It remains to verify whether the individual rationality constraint is satisfied. Note that the left-hand side of (13) is given by

$$(1 - \sigma)^{\frac{1}{\sigma} - 1}\left(\frac{\sigma}{1 - \beta} - 1\right),$$

so condition (13) is satisfied provided $\beta \geq 1 - \sigma$. In this case, the solution is given by $\pi_t = \pi^* = \beta^{-1}(1 - \sigma)$ for all $t \geq 0$.

If $\beta < 1 - \sigma$, then the maximum $\pi^* = \beta^{-1}(1 - \sigma) > 1$ is attained at a point in which the individual rationality constraint is violated. Then, it follows that $\pi_t = 1$ for all $t \geq 0$ is a solution to the monopolist's maximization problem. ∎

An immediate corollary of this proposition is that the allocation associated with any solution to the monopolist's problem is exactly the same for the nonbank public. In addition, the indirect utility of the banker is the same in any solution. Thus, the previous result indicates that a monopolistic banking system is indeed consistent with stability. If $\beta \leq 1 - \sigma$, then the equilibrium allocation under a monopolistic banking system is exactly the same as the interior stationary equilibrium allocation under perfect competition. Because the equilibrium is uniquely determined, self-fulfilling crises do not occur under a monopolistic banking structure. If $\beta > 1 - \sigma$, then the stability of the banking system under monopoly comes at the cost of a lower rate of return on bank liabilities, making the nonbank public worse off.

It is surprising to find that a monopolistic banking system is willing to pay the same rate of return on its liabilities as the return paid under a competitive banking system when $\beta \leq 1 - \sigma$. To understand why this happens, note that, assuming full commitment, a competitive banking system offers the same rate of return on its liabilities as the return on the productive technology because competition drives each banker's profit to zero. Ignoring the individual rationality constraint (13), the monopolist banker chooses a rate of return on

bank liabilities that is lower than $1/\beta$ (i.e., the competitive equilibrium return assuming full commitment). If $\beta \le 1 - \sigma$, then the optimal rate of return under monopoly lies between 1 and $1/\beta$ if we ignore the individual rationality constraint. Because this return is inconsistent with voluntary convertibility, it follows that $\pi_t = 1$ is the (constrained) optimal choice of a monopolist banker.

The relationship between banking structure and financial stability has been studied by Allen and Gale (2004) and Boyd, De Nicolo, and Smith (2004), among others. Using a variety of models, Allen and Gale (2004) study the interplay between competition and stability in the banking system. They find that the relationship between stability and competition is not uniform across different models. Boyd, De Nicolo, and Smith (2004) study the properties of a competitive and monopolistic banking system. In an environment characterized by spatial separation and limited communication, banks provide insurance to finitely lived agents against the risk of being randomly relocated. They find that the probability of a costly banking crisis is always higher under competition than under monopoly. But this advantage of a monopolistic banking system is obtained at the cost of less valuable intertemporal insurance. In my framework, I cannot characterize the probability of a banking crisis as this event depends exclusively on beliefs. But my findings are consistent with their results, despite the fundamental differences in the models.

7. FISCAL INTERVENTION

The goal of this section is to investigate whether there exists a government intervention that can ensure the stability of the private monetary system under perfect competition. Note that any successful intervention must rule out self-fulfilling beliefs involving a declining value of the note-issuing business. One way to achieve this goal is to formulate an intervention that provides a minimum revenue to note-issuing agents regardless of the private-sector demand for notes. This kind of intervention guarantees that the franchise value associated with the note-issuing business does not monotonically decline as the expected return on notes persistently falls.

Suppose the government establishes a clearinghouse in the centralized location and requires that all notes issued by bankers must be redeemed through the clearinghouse. This means that the liabilities of the clearinghouse will be given by the real value of outstanding notes. Each banker who issues notes is required to contribute to the clearinghouse at each date to ensure the redemption of private notes through the clearinghouse. The government will now set the individual limits $\left\{\bar{B}_t\right\}_{t=0}^{\infty}$ on each banker. Thus, we can now interpret the variable $\hat{n} \in \mathbb{R}_+$ in the banker's decision problem as his obligation with the clearinghouse with real value $\phi_t \hat{n}$. Recall that the constraint $\hat{n} \leq \bar{B}_t$ must hold in the banker's decision problem, which means that the government limits its exposure to each member bank.

