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INFORMATION AGGREGATION

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Dynamic Market Participation and Endogenous Information Aggregation*

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Abstract

This paper studies information aggregation in financial markets with recurrent investor exit and entry. I consider a dynamic general equilibrium model of asset trading with private information and collateral constraints. Investors differ in their aversion to Knightian uncertainty: When uncertainty is high, some investors exit the market. Since exiting investors' information is not fully revealed by prices, conditional return volatility and risk premia both increase. I use data on institutional investors' holdings of individual stocks to show that investor exits indeed move negatively with price informativeness. The model also implies that exit is more likely when wealth is more concentrated in the hands of less uncertainty-averse investors. The model thus predicts less informative prices toward the end of a long boom, as seen in the data. Moreover, economies with looser collateral constraints should see more volatility due to exit and partial revelation. Higher capital requirements can improve welfare by inducing more information revelation by prices.

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1. Introduction

Investors adjust their stock positions at both the intensive and extensive margins. For example, around 40% of changes in the stock positions of institutional investors are due to changes at the extensive margin.¹ Most standard rational expectations models with asymmetric information do not capture changes in investor participation in equilibrium. In these models, investors respond to signals by adjusting their asset positions only at the intensive margin. However, changes at the extensive margin can have important implications for information aggregation. When investors close out their positions and leave the market as opposed to simply reducing their asset positions, their private signals may not be fully reflected in equilibrium prices and thus will be lost. In this case, changes in participation play an important role in the ability of markets to aggregate information.

To study how changes in stock market participation affect information aggregation, I consider a dynamic asset market model that incorporates private signals, ambiguity aversion (or aversion to Knightian uncertainty), CRRA preferences, and borrowing constraints. In the model, investors receive private signals about the future payoff of risky assets. However, the interpretation of these signals is ambiguous. Specifically, potential investors are uncertain about the likelihood function of the true signal-generating process and evaluate probabilities according to the worst case scenario. If the uncertainty regarding the signal interpretation is high, investors may decide not to invest in risky assets. When some investors exit the stock market, a partially revealing equilibrium exists in which the private signals of these exited investors are not fully revealed. This leads to information loss and higher return volatility of the risky asset.

Why would ambiguity-averse investors exit the market at times of uncertainty? In the model, investors make portfolio decisions over purchasing a riskfree bond and a stock. They each receive an independent private signal from a finite state space about the next-period payout of the stock. They are uncertain about the correct likelihood function of the signal to use for updating their beliefs and instead update their posterior beliefs using a set of likelihood functions, where the size of the set reflects the degree of ambiguity. Ambiguity aversion is modeled as in Gilboa and Schmeidler (1989), where investors are averse to the worst case scenario. Specifically, when they hold a long position in the stock, investors pick

¹Using 13F filing data on institutional investors' stock positions, I can compute the changes in stock position (in dollars) of institutional investors for all stocks they hold every quarter. Then I compute how much those changes are due to opening new positions or closing out positions. The average is taken over all stocks and all time periods.

the likelihood function in the set that generates the most pessimistic belief about the payoff. For investors to buy the stock, its price must be low enough to be appealing even under the most pessimistic belief. Symmetrically, when they hold a short position in the stock, they pick the likelihood function that leads to the most optimistic belief about the payoffs, which is the worst case for a short position. For investors to short the stock, its price must be high enough to be appealing even under the most optimistic belief. Ambiguity aversion thus implies that there is a region of prices where investors choose zero stock. This region is wider for more ambiguous investors.

Why would non participation lead to partial revelation of information in equilibrium? In a standard rational expectations model, investors hold non-zero amounts of stock in equilibrium except for knife-edge cases. Given prices, their demand for the stock is responsive to the private signal received. A good signal about future payoffs of the stock leads to more demand for the stock, while a bad signal leads to less demand. In equilibrium, market clearing means that these changes in demand lead to equilibrium prices that are responsive to the signals that investors receive. Investors can therefore infer the private signals of other investors from the equilibrium prices. This leads to a fully revealing equilibrium in which prices reflect information from all signals, which has been shown to exist generally in the previous literature. With ambiguity aversion, however, an investor may stay out of the stock market under different realizations of her private signal, and so her demand for stock becomes non responsive to her private signal received. In equilibrium, the more ambiguity-averse investor A exits the stock market at times of high uncertainty while the less ambiguity-averse investor B stays. Thus, equilibrium prices may be the same under the different signal realizations of A , and B cannot infer the precise signal received by A from the equilibrium prices and allocations.

When partial revelation occurs, equilibrium prices convey less information about the signals received by investors, and the conditional volatility of the future stock returns becomes higher even though investors do not disagree about the volatility of the payoff of stocks. The higher conditional volatility of returns is a result of equilibrium prices being a less precise prediction of future payoffs. The conditional risk premium is also higher when an investor leaves the market completely. When there is a positive net supply of stock, at least one of the two investors needs to hold some positive amount of stock in equilibrium. The more ambiguous investor A exits the stock market, while investor B holds all of the stock. So the risk premium needs to be high enough to induce B to do so.

The requirement that investor B holds all of the stock in a partially revealing equilibrium

means that anything that impacts B 's willingness to hold all the stock will affect whether the private signal of A is revealed. This gives rise to an interesting interaction between the wealth distribution and revelation. When B 's wealth is high relative to A 's, it's more likely that B can purchase all of the stock using her wealth. Hence, a more unequal wealth distribution toward B makes partial revelation more likely.

The dynamics of the wealth distribution in response to shocks also impact information aggregation. In a dynamic setup in which B is less risk or ambiguity-averse than A , B on average holds more stock than A . A sequence of good shocks to the stock payoffs leads to more wealth for B relative to A . If this is followed by a period of increased uncertainty, partial revelation would occur more often than if the period of increased uncertainty comes after a sequence of bad shocks to the payoffs of the stock. This generates boom-bust cycles accompanied by endogenous information propagation. Capital requirements also play an important role in information aggregation in equilibrium. B finances part of her purchase of the stock through borrowing. A tighter collateral constraint makes it more difficult for B to purchase all of the stock, making partial revelation less likely.

Implications of the model are tested empirically. The model suggests that investors exit a market leads to less informative prices and that information loss is more likely when investor stock holdings are less equal. Using 13F filing data in the U.S., I observe at a quarterly frequency the stock holdings of institutional investors with more than \$100 million under management. The holding data are combined with data from CRSP, which is used to compute quarterly returns and a price informativeness measure. The informativeness measures the variance of the idiosyncratic component of stock returns, which is shown to correlate with private information (see Chen et al. (2007) for example). Regressions results show that more exits of investor from a stock are associated with less informative prices. For example, exits of 10 institutional investor from a stock are associated with a 33 percentage point decrease in the the growth rate of the price informativeness measure. The results also show that prices are less informative when the share holdings of a stock are more disperse among institutional investors or when a stock is at the end of a long boom. These are consistent with the model implications.

Related Literature This paper is related to several strands of literature. Early papers like those by Grossman and Stiglitz (1976), Radner (1979), and Allen (1981) study asymmetric information in a general equilibrium setting, usually finding that the existence of a fully

revealing equilibrium is generic.² In particular, Radner (1979) shows that, in a pure exchange economy with a finite number of signal states, a rational expectations equilibrium reveals to all traders the information possessed by all the traders taken together except for knife-edge cases. This paper shows that, in a setup with a finite set of signals, a rational expectations equilibrium with partial revelation is a robust phenomenon when investors in the economy are ambiguity-averse.

There is also a literature on limited participation. Ambiguity aversion has been considered in problems of portfolio choice. Dow and Werlang (1992) pioneer the idea that ambiguity-averse investors may not hold any risky assets in a static portfolio choice model. Epstein and Schneider (2007) study non participation and market equilibrium in a dynamic setting with ambiguity-averse investors.³ Limited participation can also be generated through other mechanisms. Constantinides (1979), Davis and Norman (1990), and Morton and Pliska (1995) show that transaction costs can lead to no-trade regions for risky assets with risk-averse investors.⁴ Vissing-Jorgensen (2002) provides empirical evidence that supports the importance of fixed transaction costs to explain household non participation in the stock market. Similarly, Reis (2006), Chien et al. (2009), and Duffie (2010) show in models with rational inattention that investors can also limit their participation in the market. When investors are faced with transaction costs or when they are rationally inattentive, they may choose not to re-balance their portfolios in response to shocks. They do not necessarily exit the market during times of uncertainty, however. This paper provides a way to model the time-varying exit behavior through state-dependent ambiguity aversion. The framework of this paper can also be used to model fixed holding costs through a simple reinterpretation of the zero-holding region of stock. This paper also studies how limited participation can play an important role in information aggregation, which is not the focus of the earlier papers.

A few papers show that partially revealing equilibria exist in a static setting with ambiguity-averse investors. Condie and Ganguli (2011a,b) demonstrate, in the tradition of Radner (1979), the existence and robustness of partially revealing rational expectations equilibria even in the absence of noise shocks. In another paper with a normal payoff distribution, Condie and Ganguli (2012) show that private information that is perceived to be ambiguous

²Allen and Jordan (1998) provide an excellent survey of this literature.

³Epstein and Schneider (2010) provide a thorough survey of how models with ambiguity-averse investors can be useful in studying financial market phenomena.

⁴A number of other papers, including those by Duffie and Sun (1990), Heaton and Lucas (1996), Vayanos (1998), Gennotte and Jung (1994), Luttmer (1996), and He and Modest (1995), study the effect of transaction costs on portfolio choice and market equilibrium.

need not be revealed by market prices in a rational expectations equilibrium, and finding that the risk premium is higher and price volatility is lower when there is unrevealed information. Easley et al. (2012) also find partial revelation in the absence of noise. The mutual fund investors in their paper are not ambiguous over the fundamentals of the stock but over the strategy used by hedge fund managers. This paper complements this literature by studying endogenous information aggregation and its interaction with the wealth distribution in a dynamic setting with an infinite horizon and finite states. In addition, I introduce a measurability requirement that puts further restriction on the existence of equilibrium.

Partial revelation is also possible under alternative models. The “noise based” approach is a widely used alternative to generate partial revelation. In these models, signal values are not fully revealed in equilibrium due to the presence of noise shocks.⁵ Dow and Gorton (2008) provide a recent discussion of this approach. Tallon (1998), Caskey (2009), Mele and Sangiorgi (2009), and Ozsoylev and Werner (2011) find partial-revelation results with ambiguity-averse investors. The partial revelation in these models relies, however, on the presence of noise in the market. Using a different approach, Hong and Stein (2003) show that partial revelation can occur when investors are faced with short-sell constraints in a three-period setting. This paper describes a dynamic model environment when information may not be revealed in the absence of noise trading.

A large literature has used institutional ownership to examine the relationship between institutional ownership and stock returns. These papers find a correlation between changes in institutional ownership, stock returns, and return volatility. See Sias (1996), Nofsinger and Sias (1999), Wermers (1999), Dennis and Strickland, 2002, Xu and Malkiel, 2003, Cai and Zheng (2004), Sias et al. (2006), Rubin and Smith, 2009, and Hong and Jiang (2011) for examples. There is also a growing literature that explores what might affect price informativeness. For example, Boehmer and Wu (2013) find that short-sell activity increases informational efficiency, while Fernandes and Ferreira (2009) show that insider trading law helps improve price informativeness. The empirical exercise in this paper also uses data on institutional ownership and a measure of price informativeness based on Chen et al. (2007), but focuses on the interaction between changes in institutional ownership at the extensive margin and stock price informativeness that is implied by the model.

