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TRADING DYNAMICS IN DECENTRALIZED MARKETS
WITH ADVERSE SELECTION

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Abstract

We study a dynamic, decentralized lemons market with one–time entry and characterize its set of non–stationary equilibria. This framework offers a theory of how a market suffering from adverse selection recovers over time endogenously; given an initial fraction of lemons, the model provides sharp predictions about how prices and the composition of assets evolve over time. Comparing economies in which the initial fraction of lemons varies, we study the relationship between the severity of the lemons problem and *market liquidity*. We use this framework to understand how asymmetric information contributed to the breakdown in trade of asset–backed securities during the recent financial crisis, and to evaluate the efficacy of one policy that was implemented in attempt to restore liquidity.

Keywords: Adverse Selection, Decentralized Trade, Liquidity, Market Freeze and Recovery

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1 Introduction

Since the seminal work of Akerlof (1970), it is well known that the introduction of low quality assets, or "lemons," into a market with asymmetrically informed buyers and sellers can disrupt trade; the typical result is that sellers with high-quality assets are unwilling to sell at depressed prices, and thus only low-quality assets are exchanged in equilibrium. Given this result, the problem of adverse selection is often used to explain why the market for high quality assets can break down or *freeze*. However, perhaps surprisingly, much less is known about how and when the exchange of these assets resumes, or how this market *thaws*.

In this paper, we develop a simple model of trade under adverse selection and use it to study how the severity of the lemons problem (i.e., the initial fraction of lemons in the market) affects the patterns of trade over time. In contrast to much of the existing literature, in which unfreezing a market requires an exogenous event or intervention, we incorporate several natural features of actual asset markets that allow this process of recovery to occur endogenously. Thus, given any initial fraction of lemons, our model delivers sharp predictions about the length of time it takes for the market to recover, and how prices and the composition of assets remaining in the market behave over this horizon.

We find that the patterns of trade depend systematically on the initial fraction of lemons. In particular, when the lemons problem is mild (i.e., this fraction is small), trades are executed quickly and at relatively uniform prices. However, when the lemons problem is more severe, trade can take a substantial amount of time and the terms of trade can vary significantly, both across agents and over time. We also characterize how the severity of the lemons problem affects the expected amount of time it takes to sell a high quality asset, which we interpret as a measure of the market's *illiquidity*; a liquid market is one where sellers can quickly find a buyer to purchase their high quality asset (at an acceptable price), whereas an illiquid market is one where this process takes a long time. In this sense, the theory presented here provides a novel theory of liquidity based on adverse selection.

Given that our framework describes explicitly how markets can recover over time on their own, it also provides a natural framework to analyze how the introduction of policies aimed

at restoring liquidity can speed up (or, perhaps, slow down) this process. We provide a specific example related to the recent financial crisis and illustrate how our environment can provide unique insights into the efficacy of such policy interventions.

We take as a starting point the classic lemons market of Akerlof (1970) and make a few simple modifications. First, in order to study how a frozen market can recover over time, the environment must be *dynamic* and equilibria must be *non–stationary*. Therefore, we consider a discrete–time, infinite–horizon model in which a fixed set of buyers and sellers have the opportunity to trade in each period. In addition, we assume that agents permanently exit the market after trading, and there are no new entrants. As a result, a central aspect of our analysis is how the composition of assets remaining in the market evolves over time, and how this interacts with agents' incentives to trade at a particular point in time or delay. Thus, in our model there is a formal sense in which trade may be sluggish because agents are waiting for market conditions to improve, which seems to be an important feature of many frozen markets that cannot be captured in a static or stationary setting.

Second, we focus our analysis on markets in which trade is *decentralized*; in contrast to the competitive paradigm, where agents are bound by the law of one price, we assume that buyers and sellers are matched in pairs, and that they decide bilaterally whether to trade and at what price. This assumption is consistent with the trading structure in many important asset markets, such as the markets for asset–backed securities, corporate bonds, derivatives, real estate, and even certain equities.[1]

There are two reasons why these modifications allow for the eventual exchange of high quality assets. First, there are two mechanisms that can adjust to facilitate trade: the price and, equally important, the time at which a transaction takes place. Second, agents with different quality assets are allowed to trade at different prices.[2] In the context of this environment, we then ask the following questions. Are all assets—and in particular high quality assets—eventually bought and sold? If so, how long does it take? How does the

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[1]By now, the literature on decentralized or "over–the–counter" asset markets has grown quite large; see, e.g., Duffie et al. (2005), Vayanos and Weill (2008), and Lagos and Rocheteau (2009).

[2]See Blouin (2003) and Moreno and Wooders (2010) for more extensive comparisons between centralized and decentralized exchange in a dynamic setting with adverse selection.

presence of low quality assets affect the expected amount of time it takes to sell high quality assets? How do prices and the composition of assets in the market evolve over time?

Before we report our findings, it is helpful to describe the model in more detail. The economy starts at $t = 0$ with an equal measure of buyers and sellers. A fraction $q_0 \in (0, 1)$ of sellers possess a single high quality asset, and the remainder possess a single low quality asset. The quality of a seller's asset is private information. In each period $t = 0, 1, 2, \ldots$, all agents receive a stochastic discount factor shock, and then buyers and sellers in the market are randomly and anonymously matched in pairs.[3] Once matched, buyers make one of two exogenously set price offers: a high price (that in equilibrium is accepted by all sellers) or a low price (that in equilibrium is only accepted by impatient sellers with the low quality asset). If a seller accepts the buyer's offer, trade ensues and the pair exits the market; if the seller rejects, the agents remain in the market. There are gains from trade in every match; in particular, the efficient outcome is for all trade to take place immediately.

Within this environment, we completely characterize the equilibrium set for all $q_0 \in (0, 1)$, and use this characterization to study the effects of asymmetric information on the patterns of trade. First, given any q_0, we show that all assets are bought and sold—the *market clears*—in a finite number of periods. The patterns of trade are such that average price offers and the average quality of assets in the market increase over time until, eventually, the average quality is high enough that all remaining buyers offer the high price, and the market clears. However, the amount of time it takes until the market clears depends crucially on the initial fraction of high quality assets: the equilibrium characterization involves partitioning the interval $(0, 1)$ based on how many periods of trade, k, it takes before all assets are bought and sold, for a given $q_0 \in (0, 1)$. Figure 1 below depicts a typical (very simple) partition.

We highlight two interesting features of this equilibrium characterization. First, there is a natural monotonicity to the equilibrium set: as q_0 gets smaller, it takes longer for the market to clear. We also derive the expected amount of time it takes to sell a high quality

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[3]The assumption of random discount factors not only captures the idea that some agents need to transact more urgently than others at a given time, but it is also technically convenient, as it allows us to focus on pure strategy equilibria.
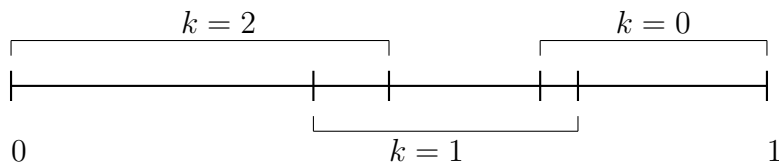
Figure 1: Number of Periods (k) Before Markets Clear for $q_0 \in (0,1)$

asset, which measures the extent to which the market for these assets is *illiquid*, and analyze the relationship between this measure of illiquidity and the initial fraction of lemons. It is in this sense that our model provides a theory of endogenous liquidity that varies systematically across states of the world and over time.

Second, note that the equilibrium regions in Figure 1 overlap: for some values of q_0, there are multiple equilibria that take different amounts of time for the market to clear. The driving force behind this multiplicity is a complementarity between buyers' actions. When other buyers offer the high price, average quality in the ensuing period does not change, since sellers with both high and low quality assets accept the high price in equal proportion. This gives buyers less incentive to wait for future periods to trade and more incentive to offer a high price now. On the other hand, when other buyers are offering the low price, a larger proportion of sellers with low quality assets accept this offer relative to sellers with high quality assets, and average quality in the future increases. This provides buyers less incentive to offer a high price and trade immediately. The existence of multiple equilibria for a given q_0 suggests that coordination failures can also contribute to illiquidity in dynamic, decentralized market settings with adverse selection.

As pointed out above, since our model provides an explicit theory of how markets recover on their own, it also provides a natural framework to analyze policies aimed at speeding up this process. As a leading example, we consider a stylized version of a policy implemented in the market for asset–backed securities in the wake of the financial crisis that began in 2007, the so–called Public–Private Investment Program for Legacy Assets.[4] This policy provided

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[4]Our model captures many of the essential features of this market: trade is decentralized, the fall of housing prices implied substantial heterogeneity in the value of these assets, and in many cases sellers had more information about these assets than potential buyers. We argue each of these points in greater detail in Section 6.

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non–recourse loans to buyers willing to purchase these securities, thus reducing the buyers' downside exposure should they discover that they acquired a lemon.

In the context of many standard models of adverse selection, a reduction of down–side risk would almost surely ease the lemons problem and help restore liquidity. Within the context of our model, we show that this policy can have an ambiguous effect on market recovery. Intuitively, this policy increases the incentive of buyers to offer the high price, thus increasing both current and future payoffs for sellers holding low quality assets. If the increase in future payoffs is greater than the increase in current payoffs, this provides the owners of low quality assets with the incentive to delay trade, thus slowing the market's recovery. As it turns out, this is more likely when q_0 is small. We believe this result highlights the importance of analyzing policies to restore liquidity within the context of an environment that models explicitly the interaction between the evolution of market conditions and the agents' incentives to delay trade.

The rest of the paper is organized as follows. After discussing the related literature below, we introduce the environment in Section 2. In Section 3, we establish some basic properties of equilibria, and in Section 4, we provide a complete characterization of the equilibrium set. In Section 5, we discuss three aspects of our equilibrium characterization: the relationship between liquidity and the lemons problem, the dynamics of trade, and the multiplicity of equilibria. In Section 6, we discuss our application to the market for asset–backed securities. In Section 7, we discuss some of our assumptions, including the restriction to exogenous prices, and also what happens to market efficiency when the time interval between trading opportunities converges to zero. Section 8 concludes.

Related Literature

Our work builds on the literature that studies dynamic, decentralized markets with asymmetric information and interdependent values.[5] The majority of this literature restricts attention

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[5]There is a parallel literature that studies dynamic, decentralized markets with asymmetric information about *private* values; most closely related to our work is Moreno and Wooders (2002), who focus on the characterization of non–stationary equilibria.

to stationary equilibria; see, for example, Inderst (2005), Moreno and Wooders (2010), and the references therein. A notable exception is Blouin (2003), who analyzes non–stationary equilibria. In all of these papers, the primary focus is to determine what happens to equilibria in a decentralized trading environment as market frictions vanish.[6] In contrast to these papers, we provide a complete characterization of the set of non–stationary equilibria, and use this characterization to study the patterns of trade over time and how these are affected by the severity of the lemons problem.

There is also a large literature that studies the lemons problem in a dynamic setting in which trade is conducted through competitive markets. Most similar to our paper is Janssen and Roy (2002), who also focus on non–stationary equilibria and the patterns of trade over time.[7] In their model, the market price at each date is the expected value of the asset to buyers, due to free entry, so that buyers are somewhat passive and receive zero payoffs in equilibrium. In contrast, the buyers in our model are quite active, and the trade–off they face between current and future payoffs is a dominant feature of the equilibrium characterization.

Our work is also related to the growing literature studying the effects of intervention in frozen markets. Perhaps most similar is Chiu and Koeppl (2009), who introduce asymmetric information into the random–matching framework of Duffie et al. (2005) and characterize steady–state equilibria in which the lemons problem is sufficiently severe to shut down trade. They, too, analyze the effect of policy intervention on trading dynamics, and show that a government purchase of low quality assets can help to restore liquidity. We highlight the crucial differences between this result and our own in Section 6.[8]

From a technical point of view, our work is related to the literature on sequential bargaining with asymmetric information and interdependent values. This literature typically studies the case of a single seller and a single buyer who bargain over time, or a single long–lived

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[6]This was an exercise first conducted in a perfect information setting by Rubinstein and Wolinsky (1985) and Gale (1986a, 1986b).

[7]Within the context of a stationary environment, there are many papers in this literature that study how introducing additional institutions, technologies, or contracts can further ease the lemons problem; see, for example, Hendel et al. (2005) and the references therein.

[8]Other recent papers studying the effects of asymmetric information on asset market liquidity and policy interventions include Guerrieri and Shimer (2010), Chari et al. (2010), Tirole (2011), Philippon and Skreta (2010), and House and Masatlioglu (2010).

seller who faces a sequence of short–lived buyers.[9] As in our framework, a feature of these models is that buyers use time to screen different types of sellers.[10] However, these models typically have a unique equilibrium, whereas we find multiple equilibria. In Section 5, we discuss how the multiplicity in our environment is driven by the fact that we have a market setting with long–lived agents.

Finally, this paper adds to the class of models that provide a theory of endogenous market liquidity based on asymmetrically informed counterparties. Rocheteau (2009) provides an excellent survey of search–based models in which information frictions interfere with exchange and thus decrease liquidity. Eisfeldt (2004), on the other hand, develops a formal relationship between the severity of the lemons problem and liquidity within a competitive market framework; in her model, an influx of low quality assets drives down the (pooling) equilibrium price of the high quality asset, thus decreasing a seller's ability to exchange the latter type of asset for cash. Finally, the dominant theory of liquidity in the finance literature, pioneered by Glosten and Milgrom (1985) and Kyle (1985), also uses informational asymmetries to generate differences in liquidity by focusing on the problem of a market–maker and treating the size of the bid–ask spread as a measure of liquidity.

2   The Environment

Time is discrete and begins in period $t = 0$. There is an equal mass of infinitely lived buyers and sellers. At $t = 0$, each seller possesses a single, indivisible asset, which is either of high (H) or low (L) quality. We refer to a seller with a type $j \in \{L, H\}$ asset as a type j seller. The fraction of sellers with a high quality asset at $t = 0$ is $q_0 \in (0, 1)$. We describe below the payoffs to a buyer and a seller from each type of asset.

In every period, each agent's discount factor δ is drawn from a continuous and strictly increasing c.d.f. F with support $[0, \bar{\delta}]$, where $\bar{\delta} < 1$. These draws are i.i.d. across both agents and time. This is meant to capture the idea that buyers and sellers have different needs at

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[9]See Vincent (1989), Evans (1989), and Deneckere and Liang (2006) for examples of the former type of model, and Daley and Green (2010), and the references therein, for an example of the latter.

[10]This basic idea goes back to, at least, Wilson (1980).

different times. At a given time, some sellers may need to sell their asset more urgently than others, while similarly some buyers may desire immediate consumption more than others. Across time, each individual agent may be more or less patient in any given period.[11]

Preferences

An asset of quality $j \in \{L, H\}$ yields flow utility y_j to a seller in each period that he holds the asset. It is convenient to denote the present discounted lifetime value of a type $j \in \{L, H\}$ asset to a seller, computed before the seller draws his discount factor, by c_j, where

$$c_j = \frac{y_j}{1 - \mathbb{E}[\delta]}, \tag{1}$$

with $\mathbb{E}[\delta] = \int \delta dF(\delta) < 1$. We normalize y_L to zero, so that $c_L = 0$. A buyer who purchases an asset of quality $j \in \{L, H\}$ receives instantaneous utility u_j. We assume that

$$u_H > y_H + \overline{\delta} c_H, \quad u_L > c_L = 0, \quad \text{and} \quad y_H > u_L. \tag{2}$$

The first two inequalities imply that there are gains from trade in every match, while the final inequality implies that the lemons problem is present. Indeed, as long as $y_H + \overline{\delta} c_H > u_L$, the price that buyers are willing to pay for a low quality asset would not be accepted by a sufficiently patient high quality seller. When $y_H > u_L$, the lemons problem is most severe, as the price that buyers are willing to pay for a low quality asset would not be accepted by even the most impatient high quality seller. Relaxing this assumption does not substantively change any of our results.

There are two aspects of this specification of preferences that warrant discussion. First, as in Duffie et al. (2005), buyers and sellers receive different levels of utility from holding a particular asset. This can arise for a multitude of reasons: for example, agents can have different levels of risk aversion, financing costs, regulatory requirements, or hedging needs. In addition, the correlation of endowments with asset returns may differ across agents. The current formulation is a reduced–form representation of such differences.[12]

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[11]Note that all types of agents draw their discount factors from the same c.d.f. F. Though non–essential, we think this is reasonable. For a deeper look at the use of random discount factors, see Higashi et al. (2009).

[12]For more discussion and examples in which these differences arise endogenously, see, e.g., Duffie et al. (2007), Vayanos and Weill (2008), and Gârleanu (2009).

Second, we assume that sellers receive flow payoffs from holding the asset, while buyers receive an instantaneous payoff upon trade. This hybrid specification is done for a number of reasons. On the one hand, we could easily adapt our analysis to the case where sellers pay a one–time production cost (c_L or c_H) when they trade with a buyer, as is standard in models of lemons markets. The current formulation is more natural for the analysis of asset markets. On the other hand, we could also assume that buyers receive flow payoffs $y_j^B > y_j$ from owning an asset of type $j \in \{L, H\}$. As we describe below, buyers exit the market upon trading, and thus the payoff from acquiring an asset would depend on the buyers' discount factor: the payoff to a buyer with discount factor δ from acquiring an asset of type j would be $u_j(\delta) = y_j^B + \delta y_j^B / (1 - \mathbb{E}[\delta])$. This heterogeneity in buyers' payoffs would make the analysis more cumbersome without providing any additional insights. The current formulation allows for sellers to receive flow payments while they own the asset, without introducing any additional heterogeneity in the buyers' payoffs.

Matching and Trade

In every period, after the agents draw their discount factors, buyers and sellers are randomly and anonymously matched in pairs.[13] Discount factors and the quality of the seller's asset are private information. Once matched, the buyer can offer one of two prices, which are fixed exogenously: a high price p_h that we assume lies in the interval $(y_H + \bar{\delta} c_H, u_H)$, or a low price p_ℓ that we assume lies in the interval $(0, u_L)$.[14] The seller can accept or reject. If a seller accepts, trade ensues and the pair exits the market; there is no entry by additional buyers and sellers. If a seller rejects, no trade occurs and the pair remains in the market. This ensures that there is always an equal measure of buyers and sellers in the market.[15]

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[13]Because of our assumption of random discount factors, our environment is a random matching model with infinitely many types. See Podczeck and Puzzello (2010) for a formalization of such models.

[14]One could imagine that buyers possess two indivisible objects that are worth p_h and p_ℓ to sellers.

[15]The use of exogenous prices is common in the literature on matching and bargaining in the presence of asymmetric information, both in stationary and non–stationary environments, as it allows for much greater tractability in the analytical characterization of equilibria; see, e.g., Wolinsky (1990), Samuelson (1992), and Blouin and Serrano (2001).

