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**AND MONETARY POLICY ON CREDIT AND INFLATION**

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# Dichotomy Between Macroprudential Policy and Monetary Policy on Credit and Inflation\*

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## Abstract

This paper examines the different effects of macroprudential policy and monetary policy on credit and inflation using a simple New Keynesian model with credit. In this model, macroprudential policy is effective in stabilizing credit but has a limited effect on inflation. Monetary policy with an interest rate rule stabilizes inflation, but this rule is ‘too blunt’ an instrument to stabilize credit. The determinacy of the model requires the interest rate’s response to inflation to be greater than one for one and independent of macroprudential policy. That is, the ‘Taylor principle’ applies to monetary policy. This dichotomy between macroprudential policy and monetary policy arises because each policy is designed to differently affect the saving and borrowing decisions of households.

**JEL Classification:** E44, E52, E59

**Keywords:** Macroprudential policy, monetary policy, inflation, credit, New Keynesian model.

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## 1. INTRODUCTION

This paper studies the impact of macroprudential policy and monetary policy on the dynamics of inflation and credit in a New Keynesian model. Macroprudential policy refers to a set of regulatory instruments, mainly imposed on financial institutions, to ex-ante limit the buildup of financial systemic risk. For example, countercyclical capital requirements on financial intermediaries and loan-to-value ratio regulations on lending contracts are considered macroprudential policy instruments. The aim of this paper is to examine how credit, a key variable for financial stability, and inflation are influenced differently by macroprudential and monetary policies.

One important issue regarding macroprudential policy is its coordination with monetary policy, including a question as to how we should use these two policies to jointly achieve financial stability and existing mandates of monetary policy such as inflation and output gap stability. A recent debate between [Woodford \(2012\)](#) and [Svensson \(2012\)](#) formalizes this concern. Using his credit model ([Curdia and Woodford \(2010\)](#)), Woodford argues that financial stability is an important policy objective, since the quadratic policy loss function consists of inflation and the output gap as well as the marginal utility gap between borrowers and savers that widens in financial crises. Even with macroprudential instruments, he argues, monetary policy should be used in response to credit conditions as long as macroprudential policy cannot provide a complete solution for financial stability. On the other hand, Svensson, based on the same policy loss function, favors separating monetary policy and macroprudential policy. Monetary policy should, in Svensson's view, be used exclusively for inflation stabilization. He wants to use macroprudential policy for stabilizing financial markets. The reason is that macroprudential policy instruments have greater effects on leverage than monetary policy.

This paper evaluates the effects of macroprudential and monetary policies on inflation and credit, using a simple New Keynesian model with a saver-borrower distinction. It shows how this class of models characterizes the effects of the two policies, a key piece of information for the policy coordination problem above. The following components of the model set up grounds for analyzing the dynamics of interest. There are two types of households, saving and borrowing households, distinguished by their time preference parameters. The difference in time preference parameters generates credit flows between these households, which are channeled through a financial intermediary. The model features price rigidity in the production sector, allowing monetary policy to affect real variables. Monetary policy, by setting the nominal interest rate, influences the intertemporal consumption decision of both savers and borrowers. The interest rate spread between the saving rate and the borrowing rate is increasing in the size of lending, reflecting the solvency risk as in [Bernanke, Gertler,](#)

and Gilchrist (1999) ('BGG' hereafter). In this paper, macroprudential policy is a counter-cyclical regulation imposed on the financial intermediary to constrain lending activity and has a direct influence only on the borrowers' decision. Based on this model, I analyze the effects of the two policies on the determinacy of the system and the dynamics of variables around the local linear approximation of the equilibrium.

This paper shows that there is a dichotomy between the two policies on inflation and credit. Monetary policy is effective in stabilizing inflation but too blunt an instrument in stabilizing credit. Macroprudential policy is effective in stabilizing credit but plays a limited role in inflation dynamics. The 'Taylor principle' still applies, since the equilibrium is indeterminate unless the interest rate responds to inflation by more than one, independent of macroprudential policy. Thus, the presence of macroprudential policy in this paper does not affect the role of monetary policy and its willingness to stabilize inflation for inflation determination. This sharp separation between monetary policy and macroprudential policy arises from the different effects these policies have on saving and borrowing decisions. Suh (2012) shows that in a medium size New Keynesian model, the optimal policy combination also separates the aims of monetary and macroprudential policies. The paper shows that there are welfare gains from inflation and credit stabilization in the model, since inflation stabilization reduces the inefficiency from price rigidity and credit stabilization increases credit supply in the presence of BGG-type financial frictions. In this paper, I show that my earlier result stems from the way monetary and macroprudential policies are characterized to influence savers and borrowers in this New Keynesian model with a saver-borrower distinction and BGG-type solvency risk.

## 2. INTUITION: DIFFERENCE BETWEEN MONETARY AND MACROPRUDENTIAL POLICY

To highlight the main intuition of the paper, this section shows the impact of monetary and macroprudential policies in an abstract credit market. This credit market simplifies the optimization decision of the original New Keynesian model of the later sections into supply and demand for credit. Assume that both households consume goods and hold real money balances that increase utility.<sup>[1]</sup> Saving households can save by holding loans that are lent to borrowing households, and credit is defined by this lending between households. Figure 1 shows the effects of monetary and macroprudential policy on a credit market. The real credit supply ( $S$ ) is savers' willingness to save, which characterizes their intertemporal

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<sup>1</sup>This environment in which agents have money in the utility function is often used in the monetary economics literature. See Woodford (2003).

consumption-saving decision. It is increasing in the real interest rate  $R$ , because when  $R$  rises, savers are willing to forgo current consumption to save for future consumption. Credit demand ( $D$ ) is borrowers' willingness to draw credit, which characterizes their intertemporal consumption-borrowing decision. It is decreasing in the interest rate, because when  $R$  rises, borrowers are less willing to borrow since the debt repayment burden in future periods is high. The initial equilibrium is given by  $(L^*, R^*)$ .

Suppose for some reason the policymaker decides to reduce credit in the economy by using monetary policy that raises the interest rate. This policy action is shown in panel (a) of figure 1. The central bank can raise the interest rate from  $R^*$  to  $R'$  by reducing the money supply. However, this money contraction simultaneously affects savers and borrowers in the economy. It increases the marginal value of money and makes it a relatively more attractive asset than loans for savers. As a result, the credit supply reduces to the left ( $S \rightarrow S'$ ). However, it increases credit demand to the right ( $D \rightarrow D'$ ) as borrowers are more willing to borrow to hold more money. Consequently, the equilibrium credit ( $L'$ ) can be anywhere between  $L^a$  and  $L^b$ , depending on the extent of the shift of the supply and (or) demand curve given monetary contraction. In other words, monetary contraction can either increase or decrease credit in equilibrium. On the other hand, panel (b) shows credit stabilization using macroprudential policy. Macroprudential policy targets only borrowers. By imposing regulations, it restricts the borrowers' demand for credit given the interest rate. As a result, it shifts credit demand to the left ( $D \rightarrow D''$ ) while keeping the supply curve unchanged. The level of credit in equilibrium reduces to  $L''$  and the interest rate decreases to  $R''$ . Macroprudential policy results in a contraction of credit in equilibrium. This example shows that macroprudential policy is a better tool to restrict credit, because this policy only influences the demand for credit. On the other hand, monetary policy affects both supply and demand for credit.

### 3. MODEL

This section presents a New Keynesian model to explore the roles of monetary policy and macroprudential policy in inflation and credit stabilization. There are saving and borrowing households of the same population in the economy, who are distinguished by their time preferences as in [Iacoviello \(2005\)](#). Borrowing households are more impatient about their future consumption, so they eventually have to borrow in the steady state. More patient households become savers in the economy. The optimization problem of the representative

saving household is

$$\max_{C_s, N_s, L^s} E_0 \sum_{t=0}^{\infty} (\beta \epsilon_t^d)^t \left( \frac{1}{1 - \sigma_C} C_{t,s}^{1 - \sigma_C} - \frac{\varphi}{1 + \sigma_N} N_{t,s}^{1 + \sigma_N} \right) \quad s.t. \quad C_{t,s} + \frac{L_t^s}{P_t} \leq R_{t-1}^s \frac{L_{t-1}^s}{P_t} + w_t N_{t,s} + Div_{t,s}, \quad (1)$$

where  $C_s$  is consumption, and  $N_s$  is labor of saving households. In the budget constraint,  $L^s$  stands for savings that are lent to borrowing households,  $R^s$  is the nominal saving interest rate,  $P$  is the price level of final consumption goods, and  $Div$  is the real dividend from the intermediate goods production firms and financial intermediaries. There is a preference shock, denoted by  $\epsilon_t^d$ , that affects the intertemporal consumption allocation decision.