Suppose the government sets the limit $\phi_t \bar{B}_t = a\left(\pi_t\right) - \alpha \tau\left(\pi_t\right)$ at each date $t \geq 0$, where $\tau : \mathbb{R}_+ \to \mathbb{R}$ is a continuously differentiable and increasing function and $\alpha \in [0, 1]$. This choice of individual limits on note issue must respect feasibility so that

$$a\left(\pi_t\right) \geq \alpha \tau\left(\pi_t\right)$$

must hold at each date $t \geq 0$. Recall that $a\left(\pi_t\right)$ represents the buyer's desired real money holdings. In the absence of intervention, this is also the amount of resources each banker obtains in exchange for his notes. Thus, we can interpret the term $\alpha \tau\left(\pi_t\right)$ as a tax on note-issuing agents collected by the clearinghouse. This means that each banker's *effective* obligation with the clearinghouse at date $t + 1$ will be given by $\left[a\left(\pi_t\right) - \alpha \tau\left(\pi_t\right)\right] \pi_t$, where $a\left(\pi_t\right) - \alpha \tau\left(\pi_t\right)$ is the net amount of good 1 that each banker receives in exchange for notes at date t and π_t is the promised return on each outstanding note. As a result, the banker's lifetime utility is now given by

$$\hat{J}_t = \left[a\left(\pi_{t-1}\right) - \alpha \tau\left(\pi_{t-1}\right)\right]\left(\beta^{-1} - \pi_{t-1}\right) + \beta \hat{J}_{t+1},$$

with $\hat{J}_t \in \mathbb{R}$ representing his continuation value in the presence of government intervention.

Note that each banker can renege on his obligation with the clearinghouse. Because the amount of good 1 invested by each banker in the productive technology is privately

observable, the following convertibility constraints must be satisfied at each date:

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[a\left(\pi_{\tau-1}\right) - \alpha\tau\left(\pi_{\tau-1}\right) \right] \left(\beta^{-1} - \pi_{\tau-1}\right)$$
$$\geq \ \left[a\left(\pi_{t-1}\right) - \alpha\tau\left(\pi_{t-1}\right)\right] \left(\beta^{-1} - \pi_{t-1}\right) + a\left(\pi_t\right) - \alpha\tau\left(\pi_t\right).$$

These constraints also ensure that each seller is willing to accept private notes as a means of payment when the government implements a tax scheme on note-issuing agents.

Finally, the government budget constraint is given by

$$v_t + \alpha\tau\left(\pi_t\right) + \left[a\left(\pi_{t-1}\right) - \alpha\tau\left(\pi_{t-1}\right)\right]\pi_{t-1} = a\left(\pi_{t-1}\right)\pi_{t-1},$$

with $v_t \in \mathbb{R}$ representing a lump-sum tax on each buyer. Note that $v_t < 0$ implies a lump-sum transfer, which is made when there is a surplus over the amount required to cover all redemptions at a particular date. The second term on the left-hand side gives the *per capita* revenue from the issuance of notes at the current date. The third term on the left-hand side is the *per capita* amount of good 1 that each banker who has issued notes at the previous date is required to contribute to the clearinghouse at the current date. The right-hand side gives the *per capita* amount of good 1 that the clearinghouse is supposed to disburse at date t to make good on notes issued at the previous date.

It is important to emphasize that, because the convertibility constraints must be satisfied at each date, the lump-sum tax v_t on nonbank agents is imposed to exclusively cover any shortfall originating from specific fiscal policies and *not* to cover defaults by member banks. This characteristic of the clearinghouse and the government fiscal policy are perfectly communicated to agents.

Definition 5 *A perfect-foresight equilibrium in the presence of intervention is a sequence* $\{\pi_t, v_t\}_{t=0}^{\infty}$ *satisfying* $0 \leq \pi_t \leq \beta^{-1}$,

$$v_t = \alpha\left[\pi_{t-1}\tau\left(\pi_{t-1}\right) - \tau\left(\pi_t\right)\right], \tag{15}$$

$$\left[a\left(\pi_{t-1}\right) - \alpha\tau\left(\pi_{t-1}\right)\right]\pi_{t-1} = a\left(\pi_t\right) - \alpha\tau\left(\pi_t\right), \tag{16}$$

and

$$a\left(\pi_t\right) \geq \alpha\tau\left(\pi_t\right) \tag{17}$$

at each date t.