We make the following assumptions:

$$u_H - p_h \quad > \quad u_L - p_\ell, \tag{3}$$

$$y_H + \overline{\delta} p_h \quad \leq \quad p_h, \tag{4}$$

$$\overline{\delta}(u_H - p_h) \quad \leq \quad u_L - p_\ell. \tag{5}$$

The first assumption implies that, in a world with no information frictions, a buyer would prefer a high quality asset to a low quality asset given the terms of trade. Though not necessary for our results, this assumption seems the most natural one. In particular, since a type H seller rejects p_ℓ, (3) implies that $u_H - p_h$ is the highest payoff possible for a buyer. As we prove in Section 3, the second assumption implies that all sellers accept an offer of p_h regardless of their discount factor; this assumption is useful for tractability, but could be relaxed without changing the main substantive results presented below. Finally, since we restrict buyers to offer either p_ℓ or p_h, we focus our attention on the region of the parameter space in which they would never prefer to simply not make an offer at all. The inequality in (5) is a sufficient condition for this to be true; it implies that a buyer would always prefer to at least make an offer of p_ℓ at time t, even if he was guaranteed to buy a high quality asset at price p_h in the following period. We return to this last assumption in Section 7, when we discuss the restriction to two prices more generally.

Strategies and Equilibrium

A history for a buyer is the set of all of his past discount factors and (rejected) price offers. However, a buyer has no reason to condition behavior on his history: this history is private information, discount factors are i.i.d., and the probability that he meets his current trading partner in the future is zero, as there is a continuum of agents. Moreover, since there is no aggregate uncertainty, the buyer's history of past offers is not helpful in learning any information about the aggregate state. Thus, a pure strategy for a buyer is a sequence $\mathbf{p} = \{p_t\}_{t=0}^{\infty}$, with $p_t : [0, \delta] \to \{p_\ell, p_h\}$ measurable for all $t \geq 0$, such that $p_t(\delta)$ is the buyer's offer in period t, conditional on still being in the market and drawing discount factor δ.

A history for a seller is the set of all of his past discount factors and all price offers that

he has rejected. The same argument as above implies that a seller has no reason to condition behavior on his history. Thus, a pure strategy for a type j seller is a sequence $\mathbf{a}_j = \{a_t^j\}_{t=0}^\infty$, with $a_t^j : [0, \overline{\delta}] \times \{p_\ell, p_h\} \to \{0, 1\}$ measurable for all $t \geq 0$, such that $a_t^j(\delta, p)$ is the seller's acceptance decision in period t, conditional on still being in the market, drawing discount factor δ, and receiving offer p. We let $a_t^j(\delta, p) = 0$ denote the seller's decision to reject and $a_t^j(\delta, p) = 1$ denote the seller's decision to accept.

We consider symmetric pure–strategy equilibria, which can be described by a list $\sigma = (\mathbf{p}, \mathbf{a}_L, \mathbf{a}_H)$.[16] In order to define equilibria, we must determine payoffs at each date t under any strategy profile σ. Though this is a standard calculation for all t in which there is a positive measure of agents remaining in the market, we must also specify what happens when there is a zero measure of agents remaining on each side of the market. More specifically, when all remaining agents trade and exit the market in the current period, we must specify the (expected) payoff to an individual should he choose a strategy that results in *not* trading.

In order to avoid imposing ad hoc assumptions, we adopt the following procedure for computing these payoffs. Consider the slightly more general version of our model in which, in each period t, agents get the opportunity to trade with probability $\alpha \in (0, 1]$, where α is independent of an agent's type and history. The environment we analyze corresponds to the case in which $\alpha = 1$. Thus, in every period t, a fraction $\alpha \in (0, 1]$ of the buyers and sellers in the market are matched in pairs, and the remainder do not get the opportunity to trade. The definition of strategies when $\alpha \in (0, 1)$ is the same as when $\alpha = 1$.[17] However, when $\alpha \in (0, 1)$, in every period t there is a strictly positive mass of agents remaining in the market, and thus payoffs are *always* well–defined; in particular, future payoffs are well–defined when all buyers and sellers who are matched trade in the current period. We define payoffs when $\alpha = 1$ as the limit as α converges to 1 of payoffs when $\alpha < 1$.

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[16]Since F has no mass points, the restriction to pure strategies is without loss of generality, as at most a zero mass of agents is indifferent between two or more actions in each period. Moreover, with a continuum of agents, two agents with the same discount factor can behave differently only if they are indifferent between the possible action choices. Thus, the restriction to symmetric equilibria is also without loss of generality.

[17]Now a player's strategy at time t is conditional on being matched. Moreover, a history for a player also includes the periods in which he was able to trade; for the same reasons given above, a player has no incentive to condition his behavior on this information, though.

More precisely, given a strategy profile σ, let $V_t^j(\mathbf{a}|\sigma, \alpha)$ be the expected lifetime payoff to a type j seller in the market in period t following the strategy $\mathbf{a}$ and $V_t^B(\mathbf{p}|\sigma, \alpha)$ be the same payoff to a buyer in the market in period t following the strategy $\mathbf{p}$ when the probability of trade in each period is $\alpha \in (0,1)$. Both payoffs are computed before discount factors are determined in period t. The payoff to a type j seller in the market in period t following the strategy $\mathbf{a}$ is then given by

$$V_t^j(\mathbf{a}|\sigma) = \lim_{\alpha \to 1} V_t^j(\mathbf{a}|\sigma, \alpha), \tag{6}$$

while the payoff to a buyer in the market in period t following the strategy $\mathbf{p}$ is

$$V_t^B(\mathbf{p}|\sigma) = \lim_{\alpha \to 1} V_t^B(\mathbf{p}|\sigma, \alpha). \tag{7}$$

See the Supplementary Appendix for the construction of $V_t^j(\mathbf{a}|\sigma, \alpha)$ and $V_t^B(\mathbf{p}|\sigma, \alpha)$ and a proof that the limits (6) and (7) are well–defined regardless of $\mathbf{a}$, $\mathbf{p}$, and σ.

For any strategy profile $\sigma = (\mathbf{p}, \mathbf{a}_L, \mathbf{a}_H)$, let

$$A_t^j(p|\sigma) = \int a_t^j(\delta, p) dF(\delta);$$

by construction, $A_t^j(p|\sigma)$ is the likelihood that a seller of type j in the market in period t accepts an offer $p \in \{p_\ell, p_h\}$. Now let $T(\sigma)$ be the period in which the market "clears," i.e., the period in which all sellers remaining in the market accept the price offers made by the buyers; we set $T(\sigma) = \infty$ if the market never clears. Moreover, let $q_t(\sigma)$ be the fraction of type H sellers in the market in period t. Finally, let $V_t^B(\sigma) = V_t^B(\mathbf{p}|\sigma)$ and $V_t^j(\sigma) = V_t^j(\mathbf{a}_j|\sigma)$. We can now define an equilibrium in our environment.[18]

Definition 1. *The strategy profile* $\sigma^* = \left(\mathbf{p}^* = \{p_t^*\}, \mathbf{a}_L^* = \{a_t^{L*}\}, \mathbf{a}_H^* = \{a_t^{H*}\}\right)$ *is an equilibrium if for each* $t \in \{0, \ldots, T(\sigma^*)\}$ *and* $j \in \{L, H\}$, *we have that:*

(i) for all $\delta \in [0, \overline{\delta}]$, $p_t^*(\delta)$ *maximizes*

$$q_t(\sigma^*) \left\{ A_t^H(p|\sigma^*)[u_H - p] + \left(1 - A_t^H(p|\sigma^*)\right) \delta V_{t+1}^B(\sigma^*) \right\}$$

$$+ (1 - q_t(\sigma^*)) \left\{ A_t^L(p|\sigma^*)[u_L - p] + \left(1 - A_t^L(p|\sigma^*)\right) \delta V_{t+1}^B(\sigma^*) \right\};$$

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[18]Note that, in the definition below, we assume that a seller accepts any offer that he is indifferent between accepting and rejecting. This is without loss since F has no mass points, and so the probability that a seller is ever indifferent between accepting and rejecting is zero. Also note that we only require sequential rationality when there is a positive mass of agents in the market. We obtain the same results if we also require sequential rationality when the mass of agents in the market is zero.

(ii) for each $p \in \{p_\ell, p_h\}$ and $\delta \in [0, \bar{\delta}]$, $a_t^{j*}(\delta, p) = 1$ if, and only if,

$$p \geq y_j + \delta V_{t+1}^j(\sigma^*).$$

In words, the strategy profile σ^* is an equilibrium if the behavior of buyers and sellers is optimal in every period $t \leq T(\sigma^*)$. Indeed, the term in (i) is the expected payoff to a buyer in the market in period t when his discount factor is δ and he offers $p \in \{p_\ell, p_h\}$: conditional on being matched to a type j seller, an offer of p is accepted with probability $A_t^j(p|\sigma^*)$, in which case the buyer's payoff is $u_j - p$, and rejected with probability $1 - A_t^j(p|\sigma^*)$, in which case the buyer's payoff is $\delta V_{t+1}^B(\sigma^*)$. Likewise, the optimal behavior for a seller in the market in period t is to accept an offer of p if, and only if, this offer is at least as high as the payoff he obtains from holding on to his asset for another period.

3   Basic Properties of Equilibria

In this section, we establish that the market clears in finite time in every equilibrium and that the fraction of type H sellers in the population is strictly increasing over time before the market clears. We start with the following result.

Lemma 1. *Suppose the market has not cleared before period t and that the fraction of type H sellers in the market is positive. The market clears in period t if, and only if, all buyers in the market offer p_h.*

Suppose a positive fraction of buyers offer p_ℓ. Since matching is random, some of them will be matched with type H sellers, who always reject an offer of p_ℓ because of (2). Hence, a positive fraction of buyers who offer p_ℓ have their offer rejected, and thus the market does not clear. We show in the Appendix that $V_t^j(\sigma) \leq p_h$ for any strategy profile σ. Thus, since $p_h \geq y_H + \bar{\delta}p_h$ by (4), in equilibrium all sellers accept an offer of p_h.

By Lemma 1, for any equilibrium σ^*, the market clears in the first period T in which all remaining buyers in the market offer p_h. For all $t < T$, a positive mass of buyers offer p_ℓ and the fraction of type L sellers who accept this offer is $F\left(p_\ell/V_{t+1}^L(\sigma^*)\right)$. Since all sellers who

receive an offer of p_h accept the offer and exit the market, we then have that if $q_t = q_t(\sigma^*)$, then $q_{t+1} = q_{t+1}(\sigma^*)$ is given by

$$q_{t+1} = \frac{q_t}{q_t + (1 - q_t)\left[1 - F\left(p_\ell / V_{t+1}^L(\sigma^*)\right)\right]}. \tag{8}$$

Now notice that $V_{t+1}^L(\sigma^*) \leq p_h$ implies that for all $t < T$, the fraction of type L sellers who accept an offer of p_ℓ is at least $F(p_\ell/p_h)$, and thus bounded away from zero. Looking at the law of motion for $\{q_t\}_{t=0}^T$, equation (8), the following result follows immediately given that $q_0 \in (0, 1)$.

Lemma 2. *For any equilibrium σ^*, the sequence $\{q_t\}_{t=0}^T$ is strictly increasing.*

This result is a common feature of dynamic models of trade with adverse selection: since the opportunity cost (the foregone dividends) of selling a high quality asset is larger than that of selling a low quality asset, type H sellers are de facto more patient and remain in the market, on average, longer than type L sellers. As a result, over time the average quality of assets in the market increases.[19] As we now show, this implies that the market eventually clears in every equilibrium. The proof is in the Appendix.

Lemma 3. *In any equilibrium, the market clears in finite time.*

Intuitively, if the market never clears, then it must be that the mass of buyers who offer p_ℓ is strictly positive in every period t. Moreover, since q_t is strictly increasing, the sequence $\{q_t\}_{t=0}^\infty$ must converge to some $q_\infty \leq 1$. Given that buyers discount the future $(\bar{\delta} < 1)$, it must be that $q_\infty < 1$; otherwise, as q_t gets sufficiently close to one, the gain from waiting for the market to improve vanishes, and it becomes a strictly dominant strategy for all buyers to offer p_h. However, if $q_\infty < 1$, the law of motion (8) implies that as t gets large, the fraction of type L sellers who accept p_ℓ gets arbitrarily close to zero, which is not possible.

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[19]Lemma 2 does not depend on the assumption of exogenous prices, nor does it depend on the assumption that $u_L < y_H$. Indeed, if the continuation payoff to a type j seller from staying in the market is V^j, then the fraction of type j sellers who accept an offer of p is $F((p - y_j)/V^j)$. Since $y_H > y_L = 0$ and a type H seller can always replicate the behavior of a type L seller, it is necessarily the case that $V^H > V^L$. Thus, $F((p - y_H)/V^H) < F(p/V^L)$, and so as long as the market does not clear, the fraction of type L sellers who exit the market in any period is greater than the fraction of type H sellers who do the same.

4 Characterizing Equilibria

In this section, we provide a complete characterization of the equilibrium set. The first step consists in characterizing the equilibria in which the market clears in the first period of trade, i.e., all buyers offer p_h in $t = 0$. We refer to such equilibria as "0–step" equilibria; more generally, we refer to equilibria in which the market clears in period k as "k–step" equilibria. Then we use the fact that a $(k+1)$–step equilibrium must be such that (i) some agents offer p_ℓ at $t = 0$, and (ii) behavior after the first period of trade is given by a k–step equilibrium to construct the set of 1–step equilibria, and so on. Since the market clears in finite time in any equilibrium, this recursive procedure exhausts the equilibrium set. All proofs in this section are relegated to the Appendix.

Zero–step equilibria

Denote by $\pi_i^B (q, \delta, v_L, v_H, v_B)$ the payoff to a buyer who offers p_i, with $i \in \{\ell, h\}$, when: (i) the fraction of type H sellers in the market is $q \in (0, 1)$; (ii) the buyer's discount factor is δ; (iii) the continuation payoff to a seller of type j who chooses not to trade is v_j; and (iv) the continuation payoff to the buyer should he not trade is v_B. Since a type j seller can hold his asset forever, $v_j \geq c_j$. Also note that $v_B \leq u_H - p_h$ and, since a buyer can always offer p_ℓ and trade at least with probability $F(p_\ell/p_h) > 0$, it follows that $v_B > 0$.

Since sellers always accept an offer of p_h, we have that

$$\pi_h^B (q, \delta, v_L, v_H, v_B) \equiv \pi_h^B(q) = q(u_H - p_h) + (1 - q)(u_L - p_h).$$

We also know that a type H seller always rejects an offer of p_ℓ. Therefore,

$$\begin{aligned}
\pi_\ell^B (q, \delta, v_L, v_H, v_B) &\equiv \pi_\ell^B (q, \delta, v_L, v_B) \\
&= (1 - q)F\left(\frac{p_\ell}{v_L}\right) [u_L - p_\ell] + \left\{ q + (1 - q) \left[1 - F\left(\frac{p_\ell}{v_L}\right) \right] \right\} \delta v_B, \quad (9)
\end{aligned}$$

where $F(p_\ell/v_L)$ is the fraction of type L sellers who accept p_ℓ. Note that $\pi_\ell^B (q, \delta, v_L, v_B)$ is strictly increasing in v_B. Since $v_B > 0$, $\pi_\ell^B (q, \delta, v_L, v_B)$ is also positive and strictly increasing in δ. Moreover, since $v_B \leq u_H - p_h$, (5) implies that $\delta v_B \leq u_L - p_\ell$, and so $\pi_\ell^B (q, \delta, v_L, v_B)$ is non–increasing in v_L.

Let $v_B^0(q_0)$ and $v_j^0(q_0)$ be the payoffs to buyers and type j sellers in a 0–step equilibrium, respectively.[20] It is easy to see that $v_B^0(q_0) = \pi_h^B(q_0)$ and $v_j^0(q_0) \equiv v_j^0 = p_h$.[21] To construct the set of 0–step equilibria, consider the strategy σ^0 in which, in every $t \geq 0$, $p_t(\delta) = p_h$ for all $\delta \in [0, \bar{\delta}]$ and type j sellers accept an offer p if, and only if, $\delta \leq (p - y_j)/p_h$. It follows from our refinement for computing payoffs when the mass of agents in the market is zero that for all $t \geq 1$, $V_t^B(\sigma^0) = v_B^0(q_0)$ and $V_t^j(\sigma^0) = v_j^0$. Indeed, under σ^0, when the fraction of buyers and sellers who are matched in each period is $\alpha < 1$, all buyers who get the opportunity to trade exit the market, and so the fraction of type H sellers among the sellers who remain in the market stays the same. Hence, the strategy profile σ^0 is an equilibrium only if $v_B^0(q_0) > 0$ and all buyers find it optimal to offer p_h in $t = 0$, which is true as long as, for all $\delta \in [0, \bar{\delta}]$,

$$\pi_h^B(q_0) \geq \pi_\ell^B\left(q_0, \delta, v_L^0, v_B^0(q_0)\right).$$

Since $v_B^0(q_0) > 0$ implies that $\pi_\ell^B(q_0, \delta, v_L^0, v_B^0(q_0))$ is strictly increasing in δ, a necessary and sufficient condition for σ^0 to be an equilibrium is that $v_B^0(q_0) > 0$ and

$$\pi_h^B(q_0) \geq \pi_\ell^B\left(q_0, \bar{\delta}, v_L^0, v_B^0(q_0)\right). \tag{10}$$

In the proof of Proposition 1, we show that there exists a unique $\underline{q}^0 \in (0, 1)$ such that (10) is satisfied if, and only if, $q_0 \geq \underline{q}^0$. Moreover, we show that $v_B^0(\underline{q}^0) > 0$, and so $v_B^0(q_0) > 0$ for all $q_0 \geq \underline{q}^0$. Thus, σ^0 is an equilibrium if, and only if, $q_0 \in [\underline{q}^0, 1)$. Finally, we also show that (10) is the loosest possible constraint on q_0 that ensures that a buyer finds it optimal to offer p_h at $t = 0$ when all other buyers in the market offer p_h as well. In other words, no strategy profile $\tilde{\sigma}^0$ such that all buyers offer p_h in $t = 0$ is an equilibrium when (10) is violated.

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[20]In general, we will adopt the convention that a numerical subscript refers to a particular time period, while a numerical superscript refers to the number of periods it takes for the market to clear in equilibrium. In addition, we will use lower case v to denote equilibrium payoffs.

[21]Note that, for a given q_0, there may be multiple 0–step equilibria that differ in how they specify behavior off the equilibrium path (i.e., for $t \geq 1$). However, for a given q_0, all such equilibria are outcome equivalent; two equilibria σ and σ' are outcome equivalent if $T(\sigma) = T(\sigma') = T$ and for all $t \leq T$, the buyers and sellers in the market behave in the same way under both strategy profiles. Since we use the set of 0–step equilibria to construct the set of 1–step equilibria, this multiplicity also exists for 1–step equilibria, but for a given q_0, the latter equilibria are outcome equivalent as well. This is true, more generally, for all $k \geq 0$. In the analysis below, we mainly ignore this trivial multiplicity; however, we will explore in great detail cases in which, for a given q_0, there exist multiple equilibria that are *not* outcome equivalent.