The rest of the paper is structured as follows: Section 2 describes the dynamic model and its computation. Section 3 discusses numerical results of the dynamic model. Section 4

⁵See Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), and Admati (1985), among others, for examples.

shows welfare calculation. Section 5 presents empirical results using data on stock holdings of institutional investors. Section 6 concludes.

2. The Dynamic Model

The endogenous information revelation in the model is driven by the extensive margin changes in investment by investors with private information. The following setup of the model uses ambiguity aversion as one explanation to the extensive margin changes. It should be emphasized that ambiguity aversion can be replaced with other setups using the framework of the paper and the results of the paper remain the same. Fixed holding costs or regulatory constraints, for example, can also lead to extensive margin changes in investment. After describing the setup with ambiguity aversion below, I will show that the setup can be easily applied to modeling fixed costs or other constraints with virtually no change.

2.1. Model Environment

There are two assets in the economy, a riskless bond and a stock. The riskless bond lives for one period and pays out one unit of consumption. The stock is long-lived and pays a dividend D_t every period. The dividend grows at a rate g_t , which follows an iid process with mean μ_t and variance σ^2 . The law of motion for the dividend is then $D_t = D_{t-1}e^{g_t}$. Each period, nature chooses a mean μ_t for the next-period mean growth rate. Investors do not know μ_t and they need to form beliefs about the distribution of the mean. This setup means that investors do not know the actual realization of g_t even when they know the true μ_t .

There are two investors, A and B , in the model. Each investor receives a private signal s_t^i , $i = A, B$ each period about the next-period mean growth rate μ_t . The signals s_t^A and s_t^B are generated independently over time and of each other from a finite state space according to the true likelihood function $\ell_0(s_t|\mu_t)$. If there is no ambiguity, a Bayesian investor who knows both the signals $\mathbf{s}_t = (s_t^A, s_t^B)$ updates her belief about the distribution of μ_t using Bayes' rule:

$$p(\mu_t|\mathbf{s}_t) = \frac{\ell_0(\mathbf{s}_t|\mu_t)p(\mu_t)}{\int_{\mu} \ell_0(\mathbf{s}_t|\mu)p(\mu)d\mu}, \text{ where } i = A, B.$$

Here, investors share a common prior $p(\mu_t)$ in every period.

An investor who is ambiguous is not sure about the correct likelihood function of the signal. Instead, she perceives a possible set of likelihood functions $\{\ell(s|\mu) : \ell \in \mathcal{L}^i\}$, $i = A, B$.

Intuitively, the size of the set \mathcal{L}^i measures the degree of ambiguity and the subscript i indicates that the set of likelihood functions can differ between investors. I assume that the true likelihood function is in the set $\ell_0 \in \mathcal{L}^i$ and that \mathcal{L}^i to be convex and compact. An investor updates her beliefs by applying Bayes' rule with each likelihood function in her set \mathcal{L}^i . If an investor observes both signals, her one-period-ahead posterior distribution set \mathcal{M}_t^i is given by

$$\mathcal{M}_t^i \equiv \{p(\mu_t|\mathbf{s}_t)\} = \left\{ \frac{\ell(\mathbf{s}_t|\mu_t)p(\mu_t)}{\int_{\mu} \ell(\mathbf{s}_t|\mu_t)p(\mu_t)d\mu} \mid \ell \in \mathcal{L}^i \right\}, \text{ where } \mathbf{s}_t = (s_t^A, s_t^B). \quad (1)$$

This signal structure is similar to those in Epstein and Schneider (2008) and Condie and Ganguli (2012). The posterior under no ambiguity is a special case when \mathcal{L}^i is a singleton that contains only the true likelihood $\ell_0(s_t|\mu_t)$. The setup here assumes that ambiguity is over the interpretation of the signal-generating process rather than over the prior belief on μ .⁶ The likelihood function set \mathcal{L}^i is assumed to be fixed over time. This assumption suggests that the investors' levels of ambiguity do not change over time. Thus an investor with ambiguity aversion believes that the mean dividend growth rate is in the set

$$\{E(\mu_t|\mathbf{s}_t)\} = \left\{ \int_{\mu} p(\mu_t|\mathbf{s}_t)d\mu \mid p(\mu_t|\mathbf{s}_t) \in \mathcal{M}_t^i \right\}. \quad (2)$$

Investors can trade on the stock and the bond. They start with initial stock holdings X_0 and initial bond holdings B_0 .⁷ Investors make consumption and savings/portfolio decisions to maximize their lifetime utility. Since investors are ambiguity-averse, utility maximization is a max-min problem following the idea in Gilboa and Schmeidler (1989) and can be written as

$$\max_{\{X_{t+1}^i, B_{t+1}^i, C_t^i\}_{t=0}^{\infty}} \min_{\ell \in \mathcal{L}^i} \left\{ E_{\ell} \left[\sum_{t=0}^{\infty} \beta^t u(C_t^i) | I_t^i \right] \right\}, \quad i = A, B. \quad (3)$$

Without the min operator, this would become a standard portfolio choice problem. The minimization reflects investors' aversion to ambiguity in the sense that they worry about the worst case scenario.⁸ Investors maximize their expected utility, where the worst case is

⁶We can also obtain sets of posterior distributions if we assume that the ambiguity is over the prior, but the interpretation would be different.

⁷Non financial endowment that grows at the same rate as dividend can be incorporated in the model.

⁸Ahn et al. (2007), Bossaerts et al. (2010), and Dimmock et al. (2012) provide direct experimental evidence supporting this multiple priors setup.

defined as minimization over the set \mathcal{L}^i .⁹

The information set I_t^i for investor i in equation (3) represents the information she directly observes and infers from the equilibrium outcomes. For example, when an investor i can observe the signal pair \mathbf{s}_t at time t perfectly, $I_t^i = \mathbf{s}_t$; when i can only observe her own signal but cannot infer any information about the other investor's signal, $I_t^i = s_t^i$. But the information set could potentially include the signal of an investor's own signal and information she can partially infer through observing equilibrium prices and quantities. Since the mean dividend growth rate follows an iid process, the signal is only informative for the next-period mean growth rate. Hence, the information set I_t^i is a function of only the current-period signal and not the whole signal history. This assumption helps to keep the size of the state space manageable.

Investors are subject to a budget constraint

$$C_t^i + P_t X_{t+1}^i + \frac{1}{R_t} B_{t+1}^i = (P_t + D_t) X_t^i + B_t^i, \quad \forall t \geq 0. \quad (4)$$

P_t denotes the stock price at time t and R_t denotes the gross interest for the one-period riskless bond. Investors are making decisions to consume or invest subject to their portfolio wealth. Investors also face a borrowing constraint

$$\frac{1}{R_t} B_{t+1}^i \geq -\bar{m} P_t X_{t+1}^i, \quad \forall t \geq 0. \quad (5)$$

Investors can only borrow up to a fraction \bar{m} of the stock value of their portfolio. This constraint can be interpreted as a margin requirement. If $\bar{m} = 0$, then they are not allowed to borrow. When $\bar{m} = 1$, their entire stock portfolio can be funded through borrowing.

I assume that the utility function follows a CRRA form $u(c) = c^{1-\gamma}/(1-\gamma)$. The CRRA utility function allows wealth effects on portfolio decision, which will lead to interesting results that information revelation interacts with changes in the wealth distribution of investors. Denoting the aggregate state vector $Z = (X^A, B^A, \mathbf{s})$, the individual problem (3) subject to constraints (4) and (5) can be written in a recursive form

$$V^i(Z, W) = \max_{\{X', B', C\}} \min_{\ell \in \mathcal{L}^i} \left\{ \frac{C^{1-\gamma_i}}{1-\gamma_i} + \beta E_\ell [V(Z', W') | I^i] \right\}, \quad i = A, B \quad (6)$$

⁹An equivalent notation is to minimize over the set \mathcal{M}_t^i

$$s.t. C + PX' + \frac{1}{R}B' = (P + D)X + B,$$

$$\frac{1}{R}B' \geq -\bar{m}PX',$$

$$D' = De^{g'}, \text{ and}$$

$$Z' = F(Z).$$

$V^i(Z, W)$ denotes the value function of investor i . With heterogeneous agents, the state vector involves keeping track of the wealth distribution of the agents. Since the model has only two investors, I choose to keep track of the stock and bond holdings of investor A for convenience. Alternatively, I could keep track of the wealth level of both agents. The function F describes how investors forecast the wealth distribution for the next period. The risk-aversion parameter γ_i can differ across investors.

2.2. Model Solution

In order to solve investor's individual optimization problem, I first simplify the dynamic problem (6). Since dividends are growing over time, I first normalize the variables in (6) by the current dividend level D . Let the price dividend ratio be $q = P/D$. Also let $c = C/D$, $w = [(P + D)X + B]/D$, $b = B/D_{-1}$, and $z = (X^A, b^A, \mathbf{s})$. After normalization, the problem is given by

$$J^i(z, w) = \max_{\{X', b'\}} \min_{\ell \in \mathcal{L}^i} \left\{ \frac{c^{1-\gamma_i}}{1-\gamma_i} + \beta E_\ell \left[e^{g'(1-\gamma_i)} J(z', w') | I^i \right] \right\}, \quad i = A, B \quad (7)$$

$$s.t. c + qX' + \frac{1}{R}b' = w \equiv (q + 1)X' + b'e^{-g'},$$

$$\frac{1}{R}b' \geq -\bar{m}qX',$$

$$z' = F(z, w).$$

With CRRA utility, the value function is a power function of wealth,

$$J^i(z, w) = w^{1-\gamma_i} \phi^i(z) / 1 - \gamma_i. \quad (8)$$

The expression $\phi^i(z)$ captures the continuation value of investing. Following Samuelson (1969), I can rewrite the optimization problem as a choice of consumption c and the portfolio weight $\theta' = qX'/w$ for the risky asset. The optimization can then be split into two steps. Investors first maximize their expected portfolio return by choosing the weight θ' on the stock independent of their wealth and consumption decision.¹⁰ Then, based on the optimized expected portfolio return, investors solve their savings problem by choosing the optimal level of consumption.

When investors are making the portfolio decision, they care about the one-period-ahead return $R'_p = \theta' e^{g'}(q'+1)/q + (1-\theta')R$ as well as the continuation value of investing $\phi^i(z')$, $i = A, B$. The optimal portfolio problem is given as

$$\begin{aligned} h^i(z) &\equiv \max_{\theta'} \min_{\ell \in \mathcal{L}} E_\ell \left[(R'_p)^{1-\gamma} \phi^i(z') \right], \quad i = A, B, \\ \text{s.t. } \theta^i &\leq \bar{\theta} \equiv \frac{1}{1 - \bar{m}}, \end{aligned} \quad (9)$$

where $h^i(z)$ defines the subjective expected return of the optimized portfolio. The continuation value of investing can then be given by

$$\phi^i(z) = \left(\frac{1}{1 + a^i(z)} \right)^{1-\gamma_i} + \beta \left(\frac{a^i(z)}{1 + a^i(z)} \right)^{1-\gamma_i} h^i(z), \quad (10)$$

where

$$a^i(z) \equiv [\beta h^i(z)]^{\frac{1}{\gamma_i}}.$$

Equations (9) and (10) motivate a solution procedure through iteration. If we have an initial guess $h^i(z)$, we can update $\phi^i(z)$ using (10), which allows us to update $h^i(z)$ using (9). The updating can be continued until convergence occurs. Once we solve for $\phi^i(z)$, the solution to the original problem in (6) can be computed accordingly using equation (8).

¹⁰If $\gamma_i > 1$, the maximization operator over θ^i becomes minimization, since utility is represented by a negative value in this case.