Borrowing households have a smaller future discount factor than saving households ( $\beta^b < \beta$ ). The optimization problem of the representative borrowing household is given by

$$\max_{C_b, N_b, L^b} E_0 \sum_{t=0}^{\infty} (\beta_b \epsilon_t^d)^t \left( \frac{1}{1 - \sigma_C} C_{t,b}^{1 - \sigma_C} - \frac{\varphi}{1 + \sigma_N} N_{t,b}^{1 + \sigma_N} \right) \quad s.t. \quad C_{t,b} + R_{t-1}^b \frac{L_{t-1}^b}{P_t} \leq w_t N_{t,b} + \frac{L_t^b}{P_t}, \quad (2)$$

where  $C_b$  and  $N_b$  are consumption and labor of borrowing households,  $L^b$  is the debt of the borrower, and  $R^b$  is the nominal borrowing interest rate. In equilibrium, the amount of borrowers' debt equals savings ( $L_t^b = L_t^s$ ). Credit ( $L_t$ ) in this paper is defined by this lending between savers and borrowers.

The production sector follows a simple New Keynesian setup. Final consumption goods are obtained by aggregating intermediate goods using the Dixit-Stiglitz aggregator ([Dixit and Stiglitz \(1977\)](#)). There is a continuum of intermediate goods producers denoted by  $i$  ( $i \in [0, 1]$ ). Their production technology is linear in the labor input,

$$Y_{i,t} = a_t N_{i,t}, \quad N_{i,t} = N_{i,t,s} + N_{i,t,b}. \quad (3)$$

In equation (3),  $a_t$  is a productivity shock that is given exogenously. Intermediate goods producers face monopolistic competition that induces a markup in price setting behavior. Also there exists price rigidity in their optimization problem as in [Calvo \(1983\)](#). The aggregate resource constraint of the economy follows from combining the budget constraints of both

saving and borrowing households,

$$Y_t = C_{t,s} + C_{t,b}. \quad (4)$$

Hereby I describe the dynamics of the linearly approximated equilibrium around the non-stochastic steady state using log-linear expressions. I show that the equilibrium condition reduces to a linear difference equation in consumption, inflation and credit. This sets up grounds on which it is possible to analyze how each policy affects inflation and credit through its influence on the intertemporal consumption, saving, and borrowing decisions of different households.

It is assumed that a financial intermediary exists to channel credit between borrowers and savers. For the reason why it should exist, I simply assume that it has an advantage in monitoring loans compared to saving households. The financial intermediary has monopolistic power, and there is a markup in financial intermediation that is increasing in the quantity of real credit ( $l_t = L_t/P_t$ ). This markup is needed for there to be a spread between the borrowing rate and the saving rate and is assumed to be increasing in credit, reflecting a higher solvency risk, as in the BGG financial accelerator mechanism. This interest rate spread is assumed to be

$$\hat{R}_t^b - \hat{R}_t^s = \omega \hat{l}_t, \omega > 0. \quad (5)$$

Monetary policy is given by an (saving) interest rate rule reacting to inflation, where parameter  $\phi_\pi$  is the parameter that represents the interest rate's reaction to inflation,

$$\hat{R}_t^s = \phi_\pi \hat{\pi}_t. \quad (6)$$

Macroprudential policy imposes a restriction that affects the borrowing decision through the borrowing rate,

$$\hat{R}_t^b = \hat{R}_t^s + \omega \hat{l}_t + \phi_L \hat{l}_t. \quad (7)$$

In equation (7),  $\phi_L$  is the macroprudential reaction to credit. A positive  $\phi_L$  implies that the

macroprudential authority imposes a restriction on borrowers, in a way that is countercyclical to the credit movement. I rule out the non-plausible cases with  $\phi_L < 0$ , because in this case the policymaker encourages borrowing when credit expands and discourages it when credit shrinks. By the way, macroprudential policy is often implemented with quantity restrictions on the financial intermediary's balance sheets or lending contracts, for example, capital requirement regulation or loan-to-value regulation. The policy design in (7) can also capture the effects of quantity restrictions when financial intermediaries facing quantity restrictions choose to raise the interest rate rather than rationing credit. Another way to rationalize this is to assume that the financial intermediary uses its monopolistic power to charge a high lending rate to make up for its loss from macroprudential regulation. From (6) and (7), it is clear that while monetary policy affects both the saving and the borrowing interest rate through  $\hat{R}^s$ , macroprudential policy only affects the borrowing interest rate through  $\hat{R}^b$ .

The first-order conditions with respect to savings and debt in the saving and borrowing household optimization problems give us the Euler equations,

$$\begin{aligned}\sigma_C \hat{C}_{t,s} &= E_t \sigma_C \hat{C}_{t+1,s} - (\hat{R}_t^s - E_t \hat{\pi}_{t+1}) + \hat{u}_t^d \\ &= E_t \sigma_C \hat{C}_{t+1,s} - (\phi_\pi \hat{\pi}_t - E_t \hat{\pi}_{t+1}) + \hat{u}_t^d \quad \text{where} \quad \hat{u}_t^d \equiv \hat{\epsilon}_t^d - E_t \hat{\epsilon}_{t+1}^d,\end{aligned}\tag{8}$$

and

$$\begin{aligned}\sigma_C \hat{C}_{t,b} &= E_t \sigma_C \hat{C}_{t+1,b} - (\hat{R}_t^b - E_t \hat{\pi}_{t+1}) + \hat{u}_t^d \\ &= E_t \sigma_C \hat{C}_{t+1,b} - (\phi_\pi \hat{\pi}_t + (\omega + \phi_L) \hat{l}_t - E_t \hat{\pi}_{t+1}) + \hat{u}_t^d.\end{aligned}\tag{9}$$

Equations (8) and (9) are separately presented rather than combined into one representative consumption Euler equation. This separate presentation is useful to see how monetary policy and macroprudential policy differently influence intertemporal saving and borrowing decisions.

Assuming a standard Calvo pricing mechanism in the production sector, the optimal price-setting behavior of intermediate good producers induces the New Keynesian Phillips curve, which represents the aggregate supply mechanism,

$$\begin{aligned}\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa \hat{m}c_t \\ &= \beta E_t \hat{\pi}_{t+1} + \kappa [(\gamma_s \hat{C}_{t,s} + \gamma_b \hat{C}_{t,b}) - (1 + \sigma_N) \hat{a}_t].\end{aligned}\tag{10}$$

In equation (10),  $\hat{m}c$  is the real marginal cost,  $\gamma_s \equiv (n^s \sigma_C + c^s \sigma_N)$ ,  $\gamma_b \equiv (n^b \sigma_C + c^b \sigma_N)$ , and  $c^s$ ,

$c^b$ ,  $n^s$ ,  $n^b$  stand for the steady-state share of borrowers and savers in aggregate consumption and labor supply ( $c^b \equiv C_b/C$ ,  $c^s \equiv C_s/C$ ,  $n^s \equiv N_s/N$ ,  $n^b \equiv N_b/N$ ). The right-hand side of (10) shows that the consumption of each household affects the marginal cost and inflation of the economy through labor supply. Lastly, the law of motion for credit is drawn by log-linearizing the budget constraint of borrowing households, then substituting out wage and labor supply,<sup>[2]</sup>

$$\begin{aligned} \frac{L}{Y}\hat{l}_t - \frac{R^b L}{Y}(\phi_\pi \hat{\pi}_{t-1} + (\omega + \phi_L)\hat{l}_{t-1} - \hat{\pi}_t + \hat{l}_{t-1}) &= c^b \hat{C}_{t,b} - \frac{wN_b}{Y}(\hat{w}_t + \hat{N}_{t,b}) \\ &= \chi_b \hat{C}_{t,b} + \chi_s \hat{C}_{t,s} + \chi_a \hat{a}_t. \end{aligned} \quad (11)$$

$$\chi_b = c^b - \frac{wN_b}{Y} \left[ \left(1 + \frac{1}{\sigma_N}\right) (n^b \sigma_C + c^b \sigma_N) - \frac{\sigma_C}{\sigma_N} \right], \chi_s = -\frac{wN_b}{Y} \left(1 + \frac{1}{\sigma_N}\right) (n^s \sigma_C + c^s \sigma_N), \chi_a = \frac{wN_b}{Y} (\sigma_N + 1).$$

The right-hand side of equation (11) is the difference between borrowers' consumption and labor income. Therefore, (11) shows that the increase in debt net of interest payments depends on borrowers' consumption net of labor income. Equations (8)-(11) form a set of linear difference equations in  $(\hat{C}_s, \hat{C}_b, \hat{\pi}, \hat{l})$ , which characterizes a local equilibrium dynamics around the steady state,

$$G_0 \begin{pmatrix} E_t \hat{C}_{t+1,s} \\ E_t \hat{C}_{t+1,b} \\ E_t \hat{\pi}_{t+1} \\ E_t \hat{l}_{t+1} \end{pmatrix} = G_1 \begin{pmatrix} \hat{C}_{t,s} \\ \hat{C}_{t,b} \\ \hat{\pi}_t \\ \hat{l}_t \end{pmatrix} + X \begin{pmatrix} \hat{u}_{t+1}^s \\ \hat{u}_{t+1}^d \end{pmatrix}. \quad (12)$$

#### 4. MODEL DETERMINACY

The determinacy of the system in (12) depends on generalized eigenvalues of  $G \equiv G_0^{-1} \cdot G_1$ . For the equilibrium solution to exist and to be unique and bounded, three out of four eigenvalues of  $G$  are required to have modulus greater than one, since there are three unpredetermined variables ( $C_b, C_s, \pi$ ) and one state variable ( $L$ ) (Blanchard and Kahn (1980)). The matrix  $G$

