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ASSESSING BANKRUPTCY REFORM IN A MODEL WITH
TEMPTATION AND EQUILIBRIUM DEFAULT**

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# Assessing Bankruptcy Reform in a Model with Temptation and Equilibrium Default

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## Abstract

A life-cycle model with equilibrium default in which consumers with and without temptation coexist is constructed to evaluate the 2005 bankruptcy law reform and other counterfactual reforms. The calibrated model indicates that the 2005 bankruptcy reform achieves its goal of reducing the number of bankruptcy filings, as seen in the data, but at the cost of loss in social welfare. The creditor-friendly reform provides borrowers with a stronger commitment to repay and thus yields lower default premia and better consumption smoothing. However, those who borrow and default due to temptation or unavoidable large expenditures suffer more under the reform due to higher costs or means-testing requirement. Moreover, those who borrow due to temptation suffer from overborrowing when the borrowing cost declines. The model indicates that the negative welfare effects dominate.

JEL Classification: D91, E21, E44, G18, K35
Keywords: consumer bankruptcy, debt, default, borrowing constraint, temptation and self-control, hyperbolic discounting, heterogeneous agents, incomplete markets

# 1 Introduction

Preferences that exhibit present bias have become widely used in economics. Based on the success of the models with present bias in replicating various dimensions of borrowing behavior, White (2007) argues that present bias is an important feature in constructing a model of bankruptcies for policy evaluation. This paper follows her claim and constructs a novel model in which agents with temptation and without coexist, and some agents optimally choose to default in equilibrium. I use the model to study macroeconomic and welfare implications of bankruptcy law reforms, in particular, the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) enacted in 2005. Using a carefully calibrated model, I can separately analyze the implications of each component of the BAPCPA as well as the effects on different types of agents.

This is the first paper that extends the quantitative macroeconomic model with equilibrium bankruptcy (Livshits et al. (2007) and Chatterjee et al. (2007)) by introducing preferences featuring temptation and self-control (Gul and Pesendorfer (2001, 2004a)). Moreover, unlike papers studying macroeconomic implications of the model where agents are subject to present bias, agents with and without temptation coexist in the current paper. I introduce the temptation preferences following the formulation provided by Krusell et al. (2010). The finite-horizon model with Gul-Pesendorfer preferences that Krusell et al. (2010) construct includes the hyperbolic-discounting model of Strotz (1956) and Laibson (1997) as a special case. I use this special case since estimates for the preference parameter that controls the degree of present bias are available for the hyperbolic-discounting model. The model is calibrated to match the facts related to recent borrowing and bankruptcy in the U.S. economy and is used for a series of counterfactual experiments.

The calibrated model implies that the 2005 bankruptcy reform achieves what it is intended for — a reduction in the number of bankruptcy filings. The model also indicates expansion of borrowing, because agents default less frequently, and thus a stronger commitment to repay yields lower default premia. However, the model implies that the overall effect on social welfare is negative. While lower default premia induces the positive welfare effect by offering lower costs of borrowing and thus allowing better consumption smoothing, there is a negative welfare effect as well because agents who are forced to default due to large expenditure shocks either suffer from a higher default cost or cannot default due to the means-testing requirement under the BAPCPA. Moreover, there is the additional negative effect among agents with temptation, as lower loan interest rates induce overborrowing. The reasonable property of the model that agents without temptation do not borrow and default as much as those with temptation is important in the overall welfare result.

Through various counterfactual policy experiments, I found two ways to improve social welfare. First is to impose a relatively high interest rate ceiling. A small welfare gain is achieved with an interest ceiling because it discourages agents with temptation from overborrowing. Second, the baseline model also indicates there is a welfare gain from significantly raising or lowering the amount of earnings that creditors can garnish upon defaulting. Raising the garnishment ratio implies a welfare gain because a creditor-friendly bankruptcy law induces a lower default premium and thus better consumption smoothing. However, the size of the welfare gain is significantly smaller than in the model with the standard preferences without temptation, in which agents do not suffer overborrowing. In general, the welfare properties of the model with tempted agents can be significantly different from the model with the standard preferences. On the other hand, lowering the garnishment ratio also implies a welfare gain because agents either borrow and default due to temptation or expenditure shocks, and for these cases there is a welfare gain by making defaulters

suffer less.

Building on earlier studies, such as those by Strotz (1956) and Pollak (1968), Laibson (1996, 1997) introduces the hyperbolic-discounting preferences into standard macroeconomic models to investigate the role of present bias. Furthermore, Laibson et al. (2003) show that the hyperbolic-discounting model can explain why the majority of households with credit cards pay interest on the cards even if they have assets as well. On the other hand, Barro (1999) finds that the neoclassical growth model with hyperbolic-discounting preferences and log utility is observationally equivalent to the same model with the standard exponential-discounting preferences. Akerlof (1991) and O'Donoghue and Rabin (1999, 2001) study cases where sophisticated agents, who are aware of the time-inconsistent nature of their preferences, and naive agents, who are not, behave very differently.

Welfare implications of macroeconomic models with preferences that exhibit present bias have been studied recently. Krusell et al. (2010) study a neoclassical growth model with Gul-Pesendorfer preferences. They find that the optimal long-run capital income tax rate in their temptation model is *negative*, as opposed to zero in the standard model because the agent undersaves, and thus it is welfare-improving to induce savings using a savings subsidy. İmrohoroğlu et al. (2003) find that unfunded social security could be welfare-improving in an overlapping-generations model with hyperbolic discounting, by mitigating undersaving. By the same logic, compulsory savings floors can be welfare-improving, as in Malin (2008). In Nakajima (2012), a relaxed borrowing constraint and associated increase in debt could imply lower welfare when agents are subject to temptation and thus overborrowing.

There has been extensive literature on the quantitative analysis of default. Athreya (2002) and Chatterjee et al. (2007) study the effects of introducing a means-testing requirement for bankruptcy. The latter find a positive welfare effect. Livshits et al. (2007) compare the model economy with "fresh start" bankruptcy, which provides a better consumption smoothing across states, and the model economy without bankruptcy, which provides a better consumption smoothing over the life cycle. Livshits et al. (2010) explore the causes of the observed rise in bankruptcies and debt since 1980s, using a calibrated life-cycle model. Narajabad (2012) and Athreya et al. (2012) study the observed rise in the number of bankruptcy filings, with a focus on the role of the improved information technology used by credit card companies. Li and Sarte (2006) construct a model with bankruptcy under both Chapters 7 and 13 and investigate their interaction. In a recent paper, Benjamin and Mateos-Planas (2013) explicitly analyze the choice between informal default (to stop repaying debt) and formal default (to file for bankruptcy). Mitman (2011) studies the interaction between bankruptcy of unsecured credit and foreclosure of secured credit. Li and White (2009) empirically show that there are interesting interactions between the two. As compared with existing literature, the model developed in this paper does not include imperfect information, general equilibrium, multiple assets, choice of default options, or informal default, but none of the existing work investigates the implications of present bias to debt and default.

The remainder of the paper is organized as follows. Section 2 gives an overview of the environment surrounding consumer bankruptcy in the U.S. Section 3 sets up the model. Section 4 describes how the model is calibrated. Section 5 comments on how the model is numerically solved. Section 6 presents the main results, studying various policy reforms that affect borrowing and bankruptcy. Section 7 conducts sensitivity analysis. Section 8 concludes. Appendix A contains detailed information on the data on U.S. credit and default. Appendix B provides more details about calibration, while Appendix C describes the computational algorithm. Appendix D describes the calibration of the alternative models.
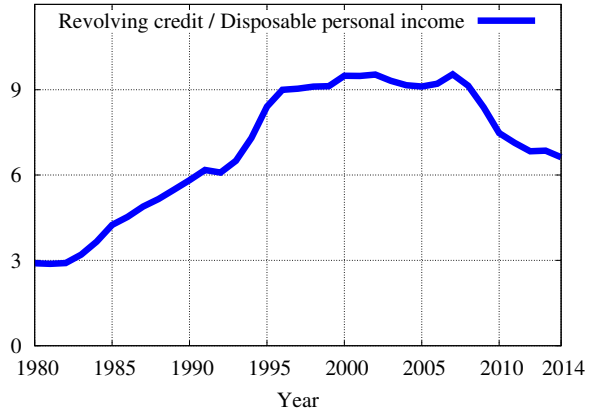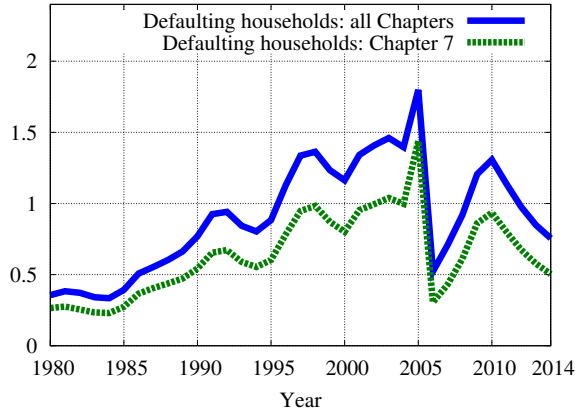
(a) Percentage of Households Filing for Bankruptcy   (b) Consumer Credit Balance over Disposable Income
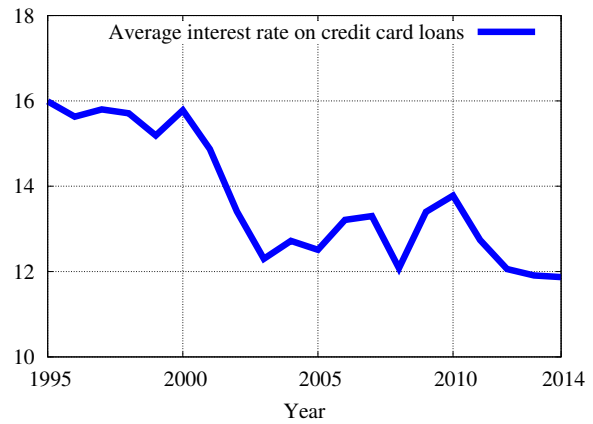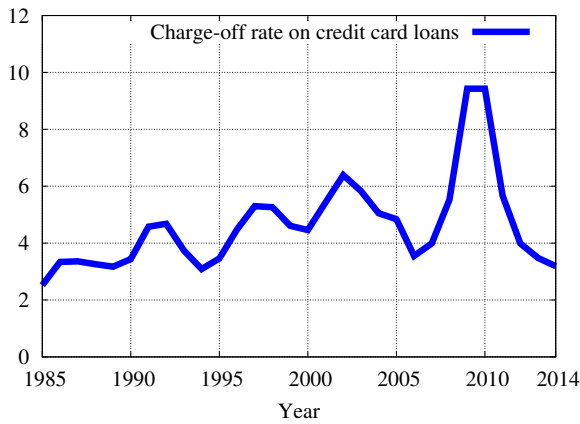
(c) Charge-Off Rate for Credit Card Loans   (d) Average Interest Rate on Credit Card Loans

Figure 1: Bankruptcy and Debt in the U.S.

## 2   Consumer Bankruptcy in the U.S.

This section provides an overview of the environment associated with consumer bankruptcy in the U.S. When a borrower of unsecured debt fails to repay his debt on schedule, creditors take various measures, such as garnishing labor income, to recover the unrepaid amount. When the borrower files for bankruptcy, these attempts to recover debt are stopped. There are two major types of consumer bankruptcy: Chapter 7 and Chapter 13. Chapter 7, which is also called liquidation, allows debtors to clean up the debt, after paying back a part of the existing debt using assets that are nonexempt. A debtor filing for Chapter 7 bankruptcy obtains a "fresh start" in the sense that once the Chapter 7 bankruptcy is in place, there is no future obligation to pay back the debt. The other major bankruptcy option is Chapter 13, an option of individual debt adjustment. Under Chapter 13, the bankrupt can draw their own repayment plan over three to five years and, upon approval by the judge, reschedule the repayment plan according to the proposed schedule.[1] The assets at the time of bankruptcy filing need not be used for immediate repayment as in Chapter 7,

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[1]   Under the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA), the bankrupt no longer draw the repayment plan themselves. See Section 6.2.

but the bankrupt have to use their future income for repayment. Once a debtor files for Chapter 7 bankruptcy, that debtor cannot file for Chapter 7 bankruptcy again for six years but can file under Chapter 13. Historically, the proportion of Chapter 7 bankruptcies remains stable at about 70 percent of total consumer bankruptcies. There is also a study reporting that many who filed for bankruptcy under Chapter 13 ended up also filing for Chapter 7 bankruptcy (Chatterjee et al. (2007)). The focus of this paper is Chapter 7 bankruptcy, and the "default" option in the model resembles the Chapter 7 bankruptcy.

Figure 1 shows data related to bankruptcies and debt in the U.S.[2] Panel (a) shows the percentage of total bankruptcy filings and Chapter 7 bankruptcy filings over the number of households in the U.S. from 1980 to 2014. There are three notable features: First, the proportion of Chapter 7 bankruptcy filings over total number of bankruptcies has remained stable, at about 70 percent. Second, the number of bankruptcy filings increased dramatically from 1980 to the early 2000s; the number of Chapter 7 bankruptcy filings increased more than fivefold, from 213,983 in 1980 to 1,117,766 in 2004. Third, there was a significant spike in 2005, followed by a plunge in 2006. This is because of the enactment of the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA), which is the focus of this paper. The BAPCPA, which made filing for bankruptcy (especially Chapter 7 bankruptcy) more difficult, became effective in fall 2005, and a large number of debtors rushed to file before the new law took effect. The dip in 2006 was a rebound from that rush to file for Chapter 7 bankruptcy. Finally, the number seems to be rising again after the dip in 2006, but because this period coincides with the Great Recession, it is impossible to tell at which level the number of bankruptcy filings stabilizes. Indeed, after the end of the Great Recession, the number of Chapter 7 bankruptcy filings has been declining, approaching the level in 2006.

In the background of the BAPCPA was a concern about the sharp increase in the number of consumer bankruptcies. The main concern behind the bankruptcy reform was the fact that many people were abusing the bankruptcy law, which was considered debtor-friendly. Naturally, the reform is intended to transform the bankruptcy scheme from a debtor-friendly one, in which the cost of defaulting is low and anybody can file for bankruptcy, to a more creditor-friendly one, in which the cost of defaulting is high and defaulting is available only to low-income borrowers. More details about the BAPCPA will be provided later when I use the models to study the implications of the reform in Section 6.2.

Behind the dramatic increase in the number of bankruptcy filings was the increase in consumer credit, which is shown in Panel (b) of Figure 1. Panel (b) shows the ratio of gross consumer credit balance and disposable personal income in the U.S. from 1980 to 2014. The ratio steadily increased from 3 percent in 1980 to 9 percent in the late 1990s, and it remained at that level before starting to decline when the Great Recession started. Panel (c) shows the changes in the charge-off rate, which is the ratio of loss and the balance of credit card loans extended, among all commercial banks. The ratio had been fluctuating around a slightly positive trend since 1985, before shooting up at the onset of the Great Recession. The rate has since come down to the level before the Great Recession. Finally, Panel (d) shows the movement of the average interest rate on credit card loans, from 1995 to 2014. The interest rate dropped from around 16 percent to stabilize at around 12 percent in the early 2000s. This decline coincides with the general trend of declining real interest rates.

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[2] Appendix A contains details on the data used to construct Figure 1.

# 3 Model

The key features of the model are overlapping generations, equilibrium default, and preferences featuring temptation and self-control. Livshits et al. (2007) feature overlapping generations and equilibrium default, while Nakajima (2012) introduces preferences with temptation and self-control, developed by Gul and Pesendorfer (2001), into an overlapping-generations model. The current paper combines all three features. Let me make two remarks about the temptation preferences. First, in the model developed in this paper, agents with and without temptation coexist. Naturally, a policy that doesn't distinguish the two types has different effects on the two types of agents. To the best of my knowledge, this is the first time that such a model has been constructed. Second, I use the preferences featuring temptation and self-control in the way formulated by Krusell et al. (2010). They show that a special case of the preferences can be interpreted as the hyperbolic-discounting preferences developed by Strotz (1956) and Laibson (1997).

## 3.1 Demographics

Time is discrete. The economy is populated by $I$ overlapping generations of agents. Each generation is populated by a mass of measure-zero agents. Agents are born at age 1 and live up to age $I$. Agents who die are replaced by the same measure of newborns, which make the total measure of agents constant over time. Agents retire at age $1 < I_R < I$. Agents with age $i \leq I_R$ are called *workers*, and those with age $i > I_R$ are called *retirees*. $I_R$ is a parameter, implying that retirement is mandatory.

## 3.2 Preferences

There are $j = 1, 2, ..., J$ types of agents, with different preference parameters. This setup allows coexistence of agents with and without temptation in the model, which is a novel feature of this paper. The proportion of type-$j$ agents is $\phi_j$, with $\sum_{\forall j} \phi_j = 1$. The preference type of an agent does not change. The preferences of type-$j$ agents are time separable and characterized by a period utility function, two discount factors, $\delta_j$ and $\beta_j$, and another parameter, $\gamma_j$. The period utility function takes the following form:

$$u\left(\frac{c_i}{\nu_i}\right), \tag{1}$$

where $u(.)$ is assumed to be strictly increasing and strictly concave. $\nu_i$ is the size of a household of age-$i$ in equivalent scale units.[3] $\delta_j$ and $\beta_j$ are called the *self-control discount factor* and the *temptation discount factor*, respectively. $\gamma_j$ represents the *strength of temptation*. $\delta_j$ is the only discount factor if the agent can exert perfect self-control and thus is not affected by temptation. In other words, in a special case in which the temptation is nonexistent (strength of temptation $\gamma_j$ is zero), the model with temptation and self-control preferences reverts to the standard exponential-discounting model with $\delta_j$ as the only discount factor. $\beta_j < 1$ is the additional discount factor with which an agent is tempted to discount future utility when making a consumption-savings decision. In other words, $\beta_j$ captures the degree of present bias.[4] I discuss the preferences in more detail when the agent's problem is formulated in Section 3.5.