One problem remains to solve is the max-min portfolio problem of equation (9). This optimal portfolio problem has no analytical solution and thus needs to be solved numerically. Alternatively, we can approximate the solution following Campbell and Viceira (2002). The approximation method involves first transforming all the variables to logs, and then applying a second-order Taylor expansion of the stock return. In addition, as we will see, the approximation also helps make the intuition of the solution clearer. Let the log next-period payoff of stock be $r'_x = \log((q' + 1)\varepsilon')$, $r_f = \log(R)$, and $p = \log(q)$, and apply a second-order Taylor approximation around the zero excess payoff point $v'_x - p - r_f = 0$. The approximated portfolio problem is

$$h^i(A, s) = e^{r_f(1-\gamma)} \max_{\theta'} \min_{\ell \in \mathcal{L}^i} \left(E_\ell [\phi'] + (1-\gamma)\theta' E_\ell [\phi'(r'_x - r_f - p)] + \frac{1}{2}(1-\gamma)\theta'(1-\gamma\theta')\sigma_x^2 \right), \quad (11)$$

where $\sigma_x^2 = E_{\ell_0} [\phi'(r'_x - r_f - p)^2]$ and is evaluated under the true likelihood function ℓ_0 . The approximated problem is now linear in r'_x , which allows for a more straightforward solution to the optimal portfolio decisions. Details of the approximation are given in the appendix.

When the borrowing constraint is not binding, the solution to problem (11) above is given by (12):

$$\theta' = \begin{cases} \frac{1}{\gamma\sigma_x^2} (\min E_\ell [\phi'(r'_x - r_f - p)] + \frac{1}{2}\sigma_x^2) & , \text{ if } \min E_\ell [\phi'(r'_x - r_f - p)] + \frac{1}{2}\sigma_x^2 > 0 \\ 0 & , \text{ if } 0 \in [\min E_\ell [\phi'(r'_x - r_f - p)] + \frac{1}{2}\sigma_x^2, \max E_\ell [\phi'(r'_x - r_f - p)] + \frac{1}{2}\sigma_x^2] \\ \frac{1}{\gamma\sigma_x^2} (\max E_\ell [\phi'(r'_x - r_f - p)] + \frac{1}{2}\sigma_x^2) & , \text{ if } \max E_\ell [\phi'(r'_x - r_f - p)] + \frac{1}{2}\sigma_x^2 < 0. \end{cases} \quad (12)$$

The last equation illustrates the intuition of the effect of ambiguity aversion on investors' portfolio decisions.¹¹ The expression $E_\ell [\phi'(r'_x - r_f - p)]$ denotes the expected excess return on the stock, taking into account its continuation value beyond the next period. When this "lifetime" risk premium of the stock evaluated at the most pessimistic belief is positive, the agent buys the stock. When the premium evaluated at the most optimistic belief is negative, the agent shorts the stock. When the sign of the risk premium could be positive or negative depending on the likelihood function ℓ , investors avoid the ambiguity by choosing to hold zero stock. The existence of this zero-holding region is important for partial revelation, as will be discussed in the following section.¹² Next, I define a recursive rational expectation

¹¹When deriving the solution, I treat the term $E_\ell [\phi']$ as independent of the likelihood function ℓ . This assumption can be verified after a solution of the value function is obtained.

¹²The existence of zero-holding regions does not depend on the approximation used previously.

equilibrium.

2.3. Equilibrium and Endogenous Information Revelation

Definition 1. A **recursive rational expectations equilibrium** is defined by a set of value functions $V^i(Z, W)$, policy functions $C^i(Z, W)$, $X^{i'}(Z, W)$, $B^{i'}(Z, W)$, $i = A, B$, pricing functions $\mathcal{P}(Z, D) = (P(Z, D), R(Z, D))$, law of motion of dividend $D' = De^{g'}$ and forecasting rule $Z' = F(Z)$, such that

1. Given the pricing functions, the law of motion, and the forecasting rules, the value functions V^A and V^B solve the recursive problem of the households with $\{C^i, X^{i'}, B^{i'}\}_{i=A,B}$ being the associated policy functions.
2. Markets clear
 - (a) $C^A + C^B = D$
 - (b) $X^{A'} + X^{B'} = Q > 0$ (positive net supply)
 - (c) $B^{A'} + B^{B'} = 0$.
3. Information is defined $I^i(\mathbf{s}) = (s^i, \mathcal{P}^{-1}(\cdot, \mathbf{s}))$, $i = A, B$.
4. Beliefs are consistent, $Z' = (X^{A'}, B^{A'}, \mathbf{s}) = F(Z)$.

An equilibrium is **fully revealing** if at least one of $P(Z, D)$ and $R(Z, D)$ is invertible in \mathbf{s} and $I^i(\mathbf{s}) = \mathbf{s}$, for all \mathbf{s} . In a fully revealing equilibrium, investors can infer the private signal from the equilibrium prices (through $\mathcal{P}^{-1}(P(\mathbf{s}), R(\mathbf{s}))$). An equilibrium is **partially revealing** if neither $P(Z, D)$ nor $R(Z, D)$ is invertible for some subset of S^2 . In other words, there exists $\mathbf{s}_1 \neq \mathbf{s}_2$, such that $\mathcal{P}(\cdot, \mathbf{s}_1) = \mathcal{P}(\cdot, \mathbf{s}_2)$. In this case, investors cannot perfectly infer the other investor's private signal by observing the market prices.

To show the intuition of the fully or partially revealing equilibrium defined above, let us consider the following case. Assume that there are two possible signal realizations, high (H) or low (L). The signal is informative in the sense that the expected excess return of the stock is higher when conditional on a high signal value under the true likelihood function, namely $E_{\ell_0} [\phi'(r'_x - r_f - p)|s = H] > E_{\ell_0} [\phi'(r'_x - r_f - p)|s = L]$. Since we have two investors and each has a private signal, there are four possible signal states $\{HH, LH, HL, LL\}$ in each period. Here the first letter refers to investor A 's signal value and the second letter refers

to investor B 's signal value. Assume further that a high signal is not ambiguous while a low signal is ambiguous. In other words, bad signals are more difficult to interpret. I also assume investor A is more ambiguity-averse than investor B .

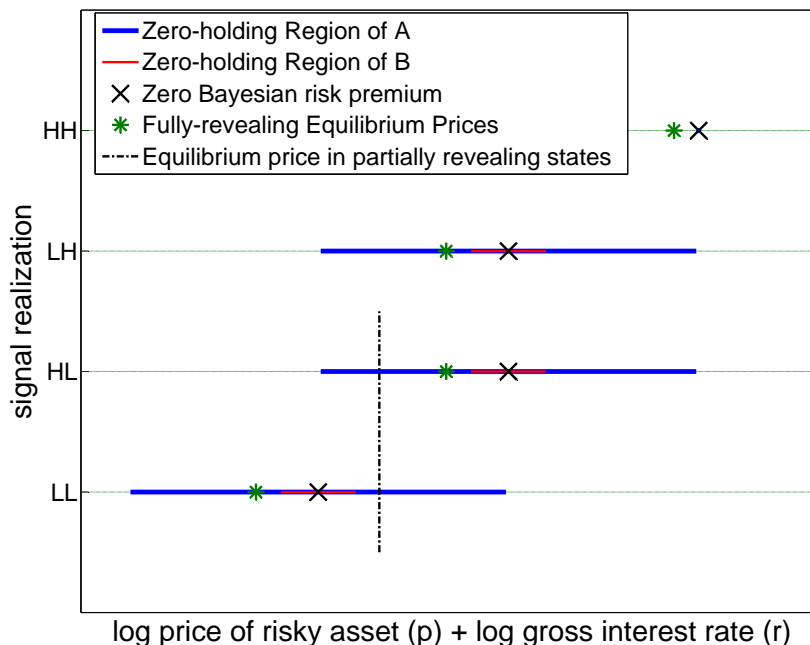
Figure 1 shows the zero-holding regions of investors against the sum of log price and log riskfree rate $p + r_f$ for each of the four signal states if signals are revealed fixing the state variables. In the actual solution to the dynamic problem, these regions are determined endogenously. We can see that there is no zero-holding region for the signal state HH for both investors since a high signal is not ambiguous. The zero-holding region of A is bigger than that of B in the ambiguous signal states, implying that investor A is more ambiguous than investor B in those states. If the investor is not ambiguous in a state, her zero-holding region becomes a point, like in the case of HH . The black crosses represent the points where a Bayesian (non-ambiguous) investor would hold zero stock. According to (12), investors would demand a positive amount of stock if the price is to the left of their respective zero-holding regions. They would demand a negative amount of stock if the the price falls to the right of their respective zero-holding regions.

If there is no ambiguity and investors are risk-neutral, the fully revealing equilibrium prices would be at the black crosses. However, with ambiguity and risk aversion, the equilibrium prices would lie to the left of the black crosses since when there is a positive net supply of stock. Any price to the right of the left end points of B 's zero-holding region would not be an equilibrium price, because both investors would want to hold zero or negative amounts of stock, and the markets would not clear. It is possible to have equilibrium prices fall in the zero-holding regions of investor A at the three ambiguous states. These are shown by the green stars in Figure 1. At these equilibrium prices, both investors hold strictly positive amounts of stock in state HH , and, in the other three states, investor B holds all of the stock and investor A holds no stock. A fully revealing equilibrium of this kind may not satisfy the measurability requirement defined in 2 below.

Definition 2. An equilibrium satisfies the **measurability requirement** if conditional on having the same signal for the other investor s^{-i} and if there exist signals for investor i , $s_1^i \neq s_2^i$ such that $\mathcal{P}(\mathbf{s}_1) \neq \mathcal{P}(\mathbf{s}_2)$, where $\mathbf{s}_1 = (s_1^i, s^{-i})$ and $\mathbf{s}_2 = (s_2^i, s^{-i})$; then $X^{i*}(\mathbf{s}_1) \neq X^{i*}(\mathbf{s}_2)$ and $B^{i*}(\mathbf{s}_1) \neq B^{i*}(\mathbf{s}_2)$.

The measurability requirement says that, if an investor's signal is revealed through the equilibrium prices, her asset holdings should be different in these different signal states. Intuitively, we interpret this as saying that an investor's signal is revealed through her

Figure 1: Zero Holding Regions of Investors



actions. In Figure 1, equilibrium prices are different in states HL and LL , but investor A , whose signal realizations are different in those two states, holds zero risky assets in both states. If investor A does not have an endowment of stocks and thus her wealth is not dependent on stock prices, her bond holdings would be identical across these two states as well. If this is the case, then the fully revealing equilibrium does not satisfy the measurability requirement. However, if investor A has an endowment of stocks and her wealth is dependent on stock prices, then her bond holdings would be different across the two states, leading to the measurability requirement being satisfied.

As mentioned before, the zero-holding regions are important in generating partial revelation as defined above. The full revelation results comes from the invertibility of the equilibrium price and signal states. If there is a one-to-one mapping from signal states to the equilibrium prices, then investors can infer the private signals through observing the equilibrium prices. The zero-holding regions break that unique mapping. If the zero-holding regions for different signal states overlap in the sense that they have a common range of prices that an investor would trade to zero and the equilibrium price falls into that region, other investors in the economy cannot tell which private signal the investor receive by looking

at the equilibrium prices (or holdings). Then we have partial revelation. This is the case for the signal states HL and LL . When the equilibrium prices fall into investor A 's overlapped zero holding regions of HL and LL , investor B cannot tell whether investor A 's signal is high or low. Investor B updates her belief using a weighted average of the states that are not fully revealed. This results in the partial revelation.