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<sup>2</sup>A detailed derivation of (10) and (11) is shown in Appendix A.2.

is given by

$$G = \begin{pmatrix} 1 + \frac{\kappa}{\beta}\gamma_s & \frac{\kappa}{\beta}\gamma_b & \phi_\pi - 1/\beta & 0 \\ \frac{\kappa}{\beta}\gamma_s & 1 + \frac{\kappa}{\beta}\gamma_b & \phi_\pi - 1/\beta & \omega + \phi_L \\ -\frac{\kappa}{\beta}\gamma_s & -\frac{\kappa}{\beta}\gamma_b & 1/\beta & 0 \\ \Xi_1 & \Xi_2 & \Xi_3 & \Xi_4 \end{pmatrix}. \quad (13)$$

$$\Xi_1 = \frac{\chi_s}{L/Y} + \frac{\kappa}{\beta}\gamma_s \left( R^b + \frac{\chi_b + \chi_s}{L/Y} \right), \quad \Xi_2 = \frac{\chi_b}{L/Y} + \frac{\kappa}{\beta}\gamma_b \left( R^b + \frac{\chi_b + \chi_s}{L/Y} \right),$$

$$\Xi_3 = \left( \phi_\pi - \frac{1}{\beta} \right) \left( R^b + \frac{\chi_b + \chi_s}{L/Y} \right), \quad \Xi_4 = R^b + (\omega + \phi_L) \left( R^b + \frac{\chi_b}{L/Y} \right).$$

The eigenvalues of the G matrix are the solutions of the characteristic equation  $P(\lambda) \equiv |G - \lambda I| = 0$ . Defining  $\gamma = \gamma_s + \gamma_b$ ,  $P$  is given as

$$\begin{aligned} P(\lambda) &= (\Xi_4 - \lambda)(1 - \lambda)[(1 - \lambda)(1/\beta - \lambda) + (\phi_\pi - \lambda)\frac{\kappa}{\beta}\gamma] + \\ &\quad (\omega + \phi_L)[(1 - \lambda)\{- (1/\beta - \lambda)\Xi_2 - \frac{\kappa}{\beta}\gamma_b\Xi_3\} + (\phi_\pi - \lambda)(\chi_s\gamma_b - \chi_b\gamma_s)\frac{\kappa}{\beta L/Y}]. \\ &= \lambda^4 + A_3\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0. \end{aligned} \quad (14)$$

$$A_3 = -(\Xi_4 + 2 + 1/\beta) - \kappa\gamma/\beta, \quad A_2 = [\Xi_4(2 + \frac{1}{\beta}) + 1 + \frac{2}{\beta}] + \frac{\kappa\gamma}{\beta}(\Xi_4 + \phi_\pi + 1) - \Xi_2(\omega + \phi_L),$$

$$\begin{aligned} A_1 &= -[(\Xi_4 + 1)/\beta + \Xi_4(1 + 1/\beta)] - [\phi_\pi(\Xi_4 + 1) + \Xi_4]\frac{\kappa}{\beta}\gamma \\ &\quad + (\omega + \phi_L)[\Xi_3\frac{\kappa}{\beta}\gamma_b + (1/\beta + 1)\Xi_2 - (\chi_s\gamma_b - \chi_b\gamma_s)\frac{\kappa}{\beta L/Y}], \end{aligned}$$

$$A_0 = \Xi_4(1/\beta + \phi_\pi\frac{\kappa}{\beta}\gamma) + (\omega + \phi_L)[-\frac{\kappa}{\beta}\gamma_b\Xi_3 - \frac{\Xi_2}{\beta} + \phi_\pi(\chi_s\gamma_b - \chi_b\gamma_s)\frac{\kappa}{\beta L/Y}].$$

The proposition below suggests how the determinacy of this equilibrium system is affected by policy parameters.

**PROPOSITION** Consider nonnegative values of  $\phi_\pi$  and  $\phi_L$ .<sup>3</sup> Suppose (i)  $\chi_s\gamma_b - \chi_b\gamma_s < 0$ , (ii)  $R^b + \frac{\chi_b}{L/Y} > 0$ . Then  $\phi_\pi > 1$  is a necessary condition for the equation  $P(\lambda) = 0$  to have exactly one (real) root with a radius smaller than 1. Provided (i),(ii) and (iii)  $P(A_0) > 0$ ,  $\phi_\pi > 1$  is also a sufficient condition.

**PROOF:** See Appendix A.3 for the proof.

*Q.E.D.*

In this proposition, conditions (i)-(iii) are satisfied in a general range of parameter values. In particular, from the definition of parameters  $\chi_s$  and  $\chi_b$ , it is understandable that conditions (i) and (ii) hold in general.  $\chi_s$  and  $\chi_b$  stand for the increase in debt for a unit increase in savers' or borrowers' consumption that appears in the borrowers' budget constraint.  $\chi_s$  is always negative, for the increase in savers' consumption reduces the borrowers' debt as borrowers' wage income increases.  $\chi_b$  can be positive or negative but small in terms of its absolute value, for the increase in debt induced by the higher consumption of borrowers is offset by the larger wage income they receive.

Since conditions (i) and (ii) are independent of both monetary ( $\phi_\pi$ ) and macroprudential ( $\phi_L$ ) policy parameters, the necessity part of the proposition does not depend on macroprudential policy. It tells us that even with macroprudential policy, a determinacy result in a simple New Keynesian model known as the Taylor principle still holds. The model does not have a unique bounded equilibrium unless  $\phi_\pi$  is greater than one, regardless of the value of  $\phi_L$ . This result shows that the addition of macroprudential policy in this paper does not affect the role of monetary policy and its willingness to stabilize inflation for inflation determination.

Figure 2 presents the determinacy space of the New Keynesian model as  $\phi_\pi$  and  $\phi_L$  vary, given the calibration shown in table 1. I let  $\phi_\pi \in (0, 10)$  and  $\phi_L \in (0, 10)$  on a grid spaced by 0.01. On this grid, the determinacy region is constrained by  $\phi_\pi$  but not by  $\phi_L$ . Figure 2 clearly shows not only the importance of satisfying the Taylor principle but also the irrelevance of the macroprudential policy parameter  $\phi_L$  for the determinacy.

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<sup>3</sup>There is a determinacy region where  $\phi_\pi \ll 0$  or  $\phi_L < 0$ , but I rule out these cases as being of little economic interest.

## 5. HOW DO MONETARY AND MACROPRUDENTIAL POLICY AFFECT INFLATION AND CREDIT?

### 5.1. FORWARD-LOOKING EXPRESSIONS: INFLATION AND CREDIT

This section derives forward-looking expressions for inflation and credit to study how their dynamics are affected by monetary and macroprudential policy parameters. To begin, define *weighted output* as  $\tilde{\gamma}\hat{Y}_t \equiv \gamma_s\hat{C}_{t,s} + \gamma_b\hat{C}_{t,b}$ . Then by combining equations (8)-(10), we can write  $\hat{\pi}_t$  as

$$\begin{aligned}
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa [(\gamma_s\hat{C}_{t,s} + \gamma_b\hat{C}_{t,b}) - (1 + \sigma_N)\hat{a}_t] \\
&= \beta E_t \hat{\pi}_{t+1} + \kappa \left[ \tilde{\gamma}\hat{Y}_{t+1} + \frac{\gamma}{\sigma_C}(-\phi_\pi\hat{\pi}_t + E_t\hat{\pi}_{t+1} + \hat{u}_t^d) - \frac{\gamma_b}{\sigma_C}(\omega + \phi_L)\hat{l}_t - (1 + \sigma_N)\hat{a}_t \right] \\
&= \frac{1}{1 + \phi_\pi\kappa\gamma/\sigma_C} E_t \left[ \underbrace{\sum_{j=0}^{\infty} \left( \frac{\beta + \kappa\gamma/\sigma_C}{1 + \phi_\pi\kappa\gamma/\sigma_C} \right)^j \kappa \{ \tilde{\gamma}\hat{Y}_{t+j+1} + \frac{\gamma}{\sigma_C}\hat{u}_{t+j}^d - (1 + \sigma_N)\hat{a}_{t+j} \}}_{(a)} \right. \\
&\quad \left. - \underbrace{\sum_{j=0}^{\infty} \left( \frac{\beta + \kappa\gamma/\sigma_C}{1 + \phi_\pi\kappa\gamma/\sigma_C} \right)^j \frac{\kappa\gamma_b}{\sigma_C}(\omega + \phi_L)\hat{l}_{t+j}}_{(b)} \right]. \tag{15}
\end{aligned}$$

According to (15), inflation is determined by the present value of future expected weighted outputs and exogenous shocks (enclosed by (a)), and financial frictions and macroprudential responses (enclosed by (b)). If the model is a simple New Keynesian model without a saver-borrower distinction, inflation would be completely explained by the future path of the output gap, which corresponds to part (a) in this model. It is observed that the monetary policy parameter ( $\phi_\pi$ ) affects inflation because of the term  $\frac{\beta + \kappa\gamma}{1 + \phi_\pi\kappa\gamma}$ , which serves to discount future variables. For example, a high value of  $\phi_\pi$  makes inflation less sensitive to the expected path of the output gap or credit. On the other hand, the effect of the macroprudential policy parameter ( $\phi_L$ ) on inflation is smaller, only captured by terms in (b) provided the effect of macroprudential policy on future  $\hat{Y}$  is limited. Moreover, (b) is scaled by  $\gamma_b$ , a weighted share of borrowers in the economy, implying that the effect of macroprudential policy on inflation is even smaller because it is a restriction applied only to borrowing households.