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[3] Changes in household size over the life cycle are found to be important in accounting for the hump-shaped life-cycle profile of consumption (Attanasio and Weber (1995)).

[4] It is straightforward to see that if $\beta_j = 1$, the preferences revert back to the standard exponential-discounting preferences with $\delta_j$ as the only relevant discount factor.

## 3.3 Endowment

Agents are born with zero assets. Working agents receive labor income $e$ each period. The labor income takes the following form:

$$e(i, p, t) = e_i \exp(p + t), \tag{2}$$

where $e_i$ captures the average life-cycle profile of labor income and is common across all age-$i$ agents. Moreover, $e_i = 0$ for retired agents (i.e., $i > I_R$). $p$ is the persistent shock to labor income and is assumed to follow a first-order Markov process with the transition probability $\pi^p_{i,p,p'}$.[5] $t$ is the transitory shock to labor income. $\pi^t_{i,t}$ represents the probability that an age-$i$ agent draws a shock $t$.[6] After retirement ($i > I_R$), an agent receives social security benefits $b(i, p, t)$. The amount of benefits does not change with age, but $i$ is an argument so that $b(i, p, t) = 0$ for working agents ($i \leq I_R$). An agent also faces shocks to compulsory expenditure $x \geq 0$. $\pi^x_{i,x}$ represents the probability that an age-$i$ agent faces a compulsory expenditure of amount $x$. $x$ is independently and identically distributed, as in Livshits et al. (2007).

## 3.4 Bankruptcy

Agents have an option to default on their debt or bills associated with expenditure shocks. The default option is modeled as in Chatterjee et al. (2007) and Livshits et al. (2007). The default option in the model resembles in procedure and consequences a Chapter 7 bankruptcy filing — in particular, before the reform of the bankruptcy law in 2005.

Suppose an agent has debt (equivalently, a negative amount of assets) or receives an expenditure shock from which the asset position becomes negative, and the agent decides to default on the debt. The following things happen:

1. The defaulting agent has to pay for a fixed cost of filing, $\xi$.

2. The debt and the expenditure shock (think of a hospital bill) are wiped out, and the agent does not have an obligation to pay back the debt or the expenditure in the future (the *fresh start*).

3. The agent cannot save during the current period. If the agent tries to save, the savings will be completely garnished by the lender.

4. Proportion $\eta$ of the current labor income is garnished by the lender. This is also intended to capture the effort of the agent to repay until finally deciding to default within a period (a year). The social security benefit is not subject to this garnishment.

5. The credit history of the agent turns bad. I use $h = 0$ and $h = 1$ to denote a good and bad credit history, respectively.

6. While the credit history is bad ($h = 1$), the agent is excluded from the loan market. In other words, the borrowing limit is zero.

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[5] $i$ is attached to the Markov transition probability, in order to accommodate the case in which the agent is retired and $p$ no longer changes.

[6] $i$ is attached in order to accommodate the case in which the agent is retired and $t$ is always zero.

7. With probability $\lambda$, the agent's bad credit history is wiped out, or $h$ turns from 1 to 0. After that, there is no longer a negative consequence of the past default.

The benefit of using the default option is to get away from debt or an expenditure shock. The default option is a means of partial insurance. The costs are (i) monetary cost of filing, (ii) income garnishment in the period of default, (iii) inability to save in the period of default (due to asset garnishment), and (iv) temporary exclusion from the loan market. (i) and (ii) are different since (ii) is received by credit card companies and thus affects (lowers) the interest rate of loans, while (i) does not directly affect the loan interest rate. Agents in debt or with an expenditure shock weigh the benefits and the costs of defaulting, and they default if it is optimal to do so or if there is no other option. The former is called *voluntary default*, and the latter is called *involuntary default*. It is possible that an agent with a bad credit history cannot have a positive consumption when hit by an expenditure shock. Only in this case (*involuntary default*) is default by agents with a bad credit history allowed. In other words, an agent with a bad credit history ($h = 1$) cannot choose *voluntary default*. In reality, a record of default remains on the credit record of an agent for 10 years. However, I use stochastic recovery of the credit status in order to reduce the size of the state space. Thanks to the stochastic recovery, I only need to have $h \in \{0, 1\}$ instead of having 11 different possibilities of $h$; in the case, one period is one year. For notational convenience, I use $\pi_0^h = \lambda$ and $\pi_1^h = 1 - \lambda$, which are the probabilities that a bad credit history is wiped out and not wiped out, respectively. I assume that the stochastic recovery does not start until the period after defaulting, but this timing assumption is not crucial. In Section 7.5, I show that the main results of the paper are robust to changing the timing assumption, in which stochastic recovery starts immediately in the period of defaulting.

## 3.5 Agent's Problem

For a clean notation, I start by defining a recursive problem of an agent with an arbitrary discount factor, $d$. Once I finish characterizing the problem of an agent given $d$, I will define the general problem, which includes preferences featuring temptation and self-control.

The individual state variables are $(j, i, h, p, t, x, a)$, where $j$ is preference type, $i$ is age, $h$ is credit history, $p$ and $t$ are persistent and transitory components of labor income shocks, $x$ is the compulsory expenditure shock, and $a$ is asset position. I will start with the problem of an agent with a good credit history ($h = 0$). Given a discount factor $d$, an agent with a good credit history chooses whether or not to default. Formally:

$$V^*(j, i, 0, p, t, x, a; d) = \max\{V^*_{\text{non}}(j, i, 0, p, t, x, a; d), V^*_{\text{def}}(j, i, 0, p, t, x, a; d)\}, \tag{3}$$

where $V^*_{\text{non}}(j, i, 0, p, t, x, a; d)$ and $V^*_{\text{def}}(j, i, 0, p, t, x, a; d)$ are values conditional on not defaulting and defaulting, respectively. The Bellman equation for an agent with a good credit history ($h = 0$), conditional on not defaulting, is as follows:

$V^*_{\text{non}}(j, i, 0, p, t, x, a; d) =$
$$\begin{cases} -\infty & \text{if } B(j, i, 0, p, t, x, a) = \emptyset \\ \max_{a' \in B(j,i,0,p,t,x,a)} \left\{ u\left(\frac{c}{\nu_i}\right) + d \, \mathbb{E}V(j, i+1, 0, p', t', x', a') \right\} & \text{if } B(j, i, 0, p, t, x, a) \neq \emptyset \end{cases} \tag{4}$$

subject to:

$$c + a'q(j, i, 0, p, t, x, a') + x = e(i, p, t) + b(i, p, t) + a, \tag{5}$$

8

where $\mathbb{E}$ is an expectation operator, taken with respect to $(p', t', x')$. $B(.)$ characterizes the budget set. For an agent with a good credit history $(h = 0)$, $B(.)$ is defined as follows:

$$B(j, i, 0, p, t, x, a) = \{a' \in \mathbb{R} | c + a'q(j, i, 0, p, t, x, a') + x = e(i, p, t) + b(i, p, t) + a, c \geq 0\}. \quad (6)$$

The first case in equation (4) takes care of the case in which the budget set is empty. In this case, since the utility from not defaulting is negative infinity, while the utility from filing is finite, the agent ends up defaulting *involuntarily*.[7] Now, let me make three remarks. First, notice that the discount factor used here is an arbitrary discount factor $d$. Second, the optimal value characterized by equation (4) is different from the future value in the same equation, $V(.)$. $V(.)$ is defined when I describe the problem featuring temptation and self-control. Third, $q(j, i, h, p, t, x, a')$ denotes the discount price of bonds and depends on the type of agent, as well as the amount saved $(a' \geq 0)$ or borrowed $(a' < 0)$. $q(.)$ depends on the individual type of borrower because I allow credit card companies to adjust the price of loans reflecting perfectly the risk associated with each loan. I will come back to the determination of $q(j, i, h, p, t, x, a')$ in Section 3.6.[8]

Given a discount factor $d$, the Bellman equation for an agent, conditional on defaulting, is defined below. Notice that this Bellman equation is valid regardless of the current credit status $(h)$, because the benefits and the costs of default are the same regardless of the current credit status of an agent. That is why the problem is defined for $\forall h$ and not only for $h = 0$:

$$V_{\text{def}}^*(j, i, h, p, t, x, a; d) = u\left(\frac{c}{\nu_i}\right) + d\, \mathbb{E}V(j, i+1, 1, p', t', x', 0) \quad (7)$$

$$c + \xi = e(i, p, t)(1 - \eta) + b(i, p, t). \quad (8)$$

Notice the following four differences from the previous case. First, the existing debt $(a)$ and the expenditure shock $(x)$ are wiped out from the budget constraint (8) as a result of default. Second, on the other hand, the agent has to pay for the default cost $\xi$, and the fraction $\eta$ of the current labor income is garnished. Third, the optimal saving level is $a' = 0$, since any assets above 0 would be garnished by assumption and the defaulting agent cannot borrow. Fourth, the credit history of the agent turns bad $(h' = 1)$.

Finally, given a discount factor $d$, the problem of an agent with a bad credit history $(h = 1)$ is defined as follows:

$$V^*(j, i, 1, p, t, x, a; d) =$$
$$\begin{cases} V_{\text{def}}^*(j, i, 1, p, t, x, a; d) & \text{if } B(j, i, 1, p, t, x, a) = \emptyset \\ \max_{a' \in B(j, i, 1, p, t, x, a)} \left\{ u\left(\frac{c}{\nu_i}\right) + d\, \mathbb{E}V(j, i+1, \hat{h}', p', t', x', a') \right\} & \text{if } B(j, i, 1, p, t, x, a) \neq \emptyset \end{cases} \quad (9)$$

subject to the budget constraint (5). $\mathbb{E}$ is an expectation operator, taken with respect to $(\hat{h}', p', t', x')$.[9] $B(.)$ characterizes the budget set, as follows:

$$B(j, i, 1, p, t, x, a) = \{a' \in \mathbb{R}^+ | c + a'q(j, i, 1, p, t, x, a') + x = e(i, p, t) + b(i, p, t) + a, c \geq 0\}. \quad (10)$$

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[7] It is possible that the utility of defaulting is not finite if the cost of defaulting is too large, but the model is calibrated such that this is not the case.

[8] In the baseline case, I assume that credit card companies can observe all individual state variables and use them to price the debt. However, this might not be the case in reality. In one of the sensitivity analyses, I study the case in which credit card companies can observe a subset of individual state variables. See Section 7.

[9] Credit status in the next period has a hat $(\hat{h}')$ in order to distinguish the future credit history that changes stochastically from the default choice $h'$.

Notice the following three differences from the problem of an agent with a good credit history. First, the agent can default only when the budget set is empty (i.e., involuntary default). In other words, there is no choice with respect to default for an agent with a bad credit history. Second, the agent with a bad credit history is excluded from the credit market (i.e., $a' \in \mathbb{R}^+$). Third, although it is contained in the expectation operator $\mathbb{E}$ and thus is not explicit, a bad credit history will be wiped out with a probability $\pi_0^h = \lambda$ and will remain with probability $\pi_1^h = 1 - \lambda$.

We are ready to define the problem with temptation and self-control. First, denote the value conditional on a discount factor $d$, a default decision $h'$, and a saving decision $a'$ as $\widetilde{V}(j, i, h, p, t, x, a, h', a'; d)$. Obviously, $V^*(j, i, h, p, t, x, a; d)$, which is the optimal value conditional on a discount factor $d$, is $\widetilde{V}(j, i, h, p, t, x, a, h', a'; d)$ associated with the optimal default and saving decision. Now, the problem of an agent with preferences featuring temptation and self-control can be defined as follows:

$$V(j, i, h, p, t, x, a) = \max_{h', a'} \Big\{ \widetilde{V}(j, i, h, p, t, x, a, h', a'; \delta_j)$$
$$+ \gamma_j \left( \widetilde{V}(j, i, h, p, t, x, a, h', a'; \beta_j \delta_j) - V^*(j, i, h, p, t, x, a; \beta_j \delta_j) \right) \Big\}, \quad (11)$$

where $h' = g_h(j, i, h, p, t, x, a) \in \{0, 1\}$ is the associated optimal default rule, and $a' = g_a(j, i, h, p, t, x, a)$ is the associated optimal saving rule. The first part in the maximand, $\widetilde{V}(.; \delta_j)$, is called *self-control utility*, while the part in the maximand multiplied by $\gamma_j$, $(\widetilde{V}(,; \beta_j \delta_j) - V^*(.; \beta_j \delta_j))$ is called *temptation utility*. In order to understand equation (11), let's assume $\gamma_j = 0$ for now. In this case, the temptation utility drops off from the maximand and the problem becomes standard: maximizing only the self-control utility using the discount factor $\delta_j$. This situation is when the agent can exert perfect self-control and is not affected by the temptation to consume or borrow more, which is represented by the discount factor $\beta_j \delta_j$ in the temptation utility. In other words, when $\gamma_j = 0$, temptation drops out of the agent's problem, and the problem collapses back to the exponential-discounting model with the discount factor $\delta_j$. Another special case is $\beta_j = 1$. When $\beta_j = 1$, even if the temptation utility is present ($\gamma_j > 0$), the problem collapses to the standard exponential-discounting model with a sole discount factor $\delta_j$. This is because when the pair $(h', a')$ is chosen to maximize $\widetilde{V}(.; \delta_j)$, the temptation utility is also maximized as well and takes the value of zero.

On the other hand, when $\gamma_j > 0$ and $\beta_j \in [0, 1)$, the agent's optimization problem includes two considerations, corresponding to the two parts in equation (11). First, the agent still benefits by maximizing the self-control utility as before. Second, at the same time, the agent suffers from deviating from the optimal decision associated with the discount factor $\beta_j \delta_j$. Remember again, $V^*(.; \beta_j \delta_j)$ is the optimal value associated with the discount factor $\beta_j \delta_j$. When the agent chooses $(h', a')$ that are different from the optimal pair associated with $V^*(.; \beta_j \delta_j)$, the agent suffers a negative temptation utility, which is multiplied by $\gamma_j$. In this sense, $\gamma_j$ represents the strength of the temptation. When $\gamma_j$ is larger, the agent is more strongly tempted to choose $(h', a')$ that are closer to the optimal pair under the discount factor $\beta_j \delta_j$ and make the utility loss from the temptation utility smaller. In an extreme case in which $\gamma_j \to \infty$, it becomes optimal for an agent to minimize the utility loss from the temptation utility by choosing $(h', a')$ that are optimal under the discount factor $\beta_j \delta_j$. This special case is shown to be equivalent to the hyperbolic-discounting preferences with the short-term discount factor $\beta_j$ and the long-term discount factor $\delta_j$, and estimates of $\beta_j$ are available for the hyperbolic-discounting model. See Krusell et al. (2010) and Nakajima (2012) for a discussion about the equivalence. Notice that when $\gamma_j \to \infty$, equation (11) becomes simplified as

follows:

$$V(j, i, h, p, t, x, a) = \widetilde{V}(j, i, h, p, t, x, a, h', a'; \delta_j), \tag{12}$$

where $h' = g_h(j, i, h, p, t, x, a) \in \{0, 1\}$ and $a' = g_a(j, i, h, p, t, x, a)$ are the optimal decision rules associated with the value $V^*(j, i, h, p, t, x, a; \beta_j \delta_j)$, which maximizes the temptation utility. In other words, when an agent completely succumbs to temptation, the agent makes the optimal default decision $h'$ and the optimal saving decision $a'$ by discounting the future with a discount factor $\beta_j \delta_j$. However, the actual value of the agent is evaluated with the discount factor $\delta_j$.

## 3.6 Credit Card Companies

The only assets available in the model are one-period discount bonds. This is a common assumption, used in Chatterjee et al. (2007) and Livshits et al. (2007). I also assume that retired agents cannot borrow, following Livshits et al. (2007). The saving interest rate is fixed at $r$. Since the only financial assets available in the model are discount bonds issued by agents, the bond price of the saving agents in equilibrium is $q(j, i, h, p, t, x, a' \geq 0) = 1/(1 + r)$. Notice that this is the only bond price for agents with a bad credit history, as they are excluded from the loan market (i.e., $a' \geq 0$). When an agent borrows, it is assumed that the agent has to pay the interest premium $\iota$ in addition to the interest rate. If there is no default premium, the borrowing interest rate is $r + \iota$ and the price of discount bonds issued by an agent who does not default is $1/(1 + r + \iota)$. However, since agents cannot commit to repaying the debt, a default premium is added to loan interest rates depending on the riskiness of loans. The unsecured loans are provided by a competitive credit sector that consists of a large number of credit card companies. Free entry is assumed. Credit card companies can target agents of one particular type with one particular level of debt. Since the credit sector is competitive, free entry is assumed, and each credit card company can target one specific level of debt, it is impossible in equilibrium to *cross-subsidize*, that is, offer agents of one type an interest rate implying a negative profit while offering agents of another type an interest rate implying a positive profit so that, in sum, the credit card company makes a positive total profit. In this case, there is always an incentive for another credit card company to offer a lower interest rate for agents of the second type and steal the profitable customers away. In equilibrium, any loans to any type of agents and any level of debt make zero profit.