If the government chooses $\alpha = 0$, then the dynamic system is exactly the same as that derived in the previous section, given by (12). Thus, I have already characterized the set of equilibrium allocations for the case $\alpha = 0$. In what follows, I want to characterize the set of equilibrium allocations under a particular form of intervention.

Assumption 1 implies that the aggregate demand for notes is given by $a(\pi) = (\beta\pi)^{\frac{1}{\sigma}-1}$ for any $\pi \in [0, \beta^{-1}]$. Now, suppose the government sets

$$\tau(\pi) = \delta\pi^{\frac{1}{\sigma}-1} - \left(\delta - \beta^{\frac{1}{\sigma}-1}\right), \tag{18}$$

with $\delta > \beta^{\frac{1}{\sigma}-1}$. In addition, suppose the government chooses a sufficiently small value for the parameter α such that $\alpha\delta < \beta^{\frac{1}{\sigma}-1}$. Given these choices, the effective tax burden on each banker is given by

$$\alpha\tau(\pi_t) = \alpha\delta\pi_t^{\frac{1}{\sigma}-1} - \alpha\left(\delta - \beta^{\frac{1}{\sigma}-1}\right)$$

at each date $t \geq 0$. Note that, for any $\pi_t < \left(1 - \frac{\beta^{\frac{1}{\sigma}-1}}{\delta}\right)^{\frac{\sigma}{1-\sigma}}$, each banker receives a subsidy from the government. If the rate of return on notes falls below the threshold value $\left(1 - \frac{\beta^{\frac{1}{\sigma}-1}}{\delta}\right)^{\frac{\sigma}{1-\sigma}}$, then the government provides an additional revenue to each banker. If the rate of return is above the threshold, then each banker is required to pay taxes when issuing notes to the public.

Given the specific fiscal intervention described above, the law of motion for the rate of return on notes is given by

$$\left(\beta^{\frac{1}{\sigma}-1} - \alpha\delta\right)\pi_{t-1}^{\frac{1}{\sigma}-1} + \alpha\left(\delta - \beta^{\frac{1}{\sigma}-1}\right)\pi_{t-1} = \left(\beta^{\frac{1}{\sigma}-1} - \alpha\delta\right)\pi_t^{\frac{1}{\sigma}-1} + \alpha\left(\delta - \beta^{\frac{1}{\sigma}-1}\right). \tag{19}$$

Figure 4 plots this law of motion. An important characteristic of the proposed fiscal intervention is that it makes any belief involving a persistent decline in the value of notes inconsistent with an equilibrium outcome. In the absence of intervention, any belief involving a persistent decline in the value of notes necessarily implies that the continuation value of the note-issuing business converges monotonically to zero. This belief becomes self-fulfilling because a declining value of the note-issuing business reduces each banker's

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incentive to maintain the convertibility of notes, so the supply of notes must decline over time to ensure voluntary convertibility.

In the case of the intervention previously described, the government provides a *minimum* revenue to note-issuing agents regardless of the private-sector demand for notes. In particular, note that the aftertax revenue function for each banker is given by

$$\left(\beta^{\frac{1}{\sigma}-1} - \alpha \delta \right) \pi^{\frac{1}{\sigma}-1} + \alpha \left(\delta - \beta^{\frac{1}{\sigma}-1} \right),$$

which is strictly increasing in the return on notes, as in Section 5. The main difference from the analysis in Section 5 is that, for each note-issuing agent, there is a minimum revenue level that is guaranteed by the government's fiscal policy even if the private-sector demand for notes converges to zero. As a result, any belief involving a persistent decline in the value of notes does not drive the continuation value of the note-issuing business to zero so that it cannot be consistent with an equilibrium outcome.

The following proposition establishes the uniqueness of equilibrium under the proposed fiscal intervention.