Similarly, a forward-looking expression for  $\hat{l}_t$  can be derived from (11),

$$\hat{l}_t = E_t \left[ \sum_{j=0}^{\infty} \left( \frac{1}{R^b(1+\omega+\phi_L)} \right)^j \left\{ \underbrace{-\frac{\phi_\pi \hat{\pi}_{t+j} - \hat{\pi}_{t+j+1}}{1+\omega+\phi_L}}_{(a)} - \underbrace{\frac{\chi_b \hat{C}_{t+j+1,b} + \chi_s \hat{C}_{t+j+1,s} + \chi_a \hat{a}_{t+j+1}}{(1+\omega+\phi_L)(R^b L/Y)}}_{(b)} \right\} \right]. \quad (16)$$

In equation (16), the current level of credit depends negatively on the expected future path of the real interest rate (enclosed in (a)) and consumption net of labor income (enclosed in (b)). The macroprudential policy parameter ( $\phi_L$ ) appears in the discount factor  $\frac{1}{R^b(1+\omega+\phi_L)}$ , where it influences  $\hat{l}_t$ . This explains the role  $\phi_L$  plays in stabilizing credit dynamics.

## 5.2. DYNAMICS

This section shows the difference between macroprudential policy and monetary policy with respect to inflation and credit dynamics. Specifically, I show the responses of inflation and credit to a preference shock ( $\hat{u}_t^d$ ) and a productivity shock ( $\hat{a}_t$ ). The response of savers' and borrowers' consumption is also presented separately rather than combined, to display how they are affected differently by macroprudential and monetary policy. To calculate these impulse responses, I calibrate preference, production, and exogenous process parameters as in table 1. In the steady state, the saving interest rate is 3.1%, the borrowing rate is 4.3% and the household debt to GDP ratio is 76.7%. Borrowing households supply 53% of total labor and consume 47% of total output. Monetary policy's reaction to inflation is given by  $\phi_\pi = 1.5$ .

In figure 3, the macroprudential reaction to credit varies at  $\phi_L = 0, 0.05, 0.15$ . Figure 3(a) shows that a positive preference shock increases borrowers' and savers' consumption and inflation. The level of credit falls as the increase in borrowers' income outweighs the increase in their consumption. A positive productivity shock increases borrowers' and savers' consumption and decreases inflation as figure 3(b) displays. In this case, the level of credit increases. These figures also reveal that a higher  $\phi_L$  drives credit dynamics to stabilize more quickly. The reason is that higher  $\phi_L$  discounts the impact of the shocks on credit more heavily, as explained by equation (16). However, inflation dynamics do not significantly vary across different values of  $\phi_L$ .

In figures 4-7, I compare the effects of macroprudential policy's and monetary policy's reaction to credit. There are three different policy specifications. In the 'Baseline' case, there is no macroprudential policy ( $\phi_L = 0$ ) and monetary policy reacts only to inflation. In the 'Macroprudential Policy' case, macroprudential policy reacts to credit ( $\phi_L = 0.1$ ) and mone-

tary policy reacts only to inflation. In the ‘Monetary Policy’ case, there is no macroprudential policy and monetary policy reacts to both inflation and credit. Monetary policy’s reaction to credit is modeled by replacing  $\phi_\pi \hat{\pi}_t$  with  $\phi_\pi \hat{\pi}_t + \phi_L \hat{L}_t$  on the right-hand side of the savers’ Euler equation (8) ( $\phi_L = 0.1$ ). Figures 4(a) and (b) show that macroprudential policy is more effective at stabilizing credit than monetary policy. Macroprudential policy stabilizes the dynamics of credit while leaving the dynamics of inflation virtually unchanged. Monetary policy’s reaction to credit does not stabilize the response of credit well, and it induces more volatile inflation dynamics. This result shows that an interest rate rule is ‘too blunt’ an instrument to be used for credit stabilization in this New Keynesian model.

In figure 5, I compare the two policies under a stronger monetary response to inflation ( $\phi_\pi = 3$ ). Thus, the monetary authority has a stronger commitment to stabilizing inflation. Similarly to figure 4, macroprudential policy stabilizes credit dynamics while leaving inflation dynamics unchanged. A monetary policy’s reaction to credit stabilizes credit, but the effect is weaker compared to macroprudential policy. Inflation dynamics are again more volatile when using monetary policy but the impact is mitigated by stronger  $\phi_\pi$ .

In figure 6, borrowing households are assumed to be less risk averse ( $\sigma_{c,s} = 2$ ,  $\sigma_{c,b} = 1$ ). In figure 7, the debt/GDP ratio is doubled to 153.4%. In both figures, macroprudential policy can stabilize credit more quickly while affecting inflation dynamics less. On the other hand, monetary policy’s reaction to credit amplifies the dynamics of credit and inflation, increasing the volatility of both variables.

To summarize, macroprudential policy is more effective than a monetary policy rule reacting to credit at controlling credit in this New Keynesian model. Not only is this type of monetary policy ineffective in controlling credit, but it also increases inflation volatility. Using a medium size New Keynesian model in which there is a welfare gain from credit stabilization, [Suh \(2012\)](#) suggests that the optimal policy combination is to separate monetary and macroprudential policy targets because the separation more efficiently achieves inflation and credit stabilization. The results in this paper support the notion that this conclusion by Suh is ‘implied by design’ given how monetary and macroprudential policies are characterized. That is, as long as the model allows monetary policy to affect both savers and borrowers and macroprudential policy to affect only borrowers, and given there is a welfare gain from credit stabilization, this separation optimality outcome is likely to follow.

## 6. CONCLUSION

This paper studies the effects of monetary and macroprudential policies in a simple New Keynesian model with a borrower-saver distinction. The model simplifies the struc-

ture of many macro-financial dynamic stochastic general equilibrium (DSGE) models that have been used for studying monetary or macroprudential policy. (For example, [Curdia and Woodford \(2010\)](#), and [Kannan, Rabanal, and Scott \(2012\)](#)). The key result of this paper is the relative advantages (and disadvantages) of macroprudential policy and monetary policy as instruments for stabilizing credit and inflation. This result stems from the characteristic that the two policies influence saving and borrowing decisions in this New Keynesian model with BGG-type solvency risk. This suggests that there is a need to explore macroprudential policies across a wider array of models. I leave this for future research.

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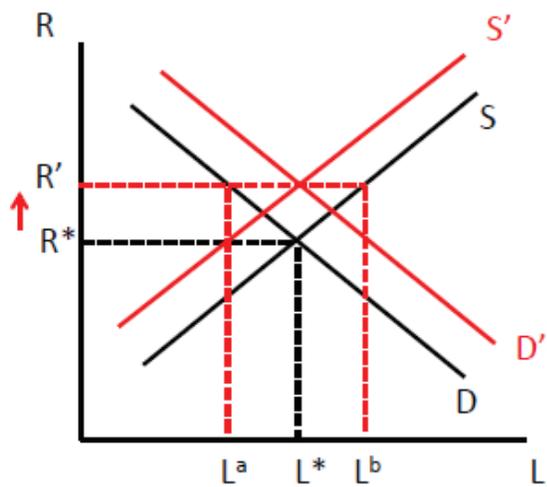
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## APPENDIX

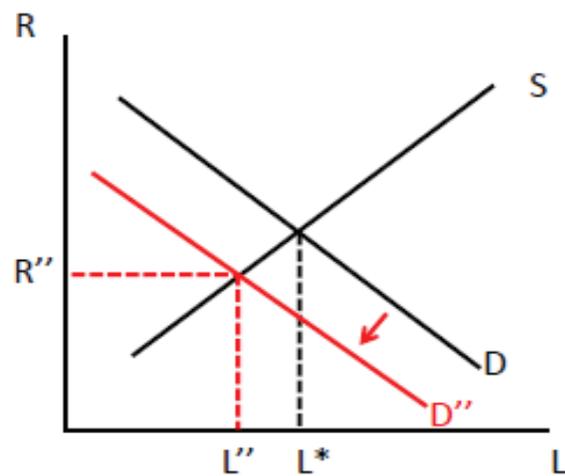
### A.1. TABLES AND FIGURES

Table 1: Parameter Calibrations

Parameters	Value	Description
$\beta$	0.9922	Discount factor, savers
$\beta_b$	0.9893	Discount factor, borrowers
$\sigma_c$	1	Intertemporal elasticity of consumption
$\sigma_N$	1	Inverse labor elasticity
$\omega$	0.02	Unit markup in financial intermediation
$\kappa$	0.17	Coefficient on the marginal cost in NK Phillips curve
$\rho_{u^d}$	0.80	Persistence, demand shock
$\rho_a$	0.80	Persistence, productivity shock
$L/Y$	0.767	Steady-state debt/GDP ratio (quarterly)
$mk_Y/Y$	0.09	Steady-state markup/GDP ratio, production
$mk_f/Y$	0.01	Steady-state markup/GDP ratio, financial intermediary