Suppose that a credit card company makes loans to type-$(j, i, 0, p, t, x)$ agents who borrow $a'$ each.[10] Remember that the current asset position of the agents, $a$, does not matter for the pricing of loans. By making loans to a mass of agents of the same type, the credit card company can exploit the law of large numbers and insure away the idiosyncratic default risks, even if the individual loans are defaultable. In other words, the credit sector provides a partial insurance by pooling risk of default across agents of the same type. Now, assume the credit card company makes loans to measure $m$ agents of the same type. The zero-profit condition associated with the loans made to type-$(j, i, 0, p, t, x)$ agents whose measure is $m$ and who borrow $a'$ each can be expressed as follows:

$$m(-a') \mathbb{E} \mathbb{1}_{g_h(j, i+1, 0, p', t', x', a') = 0} + m \mathbb{E} \mathbb{1}_{g_h(j, i+1, 0, p', t', x', a') = 1} \eta e(i+1, p', t') \frac{-a'}{x' - a'}$$
$$= m(-a' q(j, i, 0, p, t, x, a'))(1 + r + \iota), \tag{13}$$

---

[10] Notice that $h = 0$. I only need to consider the case $h = 0$, as agents with a bad credit history ($h = 1$) cannot borrow.

where $\mathbb{1}$ is an indicator function that takes the value of one (zero) if the logical statement attached to it is true (false). $\mathbb{E}$ is an expectation operator and is taken with respect to $(p', t', x')$. The two terms on the left-hand side represent the total income from the loans. In particular, if an agent repays the loan ($g_h(.) = 0$), the credit card company receives the amount $-a$. If an agent defaults on its loan, $\eta e(i + 1, p', t')$ is garnished, but the garnished amount is shared proportionally between the issuer of the bill $x'$ and the credit card company that extended the loan of amount $-a'$. The right-hand side is the total cost of the loans. Specifically, the discount value of a loan $-a'q(.)$ is the principal, and the credit card company has to pay for the interest and the premium $r + \iota$. By solving equation (13) for $q(j, i, 0, p, t, x, a')$, one can obtain the formula for the equilibrium discount price of loans, as follows:

$$q(j, i, 0, p, t, x, a') = \frac{\mathbb{E}\left\{\mathbb{1}_{g_h(j,i+1,0,p',t',x',a')=0} + \mathbb{1}_{g_h(j,i+1,0,p',t',x',a')=1}\frac{\eta e(i+1,p',t')}{x'-a'}\right\}}{1 + r + \iota}. \tag{14}$$

Finally, I assume there is a maximum limit on the interest rate charged by credit card companies, which is denoted by $\bar{r}$. Since the price of the bond $q(.)$ is used instead of interest rate $r(.)$ for loans, the upper bound of the interest rate $\bar{r}$ is converted into the lower bound of the bond price by $\underline{q} = \frac{1}{1+\bar{r}}$. In the U.S., since the *Marquette* decision in 1978, which basically eliminated the usury law, nationally operating credit card companies are no longer subject to the usury law of the states in which they operate.[11] In other words, currently, there is no effective limit on the interest rate. Therefore, I will set $\bar{r}$ at a level that is virtually non-binding in the baseline calibration and later, in Section 6.3, investigate macroeconomic and welfare implications of introducing a binding interest rate ceiling.

In order to better understand the pricing of unsecured loans, let's look at some of the special cases. If the default probability is zero, the price of loans will be

$$q(j, i, 0, p, t, x, a') = \frac{1}{1 + r + \iota}. \tag{15}$$

If all agents default on the debt in the next period, the price of loans will be

$$q(j, i, 0, p, t, x, a') = \frac{\mathbb{E}\frac{\eta e(i+1,p',t')}{x'-a'}}{1 + r + \iota}. \tag{16}$$

Consider the special case in which there is no garnishment (i.e., $\eta = 0$). If the loan is defaulted with probability one, $q(j, i, 0, p, t, x, a') = 0$. This is because, when $\eta = 0$, credit card companies cannot receive anything from defaulters. In this case, if $q(j, i, 0, p, t, x, a')$ is monotonically increasing with respect to $a'$, one can define $\underline{a}(j, i, 0, p, t, x)$, which satisfies

$$\underline{a}(j, i, 0, p, t, x) = \max\{a'|q(j, i, 0, p, t, x, a') = 0\}, \tag{17}$$

where $\underline{a}(j, i, 0, p, t, x)$ is the endogenous borrowing limit for agents of type $(j, i, 0, p, t, x)$. For an agent with a bad credit history, $\underline{a}(j, i, 1, p, t, x) = 0$. By construction, the constraint is less strict than the not-too-tight borrowing constraint of Alvarez and Jermann (2000). This is because the not-too-tight borrowing constraint is associated with no default in equilibrium, while the constraint here allows default in equilibrium. See Chatterjee et al. (2007) for further characterization of the equilibrium loan price function.

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[11] See Supreme Court decision on *Marquette National Bank of Minneapolis v. First of Omaha Service Corp.*

Finally, let me make three remarks. First, although the bond price function $q(.)$ takes $(j, i, h, p, t, x, a')$ as arguments, this is intended for completeness. Actually, $q(.)$ does not depend on $t$ and $x$ because both are assumed to be i.i.d. In other words, knowing $(t, x)$ today does not tell us anything about the default probability of a loan that is extended today and could be defaulted on in the next period. Second, an implicit assumption is that credit card companies can observe and use all the information about individual types. In reality, some information cannot be observe precisely, but this is probably a good approximation of the credit sector of the current U.S. economy. In Section 7, an alternative model in which credit card companies cannot observe some of the agent types, and thus the price of discount loans does not depend on some of the agent types, is studied. Finally, the assumption that only one-period discount bonds are available is common in the literature, but it is probably less innocuous in the case with temptation preferences. When agents with preferences featuring temptation and self-control can restrict future borrowing, they might want to trade bonds for more than one period ahead. Basically, multi-period bonds could be used as a commitment device against overborrowing in the future. By assuming that only one-period bonds are traded, such a possibility is assumed away.

### 3.7 Equilibrium

I define the steady-state recursive equilibrium, in which the type distribution of agents is time-invariant while individual states of agents change over time.[12] Let $\mathbf{M}$ be the space of the individual state. $(j, i, h, p, t, x, a) \in \mathbf{M}$. Let $\mathbb{M}$ be the Borel $\sigma$-*algebra* generated by $\mathbf{M}$ and $\mu$ a probability measure defined over $\mathbb{M}$. I will use a probability space $(\mathbf{M}, \mathbb{M}, \mu)$ to represent a type distribution of agents.

**Definition 1 (Steady-state recursive equilibrium)** *A steady-state recursive equilibrium consists of loan pricing function $q(j, i, h, p, t, x, a')$, value function $V(j, i, h, p, t, x, a)$, optimal decision rules $g_a(j, i, h, p, t, x, a)$ and $g_h(j, i, h, p, t, x, a)$, and the type distribution $\mu$, such that:*

1. *Given the loan price function, $V(j, i, h, p, t, x, a)$ is a solution to the agent's optimization problem defined in Section 3.5, and $g_a(j, i, h, p, t, x, a)$ and $g_h(j, i, h, p, t, x, a)$ are the associated optimal decision rules.*

2. *Loan price function $q(j, i, h, p, t, x, a')$ satisfies the zero-profit conditions for all types. Specifically, the loan price function is characterized by equation (14).*

3. *Measure of agents $\mu$ is time-invariant and consistent with the demographic transition, the stochastic process of shocks, and the optimal decision rules.*

## 4  Calibration

This section describes how the baseline models are calibrated. The baseline model is calibrated to capture salient characteristics of the U.S. economy, especially in terms of debt and default, around 2000. This includes the environment surrounding consumer bankruptcy before the 2005 bankruptcy law reform. The main experiment is to introduce a stylized version of the 2005 reform and analyze its macroeconomic and welfare implications using the calibrated model. Table 1 summarizes the parameter values.

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[12] In this sense, this equilibrium concept is also referred to as stationary equilibrium.

## Table 1: Summary of Calibration

| Parameter | Value | Remark |
|---|---|---|
| $I$ | 54 | Maximum age (corresponding to 73 years old). |
| $I_R$ | 45 | Last working age (corresponding to 64 years old). |
| $\sigma$ | 2.000 | Coefficient of relative risk aversion. |
| $\{\nu_i\}$ | Fig 7 | Household size in family equivalence scale. |
| | | |
| $\phi_1 = \phi_2$ | 0.5000 | Equal measure for both preference types. |
| $\gamma_1$ | 0.0000 | $j = 1$ has no temptation. |
| $\gamma_2$ | $\infty$ | $j = 2$ succumbs to temptation. |
| $\beta = \beta_1 = \beta_2$ | 0.7000 | Temptation discount factor. |
| $\delta = \delta_1 = \delta_2$ | 0.9544 | Self-control discount factor. Calibrated to match $D/Y = 0.09$. |
| | | |
| $\{e_i\}$ | Fig 8 | Average labor income profile. Following Gourinchas and Parker (2002). |
| $\rho_p$ | 0.9500 | Persistence of persistent shocks to earnings. From Livshits et al. (2010). |
| $\sigma_p^2$ | 0.0250 | Variance for persistent shocks to earnings. From Livshits et al. (2010). |
| $\sigma_t^2$ | 0.0500 | Variance of transitory shock to earnings. From Livshits et al. (2010). |
| $\psi_e$ | 0.2000 | Parameter for social security benefits. From Livshits et al. (2010). |
| $\psi_p$ | 0.3500 | Parameter for social security benefits. From Livshits et al. (2010). |
| | | |
| $\pi_1^x$ | 0.02367 | Probability of a small expenditure shock. From Livshits et al. (2010). |
| $\pi_2^x$ | 0.00153 | Probability of a large expenditure shock. From Livshits et al. (2010). |
| $x_1$ | 0.3960 | Magnitude of a small expenditure shock. From Livshits et al. (2010). |
| $x_2$ | 1.2327 | Magnitude of a large expenditure shock. From Livshits et al. (2010). |
| | | |
| $\lambda$ | 0.1000 | On average, 10 years of exclusion from loan market upon default. |
| $\xi$ | 0.0280 | Cost of a bankruptcy filing is \$600. |
| $\eta$ | 0.3064 | Garnishment ratio; calibrated to match number of bankruptcies= 0.84%. |
| | | |
| $r$ | 0.0200 | Annual real risk-free interest rate. |
| $\iota$ | 0.0600 | Transaction cost of loans. |
| $\bar{r}$ | 1.0000 | Non-binding ceiling for interest rate in the baseline. |

### 4.1 Demographics

One period is set as one year in the model. Age 1 in the model corresponds to the actual age of 20. $I$ is set at 54, as in Livshits et al. (2007), meaning that the maximum actual age is 73. $I_R$ is set at 45, implying that the agents become retired at the actual age of 65.

### 4.2 Preferences

For the period utility function, the following constant relative risk aversion (CRRA) functional form is used:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}. \tag{18}$$

$\sigma$ is set at 2.0, which is the commonly used value in macroeconomics, and the baseline choice of Laibson et al. (2007). The household size in equivalent scale units, $\{\nu_i\}$, is constructed using the average household size in the 2006 Current Population Survey (CPS), converted into equivalence scale units following Fernández-Villaverde and Krueger (2007). Figure 7 in Appendix B shows $\{\nu_i\}$ used here.

I assume $J = 2$, i.e., two preference types for agents. $j = 1$ and $j = 2$ represent agents without temptation and those with temptation, respectively. There are equal measures of agents with and without temptation ($\phi_1 = \phi_2 = 0.5$), since there is no strong prior for the distribution of the preference types.

As for the rest of the preference parameters that can be different for each preference type $j$, I assume that discount factors $\beta_j$ and $\delta_j$ are the same between the two types, and use $\beta = \beta_1 = \beta_2$ and $\delta = \delta_1 = \delta_2$ below. However, the strength of temptation is different. In particular, agents without temptation ($j = 1$) have $\gamma_1 = 0$, by definition, while agents with temptation ($j = 2$) have $\gamma_2 = \infty$. $\gamma_2 = \infty$ implies that the preferences with temptation are equivalent to quasi-hyperbolic-discounting preferences, and thus available estimates for $\beta$ with quasi-hyperbolic discounting can be utilized. For the baseline calibration, I set $\beta_2 = 0.70$. The temptation discount factor, or the long-term discount facto in the language of hyperbolic-discounting, of 0.7 corresponds to the discount rate of 40 percent, which is the point estimate obtained by Laibson et al. (2007) with the hyperbolic-discounting model. Angeletos et al. (2001) argue that $\beta = 0.7$ "corresponds to the one-year discount factor typically measured in laboratory experiments." By assumption, $\beta = \beta_1 = \beta_2 = 0.7$, but $\beta_1$ does not matter since $\gamma_1 = 0$ implies that the temptation discount factor does not affect decision of type-$j = 1$ agents. Finally, for the self-control discount factor, or long-term discount factor in the language of hyperbolic-discounting, $\delta = \delta_1 = \delta_2$ are calibrated, jointly with other parameters (see Section 4.6), to match the aggregate debt-to-income ratio, which is 9.0 percent in recent years (Livshits et al. (2010)).

For a sensitivity analysis, I use $\beta = 0.6667$. The discount factor of 0.6667 corresponds to the 50 percent annual discount rate. In the alternative specification with $\beta = 0.6667$, the self-control discount factors $\delta$ are calibrated to match the same target: the debt-to-income ratio of 9 percent. Of course, $\delta$ will be different from their baseline values, but the models are calibrated to match the same set of targets such that all models are observationally equivalent with respect to the chosen targets.

### 4.3 Endowment

The average life-cycle profile of the individual labor productivity $\{e_i\}_{i=1}^{I}$ is taken from the estimates of Gourinchas and Parker (2002). Figure 8 in Appendix B shows the life-cycle profile of the average labor productivity used in the model. Since mandatory retirement at the model age is $I_R$, $e_i = 0$ for $i > I_R$. The persistent shock to labor income, $p$, is constructed by discretizing an AR(1) process with the persistence parameter of $\rho_p = 0.95$ and the variance of the normally distributed innovation of $\sigma_p^2 = 0.025$. I use the discretization method of Adda and Cooper (2003) with 15 grid points to approximate the AR(1) process using a first-order Markov process. For the transitory shock to labor income, I discretize a normal distribution with variance of $\sigma_t^2 = 0.05$, again using the method of Adda and Cooper (2003), with three grid points. These parameter values are within the range of values estimated in the literature and also used in Livshits et al. (2010). For the social security benefits, I use the same formula as Livshits et al. (2010), which is the sum of $\psi_e = 0.2$ of the average labor income of the economy and $\psi_p = 0.35$ of the persistent component of the individual labor

income just before retirement ($i = I_R$).

Livshits et al. (2010) construct the compulsory expenditure shocks using a three-point distribution, characterized by the three different sizes of expenditures $\{x_0, x_1, x_2\}$ and the probabilities attached to each size $\{1 - \pi_1^x - \pi_2^x, \pi_1^x, \pi_2^x\}$. The first point is associated with zero expenditure ($x_0 = 0$). The second point is a smaller expenditure shock and captures three kinds of events: unwanted births, divorces, and smaller medical expenditures. The size of the shock ($x_1$) is calibrated to be 26.4 percent of the average income, and the probability attached to the shock is 7.1 percent. However, since the model period in Livshits et al. (2010) is three years, I use $\pi_1^x = 0.0237$, which is one-third of the probability they used. For the size of the shock, I use half of the value used by Livshits et al. (2010). The adjustment to the size of the shock is not straightforward, since the size of the shock is computed by calculating the expenditures across a three-year period when an agent is hit by one of the events that the expenditure shock captures. Dividing the size of the shock used by Livshits et al. (2010) by three (since one period is one year) ignores the persistence of the expenditures, while not dividing by anything overstates the size of expenditures per year. Dividing by two is a compromise between the two considerations. The large shock ($x_2$) captures a large medical expenditure. Livshits et al. (2010) calculate that the size of such a shock is 82.2 percent of the average income, and the probability of such an occurrence is 0.46 percent. I adjusted their parameter values in the same way as I did for the smaller expenditure shock ($x_1$).

## 4.4 Bankruptcy

There are three parameters associated with defaulting: $\lambda$, which represents the average length of punishment; $\xi$, which represents the filing cost of defaulting; and $\eta$, which defines the amount of labor income garnished during the period of filing. $\lambda$ is set at 0.1, implying that, on average, defaulters cannot obtain new debt for 10 years after defaulting. This average punishment period corresponds to a 10-year period during which a bankruptcy filing stays on a person's credit record, in accordance with the Fair Credit Reporting Act. According to White (2007), the average cost of filing for Chapter 7 bankruptcy was $600 before the BAPCPA was introduced in 2005. $\xi$ is pinned down by converting $600 into the unit in the model. I obtain $\xi = 0.028$, meaning 2.8 percent of the average annual labor income. $\eta$ is calibrated such that the number of defaults in the model matches the same number in the U.S. economy (0.84 percent of households per year, according to Livshits et al. (2007)). However, notice that the parameter will be calibrated jointly. I will come back to the calibration of $\eta$ in Section 4.6.