Proposition 3 shows that a necessary condition for an investor's signal to be partially revealed is that she needs to hold no risky assets in states that are not revealing. Intuitively, if an investor is not in the zero-holding zone, her demand for stock would be responsive to the signals received. This would lead to different prices in equilibrium, which contradicts the definition of a partially revealing equilibrium. Second, the signal of the less ambiguous investor is revealed in equilibrium, because if the less ambiguous investor's signal is not revealed in a state, according to Proposition 3, investor B would be holding no stock. Since the other investor is more ambiguity-averse, she would hold no stock either. This means that the stock market would not clear and we reach a contradiction. The idea is formalized in Corollary 4.

Proposition 3. *If investor i 's signal is not revealed at a pair of states $\mathbf{s}_1 \neq \mathbf{s}_2$, where $\mathbf{s}_1 = (s_1^i, s^{-i})$, $\mathbf{s}_2 = (s_2^i, s^{-i})$, and $s_1^i \neq s_2^i$, then investor i must be holding zero risky assets at both states.*

Proof. Suppose there exists a partially revealing equilibrium such that $\mathcal{P}(\mathbf{s}_1) = \mathcal{P}(\mathbf{s}_2)$. Then $\mathcal{P}^{-1}(P^*(\mathbf{s}_1), R^*(\mathbf{s}_1)) = \mathcal{P}^{-1}(P^*(\mathbf{s}_2), R^*(\mathbf{s}_2)) = \{\mathbf{s}_1, \mathbf{s}_2\}$. $\theta^{-i}(P^*(\mathbf{s}_1), R^*(\mathbf{s}_1), \{\mathbf{s}_1, \mathbf{s}_2\}) = \theta^{-i}(P^*(\mathbf{s}_2), R^*(\mathbf{s}_2), \{\mathbf{s}_1, \mathbf{s}_2\})$. However, $\theta^i(P^*(\mathbf{s}_1), R^*(\mathbf{s}_1), \{\mathbf{s}_1, \mathbf{s}_2\}) \neq \theta^i(P^*(\mathbf{s}_2), R^*(\mathbf{s}_2), \{\mathbf{s}_1, \mathbf{s}_2\}) \neq 0$. Using the market-clearing condition, we would have $\theta^{-i}(P^*(\mathbf{s}_1), R^*(\mathbf{s}_1), \{\mathbf{s}_1, \mathbf{s}_2\}) \neq \theta^{-i}(P^*(\mathbf{s}_2), R^*(\mathbf{s}_2), \{\mathbf{s}_1, \mathbf{s}_2\})$. This is a contradiction. \square

Corollary 4. *If investor i is less ambiguous than investor $-i$, then i 's signal must be revealed in an equilibrium state.*

Proof. Suppose that this is not true, which means investor i would be holding no risky assets in the state. Since investor $-i$ is more ambiguous than i as measured by a larger zero-holding region, investor $-i$ would hold no risky assets either. Therefore, the stock market would not clear, which contradicts the definition of an equilibrium. \square

Going back to the example before, investor B 's signal is always revealed in equilibrium due to her low ambiguity aversion. So we can focus on whether investor A 's signal is revealed. In signal states LH and HH , A 's signal is revealed because in state HH , investor A holds a

non-zero amount of risky assets in equilibrium and, according to Proposition 3, A 's signal will be revealed. In signal states HL and LL , it is possible to have partial revelation as defined previously. At HL and LL , investor A 's signal may not be revealed to investor B . Investor B can observe her own signal L , but is not sure of A 's signal. Therefore, B 's belief of the signal is a weighted average of the signal states HL and LL . In this case, if the market clears at a price within the zero-holding region of investor A , as marked by the dotted vertical line in Figure 1, the equilibrium prices in states HL and LL are non-revealing. At these prices, investor B holds all of the stocks and investor A holds only riskfree bonds. It is important to note that this partially revealing equilibrium satisfies the measurability requirement.

There are a few things that might affect the existence of a partially revealing equilibrium. For a partially revealing equilibrium to exist, investor B needs to hold all of the stocks in the ambiguous states. Anything that reduces the willingness of B to hold all of the stocks would make partial revelation less possible. For example, if investor B 's risk aversion is high, she would demand the stock price to be low in order to hold all the stocks. This would push the prices outside of the zero-holding region of investor A . Then according to Proposition 3, investor A 's signal would be revealed. Another factor that might affect the existence of a partially revealing state is the distribution of wealth. The lower the wealth of investor B , the less she would demand the stock given prices, which makes the existence of a partially revealing equilibrium less likely. Since the wealth distribution changes over time in the model, this leads to changes of information propagation over time.

2.4. Alternatives to Ambiguity Aversion

As mention earlier, we observe exits of investors in asset positions. When investors have private information, their exits may lead to less information being revealed in equilibrium. In the previous section, investors' exits (or investors' zero-holding region) are a result of the ambiguity of the signal and the agents' aversion to this ambiguity, but the partial-revelation results do not rely on ambiguity aversion so long as the model can generate exits of investors. Other model assumptions may also generate zero-holding regions.

Fixed holding costs, for example, can also generate zero-holding regions. The model in this paper can be easily used to study partial revelation and endogenous information propagation with fixed holding costs. Suppose that we have a similar setup as in the previous section, but investors are not ambiguity-averse and thus are Bayesian investors. They face a per period fixed cost $z^i(s)$, $i = A, B$, when they hold non-zero amounts of a risky asset. Examples

of this fixed cost may include fixed account maintenance costs, or fixed monitoring costs involved in having non-zero amounts of the risky asset (see Vissing-Jorgensen (2002) for example). Under this alternative setup with fixed costs but no ambiguity aversion, we can show that the optimal portfolio decision of investors is of a similar form to (11) when $z^i(s)$ is equal to half of the length of their respective zero-holding regions in Figure 1. These optimal portfolio decisions will lead to same equilibria as in the previous section. Similar to fixed holding costs, any mechanism that can generate zero-holding regions in investment decisions can use the framework described in this paper to get partial-revelation results.

In this paper, I focus on the setup with ambiguity aversion. Ambiguity aversion provides an intuitive way to model state-dependent exits and entries. In addition, I focus on institutional investors in the empirical section of the paper. The fixed costs may have to be very large in order to justify institutional investors' exit and entry decisions.¹³ The results of the paper, however, remain the same with alternative setups as long as there are exits and entries.

2.5. Computation and Equilibrium Selection

I start solving the problem by an initial guess $h = 0$. Investors would consume all of their endowment and leave no assets for the next period. The forecasting rule F_T would give $z' = (0, 0, \mathbf{s})$. This also gives $\phi = 1$. Given that we have a guess for ϕ and z' , we can start the iteration process defined in the following steps.

1. Discretize over the range of z .
2. Find equilibrium prices
 - (a) Given $\phi^i(z')$, $p(z')$, and $F(z)$ from the last iteration, solve the individual investor problem in (12) by assuming that the borrowing constraint is not binding and the signal of the more ambiguous investor A is not revealed at the ambiguous states. Find the market-clearing prices $p(z)$ and $R(z)$ at each grid point under these two assumptions.
 - (b) Check whether the borrowing constraint is violated under the unconstrained problem at the market-clearing prices at each grid point. If this is the case for at least

¹³Using household data, Naudon et al. (2004) also find evidence that ambiguity aversion plays an important role in non participation.

one grid point, go back to step (a) and solve the problem by imposing the borrowing constraint at the grid point at which the borrowing constraint was previously violated. If the borrowing constraint is not violated for all grid points, go to step (c).

- (c) Check whether prices are indeed not revealing at the ambiguous states, namely whether the market-clearing prices are the same across the states where A 's signal is assumed to be not revealed. If prices are revealing at some ambiguous state, this means that the assumption in (a) does not lead to a partially revealing equilibrium. Go back to step (a) and solve the problem under full revelation for the relevant grid points. If prices are the same across the ambiguous signal states, keep those market-clearing prices.
- (d) Update $p(z')$ using the market-clearing prices. Update $\phi^i(z')$ using equations (10) and (9), and $z' = F(z) = (X^A(z), b^A(z), \mathbf{s}')$.

3. Repeat step 2 until convergence.

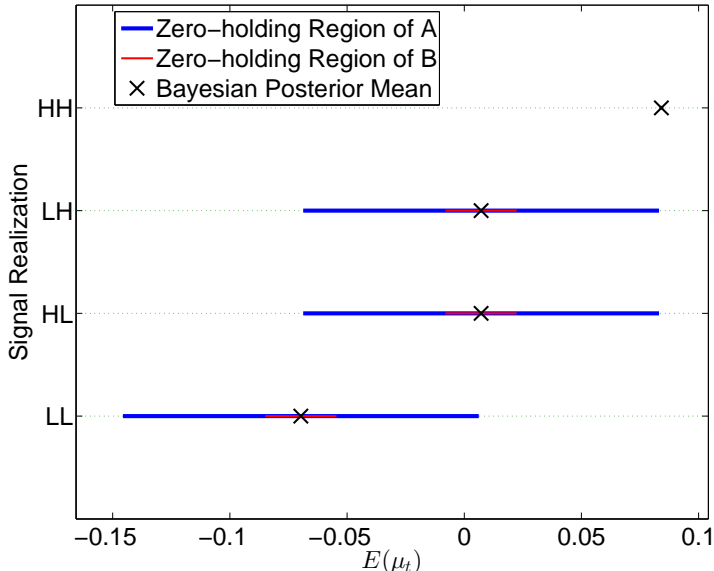
The procedure above attempts to obtain a partially revealing equilibrium first at each step. When that fails, then we resort back to the fully revealing equilibrium. This is to maximize the occurrence of a partially revealing equilibrium, which is the interest of the paper. The iteration is used to obtain an equilibrium under a stationary distribution.

3. Results

This section shows numerical results following the example outlined here. Since the growth rate grows at an iid rate, we can formulate the setup for each period. In each period, nature chooses one of the two potential mean growth rates of dividend $\{\mu_L, \mu_H\}$, where $\mu_L < \mu_H$. Conditional on the mean, the actual dividend growth follows a discretized normal distribution $g_t \stackrel{iid}{\sim} N_d(\mu, \sigma_\eta^2)$.¹⁴ Nature also picks signals about the mean growth rate with noise. Investors A and B each receive one signal, and the two signals are generated independently. The signal can be either high (L) or low (H). The likelihood of the signal can be conveniently described by the probability pair $P(s = L|\mu_L)$ and $P(s = H|\mu_H)$. For symmetry, I assume that $P(s = L|\mu_L) = P(s = H|\mu_H)$. I also want a low signal to indicate a higher probability

¹⁴If a random variable Y follows a discretized normal with mean μ and variance σ^2 , then its pmf is $Pr(Y = y) = \frac{e^{-0.5(y-\mu)^2/\sigma^2}}{\sum_Y e^{-0.5(Y-\mu)^2/\sigma^2}}$ (Harris et al. (2001)).

Figure 2: Configuration of Belief for the Dynamic Problem



for a low mean growth rate and a high signal to suggest a higher probability for a high mean growth rate. This means that $P(s = L|\mu_L) = P(s = H|\mu_H) \in [0.5, 1]$. When these probabilities are equal to 1, the signals are perfect and reveal the mean growth rate without noise. When these probabilities are equal to 0.5, the signals are non-informative.