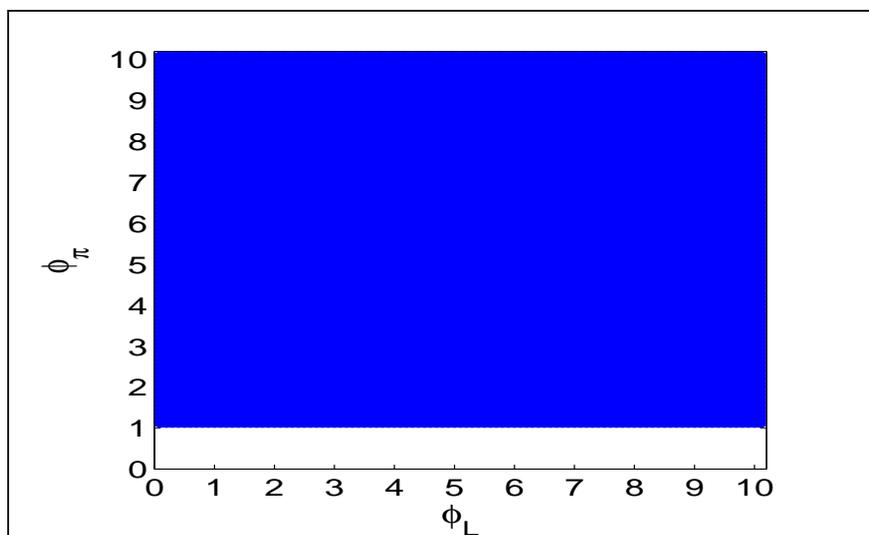


(a) Credit control using monetary policy



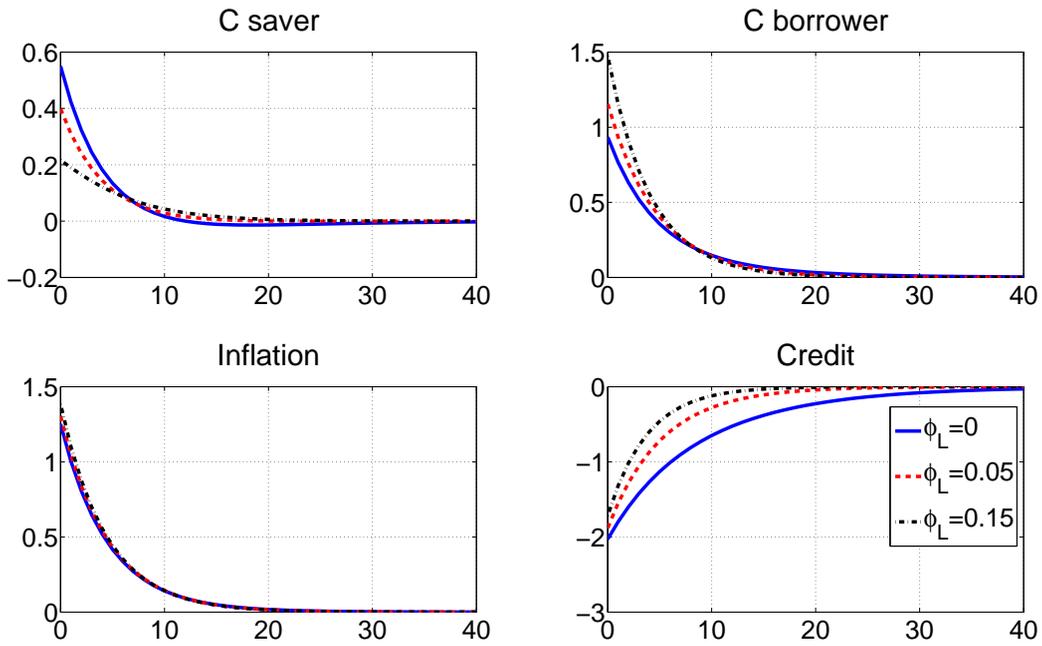
(b) Credit control using macroprudential policy

Figure 1: Equilibrium for private credit

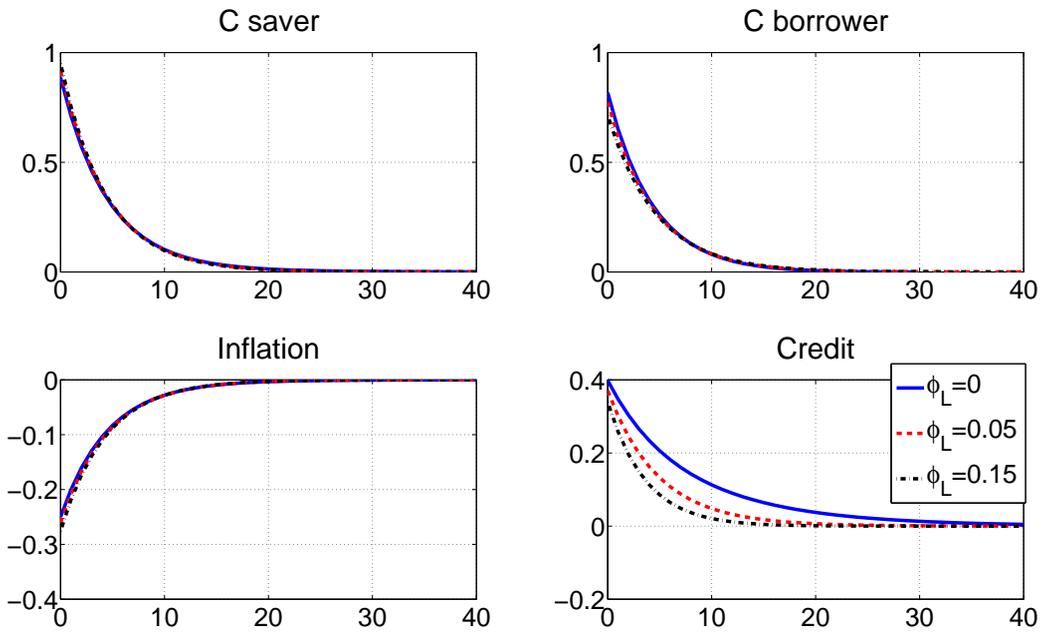


\*  $\phi_\pi$  : reaction of monetary policy to inflation.  $\phi_L$  : reaction of macroprudential policy to credit.

Figure 2: Determinacy (colored) and indeterminacy (white) region.

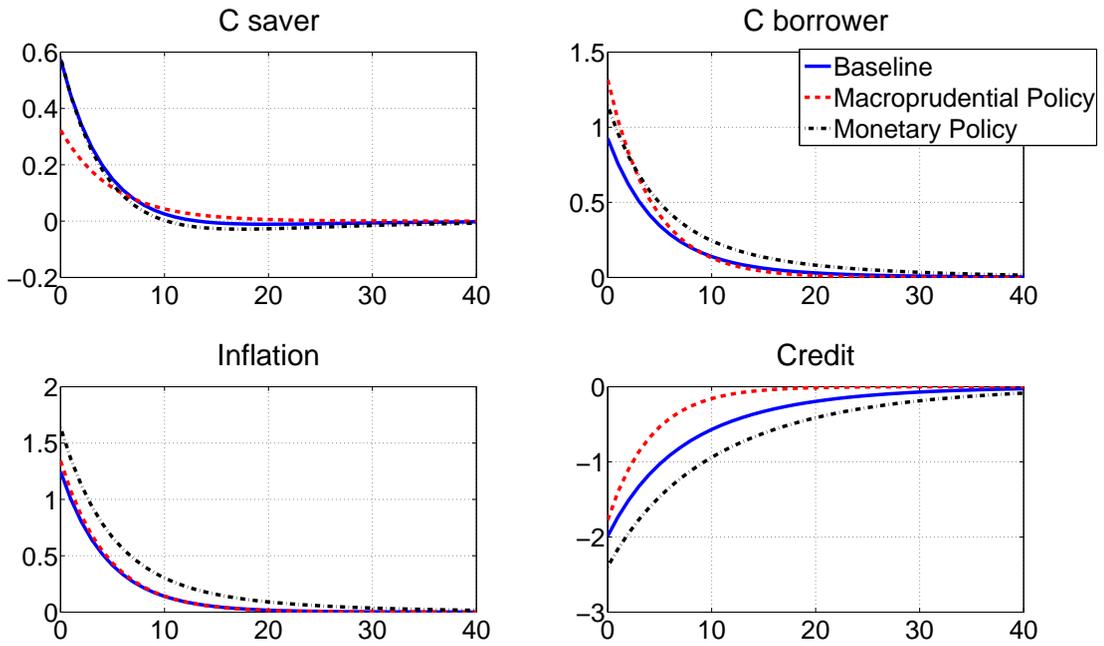


(a) Preference shock. Monetary policy reaction to inflation  $\phi_\pi = 1.5$ .