## 4.5 Credit Card Companies

The interest rate is set at 2 percent per year ($r = 0.02$). This is the average of real interest rates between 1996 and 2005. The real interest rate is constructed as the difference between the market yield on the one-year U.S. Treasury and the inflation rate of personal consumption expenditures (PCE). Neely and Rapach (2008) compute the U.S. average real interest rate between 1989:Q4 and 2007:Q2 to be 1.82 percent. The interest rate is calibrated to be 4 percent in Livshits et al. (2007), but, as shown in Neely and Rapach (2008), the real interest rate has shifted down significantly since the 1980s. I will investigate an alternative model with a higher real interest rate in Section 7. The cost of making loans, $\iota$, is set at 6 percent, following the calibration strategy of Livshits et al. (2007). As will be shown in Section 6.1, the average credit card interest rate in the baseline model turns out to be close to its empirical counterpart with $\iota = 0.06$. Davis et al. (2006) report the existence of the wedge between the saving and borrowing rates, and they argue that the wedge is

important for their life-cycle model to replicate the observed pattern of equity holding. The upper bound of the lending interest rate is set at 100 percent ($\bar{r} = 1.0$) so that it is virtually not binding in the baseline model. I will lower $\bar{r}$ to investigate the effects of the usury law in Section 6.3.

### 4.6 Simultaneously Calibrated Parameters

As mentioned, there are two parameters, $\delta$ and $\eta$, which cannot be pinned down independently from the model. I calibrate the two parameters such that two closely related targets — the aggregate debt-to-income ratio of 9 percent and the proportion of defaulters each year at 0.84 percent — are achieved in the steady-state equilibrium of the model. Notice two things. First, in order to find such parameter values, it is necessary to run the model many times while trying different combinations of $(\delta, \eta)$. Basically, this is a simulated method of moments with exact identification. Second, the values of $(\delta, \eta)$ are different, depending on the model specification. At the end, parameter values are different, depending on the preference specifications of the model, but the targets are the same across different versions of the model.

In the baseline model with the temptation discount factor of 0.7, $\delta$ is calibrated to be 0.9544, which is close to 0.9588, the point estimate of Laibson et al. (2007).[13] The garnishment parameter $\eta$ is calibrated to be 0.3064 for the baseline model. This value is between the federal limit of the garnishment ratio (25 percent) and the calibrated value of Livshits et al. (2007) (35.5 percent). The latter is obtained by fixing the discount factor and using $\eta$ to match the debt-to-income ratio.

## 5   Computation

Since the model cannot be solved analytically, numerical methods are employed. I solve the individual agent's problem using backward induction, starting from the last period of life, with discretized state space. Details about the solution algorithm can be found in Appendix C, but one feature of the model is worth pointing out. The equilibrium price of loans, $q(j, i, h, p, t, x, a')$, is solved simultaneously with the agent's optimization problem. Once the optimal decision rules for age-$i$ agents are obtained, the price of debt for age-$i-1$ agents, $q(j, i-1, h, p, t, x, a')$, can be computed, using the optimal default policy $g_h(j, i, h, p, t, x, a)$. $q(j, i-1, h, p, t, x, a')$ in turn is used to solve the optimization problem of agents of age $i-1$. In short, there is no need to use iteration to find an equilibrium loan price $q(j, i, h, p, t, x, a')$ as in Chatterjee et al. (2007), which is a model with infinitely-lived agents and an option to default.

## 6   Main Results

This section presents the main results. Section 6.1 presents the properties of the calibrated baseline model. Section 6.2 investigates the macroeconomic and welfare implications of the 2005 bankruptcy law reform. Sections 6.3 and 6.4 analyze implications of introducing the usury law and changing the income garnishment ratio, respectively.

### 6.1   Properties of the Baseline Model

Before analyzing bankruptcy policy reforms, let us investigate properties of the baseline model. Table 2 presents aggregate statistics of the baseline model (second column), together with the U.S. data (first column). The table also shows the aggregate statistics for agents without and with

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[13] Note, however, that Laibson et al. (2007) assume that all agents have hyperbolic-discounting preferences with the same discount factors.

**Table 2: Debt and Bankruptcies: Baseline Model vs. Data**

| (Percent) | U.S. 1995-1999[1] | Baseline Model | | |
|---|---|---|---|---|
| | | All Agents | Agents without Temptation | Agents with Temptation |
| Proportion in debt | 11.0-48.4 | 30.8 | 18.4 | 43.1 |
| Total debt over income | 9.0 | 9.0 | 3.9 | 14.2 |
| Charge-off rate | 4.8 | 4.5 | 5.7 | 4.2 |
| Average borrowing rate | 10.9-12.8 | 10.1 | 9.9 | 10.2 |
| | | | | |
| Total bankruptcies | 0.84 | 0.84 | 0.46 | 1.22 |
| Due to expenditure shock | – | 0.71 | 0.45 | 0.98 |
| Due to income shock only | – | 0.13 | 0.01 | 0.25 |

[1] Sources: Livshits et al. (2007, 2010) except for proportion in debt. Proportion in debt is calculated using the 1998 Survey of Consumer Finances.
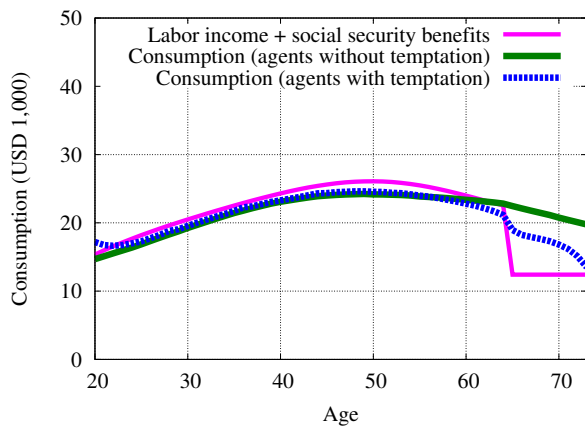


(a) Income and Consumption
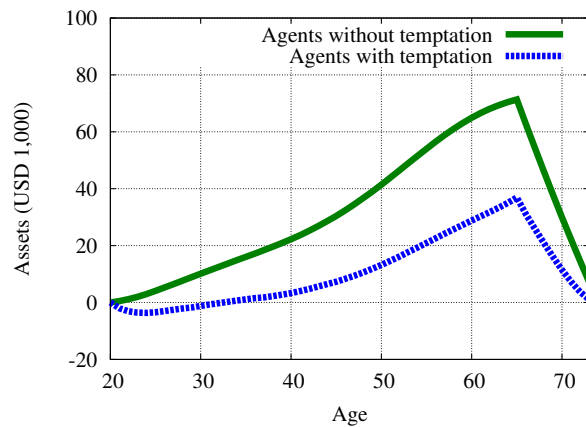
(b) Mean Asset Holdings

(c) Proportion of Debtors

(d) Proportion of Defaulters

**Figure 2: Life-Cycle Profiles of the Baseline Model**

temptation (third and fourth columns). By comparing the data (first column) and the aggregate statistics of the baseline model (second column), it is easy to see that the calibration is successful; the model replicates both the number of bankruptcy filings (0.84 percent per year) and the aggregate amount of unsecured debt relative to disposable personal income (9.0 percent). Moreover, the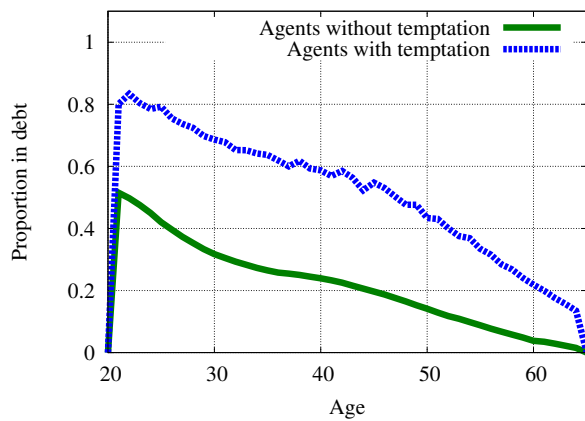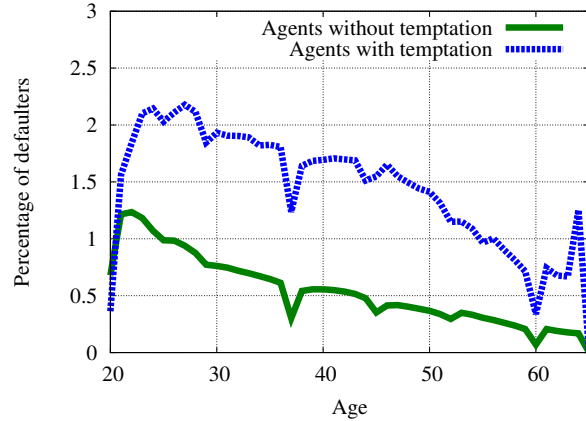 parameter representing the cost of making loans ($\iota$) is calibrated such that the average credit card loan rate in the baseline model (10.1 percent) is close to the data counterpart (10.9 to 12.8 percent).

In the baseline model, 30.8 percent of agents are in debt. The data counterpart has a wide range, between 11.0 percent and 48.4 percent.[14] Probably there are two reasons. First, as carefully discussed in Livshits et al. (2010), most households in the data have debt and assets simultaneously. If we look at the proportion of households with a positive amount of gross debt, the proportion is 48.4 percent in the 1998 Survey of Consumer Finances (SCF). This definition of debt is consistent with the target of the aggregate amount of debt (9 percent of disposable personal income). On the other hand, many households do not fully benefit from defaulting on their debt because (some of) their assets would be confiscated upon filing for bankruptcy. If one assumes that all of the assets that borrowers own could be confiscated upon bankruptcy filing, the right proportion to look at is the proportion of households who have more debt than assets, which is 11.0 percent in the SCF. As Livshits et al. (2010) argue, since many assets are exempt from confiscation when a debtor defaults, the number that can be comparable to the proportion in debt in the model is somewhere between the two bounds. And the proportion of agents in debt in the model (30.8 percent) is right in the middle of the range. The second potential reason is related to the timing assumption. In reality, households might need to use credit to pay for some expenditures and, for some reason, fail to repay the debt in a timely manner and end up paying interest.

In the model, agents default on their debt due to either a series of bad income shocks, or an expenditure shock. In the baseline model, among the 0.84 percent of agents that default every year, 0.71 percent (85 percent of total defaults) default mainly due to a large expenditure shock, while 0.13 percent (15 percent) default from (a series of) bad income shocks. In the data, Chakravarty and Rhee (1999) classify the reasons for bankruptcy filings into five categories using the Panel Study of Income Dynamics (PSID) for the period 1984-1995.[15] According to their results, 46.6 percent of total filers cite marital disruption, healthcare bills, or lawsuit and harassment as the primary reasons for filing for bankruptcy, while 12.2 percent cite job loss and 41.3 percent mention credit misuse. The proportion of bankruptcy filings due to a large expenditure shock in the model (85 percent) is higher than the proportion of reasons reasonably considered to be associated with expenditure shocks (46.6 percent). However, it is also reasonable to think that those who report credit misuse as the primary reason for filing for bankruptcy might be affected by some large expenditure as well, which makes the proportion in the data that directly corresponds to the proportion of filings due to a large expenditure shock to be larger than 46.6 percent. Besides, in one of the sensitivity exercises, probabilities of expenditure shocks or sizes of expenditure shocks are calibrated to be smaller than in the baseline model, which make defaults due to expenditure shocks fewer, but the main results are found to be robust to such changes.

One problem of the calibrated baseline model is that the model cannot replicate the total amount of assets or its distribution in the economy. For example, Laibson et al. (2007) report that the average asset-to-income ratio among households headed by individuals in their 50s is 2.60. The

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[14] Laibson et al. (2007) report 67.8 percent using the 1995 and 1998 waves of the SCF, but they only look at a subset of households whose head has a high school degree but not a college degree.

[15] Chatterjee et al. (2007) summarize their results in Table 1.

corresponding number in the baseline model is 1.42. Technically this is because the discount factor is used to match the debt-to-income ratio rather than the aggregate amount of savings. However, there are missing features that would enable the model to match the aggregate savings in the data. For example, permanent differences in earnings (due to differences in educational attainment), heterogeneity in the discount factor (Krusell and Smith (1998)), existence of agents with extremely high productivity (Castañeda et al. (2003)), which captures entrepreneurs, and multiple assets, especially housing, would help bring the model closer to the data in this regard.

The last two columns of Table 2 compare the statistics among agents without temptation (third column) and those with temptation (fourth column) in the baseline model. Since both types of agents share the same value for the self-control discount factor ($\delta$) but only the agents with temptation are affected by temptation associated with the discount factor $\beta$, it is not surprising that agents with temptation save less, borrow more, and, as a result, default more often. Agents with temptation borrow at a higher interest rate on average because they borrow more. The numbers in Table 2 are consistent with the intuition: 43.1 percent of agents with temptation are in debt and their average debt is 14.2 percent of income, while only 18.4 percent of agents without temptation are in debt and the average debt-to-income ratio for them is 3.9 percent. Every year 1.22 percent of agents with temptation default, while the default rate is 0.46 percent among agents without temptation. Interestingly, most of the defaults among agents without temptation are due to expenditure shocks (0.45 percent out of 0.46 percent, or 99 percent of total defaults), while only 80 percent of defaults among agents with temptation are due to expenditure shocks. The remaining 20 percent default due to a series of bad income shocks. The difference highlights the important distinction between agents with and without temptation. It is considered a puzzle as to why agents without temptation, i.e., agents with the standard exponential-discounting preferences, borrow at an interest rate as high as 10 percent per year when the discount factor (rate) is reasonably high (low). For agents with temptation, this is not a puzzle since those agents discount the immediate future at a very low discount factor, such as the 0.70 temptation discount factor used in the baseline calibration. Finally, somewhat unintuitively, the charge-off rate is lower for loans to agents with temptation (4.2 percent) compared with the charge-off rate for loans to agents without temptation (5.7 percent). This is because of the assumption that, if an agent defaults due to an expenditure shock, part of the confiscated income of the defaulter is used to partially pay back the expenditure. Therefore, the lending credit card company receives less upon default by agents with temptation on average, even if they are more likely to default. See equation (14).

Figure 2 compares life-cycle profiles of the two types of agents. Panel (a) shows life-cycle profiles of non-financial income (labor income and social security benefits) and consumption of agents without and with temptation. Notice that the two types of agents have the same non-financial income. Both types of agents save to smooth out consumption over the life cycle. However, since agents with temptation tend to save less during working years, their consumption declines more sharply after retirement. Panel (b) compares life-cycle profiles of mean asset holdings. As discussed already, agents without temptation save more or borrow less than those with temptation, on average, over the life cycle. Panel (c), which shows life-cycle profiles of the proportion of agents in debt, presents a consistent picture. The proportion is consistently higher among agents with temptation. Finally, not surprisingly, the proportion of defaulting agents is higher for all ages among agents with temptation.

## 6.2 Assessing the 2005 Bankruptcy Reform

This section investigates the effects of the (stylized version of the) BAPCPA using the baseline model. Section 6.2.1 presents the positive implications of introducing the BAPCPA, while Section 6.2.2 discusses welfare implications. Section 6.2.3 looks at the transition dynamics after the introduction of the bankruptcy law reform. Finally, Section 6.2.4 investigates whether there is a way to improve the BAPCPA.

### 6.2.1 Positive Implications of the BAPCPA

In 2005, the U.S. government enacted the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) in response to the significant increase in consumer bankruptcy filings (see Figure 1). According to White (2007), the main elements of the BAPCPA are the following three:

(1) Means-testing: Under the BAPCPA, if a debtor's household income over the past six months prior to the filing is over the median income of the state in which he or she lives, the borrower cannot file for Chapter 7 (fresh start) bankruptcy and can only file for Chapter 13 bankruptcy, which is basically debt restructuring and repayment rescheduling.

(2) Higher cost of filing: Under the BAPCPA, in order for his debt to be discharged, a debtor is required to take credit counseling, complete a financial management course, and submit detailed financial information that has to be certified by a lawyer. The typical cost of filing for Chapter 7 bankruptcy is raised from \$600 to \$2,500.

(3) Repayment schedule: Under the BAPCPA, a debtor filing for a Chapter 13 bankruptcy can no longer propose a repayment schedule. Instead, the law determines how much a filer has to pay back.

Since the Chapter 13 is not explicitly modeled, I focus on the effects of (1) means-testing and (2) the higher cost of bankruptcy filing. As for the means-testing, a borrower cannot default if his current income is above the median income of the model economy.[16] The exception is when the budget set is empty, i.e., the borrower cannot consume a positive amount without defaulting. As for the higher cost of defaulting, I change $\xi$, the fixed cost of defaulting, from \$600 (converted into the model unit) to \$2,500. I also implement exercises in which only one of the two components of the BAPCPA is enacted, in order to evaluate separately the effects of each of the two components. I investigate the aggregate effects of the bankruptcy reform, as well as heterogeneous effects for agents with and without temptation.

I also compare the model predictions with the observed changes in the U.S. However, the numbers for the U.S. have to be examined carefully, especially when the U.S. numbers are compared to the numbers in the steady-state of the model. Because the year 2005 saw a surge in bankruptcy filings before the BAPCPA became effective and 2006 observed a rebound from the spike in 2005, I will not use the data from 2005 to 2006. On the other hand, since the Great Recession started at the end of 2007, it is difficult to disentangle the effects from the BAPCPA and the cyclical effects from the Great Recession, especially when the economy seems to be still on its recovery from the recession, as seen in Figure 1. In order to deal with the issue, I compare the model predictions to three sets

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[16] In the actual law, what matters is the median income of the state in which the debtor resides. However, since there is no across-state income heterogeneity in the model, I use the overall median income in the model for the means-testing.

