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Bankruptcy: Is It Enough to Forgive or Must We Also Forget?*

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Abstract

In many countries, lenders are restricted in their access to information about borrowers' past defaults. We study this provision in a model of repeated borrowing and lending with moral hazard and adverse selection. We analyze its effects on borrowers' incentives and access to credit, and identify conditions under which it is optimal. We argue that "forgetting" must be the outcome of a regulatory intervention by the government. Our model's predictions are consistent with the cross-country relationship between credit bureau regulations and the provision of credit, as well as the evidence on the impact of these regulations on borrowers' and lenders' behavior.

Keywords: Bankruptcy, Information, Incentives, Fresh Start
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I Introduction

In studying the "fresh start" provisions of personal bankruptcy law, economists typically focus on the *forgiveness* of debts. However, another important feature is the *forgetting* of past defaults. In many countries, lenders are not permitted to use information about past defaults after a specified period of time has elapsed.

In the United States, the Fair Credit Reporting Act (FCRA) prescribes that a personal bankruptcy filing may be reported by credit bureaus for up to 10 years, after which it must be removed from the records made available to lenders.[1] Similar provisions exist in most other countries. In Figure 1 we summarize the distribution of credit bureau regulations governing the time period of information transmission across countries.[2] Of the 113 countries with credit bureaus as of January 2007, over 90 percent of them had provisions for restricting the reporting of adverse information after a certain period of time.[3]

Differences in information-sharing regimes across countries — whether a credit-reporting system exists, and whether there are time limits on reporting past defaults — are associated with differences in the provision of credit. In Figure 1 we also graph the average ratio of private credit to GDP according to whether the country restricts the time period of information sharing. It is interesting to note that countries in which defaults are always reported tend to have *lower* provision of credit than those countries in which defaults are not reported ("erased") after a certain period of time.[4]

Musto (2004) studies the effect on lenders and individual borrowers of restrictions on the reporting of past defaults, using U.S. data. He shows that (i) these restrictions are binding — access to credit increases significantly when the bankruptcy "flag" is dropped from credit files;[5] and (ii) these individuals who subsequently obtain new credit are subsequently likelier to default than those with similar credit scores.

In this paper we analyze these restrictions in the framework of a model of repeated borrowing and lending, and determine conditions under which they are welfare improving.

[1] Other derogatory information can be reported for a maximum of seven years; see Hunt (2006) for a discussion of the history and regulation of consumer credit bureaus in the United States. This time period is often even shorter in other countries; Jappelli and Pagano (2004) report several specific examples.

[2] Source: Doing Business Database, World Bank, 2008. Throughout, we use the term "credit bureau" to refer to both private credit bureaus and public credit registries.

[3] See also Jappelli and Pagano (2006).

[4] Private credit/GDP is constructed from the IMF International Financial Statistics for year-end 2006. As in Djankov, McLiesh, and Shleifer (2007), private credit is given by lines 22d and 42d (claims on the private sector by commercial banks and other financial institutions). The credit bureau regulations are current as of January 2007 (source: Doing Business Database 2008).

[5] That is, after 10 years.

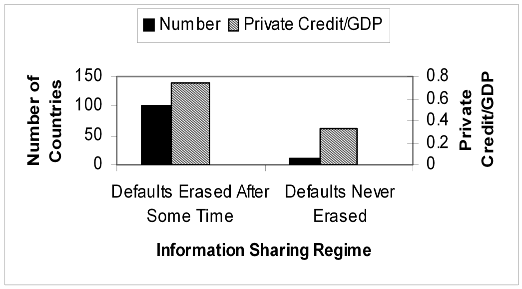

Figure 1: Information-Sharing Regime and the Provision of Credit

In particular, we study an environment where entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. We have in mind a world of small entrepreneurs who finance their business ventures with loans for which they are personally liable.[6] In this setup, an entrepreneur's reputation, or *credit history*, as captured by the past history of successes and failures of his projects, can affect the terms at which he can get credit and, hence, his incentives.

In a typical equilibrium of the model, entrepreneurs whose projects fail will see a significant deterioration in their reputation and, hence, in their incentives; they will thus no longer be able to obtain financing. On the other hand, the success of a project improves the entrepreneur's reputation and allows him to get credit at a lower interest rate. Hence the higher an entrepreneur's reputation, the costlier a failure, and the stronger his incentives.