Proposition 6 *It is possible to choose a function $\tau : \mathbb{R}_+ \to \mathbb{R}$ and a scalar $\alpha \in [0, 1]$ such that the government intervention implies that the stationary solution $\pi_t = 1$ for all $t \geq 0$ is the unique equilibrium allocation.*

Proof. Suppose $\tau : \mathbb{R}_+ \to \mathbb{R}$ is given by (18) and choose α sufficiently small such that $\alpha \delta < \beta^{\frac{1}{\sigma}-1}$. First, note that $\pi_{t-1} = \pi_t = 1$ satisfies (19), so $\pi_t = 1$ for all $t \geq 0$ is a stationary solution. Second, when $\pi_t = 0$, we must have $\pi_{t-1} > 0$ according to (19). When $\pi_t = \beta^{-1}$, it follows from (19) that $\pi_{t-1} < \beta^{-1}$. Finally, using the *Implicit Function Theorem*, we find that

$$\frac{d\pi_{t-1}}{d\pi_t} = \frac{\left(\beta^{\frac{1}{\sigma}-1} - \alpha \delta \right) \left(\frac{1}{\sigma} - 1 \right) \pi_t^{\frac{1}{\sigma}}}{\left(\beta^{\frac{1}{\sigma}-1} - \alpha \delta \right) \left(\frac{1}{\sigma} - 1 \right) \pi_{t-1}^{\frac{1}{\sigma}} + \alpha \left(\delta - \beta^{\frac{1}{\sigma}-1} \right)} > 0$$

for any $\pi_t \in \left[0, \beta^{-1} \right]$. This means that the law of motion for π_t is also strictly increasing.

Using the *Inverse Function Theorem*, we have

$$\frac{d\pi_t}{d\pi_{t-1}} = \frac{\left(\beta^{\frac{1}{\sigma}-1} - \alpha\delta\right)\left(\frac{1}{\sigma} - 1\right)\pi_{t-1}^{\frac{1}{\sigma}} + \alpha\left(\delta - \beta^{\frac{1}{\sigma}-1}\right)}{\left(\beta^{\frac{1}{\sigma}-1} - \alpha\delta\right)\left(\frac{1}{\sigma} - 1\right)\pi_t^{\frac{1}{\sigma}}} > 0$$

In particular, we have

$$\left.\frac{d\pi_t}{d\pi_{t-1}}\right|_{\pi_{t-1}=\pi_t=1} = \frac{\left(\beta^{\frac{1}{\sigma}-1} - \alpha\delta\right)\left(\frac{1}{\sigma} - 1\right) + \alpha\left(\delta - \beta^{\frac{1}{\sigma}-1}\right)}{\left(\beta^{\frac{1}{\sigma}-1} - \alpha\delta\right)\left(\frac{1}{\sigma} - 1\right)} > 1,$$

which means that the law of motion for π_t must cross the 45-degree line from below at $(\pi_{t-1}, \pi_t) = (1, 1)$. Thus, $\pi_t = 1$ for all $t \geq 0$ is the unique solution to the dynamic system (19). ∎

The equilibrium allocation for each buyer and each seller is exactly the same as the (interior) stationary equilibrium allocation obtained in the absence of intervention. Because $\tau(1) = a(1)$, the consumption of each banker is now given by

$$(1 - \alpha)\, a(1)\left(\beta^{-1} - 1\right) = (1 - \alpha)\,\beta^{\frac{1}{\sigma}}\,(1 - \beta)$$

at each date. Thus, for α sufficiently small, the consumption of each banker is approximately the same as the amount he gets in the interior stationary equilibrium in the absence of intervention.

Note that, on the equilibrium path, the government does not need to levy any tax on nonbank agents. The anticipation of an active fiscal intervention off the equilibrium path is sufficient to rule out self-fulfilling beliefs involving a persistent decline in the value and volume of private money. As we have seen, the government's subsidy to note-issuing agents guarantees a minimum franchise value if the private-sector demand for notes declines below a certain level. Such a subsidy is financed by a lump-sum tax on each buyer, who is a producer of good 1, so that the government's fiscal policy is sustainable. A crucial aspect of this intervention is that it specifies a fiscal rule contingent on the (publicly observable) rate of return on notes that is perfectly communicated to agents.