Proposition 1. *Let $\underline{q}^0 \in (0, 1)$ denote the unique value of q_0 satisfying (10) with equality. There exists a 0–step equilibrium if, and only if, $q_0 \geq \underline{q}^0$.*

Notice that $q_0 u_H + (1 - q_0) u_L \geq p_h > y_H + \bar{\delta} c_H$ for any q_0 in the interval $[\underline{q}^0, 1)$, since a buyer is only willing to offer p_h if his payoff from doing so is non–negative. Hence, p_h corresponds to a market–clearing price in a competitive equilibrium. Thus, when the lemons problem is relatively small, i.e., when q_0 is sufficiently large, the equilibrium outcome in this dynamic decentralized market coincides with that of a static, frictionless market: trade occurs instantaneously at a single market–clearing price. We will now show, however, that as the lemons problem becomes more severe, equilibrium outcomes no longer resemble those of a centralized competitive market. Instead, these outcomes appear more consistent with models of decentralized trade with search frictions, in the sense that it takes time for buyers and sellers to trade, and they do so at potentially different prices.

One–step equilibria

To characterize the set of 1–step equilibria, the following convention will be useful: for any strategy profile σ, let σ_+ be the strategy profile such that for all $t \geq 0$, the agents' behavior in period t is given by their behavior in period $t + 1$ under σ. In addition, for $q \in (0, 1)$, let

$$q^+ (q, v_L) = \frac{q}{q + (1 - q) [1 - F (p_\ell/v_L)]}. \tag{11}$$

By construction, $q^+(q, v_L)$ is the fraction of type H sellers in the market in the next period if this fraction is q in the current period, a positive mass of buyers offer p_ℓ, and the continuation payoff to a type L seller in case he rejects a price offer is v_L. Since $v_L \leq p_h$, we have that $F(p_\ell/v_L) \geq F(p_\ell/p_h) > 0$, and so $q^+(q, v_L) > q$ for all $q \in (0, 1)$. Also note that $q^+(q, v_L)$ is strictly increasing in q if $p_\ell/v_L < \bar{\delta}$ and $q^+(q, v_L) \equiv 1$ if $p_\ell/v_L \geq \bar{\delta}$.

Consider a strategy profile σ^1 such that a positive mass of buyers offer p_ℓ in $t = 0$ and all buyers offer p_h in $t = 1$. In order for σ^1 to be an equilibrium, it must be that: (i) σ^1_+ is a 0–step equilibrium when the initial fraction of type H sellers is $q' = q^+(q_0, v^0_L)$; and (ii) a positive mass of buyers find it optimal to offer p_ℓ in $t = 0$ when the market clears in $t = 1$.[22]

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[22] It must also be the case that the type j sellers accept an offer of p in $t = 0$ if, and only if, $\delta \leq (p - y_j)/p_h$.

Formally, the following conditions are necessary and sufficient for σ^1 to be an equilibrium:

$$q^+(q_0, v_L^0) = q' \tag{12}$$

$$q' \geq \underline{q}^0 \tag{13}$$

$$\pi_h^B(q_0) < \pi_\ell^B\left(q_0, \overline{\delta}, v_L^0, v_B^0(q')\right). \tag{14}$$

The first condition is simply the law of motion for q_t from $t = 0$ to $t = 1$. Since $v_L^0 = p_h$ is a constant, the law of motion $q^+(q_0, v_L^0)$ is a continuous, strictly increasing function of q_0 specifying the unique implied value of q_1 in a candidate 1–step equilibrium. The second condition follows from Proposition 1. It ensures that the fraction of type H sellers in $t = 1$ falls in the region of 0–step equilibria. The third condition ensures that a positive mass of buyers find it optimal to offer p_ℓ in $t = 0$ when the strategy profile under play is σ^1. Since $q' \geq \underline{q}^0$ implies that $v_B^0(q') > 0$, $\pi_\ell^B\left(q_0, \delta, v_L^0, v_B^0(q')\right)$ is strictly increasing in δ. Thus, the incentive of a buyer to offer p_ℓ in $t = 0$ when the market clears in $t = 1$ increases with the buyer's patience. As it turns out, combining (12) and (13) provides a lower bound on the values of q_0 for which a 1–step equilibrium exists, and (14) provides an upper bound. Proposition 2 below formalizes these results.

Proposition 2. *Let $\overline{q}^1$ denote the unique value of q_0 satisfying (14) with equality, and define $\underline{q}^1$ to be such that $q^+(\underline{q}^1, v_L^0) = \underline{q}^0$ if $p_\ell / v_L^0 < \overline{\delta}$ and $\underline{q}^1 = 0$ otherwise. Then $\underline{q}^1 < \underline{q}^0 < \overline{q}^1 < 1$ and there exists a 1–step equilibrium if, and only if, $q_0 \in [\underline{q}^1, \overline{q}^1) \cap (0, 1)$. Moreover, for each $q_0 \in [\underline{q}^1, \overline{q}^1) \cap (0, 1)$, there exists a unique $q' \in [\underline{q}^0, 1]$ such that q' is the value of q_1 in any 1–step equilibrium when the initial fraction of type H sellers is q_0.*

In words, if $q_0 = \overline{q}^1$, then the most patient buyer is exactly indifferent between offering p_ℓ and p_h when a positive mass of other buyers are offering p_ℓ. For any $q_0 > \overline{q}^1$, the payoff to such a buyer from immediately trading at price p_h is greater than the payoff from offering p_ℓ and not trading with positive probability, in which case the buyer trades at price p_h in the ensuing period (when the fraction of type H sellers in the market is larger). When $p_\ell / v_L^0 < \overline{\delta}$, $\underline{q}^1$ is the unique value of q_0 such that, if a positive mass of buyers offer p_ℓ, then the fraction of

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This optimal behavior of sellers will be implicitly assumed throughout the analysis.

high quality sellers in the next period is $\underline{q}^0$, the minimum value required for market clearing; notice that $\underline{q}^1 > 0$ in this case. If even the most patient type L seller would rather accept an offer of p_ℓ today than wait one period for an offer of p_h, i.e., if $p_\ell/v_L^0 \geq \overline{\delta}$, then $\underline{q}^1 = 0$.

The fact that $\underline{q}^0 < \overline{q}^1$ implies that there are both 0–step and 1–step equilibria when $q_0 \in [\underline{q}^0, \overline{q}^1)$. In this region, if all other buyers are offering p_h, the payoffs to trading at $t = 0$ and at $t = 1$ are the same, and so it is optimal for an individual buyer to offer p_h no matter his discount factor. However, if a positive mass of other buyers are offering p_ℓ, the market does not clear at $t = 0$ and the payoff to trading at $t = 1$ increases (since $q_1 > q_0$), rendering it optimal for patient buyers to offer p_ℓ and incur a chance that they trade only in the next period. We return to this point in Section 5.

We know from above that in every 1–step equilibrium, $Q_+^1(q_0) \equiv q^+(q_0, v_L^0)$ is the value of q_1 corresponding to each initial value q_0. Moreover, for each value of q_1, payoffs at $t = 1$ are uniquely defined, since all 0–step equilibria are outcome equivalent. Therefore, it follows that the payoffs in a 1–step equilibrium are uniquely defined for each q_0. The payoff to a buyer in a 1–step equilibrium is

$$v_B^1(q_0) = \int \max \left\{ \pi_h^B(q_0), \pi_\ell^B\left(q_0, \delta, v_L^0, v_B^0(Q_+^1(q_0))\right) \right\} dF(\delta).$$

We denote the fraction of buyers that offer p_h at $t = 0$ in a 1–step equilibrium by

$$\xi^1(q_0) = \int \mathbb{I}\{\pi_h^B(q) \geq \pi_\ell^B\left(q, \delta, v_L^0, v_B^0(Q_+^1(q_0))\right)\} dF(\delta),$$

where $\mathbb{I}$ represents the indicator function. Thus, the payoff to a type L seller is

$$v_L^1(q_0) = \xi^1(q_0)p_h + (1 - \xi^1(q_0)) \int \max \left\{ p_\ell, \delta v_L^0 \right\} dF(\delta).$$

It turns out that ξ^1 is continuous and increasing in q_0, with $\lim_{q_0 \to \overline{q}^1} \xi^1(q_0) = 1$, which implies that v_L^1 is also continuous and increasing in q_0, with $\lim_{q_0 \to \overline{q}^1} v_L^1(q_0) = v_L^0$. Lemma 4 in the appendix establishes these results formally, as well as some additional properties of v_B^1 that are useful in constructing 2–step equilibria. In what follows, we let $v_L^1(\overline{q}^1) = \lim_{q_0 \to \overline{q}^1} v_L^1(q_0)$.

In addition, note that the payoff to a high quality seller is simply $v_H^1(q_0) = \xi^1(q_0)p_h + (1 - \xi^1(q_0)) \int \delta p_h dF(\delta)$. Since the behavior of type H sellers is trivial, we will not explicitly derive their payoffs in what follows.

Two–step equilibria

We now provide a complete characterization of 2–step equilibria. As it turns out, the process of characterizing k–step equilibria is nearly identical for all $k \geq 2$. Thus, the methodology developed here will allow for a complete characterization of equilibria in the next subsection.

Consider a strategy profile σ^2 such that a positive mass of buyers offer p_ℓ in $t = 0$ and $t = 1$, and then all buyers offer p_h in $t = 2$. In order for σ^2 to be an equilibrium, it must satisfy the following three necessary and sufficient conditions:

$$q^+ \left(q_0, v_L^1(q')\right) = q' \tag{15}$$

$$q' \in \left[\underline{q}^1, \overline{q}^1\right) \cap (0, 1) \tag{16}$$

$$\pi_h^B(q_0) < \pi_\ell^B \left(q_0, \overline{\delta}, v_L^1(q'), v_B^1(q')\right). \tag{17}$$

The first condition is the analog of (12); it is the law of motion for q_t from $t = 0$ to $t = 1$, conditional on a 1–step equilibrium beginning at $t = 1$. Unlike (12), the fraction q' in (15) is the solution to a fixed point problem: if the type L sellers expect continuation payoffs to be that of a 1–step equilibrium in which the initial fraction of type H sellers is q', then the fraction of type L sellers who accept an offer of p_ℓ in $t = 0$ must be such that this conjecture is correct. This fixed point problem does not appear in (12) since $v_L^0(q)$ is independent of q. The second condition ensures that there exists a 1–step equilibrium at $t = 1$ given an initial fraction q' of high quality assets. The final condition ensures that a positive mass of buyers find it optimal to offer p_ℓ in $t = 0$ when σ_+^2 is a 1–step equilibrium.

Since $v_L^1(q') \leq v_L^1(\overline{q}^1) = v_L^0$ for all $q' \in [\underline{q}^1, \overline{q}^1) \cap (0, 1)$, we have that $p_\ell/v_L^0 \geq \overline{\delta}$ implies that $q^+(q_0, v_L^1(q')) = 1$ for all $q' \in [\underline{q}^1, \overline{q}^1) \cap (0, 1)$. Thus, no 2–step equilibrium exists if $p_L/v_L^0 \geq \overline{\delta}$. Intuitively, when $p_\ell/v_L^0 \geq \overline{\delta}$, all type L sellers with $\delta < \overline{\delta}$ strictly prefer to accept on offer of p_ℓ if continuation payoffs are that of a 1–step equilibrium. Therefore, the fraction of type H sellers in the market at $t = 1$ is one, and the market clears in two periods.

Suppose then that $p_\ell/v_L^0 < \overline{\delta}$. We show in the proof of Proposition 3 that (15) and (16) imply (17). Intuitively, the incentive of the most patient buyer to choose p_ℓ in $t = 0$ is even greater than his incentive to choose p_ℓ in $t = 1$, when the fraction of type H sellers in the

market is $q' > q_0$. Hence, if the most patient buyer strictly prefers to choose p_ℓ in $t = 1$, which is true by (16), then he also strictly prefers to offer p_ℓ at $t = 0$ and (17) is satisfied. Therefore, (15) and (16) are necessary and sufficient conditions for a 2–step equilibrium.

Let $Q_+^2 : q_0 \mapsto q'$ denote the map defined by (15); in words, $Q_+^2(q_0)$ is the value of q_1 in a 2–step equilibrium, given q_0. In the proof of Proposition 3, we show that $Q_+^2(q_0)$ is a well–defined function that is both continuous and strictly increasing in q_0. Therefore, for any q_0, there is a unique value of q_1 in any candidate 2–step equilibrium. These properties of $Q_+^2(q_0)$ greatly simplify the characterization of 2–step equilibria: the necessary and sufficient conditions (15) and (16) become $Q_+^2(q_0) \geq \underline{q}^1$ and $Q_+^2(q_0) < \overline{q}^1$. Hence, the lower bound on q_0 for which a 2–step equilibrium exists is the value of q_0 such that $Q_+^2(q_0) = \underline{q}^1$, while the upper bound is the value of q_0 such that $Q_+^2(q_0) = \overline{q}^1$. The proposition below summarizes.

Proposition 3. *Suppose that $\overline{\delta} > p_\ell/v_L^0$. Let $\overline{q}^2$ be the unique solution to $q^+(\overline{q}^2, v_L^1(\overline{q}^1)) = \overline{q}^1$, and define $\underline{q}^2$ to be such that $q^+(\underline{q}^2, v_L^1(\underline{q}^1)) = \underline{q}^1$ if $p_\ell/v_L^1(\underline{q}^1) < \overline{\delta}$ and $\underline{q}^2 = 0$ otherwise. Then $\underline{q}^2 < \underline{q}^1 < \overline{q}^2 < \overline{q}^1$ and there exists a 2–step equilibrium if, and only if, $q_0 \in [\underline{q}^2, \overline{q}^2) \cap (0, 1)$. Moreover, for each $q_0 \in [\underline{q}^2, \overline{q}^2) \cap (0, 1)$, there exists a unique $q' \in [\underline{q}^1, \overline{q}^1)$ such that q' is the value of q_1 in any 2–step equilibrium when the initial fraction of type H sellers is q_0.*

Figure 2 below provides some intuition for the equilibrium characterization so far. After deriving $\underline{q}^0$ and $\overline{q}^1$, we identified $\underline{q}^1$ as the value of q_0 that would "land" exactly on $\underline{q}^0$ at $t = 1$ given the law of motion $Q_+^1(q_0)$. Since this law of motion is continuous and strictly increasing in q_0 (for $\overline{\delta} > p_\ell/v_L^0$), we are assured that any $q_0 > \underline{q}^1$ will "land" at $q' > \underline{q}^0$ in a candidate 1–step equilibrium. Moving backwards, we then identified $\underline{q}^2$ and $\overline{q}^2$ as the values of q_0 that would "land" exactly on $\underline{q}^1$ and $\overline{q}^1$, respectively, given the law of motion $Q_+^2(q_0)$. Though this law of motion is slightly more complicated, the fact that it remains continuous and strictly increasing assures us that any $q_0 \in [\underline{q}^2, \overline{q}^2)$ will "land" within the region of 1–step equilibrium. Finally, since $v_L^1(\overline{q}^1) = v_L^0$, $\overline{q}^1 > \underline{q}^0$, and

$$q^+(\overline{q}^2, v_L^0) = \overline{q}^1 > \underline{q}^0 = q^+(\underline{q}^1, v_L^0),$$

the fact that $\overline{q}^2 > \underline{q}^1$ follows immediately from the fact that $q^+(q_0, v_L)$ is strictly increasing in q_0 for any v_L such that $p_\ell/v_L < \overline{\delta}$.
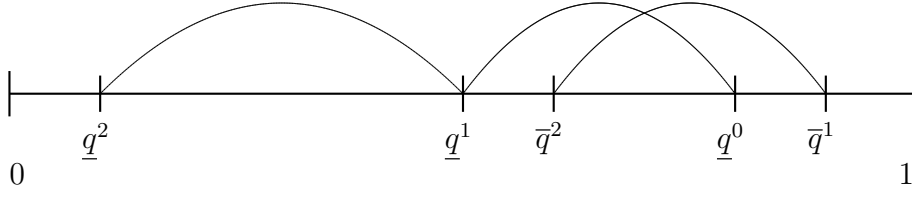
Figure 2: Deriving Bounds on Equilibrium Regions

Since $Q_+^2(q_0)$ is uniquely defined, so too are payoffs in a 2–step equilibrium: the payoff to a buyer in a 2–step equilibrium is

$$v_B^2(q_0) = \int \max \left\{ \pi_h^B(q_0), \pi_\ell^B \left(q_0, \delta, v_L^1(Q_+^2(q_0)), v_B^1(Q_+^2(q_0)) \right) \right\} dF(\delta).$$

If we denote the fraction of buyers that offer p_h at $t = 0$ in a 2–step equilibrium by

$$\xi^2(q_0) = \int \mathbb{I}\{\pi_h^B(q) \geq \pi_\ell^B \left(q, \delta, v_L^1(Q_+^2(q_0)), v_B^1(Q_+^2(q_0)) \right)\} dF(\delta),$$

then the payoff to a type L seller in a 2–step equilibrium is given by

$$v_L^2(q_0) = \xi^2(q_0)p_h + (1 - \xi^2(q_0)) \int \max \left\{ p_\ell, \delta v_L^1(Q_+^2(q_0)) \right\} dF(\delta).$$

As in the case of 1–step equilibria, it turns out that ξ^2 is continuous and increasing in q_0, with $\lim_{q_0 \to \bar{q}^2} \xi^2(q_0) = \xi^1(\bar{q}^2)$, so that the payoff v_L^2 is also continuous and increasing in q_0, with $v_L^2(\bar{q}^2) \equiv \lim_{q_0 \to \bar{q}^2} v_L^2(q_0) = v_L^1(\bar{q}^2)$. Lemma 5 in the appendix establishes these properties formally, as well as some additional properties of v_B^2 and v_L^2 that are useful in what follows.

A Full Characterization

The characterization of k–step equilibria for $k \geq 3$ proceeds by induction and follows almost exactly the characterization of 2–step equilibria. Hence, for ease of exposition, we just sketch the process here and leave the details for the Appendix.