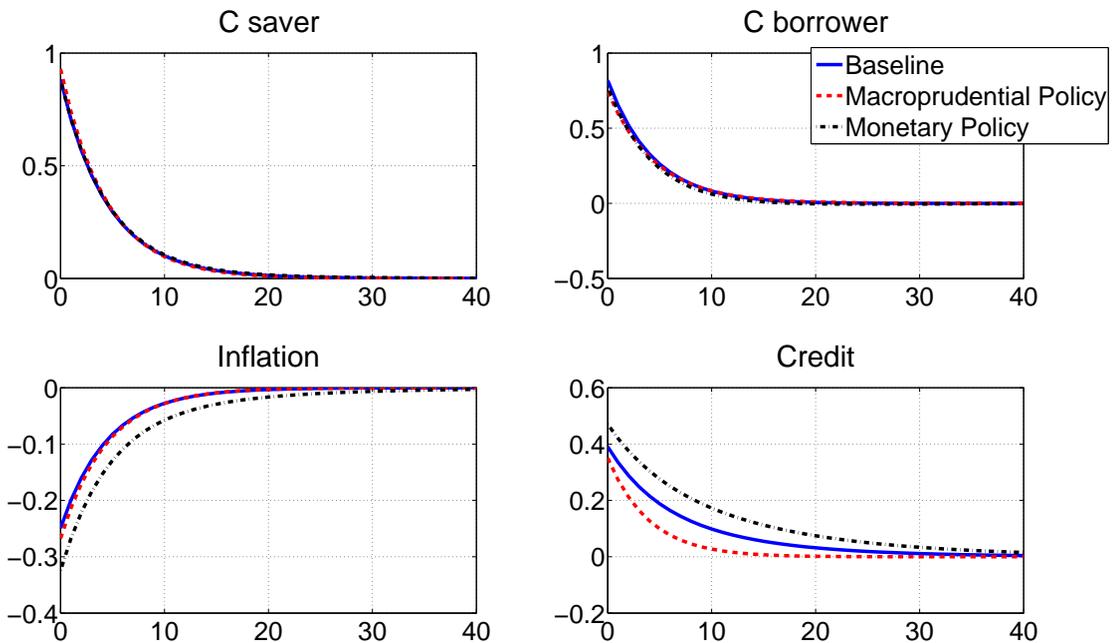


(b) Productivity shock. Monetary policy reaction to inflation  $\phi_\pi = 1.5$ .

Figure 3: The effect of macroprudential policy parameter ( $\phi_L$ ) to credit

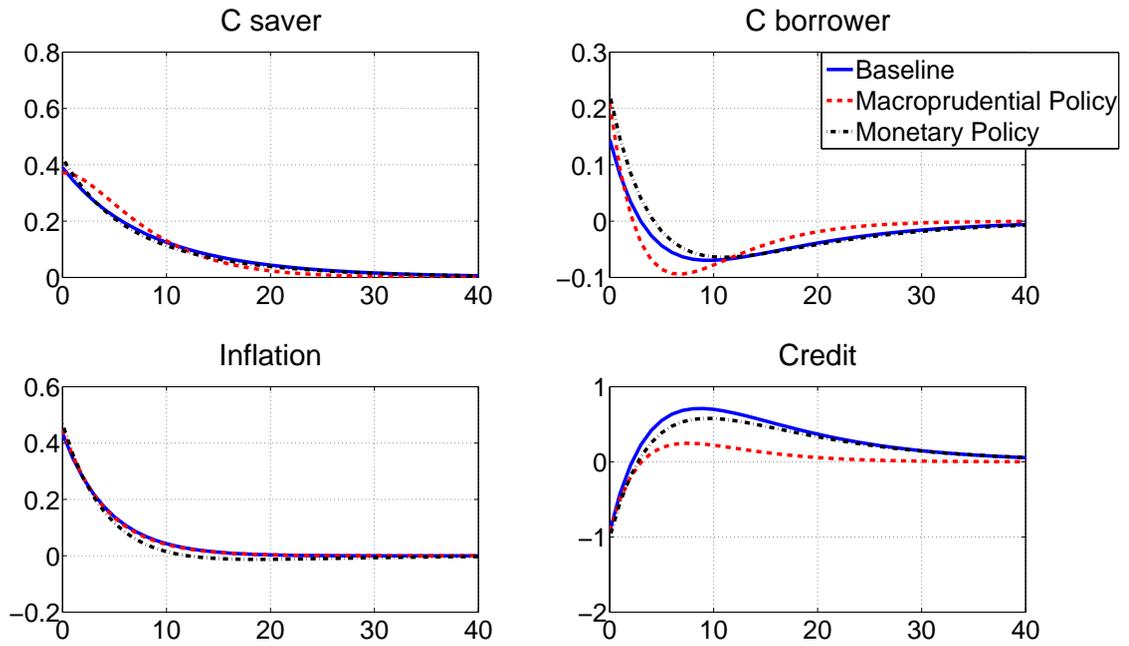


(a) Preference shock. Monetary policy reaction to inflation  $\phi_\pi = 1.5$ .

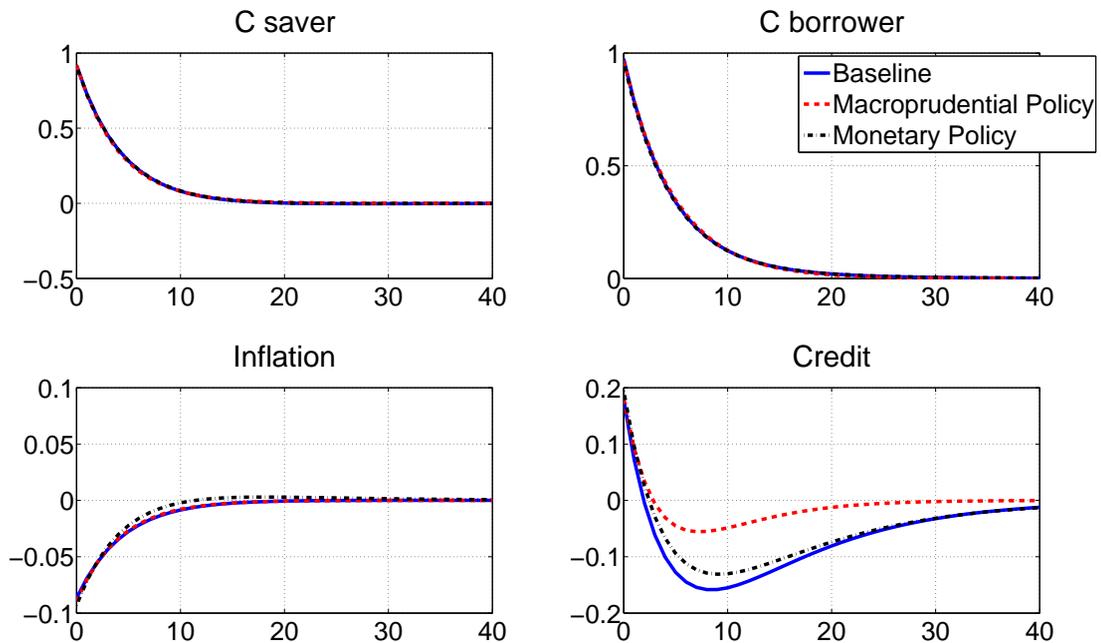


(b) Productivity shock. Monetary policy reaction to inflation  $\phi_\pi = 1.5$ .

Figure 4: Comparison between macroprudential policy and monetary policy reacting to credit

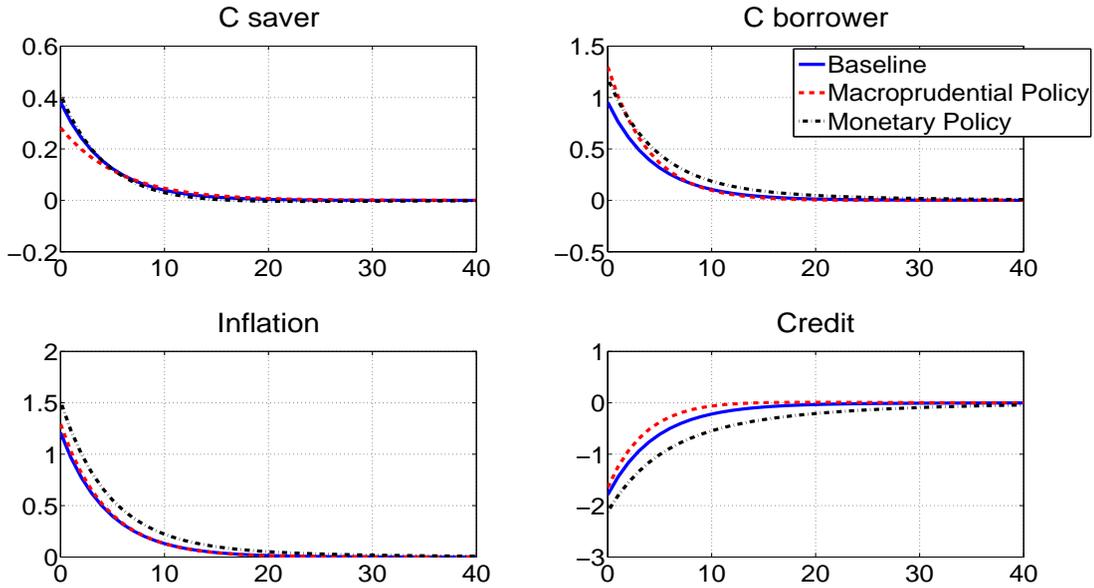


(a) Preference shock. Monetary policy reaction to inflation  $\phi_\pi = 3$ .