Table 3: Effects of the 2005 Bankruptcy Reform[1]

| | % Default | % in Debt | D/Y % | Charge-off % | Avg r % | Welfare % |
|---|---|---|---|---|---|---|
| **U.S. Economy** | | | | | | |
| Avg of 1999-2004 | 0.94 | – | 9.4 | 5.3 | 14.0 | – |
| 2007 | 0.43 | – | 9.5 | 4.0 | 13.3 | – |
| 2014 | 0.50 | | 6.6 | 3.2 | 11.9 | – |
| Avg of 2007-2014 | 0.67 | – | 7.7 | 5.6 | 12.6 | – |
| **Baseline Model: All Agents** | | | | | | |
| Baseline | 0.84 | 30.8 | 9.0 | 4.5 | 10.1 | – |
| BAPCPA | 0.35 | 33.0 | 11.1 | 2.4 | 9.4 | −0.34 |
| Means-testing | 0.65 | 31.5 | 9.5 | 3.8 | 10.2 | −0.05 |
| Default cost | 0.49 | 31.9 | 10.6 | 3.2 | 9.7 | −0.31 |
| **Baseline Model: Agents without Temptation** | | | | | | |
| Baseline | 0.46 | 18.4 | 3.9 | 5.7 | 9.9 | – |
| BAPCPA | 0.17 | 19.0 | 4.4 | 2.8 | 9.2 | −0.34 |
| Means-testing | 0.38 | 18.5 | 3.9 | 4.8 | 9.8 | −0.05 |
| Default cost | 0.21 | 19.0 | 4.3 | 3.4 | 9.3 | −0.31 |
| **Baseline Model: Agents with Temptation** | | | | | | |
| Baseline | 1.22 | 43.1 | 14.2 | 4.2 | 10.2 | – |
| BAPCPA | 0.54 | 47.0 | 18.0 | 2.3 | 9.4 | −0.34 |
| Means-testing | 0.92 | 44.5 | 15.2 | 3.5 | 10.3 | −0.05 |
| Default cost | 0.77 | 44.8 | 17.0 | 3.1 | 9.8 | −0.32 |
| **Alternative Model: Only Agents without Temptation** | | | | | | |
| Baseline | 0.84 | 36.9 | 9.0 | 4.8 | 9.9 | – |
| BAPCPA | 0.38 | 41.0 | 12.5 | 2.3 | 9.2 | −0.04 |
| **Alternative Model: Only Agents with Temptation** | | | | | | |
| Baseline | 0.84 | 31.7 | 9.0 | 4.5 | 10.1 | – |
| BAPCPA | 0.36 | 33.3 | 10.3 | 2.5 | 9.4 | −0.31 |

[1] The six columns show the proportion defaulting, the proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption.

of data. First, I use only the data in 2007. Second, I also use the data in 2014, which are the latest available. Finally, I use the average between 2007 and 2014. Admittedly, neither is perfect, but this is the best among the feasible options.

Table 3 compares the U.S. data (top panel) with the effects of the BAPCPA implied by the baseline model (second panel). The third and fourth panels show the changes induced by the BAPCPA among agents without and with temptation, respectively. The last two panels show the effects of the BAPCPA implied by the alternative models where only agents without or with temptation inhabit. The calibration of these alternative models is discussed in Appendix D.

In response to introduction of the BAPCPA, the baseline model implies a large decline in the number of bankruptcies, from 0.84 to 0.35 percent (first column). In the sense that the BAPCPA aims to reduce the number of bankruptcy filings, the model implies that the reform is successful. The number of bankruptcy filings also declined in the data. The number is 0.94 percent during 1999

and 2004, but dropped to 0.43 in 2007, before temporarily going up during the Great Recession. In 2014, the number is 0.50. The average between 2007 and 2014 is 0.67. Considering that the starting point is higher in the data (0.94 versus 0.84 percent), the size of the decline was generally in line with the model's prediction.

Both of the main components of the BAPCPA contributed to the decline, as can be seen in the bottom two lines in the panel. However, notice that the two components of the BAPCPA affect different types of agents. Means-testing prevents borrowers whose income is above median from filing for bankruptcy, while higher default costs affect all borrowers. Indeed, the higher filing costs discourage lower-income agents more, because the filing cost is fixed, and thus the cost is relatively larger for agents with lower income. The effect is stronger with the higher default cost; the proportion of agents filing for bankruptcy declines to 0.65 percent if only means-testing is introduced, while the number drops to 0.49 percent if only the higher default cost is introduced. This is because higher-income agents default less from the beginning, while default costs affect more strongly lower-income agents, who tend to borrow and default more frequently. The fact that the two components of the BAPCPA affect different segments of agents is why the combined effect of the two components to the number of bankruptcy filings is substantially stronger than the effect of each component.

Consistent with the decline in the number of bankruptcy filings, the baseline model predicts a decline in the charge-off rate of credit card loans (fourth column), from 4.5 percent to 2.4 percent. Both the means-testing and the higher default cost contributed to the decline. In the data, the charge-off rate declined as well, if one looks at the number before or after the Great Recessions: The charge-off rate was 5.3 percent during 1999-2004 but went down to 4.0 percent in 2007 and 3.2 percent in 2014. These numbers are consistent with the size of the decline implied by the baseline model. However, the charge-off rate shot up during the Great Recession (see Panel (c) of Figure 1), making the average charge-off rate during 2007-2014 higher (5.6 percent) than the initial value. Since the higher charge-off rate seems to be temporary only during the Great Recession, it is reasonable to think that the data imply a slight decline after implementation of the BAPCPA in 2005, which the baseline model also predicts.

Also consistent with the decline in the number of bankruptcy filings, the average loan interest rate in the baseline model declines from 10.1 to 9.4 percent. If the effect of each of the two components of the BAPCPA is investigated separately, the average loan rate declines with the higher default cost, but *rises* with the introduction of means-testing. The latter is due to the composition effect; with means-testing, borrowing among higher-income agents declines relative to borrowing by lower-income agents, resulting in an increase in the proportion of borrowing by lower-income agents, who are typically charged a higher interest rate reflecting a higher risk. In the data, the average credit card interest rate declined as well compared with the average of 1999-2004. However, one has to be careful because, as can be seen in Panel (d) of Figure 1, the majority of the decline happened before the introduction of the BAPCPA in 2005. One could say there is no significant change in the average loan interest rate before and after the 2005 reform. However, as shown by Neely and Rapach (2008), the real interest rate in the U.S. shifted up after 2005. Therefore, the fact that the average loan interest rate did not increase after the 2005 reform is consistent with the model's prediction, considering there was an upward trend of the risk-free real interest rate.

Figure 3 confirms the response of borrowing interest rates to the BAPCPA. The figure compares the loan rate schedules under the baseline model economy and the alternative economy under the BAPCPA, for agents with and without temptation. The loan rate schedules for age-30 agents with
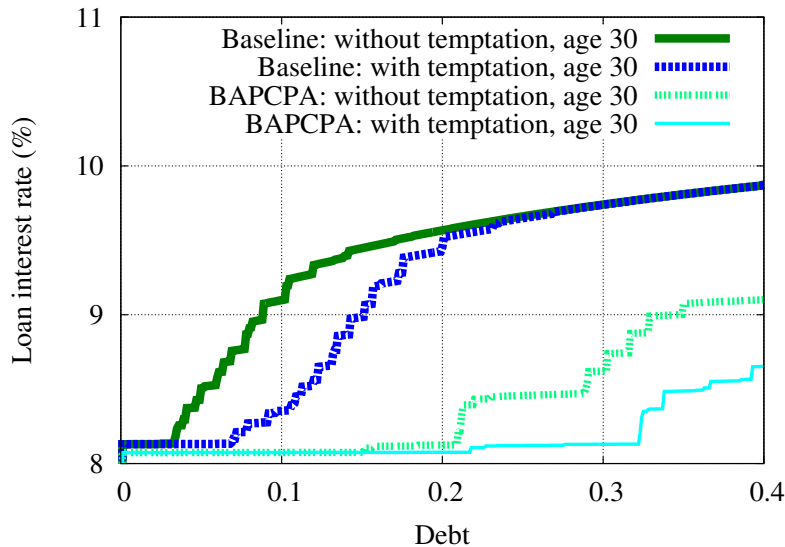
**Figure 3: Loan Interest Rate: Baseline Model and under BAPCPA**

the median productivity shock and zero expenditure shock are drawn. Clearly, for both agents with and without temptation, the BAPCPA lowers the loan interest rate since the BAPCPA makes defaulting more difficult, either by charging a higher cost of defaulting or imposing a means-testing requirement.[17]

Although both components of the BAPCPA work to lower the number of bankruptcies, lower loan rates, which reflect the lower default risk among borrowers under the BAPCPA, make it less costly to borrow, and thus the model predicts the total amount of borrowing to increase. In the baseline model, the aggregate debt-to-income ratio increased from 9.0 percent without the BAPCPA to 11.1 percent with the BAPCPA (third column). Both components of the BAPCPA contribute to the increase. The proportion of agents in debt increased as well, from 30.8 percent to 33.0 percent. On the other hand, the data seem to imply the opposite, although the debt-to-income ratio slightly edged up right after 2005, before dropping significantly at the onset of the Great Recession (see Panel (b) of Figure 1). All in all, in the data, the aggregate debt-to-income ratio edged up from 9.4 percent in the pre-BAPCPA years to 9.5 percent in 2007, before dropping to 6.6 percent in 2014. The average ratio during 2007-2014 is 6.6 percent. One can say that the model's prediction is consistent with the data in 2007, but the model's prediction is inconsistent with the data if one focuses on the data after the Great Recession. The discrepancy might be due to the financial crisis having a strong negative effect on credit supply.

The third and fourth panels separately look at the changes induced by the BAPCPA, for agents without and with temptation in the baseline model. In general, both types of agents react to the BAPCPA similarly. With the introduction of the BAPCPA, the number of bankruptcy filings declined among both types of agents; the charge-off rate declined, close to half, for both types; the average loan interest rate declined slightly; and the aggregate debt-to-income ratio increased.

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[17] One interesting feature of Figure 3 is that agents without temptation are charged higher interest rates controlling the type of agents and amount of debt. Since age-30 agents with temptation tend to be saving more than agents without, they value less the future option to borrow, making them more likely to default on their debt.

### 6.2.2 Welfare Implications of the BAPCPA

Throughout the paper, welfare is evaluated as the ex-ante expected lifetime utility. Formally, social welfare $\mathbb{E}V$ is defined as

$$\mathbb{E}V = \sum_j \sum_p \sum_t \sum_x \phi_j \pi_p^0 \ \pi_t^t \ \pi_x^x \ V(j, 1, 0, p, t, x, 0), \tag{19}$$

where $\pi_p^0$, $\pi_t^t$, and $\pi_x^x$ denote the initial distribution of the persistent income shock $p$, the distribution of the transitory income shock $t$, and the distribution of the expenditure shock $x$, respectively. $\phi_j$ is the measure of type-$j$ agents. Also notice that an agent is born into the model economy with the initial age ($i = 1$), a good credit history ($h = 0$), and zero assets ($a = 0$). This definition of welfare is the standard in the life-cycle model with heterogeneous agents in evaluating the welfare effects of policy changes (e.g., Conesa et al. (2009)). Similarly, welfare of type-$j$ agents $\mathbb{E}V_j$ is defined as follows:

$$\mathbb{E}V_j = \sum_p \sum_t \sum_x \pi_p^0 \ \pi_t^t \ \pi_x^x \ V(j, 1, 0, p, t, x, 0). \tag{20}$$

The difference in welfare between the two economies is measured by consumption equivalent variation (CEV), which is the percentage change in consumption every period.

Remember that the model with temptation and self-control has the same behavioral implications as the hyperbolic-discounting model. Although there could be differences in how to conduct welfare analysis under the two formulations, both formulations support the use of $V(.)$ as the basis of welfare analysis. As for the preferences featuring temptation and self-control, as argued in Gul and Pesendorfer (2004b), the preference of an agent is dynamically consistent. Therefore, the welfare of an agent can be defined in a straightforward manner, using $V(.)$. On the other hand, since a hyperbolic-discounting agent is dynamically inconsistent, and the same agents in different time periods are considered as different selves, there is no naturally accepted notion of agent's welfare. However, O'Donoghue and Rabin (2001) advocate conducting welfare analysis by discounting continuation payoffs exponentially. This basically means the use of $V(.)$ as well.[18]

The last column of Table 3 shows changes in welfare by moving from the baseline model economy without the BAPCPA to the economy with it. The baseline model indicates that, even though the BAPCPA achieves what it is designed for (reducing the number of bankruptcy filings), the policy reform causes welfare loss on average among all agents. The size of the average welfare loss is equivalent to 0.34 percent increase in consumption every period. Both of the two main components of the BAPCPA are welfare-reducing, but the higher default cost has a stronger welfare effect. The welfare effect of introducing means-testing alone is equivalent to 0.05 percent of consumption reduction every period, while the welfare effect of the higher default cost alone is equivalent to 0.31 percent of consumption loss. A higher default cost affects more agents (the number of defaults declines more) and hurts agents with lower income and consumption. Notice, however, that an important assumption here is that the default cost is completely a waste.

The third and fourth panels show that the welfare effects are very similar for both types of agents. For agents both with and without temptation, the welfare effect of introducing the BAPCPA is equivalent to 0.34 percent of consumption reduction. This is somewhat surprising. With the standard preferences without temptation, the BAPCPA has two kinds of welfare effects. First

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[18] I thank a referee for pointing this out.

is the negative effect. A higher default cost makes defaulting agents suffer more. Means-testing prevents high-income agents from filing, even if filing for bankruptcy is the better option than repaying. The second effect is positive. Since agents default less, credit card companies offer lower interest rates because the default premia should be brought down, reflecting lower default risks. For agents with temptation, there is the third effect. When borrowing becomes less costly, agents with temptation might overborrow in the sense of borrowing more than what maximizes the self-control utility. In the end, agents with temptation might not value the ability to borrow at a lower cost as much as agents without temptation. Nakajima (2012) shows that the welfare effect of relaxing borrowing constraint is positive for agents without temptation, but could be negative for those with the temptation. However, in the current experiment, the welfare effects of introducing the BAPCPA are almost identical between agents with and without temptation. Why? This is because not many agents without temptation are in debt in the current baseline model. Remember that, because of the way the discount factors are calibrated, agents without temptation, who have the same self-control (long-term) discount factor $\delta$ as agents with temptation but are not affected by temptation (short-term) discount factor $\beta$, end up saving more and borrowing less. Therefore, even though they potentially benefit from lower loan interest rates, they do not in equilibrium, because not many agents without temptation are constrained from the beginning. When they are hit by expenditure shocks and find it optimal to default, they suffer from a higher borrowing cost or means-testing requirement. But they do not benefit from the lower default premia much because not many of them are borrowing. This is why the negative welfare effects of the BAPCPA dominate even for agents without temptation. For agents with temptation, even though many are borrowing, they do not benefit significantly from lower default premia because of the disutility from overborrowing.

The above argument is clearer if the welfare results of the baseline model are compared with the welfare results of alternative models inhabited only by agents with or without temptation.[19] Those welfare results are shown in the last two panels of Table 3. In the alternative model economy inhabited only by agents without temptation, i.e., agents with the standard preferences, the welfare effect of introducing the BAPCPA is still negative but significantly smaller at 0.04 percent of flow consumption loss. This is because the negative effect is offset significantly by the positive effect of lower loan interest rates. On the other hand, in the other alternative model inhabited only by agents with temptation, the welfare effect of the BAPCPA is similar to the baseline result, at 0.31 percent of consumption loss every period.

### 6.2.3   Transition Dynamics

Figure 4 shows the transition dynamics after introduction of the BAPCPA in 2005. It is assumed that the economy is in its initial steady-state in 2004, and suddenly the bankruptcy reform is introduced in 2005. No other change is introduced after 2005. The four panels of Figure 4 present (a) the percentage of agents filing for bankruptcy, (b) the aggregate debt-to-income ratio, (c) the charge-off rate of loans, and (d) the average loan interest rate. Each of the four panels of Figure 4 corresponds to each of the panels in Figure 1, respectively. According to the model, the number of defaults, the charge-off rate, and the average interest rate react immediately in 2005 and change little as the economy converges to the new steady-state. On the other hand, the debt-to-income ratio gradually increases to the new steady-state level.