A final and interesting question is whether a government intervention is necessary for achieving stability. So far, I have shown that a government intervention is sufficient to obtain

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determinacy. Is it possible to have a purely private arrangement within the group of bankers with the property that the equilibrium is globally determinate? For instance, is it possible to interpret the taxation scheme previously described as a membership fee imposed by a private clearinghouse association? The scheme implemented within the group of bankers can legitimately be interpreted as a purely private arrangement (for instance, a mutually owned clearinghouse) given that it is designed to be incentive-compatible. However, a successful scheme requires the taxation of nonbank agents off the equilibrium path to provide the necessary subsidy to note-issuing agents, which makes it inconsistent with the interpretation of a mutually owned clearinghouse association without any government intervention. For this reason, I believe that the kind of intervention presented in this section has to be implemented by a central authority with the legal power to tax agents in the economy.

8. CONCLUSION

In this paper, I have characterized the properties of a purely private monetary system. The key frictions in the environment are agents' inability to commit to their promises and to verify the amount of collateral pledged as reserves to secure privately issued claims that circulate as a medium of exchange. As a result, agents distrust those who have the ability to issue these claims, giving rise to endogenous limits on money creation. Under a competitive regime, multiple beliefs regarding the value of the note-issuing business are consistent with an equilibrium outcome, including self-fulfilling beliefs that result in the collapse of the banking system. These equilibria have undesirable properties: The amount of private money in circulation persistently declines over time, agents continuously reduce their demand for money, and trading activity collapses.

In view of these difficulties, I have characterized a government intervention that results in the global determinacy of equilibrium, which is a desirable property of any monetary system. Specifically, it is possible to formulate an intervention that results in a stable value of private money so that trading activity does not contract as a result of a self-fulfilling collapse. Thus, private agents are able to provide useful liquidity services consistent with