Suppose there exists $k \geq 3$ such that for all $s \leq k - 1$, a s–step equilibrium exists if, and only if, $q_0 \in [\underline{q}^s, \bar{q}^s) \cap (0, 1)$, where $\underline{q}^s \leq \underline{q}^{s-1} < \bar{q}^s < \bar{q}^{s-1}$. Let v_L^s and v_B^s be, respectively,

the payoff functions to type L sellers and buyers in a s–step equilibrium. The functions v_L^s and v_B^s satisfy the properties of v_L^2 and v_B^2 described in Lemma 5 (in the Appendix); in particular, v_L^s is continuous and strictly increasing in q_0. As in the case of 2–step equilibria, the necessary and sufficient conditions for the existence of a k–step equilibrium are

$$q^+\left(q_0, v_L^{k-1}(q')\right) = q', \tag{18}$$

$$q' \in \left[\underline{q}^{k-1}, \overline{q}^{k-1}\right) \cap (0,1), \tag{19}$$

$$\pi_h^B(q_0) < \pi_\ell^B\left(q_0, \overline{\delta}, v_L^{k-1}(q'), v_B^{k-1}(q')\right). \tag{20}$$

Since v_L^{k-1} is increasing in q_0, $p_\ell/v_L^{k-1}(q') \geq p_\ell/v_L^{k-1}(\overline{q}^{k-1})$ for all $q' \in [\underline{q}^{k-1}, \overline{q}^{k-1}) \cap (0,1)$. Hence, $p_\ell/v_L^{k-1}(\overline{q}^{k-1}) \geq \overline{\delta}$ implies that $q^+(q_0, v_L^{k-1}(q')) = 1$ for all $q' \in [\underline{q}^{k-1}, \overline{q}^{k-1}) \cap (0,1)$, in which case there exists no k–step equilibrium.

Suppose then that $p_\ell/v_L^{k-1}(\overline{q}^{k-1}) < \overline{\delta}$. As we prove in the Appendix, given the characteristics of $(k-1)$–step equilibria, all the crucial features of 2–step equilibria are true for k–step equilibria. First, (18) and (19) imply (20). Second, if we define $Q_+^k : q_0 \mapsto q'$ as the map implied by (18), then Q_+^k is continuous and strictly increasing. This implies that a k–step equilibrium exists if, and only if, $q_0 \in [\underline{q}^k, \overline{q}^k) \cap (0,1)$, where the lower bound $\underline{q}^k$ is such that $q^+(\underline{q}^k, v_L^{k-1}(\underline{q}^{k-1})) = \underline{q}^{k-1}$ if $p_\ell/v_L^{k-1}(\underline{q}^{k-1}) < \overline{\delta}$ and $\underline{q}^k = 0$ otherwise, and the upper bound $\overline{q}^k$ satisfies $q^+(\overline{q}^k, v_L^{k-1}(\overline{q}^{k-1})) = \overline{q}^{k-1}$, where $v_L^{k-1}(\overline{q}^{k-1}) \equiv \lim_{q \to \overline{q}^{k-1}} v_L^{k-1}(q)$; note that $\underline{q}^k \leq \underline{q}^{k-1}$ and $\overline{q}^k < \overline{q}^{k-1}$ by definition. Finally, we have that $\underline{q}^{k-1} < \overline{q}^k$.

We know from above that given $q_0 \in [\underline{q}^k, \overline{q}^k) \cap (0,1)$, $Q_+^k(q_0) \in [\underline{q}^{k-1}, \overline{q}^{k-1}) \cap (0,1)$ is the value of q_1 in any k–step equilibrium when the initial fraction of type H sellers is q_0. Thus, the payoffs to buyers and type L sellers in a k–step equilibrium are well–defined and given, respectively, by

$$v_B^k(q_0) = \int \max\left\{\pi_h^B(q_0), \pi_\ell^B\left(q_0, \delta, v_L^{k-1}(Q_+^k(q_0)), v_B^{k-1}(Q_+^k(q_0))\right)\right\} dF(\delta), \tag{21}$$

and

$$v_L^k(q_0) = \xi^k(q_0)p_h + (1 - \xi^k(q_0)) \int \max\left\{p_\ell, \delta v_L^{k-1}(Q_+^k(q_0))\right\} dF(\delta), \tag{22}$$

where

$$\xi^k(q_0) = \int \mathbb{I}\{\pi_h^B(q) \geq \pi_\ell^B\left(q, \delta, v_L^{k-1}(Q_+^k(q_0)), v_B^{k-1}(Q_+^k(q_0))\right)\} dF(\delta)$$

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is the fraction of buyers who offer p_h at $t = 0$. Crucially, v_B^k, v_L^k, and ξ^k have the same properties that we establish for v_B^2, v_L^2 and ξ^2 in Lemma 5, which allows us to proceed by induction. We begin this inductive process for $k = 3$, and continue as long as $p_\ell / v_L^{k-1}(\overline{q}^{k-1}) < \overline{\delta}$, which ensures $\overline{q}^k > 0$ and thus the existence of a k–step equilibrium. The following theorem provides a full characterization of the equilibrium set.

Theorem 1. *There exists $1 \leq K < \infty$ and sequences $\{\underline{q}^k\}_{k=0}^K$ and $\{\overline{q}^k\}_{k=0}^K$, with $\overline{q}^0 = 1$, $\underline{q}^K = 0$, and $\underline{q}^k \leq \underline{q}^{k-1} < \overline{q}^k < \overline{q}^{k-1}$ for all $k \in \{1, \ldots, K\}$, such that a k–step equilibrium exists if, and only if, $q_0 \in [\underline{q}^k, \overline{q}^k) \cap (0, 1)$. Moreover, for each $q_0 \in [\underline{q}^k, \overline{q}^k) \cap (0, 1)$, there exists a unique $q' \in [\underline{q}^{k-1}, \overline{q}^{k-1})$ such that $q' = Q_+^k(q_0)$ is the value of q_1 in any k–step equilibrium when the initial fraction of type H sellers is q_0.*

The payoffs for buyers and type L sellers are uniquely defined in every equilibrium and are determined recursively as follows: (i) $v_B^0(q_0) = \pi_h^B(q_0)$ and $v_L^0(q_0) \equiv p_h$; (ii) for each $k \in \{1, \ldots, K\}$, v_B^k and v_L^k are given by (21) and (22), respectively.

The cutoffs $\{\underline{q}^k\}_{k=0}^{K-1}$ and $\{\overline{q}^k\}_{k=1}^K$ are defined recursively as follows: (i) $\underline{q}^0$ is the unique value of q_0 for which $\pi_h^B(q_0) = \pi_\ell^B\left(q_0, \overline{\delta}, v_L^0, v_B^0(q_0)\right)$ and, for each $k \in \{1, \ldots, K\}$, $\underline{q}^k$ is such that $q^+(\underline{q}^k, v_L^{k-1}(\underline{q}^{k-1})) = \underline{q}^{k-1}$ if $p_\ell / v_L^{k-1}(\underline{q}^{k-1}) < \overline{\delta}$, and $\underline{q}^k = 0$ otherwise; (ii) $\overline{q}^1$ is the only value of q_0 for which $\pi_h^B(q_0) = \pi_\ell^B\left(q_0, \overline{\delta}, v_L^0, v_B^0(q^+(q_0, v_L^0))\right)$ and, for each $k \in \{2, \ldots, K\}$, $\overline{q}^k$ is such that $q^+(\overline{q}^k, v_L^{k-1}(\overline{q}^{k-1})) = \overline{q}^{k-1}$. Finally, $K = \max\{k : p_\ell / v_L^{k-1}(\overline{q}^{k-1}) < \overline{\delta}\}$.

Theorem 1 offers a complete characterization of the equilibrium set. In particular, it specifies a sequence of cutoffs that partition the interval $(0, 1)$ into regions such that, for all q_0 in one such region, there exists an equilibrium in which the market takes the same number of periods k to clear. Figure 3 below illustrates these cutoffs for the case in which $u_H = 1$, $p_h = 0.6$, $u_L = 0.42$, $p_\ell = 0.05$, and F is uniformly distributed over $[0, 0.5]$. In addition to plotting these cutoffs, we have also highlighted the maximum and minimum number of periods it takes before the market clears for each $q_0 \in (0, 1)$.

Notice that there is a natural monotonicity to the equilibria in the above example: for any $0 < q_0 < q_0' < 1$, if there exists a k–step equilibrium when the initial fraction of high quality assets is q_0, then there exists a k'–step equilibrium with $k' \leq k$ when the initial fraction
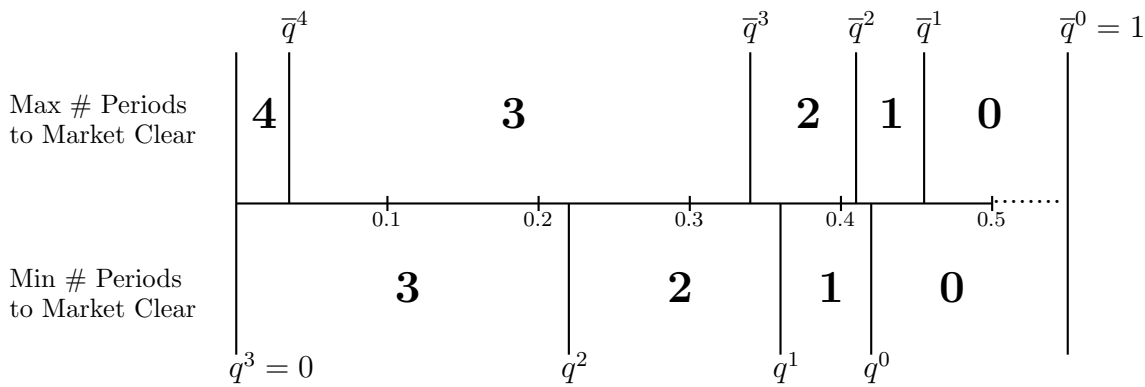
25

Figure 3: Equilibrium Cutoffs

of high quality assets is q_0'. This is true in general since $\bar{q}^k$ is strictly decreasing in k by Theorem 1, and so an increase in q_0 reduces the maximum number of periods it takes for the market to clear. Also notice that in the example, the market clears in at most four periods. However, depending on the distribution F and parameters of the model, market clearing can take a large number of periods when q_0 is small. We show this in the Supplementary Appendix.

5 Discussion

We now illustrate how the theory developed above can provide insight into a number of important issues. First we study how the initial composition of assets in the market affects the expected amount of time it takes to sell—or the *illiquidity* of—high quality assets. Then we study the dynamics of trade for a given value of q_0, exploring the model's implications for how prices, trading volume, and average quality evolve over time in this type of environment. Establishing such a benchmark is important, as it not only allows us to understand how frozen markets can thaw over time on their own, but also provides a framework to formally analyze the effects of various policies intended to unfreeze such markets; we discuss one particular policy intervention in the next section. Finally, the existence of multiple equilibria for a given value of q_0 suggests that coordination failures can exacerbate liquidity problems in dynamic, decentralized markets with adverse selection. Since such multiplicity does *not* arise in several

closely related (and well–known) environments, we end this section with a discussion of those features of our framework that are crucial for generating these coordination failures.

Liquidity and Lemons

Here we study how the fraction of lemons in the market affects the liquidity of high quality assets.[23] An asset is typically considered liquid if it can be sold quickly and at little discount. In many models, trade is instantaneous by construction, and thus the only measure of liquidity is the difference between the actual market price and the price in some frictionless benchmark; in these models, time is a margin that simply cannot adjust.[24] In the current model, the opposite is true: since p_h is the only price that type H sellers accept, the appropriate measure of liquidity for these assets is the expected amount of time it takes to sell them. We derive this statistic below and use it to study the relationship between the severity of the lemons problem (i.e., the value of q_0) and the liquidity of high quality assets.

Consider a k–step equilibrium with initial fraction $q_0 \in [\underline{q}^k, \overline{q}^k) \cap (0,1)$ of high quality assets, and define the sequence $\{q_t\}_{t=1}^{k}$ by $q_t = Q_{+}^{k-t+1}(q_{t-1})$ for $t \in \{1, \ldots, k\}$. By construction, q_t is the fraction of high quality assets in the market in period t. Therefore, the probability that a type H seller trades his asset in period $t \in \{0, \ldots, k\}$ is

$$\lambda^k(t|q_0) = \left\{ \prod_{s=0}^{t-1} \left[1 - \xi^{k-s}(q_s) \right] \right\} \xi^{k-t}(q_t),$$

where $\xi^k(q)$ is the fraction of buyers who offer p_h in the first period of trade in a k–step equilibrium when the starting fraction of type H sellers is q. The expected number of periods it takes to sell a high quality asset in the equilibrium under consideration is then

$$E_H^k(q_0) = \sum_{t=0}^{k} \lambda^k(t|q_0)t.$$

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[23]Focusing on the ability to sell high quality assets is standard in this literature, going back to Akerlof (1970). Of course, a seller can always sell a low quality asset instantaneously at price p_ℓ.

[24]In the finance literature, the typical measure of liquidity is the (inverse of) the bid–ask spread, which can be generated by exogenous transaction costs (see, e.g., Amihud and Mendelson (1986) and Constantinides (1986)), asymmetric information (see Kyle (1985) and Glosten and Milgrom (1985)), or search frictions (see Duffie et al. (2005)), among other things. Eisfeldt (2004) provides an alternative definition, but also in a model in which trade is instantaneous; see the discussion of this model in the Related Literature section.

We know from the proof of Theorem 1 that, in any k–step equilibrium, both $\xi^k(q_0)$ and $Q_+^k(q_0)$ are increasing in q_0. Hence, an increase in q_0 implies an increase in the fraction of buyers who offer p_h in the first period of trade. Moreover, an increase in q_0 also leads to an increase in q_t for all $t \in \{1, \ldots, k\}$, which in turn implies an increase in the fraction of buyers who offer p_h in every period before the market clears. Taken together, these two facts help to establish that $E_H^k(q_0)$ is a decreasing function of q_0; we present a formal proof of this result in Lemma 6 in the Appendix.

As we established earlier, for some values of q_0 there exist multiple equilibria that take a different number of periods for the market to clear. This, of course, makes comparing the liquidity of high quality assets across different values of q_0 difficult. Here we do not take a stance on equilibrium selection and instead focus on the relationship between the minimum expected number of periods it takes for a type H seller to sell his asset and q_0. Let $\underline{\mathcal{E}}(q_0)$ be given by

$$\underline{\mathcal{E}}(q_0) = \min \left\{ E_H^k(q_0) : \exists \text{ a } k\text{–step equilibrium given } q_0 \right\}.$$

In Lemma 7 in the Appendix we use the fact that $E_H^k(q_0)$ is decreasing in q_0 to show that $\underline{\mathcal{E}}(q_0)$ is decreasing in q_0. Thus, a reduction in the initial fraction of high quality assets reduces their liquidity in the sense that it increases the smallest expected amount of time it takes to sell them.[25]

The Dynamics of Trade

We now illustrate typical market dynamics for a given value of q_0. The numerical example of Section 4 is a convenient vehicle for conveying the intuition; we choose $q_0 = 0.1$, which falls within the set of 3–step equilibria. The average price in period $t \in \{0, \ldots, k\}$ in a k–step equilibrium is given by

$$p_t^{avg} = \xi^{k-t}(q_t)p_h + \left[1 - \xi^{k-t}(q_t)\right] p_\ell,$$

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[25]Alternatively, one could compare the liquidity of high quality assets across different values of q_0 by applying a rule that selects a particular value of k for each q_0. For example, if we let $k_{\max}(q_0) = \max \{k : \exists \text{ a } k\text{–step equilibrium given } q_0\}$ and define $\mathcal{E}_{\max}(q_0) = E_H^{k_{\max}(q_0)}(q_0)$, it is possible to show that $\mathcal{E}_{\max}(q_0)$ is also decreasing in q_0.

where, as above, $\{q_t\}_{t=1}^k$ is the sequence such that $q_t = Q_+^{k-t+1}(q_{t-1})$ for $t \in \{1, \ldots, k\}$. In figure 4 below, we plot the evolution of q_t and p_t^{avg} in the example.

In the first two periods of trade, the fraction of high quality assets is sufficiently low that all buyers offer p_ℓ. All type H sellers and patient type L sellers reject this offer, but sufficiently impatient type L sellers accept, causing the average quality of assets in the market to be higher in the following period. In the third period of trade, the fraction of high quality assets is sufficiently high that some impatient buyers offer p_h, increasing the average price. Still, patient buyers continue to offer p_ℓ and (perhaps) wait for market conditions to improve. In the fourth period of trade, all remaining buyers offer p_h and the market clears. Thus, average prices increase over time along with average quality. In the example, the price path exhibits an S–shape: prices are persistently low in early periods, and then quickly increase in the latter stages of trade.

<div align="center">INSERT FIGURE 4 HERE</div>

Many of the features of the above example are true in general. We know from the proof of Theorem 1 that $\xi^k(q) \le \xi^{k-1}(Q_+^k(q))$ for any k–step equilibrium. Hence, the fraction of buyers who offer p_h increases over time, so that p_t^{avg} increases over time as well. Now note that if

$$\pi_h^B(q_0) \le (1 - q_0) F\left(\frac{p_\ell}{v_L^{k-1}(q_1)}\right)[u_L - p_\ell],$$

then only low quality assets are exchanged in the first period of trade. The trade of high quality assets remains frozen until the first period in which myopic buyers find it strictly optimal to offer p_h.

Multiplicity of Equilibria

The presence of multiple equilibria for some values of q_0 suggests that liquidity problems can be exacerbated by coordination failures. At the heart of this multiplicity is the fact that the behavior of an individual buyer depends on the future composition of assets in the market, which in turn is determined by the aggregate behavior of buyers.

Identifying the ingredients of our framework that lead to multiple equilibria—that there are many buyers, and that these buyers are forward–looking—is helpful in understanding why such multiplicity does *not* typically arise in certain related environments. For example, in models of bargaining with asymmetric information in which there is only one buyer and one seller (see, e.g., Vincent (1989), Evans (1989), and Deneckere and Liang (2006)), clearly there is no scope for coordination between buyers' actions; as a result, there is typically a unique sequential equilibrium in these models. Alternatively, in similar frameworks in which a single seller with private information meets a sequence of buyers (see, e.g., Hörner and Vieille (2009) and the references therein), the buyers are typically assumed to be myopic. As a result, there is no potential for buyers to coordinate their behavior based on future payoffs, and again the type of multiplicity that we find here does not emerge.[26]

6 Application: The Market for Legacy Assets

The theory developed above provides a parsimonious framework to study the role that asymmetric information played in disrupting trade in the market for asset–backed securities during the financial crisis that began in 2007. Since our model provides a formal treatment of how a frozen market can thaw over time on its own, it also provides an ideal environment to analyze how various government interventions affect this process. To illustrate this point, in this section we assess the theoretical implications of a policy that was recently implemented in an attempt to restore liquidity in the market for asset–backed securities. We should note that there is nothing special per se about the policy we consider, over and above the fact that it was actually implemented. However, as our exercise produces some surprising results, we believe it underscores the need for explicit models to formally analyze the implications of intervention in these types of markets.

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[26]The two ingredients we identify are not sufficient for multiplicity. For example, in Janssen and Roy (2002), there is a continuum of forward–looking buyers and sellers who trade in a sequence of centralized markets in the presence of asymmetric information. However, their equilibrium requires that buyers receive zero expected payoffs from trading at any date, thus precluding the possibility of the multiplicity we find in our model. The two ingredients are also not necessary. Gerardi et al. (2010) show that multiple equilibria arise in sequential bargaining with asymmetric information when the party that makes the offers is the informed one; signalling is the source of multiplicity in their environment.