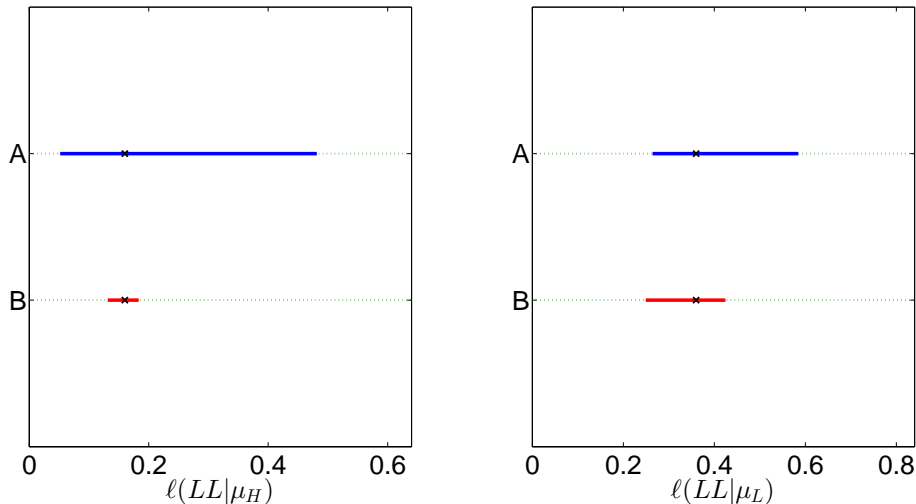
Investors are ambiguity-averse, while investor A is more ambiguous than investor B . Figure 2 plots the zero-holding regions for both investors as if they lived for two periods. It is important to note that investors live infinitely in the model. The assumption that investors live for two periods are only used to guide the configuration of the belief structure. So the zero-holding regions in Figure 2 are not the actual zero-holding regions in the stationary distribution, because investors care about the future value of investing. The actual zero-holding regions need to be solved in the dynamic problem. The reason of doing this is for its simplicity. Figure 2 specifies a range of mean growth rates that investors may believe. Once we have these beliefs, we can compute the distribution of g_t under each of the means in the range since the distribution of g_t depends on only the mean and variance, and the variance σ^2 is assumed to be known. Hence, specifying the mean growth rate range as in Figure 2 determines the set of expectations $E_\ell [\phi'(r'_x - r_f - p)]$ that investors can have without going through computing the underlying likelihood sets \mathcal{L}^i . This configuration also allows an easy adoption of the alternative fixed cost assumption, as discussed in the previous section.

To obtain the belief structure in Figure 2, I first compute the zero Bayesian risk premium points depicted as the black crosses. Then I specify the zero-holding regions of investor A as follows. Investor A is not ambiguous over the interpretation of the signal pair HH . Investor A , however, is ambiguous over the interpretation of a signal pair if it contains at least one L signal. This captures the feature that a bad signal usually comes at times when uncertainty is higher, and thus a bad signal is more difficult to interpret. In the other three signal states where there is at least one L signal, investor A is ambiguous. The level of ambiguity is reflected in the length of the zero-holding regions. I set the zero-holding region in states LH and HL to be between the two zero Bayesian risk premium points in states LL and HH . Due to symmetry, the zero-holding regions in states LH and HL are of equal length from the zero Bayesian risk premium point to the edges. The length of the zero-holding region in state LL is set to be the same as in the middle two states. Investor B is assumed to be 20% as ambiguous as investor A , in the sense that B 's zero-holding regions are 20% of the length of those of A .

The corresponding likelihood function sets \mathcal{L}^i can be computed accordingly using the procedures in the appendix. In particular, since investors are not ambiguous for the signal pair HH , I assume that the likelihood functions of investors for HH are identical to the true likelihood function value, $\ell^i(\mathbf{s} = HH|\mu_j) = \ell_0(\mathbf{s} = HH|\mu_j)$, $i = A, B$, $j = H, L$. I also impose symmetry on the likelihood functions of the investors $\ell^i(\mathbf{s} = HL|\mu_j) = \ell^i(\mathbf{s} = LH|\mu_j)$. Given that the likelihoods sum to 1 ($\sum_{\mathbf{s}} \ell^i(\mathbf{s}|\mu_j) = 1$), the likelihood function sets \mathcal{L}^i can be summarized by the sets of possible likelihood values for LL . The appendix provides a way to obtain the likelihood function set when given the required zero-holding regions. The resulting likelihood function sets are shown in Figure 3. We can see that the range of possible likelihoods $\ell^i(\mathbf{s} = LL|\mu_j)$ is bigger for investor A than for investor B .

Other parameters used in computing the dynamic model are given in Table 1. The mean log dividend growth rates are set to -4% and 13% for μ_L and μ_H , respectively. I sort the data from Chen (2009) on annual dividend growth rates for 1872-2008, split them in half, and compute the averages for each half. The mean for the lower half is equal to -4% and is set to be μ_L , while the mean for the upper half is equal to 13% and is set to be μ_H . Since I split the data in half to compute the mean, the prior distribution of μ is accordingly set to $Pr(\mu_L) = Pr(\mu_H) = 50\%$. The support of the log dividend growth rate g_t is set to be an equally spaced grid between -40% and 50% . These numbers are taken as the respective minimum and maximum of the observed dividend growth rates in the same data set from Chen (2009). The standard deviation of the dividend growth is set to be the standard

Figure 3: Likelihood Function Set



deviation of the observed data. I assume that the signals are moderately informative by setting $P(s = L|\mu_L) = P(s = H|\mu_H) = 0.6$. The discount factor β is set to 0.8, which is quite low for an annual frequency. In order to match the riskfree rate, β is usually calibrated to a higher number (for example, 0.98). Having a high discount factor would make the equilibrium prices very sensitive to the signal realized, however. Dividend growth shocks are iid in my model, and having a high β would make investors care little about the dividend growth for one period. Both investors are risk-averse and investor B is less risk-averse than investor A . The goal of this section is to illustrate the qualitative results of the model.

The model is then calculated according to the procedure outlined in the previous section to obtain a stationary equilibrium. Revelation depends on the wealth distribution at the beginning of each period. Since investor B needs to hold all stock in order for non-revelation to occur, the higher investor B 's wealth, the more likely that non-revelation will happen. In Table 2, I show the equilibrium wealth cutoff above which the signal of investor A is not revealed in state LL . When the margin requirement is 10%, non-revelation happens when investor B 's equilibrium wealth is above 57% of the total wealth in the economy. It should be noted that since wealth depends on the equilibrium prices, the cutoff given in Table 2 is also a quantity that is determined in equilibrium. The wealth distribution of investors is changing over time in a dynamic setting so we would see investor A go in and out of the stock market, which is accompanied by partial revelation.

If we tighten the collateral constraint in the economy, less borrowing is allowed. This

Table 1: Parameter Values

Parameters	Parameter Values (annual frequency)
Mean dividend growth: high or low	$\mu_L = -4\%, \mu_H = 13\%$
Prior distribution of μ	$Pr(\mu_L) = Pr(\mu_H) = 50\%$
Realization of dividend growth g_t	uniform grid over $\{-40\%, \dots, 50\%\}$
Volatility of dividend growth σ_ε	12%
Informativeness of the signals	$Pr(s = L \mu = \mu_L) = 0.6,$ $Pr(s = H \mu = \mu_H) = 0.6$
Discount factor β	0.8
Risk aversion	$\gamma_A = 0.8, \gamma_B = 0.3$
Total net supply of stock Q	1

Table 2: Wealth Bounds for Revelation

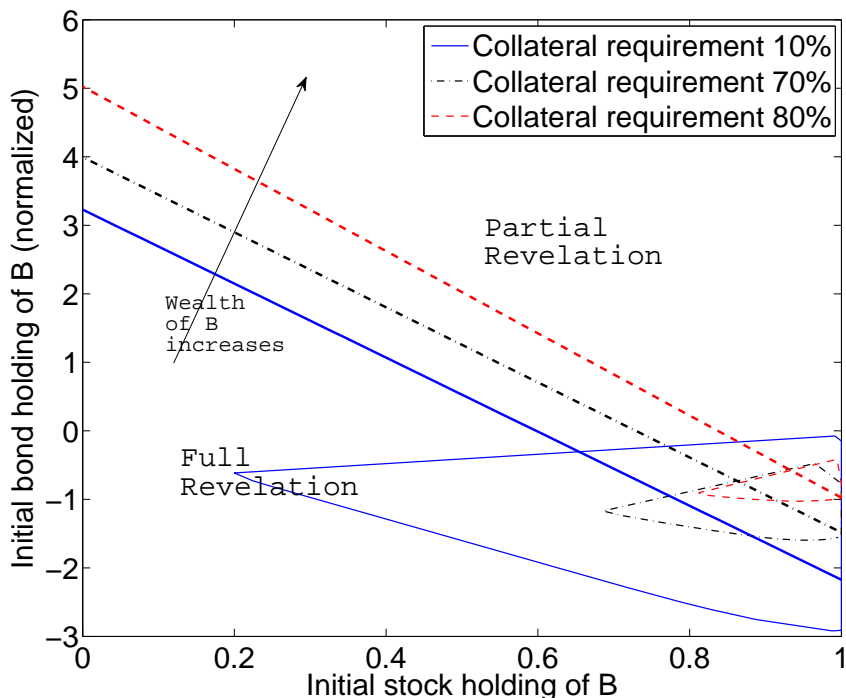
Margin Requirement	10%	50%	60%	70%	80%	90%
Wealth of A	2.4	2.4	2.2	1.7	1.0	0.5
Wealth of B	3.2	3.2	3.4	3.9	4.6	5.1
Wealth of B as a fraction of total	57%	58%	61%	70%	82%	91%

would impede the ability of investor B to hold all of the stock through borrowing. Alternatively, investor A cannot save all of her non consumed wealth in bonds because investor B cannot absorb all of the borrowing due to the tighter borrowing constraint. For either interpretation, partial revelation happens only when investor B has higher wealth at the beginning of a period. As can be seen in Table 2, the wealth cutoff increases from 57% to 91% as the collateral constraint moves from 10% to 90%.

Figure 4 shows a similar message. As the collateral constraint gets tighter, the region of partial revelation shrinks. The areas in the lower-right corner of Figure 4 show the support of the random wealth distribution under the stationary distribution for each collateral constraint. We can see that the supports of the wealth distribution cover both the full and partial revelation regions. As the wealth distribution changes over time, the private signal of A can be partially revealed, depending on the wealth distribution dynamics.