macroeconomic stability provided that the government is willing to intervene to guarantee a minimum franchise value associated with private money creation.
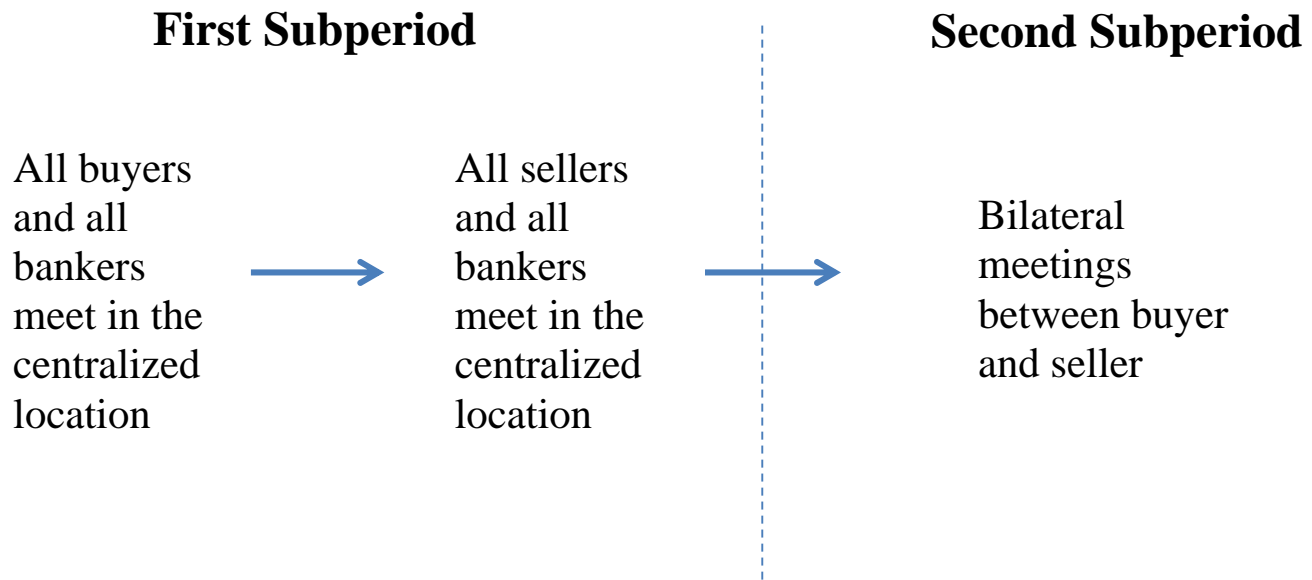