As discussed, comparing the model dynamics to the data is not straightforward, for two reasons. First, it is reasonable to think that, starting from 2008, the Great Recession significantly affects

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[19] See Appendix D for details about these alternative models.
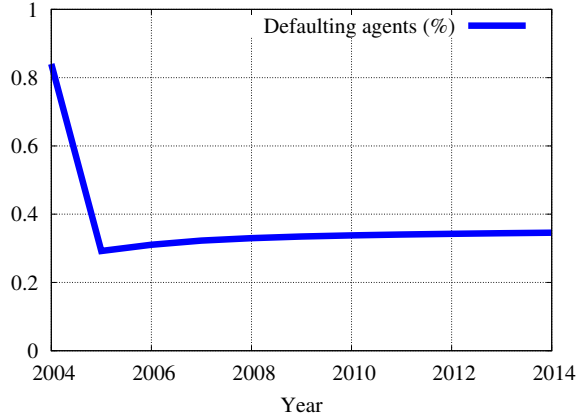
|  | |
|---|---|
| (a) Percentage of Defaulting Agents | (b) Debt-to-Income Ratio |
| (c) Charge-Off Rate of Loans | (d) Average Loan Interest Rate |
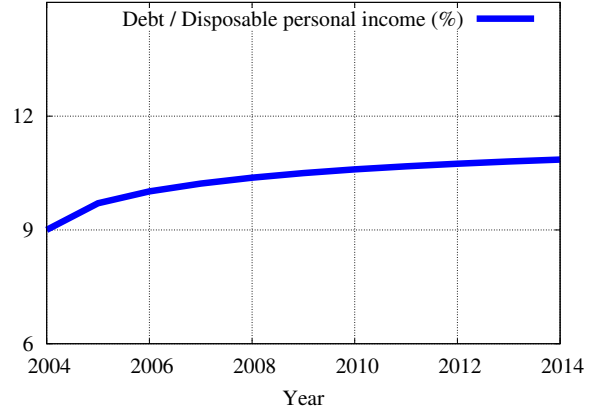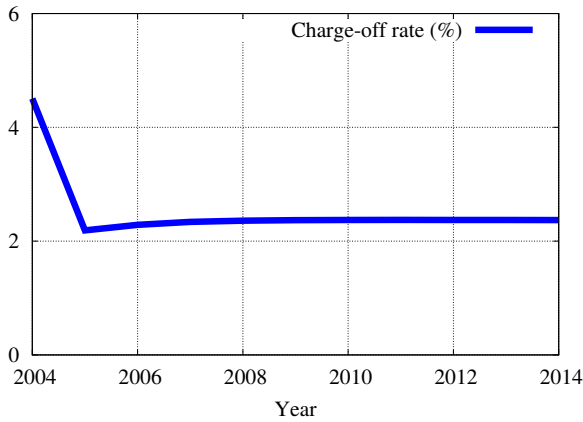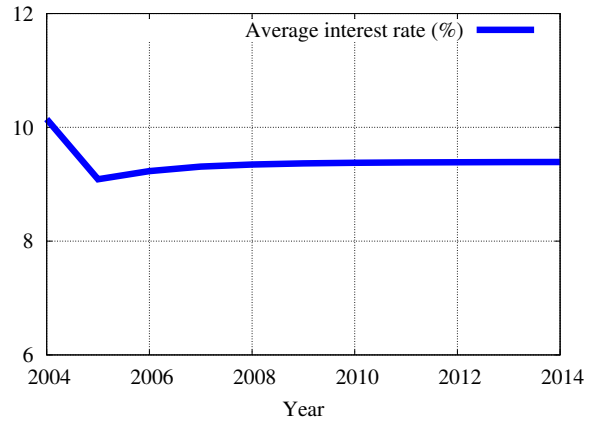
Figure 4: Transition Dynamics after Introducing the BAPCPA

the data.[20] Second, since the bankruptcy law reform was announced before its introduction, there was a spike upward in 2005, reflecting the rush to file before the reform was implemented in late 2005. On the other hand, since it is assumed that the reform is a complete surprise in the model experiment, the model does not generate the rush.

### 6.2.4 Adjusting the BAPCPA

In this section, I implement a preliminary investigation as to whether there is a way to improve the BAPCPA. In particular, I change (1) the level of threshold income for means-testing and (2) the cost of bankruptcy filing, independently.

Table 4 summarizes the results. In the top panel, I change the threshold income levels for the means-testing, while the default cost is fixed at the baseline level of $600. The numbers are shown as percentage of median income. The baseline model economy corresponds to the threshold income level of infinity because the threshold is never binding with it. As the threshold level is lowered, the number of bankruptcy filings declines, since more and more agents cannot satisfy the means-testing

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[20] Since the Great Recession started in December 2007, it is probably safe to assume that the data in 2007 were not significantly affected by the Great Recession.

<div align="center">

**Table 4: Adjusting the BAPCPA**[1]

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|  | % Default | % in Debt | D/Y % | Charge-off % | Avg r % | Welfare % |
|---|---|---|---|---|---|---|
| **Changing Means-Testing Threshold** | | | | | | |
| 0% | 0.02 | 39.6 | 26.2 | 0.1 | 8.1 | +0.55 |
| 50% | 0.29 | 33.5 | 11.5 | 1.6 | 9.3 | −0.28 |
| 100% (BAPCPA) | 0.65 | 31.5 | 9.5 | 3.8 | 10.2 | −0.05 |
| ∞% (Baseline) | 0.84 | 30.8 | 9.0 | 4.5 | 10.1 | – |
| **Changing Default Cost** | | | | | | |
| $0 | 1.02 | 30.2 | 8.1 | 5.1 | 10.4 | +0.11 |
| $600 (Baseline) | 0.84 | 30.8 | 9.0 | 4.5 | 10.1 | – |
| $1,200 | 0.72 | 31.1 | 9.7 | 4.1 | 10.0 | −0.11 |
| $2,500 (BAPCPA) | 0.49 | 31.9 | 10.6 | 3.2 | 9.7 | −0.31 |

[1] The six columns show the proportion defaulting, the proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption.

constraint. The social welfare declines when the threshold income level is lowered from 100 percent to 50 percent of median income, because more and more agents cannot file even if it is optimal to do so. However, when the threshold income level for the means-testing is set at a very low level, and only agents who cannot consume a positive amount of consumption without filing can file, the number of bankruptcy filings becomes extremely low (0.02 percent per year) but the welfare effect turns positive. It is basically an economy without default. If defaults are not allowed except for involuntary ones, agents can benefit from borrowing at a near risk-free loan rate (see column five). This welfare gain seems to dominate the loss from being unable to file for bankruptcy even after a series of bad shocks to income or expenditures.

In the bottom panel, I start from the baseline model economy and change the default cost to various levels. Not surprisingly, a lower default cost implies a higher social welfare, because default cost is a waste by assumption. When the default cost is eliminated, even though the number of bankruptcy filings increases to 1.02 percent per year, social welfare increases by 0.11 percent in consumption equivalent variation. However, this result should be taken with a grain of salt, because there is probably a positive role for goods or services purchased with the positive default cost in the real world, which is abstracted in the current model. Although it is outside of the current model, the result implies that, if the default cost is financed by tax, a welfare gain might be attainable.

## 6.3 Assessing Usury Law

Until the early 1980s, banks and other lending institutions were subject to limits on the interest rates they could charge. This usury law was imposed by the state in which each loan was made. However, the *Marquette* decision in 1978 and the Depository Institutions Deregulation and Monetary Control Act virtually freed banks and lending institutions of interest rate limits and allowed them to charge any rate they chose. In the U.S., there is currently no upper bound for the loan interest rate that financial institutions can charge, but it is reasonable to think that a usury law, by discouraging loans to risky borrowers who require high default premia, could also help achieve the same goal

**Table 5: Effects of Usury Law[1]**

|  | % Default | % in Debt | D/Y % | Charge-off % | Avg r % | Welfare % |
|---|---|---|---|---|---|---|
| **Baseline Model: All Agents** | | | | | | |
| Baseline | 0.84 | 30.8 | 9.0 | 4.5 | 10.1 | – |
| Usury law: 20% | 0.83 | 30.6 | 9.0 | 4.5 | 10.1 | +0.02 |
| Usury law: 10% | 0.74 | 25.2 | 4.8 | 6.0 | 9.6 | −0.98 |
| **Baseline Model: Agents without Temptation** | | | | | | |
| Baseline | 0.46 | 18.4 | 3.9 | 5.7 | 9.9 | – |
| Usury law: 20% | 0.46 | 18.4 | 3.9 | 5.7 | 9.9 | −0.00 |
| Usury law: 10% | 0.46 | 10.7 | 1.7 | 10.0 | 9.5 | −1.08 |
| **Baseline Model: Agents with Temptation** | | | | | | |
| Baseline | 1.22 | 43.1 | 14.2 | 4.2 | 10.2 | – |
| Usury law: 20% | 1.21 | 42.8 | 14.1 | 4.2 | 10.2 | +0.03 |
| Usury law: 10% | 1.02 | 39.6 | 7.9 | 5.1 | 9.6 | −0.89 |

[1] The six columns show the proportion defaulting, the proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption.

as the recent bankruptcy law reform — reducing the number of bankruptcies.[21] What are the effects of this usury law? How are the effects of this law different between agents with and without temptation? In order to answer these questions, I introduce a usury law with various interest rate limits to the baseline model.

Table 5 summarizes the results. The first panel shows the effects of introducing the upper bound of a loan interest rate for the entire economy. The second and third panels show the effects on agents without and with temptation, respectively. First, when the upper bound of the loan interest rate is set at 20 percent, there is little effect on macroeconomic aggregates. The number of bankruptcy filings edges down from 0.84 percent per year to 0.83 percent. The proportion of agents in debt slightly declines, from 30.8 percent to 30.6 percent. The debt-to-income ratio, charge-off rate, and average loan interest rate barely move in response to the introduction of a 20 percent interest rate upper bound. The effects are similarly minor for both agents with and without temptation, which are shown in the second and third panels. However, interestingly, although the welfare effect for agents without temptation is basically nil, there is a small but positive welfare gain among agents with temptation. The size of the gain is equivalent to 0.03 percent increase in consumption every period. This can be interpreted as the tighter borrowing constraint benefiting agents with temptation by helping them not to overborrow. On the other hand, the welfare effect is always negative among agents without temptation since they do not have the gain from preventing overborrowing. And the welfare effect is negligible since only a small number of agents without temptation borrow substantially and thus are affected by the relatively high interest rate limit.

However, when the upper bound of the loan interest rate is further lowered to 10 percent, the negative effect of a tighter borrowing constraint dominates for both types of agents. When the tight usury law is introduced in the baseline model, the debt-to-income ratio declines almost by

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[21] For example, in Japan, a law that prohibits loans with an interest rate higher than 20 percent per year was implemented in 2007. Indeed, the 20 percent upper bound existed even before the reform, but it wasn't virtually effective, and only the 29.2 percent limit was valid. The reform was intended to discourage *predatory* lending.
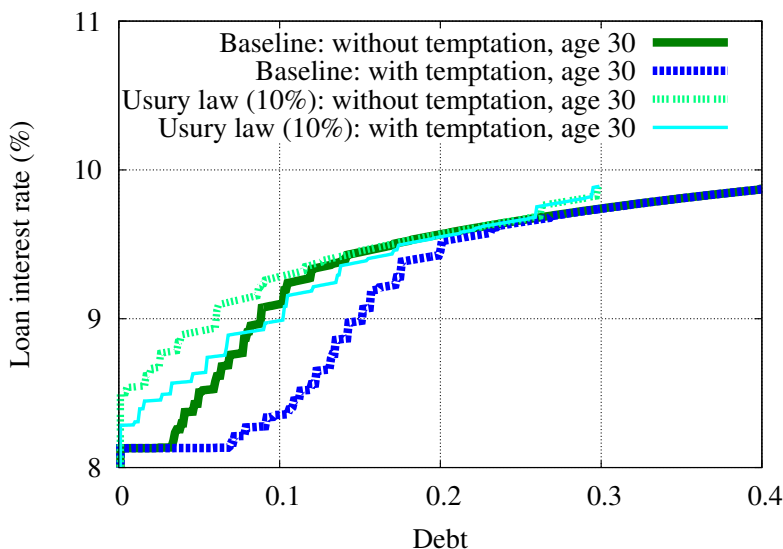
**Figure 5: Loan Interest Rate: Baseline Model and under Usury Law of 10%**

half, from 9.0 percent to 4.8 percent. Naturally, the proportion in debt declines as well. The number of defaults per year drops from 0.84 percent to 0.74 percent. The average interest rate declines, reflecting the improvement in the riskiness of the pool of borrowers. The charge-off rate rises, because the defaults due to expenditure shocks, which often cause a large loss to credit card companies, increase relatively. Again, the effects are similar between agents with and without temptation. Unlike the case with the mild usury law of 20 percent, the welfare effects are similar between the two types of agents as well. The overall welfare effect is 0.98 percent of consumption loss. The welfare loss is 1.08 percent among agents without temptation, while the welfare loss is slightly smaller, at 0.89 percent, among agents with temptation. The welfare effects are similar because the negative welfare effect of a tightened borrowing constraint dominates other effects for all agents.

The tightening credit can be seen in Figure 5, which compares the loan rate schedules of the baseline model economy and the alternative economy with the interest rate ceiling of 10 percent. Interest rate schedules for both agents with and without temptation are drawn. There are two significant differences between the interest rate schedules under the two model economies. First, because of the interest rate ceiling, a large amount of loans become unavailable; it is simply too risky to offer such large loans. In Figure 5, the interest rate schedule under the usury law disappears at around the loan size of 3.0 for both types of agents. This is because loans of such a large amount require an interest rate of above 10 percent to be profitable, which violates the usury law. Second, because of the tightening credit, agents default with smaller loan amounts, which pushes the interest rate schedule upward in general, reflecting the higher default premium, and makes borrowing more costly even for a loan of a smaller amount.

## 6.4 Changing the Level of Income Garnishment

The main purpose of the 2005 bankruptcy law reform was to discourage bankruptcy filing among those who do not really *need* to default. Another way to discourage bankruptcy, albeit less directly, is to allow credit card companies to garnish more earnings upon default. Currently, federal law

|  | % Default | % in Debt | D/Y % | Charge-off % | Avg r % | Welfare % |
|---|---|---|---|---|---|---|
| **Baseline Model: All Agents** | | | | | | |
| $\eta = 0.000$ | 1.19 | 25.6 | 3.0 | 18.2 | 11.6 | +0.31 |
| $\eta = 0.306$ (baseline) | 0.84 | 30.8 | 9.0 | 4.5 | 10.1 | – |
| $\eta = 0.840$ (optimal) | 0.02 | 38.8 | 23.8 | 0.2 | 8.1 | +0.39 |
| **Baseline Model: Agents without Temptation** | | | | | | |
| $\eta = 0.000$ | 0.84 | 13.5 | 1.8 | 22.2 | 11.0 | +0.09 |
| $\eta = 0.306$ (baseline) | 0.46 | 18.4 | 3.9 | 5.7 | 9.9 | – |
| $\eta = 0.840$ | 0.01 | 24.6 | 10.5 | 0.1 | 8.1 | +0.52 |
| **Baseline Model: Agents with Temptation** | | | | | | |
| $\eta = 0.000$ | 1.55 | 37.8 | 4.3 | 16.5 | 11.7 | +0.52 |
| $\eta = 0.306$ (baseline) | 1.22 | 43.1 | 14.2 | 4.2 | 10.2 | – |
| $\eta = 0.840$ | 0.04 | 53.0 | 37.4 | 0.2 | 8.1 | +0.26 |
| **Alternative Model: Only Agents without Temptation** | | | | | | |
| $\eta = 0.000$ | 1.27 | 25.3 | 3.0 | 20.7 | 11.0 | −0.66 |
| $\eta = 0.269$ (baseline) | 0.84 | 36.9 | 9.0 | 4.8 | 9.9 | – |
| $\eta = 0.840$ (optimal) | 0.03 | 52.1 | 38.8 | 0.1 | 8.1 | +3.35 |
| **Alternative Model: Only Agents with Temptation** | | | | | | |
| $\eta = 0.000$ (optimal) | 1.27 | 27.8 | 3.1 | 17.9 | 12.0 | +0.50 |
| $\eta = 0.320$ (baseline) | 0.84 | 31.7 | 9.0 | 4.5 | 10.1 | – |
| $\eta = 0.840$ | 0.02 | 37.7 | 19.4 | 0.1 | 8.1 | −0.14 |

[1] The six columns show the proportion defaulting, the proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption.

sets an upper bound on how much a creditor can garnish its debtor's wage per week: It is either 25 percent of wages, or the wage amount exceeding 30 times the federal minimum wage, whichever is smaller.[22],[23] By raising the amount that creditors can garnish, the government can make loans more creditor-friendly and discourage defaulting. This section explores the implications of changing the garnishment ratio in the model economy and discusses the *optimal* level of income garnishment.

Two remarks are worth making. First, I use the word *optimal* in a very specific manner, in the sense that I change $\eta$ without changing other elements of the bankruptcy law and call $\eta$ optimal when the social welfare $\mathbb{E}V$ is maximized. The social welfare, defined in equation (19), is ex-ante expected lifetime utility. I leave the problem of designing the optimal bankruptcy law in a less restricted policy space for future research. Second, the general equilibrium effect is not considered here. Nakajima (2012) considers the general equilibrium effect when the *optimal* severity of the borrowing constraint is investigated. As expected, the general equilibrium effect lowers (increases) welfare when the borrowing constraint is relaxed (tightened), due to capital decumulation (accumulation).

Table 6 compares the calibrated baseline model with alternative models with different values of the garnishment rate ($\eta$). The first panel shows the results for the baseline model with all agents.

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[22] States can set a lower limit for garnishment, providing better protection for debtors. For example, in Massachusetts, the garnishment limit is typically 15 percent of wages.

[23] In the model, the upper bound of the amount of garnishment associated with the minimum wage is abstracted.

The second and third panels show the results of the baseline model but only for agents without or with temptation, respectively. The fourth panel shows the results for the alternative model economy inhabited only by agents without temptation.[24] The last panel is associated with the other alternative model economy containing only agents with temptation. For each panel, the six columns show the number of bankruptcy filings, the proportion of agents in debt, debt-to-income ratio, charge-off rate, the average loan interest rate, and the change in welfare from the model with the baseline (calibrated) value of $\eta$, represented as percentage change in consumption every period, respectively. Each panel contains the results regarding the calibrated value of $\eta$ and those associated with the lower and upper bounds of $\eta$. The lower bound is $\eta = 0$. The upper bound of $\eta$ is 0.84 in all models, because of the fixed cost of filing, $\xi$; with $\eta > 0.84$, the budget set for a filing agent with the lowest income shock becomes empty.

The reason why only the cases with lower and upper bounds of $\eta$ are shown is that the optimal $\eta$ for the three model economies considered ends up being one of the two bounds. For the baseline model, the social welfare is maximized with $\eta = 0.84$, its highest feasible value. Since the punishment for defaulting is maximized, the number of bankruptcy filings becomes tiny, at 0.02 percent each year as opposed to 0.84 percent in the baseline case. The average loan interest rate (8.1 percent) is very close to the risk-free loan rate of 8.0 percent, because the default premium is close to zero. Naturally, the charge-off rate of consumer debt is also very low (0.2 percent compared with 4.5 percent in the baseline case). On the other hand, reflecting the low loan interest rates, agents increase borrowing significantly. The debt-to-income ratio rises from 9.0 percent to 23.8 percent, and the proportion in debt rises from 30.8 percent to 38.8 percent.