Our model shares many features of the markets for asset–backed securities. For one, buyers and sellers in this market negotiate bilaterally, as opposed to trading against their budget constraint in a competitive, centralized market where the law of one price prevails. Moreover, the market is inherently dynamic and non–stationary: there is a relatively fixed stock of assets of a particular vintage, and the manner in which the composition of assets remaining in the market evolves over time affects both prices and the incentive of market participants to delay trade. Finally, many believe that the presence of asymmetric information contributed to the illiquidity in this market. The decline of housing prices in various parts of the country introduced considerable heterogeneity into the quality of residential mortgage–backed securities, and many of the usual buyers of these assets did not possess the expertise to comfortably value the assets that were being offered by financial institutions.[27] As a result, both the prices and the volume of these assets being sold quickly dropped.[28]

The lack of liquidity in this market posed a threat to the economy at large. The Treasury department described the "challenge of legacy assets" as follows:

> One major reason [for the prolonged recession] is the problem of "legacy assets"—both real estate loans held directly on the books of banks ("legacy loans") and securities backed by loan portfolios ("legacy securities"). These assets create uncertainty around the balance sheets of these financial institutions, compromising their ability to raise capital and their willingness to increase lending... As a result, a negative cycle has developed where declining asset prices have triggered further deleveraging, which has in turn led to further price declines. The excessive discounts embedded in some legacy asset prices are now straining the capital of U.S. financial institutions, limiting their ability to lend and increasing the cost of credit throughout the financial system.

In response to this problem, the Treasury department introduced the Public–Private Invest-

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[27]The financial institutions that were selling these assets often had a team of analysts that had purchased the underlying assets (e.g., mortgages), studied their properties, and worked closely with the rating agencies to bundle them into more opaque final products. An extreme example of this asymmetric information is the "Abacus" deal, in which Goldman Sachs created and sold collateralized debt obligations to investors while simultaneously betting against them. In general, there are many reasons to believe that financial institutions often have better information about the quality of their assets than potential buyers, perhaps because they learn about the asset while they own it (as argued by Bolton et al. (2011)), or because they conduct research about the asset in anticipation of selling it (as argued by Guerrieri and Shimer (2010)). By now, there is a large literature on the role of asymmetric information in the financial crisis; see, e.g., Gorton (2009) and the references therein.

[28]For a detailed analysis, see Krishnamurthy (2010).

ment Program for Legacy Assets. Under this program, the government issued non–recourse loans to private investors to assist in buying legacy assets, with a minimum fraction of the purchase price being financed by the investor's own equity. This program essentially subsidizes the buyer's purchase and partially insures his downside loss; if the asset turns out to be a lemon, the buyer can default and incur only a fraction of the total loss from the purchase (his equity investment). The Treasury department described the "merits of this approach" as follows:

> This approach is superior to the alternatives of either hoping for banks to gradually work these assets off their books or of the government purchasing the assets directly. Simply hoping for banks to work legacy assets off over time risks prolonging a financial crisis, as in the case of the Japanese experience. But if the government acts alone in directly purchasing legacy assets, taxpayers will take on all the risk of such purchases—along with the additional risk that taxpayers will overpay if government employees are setting the price for those assets.

In an attempt to capture this policy response, suppose now that a buyer who pays price p for an asset can borrow $(1 - \gamma)p$ from the government. For simplicity, assume the buyer observes the quality of the asset immediately after buying it and then faces the following choice: either pay back the loan to the government, or default on the loan and surrender the asset. A buyer who pays price p for a type j asset repays his loan if, and only if, $u_j - (1 - \gamma)p > 0$. Thus, a buyer who receives a high quality asset always repays his loan, as does a buyer who pays p_ℓ for a low quality asset. However, a buyer who pays p_h for a low quality asset defaults if $\gamma \leq 1 - u_L/p_h$. Therefore, this policy amounts to a transfer $\tau = (1 - \gamma)p_h - u_L \in [0, p_h - u_L]$ to the buyers who pay p_h for a low quality asset.

Denote the payoff to a buyer from offering p_h given a transfer τ by

$$\pi_h^B(q_0, \tau) = q_0(u_H - p_h) + (1 - q_0)(u_L - p_h + \tau).$$

The payoff to a buyer from offering p_ℓ is still given by (9), and the characterization of the equilibrium set proceeds in exactly the same way as in Section 4. In particular, Theorem 1 is still valid with the only difference that now, in the recursive procedure that determines the equilibrium payoffs, the payoff to a buyer in a 0–step equilibrium is $\pi_h^B(q_0, \tau)$.

Let $v_B^k(q_0, \tau)$ and $v_L^k(q_0, \tau)$ be, respectively, the payoffs to buyers and type L sellers in a k–step equilibrium when the transfer is τ. Moreover, let $\underline{q}^k(\tau)$ and $\overline{q}^k(\tau)$ be, respectively, the lower and upper cutoffs for a k–step equilibrium as a function of τ. The cutoff $\underline{q}^0(\tau)$ is the unique value of q_0 such that

$$\pi_h^B(q_0, \tau) = \pi_\ell^B\left(q_0, \overline{\delta}, v_L^0, v_B^0(q_0, \tau)\right),$$

where $v_L^0 = v_L^0(q_0, \tau) \equiv p_h$. The cutoffs $\underline{q}^1(\tau)$ and $\overline{q}^1(\tau)$ are the unique values of q_0 satisfying the following two equations, respectively:

$$q^+\left(q_0, v_L^0\right) = \underline{q}^0;$$
$$\pi_\ell^B\left(q_0, \overline{\delta}, v_L^0, v_B^0(q^+(q_0, v_L^0), \tau)\right) = \pi_h^B(q_0, \tau).$$

It is straightforward to show that $\underline{q}^0(\tau)$, $\underline{q}^1(\tau)$, and $\overline{q}^1(\tau)$ are decreasing in τ. Therefore, if the initial fraction of high quality assets is sufficiently large, an increase in τ can decrease the amount of time it takes for the market to clear, and thus increase market liquidity. For example, for $\tau \in (0, p_h - u_L)$, there exists a 0–step equilibrium when $q_0 \in (\underline{q}^0(\tau), \underline{q}^0(0))$, whereas the market would take at least one additional period to clear if $\tau = 0$. Intuitively, since the transfer τ increases the payoff from offering p_h, buyers are more willing to offer the high price given any fraction of lemons in the market.

However, the policy under consideration has a second, opposing effect. Since buyers are more willing to offer p_h when they are partially insured against buying a lemon, the average price sellers receive in the future increases as τ grows larger. Ceteris paribus, this makes sellers more likely to reject offers of p_ℓ in early rounds of trade, opting instead to wait for larger payoffs later in the game. To see this, let $\xi^1(q_0, \tau)$ be the mass of buyers who offer p_h in the first period of trade in a 1–step equilibrium when the transfer is τ. The payoff to a type L seller in a 1–step equilibrium is then given by

$$v_L^1(q_0, \tau) = \xi^1(q_0, \tau)p_h + (1 - \xi^1(q_0, \tau))\int \max\left\{p_\ell, \delta v_L^0\right\} dF(\delta).$$

Straightforward algebra shows that $\xi^1(q_0, \tau)$, and thus $v_L^1(q_0, \tau)$, are increasing in τ.

Now observe that $\underline{q}^2(\tau)$ satisfies

$$q^+\left(\underline{q}^2(\tau), v_L^1(\underline{q}^1(\tau), \tau)\right) = \underline{q}^1(\tau). \tag{23}$$

Thus, as τ increases, two opposing forces are at work. On the one hand, since $\underline{q}^1$ is decreasing in τ, this tends to decrease $\underline{q}^2$ as well; holding v_L^1 constant in (23), $\underline{q}^2$ is decreasing in $\underline{q}^1$. On the other hand, holding $\underline{q}^1$ fixed, v_L^1 is increasing in τ, which tends to make sellers more likely to reject an offer of p_ℓ at $t = 0$. This implies a smaller increase in the fraction of high quality assets, and hence a larger value of $\underline{q}^2$. This second effect is not present in a 1–step equilibrium since v_L^0 is constant in τ, which explains why $\underline{q}^1$ is unambiguously decreasing in τ. However, when the market is two or more periods away from market–clearing, the second effect is active, and can even dominate the first effect. In other words, subsidizing the purchase of assets can increase the time required for market clearing, thus making the market *less liquid*. These considerations extend to k–step equilibria, with $k \geq 3$.

Using the numerical example from the previous section, Table 1 below summarizes the effect of a transfer τ that is equal to 25% of the loss from purchasing a lemon at price p_h, relative to the benchmark of $\tau = 0$.

Policy	$\underline{q}^3$	$\overline{q}^4$	$\underline{q}^2$	$\overline{q}^3$	$\underline{q}^1$	$\overline{q}^2$	$\underline{q}^0$	$\overline{q}^1$
$\tau = 0$	0	.036	.206	.344	.379	.410	.422	.455
$\tau = .25(p_h - u_L)$	0	.049	.231	.301	.340	.369	.382	.412

Table 1: Policy Analysis

One can see immediately that this policy allows markets to clear faster if the initial fraction of high quality assets is large, but it has little effect on (and can even increase) the time to market clearing if this fraction is small. Consider, for example, an economy with $q_0 = .4$: under this policy there exists an equilibrium that clears at $t = 0$, whereas it takes until at least $t = 1$ for the market to clear in the absence of this policy. However, the opposite is true for, say, $q_0 = .22$: the policy increases the minimum number of periods before the market clears from two to three. Thus, even without considering the *cost* of this type of intervention, we see that its efficacy depends crucially on the underlying severity of the lemons problem. More generally, this exercise highlights that the *timing* of any intervention is crucial in an environment with forward–looking agents; a policy that increases future payoffs relative to current payoffs will give agents the incentive to delay trade in early periods.[29]

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[29]Note that there would be a second effect if we introduced speculators that were more patient than sellers;

7 Assumptions and Market Efficiency

We make several assumptions in our model that allow for a complete analytical characterization of the equilibrium set. In this section, we discuss two important restrictions, and how our results might change if they were relaxed. The first is the assumption that a buyer is restricted to offer one of two exogenously specified prices, and the second is the assumption that all agents exit the market after trading. We close the section with a discussion of market efficiency as the time interval between trading opportunities converges to zero.

Prices

We begin with the assumption of two fixed prices. Though this restriction certainly has implications for the dynamics of trade, we argue here that it captures the key trade–off that buyers face when deciding on an offer. We also discuss the features of our equilibrium characterization that are preserved when this restriction is relaxed.

Suppose buyers are free to choose any price p when matched with a seller. Also, consider a variant of our model in which assets yield no flow dividends to a seller, but instead a type j seller pays a cost c_j for producing his asset, where $0 = c_L < u_L < c_H < u_H$. This change makes little difference in our benchmark model, but it makes the analysis less cumbersome when we place no restrictions on prices. In particular, since agents discount the future, it is easy to show that all sellers accept any offer $p \geq c_H$, so that no buyer offers more than c_H in equilibrium. In an abuse of notation, denote the payoff to a buyer from offering $p = c_H$ by

$$\pi_h^B(q) = q[u_H - c_H] + (1 - q)[u_L - c_H].$$

Now note that a buyer never offers $p \in (u_L, c_H)$ in equilibrium, as only type L sellers accept such an offer, and the buyer would receive a negative payoff. Therefore, a buyer effectively chooses between offering $p = c_H$ and a price that solves

$$\pi_\ell^B(q, \delta, v_L, v_B) = \max_p (1 - q)F\left(\frac{p}{v_L}\right)[u_L - p] + \left\{q + (1 - q)F\left(\frac{p}{v_L}\right)\right\}\delta v_B, \qquad (24)$$

in this case, these agents might purchase assets early from sellers in order to take advantage of increased prices at a later date. This is the "announcement effect" identified in Chiu and Koeppl (2009).

where, as before, $v_L > 0$ and $v_B > 0$ are the continuation payoffs to type L sellers and buyers, respectively. Further abusing notation, let $p_\ell(\delta, v_L, v_B)$ denote the solution to (24); one can easily show that this solution is independent of q. Moreover, if, for example, F is concave, this solution is unique.[30]

Thus, in this environment, when a buyer contemplates an optimal price offer, he faces the same trade-off captured in the model with two exogenous prices: either offer a high price that is accepted by all sellers and trade immediately, or a low price that will be accepted only by sufficiently impatient type L sellers. The crucial difference here is that the low price is sensitive to the buyer's discount factor, as well as the continuation payoffs of both buyers and type L sellers. These continuation values, in turn, depend on future prices, which again depend on future continuation payoffs. This makes it quite difficult to derive analytical results.

Nevertheless, several crucial results from our benchmark model can be established in this more general environment. First, one can show that the fraction of high quality assets increases over time until the market clears (see footnote 19). As in the case with exogenous prices, this implies that the market clears in finite time in every equilibrium. Second, using the same argument as in Section 4, one can show that there exists $\underline{q}^0 \in (0,1)$ such that a 0–step equilibrium exists if, and only if, $q_0 \in [\underline{q}^0, 1)$. Then, using the revised law of motion

$$q^+(q, v_L, v_B) = \frac{q}{q + (1-q)\left[1 - \int F\left(p^*(q, \delta, v_L, v_B)/v_L\right) dF(\delta)\right]},$$

where $p^*(q, \delta, v_L, v_B) = p_\ell(\delta, v_L, v_B)$ if $\pi_h^B(q) < \pi_\ell^B(q, \delta, v_L, v_B)$ and p_h otherwise, one can follow the recursive procedure in Section 4 to derive the necessary and sufficient conditions for a k–step equilibrium for $k \geq 1$:

$$q^+\left(q_0, v_L^{k-1}(q'), v_B^{k-1}(q')\right) = q'; \tag{25}$$

$$q' \in \left[\underline{q}^{k-1}, \overline{q}^{k-1}\right) \cap (0,1); \tag{26}$$

$$\pi_h^B(q) < \pi_\ell^B\left(q_0, \overline{\delta}, v_L^{k-1}(q'), v_B^{k-1}(q')\right). \tag{27}$$

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[30]Note that concavity of F is not necessary for uniqueness; even without this assumption, one can show that $p_\ell(\delta, v_L, v_B)$ is unique almost everywhere (i.e., at all but a countable number of values of δ).

Conditions (25) to (27) are analogous to conditions (18) to (20) in our benchmark model with two fixed prices. Crucially, the fixed point mapping described in (18) was shown to be single–valued, continuous, and strictly increasing in q_0, which greatly simplified the equilibrium characterization. Establishing these properties for the analogous mapping described in (25) would allow for the same clean characterization of equilibria in this more general setting. Unfortunately, though these properties appear to be satisfied in a variety of numerical simulations, we cannot verify them analytically; since the law of motion depends explicitly on buyers' offers, and these offers in turn depend on future payoffs (which depend on both q' and future offers), the analysis quickly becomes considerably more complex.

However, if for each $k \geq 1$, we simply define $\underline{q}^k$ and $\overline{q}^k$ by

$$\underline{q}^k = \inf\{q_0 \in (0,1) : \exists \text{ a } k\text{–step equilibrium given } q_0\}$$
$$\overline{q}^k = \sup\{q_0 \in (0,1) : \exists \text{ a } k\text{–step equilibrium given } q_0\},$$

it is possible to show that: (i) $\underline{q}^k > 0$ implies that $\underline{q}^{k+1} < \underline{q}^k$; and ($ii$) $\overline{q}^k > 0$ implies that $\overline{q}^{k+1} < \overline{q}^k$. Thus, even in the absence of a complete characterization, there is a sense in which a reduction in q_0 increases the amount of time for markets to clear.

One–Time Entry

In order to study how markets clear on their own in the simplest possible environment, we assume there is a fixed stock of buyers and sellers, and that these agents leave the market after trading. In doing so, we abstract from several interesting issues. For example, one might want to allow buyers to re–sell their asset, either because they discover it is of low quality or because of some stochastic, exogenous taste shock (as in Chiu and Koeppl (2009)). The effect of allowing re–sale depends crucially on the information structure.

Suppose, for instance, that agents observe the history of trade for a particular asset.[31] Consider first the case in which there are no taste shocks, and so buyers who purchase a

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[31]This is possible in many markets, either because there are relatively few agents in the market (as in some markets for very specific financial assets), or because there exists a technology that keeps track of such histories (such as Carfax, in the used car market).

high quality asset have no reason to re–sell it. In this case, agents are able to infer that a particular asset is being re–sold precisely because it is low quality, leaving no expected gains from trade. Thus, buyers who purchase a lemon have no incentive to attempt to re–sell it even if this is feasible. Consider now the case in which there are taste shocks, so that the decision to re–sell an asset does not signal it as being of low quality. While the analysis in this case would certainly be more complicated, we conjecture that allowing re–sale makes markets less liquid than they are in the baseline model. Intuitively, since buyers may need to re–sell a high quality asset that they purchase, they should be less willing to offer p_h to begin with (because of adverse selection in the re–sale market), thus decreasing the liquidity of these assets.

In addition to allowing buyers to re–enter the market, one might also want to consider what would happen if new sellers could generate assets at some cost and enter the market over time. Again, the consequences of this extension depend heavily on assumptions about the properties of an asset that are observable. For instance, if an asset's date of creation or "vintage" is observable, one could imagine a constant inflow of new vintages at every date, where the trading dynamics of each vintage resembles those of the single vintage we consider in our baseline model. Alternatively, if vintages are not observable, an entry condition could be used to endogenously determine the composition of high and low quality assets in the market. In fact, we think the question of asset generation is extremely important— particularly in the context of policy analysis—and our model is ideally situated to address this issue. This is the focus of current work.

Market Efficiency

Much of the literature on sequential bargaining and trade in dynamic, decentralized markets focuses on whether equilibrium outcomes become efficient as trading frictions vanish, i.e., as the time interval between two consecutive trading opportunities converges to zero. We know that when adverse selection is present, real inefficiencies can persist as trading frictions vanish; see Janssen and Roy (2002), Deneckere and Liang (2006), and Hörner and Vieille (2009). Given the assumption of exogenous prices, our framework is not ideally suited to

address this question, though. Indeed, since we require that $\overline{\delta}(u_H - p_h) \leq u_L - p_\ell$, we cannot consider the limit as $\overline{\delta}$ converges to one (and $F(\delta)$ converges to zero for all $\delta < \overline{\delta}$) unless we either drop the assumption that $u_H - p_h > u_L - p_\ell$ or allow p_h and p_ℓ to change as $\overline{\delta}$ increases. The assumption that $u_H - p_h > u_L - p_\ell$ was not necessary for our analysis, though. In the Supplementary Appendix we show that if $u_H - p_h \leq u_L - p_\ell$, then under mild assumptions about the distribution of discount factors, the amount of time it takes for the market to clear does not converge to zero as trading frictions vanish.[32]

8   Conclusion

This paper provides a theory of how markets suffering from adverse selection can recover over time on their own. Sellers with low quality assets exit the market relatively more quickly than those with high quality assets, causing the average quality of assets in the market to increase over time. Eventually, all assets are exchanged. The model delivers sharp predictions about how long this process takes, or the extent to which the market is *illiquid*, as well as the behavior of prices over time. Interestingly, we find multiple equilibria, which suggests that there is scope for coordination failures in dynamic, decentralized markets with adverse selection. We argue that this model serves as a useful benchmark for understanding how exogenous events or interventions will affect the speed with which markets recover. We provide a specific example from the recent financial crisis, and show how accounting for dynamic considerations can shed light on potentially harmful, unintended effects of policies aimed at restoring liquidity in frozen markets. Natural extensions include allowing sellers the choice of what type of asset to generate and when to enter the market, allowing buyers to acquire costly information about an asset's quality, and introducing aggregate uncertainty and learning. These are left for future work.