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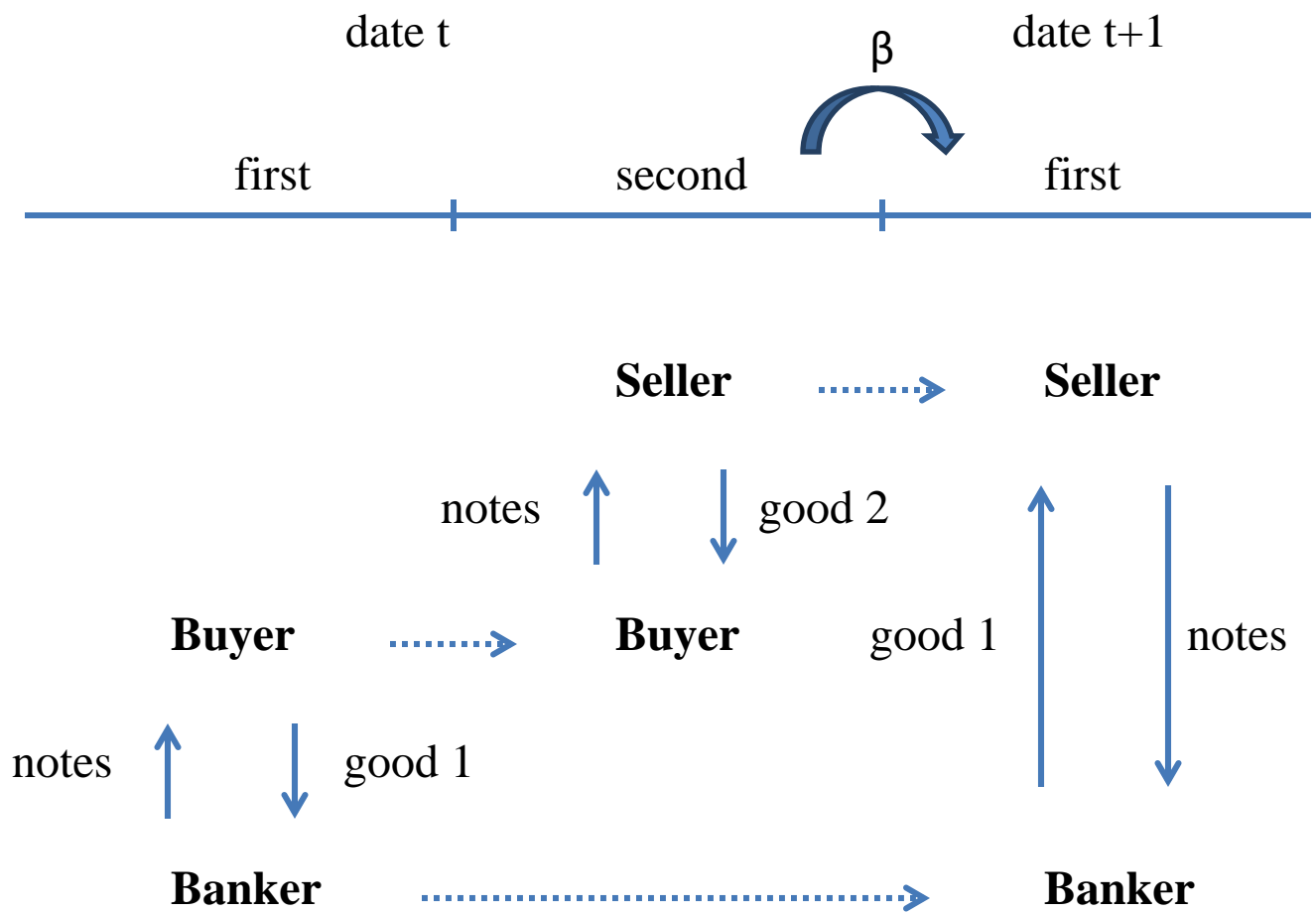
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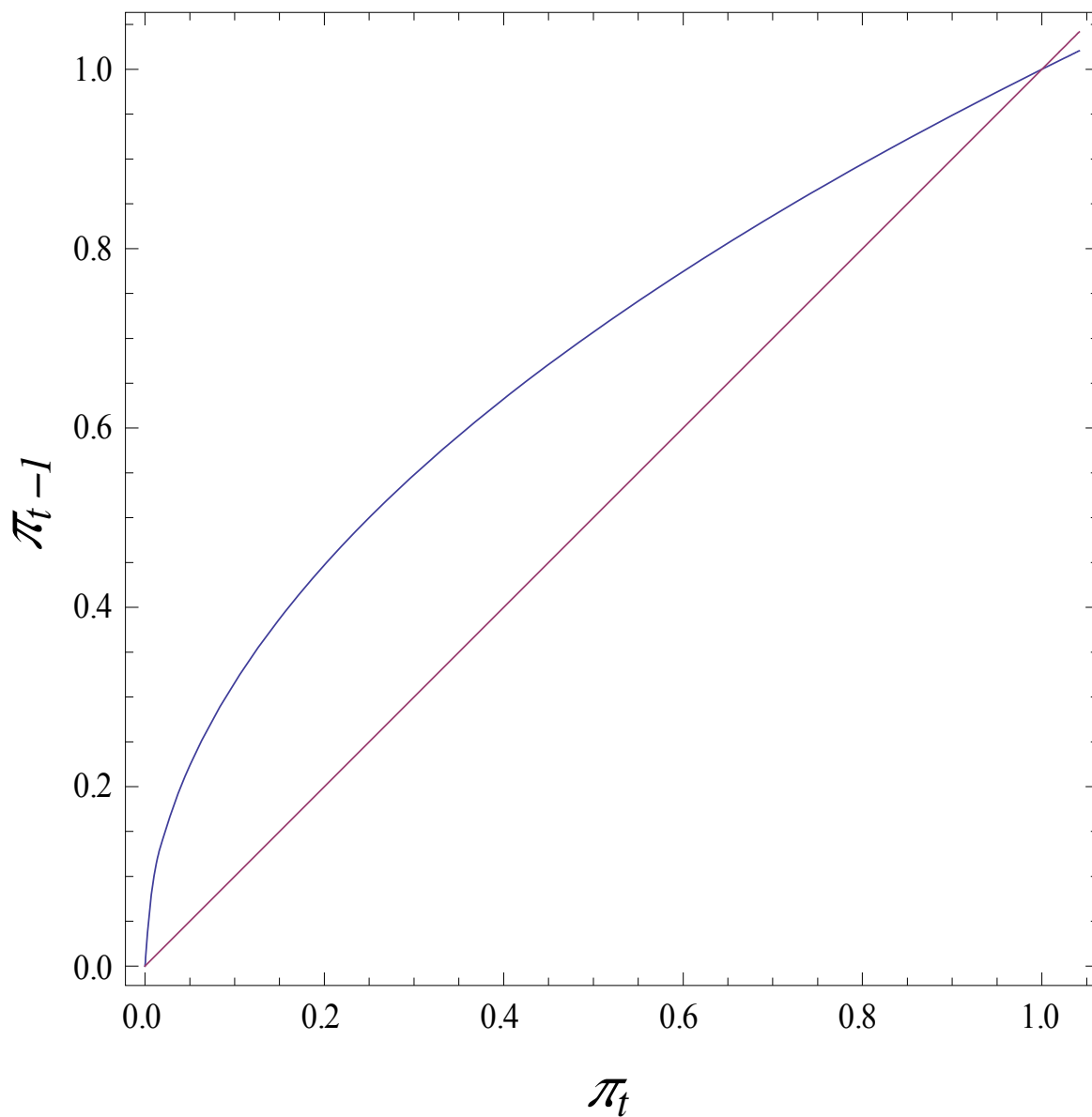
Figure 1: Sequence of Events Within a Period