The overall welfare effect is a combination of the negative effect due to a higher punishment when defaulting, the positive effect of a lower cost of credit, and (in the case of agents with temptation) the negative effect of induced overborrowing. In the baseline model, the positive effect ends up dominating the negative effect slightly. The overall welfare gain is equivalent to a 0.39 percent increase in consumption every period. The response is similar between agents with and without temptation, as seen in the second and third panels of Table 6. Agents without temptation gain more (0.52 percent) than agents with temptation (0.26 percent), because the latter tend to default more frequently (0.04 percent of agents with temptation default each period, against 0.01 percent of agents without), and cheaper credit could hurt the latter by encouraging overborrowing.

What is interesting in Table 6 is that, in the baseline model, social welfare *increases* when the garnishment ratio $\eta$ is lowered as well. Indeed, the gain in social welfare by lowering $\eta$ from its baseline value of 0.306 to 0 is equivalent to a 0.31 percent increase in consumption every period, which is not very different from the welfare gain associated with the optimal $\eta$ (0.39 percent). Panel (a) of Figure 6 shows this point more clearly. Panel (a) has $\eta$ on the x-axis and shows the welfare gain associated with various values of $\eta$ on the y-axis. The welfare gain for all agents in the baseline model and the gains for agents without and with temptation are shown in the panel. The panel implies not only that the change in welfare is non-monotonic with respect to $\eta$, but also that the calibrated value of $\eta = 0.306$ achieves the level of welfare close to the lowest. Why isn't the response of welfare monotonic? The decomposition shown in panels (c) and (d) in Figure 6 helps. In panel (c), I do the same exercise as in panel (a), but use the model without expenditure shocks. In panel (d), income shocks are shut down instead. When there are only income shocks (panel (c)), since agents are not forced to default with a large expenditure shock, the welfare gain from lower borrowing interest rates when the garnishment rate is increased becomes strong. Indeed,

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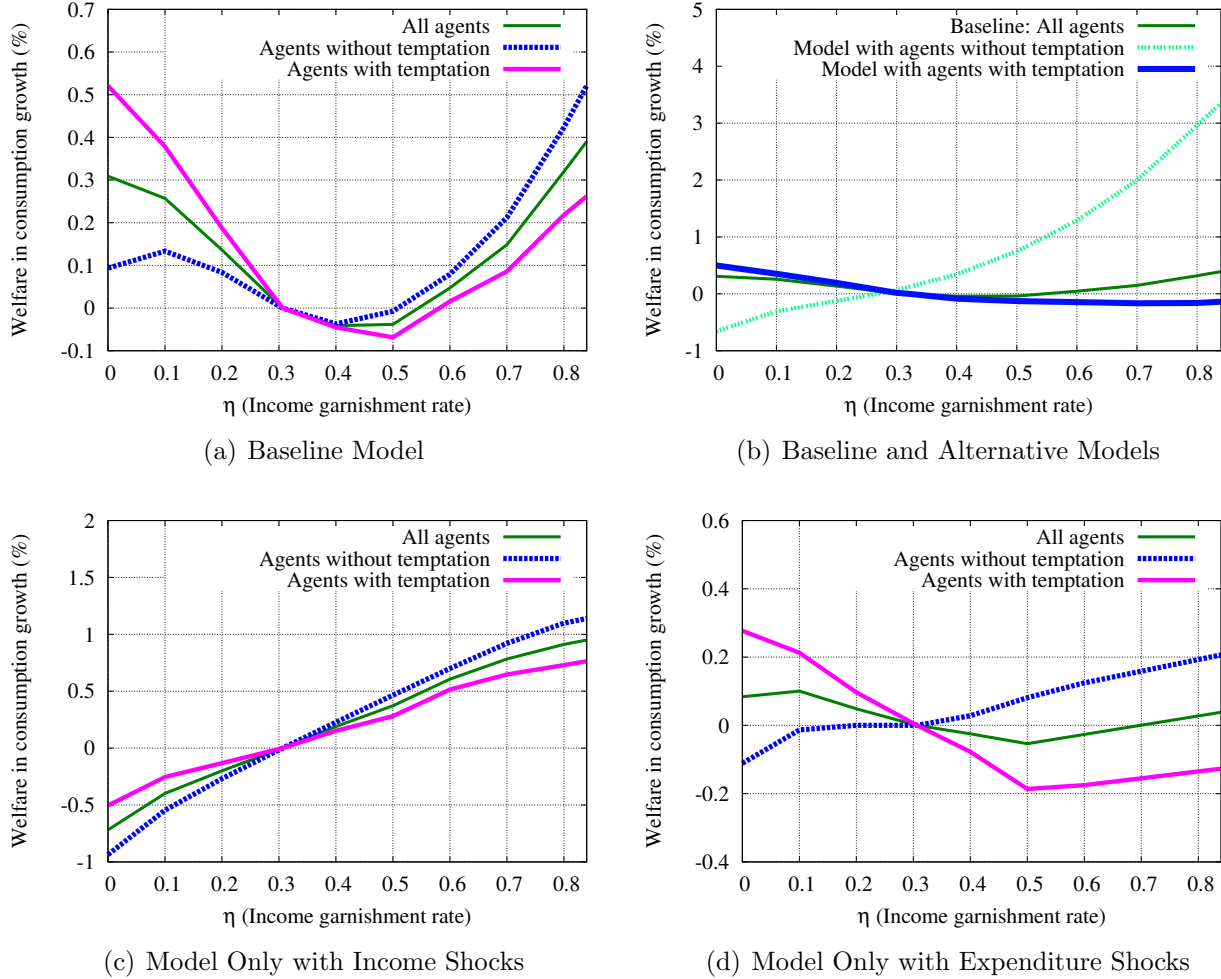[24] See Appendix D for details on the alternative models.

Figure 6: **Welfare Effects of Changing the Income Garnishment Rate**

the welfare increases monotonically with $\eta$ for all agents. On the other hand, when there are only expenditure shocks, agents with and without temptation are affected very differently. The welfare effect for agents without temptation is similar to the case only with income shocks (panel (c)). On the other hand, agents with temptation benefit from a lower $\eta$. This is because they tend to save less or borrow more and thus are more likely to be forced into default by expenditure shocks. The overall welfare effect in the economy only with expenditure shocks is monotonic because of the heterogeneous welfare effects to the two types of agents. The non-monotonic relationship between $\eta$ and welfare in the baseline economy with both shocks is generated by the combination of the effects in the two economies with income or expenditure shocks.

What is also interesting is that, in the baseline model, the welfare effects are similar between agents with and without temptation. This is in contrast to findings in Nakajima (2012), in which the welfare effects of the relaxing borrowing constraint are contrasting between agents with and without temptation, because agents with temptation suffer overborrowing when the borrowing constraint is relaxed. The reason why the welfare effects are similar between agents with and without temptation is that, in the baseline model, agents without temptation do not borrow much, and thus the positive welfare effect from lower loan interest rates is weak among agents without temptation in the baseline

model. To show this point more clearly, in panel (b) of Figure 6, the welfare effects of changing the garnishment ratio $\eta$ for the alternative models only with agents with or without temptation are compared with the welfare effects of the baseline model (same as in panel (a)). It is easy to see that, when there are only agents without temptation in the model and many of them are borrowing-constrained, there is a strong welfare effect from increasing the punishment of defaulting (a higher $\eta$), which yields lower borrowing costs. And the magnitude of the positive welfare effect is significantly larger than the effects in the baseline model. As shown in the fourth panel of Table 6, when the garnishment ratio $\eta$ is increased to the highest level of 0.84, agents without temptation gain as much as 3.4 percent of flow consumption. The welfare effect of changing $\eta$ is monotonic in the model only for agents without temptation. Those agents suffer a welfare loss of 0.66 percent in consumption equivalent variation when the garnishment ratio is lowered to zero.

On the other hand, the welfare effect of changing $\eta$ is the opposite according to the model only with tempted agents. Tempted agents in the model suffer a welfare loss of 0.14 percent in consumption equivalent variation when the garnishment ratio is raised to its maximum of 0.84, while they gain as much as 0.50 percent of flow consumption when the income garnishment is eliminated, by setting $\eta = 0$. The property that agents with temptation gain more with $\eta = 0$ than with $\eta = 0.84$ is shared in the baseline model. Agents with temptation benefit from both lower default costs and higher borrowing interest rates, the latter of which discourage overborrowing, more than agents without temptation. Moreover, for agents with temptation, temporary exclusion from the unsecured loan market after defaulting has some value as a commitment device against overborrowing.

The contrast between the two alternative models is consistent with the finding of Nakajima (2012), although the contrast does not show up clearly in the baseline model, because the majority of agents without temptation are not borrowing and thus do not benefit from lower interest rates. Generally put, the composition of different types of agents that are borrowing and defaulting is crucial in evaluating the social welfare of bankruptcy law reform.

# 7    Sensitivity Analysis

This section investigates the sensitivity of the main results of the paper. Due to space restrictions, only the effects of implementing the BAPCPA in the model are presented. I investigate (i) the model with naive agents, (ii) the model with a lower (higher) temptation discount factor (rate), (iii) the model with information frictions, (iv) the model with a bequest motive, and (v) the model with different parameter values. Table 7 summarizes all the results. Appendix D.2 provides further details about the alternative models used in this section.

## 7.1    Model with Naive Agents

Agents in the baseline model presented above are called "sophisticated," because they correctly see themselves as subject to temptation in the future. In the language of hyperbolic discounting, agents are aware that they are time-inconsistent, i.e., they correctly expect their future selves to also use hyperbolic discounting. The alternative assumption is what is called "naive." Naive agents are not aware that they are time-inconsistent, or they do not recognize they will suffer temptation in the future. In this sensitivity experiment, I assume agents with temptation are naive. Notice that agents without temptation are not affected. I recalibrate the model parameters $\delta$ and $\eta$, leaving all the other parameters unchanged.

The second panel of Table 7 shows the results with naive agents. It is clear that the response of

**Table 7: Sensitivity Analysis: Effects of the 2005 Bankruptcy Reform[1]**

|  |  | % Default | % in Debt | D/Y % | Charge-off % | Avg r % | Welfare % |
|---|---|---|---|---|---|---|---|
| 1. | Baseline Model | 0.84 | 30.8 | 9.0 | 4.5 | 10.1 | – |
|  | BAPCPA | 0.35 | 33.0 | 11.1 | 2.4 | 9.4 | −0.34 |
| 2. | Naive Agents | 0.84 | 26.5 | 9.0 | 4.5 | 10.6 | – |
|  | BAPCPA | 0.32 | 29.1 | 11.0 | 2.3 | 9.5 | −0.39 |
| 3. | $\beta = 0.6667$ | 0.84 | 30.7 | 9.0 | 4.4 | 10.3 | – |
|  | BAPCPA | 0.33 | 31.4 | 11.0 | 2.3 | 9.4 | −0.35 |
| 4. | Information Frictions | 0.84 | 30.9 | 9.0 | 4.4 | 9.8 | – |
|  | BAPCPA | 0.66 | 31.2 | 11.1 | 6.7 | 11.9 | −0.25 |
| 5. | Bequest Motive | 0.84 | 30.9 | 9.0 | 4.5 | 10.1 | – |
|  | BAPCPA | 0.36 | 32.9 | 11.2 | 2.4 | 9.4 | −0.35 |
| 6. | $\psi_e = 0.55,\ \psi_p = 0$ | 0.84 | 31.0 | 9.0 | 4.5 | 10.1 | – |
|  | BAPCPA | 0.33 | 33.3 | 11.1 | 2.2 | 9.3 | −0.35 |
| 7. | $\psi_e = 0,\ \psi_p = 0.55$ | 0.84 | 30.3 | 9.0 | 4.5 | 10.2 | – |
|  | BAPCPA | 0.36 | 32.4 | 11.3 | 2.4 | 9.4 | −0.34 |
| 8. | $\sigma = 3.0$ | 0.84 | 35.5 | 9.0 | 6.0 | 10.4 | – |
|  | BAPCPA | 0.44 | 42.4 | 15.3 | 2.5 | 9.6 | +1.36 |
| 9. | $r = 0.034$ | 0.84 | 29.6 | 9.0 | 4.6 | 8.2 | – |
|  | BAPCPA | 0.43 | 31.7 | 11.6 | 2.7 | 7.6 | −0.27 |
| 10. | $\lambda = 0.20$ | 0.84 | 30.8 | 9.0 | 4.3 | 10.1 | – |
|  | BAPCPA | 0.32 | 32.3 | 10.9 | 2.2 | 9.3 | −0.39 |
| 11. | Immediate Recovery | 0.84 | 30.5 | 9.0 | 4.5 | 10.1 | – |
|  | BAPCPA | 0.35 | 33.0 | 11.2 | 2.3 | 9.4 | −0.37 |
| 12. | Proportional $\xi$ | 0.84 | 30.9 | 9.0 | 4.4 | 10.1 | – |
|  | BAPCPA | 0.49 | 32.8 | 11.0 | 2.8 | 9.6 | −0.14 |
| 13. | Half $\pi^x$ | 0.84 | 38.3 | 9.0 | 3.4 | 9.5 | – |
|  | BAPCPA | 0.43 | 41.5 | 14.1 | 2.0 | 9.4 | −0.09 |
| 14. | Smaller $x$ | 0.84 | 33.3 | 9.0 | 3.2 | 10.1 | – |
|  | BAPCPA | 0.32 | 37.2 | 12.4 | 1.6 | 9.1 | −0.00 |
| 15. | $\rho_p = 0.93$ | 0.84 | 30.3 | 9.0 | 4.5 | 10.1 | – |
|  | BAPCPA | 0.40 | 31.9 | 11.2 | 2.6 | 9.6 | −0.22 |
| 16. | $\sigma_p^2 = 0.0275$ | 0.84 | 30.9 | 9.0 | 4.5 | 10.1 | – |
|  | BAPCPA | 0.35 | 32.9 | 11.1 | 2.3 | 9.4 | −0.77 |
| 17. | $\sigma_t^2 = 0.075$ | 0.84 | 31.8 | 9.0 | 4.5 | 10.1 | – |
|  | BAPCPA | 0.36 | 35.0 | 12.4 | 2.2 | 9.3 | −0.21 |

[1] The six columns show the proportion defaulting, the proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption. The first row in each panel represents the model without the BAPCPA, while the second row represents the same model but with the BAPCPA.

the model with naive agents to the introduction of the BAPCPA is similar to that of the baseline model with sophisticated agents. With the BAPCPA, the number of defaults declines by about 60 percent, while the debt-to-income ratio increases from 9.0 percent to 11.0 percent. The charge-off

rate declines almost by half, and, naturally, the average loan interest rate declines from 10.6 percent to 9.5 percent. The welfare effect of introducing the BAPCPA is also similar. The size of the loss in social welfare is 0.39 percent in consumption equivalent variation in the model with naive agents, while the welfare loss is 0.34 percent in the baseline model.

Akerlof (1991) and O'Donoghue and Rabin (1999, 2001) study cases where sophisticated and naive agents behave in very different ways, while Angeletos et al. (2001) find that the two types of agents produce similar results in their estimated life-cycle model. It is not surprising that the two types of agents generate similar results in the current experiment, because the current model shares a lot with the latter.

## 7.2 Model with Stronger Present Bias

In the next experiment, I change the temptation (short-term) discount factor $\beta$ from its baseline value of 0.70 to 0.6667. This change is equivalent to raising the temptation discount rate from the baseline value of 40 percent annually to 50 percent. The third panel of Table 7 shows the results. Basically, the effects of introducing the BAPCPA are the same as in the baseline.

## 7.3 Model with Information Frictions

The fourth panel of Table 7 shows the results from the model in which, due to some information frictions, credit card companies cannot tailor the bond prices for different types of agents and instead offer the same bond price schedules for agents of the same age and with the same amount of debt. In other words, agents with different preference types ($j$) and persistent income shocks ($p$) are pooled.[25] Athreya et al. (2012) argue that an economy where credit card companies cannot distinguish borrower types and thus pool the borrowers when pricing loans better replicates credit conditions in the U.S. economy in the 1980s.

Due to a technical issue, however, I assume that credit card companies use the unconditional distribution of different types, instead of the actual distribution for a given $(i, a')$.[26] Still, the model with information frictions implies that the number of defaults declines (from 0.84 percent to 0.66), while the debt-to-income ratio increases from 9.0 percent to 11.1 percent, due to the BAPCPA. The size of the welfare loss is 0.25 percent in consumption equivalent variation. All are similar to the baseline results.

## 7.4 Model with Bequest Motive

Next, I introduce a warm-glow bequest motive to the model. Since agents are assumed to live up to age $I$ with certainty, adding a bequest motive assumes that agents gain utility from leaving assets in age-$I$. As for the functional form, I follow De Nardi et al. (2010) and assume the following utility function from bequests:

$$\widetilde{u}(a) = \zeta_0 \frac{(\zeta_1 + a)^{1-\sigma}}{1 - \sigma}, \tag{21}$$

where $\zeta_0$ represents the strength of the bequest motive and $\zeta_1$ represents the curvature of the motive. When $\zeta_1$ is larger, the bequest becomes more luxurious goods. De Nardi et al. (2010) also use the

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[25] Since transitory income shocks $t$ and expenditure shocks $x$ are i.i.d. and thus do not contain any information about the probability of default in the next period, they do not affect the bond prices.