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[32]More generally, if we allow endogenous prices as in Subsection 7, then an argument very similar to the one in the Appendix shows that under the same assumptions on the distribution of discount factors, the amount of time it takes for the market to clear is bounded away from zero as the time between two consecutive trading opportunities goes to zero.

Appendix A: Omitted Proofs and Intermediate Results

Proof of Lemma 3

Let σ^* be an equilibrium and assume, toward a contradiction, that $T(\sigma^*) = \infty$. First notice that there exists $q^* \in (0,1)$ such that

$$q^*[u_H - p_h] + (1 - q^*)[u_L - p_h] = u_L - p_\ell. \tag{28}$$

Since $\overline{\delta}[u_H - p_h] \leq u_L - p_\ell$, the right side of (28) is an upper bound for the payoff a buyer can obtain if he offers p_ℓ. Hence, if the fraction of type H sellers in the market is above q^*, then all buyers offer p_h and the market clears. Consequently, for all $t \geq 0$, q_t is bounded above by q^*, and so is the limit of the sequence $\{q_t\}_{t=0}^{\infty}$. Now observe that since $V_t^L(\sigma^*) \leq p_h$, we have that $F(p_\ell / V_t^L(\sigma^*)) \geq F(p_\ell / p_h)$ for all $t \geq 1$. Thus, the law of motion (8) implies that

$$q_t \geq \frac{q_0}{q_0 + (1 - q_0)[1 - F(p_\ell/p_h)]^t}.$$

However, the right side of the above equation converges to one, a contradiction. ∎

Proof of Proposition 1

Let $\eta^0(q_0, \delta) = \pi_h^B(q_0) - \pi_\ell^B(q, \delta, v_L^0, v_B^0(q_0))$. Note that

$$\eta^0(q_0, \delta) = (1 - \delta)v_B^0(q_0) - (1 - q_0)F\left(\frac{p_\ell}{p_h}\right)\left[u_L - p_\ell - \delta v_B^0(q_0)\right].$$

Since $v_B(q) \leq u_H - p_h$ and $u_L - p_\ell \geq \overline{\delta}[u_H - p_h]$, we then have that

$$\frac{\partial \eta^0}{\partial q_0}(q, \delta) = (1 - \delta)[u_H - u_L] + F\left(\frac{p_\ell}{p_h}\right)[u_L - p_\ell - \delta v_B^0(q)] + (1 - q)F\left(\frac{p_\ell}{p_h}\right)\delta[u_H - u_L] > 0.$$

Now observe that $\eta^0(0) < 0 < \eta^0(1)$ and η^0 is continuous. So, there exists a unique $\underline{q}^0 \in (0,1)$ such that $\eta^0(q_0, \overline{\delta}) \geq 0$ if, and only if, $q_0 \geq \underline{q}^0$. Moreover, $\eta(\underline{q}^0, \overline{\delta}) = 0$ implies that

$$v_B^0(q_0)\left[1 - \overline{\delta} + (1 - q_0)F\left(\frac{p_\ell}{p_h}\right)\overline{\delta}\right] = (1 - q_0)F\left(\frac{p_\ell}{p_h}\right)[u_L - p_\ell]$$

when $q_0 = \underline{q}^0$, and so $v_B(\underline{q}^0) > 0$. Thus, σ^0 is an equilibrium if, and only if, $q_0 \in [\underline{q}^0, 1)$.

Suppose now $q_0 < \underline{q}^0$ and consider a strategy profile $\widetilde{\sigma}^0$ with the necessary property that all buyers offer p_h in $t = 0$. One alternative for a buyer is to offer p_h in every period regardless

of his discount factor. Let $\widetilde{\mathbf{p}}$ denote this strategy. It must be that $V_t^B(\widetilde{\sigma}^0) \geq V_t^B(\widetilde{\mathbf{p}}|\widetilde{\sigma}^0)$ for all $t \geq 0$ if $\widetilde{\sigma}^0$ is to be an equilibrium. Now observe that if the probability of trade in each period is $\alpha \in (0,1)$, then

$$V_t^B(\widetilde{\mathbf{p}}|\widetilde{\sigma}^0, \alpha) = \sum_{\tau=1}^{\infty} \alpha(1-\alpha)^{\tau-1}(\mathbb{E}[\delta])^{\tau-1} v_B^0(q_{t+\tau-1}^{\alpha})$$

for all $t \geq 0$, where $q_{t+\tau-1}^{\alpha}$ is the fraction of type H sellers in the market in period $t+\tau-1$. It is easy to see that the sequence $\{q_t^{\alpha}\}_{t=0}^{\infty}$ is non–decreasing. Hence,

$$V_t^B(\widetilde{\mathbf{p}}|\widetilde{\sigma}^0, \alpha) \geq \sum_{\tau=1}^{\infty} \alpha(1-\alpha)^{\tau-1}(\mathbb{E}[\delta])^{\tau-1} v_B^0(q_0),$$

which implies that $V_t^B(\widetilde{\mathbf{p}}|\widetilde{\sigma}^0) \geq v_B^0(q_0)$. Thus, $\widetilde{\sigma}^0$ is an equilibrium only if $V_1^B(\widetilde{\sigma}^0) \geq v_B^0(q_0)$. However, since $V_1^L(\widetilde{\sigma}^0) \leq p_h$ and $q_0 < \underline{q}^0$ implies that $\eta^0(q_0) < 0$, we have that

$$\pi_{\ell}^B\left(q_0, \overline{\delta}, V_1^L(\widetilde{\sigma}^0), V_1^B(\widetilde{\sigma}^0)\right) \geq \pi_{\ell}^B\left(q_0, \overline{\delta}, v_L^0, v_B^0(q_0)\right) > \pi_h^B(q_0)$$

for all $q_0 < \underline{q}^0$. Therefore, there exists $\delta' < \overline{\delta}$ such that it can *not* be optimal for a buyer with discount factor in $(\delta', \overline{\delta}]$ to offer p_h at $t = 0$, so that the market clearing immediately cannot be an equilibrium outcome. ∎

Proof of Proposition 2

Recall that $q^+(q, v_L^0)$ is strictly increasing in q when $p_{\ell}/v_L^0 < \overline{\delta}$ and that $q^+(q, v_L^0) \equiv 1$ otherwise. From this it is immediate to see that there exists $\underline{q}^1 < \underline{q}^0$ such that $q^+(q_0, v_L^0) \geq \underline{q}^0$ if, and only if, $q_0 \in [\underline{q}^1, 1]$. Note that $\underline{q}^1 = 0$ if $p_{\ell}/v_L^0 \geq \overline{\delta}$ and $\underline{q}^1$ is such that $q^+(\underline{q}^1, v_L^0) = \underline{q}^0$ otherwise. Now let $\eta^1(q_0, \delta) = \pi_h^B(q_0) - \pi_{\ell}^B\left(q_0, \delta, v_L^0, v_B^0[q^+(q_0, v_L^0)]\right)$. Straightforward algebra shows that

$$\frac{\partial \eta^1}{\partial q_0}(q, \delta) = F\left(\frac{p_{\ell}}{p_h}\right)\left\{u_L - p_{\ell} - \delta v_B^0[q^+(q, p_h)]\right\}$$

$$+(u_H - u_L)\left\{1 - \delta\left\{q + (1-q)\left[1 - F\left(\frac{p_{\ell}}{p_h}\right)\right]\right\}\frac{\partial q^+}{\partial q}\right\}.$$

Thus, since $u_L - p_{\ell} \geq \overline{\delta}[u_H - p_h]$ and

$$\left\{q + (1-q)\left[1 - F\left(\frac{p_{\ell}}{p_h}\right)\right]\right\}\frac{\partial q^+}{\partial q} = 1 - \frac{qF(p_{\ell}/p_h)}{q + (1-q)[1 - F(p_{\ell}/p_h)]} < 1,$$

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we can then conclude that η^1 is strictly increasing in q_0 regardless of the value of p_ℓ/p_h. Since $\eta^1(0,\bar\delta) < 0 < \eta^1(1,\bar\delta)$ and η^1 is continuous in q_0, there exists a unique $\bar q^1 \in (0,1)$ such that $\eta^1(q_0,\bar\delta) < 0$ if, and only if $q_0 \in [0,\bar q^1)$. Hence, $\pi_h^B(q_0) < \pi_\ell^B\left(q_0, \bar\delta, v_L^0, v_B^0\left[q^+(q_0, v_L^0)\right]\right)$ if, and only if $q_0 \in [0,\bar q^1)$. Next, observe that since $v_B^0[q^+(q_0, p_h)] > v_B^0(q_0)$ for all $q_0 \in (0,1)$,

$$\pi_\ell^B\left(\underline q^0, \bar\delta, v_L^0, v_B^0[q^+(\underline q^0, p_h)]\right) > \pi_\ell^B\left(\underline q^0, \bar\delta, v_L^0, v_B^0(\underline q^0)\right) = \pi_h^B(\underline q^0).$$

Thus, $\eta^1(\underline q^0, \bar\delta) < 0$, from which we obtain that $\bar q^1 > \underline q^0$. ∎

Lemma 4 and Proof

Lemma 4. *The payoff v_B^1 is continuous in q_0 and $v_B^1(q_0') - v_B^1(q_0) \le (q_0' - q_0)[u_H - u_L]$ for all $q_0' > q_0$. The fraction ξ^1 is continuous and increasing in q_0, with $\lim_{q_0 \to \bar q^1} \xi^1(q_0) = 1$. The payoff v_L^1 is continuous and increasing in q_0, with $\lim_{q_0 \to \bar q^1} v_L^1(q_0) = v_L^0$.*

Proof: Note that $q^+(q_0, v_L^0)$ continuous in q_0 implies that $v_B^1(q_0)$ is also continuous in q_0. We now prove that $v_B^1(q_0') - v_B^1(q_0) \le (q_0' - q_0)[u_H - u_L]$ for all $q_0' > q_0$. First, note that

$$\pi_\ell^B\left(q, \delta, v_L, \pi_h^B[q^+(q, v_L)]\right) = \delta\pi_h^B(q) + (1-q)F\left(\frac{p_\ell}{v_L}\right)[u_L - p_\ell - \delta(u_L - p_h)] \tag{29}$$

for all $q \in (0,1)$ and $\delta \in [0,\bar\delta]$. This fact is useful in what follows. Now let

$$v_B^1(q_0, \delta) = \pi_\ell^B\left(q_0, \delta, v_L^0, v_B^0(Q_+^1(q_0))\right) + \max\{\eta^1(q_0,\delta), 0\}.$$

Since $v_B^1(q_0) = \int v_B^1(q_0, \delta)dF(\delta)$, we are done if we show that $q_0' > q_0$ implies that

$$v_B^1(q_0', \delta) - v_B^1(q_0, \delta) \le (q_0' - q_0)[u_H - u_L] \tag{30}$$

regardless of δ. We know from the proof of Proposition 2 that η^1 is strictly increasing in q_0. Since $\eta^1(0,\delta) < 0 < \eta^1(0,\delta)$ for all $\delta \in [0,\bar\delta]$, for each $\delta \in [0,\bar\delta]$ there exists a unique $q^* = q^*(\delta) \in (0,1)$ such that $\eta^1(q^*,\delta) \ge 0$ if, and only if, $q \ge q^*$; note that $q^*(\bar\delta) = \bar q^1$. Now let $q_0' > q_0$. Since

$$v_B^1(q_0', \delta) - v_B^1(q_0, \delta) \le v_B^1(q_0', \delta) - \pi_h^B(q_0),$$

we have that (30) holds if $q_0' > q^*$. Suppose then that $q_0' \le q^*$. In this case, by (29),

$$v_B^1(q_0', \delta) - v_B^1(q_0, \delta) = \delta[\pi_h^B(q_0') - \pi_B^h(q_0)] + (q_0 - q_0')F\left(\frac{p_\ell}{v_L^0}\right)[u_L - p_\ell - \delta(u_L - p_h)],$$

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from which the desired result follows given that the second term on the right side of the above equation is negative.

Next, we prove that $\xi^1(q_0)$ is continuous and increasing in q_0, with $\lim_{q_0 \to \overline{q}^1} \xi^1(q_0) = 1$. Given that $\pi_\ell^B\left(q_0, \delta, v_L^0, v_B^0(Q_+^1(q_0))\right)$ is strictly increasing in δ, η^1 is strictly decreasing in δ. Let $\delta^1(q_0)$, with $q_0 \in [\underline{q}^1, \overline{q}^1) \cap (0, \overline{q}^1)$, be such that: (i) $\delta^1(q_0) = 0$ if $\eta^1(q_0, \delta^1(q_0)) = 0$; and (ii) $\eta^1(q_0, \delta^1(q_0)) = 0$ if $\eta^1(q_0, 0) > 0$. Since $\eta^1(\overline{q}^1, \overline{\delta}) = 0$ and η^1 is strictly increasing in q, $\delta^1(q_0)$ is uniquely defined. By construction, δ^1 is the cutoff discount factor below which a buyer finds it optimal to offer p_h in $t = 0$. Hence, the probability $\xi^1(q_0)$ that a buyer offers p_h in $t = 0$ is equal to $F(\delta^1(q_0))$. Given that η^1 is jointly continuous, a standard argument shows that δ^1 depends continuously on q_0. Moreover, the cutoff $\delta^1(q_0)$ is strictly increasing in q_0 if $\eta^1(q_0, 0) > 0$, as η^1 is strictly increasing in q. The desired result follows from the fact that F is continuous and strictly increasing and $\lim_{q_0 \to \overline{q}^1} \delta^1(\overline{q}^1) = \overline{\delta}$ (as $\eta^1(\overline{q}^1, \overline{\delta}) = 0$).

To finish the proof, note that the continuity of $\xi^1(q_0)$ and the fact that $\lim_{q_0 \to \overline{q}^1} \xi^1(q_0) = 1$ imply that $v_L^1(q_0)$ is continuous in q_0, with $\lim_{q_0 \to \overline{q}^1} v_L^0(q_0) = p_h = v_L^0$. $\blacksquare$

Proof of Proposition 3

We first show that (15) and (16) imply (17), so that the first two conditions completely determine the range of values of q_0 for which there exists a 2–step equilibrium. Suppose that $q' \in [\underline{q}^1, \overline{q}^1) \cap (0, 1)$. In order to prove that (17) is satisfied, it is sufficient to show

$$\pi_h^B(q') - \pi_h^B(q_0) \geq \pi_\ell^B\left(q', \delta, v_L^0, v_B^0[q^+(q', v_L^0)]\right) - \pi_\ell^B\left(q_0, \delta, v_L^1(q'), v_B^1(q')\right) \tag{31}$$

for all $\delta \in [0, \overline{\delta}]$. Condition (31) implies that, no matter his discount factor, the incentive of a buyer to choose p_ℓ in $t = 0$ is even greater than his incentive to choose p_ℓ in $t = 1$, when the fraction of type H sellers in the market is $q' > q_0$. First, note from (29) that

$$\pi_\ell^B\left(q', \overline{\delta}, v_L^0, v_B^0[q^+(q', v_L^0)]\right) = \delta \pi_h^B(q') + (1 - q') F\left(\frac{p_\ell}{v_L^0}\right) \left[u_L - p_\ell - \overline{\delta}(u_L - p_h)\right].$$

Second, since $v_B^1(q') \geq \pi_h^B(q')$, we have

$$\pi_\ell^B\left(q_0, \overline{\delta}, v_L^1(q'), v_B^1(q')\right) \geq \pi_\ell^B\left(q_0, \overline{\delta}, v_L^1(q'), \pi_h^B(q')\right)$$
$$= \delta \pi_h^B(q_0) + (1 - q_0) F\left(\frac{p_\ell}{v_L^1(q')}\right) \left[u_L - p_\ell - \overline{\delta}(u_L - p_h)\right];$$

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the second equality follows from (15) and (29). Therefore,

$$\pi_\ell^B\left(q', \bar\delta, v_L^0, v_B^0[q^+(q', v_L^0)]\right) - \pi_\ell^B\left(q_0, \bar\delta, v_L^1(q'), v_B^1(q')\right)$$
$$\leq \bar\delta\left[\pi_h^B(q') - \pi_h^B(q_0)\right] + \left[(1-q')F\left(\frac{p_\ell}{v_L^0}\right) - (1-q_0)F\left(\frac{p_\ell}{v_L^1(q')}\right)\right]\left[u_L - p_\ell - \bar\delta(u_L - p_h)\right].$$

Since $v_L^0 > v_L^1(q')$ for all $q' \in [\underline{q}^1, \bar{q}^1) \cap (0,1)$, $u_L < p_h$, and $q' > q_0$, the second term on the right side of the above inequality is negative, which confirms (31).

We now show that there exists a 2–step equilibrium if, and only if, $q_0 \in [\underline{q}^2, \bar{q}^2) \cap (0,1)$. First note that since $\bar{q}^1 < 1$, (15) and (16) can be satisfied only if the denominator of

$$q^+(q_0, v_L^1(q_0)) = \frac{q_0}{q_0 + (1-q_0)[1 - F(p_\ell/v_L^1(q'))]}$$

is greater than q_0, i.e., only if $p_\ell/v_L^1(q') < \bar\delta$. Now observe that if $p_\ell/v_L^1(q') < \bar\delta$, then

$$q^-(q') = \frac{q'\left[1 - F\left(p_\ell/v_L^1(q')\right)\right]}{1 - q'F\left(p_\ell/v_L^1(q')\right)}$$

belongs to the interval $(0,1)$ and is such that $q^+(q^-(q'), v_L^1(q')) = q'$. Thus, (15) is satisfied for $q' \in [\underline{q}^1, \bar{q}^1) \cap (0,1)$ if, and only if, $p_\ell/v_L^1(q') < \bar\delta$. Moreover, it is immediate to see that $q^-(q')$ is the only possible value of q_0 for which (15) and (16) can hold.