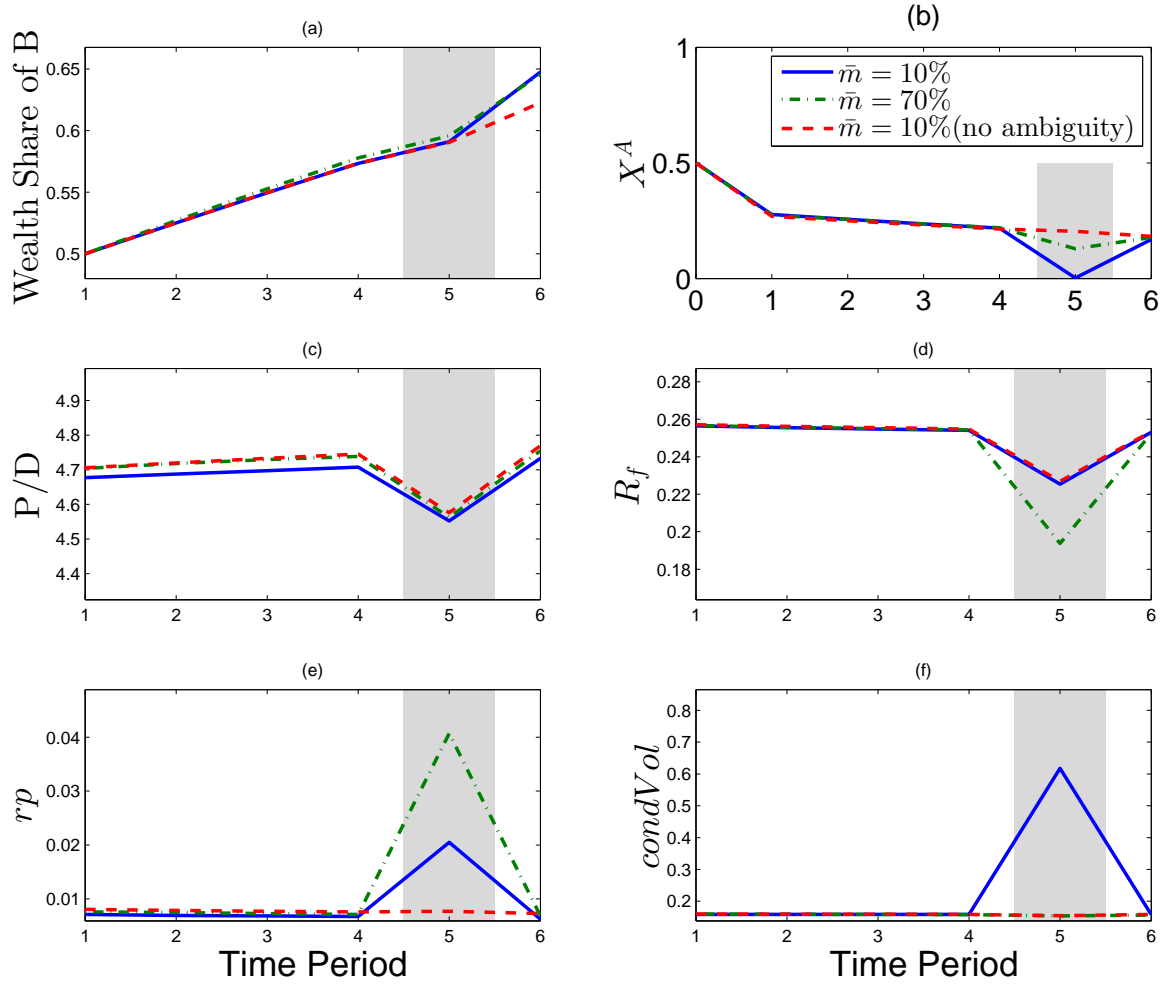
In order to see how the dynamics of the wealth distribution can affect revelation, I conduct two numerical experiments based on the stationary distribution. Both experiments last for six periods and start with each investor holding 50% of the stock and no bonds. The realized signals in all periods are HH , but in period 5, the signal is LL in both experiments. In this case, partial revelation is only possible in period 5. The two experiments differ

Figure 4: Wealth Bounds for Revelation



in the realizations of dividend growth. In experiment 1, the dividend grows at 13% per period, which corresponds to the value for μ_H in Table 1. In experiment 2, the dividend declines 10% per period. This difference in dividend growth would result in different wealth distribution dynamics. Since investor B is less ambiguity and risk-averse than investor A , investor B holds more stock than investor A on average. A sequence of high dividend growth would lead to an increase in the relative wealth of investor B . This is shown in panel (a) of Figure 5. I simulated three series for each experiment: an unconstrained case (10% collateral requirement), a constrained case (70% collateral requirement), and a no-ambiguity case (10% collateral requirement without ambiguity aversion). Panel (a) shows that the wealth fraction of investor B grows from 50% to about 58% in period 4 for all three series. The shaded area in period 5 indicates partial revelation for the 10% collateral requirement case. Panel (b) shows investor A 's holdings of stock at the end of each period. Investor A holds a strictly positive amount of stock for all periods for the no-ambiguity case. For the unconstrained case, investor A exits the stock market in period 5 when the bad signal LL hits, resulting in partial revelation. In the constrained case, however, the binding collateral constraint makes partial revelation not possible in period 5. Investor A lowers her holdings of stock but does

Figure 5: Simulation 1: High Dividend Growth

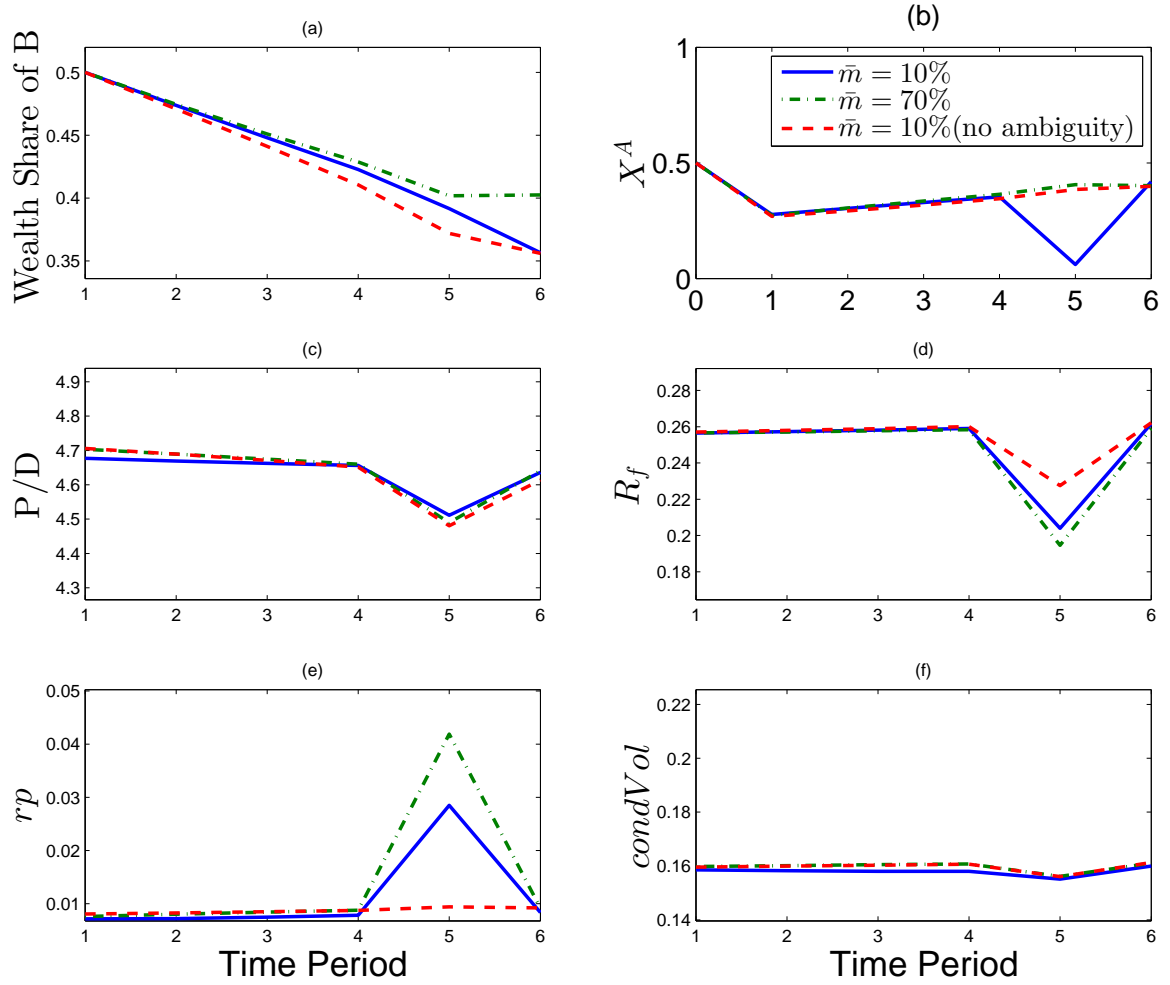


not stay out of the market completely. We can see that the reduction in A 's stock holdings also affects the wealth distribution for period 6 after the bad signal in period 5. The wealth fraction of B increases more in the two cases with ambiguity relative to the case without ambiguity, since investor A holds no or less stock in period 5 under ambiguity, leading the wealth to diverge when good dividend growth hits in period 6.

Both the price dividend ratio (P/D) and the riskfree rate (r_f) drop in period 5 across all three series due to the bad signal realization. The interest rate drops much more for the constrained case since the riskfree rate needs to be low enough that investor A would be willing to not put all of her non consumed wealth in bonds and to hold some stock. In addition, we see that the price dividend ratio and riskfree rate drop together, suggesting that the effect of ambiguity over dividend growth on the stock price outweighs the indirect effect of a lower interest rate on the price. The conditional risk premium is computed from the point of view of an econometrician observing the equilibrium quantities but not the true states of nature. When partial revelation occurs, the econometrician cannot tell the difference between states LL and HL . The conditional risk premium is computed as $rp_t = E((P_{t+1} + D_{t+1})/P_t R_t)$. The conditional risk premium is higher for the two cases with ambiguity since a higher premium is required for investor B to hold more risky assets than she would under no ambiguity. The drops in both the stock price and interest rate help contribute to this rise in the conditional risk premium under ambiguity. The conditional risk premium is higher for the constrained case because the interest rate is lower, as mentioned before. The conditional volatility of the stock return is computed as $condVol_t = Var [(P_{t+1} + D_{t+1})/P_t]$. The conditional volatility of the stock return rises in the period with partial revelation. This is because the equilibrium price incorporates less information due to partial revelation and becomes a less accurate prediction for the payoff of the stock next period.

In the experiment with a sequence of low dividend growth, the wealth share of investor B declines over time from 50% to around 35% in the unconstrained case. When the bad signal LL arrives in period 5, the wealth fraction of investor B falls below the wealth cutoff needed for partial revelation. There is no shaded region in period 5 in Figure 6 for this second experiment, indicating that partial revelation does not occur. Investor A reduces her stock holdings in period 5 but does not leave the market completely as seen in panel (b) of Figure 6. The conditional risk premium rises in period 5 for both cases with ambiguity, with the premium being highest in the constrained case. The conditional volatility of the stock return, on the other hand, is not higher in the cases with ambiguity relative to the benchmark. This is because the equilibrium is fully revealing and prices incorporate all of

Figure 6: Simulation 2: Low Dividend Growth



the information that investors have.

To show the time-series properties of the model, I simulate time series for 3,000 periods and compute the moments of various variables. Table 3 summarizes the findings. Panel (a) shows the unconstrained case when the collateral requirement is 10%, while panel (b) shows the case when the collateral requirement could be binding in some periods. For each variable, I show the mean, the standard deviation, and the first-order autocorrelation coefficient (*acf*). The price dividend ratio and interest rate are lower under ambiguity relative to the benchmark without ambiguity for both the unconstrained and constrained cases. The numerical differences are small, though, due to the iid shocks of the model. High or low dividend growth matters little when the shock is not persistent relative to the investors' infinite lifespan. The models with ambiguity can generate a higher and more volatile conditional risk premium. This is particularly true for the constrained case, as we saw in the simulations in the previous section. i_p is an indicator variable that takes value one if partial revelation occurs in a period. We see that partial revelation does not occur in the benchmark cases as expected. Partial revelation occurs when there is ambiguity. About 48% of periods are partially revealed in the unconstrained case. This percentage drops to about 21% when the collateral constraint is tighter. As mentioned before, the goal of this section is to illustrate the qualitative results of the model. In the appendix, I provide results of simulated data that are more reasonable quantitatively using an alternative set of parameters.

4. Welfare Comparison

The analysis in the previous sections suggests that one policy instrument, collateral constraints, could be used to help the market reveal information. In this section, I compute the welfare of the two investors for different levels of collateral constraints. The parameters used for the calculation in this section are the same as in Table 1. Figure 7 reports the expected welfare under the stationary distribution for both investors. The numbers are normalized to 1 for the case with a 10% collateral constraint. As we tighten the collateral constraint from 10% to 90%, the welfare of the investors does not move in a monotonic fashion. A tighter collateral requirement could lead to a Pareto improvement. For example, the welfare of both investors is higher at an 80% collateral constraint than at the 10% level.