[26] It turned out that an equilibrium cannot be obtained consistently, since agents of good types try to escape from the pooling and enjoy lower interest rates on loans.

same curvature parameter $\sigma$ as in the period utility function. In calibrating the parameters, I use the estimates of De Nardi et al. (2010) and convert them into the parameter values in the current model.[27] The fourth panel of Table 7 shows the results. Basically, the results from the model with bequest motives are the same as in the baseline model.

## 7.5    Alternative Parameter Values

The remaining twelve panels of Table 7 show the results from the models, in each of which one parameter value is changed from the baseline value. For each case, I recalibrate the model parameters $\delta$ and $\eta$, leaving all the other parameters unchanged. In the sixth panel (labeled "$\psi_e = 0.55$, $\psi_p = 0$" in Table 7), I assume that the amount of the social security benefits is the same across all agents, and it is $\psi_e = 0.55$ of the average labor income of the economy. In the seventh panel ("$\psi_e = 0$, $\psi_p = 0.55$"), I assume the opposite: the amount of social security benefits does not depend on the average labor income and instead depends entirely on the realization of the persistent income shock in the last working age. Therefore, the amount of social security benefits differs across agents, depending on the last realization of $p$, more than in the baseline model. In the eighth panel ("$\sigma = 3.0$"), I set the coefficient of relative risk aversion to 3.0 instead of the baseline value of 2.0. In the ninth panel ("$r = 0.034$"), I follow Livshits et al. (2010) and assume that the annual risk-free real interest rate is 3.4 percent instead of the baseline value of 2 percent. I also set the transaction cost of loans at 2.6 percent instead of 6 percent, again following Livshits et al. (2010). This higher interest rate is consistent with the real interest rate in the late 1990s. In the tenth panel ("$\lambda = 0.20$"), I assume that, on average, a defaulter can have a history of the past default erased five years after default. Although credit card companies can retain the history of past default for 10 years, some studies such as Han and Li (2011) find that households can use unsecured credit again less than 10 years after filing for bankruptcy.[28] In the eleventh panel ("Immediate Recovery"), I assume that agents could recover from a bad credit history immediately after a bankruptcy filing with probability $\lambda$. In the baseline model, to the contrary, I assume that agents have a bad credit history with certainty in the period right after filing for bankruptcy. In the twelfth panel ("Proportional $\xi$"), I assume that the cost of filing for bankruptcy is proportional to earnings instead of a fixed cost. I set the parameter representing the proportion ($\xi$) such that the default cost is the same for agents with mean earnings. In the thirteenth panel ("half $\pi^x$"), I assume that the probabilities of experiencing expenditure shocks are half of their baseline probabilities. Since the total number of defaults is the same as in the baseline model, this change yields a relatively larger proportion of bankruptcies due to bad income shocks rather than expenditure shocks. In the fourteenth panel ("Smaller $x$"), I assume that the size of the expenditure shocks is two-thirds of their baseline values. In the fifteenth panel ("$\rho_p = 0.93$"), the persistence parameter of persistent income shocks $p$ is lowered from the baseline value of 0.95 to 0.93. In the sixteenth panel ("$\sigma_p^2 = 0.0275$"), the variance of the innovation of the persistent income shocks is raised from its baseline value of 0.025 to 0.0275. In the last panel ("$\sigma_t^2 = 0.075$"), variance of transitory income shocks $t$ is raised from its baseline value of 0.05 to 0.075.

The findings from the sensitivity analysis are summarized below:

1. With the exception of changing the risk aversion parameter $\sigma$, the main results are found to be robust to changing parameter values. The BAPCPA reduces the number of bankruptcies,

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[27] See Appendix D of the working paper version of De Nardi et al. (2010) for details on converting their estimated parameter values into the current model.

[28] I thank a referee for pointing out this study.

but at the same time increases the total amount of loans. The overall welfare effect is negative because many agents who are forced to default with expenditure shocks suffer from a higher default cost or the means-testing requirement. The potentially large welfare gain from lower loan interest rates is dominated because not many agents without temptation borrow, and agents with temptation, who could suffer from overborrowing, do not value lower interest rates as much.

2. When agents are more risk averse (eighth panel of Table 7), they have a significantly weaker incentive to default at the expense of low consumption in the filing period and a larger volatility of consumption while being excluded from consumer credit markets. Therefore, the garnishment parameter is calibrated to be significantly lower than in the baseline ($\eta = 0.0593$ instead of the baseline value of 0.3064; see Appendix D.2). Therefore, the welfare cost of the higher default cost, or the binding means-testing requirement when an expenditure shock forces an agent to default, is lower. On the other hand, improved consumption smoothing due to lower loan interest rates, which is induced by the stronger commitment not to default under the BAPCPA, generates a significantly higher welfare gain than in the baseline model. Both agents with and without temptation benefit significantly from the BAPCPA.

3. Proportional default cost (twelfth panel) implies a smaller welfare loss from introducing the BAPCPA. This is because lower-income filers suffer less when default costs are proportional to their income.

4. The welfare loss due to the BAPCPA is smaller in cases where the probabilities or the sizes of expenditure shocks are smaller (thirteenth and fourteenth panels). In those cases, more agents default without expenditure shocks. Therefore, the negative welfare effect of the higher default cost, or the binding means-testing requirement when agents are forced to default due to expenditure shocks, is weaker than in the baseline model. Indeed, in those cases, welfare for agents without temptation improves, albeit slightly.

# 8 Conclusion

In this paper, I develop a novel model in which some agents suffer from temptation to consume now and others have self-control against such temptation. I use the model to evaluate the recent bankruptcy law reform, which was implemented to discourage abusers of the debtor-friendly bankruptcy law from filing for bankruptcy. The model indicates that, although the bankruptcy law reform achieves what it is intended for — a reduction in the number of bankruptcies by about 60 percent — the overall welfare effect is negative, when those who file for bankruptcy are forced into it due to some unexpected large expenditures or temptation. There is a welfare gain from improved consumption smoothing, when agents default less under the reform and thus enjoy lower default risk premia when borrowing. However, agents without temptation tend to borrow less frequently, which makes lower default premia not appealing to them, while agents with temptation suffer overborrowing at the same time. Although it is hard to eliminate from the data the effects of the Great Recession, which happened soon after the reform, the model's implications for the bankruptcy reform are generally consistent with the data. I also find that the proportion of agents with and without temptation is important when investigating the optimal design of the bankruptcy law. If the majority of agents are not subject to temptation, a stronger punishment for default and subsequently lower default premia generate a large gain in social welfare. On the other hand,

when the majority of agents suffer overborrowing, most of the gain from lower default premia is eliminated.

Let me conclude by pointing out three promising directions for future research. First, more micro data should be used to improve estimates of parameter values for different preference types and their distribution. In particular, although I assume no correlation between preference types and other individual characteristics, such as income, these are likely to be correlated. Second, carefully calibrated macroeconomic models developed in this paper could be applied to analyze other types of policies. Finally, the analysis in the current paper can be expanded to investigate the optimal design of bankruptcy law in a less restricted policy space. Interaction with the tax system is an important thing to consider.

# Appendix (Not for Publication)

Appendix A includes details about the data used to construct Figure 1. Appendix B contains additional details about calibration. Appendix C presents the computation algorithm of the baseline model. Finally, Appendix D includes a description of the alternative models, in which only agents with or without temptation inhabit.

## A  Data Appendix

This appendix contains the detailed description of the data used to construct Figure 1. The four panels in Figure 1 show the number of bankruptcies, the debt-to-income ratio, the charge-off rate, and the average interest rate. The number of bankruptcies is computed by dividing the number of total consumer bankruptcy filings and the Chapter 7 bankruptcy filings by the number of households in respective years. The data on bankruptcy filings are obtained from the U.S. Courts. The total number of households is from the U.S. Bureau of the Census.[29] The number of bankruptcy filings can be considered as the upper bound because multiple persons in a single household could file for bankruptcy simultaneously. The debt-to-income ratio is computed by dividing the balance of the revolving credit by disposable personal income. The former is constructed by the Federal Reserve Board (FRB, G.19). The revolving credit differs from unsecured credit in the sense that the revolving credit does not capture nonauto nonrevolving credit. However, after constructing the corrected measure of unsecured credit, Livshits et al. (2010) find that the gap between the two measures has been shrinking (see Figure 3 of their paper). Disposable personal income is obtained from the Bureau of Economic Analysis (BEA). The charge-off rate for all credit card loans is obtained from the FRB (G.19). The average interest rate on credit card loans is also obtained from the FRB (G.19). This is an account-weighted average.
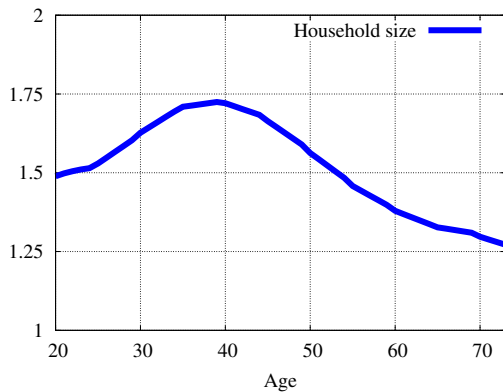
## B  Calibration Appendix



Figure 7: Household Size in Family Equivalence Scale



Figure 8: Average Life-Cycle Profile of Labor Productivity

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[29] Since the number of households in 2014 is not yet available, I extrapolate the number in 2013 using the growth rate of the number of households between 2012 and 2013.

# C  Computational Appendix

I describe below the computational algorithm to solve the steady-state equilibrium of the baseline model inhabited by agents with and without temptation.

**Algorithm 1 (computation algorithm for solving steady-state equilibrium)**

1. *Obtain the optimal value function $V(j, i, h, p, t, x, a)$ and the optimal decision rules $g_h(j, i, h, p, t, x, a)$ and $g_a(j, i, h, p, t, x, a)$ by solving the optimization problem backwards.*

   (a) *Start from the problem of age-I agents.*

   (b) *If $i = I$, set $V(j, i+1, h, p, t, x, a) = 0$ for all $(j, h, p, t, x, a)$. In the case $i < I$, $V(j, i+1, h, p, t, x, a)$ is already obtained in the previous step.*

   (c) *If $i = I$, set $q(j, i, h, p, t, x, a') = 0$ for all $(j, h, p, t, x, a' < 0)$ and $q(j, i, h, p, t, x, a') = 1/(1+r)$ for all $(j, h, p, t, x, a' \geq 0)$. If $i < I$, $q(j, i, h, p, t, x, a')$ is already obtained in the previous step.*

   (d) *For agents with temptation, the temptation problem is solved first. In case of $h = 0$ (clean credit history), using the discount factor $d = \beta_j \delta_j$, and given $V(j, i+1, h, p, t, x, a)$ and $q(j, i, h, p, t, x, a')$, values conditional on non-defaulting and defaulting are obtained from Bellman equations (4) and (7). The optimal default decision $g_h(j, i, h, p, t, x, a)$ is characterized by equation (3). The optimal saving decision $g_a(j, i, h, p, t, x, a)$ is the one conditional on not defaulting if $g_h(j, i, h, p, t, x, a) = 0$ and is zero if $g_h(j, i, h, p, t, x, a) = 1$. The optimal value of the temptation problem $V^*(j, i, 0, p, t, x, a; \beta_j \delta_j)$ is obtained.*

   (e) *In case of $h = 1$ (bad credit history), using the discount factor $\beta_j \delta_j$, and given $V(j, i+1, h, p, t, x, a)$ and $q(j, i, h, p, t, x, a')$, the optimal default decision $g_h(j, i, h, p, t, x, a)$ and the optimal saving decision $g_a(j, i, h, p, t, x, a)$ are obtained from equations (9) and (7). Notice that there is no optimal default decision because only involuntary default is allowed. The optimal value of the temptation problem $V^*(j, i, 1, p, t, x, a; \beta_j \delta_j)$ is obtained.*

   (f) *Solve the self-control problem. In general, this step requires solving equation (11). However, this step becomes trivial because, by assumption of $\gamma = \infty$, an agent completely succumbs to temptation. Formally, this step only requires updating the value function using equation (12).*

   (g) *Once the optimal default decision rule for age-i agents is obtained, the loan price for age-$i-1$, $q(j, i-1, h, p, t, x, a')$, can be computed using equation (14).*

   (h) *If $i > 1$, go back to step (b) and solve the problem of age-$i-1$ agents. If $i = 1$ (initial age), this step is over.*

2. *Using the obtained optimal decision rules $g_h(j, i, h, p, t, x, a)$ and $g_a(j, i, h, p, t, x, a)$, simulate the model forward, starting from the type distribution of age-1 agents.*

   (a) *Set the type distribution for the newborns, which is exogenously given. In particular, all newborns have $i = 1$, $h = 1$ (no default history), and $a = 0$ (no asset/debt). The initial distributions of $j$, $p$, $t$, and $x$ are also exogenously given.*

   (b) *Update the type distribution using the stochastic process for $(p, t, x)$ and the optimal decision rules $g_h(j, i, h, p, t, x, a)$ and $g_a(j, i, h, p, t, x, a)$.*

*(c) Keep updating until age I (last age).*

3. *Once the type distribution of agents is obtained, aggregate data can be computed by aggregating the individual data.*

# D   Description of the Alternative Models

## D.1   Models Containing Only Agents with or without Temptation

In the alternative model with only agents without temptation, by definition $J = 1$ (number of preference types is one) and $\phi_1 = 1.0$ (all agents are type-1). Since the agents do not have temptation, $\beta_1 = 1.0$ or $\gamma_1 = 0.0$. All the other parameters, except for self-control (or long-term) discount factor $\delta_1$ and the garnishment rate parameter $\eta$, are kept at the same values as in the baseline model economy. $\delta_1$ and $\eta$ are calibrated to match the same set of targets as the baseline model economy: namely, the measure of defaulters (0.84 percent annually) and the aggregate debt-to-income ratio (9.0 percent). The calibration procedure yields $\delta_1 = 0.9073$ and $\eta = 0.2688$. $\delta_1$ is lower than the typical value used in the literature because the parameter is used mainly to match the large amount of debt. $\eta$ is calibrated to be lower than the baseline because agents without temptation default less than agents with temptation, and thus a weaker punishment for defaulting is needed to match the same number of defaults.

The other alternative model is the one with only agents with temptation. Again, by definition, $J = 1$ and $\phi_1 = 1.0$. Since agents are assumed to completely succumb to temptation, $\gamma_1$ is set to be infinity. I use the same temptation (short-term) discount factor as in the baseline model, i.e., $\beta_1 = 0.7$. Again, $\delta_1$ and $\eta$ are calibrated to match the number of defaults and the aggregate debt-to-income ratio. In the end, all model economies are observationally identical in terms of the statistics targeted. The calibration procedure yields $\delta_1 = 0.9544$ and $\eta = 0.3200$, both slightly higher than the baseline values.

## D.2   Model for Sensitivity Analysis

Section 7 contains descriptions of each of the alternative models used for sensitivity analysis. Notice that endogenously calibrated parameters $\delta$ and $\eta$ are recalibrated for all economies so that all the models yield the same proportion of agents filing for bankruptcy (0.84 percent) and the same aggregate debt-to-income ratio (9.0 percent). Table 8 summarizes the calibrated values of $(\delta, \eta)$ in all alternative model economies studied in Section 7. Other than the alternative model with $\sigma = 3$, calibrated parameter values are similar to their respective values in the baseline model.

**Table 8: Calibration of Models for Sensitivity Analysis**

|     |                              | $\delta$ | $\eta$ | Remarks |
|-----|------------------------------|----------|--------|---------|
| 1.  | Baseline Model               | 0.9544   | 0.3064 | – |
| 2.  | Naive Agents                 | 0.9648   | 0.3490 | Agents do not consider temptation in the future. |
| 3.  | $\beta = 0.6667$             | 0.9614   | 0.3286 | Temptation discount rate is 50% instead of 40%. |
| 4.  | Information Frictions         | 0.9558   | 0.3426 | Loan rate only depends on $(i, a')$ |
| 5.  | Bequest Motive               | 0.9543   | 0.3064 | With warm-glow bequest motive. |
| 6.  | $\psi_e = 0.55, \psi_p = 0$  | 0.9565   | 0.3106 | – |
| 7.  | $\psi_e = 0, \psi_p = 0.55$  | 0.9510   | 0.3053 | – |
| 8.  | $\sigma = 3.0$               | 0.8430   | 0.0593 | – |
| 9.  | $r = 0.034$                  | 0.9533   | 0.2803 | $r = 0.0344$ and $\iota = 0.0256$ as in Livshits et al. (2010). |
| 10. | $\lambda = 0.20$             | 0.9573   | 0.3341 | – |
| 11. | Immediate Recovery           | 0.9545   | 0.3168 | History of defaulting immediately erased with prob $\lambda$. |
| 12. | Proportional $\xi$           | 0.9540   | 0.3228 | Default cost is proportional to earnings. |
| 13. | Half $\pi^x$                 | 0.9364   | 0.2191 | $\pi_1^x = 0.01184$ and $\pi_2^x = 0.00077$. |
| 14. | Smaller $x$                  | 0.9466   | 0.3020 | $x_1 = 0.2640$ and $x_2 = 0.8218$. |
| 15. | $\rho_p = 0.93$              | 0.9590   | 0.2805 | – |
| 16. | $\sigma_p^2 = 0.0275$        | 0.9515   | 0.3179 | – |
| 17. | $\sigma_t^2 = 0.075$         | 0.9467   | 0.3009 | – |

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