Since $v_L^1(q')$ is increasing in q', $p_\ell/v_L^1(\widetilde{q}) < \bar\delta$ implies that $p_\ell/v_L^1(q') < \bar\delta$ for all $q' > \widetilde{q}$. Then let $\widetilde{q}^1$ be such that $\widetilde{q}^1 = 0$ if $p_\ell/v_L^1(\underline{q}^1) < \bar\delta$ and $\widetilde{q}^1 = \sup\{q' \in [\underline{q}^1, \bar{q}^1) : p_\ell/v_L^1(q') = \bar\delta\}$ if $p_\ell/v_L^1(\underline{q}^1) \geq \bar\delta$; note that $\widetilde{q}^1$ is well–defined since $p_\ell/v_L^1(\bar{q}^1) = p_\ell/v_L^0 < \bar\delta$. By construction, there exists $q_0 \in (0,1)$ such that (15) and (16) are satisfied if, and only if, $q' \in [\underline{q}^1, \bar{q}^1) \cap (\widetilde{q}^1, 1)$, in which case $q_0 = q^-(q')$. Given that F and v_L^1 are continuous, it is easy to see that q^- is continuous. Moreover, since v_L^1 is increasing in q', the map q^- is also strictly increasing in q'. Thus, q^- is invertible and its inverse $Q_+^2 : [\underline{q}^1, \bar{q}^1) \cap (\widetilde{q}^1, 1) \to (0,1)$ is continuous and strictly increasing. By construction, we have that: (i) when $p_\ell/v_L^1(\underline{q}^1) < \bar\delta$, a 2–step equilibrium exists if, and only if, $Q_+^2(q_0) \in [\underline{q}^1, \bar{q}^1) \cap (0,1)$; (ii) when $p_\ell/v_L^1(\underline{q}^1) \geq \bar\delta$, a 2–step equilibrium exists if, and only if, $Q_+^2(q_0) \in (\widetilde{q}^1, \bar{q}^1)$. We are done if we show that $\lim_{q' \to \widetilde{q}^1} q^-(q') = 0$ when $p_\ell/v_L^1(\widetilde{q}) \geq \bar\delta$. This follows from the fact that $\lim_{q' \to \widetilde{q}^1} F(p_\ell/v_L^1(q')) = 1$. ∎

Lemma 5 and Proof

Lemma 5. *The payoff v_B^2 is continuous in q_0 and $v_B^2(q_0') - v_B^2(q_0) \leq (q_0' - q_0)[u_H - u_L]$ for all $q_0' > q_0$. The fraction ξ^2 is continuous and increasing in q_0, with $\lim_{q_0 \to \bar{q}^2} \xi^2(q_0) = \xi^1(\bar{q}^2)$. The payoff v_L^2 is continuous and increasing in q_0, with $v_L^2(\bar{q}^2) \equiv \lim_{q_0 \to \bar{q}^2} v_L^2(q_0) = v_L^1(\bar{q}^2)$ and $v_L^2(q_0) \leq v_L^1(Q_+^2(q_0))$ for all q_0.*

Proof: We start by showing that $\eta^2(q_0, \delta) = \pi_h^B(q_0) - \pi_\ell^B\left(q_0, \delta, v_L^1(Q_+^2(q_0)), v_B^1(Q_+^2(q_0))\right)$ is strictly increasing in q_0; this is important for what follows. Let $q_0' > q_0$ and note that

$$
\begin{aligned}
&\pi_\ell^B\left(q_0', \delta, v_L^1(Q_+^2(q_0')), v_B^1(Q_+^2(q_0'))\right) - \pi_\ell^B\left(q_0, \delta, v_L^1(Q_+^2(q_0)), v_B^1(Q_+^2(q_0))\right) \\
&= \left\{q_0' + (1 - q_0')\left[1 - F\left(\frac{p_\ell}{v_L^1(Q_+^2(q_0'))}\right)\right]\right\} \delta\left[v_B^1(Q_+^2(q_0')) - v_B^1(Q_+^2(q_0))\right] \\
&\quad + \left[\delta v_B^1(Q_+^2(q_0)) - (u_L - p_\ell)\right]\left\{(1 - q_0)F\left(\frac{p_\ell}{v_L^1(Q_+^2(q_0))}\right) - (1 - q_0')F\left(\frac{p_\ell}{v_L^1(Q_+^2(q_0'))}\right)\right\} \\
&\leq \left\{q_0' + (1 - q_0')\left[1 - F\left(\frac{p_\ell}{v_L^1(Q_+^2(q_0'))}\right)\right]\right\} \delta\left[Q_2^+(q_0') - Q_2^+(q_0)\right](u_H - u_L) \\
&\leq \delta(q_0' - q_0)(u_H - u_L); \quad\quad\quad (32)
\end{aligned}
$$

the first inequality follows from Lemma 4 and the fact that $Q_+^2(q_0)$ is increasing in q_0. Hence, $\eta^2(q_0', \delta) - \eta^2(q_0, \delta) \geq (1 - \delta)(q_0' - q_0)(u_H - u_L) > 0$, which proves the desired result.

We first establish the properties of v_B^2. Since Q_+^2 is continuous in q_0, the continuity of v_B^2 follows from the continuity of v_B^1 and v_L^1. We now prove that if $q_0' > q_0$, then $v_B^2(q_0') - v_B^2(q_0) \leq (q_0' - q_0)[u_H - u_L]$. For this, let

$$
v_B^2(q_0, \delta) = \pi_\ell^B\left(q_0, \delta v_L^1(Q_+^2(q_0)), v_B^1(Q_+^2(q_0))\right) + \max\{\eta^2(q_0, \delta), 0\},
$$

As in the proof of Lemma 4, we know that for each $\delta \in [0, \bar{\delta}]$, there exists a unique $q^* = q^*(\delta) \in (0, 1)$ such that $\eta^2(q_0, \delta) \geq 0$ if, and only if, $q_0 \geq q^*$; by construction, $q^*(\bar{\delta}) = \bar{q}^2$. Then let $q_0' > q_0$. The same argument as in the proof of Lemma 4 shows that

$$
v_B^2(q_0, \delta) - v_B^2(q_0, \delta) \leq (q_0' - q_0)[u_H - u_L]
$$

if $q_0' > q^*$. Suppose then that $q_0' \leq q^*$. In this case, the above inequality follows from (32), and the desired result holds from the fact that $v_B^2(q_0) = \int v_B^2(q_0, \delta) dF(\delta)$.

Now, we establish the properties of ξ^2. Since η^2 is strictly increasing in q_0 and strictly decreasing in δ, an argument similar to the one used in the proof of Proposition 2 shows that for each $q_0 \in [\underline{q}^2, \overline{q}^2) \cap (0,1)$, there is a unique $\delta^2 = \delta^2(q_0) \in [0, \overline{\delta})$, which is continuous and increasing in q_0, such that $\eta^2(q_0, \delta) \geq 0$ if, and only if $\delta \leq \delta^2(q_0)$. Thus, $\xi^2(q_0) = F(\delta^2(q_0))$ is continuous and increasing in q_0. Now notice that $\lim_{q_0 \uparrow \overline{q}^2} \xi^2(\overline{q}^2) = \xi^1(\overline{q}^2)$, given that

$$\lim_{q_0 \uparrow \overline{q}^2} \eta^2(q_0, \delta) = \pi_h^B(\overline{q}^2) - \pi_\ell^B(\overline{q}^2, \delta, v_L^1(\overline{q}^1), v_B^1(\overline{q}^1))$$
$$= \pi_h^B(\overline{q}^2) - \pi_\ell^B(\overline{q}^2, \delta, v_L^0, v_B^0[q^+(\overline{q}^2, v_L^0)]) = \eta^1(\overline{q}^2, \delta).$$

For the properties of v_L^2, first notice that Q_+^2 and v_L^1 continuous in q_0 imply that v_L^2 is also continuous in q_0. Moreover,

$$\lim_{q_0 \uparrow \overline{q}^2} v_L^2(q_0) = \xi^1(\overline{q}^2) p_h + (1 - \xi^1(\overline{q}^2)) \int \max\left\{p_\ell, \delta v_L^0\right\} dF(\delta) = v_L^1(\overline{q}^1).$$

To finish, note that $v_L^1(q) \leq v_L^0[q^+(q, v_L^0)] = v_L^0$ implies that

$$v_L^2(q_0) \leq \xi^2(q_0) p_h + (1 - \xi^2(q_0)) \int \max\{p_L, \delta v_L^0\} dF(\delta).$$

Moreover, by (31), we have that $\xi^2(q_0) \leq \xi^1(Q_+^2(q_0))$. From this, it is immediate to see that $v_L^2(q_0) \leq v_L^1(Q_+^2(q_0))$ for all q_0. ∎

Proof of Theorem 1

We proceed by induction. Suppose there exists $k \geq 3$ and sequences of cutoffs $\{\underline{q}^s\}_{s=0}^{k-1}$ and $\{\overline{q}^s\}_{s=0}^{k-1}$ such that:

(A1) $\overline{q}^0 = 1$ and $\underline{q}^s \leq \underline{q}^{s-1} < \overline{q}^s < \overline{q}^{s-1}$ for all $s \in \{1, \ldots, k-1\}$;

(A2) for each $s \in \{0, \ldots, k-1\}$, a s–step equilibrium exists if, and only if, $q_0 \in [\underline{q}^s, \overline{q}^{s-1}) \cap (0,1)$.

Moreover, suppose that for each $s \in \{0, \ldots, k-1\}$, there exist functions $v_B^s(q_0)$ and $v_L^s(q_0)$, and a map $Q_+^s(q_0)$, such that:

(A3) $Q_+^s(q_0)$ is the value of q_1 in any s–step equilibrium when the initial fraction of type H sellers is q_0;

(A4) given $q_0 \in [\underline{q}^s, \overline{q}^s) \cap (0,1)$, the payoffs to buyers and type L sellers in a s–step equilibrium are $v_B^s(q_0)$ and $v_L^s(q_0)$, respectively;

(A5) for all $s \in \{2, \ldots, k-1\}$, if $q' = Q_+^s(q_0)$, then

$$\eta^{s-1}(q', \delta) = \pi_h^B(q') - \pi_\ell^B\left(q', \delta, v_L^{s-2}(Q_+^{s-1}(q')), v_B^{s-2}(Q_+^{s-1}(q'))\right)$$
$$\geq \eta^s(q_0, \delta) = \pi_h^B(q_0) - \pi_\ell^B\left(q_0, \delta, v_L^{s-1}(q'), v_B^{s-1}(q')\right) \qquad (33)$$

for all $q_0 \in [\underline{q}^s, \overline{q}^s)$ and $\delta \in [0, \overline{\delta}]$;

(A6) v_B^s is continuous in q_0 and such that $v_B^s(q_0') - v_B^s(q_0) \leq (q_0' - q_0)[u_H - u_L]$ for all $q_0' > q_0$;

(A7) v_L^s is continuous and increasing in q_0, with $v_L^s(\overline{q}^s) \equiv \lim_{q_0 \to \overline{q}^s} v_L^s(q_0) = v_L^{s-1}(\overline{q}^s)$ and $v_L^s(q_0) \leq v_L^{s-1}(Q_+^s(q_0))$ for all q_0.

Finally, suppose that:

(A8) for each $s \in \{1, \ldots, k-1\}$, $\underline{q}^s = 0$ if, and only if, $p_\ell / v_L^{s-1}(\underline{q}^{s-1}) \geq \overline{\delta}$.

Note that the payoffs $v_B^s(q_0)$ and $v_L^s(q_0)$ must be such that

$$v_B^s(q_0) = \int \max\{\pi_h^B(q_0), \pi_\ell^B\left(q_0, \delta, v_L^{s-1}(Q_+^s(q_0)), v_B^{s-1}(Q_+^s(q_0))\right)\} dF(\delta)$$

and

$$v_L^s(q_0) = \xi^s(q_0) + (1 - \xi_s(q_0)) \int \max\{p_\ell, \delta v_L^{s-1}(Q_+^s(q_0))\} dF(\delta),$$

where $\xi^s(q_0) = \int \mathbb{I}\{\eta^s(q_0, \delta) > 0\} dF(\delta)$ is the mass of buyers who offer p_h in the first period of trade in a s–step equilibrium when the initial fraction of type H sellers is q_0.

Conditions (A1) to (A8) are true when $k = 3$ by Propositions 1 to 3 and Lemmas 4 and 5 (equation (33) reduces to (31) when $s = 2$). In what follows we show that $p_\ell / v_L^{k-1}(\overline{q}^{k-1}) < \overline{\delta}$ implies that there exist cutoffs $\underline{q}^k$ and $\overline{q}^k$, payoff functions $v_B^k(q_0)$ and $v_L^k(q_0)$, and a map $Q_+^k(q_0)$ such that (A1) to (A8) are also satisfied when $s = k$.

We know from the main text that conditions (18) to (20) are necessary and sufficient for a k–step equilibrium to exist. We first show that (18) and (19) imply (20), so that the

first two conditions are necessary and sufficient for a k–step equilibrium to exist. For this, suppose that $q' = q^+(q_0, v_L^{k-1}(q'))$ and $q' \in [\underline{q}^{k-1}, \overline{q}^{k-1}) \cap (0,1)$, and let

$$\eta^k(q_0, \delta) = \pi_h^B(q_0) - \pi_\ell^B(q_0, \delta, v_L^{k-1}(q'), v_B^{k-1}(q')).$$

Note that

$$
\begin{aligned}
\pi_\ell^B &\left(q_0, \delta, v_L^{k-1}(q'), v_B^{k-1}(q')\right) \\
&= (1 - q_0)F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right)[u_L - p_\ell] + \delta\left\{q_0 + (1 - q_0)\left[1 - F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right)\right]\right\}\pi_h^B(q') \\
&\quad + \delta\left\{q_0 + (1 - q_0)\left[1 - F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right)\right]\right\}[v_B^{k-1}(q') - \pi_h^B(q')] \\
&= \delta\pi_h^B(q_0) + (1 - q_0)F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right)[u_L - p_\ell - \delta(u_L - p_h)] \\
&\quad + \delta\left\{q_0 + (1 - q_0)\left[1 - F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right)\right]\right\}[v_B^{k-1}(q') - \pi_h^B(q')],
\end{aligned}
$$

where the last equality follows from (29). Similarly, one can show that if $q'' = Q_+^{k-1}(q')$, then

$$
\begin{aligned}
\pi_\ell^B\left(q', \delta, v_L^{k-2}(q''), v_B^{k-2}(q'')\right) &= \delta\pi_h^B(q') + (1 - q')F\left(\frac{p_\ell}{v_L^{k-2}(q'')}\right)[u_L - p_\ell - \delta(u_L - p_h)] \\
&\quad + \delta\left\{q' + (1 - q')\left[1 - F\left(\frac{p_\ell}{v_L^{k-2}(q'')}\right)\right]\right\}[v_B^{k-2}(q'') - \pi_h^B(q'')].
\end{aligned}
$$

Now observe that

$$
\begin{aligned}
v_B^{k-1}(q') - \pi_h^B(q') &= \int \max\{0, \pi_h^B(q') - \pi_\ell^B(q', \delta, v_L^{k-2}(q''), v_B^{k-2}(q''))\}dF(\delta) \\
&\geq \int \max\{0, \pi_h^B(q'') - \pi_\ell^B\left(q'', \delta, v_L^{k-3}(Q_+^{k-2}(q'')), v_B^{k-2}(Q_+^2(q''))\right)\}dF(\delta) \\
&= v_B^{k-2}(q'') - \pi_h^B(q''),
\end{aligned}
$$

where the inequality follows from (33). Therefore,

$$
\begin{aligned}
\pi_\ell^B &\left(q_0, \delta, v_L^{k-1}(q'), v_B^{k-1}(q')\right) - \pi_\ell^B\left(q', \delta, v_L^{k-2}(q''), v_B^{k-2}(q'')\right) \\
&\geq \delta\left[\pi_h^B(q_0) - \pi_h^B(q')\right] + \lambda\left[(1 - q_0)F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right) - (1 - q')F\left(\frac{p_\ell}{v_L^{k-2}(q'')}\right)\right],
\end{aligned}
$$

where $\lambda = \left\{u_L - p_\ell - \delta(u_L - p_h) - \delta\left[v_B^{k-1}(q'') - \pi_h^B(q'')\right]\right\}$. Given that $v_L^{k-1}(q') < v_L^{k-2}(q'')$ by (A6), $q' \geq q_0$, and $\lambda > 0$ (as $\delta v_B^{k-1}(q'') \leq \overline{\delta}[u_H - p_h]$), we can then conclude that

$$\pi_\ell^B\left(q', \delta, v_L^{k-2}(q''), v_B^{k-2}(q'')\right) - \pi_\ell^B\left(q_0, \delta, v_L^{k-1}(q'), v_B^{k-1}(q')\right) < \pi_h^B(q') - \pi_h^B(q_0).$$

Consequently, $\eta^{k-1}(q', \delta) \geq \eta^k(q_0, \delta)$ for all $\delta \in [0, \bar{\delta}]$. In particular, since $\eta^{k-1}(q', \bar{\delta}) \leq 0$ for all $q' \in [\underline{q}^{k-1}, \bar{q}^{k-1}) \cap (0, 1)$, we have that $\eta^k(q_0, \bar{\delta}) \leq 0$ as well, so that (20) is indeed satisfied.

Suppose now that $p_\ell / v_L^{k-1}(\bar{q}^{k-1}) < \bar{\delta}$ and define the cutoffs $\underline{q}^k$ and $\bar{q}^k$ to be such that: (i) $q^+(\underline{q}^k, v_L^{k-1}(\underline{q}^{k-1})) = \underline{q}^{k-1}$ if $p_\ell / v_L^{k-1}(\underline{q}^{k-1}) < \bar{\delta}$ and $\underline{q}^k = 0$ otherwise; (ii) $q^+(\bar{q}^k, v_L^{k-1}(\bar{q}^{k-1})) = \bar{q}^{k-1}$. It is immediate to see $0 < \bar{q}^k < \bar{q}^{k-1}$. Since $\underline{q}^{k-1} = 0$ if, and only if, $p_\ell / v_L^{k-2}(\underline{q}^{k-2}) \geq \bar{\delta}$ (by (A8)) and $v_L^{k-1}(\underline{q}^{k-1}) \leq v_L^{k-2}(\underline{q}^{k-2})$ (by (A6)), we have that $\underline{q}^k \leq \underline{q}^{k-1}$. Now note that if $\underline{q}^{k-1} = 0$, then (trivially) $\bar{q}^k > \underline{q}^{k-1}$. Suppose then that $\underline{q}^{k-1} > 0$. Given that $\bar{q}^{k-1} > \underline{q}^{k-2}$, we have that $v_L^{k-1}(\bar{q}^{k-1}) = v_L^{k-2}(\bar{q}^{k-1}) \geq v_L^{k-2}(\underline{q}^{k-2})$. Thus,

$$q^+(\bar{q}^k, v_L^{k-2}(\underline{q}^{k-2})) \geq q^+(\bar{q}^k, v_L^{k-1}(\bar{q}^{k-1})) = \bar{q}^{k-1} > \underline{q}^{k-2} = q^+(\underline{q}^{k-1}, v_L^{k-2}(\underline{q}^{k-2})),$$

from which we obtain that $\bar{q}^k > \underline{q}^{k-1}$; recall that $q^+(q, v_L)$ is strictly increasing in q when $p_\ell / v_L < \bar{\delta}$. Finally, the same argument used in the proof of Proposition 3—just replace the superscripts "1" and "2" with "$k - 1$" and "k," respectively—shows that: (i) there exists a k–step equilibrium if, and only if, $q_0 \in [\underline{q}^k, \bar{q}^k) \cap (0, 1)$; (ii) for each $q_0 \in [\underline{q}^k, \bar{q}^k) \cap (0, 1)$, there exists a unique $q' = Q_+^k(q_0) \in [\underline{q}^{k-1}, \bar{q}^{k-1}) \cap (0, 1)$ such that q' is the value of q_1 in any k–step equilibrium when the initial fraction of type H sellers is q_0; (iii) the map Q_+^k is continuous and strictly increasing. Thus, (A1), (A2), (A3), (A5), and (A8) are valid for $s = k$.