First Subperiod

Second Subperiod

All buyers and all bankers meet in the centralized location

→

All sellers and all bankers meet in the centralized location

→

Bilateral meetings between buyer and seller

Figure 2: Circulation of Private Notes

date t   β   date t+1

first   second   first

Seller ┈┈> **Seller**

notes ↑   ↓ good 2

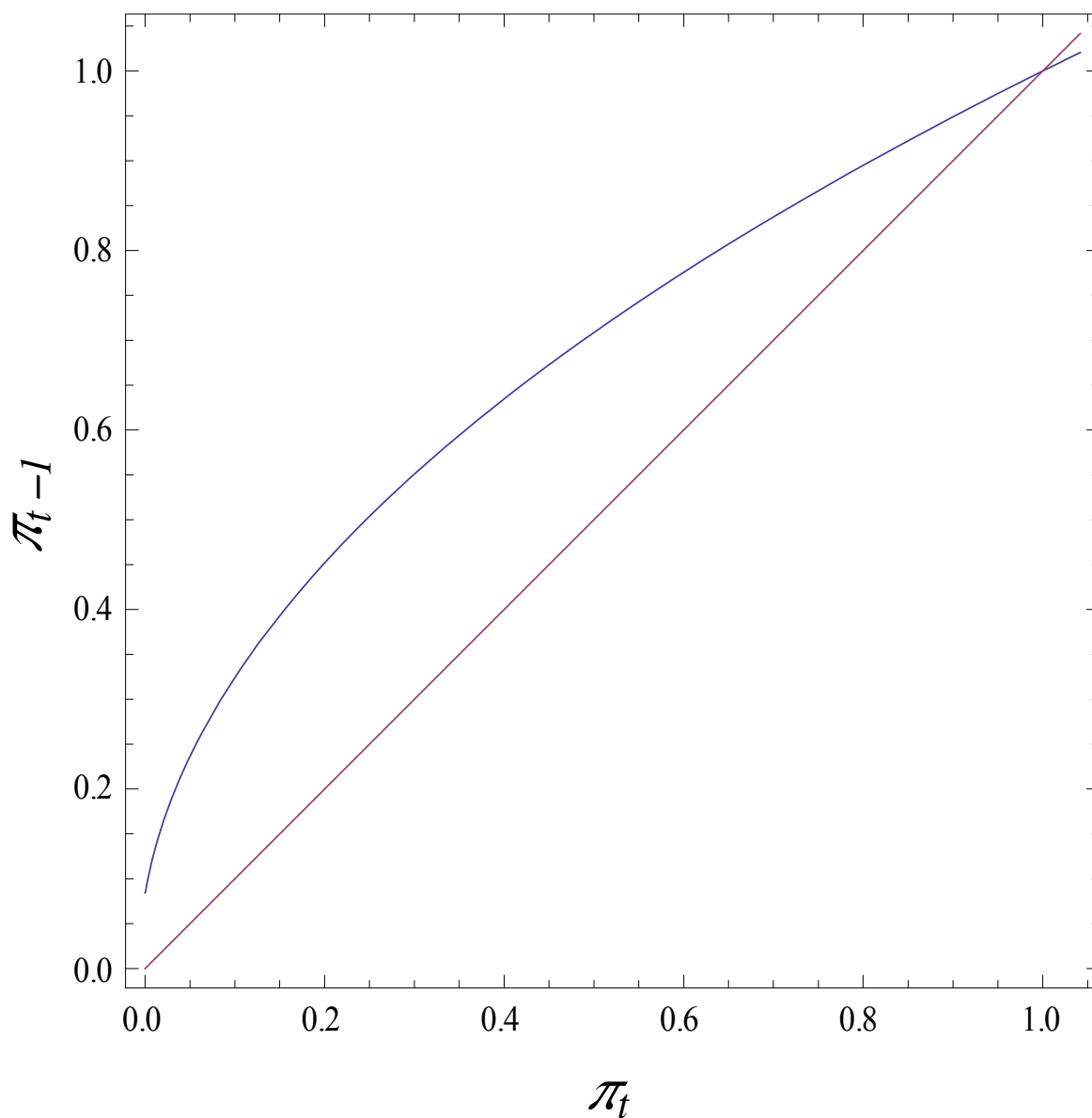**Buyer** ┈┈> **Buyer**   good 1   notes

notes ↑   ↓ good 1

Banker ┈┈┈┈┈> **Banker**

Figure 3: Self-Fulfilling Banking Crises

$u(q)=(1-\sigma)^{-1}q^{1-\sigma}$; $c(q)=q$; $\beta=0.96$; $\sigma=0.5$

Figure 4: Fiscal Intervention and Global Determinacy

$u(q)=(1-\sigma)^{-1}q^{1-\sigma}$; $c(q)=q$; $\beta=0.96$; $\alpha=0.05$; $\delta=1.1$; $\sigma=0.5$