A tighter collateral constraint generally has an ambiguous effect on welfare. This is because tighter collateral constraints affect welfare through two channels. As seen in the

Table 3: Summary of Simulated Data

(a) $\bar{m} = 10\%$						
	With ambiguity			No ambiguity		
	mean	std	acf	mean	std	acf
P/D	4.72	0.07	0.00	4.79	0.08	0.00
r_f	23.5%	0.6%	-2.3%	23.6%	0.7%	-0.7%
rp	0.9%	0.4%	-0.3%	0.5%	0.21%	-3.0%
$condVol$	38.3%	22.4%	-2.2%	15.4%	0.1%	-0.9%
i_p	51%	50%	-2%	0%	0%	NA

(b) $\bar{m} = 70\%$						
	With ambiguity			No ambiguity		
	mean	std	acf	mean	std	acf
P/D	4.66	0.07	0.02	4.79	0.08	0.00
r_f	23.0%	1.3%	14.3%	23.6%	0.7%	-0.7%
rp	1.6%	0.9%	34.8%	0.5%	0.21%	-3.1%
$condVol$	32.4%	21.8%	10.1%	15.4%	0.1%	-0.9%
i_p	37%	48%	10%	0%	0%	NA

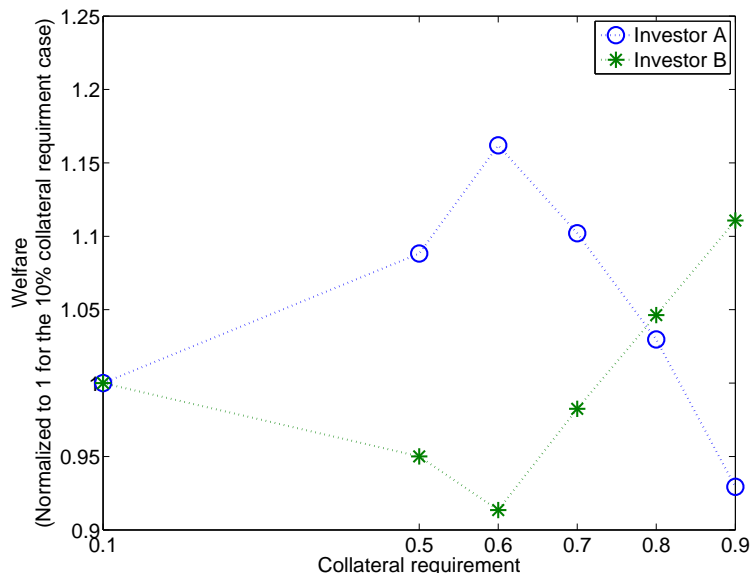
previous section, a tighter collateral requirement makes the market more transparent by reducing exits of investor A . This improves the information content of prices and lowers volatility in asset returns. However, a tighter collateral requirement also affects the sharing of uncertainty. Investors cannot share uncertainty as well when the collateral requirement is tighter, which decreases efficiency. Hence, the overall effect of the collateral requirement on welfare is ambiguous.

5. Empirical Evidence

In this section, I investigate the empirical evidence against the findings of the model. The model suggests that prices are less informative when investors exit and when there is higher concentration of ownership of a stock. Specifically, I attempt to check whether a price informativeness measure of stocks respond to the extensive margin changes in investment by institutional investors.

Some explanation is needed when I map the data with a panel of stocks and investors to a model with two investors and one stock. First, there are only two investors in the model. In the data, however, a stock is usually held by more than two investors. The rational of

Figure 7: Welfare under Various Collateral Requirements



the two-investor model applies to the case with more than two investors, as long as each investor has private information that is not perfectly correlated with each other. The exit of an institutional investor of a stock could lead to her private information not fully revealed in the price. Since her private information is not perfectly correlated with that of the other investors, some information is lost when she exits. Hence, we would expect the price of a stock becomes less informative when more investors exit the stock.

Second, there is only one stock in the model, but there are many stocks in the data. This will not be a problem if investors have private information of the idiosyncratic returns of stocks and the idiosyncratic returns of stocks are orthogonal to each other. To see this point, consider the CAPM model, where the return of a stock can be decomposed into two components: the systemic component and the idiosyncratic component. By definition, the idiosyncratic components of stock returns are independent of each other. If investors have private information about the idiosyncratic returns, then an investor's private information would not be fully revealed regardless of her trading action of other stocks. The partial-revelation result of the model survives even in a market with more than one stock. So the empirical analysis based on a panel of stock returns and institutional investors will be useful to study the theoretical findings of the paper.

5.1. Data

To get a measure of extensive margin changes in investment of stocks, I obtain information on institutional stock holdings. Institutions with assets under management of over \$100 million are required to file form 13F every quarter to the SEC. The filing information includes the amount of stocks held the end of each quarter. This gives us a snapshot of institutional investors' stock positions each quarter. Institutional investors include hedge funds, banks, insurance companies, mutual funds, pension funds, and endowment foundations. This group of investors maps naturally to the traders in the model. Institutional investors account for most market activity and frequently enter and exit individual stocks. They also do market research to gather information and then choose optimal portfolios. The stock-holding data are at the parent-firm level.

Since the 13F data contains all the institutional owners of a stock, we can count the number of institutional investors holding each stock for each quarter. Let $N_{i,t}$ be the number of institutional investors who are holding any positive amount of stock i in quarter t . I count the number of net exits of institutional investors in quarter t for a given stock i as $n_{i,t} = N_{i,t} - N_{i,t-1}$. The number of exits is a measure of non participation of institutional investors for a stock i . The main data set is then created through merging the 13F and CRSP data by CUSIP at the stock level. The sample period is from 1980 Q2 to 2011 Q4. The unit of observation of the data set is a stock-quarter pair.

I obtain Data on stock returns and prices from CRSP at a daily frequency for US stocks to construct a price informativeness measure as in Chen et al. (2007). The price informativeness measure for stock i in quarter t is constructed as $INFO_{it} = 1 - R_{it}^2$, where the R_{it}^2 is estimated through the regression of daily firm returns on market and industry returns given by

$$r_{ijd} = \beta_{i0} + \beta_{im}r_{md} + \beta_{ij}r_{jd} + \varepsilon_{id}.$$

r_{ijd} is the log returns of the stock of firm i in industry j on day d . r_{md} is the value weighted log market return. And r_{jd} is the value weighted log industry returns at the SIC 3-digit level. R_{it}^2 is obtained from running the regression on daily data for each stock i and for each quarter t . Then the informativeness measure $INFO_{it}$ is computed for each stock for every quarter as $1 - R_{it}^2$. The idea is that high variation of idiosyncratic changes in prices are correlated with more private information. In other words, stock prices contain less private information if the returns are mostly following market and industry returns, as reflected by a high R_{it}^2 . The mean of $INFO_{it}$ in the sample is 0.89 with a standard deviation of 0.14. The

numbers are similar to those that are reported in Roll (1988) and Chen et al. (2007). The $INFO_{it}$ exhibits some overall trend, with a moderate decline after 2000. Since the model implication is on the time-series dimension, I detrend the series by focusing on the growth rate of $INFO_{it}$ to avoid spurious regression results.¹⁵ The change in price informativeness is defined to be the log growth rate of $INFO_{it}$ over time, $g_{it}^{INFO} = \log(INFO_{it}/INFO_{it-1})$.

To measure the degree of unequal holdings of shares of stock among institutional investors, I compute the coefficient of variance of holdings $CoV_{it} = \text{mean}(\text{shares}_{ikt}) / \text{sd}(\text{shares}_{ikt})$, where shares_{ikt} is the number of shares of stock i held by institutional investor k in quarter t . The mean and standard deviation are taken over institutional investors for each stock i and quarter t . CoV_{it} is higher when stock holdings are less equal. If all investors hold the same amount of shares of a stock, the coefficient of variance is zero. In the regression, the log growth rate of CoV_{it} is used to eliminate the trend.

Other control variables, including realized volatility, firm age, market capitalization, and return on assets are defined in Table 4. Log growth rates of some variables are used to detrend the series in order to avoid spurious regression results. Summary statistics of all variables are also shown in the lower panel of Table 4. Since I am interested in testing the time-series implication of the model, I restrict the sample to stocks that have at least 40 quarters of observations. Stocks in the financial industries (SIC code 6000-6999) are also excluded from the sample.

5.2. Results

To see whether exits of investors lead to reduced informativeness of price of a stock, I run a regression of informativeness growth g_{it}^{INFO} on net exits of investors $n_{i,t}$. Column 1 in Table 5 shows that more exits of investors are associated with lower informativeness. For example, a one standard deviation increase in the number of exits (about 11) is associated with about 0.1 percentage point lower growth rate of the informativeness measure. This is a 33% reduction from the mean growth rate. Similar result is reported in column 2 when stock fixed effects are included in the regression to control for time invariant stock characteristics. To control for the underlying increase in volatility, realized return volatility is included in the regression in column 3. The coefficient estimate of the realized volatility is negative, indicating higher volatility may lead to less informative prices, but the estimate is not statistically significant. The coefficient estimate of net exits remain significant and its

¹⁵Appendix D plots the average of $INFO_{it}$ for each quarter across stocks and that of the log growth rate of $INFO_{it}$. There does not seem to be a trend for the log growth rate of $INFO_{it}$.

Table 4: Variable Definitions and Summary Statistics

Variable	Definition
g_{it}^{INFO}	The log growth rate of the informativeness measure. The informativeness measure is computed as 1-R2, where R2 is from regressions of daily log returns on weighted average market returns and industry (SIC 3-digit) returns. (%)
Net exits	The net decrease in the number of institutional investors holding a stock relative to the previous quarter.
CoV of holdings growth	Log growth rate of the coefficient of variance of the number of shares of a stock held by each institutional investor in a quarter. The coefficient of variance is defined to be the ratio of the standard deviation to the mean of the variable of interest.
Realized volatility	Realized volatility of a stock over quarters. This is computed by the sample standard deviation of daily log returns. Numbers are annualized.
Firm age	The number of quarters that a stock appears on the CRSP data set.
Market cap growth	Log growth rate of market capitalization of a stock over quarters
Institutional ownership growth	The log growth rate of the percentage shares of a stock held by all institutional investors over quarters. (%)
Return on assets	This is computed as net income over total assets by using data from Compustat. (%)

Variable	Number of observations	MEAN	SD	5%	25%	50%	75%	95%
g_{it}^{INFO}	466810	-0.3	17.3	-26.8	-4.7	0	4.7	25.5
Net exits	439402	-1	11	-17	-3	0	2	11
CoV of holdings	341910	0.01	0.22	-0.22	-0.05	0	0.07	0.25
Realized volatility	377523	0.61	0.45	0.19	0.33	0.49	0.74	1.43
Firm age	466810	48	39	4	16	38	71	128
Market cap growth	463603	0.00	0.34	-0.53	-0.14	0.01	0.16	0.48
Institutional ownership growth	435642	0.02	0.45	-0.37	-0.05	0.00	0.08	0.44
Return on assets	395926	-0.01	0.19	-0.14	-0.01	0.01	0.02	0.05

magnitude becomes larger. Regression in column 4 includes more control variables like firm age, market cap growth rate, institutional ownership growth and return on assets. The result that net exits is negatively associated with the informativeness remains the same. This is exactly what the model implies.

The model suggests that more unequal holding of the stock at the beginning of a period can lead to less information being revealed. In column 5 of Table 5, the lagged growth rate of the coefficient of variance growth of share holdings is included in the regression. A lag value of the variable is consistent with the timing of the model. The result indeed confirms that growing dispersion of share holdings is associated with lower informativeness. A one standard deviation increase (0.22) in the lagged CoV is related to a 0.26 percentage point decrease in information growth. The regression results are consistent with what the model implies.