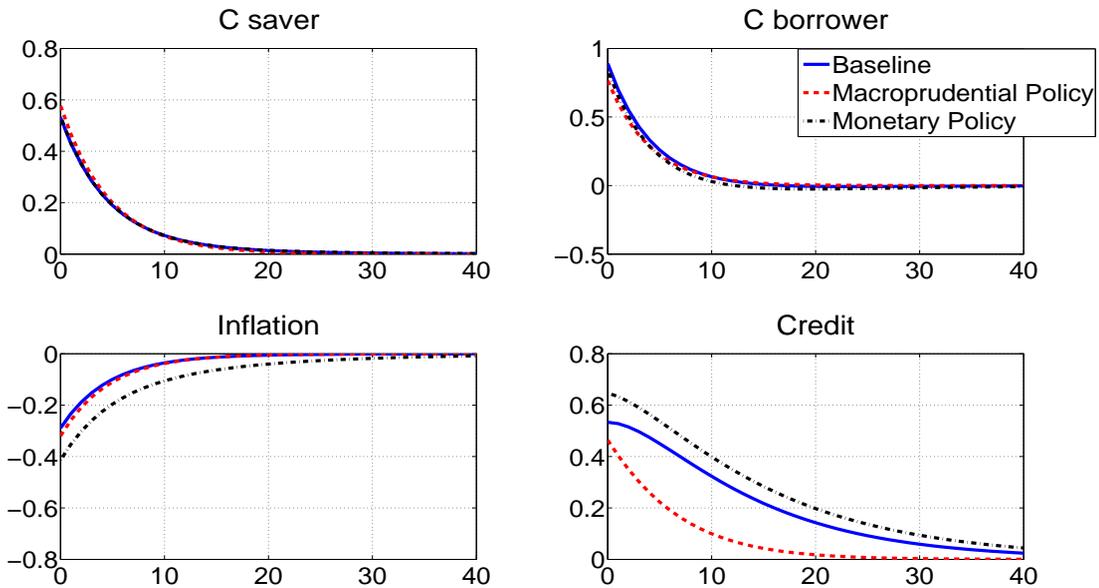


(b) Productivity shock. Monetary policy reaction to inflation  $\phi_\pi = 3$ .

Figure 5: Comparison between macroprudential policy and monetary policy reacting to credit

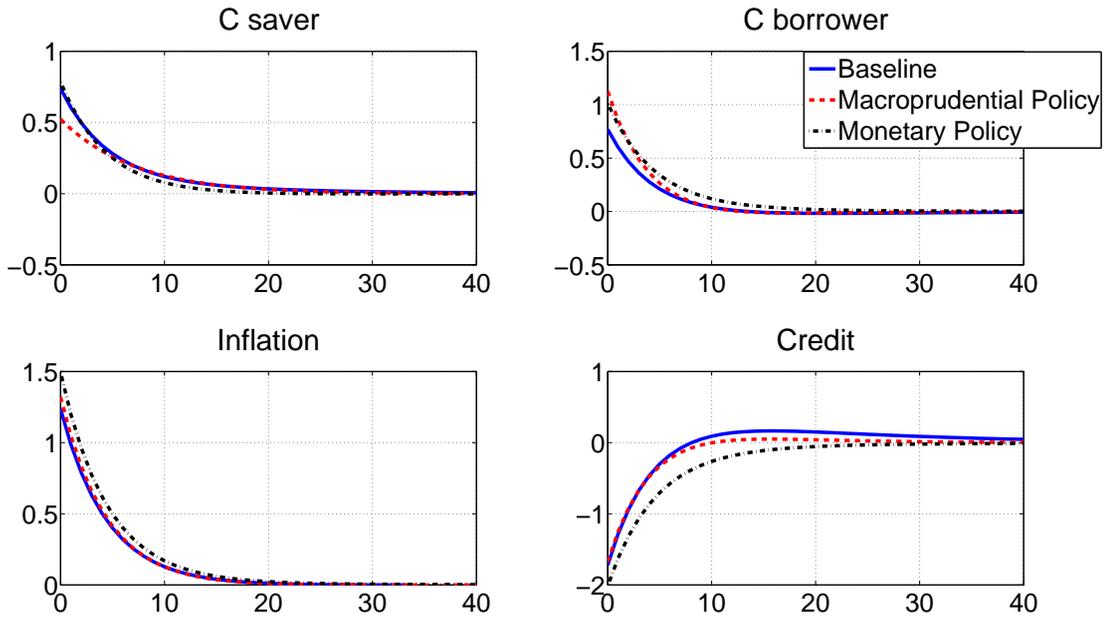


(a) Preference shock. Monetary policy reaction to inflation  $\phi_\pi = 1.5$ . Saving households have lower elasticity of intertemporal substitution ( $\sigma_{c,s} = 2, \sigma_{c,b} = 1$ ).

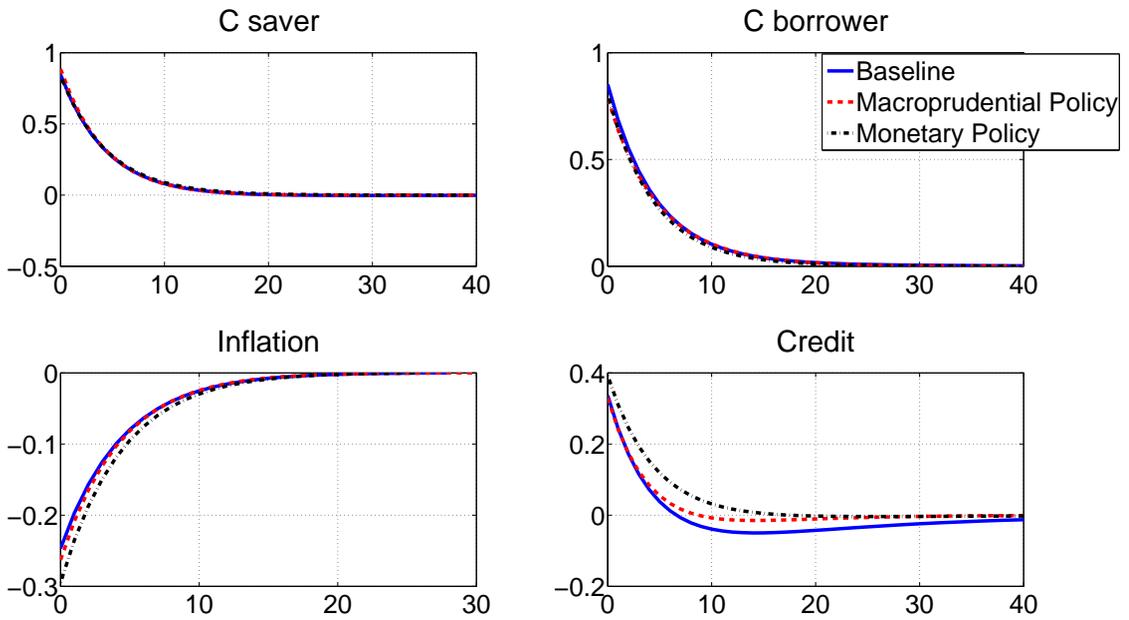


(b) Productivity shock. Monetary policy reaction to inflation  $\phi_\pi = 1.5$ . Saving households have lower elasticity of intertemporal substitution ( $\sigma_{c,s} = 2, \sigma_{c,b} = 1$ ).

Figure 6: Comparison between macroprudential policy and monetary policy reacting to credit



(a) Preference shock. Monetary policy reaction to inflation  $\phi_\pi = 1.5$ . Debt/GDP ratio is higher (153.4%).



(b) Productivity shock. Monetary policy reaction to inflation  $\phi_\pi = 1.5$ . Debt/GDP ratio is higher (153.4%).