To finish the induction step, let v_B^k and v_L^k be given by (21) and (22), respectively, where $\xi^k(q_0) = \int \mathbb{I}\{\eta^k(q_0, \delta) \geq 0\} dF(\delta)$. By construction, for each $q_0 \in [\underline{q}^k, \bar{q}^k) \cap (0, 1)$, $v_B^k(q_0)$ and $v_L^k(q_0)$ are, respectively, the payoffs to buyers and type L sellers in a k–step equilibrium (so that (A4) holds when $s = k$), and $\xi^k(q_0)$ is the fraction of buyers who offer p_h in the first period of trade in a k–step equilibrium. The same argument used in the proof of Lemma 5 shows that $\xi^k(q_0)$ is increasing in q_0 and that (A6) and (A7) hold when $s = k$; once more just replace the superscripts "1" and "2" with "$k - 1$" and "k," respectively.

The induction process described above continues until k is such that $p_\ell / v_L^k(\bar{q}^k) \geq \bar{\delta}$, if such a k exists. We conclude the proof by showing that such a k indeed exists, so that $K = \max\{k : p_\ell / v_L^{k-1}(\bar{q}^{k-1}) < \bar{\delta}\}$. Suppose not. In this case, there exists a strictly decreasing sequence $\{\bar{q}^k\}_{k=0}^\infty$ such that if $q_0 < \bar{q}^k$, then there exists a s–step equilibrium with $s \geq k$ when the initial fraction of type H sellers in the market is q_0. Since the market clears in a finite

number of periods in any equilibrium, it must then be that $\lim_{k \to \infty} \overline{q}^k = 0$. In particular, there exists $k_0 \in \mathbb{N}$ such that $\pi_h^B(\overline{q}^k) < 0$ for all $k \geq k_0$. This implies that $\xi^{k-1}(\overline{q}^k) = 0$ for all $k \geq k_0$; not even a myopic buyer finds it optimal to offer p_h when the expected payoff from doing so is negative. Therefore, $\lim_{k \to \infty} v_L^k(\overline{q}^k) = \lim_{k \to \infty} v_L^{k-1}(\overline{q}^k) = p_\ell$, a contradiction. ∎

Lemmas 6 and 7 and Proofs

Lemma 6. $E_H^k(q_0)$ *is decreasing in q_0 for all $k \in \{0, \ldots, K\}$.*

Proof: For each $q_0 \in [\underline{q}^k, \overline{q}^k) \cap (0, 1)$, let $\Lambda_{q_0}^k : \{0, \ldots, k\} \to [0, 1]$ be the c.d.f. given by

$$\Lambda_{q_0}^k(s) = \sum_{r=0}^{s} \lambda^k(s|q_0).$$

By construction, $\Lambda_{q_0}^k(s)$ is the probability that a type H seller trades his asset on or before period $s \in \{0, \ldots, k\}$ in a k–step equilibrium when the initial fraction of high quality assets is q_0. A straightforward induction argument shows that

$$\Lambda^k(s|q_0) = 1 - \prod_{r=0}^{s} \left[1 - \xi^{k-r}(q_r)\right];$$

recall that $\{q_t\}_{t=1}^k$ is the sequence such that $q_t = Q_+^{k-t+1}(q_{t-1})$. We know from the main text that an increase in q_0 increases $\xi^{k-r}(q_r)$ for all $r \in \{0, \ldots, k\}$. Thus, $q_0 < q_0'$ in $[\underline{q}^k, \overline{q}^k) \cap (0, 1)$ implies that $\Lambda_{q_0'}^k(s) \geq \Lambda_{q_0}^k(s)$ for all $s \in \{0, \ldots, k\}$, in which case $\Lambda_{q_0}^k$ dominates $\Lambda_{q_0'}^k$ in the first–order stochastic sense; the desired result follows from this. ∎

Lemma 7. $\underline{\mathcal{E}}(q_0)$ *is decreasing in q_0.*

Proof: Let $q_0, q_0' \in (0, 1)$ be such that $q_0 < q_0'$. By construction, there exist $k_1, k_2 \in \{0, \ldots, K\}$ such that $\underline{\mathcal{E}}(q_0) = E_H^{k_1}(q_0)$ and $\underline{\mathcal{E}}(q_0') = E_H^{k_2}(q_0')$.

(*i*) Suppose that $k_2 \geq k_1$. In this case, there exists a k_1–step at q_0'. Indeed, if $q_0' \geq \overline{q}^{k_1}$, then $q_0' \geq \overline{q}^{k_2}$, which implies that no k_2–step equilibrium exists at q_0', a contradiction. Moreover, if $q_0' < \underline{q}^{k_1}$, then $q_0 < \underline{q}^{k_1}$, in which case no k_1–step equilibrium exists at q_0, a contradiction as well. By Lemma 6, we then have that $\underline{\mathcal{E}}(q_0') \leq E_H^{k_1}(q_0') \leq E_H^{k_1}(q_0) = \underline{\mathcal{E}}(q_0)$;

(*ii*) Suppose now that $k_2 < k_1$ and let k' be the greatest value of k such that a k–step equilibrium exists at q_0'. Note that $k' \geq k_2$ and $\overline{q}^{k'+1} \leq q_0$. Moreover, note that if $k' \geq k_1$,

then $\underline{\mathcal{E}}(q'_0) \leq E_H^{k_1}(q'_0) \leq E_H^{k_1}(q_0) = \underline{\mathcal{E}}(q_0)$. Suppose then that $k' < k_1$. We know from the proof of Theorem 1 that $\lim_{q_0 \to \overline{q}^k} \xi^k(q_0) = \xi^{k-1}(\overline{q}^k)$ for all $k \in \{1, \ldots, K\}$, from which it is easy to see that $\lim_{q_0 \to \overline{q}^k} E_H^k(q_0) = E_H^{k-1}(\overline{q}^k)$ for all $k \in \{1, \ldots, K\}$. Hence, using Lemma 6 one more time, we have that

$$\underline{\mathcal{E}}(q_0) \geq \lim_{q_0 \to \overline{q}^{k_1}} E_H^{k_1}(q_0) \geq E_H^{k_1-1}(\overline{q}^{k_1}) \geq \cdots \geq E_H^{k'}(\overline{q}^{k'+1}) \geq E_H^{k'}(q'_0) \geq \underline{\mathcal{E}}(q'_0),$$

which establishes the desired result. ∎

Appendix B: Supplemental Material (Online Appendix)

Section 2: Constructing Payoffs

Here we show how to derive payoffs using our refinement. We make use of the following result, the proof of which we omit; uniform continuity is crucial for the limit to exist.

Lemma 8. If $f : (0, 1) \to \mathbb{R}$ is bounded and uniformly continuous, then $\lim_{x \to 1} f(x)$ exists.

Fix a strategy profile σ and let $\mathbf{a} = \{a_t^j\}$ be a strategy for a type j seller. For each $n \geq 0$ and $\alpha \in (0, 1)$, define the sequence $\{V_{t,n}^j(\mathbf{a}|\sigma, \alpha)\}_{t=0}^{n+1}$ of payoffs recursively as follows. Let $V_{n+1,n}^j(\mathbf{a}|\sigma, \alpha) = c_j$ and for each $t \leq n$, let

$$V_{t,n}^j(\mathbf{a}|\sigma, \alpha) = (1 - \alpha) \int \left[y_j + \delta V_{t+1,n}^j(\mathbf{a}|\sigma, \alpha) \right] dF(\delta)$$

$$+ \alpha \sum_{p \in \{p_\ell, p_h\}} \xi_t(p|\sigma) \int \left\{ a_t^j(\delta, p) p + \left[1 - a_t^j(\delta, p) \right] \left[y_j + \delta V_{t+1,n}^j(\mathbf{a}|\sigma, \alpha) \right] \right\} dF(\delta),$$

where $\xi_t(p|\sigma)$ is the fraction of buyers who offer p in period t; $\xi_t(p|\sigma)$ is the probability that a buyer who can trade in period t draws a discount factor δ with $p_t(\delta) = p$. By construction, given σ, $V_{t,n}^j(\mathbf{a}|\sigma, \alpha)$ is the expected lifetime payoff to a type j seller who is in the market in period $t \leq n + 1$ following strategy $\mathbf{a}$ when the market stops operating in period n. The payoff $V_t^j(\mathbf{a}|\sigma, \alpha)$ is the limit $\lim_{n \to \infty} V_{t,n}^j(\mathbf{a}|\sigma, \alpha)$; it is easy to see that $\{V_{t,n}^j(\mathbf{a}|\sigma, \alpha)\}$ is an increasing, and thus convergent, sequence.

Now let $\widehat{\sigma}$ be the strategy profile that differs from σ only in that buyers offer p_h in every period t no matter their discount factor. It is immediate to see that $V_t^j(\mathbf{a}|\sigma, \alpha) \leq V_t^j(\mathbf{a}|\widehat{\sigma}, \alpha)$

for all $t \geq 0$ and $\alpha \in (0,1)$. Moreover, since $p_h \geq y_H + \bar{\delta}p_h$ by (4), if the behavior of buyers is given by $\hat{\sigma}$, then the optimal decision for a seller is to accept p_h immediately. Therefore, $V_t^j(\mathbf{a}|\hat{\sigma},\alpha) \leq p_h$, and thus $V_t^j(\mathbf{a}|\sigma,\alpha) \leq p_h$, for all $t \geq 0$ and $\alpha \in (0,1)$.

To finish, for each $t \leq n$, let $\lambda_n^t(t|\sigma,\mathbf{a},\alpha)$ be the probability that a type j seller in the market in period t following strategy $\mathbf{a}$ stays in the market until period $n+1$ when the other agents behave according to σ. Since $V_{n+1}^j(\mathbf{a}|\sigma,\alpha) \leq p_h$, we have that for all $t \leq n$,

$$V_{t,n}^j(\mathbf{a}|\sigma,\alpha) \leq V_t^j(\mathbf{a}|\sigma,\alpha) \leq V_{t,n}^j(\mathbf{a}|\sigma,\alpha) + \lambda_n^j(t|\sigma,\mathbf{a},\alpha)\mathbb{E}[\delta]^{n+1-t}\left[p_h - V_{n+1,n}^j(\mathbf{a}|\sigma,\alpha)\right].$$

Hence, for each $t \geq 0$, $V_{t,n}^j(\mathbf{a}|\sigma,\alpha)$ converges to $V_t^j(\mathbf{a}|\sigma,\alpha)$ uniformly in α. It is straightforward to see that if $V_{t+1,n}^j(\mathbf{a}|\sigma,\alpha)$ is uniformly continuous in α for $\alpha \in (0,1)$, then $V_{t+1,n}^j(\mathbf{a}|\sigma,\alpha)$ also is. Given that $V_{n+1,n}^j(\mathbf{a}|\sigma,\alpha)$ is (trivially) uniformly continuous in α, we can then conclude (since uniform continuity is preserved by uniform convergence) that for all $t \geq 0$, $V_t^j(\mathbf{a}|\sigma,\alpha)$ is uniformly continuous in α for $\alpha \in (0,1)$. Consequently, since $V_t^j(\mathbf{a}|\sigma,\alpha)$ is bounded above by p_h, the limit $\lim_{\alpha \to 1} V_t^j(\mathbf{a}|\sigma,\alpha)$ is well–defined for all $t \geq 0$.

For any strategy $\mathbf{p}$ for a buyer, the payoffs $V_t^B(\mathbf{p}|\sigma,\alpha)$, with $t \geq 0$ and $\alpha \in (0,1)$, can be computed in a similar way and the same argument as above shows that $V_t^B(\mathbf{p}|\sigma,\alpha)$ is bounded (by $u_H - p_h$) and uniformly continuous in α, in which case $\lim_{\alpha \to 1} V_t^B(\mathbf{p}|\sigma,\alpha)$ exists.

Section 4: Time to Market Clearing

Let q^* be such that $\pi_h^B(q^*) = 0$. Clearly $\underline{q}^0 > q^*$ regardless of F, as the payoff from offering p_ℓ is positive, and so no buyer offers p_h when $q \leq q^*$. Suppose then that $q_0 < q^*$ and let $N \geq 1$ be the value of k such that

$$\bar{\delta}^N p_h > p_\ell \geq \bar{\delta}^{N+1}p_h. \tag{34}$$

Now define $\{\underline{v}_L^k\}_{k=0}^N$ to be the sequence such that $\underline{v}_L^0 = p_h$ and $\underline{v}_L^k = \int \max\{p_\ell, \delta\underline{v}_L^{k-1}\}dF(\delta)$ for all $k \in \{1,\ldots,N\}$. By construction, the payoff to sellers in a k–step equilibrium is bounded below by $\underline{v}_L^k$. Since $\underline{v}_L^k$ is decreasing in k, the fraction of high quality assets in the market after N periods of trade is bounded above by

$$q_{\mathrm{max},N} = \frac{q_0}{q_0 + (1-q_0)[1 - F(p_\ell/\underline{v}_L^{N-1})]^N}.$$

Thus, the market takes at least N periods to clear if $q_{\max,N} \leq q^*$, which holds if

$$\left[1 - F\left(\frac{p_\ell}{\underline{v}_L^{N-1}}\right)\right]^N > \frac{q_0(1-q^*)}{(1-q_0)q^*}. \tag{35}$$

Note that the right side of (35) is smaller than one since $q_0 < q^*$. Finally, given that $\underline{v}_L^{N-1} \geq \overline{\delta}^{N-1}p_h$, we have that $F(p_\ell/\underline{v}_L^{N-1})$ converges to zero as $F(p_\ell/\overline{\delta}^{N-1}p_h)$ converges to zero, in which case the left side of (35) converges to one. Therefore, if the distribution F puts sufficient mass on discount factors close enough to $\overline{\delta}$, the market takes at least N periods to clear when $q_0 < q^*$. It is easy to see that there are values of the model's parameters for which N can be very large.

Section 7: Market Efficiency as Time Between Trades Vanishes

In order to study market efficiency as trading frictions vanish, it is convenient to embed our framework in a continuous time environment. Suppose now that time runs continuously and the market opens for trade every $\Delta > 0$ units of time, i.e., the market opens in $t = 0, \Delta, 2\Delta$, and so on. The agents' discount rate r in each time interval between two consecutive trading opportunities is a draw from a c.d.f. G with support $[\underline{r}, +\infty)$, where $\underline{r} > 0$ and the draws are independent across agents and over time. Thus, the c.d.f. F_Δ describing the distribution of the agents' discount factors in each period has support $[0, \overline{\delta}]$, where $\overline{\delta} = \overline{\delta}(\Delta) = e^{-\underline{r}\Delta} < 1$, and is such that $F_\Delta(\delta) = 1 - G(-\ln\delta/\Delta)$. Note that $\lim_{\Delta \to 0} \overline{\delta} = 1$ and that for all $\delta < 1$, $\lim_{\Delta \to 0} F_\Delta(\delta) = 0$. Thus, the agents become infinitely patient in the limit as the time interval between two consecutive trading periods goes to zero.

Let $q_0 < q^*$ and define T_Δ to be the number of trading periods it takes for the market to clear; recall that q^* is the unique value of q such that $\pi_h^B(q) = 0$. Note that an option for a type L seller is to reject any offer he receives until the period in which the market clears. Thus, a lower bound for his payoff is $\underline{v}_L = \mathbb{E}[e^{-rT_\Delta\Delta}]p_h$. Now assume that $\chi = \underline{v}_L/p_\ell > 1$ (which is true if $T_\Delta\Delta$ is small enough) and define N_Δ to be such that

$$G\left(\frac{\ln\chi}{\Delta}\right)^{N_\Delta} = \frac{q_0(1-q^*)}{(1-q_0)q^*} = \gamma(q_0);$$

here we implicitly assume that $\ln\chi > \underline{r}\Delta$, which is satisfied if Δ is small enough. Since an

upper bound for the fraction of high quality assets in the market after N periods of trade is

$$q(N) = \frac{q_0}{q_0 + (1 - q_0)[1 - F_\Delta(p_\ell/\underline{v}_L)]^N} = \frac{q_0}{q_0 + (1 - q_0)[G(\ln \chi/\Delta)]^N},$$

and no buyer offers p_h if the fraction of type H sellers in the market is smaller than q^*, a necessary condition for the market to clear after T_Δ periods of trade is that

$$q(T_\Delta) \geq q^* \Leftrightarrow \gamma(q_0) \geq G\left(\frac{\ln \chi}{\Delta}\right)^{T_\Delta}.$$

Since $G(\ln \chi/\Delta) \in (0, 1)$, we can then conclude that $T_\Delta \geq N_\Delta$. Therefore,

$$T_\Delta \Delta \geq N_\Delta \Delta = \frac{\ln(\gamma(q_0))\Delta}{\ln(G(\ln \chi/\Delta))} \tag{36}$$

Suppose now that G has a density g and observe, by L'Hospital's rule, that

$$\frac{\ln(\gamma(q_0))\Delta}{\ln(G(\ln \chi/\Delta))} \propto \frac{-\ln(\gamma(q_0))}{\ln \chi} \cdot \frac{\Delta^2}{g(\ln \chi/\Delta)}$$

when $\Delta \approx 0$. Thus, if $g(r) = O(1/r^k)$ with $k \geq 2$ when $r \to \infty$, we have that

$$\frac{\ln(\gamma(q_0))\Delta}{\ln(G(\ln \chi/\Delta))} \propto -\ln(\gamma(q_0))(\ln \chi)^{k-1}\Delta^{2-k}$$

when $\Delta \approx 0$. Suppose then, by contradiction, that $\lim_{\Delta \to 0} T_\Delta \Delta = 0$. Since $\chi > 1$ implies that $-\ln(\gamma(q_0)) \ln \chi > 0$, we can then conclude from (36) that $\lim_{\Delta \to 0} T_\Delta \Delta > 0$, a contradiction. The assumption that $g(r) = O(1/r^k)$ with $k \geq 2$ when $r \to \infty$ is fairly mild. In fact, a necessary condition for $\int_{\underline{r}}^\infty g(r)dr$ to be finite is that $g(r)$ converges to zero faster than $1/r$ as $r \to \infty$, i.e., the density g is such that $g(r) = O(1/r^k)$ with $k > 1$ as $r \to \infty$.

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Figure 4: The Dynamics of Trade