The model also predicts less informative prices toward the end of a long boom. To test this implication in the data, I first create a dummy variable to capture a boom for a stock. The dummy variable $D4_{it}$ is equal to one if the returns for a stock i are positive for all of the last 4 quarters and zero otherwise.¹⁶ Then regressions are run on realized volatility, the dummy variable, and the interaction term of the two in addition to the control variables. The coefficient for the interaction term should be positive if volatility (a measure of underlying uncertainty) has a stronger effect on information loss after four consecutive periods of positive returns. Column 7 of the table shows that this is true in the data. If volatility is 10 percentage points higher after four quarters of positive returns, the informativeness measure growth rate is about 1 percentage points lower. However, if volatility is higher but not after a boom period ($D_{it} = 0$), the coefficient is not statistically significant. Column 8 shows similar results when net exits and lag CoV are included in the regression.

6. Conclusion

This paper provides a useful framework to study endogenous market participation and information aggregation. The model in this paper generates endogenous co-movement of non participation, conditional risk premia, and information efficiency. In addition, it demonstrates the important roles of the wealth distribution and collateral requirements in information aggregation. Empirical evidence supports that exits of investors move negatively with

¹⁶The results are not sensitive to the choice of the length of boom. Similar results are found with a different choice of the number of consecutive positive return periods.

Table 5: Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	g_{it}^{INFO}	g_{it}^{INFO}	g_{it}^{INFO}	g_{it}^{INFO}	g_{it}^{INFO}	g_{it}^{INFO}	g_{it}^{INFO}
Net exits	-0.088* (-2.32)	-0.097* (-2.29)	-0.11* (-2.36)	-0.097* (-2.08)	-0.10* (-2.15)		-0.11* (-2.34)
Lag CoV of holdings growth					-1.22** (-4.89)		-1.06** (-4.37)
Realized volatility			-2.20 (-1.48)	-2.47 (-1.48)	-2.66 (-1.47)	-2.47 (-1.50)	-2.47 (-1.42)
D4 = 1 if past 4 quarters returns are positive						-0.22 (-0.11)	-0.32 (-0.16)
D4 × realized volatility						-10.1* (-2.18)	-10.6* (-2.21)
Firm age				-0.022 (-0.54)	-0.022 (-0.51)	-0.023 (-0.55)	-0.024 (-0.57)
Market cap growth				4.58** (3.08)	4.92** (3.09)	5.71** (3.21)	4.86** (3.12)
Insituational owernship growth				-1.22** (-3.92)	-1.84** (-3.95)	-0.78** (-2.87)	-1.74** (-3.75)
Return on assets				-1.44* (-2.23)	-2.32* (-2.53)	-1.22* (-2.11)	-1.95* (-2.47)
Fixed Effects		stock	stock	stock	stock	stock	stock
Observations	439402	439402	352402	304233	288455	286620	276982
R squared	0.003	0.007	0.011	0.019	0.020	0.022	0.027

This table reports the regression results of the log growth rate of the informativeness measure on various sets of variables. The definition of each of the variables is provided in the previous table. Observations are stock quarter pairs. All standard errors are clustered by quarter to adjust for common shocks. Columns (2) - (5) include stock fixed effects to account for time invariant stock characteristics. One and two stars indicate 5% and 1% statistical significance, respectively. Constants and fixed effects coefficient estimates are not reported in the table.

informativeness of prices.

A couple of extensions can be incorporated using the framework of the model in this paper. The model could use more persistent dividend growth shocks rather than the iid structure it currently has. Persistent dividend growth will lead to amplified effects that are not examined in the paper, since lower dividend growth or a bad signal means not just lower dividend growth in the next period, but for many periods to come. This increases the chance that the more ambiguity-averse investor stays out of the market for a longer period of time, which leads to more persistence in information loss.

Investors' belief (likelihood function) set is fixed and exogenous in the model. An extension that endogenizes the likelihood function set will provide another interesting channel of endogeneity for information aggregation. One can incorporate information production decision by investors. The model can also include feature of learning by investors as in Epstein and Schneider (2007). Endogenous information acquisition could potentially lead to more or less information loss depending on how information production is correlated with exogenous uncertainty.

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Appendix

A. Optimal Portfolio Weight for the Infinite-horizon Case

After letting $v'_x = \log((p' + 1)\varepsilon')$, $r_f = \log(R)$, and $p = \log(q)$, we can apply a second-order Taylor approximation around the point $r'_x - r_f - p = 0$,

$$\begin{aligned}
h(z) &= \max_{\theta'} \min_{\ell \in \mathcal{L}} E_\ell \left[(R'_p)^{1-\gamma} \phi(z') \right] \\
&= e^{r_f(1-\gamma)} \max_{\theta'} \min_{\ell \in \mathcal{L}} E_\ell \left[(\theta e^{v'_x - r_f - p} + (1-\theta))^{1-\gamma} \phi(z') \right] \\
&\approx e^{r_f(1-\gamma)} \max_{\theta'} \min_{\ell \in \mathcal{L}} E_\ell \left[\phi' \left(1 + (1-\gamma)\theta'(r'_x - r_f - p) + \frac{1}{2}(1-\gamma)\theta'(1-\gamma\theta')(r'_x - r_f - p)^2 \right) \right] \\
&= e^{r_f(1-\gamma)} \max_{\theta'} \min_{\ell \in \mathcal{L}} \left(E_\ell[\phi'] + (1-\gamma)\theta' E_\ell \left[\phi'(r'_x - r_f - p) \right] + \frac{1}{2}(1-\gamma)\theta'(1-\gamma\theta') E_\ell \left[\phi'(r'_x - r_f - p)^2 \right] \right) \\
&= e^{r_f(1-\gamma)} \max_{\theta'} \min_{\ell \in \mathcal{L}} \left(E_\ell[\phi'] + (1-\gamma)\theta' E_\ell \left[\phi'(r'_x - r_f - p) \right] + \frac{1}{2}(1-\gamma)\theta'(1-\gamma\theta') \sigma_x^2 \right).
\end{aligned}$$

The last equality uses the assumption that investors do not perceive ambiguity on the second moment term $\phi'(r'_x - r_f - p)^2 \sigma_x^2 = E_\ell \left[\phi'(r'_x - r_f - p)^2 \right]$, independent of the signal realization.

B. Finding the Likelihood Function Set

1. Find the likelihood function that can generate the boundary expected values for the zero-holding regions. For each state \mathbf{s} , find the likelihood function ℓ that can generate the required $\mu^* \in \{\underline{\mu}(\mathbf{s}), \bar{\mu}(\mathbf{s})\}$ by solving the following problem:

$$\min_{\ell(\tilde{s}|\mu)} \|\ell(\tilde{s}|\mu) - \ell_0(\tilde{s}|\mu)\|$$

subject to

$$\begin{aligned}
\sum_{\tilde{s}} \ell(\tilde{s}|\mu) &= 1, \forall \mu \\
\ell(\tilde{s}|\mu) &\geq 0, \forall \mu, \tilde{s} \\
\sum_{\mu} \ell(s|\mu) p(\mu) (\mu - \mu^*) &= 0 \\
\sum_{\mu} \ell(s|\mu) p(\mu) (\mu - \underline{\mu}^A(\tilde{s})) &\geq 0, \forall \tilde{s} \\
\sum_{\mu} \ell(s|\mu) p(\mu) (\mu - \bar{\mu}^A(\tilde{s})) &\leq 0, \forall \tilde{s}.
\end{aligned}$$

The summation dummy $\tilde{s} \in S^2$ is used to differentiate the state \mathbf{s} for which I generated the expectation. The third last constraint ensures that the likelihood function from the solution of the minimization problem generates the required mean μ^* in state \mathbf{s} . More formally,

$$\begin{aligned} \sum_{\mu} p(\mu|\mathbf{s})\mu &= \mu^* \\ \sum_{\mu} \frac{\ell(\mathbf{s}|\mu)p(\mu)\mu}{\sum_{\mu} \ell(\mathbf{s}|\mu)p(\mu)} &= \mu^* \\ \sum_{\mu} \ell(\mathbf{s}|\mu)p(\mu)\mu &= \sum_{\mu} \ell(\mathbf{s}|\mu)p(\mu)\mu^* \\ \sum_{\mu} \ell(\mathbf{s}|\mu)p(\mu)(\mu - \mu^*) &= 0. \end{aligned}$$

The two last constraints ensure that the likelihood function from the solution of the minimization problem generates means that are within the required range of expectations for all states $\tilde{s} \in S^2$. The two constraints are obtained from $\sum_{\mu} \frac{\ell(\mathbf{s}|\mu)p(\mu)\mu}{\sum_{\mu} \ell(\mathbf{s}|\mu)p(\mu)} \in [\underline{\mu}^A(\tilde{s}), \bar{\mu}^A(\tilde{s})], \forall \tilde{s}$ in a similar style as the equality constraint. In an application as in Section 3, we might want to put other restrictions, such as $\ell(\mathbf{s} = LH|\mu) = \ell(\mathbf{s} = HL|\mu)$. The minimization problem tries to find a likelihood function that has the least deviation from the true likelihood function measured by the Euclidean norm such that the resulting likelihood function generates the expected values required. Repeat step 2 until every single \mathbf{s} is covered.

2. Define a likelihood function set that investor A needs to have in order to generate the ranges of expectations. Denote the solution to the problems in step 2 by $\ell(\underline{\mu}(\mathbf{s}))$ and $\ell(\bar{\mu}(\mathbf{s}))$ for each state \mathbf{s} . We can show that, for any expectation $\tilde{\mu}$ within the range $[\underline{\mu}(\mathbf{s}), \bar{\mu}(\mathbf{s})]$, we can find a unique $\alpha \in [0, 1]$ such that $\tilde{\mu}$ can be generated by the likelihood function $\ell(\alpha) = \alpha\ell(\underline{\mu}^A(\mathbf{s})) + (1 - \alpha)\ell(\bar{\mu}^A(\mathbf{s}))$.

C. Alternative Set of Parameters

As discussed in Section 3, with the parameter values in Table 1, the model yields a very high riskfree rate. The high riskfree rate is primarily the result of a low discount factor β and a high dividend growth rate. As the investors in the model are close to being risk-neutral, they are less concerned about risk. Hence, the high dividend growth rate means that they would like to hold a lot of stocks. The riskfree rate then needs to be very high to induce the

Table 6: Summary of Simulated Data with Alternative Parameters

	(a) $\bar{m} = 10\%$			(b) $\bar{m} = 80\%$		
	mean	std	acf	mean	std	acf
P/D	3.53	0.05	0.05	3.66	0.06	0.03
r_f	4.3%	0.6%	-0.4%	3.9%	0.8%	22.8%
rp	1.2%	0.5%	-1.9%	0.9%	0.6%	20.1%
$condVol$	12.7%	0.1%	-0.7%	12.3%	0.2%	14.0%
i_p	50.0%	50.0%	0.3%	40.5%	49.1%	6.7%

investors to hold riskfree bonds for the asset markets to clear in equilibrium.

The results in Table 6 are obtained with almost identical parameters in Table 1. There are two exceptions. First, I use a higher discount factor $\beta = 0.9$. Second, the perceived dividend mean growth rate is 20% lower. The actual dividend growth rate used for simulation is still the same as in Table 1. The difference is that investors perceive the mean dividend growth rate to be 20% lower when they solve their optimization problem. These two changes help reduce the interest rate to a more reasonable level, which is comparable to the data.

D. Detrending the Informativeness Measure

Panel (A) plots the average of $INFO_{it}$ for each quarter across stocks, while Panel (B) plots the average log growth rate of $INFO_{it}$. The average is taken over all stocks for each quarter. The plots show that there is a moderate overall decline in the level of the informativeness measure. The trend disappears when the log growth rate is used.