Figure 7: Comparison between macroprudential policy and monetary policy reacting to credit

## A.2. DERIVATION OF THE LOG-LINEARIZED EQUILIBRIUM

Marginal cost  $\hat{m}c_t$  in equation (10) is obtained from

$$\begin{aligned}
\hat{m}c_t &= \hat{w}_t - \hat{m}\hat{p}n_t = (n^s + n^b)\hat{w}_t - \hat{m}\hat{p}n_t \\
&= (n^s\sigma_C\hat{C}_{t,s} + n^s\sigma_N\hat{N}_{t,s} + n^b\sigma_C\hat{C}_{t,b} + n^b\sigma_N\hat{N}_{t,b}) - (\hat{Y}_t - \hat{N}_t) \\
&= (n^s\sigma_C\hat{C}_{t,s} + n^b\sigma_C\hat{C}_{t,b} + \sigma_N\hat{N}_t) - (\hat{Y}_t - \hat{N}_t) \\
&= [(n^s\sigma_C + c^s\sigma_N)\hat{C}_{t,s} + (n^b\sigma_C + c^b\sigma_N)\hat{C}_{t,b}] - (1 + \sigma_N)\hat{a}_t.
\end{aligned} \tag{17}$$

The second line of (17) uses the first-order conditions of households with regard to their labor supply

$$\hat{w}_t = \sigma_C\hat{C}_{t,s} + \sigma_N\hat{N}_{t,s} = \sigma_C\hat{C}_{t,b} + \sigma_N\hat{N}_{t,b}, \tag{18}$$

and the fourth line of (17) uses the production technology and the aggregate resource constraint

$$\hat{N}_t = \hat{Y}_t - \hat{a}_t = c^s\hat{C}_{t,s} + c^b\hat{C}_{t,b} - \hat{a}_t. \tag{19}$$

The law of motion for credit (11) is derived by first log-linearizing the resource constraint of the borrowing households

$$\frac{L}{Y}\hat{l}_t - \frac{R^b L}{Y}(\hat{R}^b_{t-1} - \hat{\pi}_t + \hat{l}_{t-1}) = c^b\hat{C}_{t,b} - \frac{wN_b}{Y}(\hat{w}_t + \hat{N}_{t,b}), \tag{20}$$

then by substituting out  $\hat{w}_t$  and  $\hat{N}_{t,b}$  using labor market equilibrium conditions

$$\sigma_N\hat{N}_{t,b} = \hat{w}_t - \sigma_C\hat{C}_{t,b}, \tag{21}$$

$$\hat{w}_t = (n^s\sigma_C + c^s\sigma_N)\hat{C}_{t,s} + (n^b\sigma_C + c^b\sigma_N)\hat{C}_{t,b} - \sigma_N\hat{a}_t. \tag{22}$$

## A.3. PROOF OF PROPOSITION

**PROPOSITION** Consider nonnegative values of  $\phi_\pi$  and  $\phi_L$ . Suppose (i)  $\chi_s\gamma_b - \chi_b\gamma_s < 0$ , (ii)  $R^b + \frac{\chi_b}{L/Y} > 0$ . Then  $\phi_\pi > 1$  is a necessary condition for the equation  $P(\lambda) = 0$  to have exactly one (real) root with a radius smaller than 1. Provided (i), (ii) and (iii)  $P(A_0) > 0$ ,  $\phi_\pi > 1$  is also a sufficient condition.

PROOF:

1. Necessity. Suppose  $P(\lambda) = 0$  has exactly one real root with a radius smaller than 1. Then  $P(1) \cdot P(-1) < 0$ , since otherwise it has zero, two or four roots inside the unit circle. Next, to examine the signs of  $P(1)$  and  $P(-1)$ , calculate  $A_3 + A_1 = (P(1) - P(-1))/2$ .

$$\begin{aligned}
A_3 + A_1 &= (\Xi_4 + 1) \left[ -2\left(1 + \frac{1}{\beta}\right) - \frac{\kappa\gamma}{\beta}(\phi_\pi + 1) \right] \\
&\quad + (\omega + \phi_L) \left[ \phi_\pi \left( R^b + \frac{\chi_b + \chi_s}{L/Y} \right) \frac{\kappa}{\beta} \gamma_b + \frac{\kappa}{\beta} \left( \gamma_b R^b + \gamma \frac{\chi_b}{L/Y} \right) + \left(1 + \frac{1}{\beta}\right) \frac{\chi_b}{L/Y} \right] \\
&= (1 + R^b) \left[ -2\left(1 + \frac{1}{\beta}\right) - \frac{\kappa\gamma}{\beta}(\phi_\pi + 1) \right] \\
&\quad + (\omega + \phi_L) \left[ -\frac{\kappa\gamma_s}{\beta} R^b (1 + \phi_\pi) - \frac{\kappa\phi_\pi}{\beta} \left( \frac{\gamma_s \chi_b - \gamma_b \chi_s}{L/Y} \right) - \left(2R^b + \frac{\chi_b}{L/Y}\right) \left(1 + \frac{1}{\beta}\right) \right] \\
&< 0 \Leftrightarrow P(1) - P(-1) < 0.
\end{aligned} \tag{23}$$

Since  $P(1) < P(-1)$ , it follows that  $P(1) = (\omega + \phi_L)(\phi_\pi - 1)(\chi_s \gamma_b - \chi_b \gamma_s) \frac{\kappa}{\beta L/Y} < 0$ . By assumption (i),  $(\omega + \phi_L)(\chi_s \gamma_b - \chi_b \gamma_s) \frac{\kappa}{\beta L/Y} < 0$  and  $\phi_\pi > 1$ .

2. Sufficiency. Suppose (i)-(iii). First note  $P(0) > 1$ . To see this, write  $\beta P(0)$  as

$$\begin{aligned}
\beta P(0) &= \Xi_4(1 + \phi_\pi \kappa \gamma) + (\omega + \phi_L) \left[ -\kappa \gamma_b \Xi_3 - \Xi_2 + \phi_\pi (\chi_s \gamma_b - \chi_b \gamma_s) \frac{\kappa}{L/Y} \right] \\
&= R^b(1 + \phi_\pi \kappa \gamma) + (\omega + \phi_L) \left( R^b + \frac{\chi_b}{L/Y} \right) (1 + \phi_\pi \kappa \gamma) \\
&\quad + (\omega + \phi_L) \left[ -\kappa \gamma_b \phi_\pi \left( R^b + \frac{\chi_b + \chi_s}{L/Y} \right) - \frac{\chi_b}{L/Y} + \phi_\pi (\chi_s \gamma_b - \chi_b \gamma_s) \frac{\kappa}{L/Y} \right] \\
&= R^b(1 + \phi_\pi \kappa \gamma) \\
&\quad + (\omega + \phi_L) \left[ \left( R^b + \frac{\chi_b}{L/Y} \right) (1 + \phi_\pi \kappa \gamma) - \kappa \phi_\pi \left( R^b \gamma_b + \frac{\chi_b}{L/Y} \gamma \right) - \frac{\chi_b}{L/Y} \right] \\
&= R^b(1 + \phi_\pi \kappa \gamma) + (\omega + \phi_L) R^b(1 + \phi_\pi \kappa \gamma_s).
\end{aligned} \tag{24}$$

$$P(0) = \frac{R^b}{\beta} (1 + \phi_\pi \kappa \gamma) + (\omega + \phi_L) \frac{R^b}{\beta} (1 + \phi_\pi \kappa \gamma_s) > 1. \tag{25}$$

Next, we can show that  $P(-1) > 0$ .

$$\begin{aligned}
P(-1) &= (\Xi_4 + 1)2\{2(1 + \frac{1}{\beta}) + (\phi_\pi + 1)\frac{\kappa}{\beta}\gamma\} \\
&\quad + (\omega + \phi_L) \left[ 2\{-(1 + \frac{1}{\beta})\Xi_2 - \frac{\kappa}{\beta}\gamma_b\Xi_3\} + (\phi_\pi + 1)(\chi_s\gamma_b - \chi_b\gamma_s)\frac{\kappa}{\beta L/Y} \right] \\
&= \underbrace{(1 + R^b)2\{2(1 + \frac{1}{\beta}) + (\phi_\pi + 1)\frac{\kappa}{\beta}\gamma\}}_{(a)} \\
&\quad + \underbrace{(\omega + \phi_L)(R^b + \frac{\chi_b}{L/Y})2\{2(1 + \frac{1}{\beta}) + (\phi_\pi + 1)\frac{\kappa}{\beta}\gamma\}}_{(b)} \\
&\quad + \underbrace{(\omega + \phi_L) \left[ -2(1 + \frac{1}{\beta})\frac{\chi_b}{L/Y} - (1 + 1/\beta)\frac{\kappa}{\beta}\{\gamma_b(R^b + \frac{\chi_b + \chi_s}{L/Y})\} - (\phi_\pi + 1)\{\gamma_b R^b + \gamma\frac{\chi_b}{L/Y}\} \right]}_{(c)}.
\end{aligned} \tag{26}$$

In equation (26), (a) is clearly positive, and one can show that (b) + (c) > 0. Knowing  $P(0) = A_0 > 1$ ,  $P(-1) > 0$ , and provided  $P(A_0) > 0$ , a sufficient condition for  $P(\lambda) = 0$  to have exactly one real root with a radius smaller than 1 is  $P(1) < 0$ , which holds if and only if  $\phi_\pi > 1$ . First,  $P(0) > 0$ ,  $P(1) < 0$ ,  $P(A_0) > 0$  guarantees that there is at least one root between 0 and 1, and at least one root between 1 and  $A_0$ . Since  $P(-1) > 0$ , it is only possible that the other two roots both have radii greater than 1 or smaller than 1. Suppose the other two roots have radii smaller than 1. Then there is only one root with a radius greater than 1 (denoted by  $\lambda_1$ ) that satisfies  $1 < \lambda_1 < A_0$ . However, then the product of all roots ( $\lambda_1\lambda_2\lambda_3\lambda_4$ ) becomes smaller than  $A_0$ , a contradiction because  $\lambda_1\lambda_2\lambda_3\lambda_4 = A_0$ . Therefore, the radii of the other two roots must be greater than 1.

*Q.E.D.*