We then consider the impact of restricting the availability to lenders of information on entrepreneurs' past defaults. Such a restriction leads to a trade-off in our model. On the one hand, "forgetting" a default makes incentives weaker, *ex-ante*, because it reduces the punishment from failure. On the other hand, forgetting a default improves an entrepreneur's reputation, *ex-post*. This improvement in his reputation allows him to obtain financing when he otherwise would not be able to. It also strengthens his incentives, since this improved reputation would be jeopardized by a project failure. To put it another way, those entrepreneurs who have their failure forgotten are pooled again with those who have not failed; as we discuss below, this plays a central role in our model.

[6]And indeed, Avery, Bostic, and Samolyk (1998) use the NSSBF and SCF to show that "[l]oans with personal commitments comprise a majority of small business loans."

Our key result is that if either borrowers' incentives are sufficiently strong or if their average risk-type is not too low, welfare is higher in the presence of a limited amount of forgetting, that is, by restricting the information available to lenders on borrowers' credit history. The same result holds even if these conditions do not hold when the output loss from poor incentives is not too large and agents are sufficiently patient. We also argue that forgetting must be the outcome of a regulatory intervention by the government — no lender would willingly agree to ignore the information available to him.

The effects of "forgetting" on lenders' and individual borrowers' behavior in our model are consistent with the empirical evidence presented by Musto (2004). However, while Musto interprets this evidence as an indication that laws imposing restrictions on memory are suboptimal, we argue that these restrictions may be optimal. In addition, our results on the relation between the presence of a forgetting clause and the aggregate volume of credit are consistent with the international evidence reported above.

In the congressional debate surrounding the adoption of the FCRA (U.S. House, 1970, and U.S. Senate, 1969), the following arguments were put forward in favor of forgetting past defaults: (i) if information was not erased, the stigmatized individual would not obtain a "fresh start" and so would be unable to continue as a productive member of society, (ii) old information might be less reliable or salient, and (iii) there is limited computer storage capacity. On the other hand, the arguments raised against forgetting this information were (i) it discourages borrowers from repaying their debts by reducing the penalty for failure, (ii) it increases the chance of costly fraud or other crimes by making it harder to identify seriously bad risks, (iii) it could lead to a tightening of credit policies (which would affect the worst risks disproportionately), and finally, (iv) it forces honest borrowers to subsidize the dishonest ones. We will show that our model, while admittedly quite stylized, allows us to capture many of these arguments and will use it to assess the trade-offs between the positive and negative effects of forgetting.

The paper is organized as follows. In section II we present the model and the strategy sets of entrepreneurs and lenders. In the following section we show that a Markov Perfect Equilibrium (MPE) of the model exists and characterize the equilibrium strategies at the most efficient MPE. In section IV we study the effects of introducing a forgetting clause on equilibrium outcomes and welfare. We derive conditions under which forgetting defaults is socially optimal and relate them to the empirical evidence and the policy debate surrounding the adoption of the FCRA. Section V concludes, and the proofs are in the Appendix.

Related Literature

Our basic model is one of reputation and incentives, like those of Diamond (1989), Mailath and Samuelson (2001), and Fishman and Rob (2005). In these models, principals and agents interact repeatedly under conditions of both adverse selection and moral hazard. The equilibrium in our model shares many similarities with the ones in these papers, in that agents build reputations over time. There are nevertheless some key differences between our model and theirs — in both the setting and in the structure of markets and information — which are discussed below (see Remarks 1 and 4).

The positive effects that a credit bureau can have through increasing the information publicly available on borrowers' histories have been widely discussed. One noteworthy paper that focuses on lenders' incentives to voluntarily share information is Pagano and Jappelli (1993). In recent empirical work, Djankov, McLiesh, and Shleifer (2007) and Brown, Jappelli, and Pagano (2007) have found that credit bureaus are positively associated with increased credit.

Our main focus, however, is on the possible benefits of limiting the information available on borrowers past histories. The paper that is closest in spirit to ours is Vercammen (1995). Like us, he studies the effect on incentives of restricting the information available on borrowers credit histories within a model of repeated lending under moral hazard and adverse selection. In his model, however, the primary benefit of forgetting is to prevent the negative effect on incentives arising from reputation becoming too good (see also Mailath and Samuelson, 2001. Along these lines, Moav and Neeman, 2010, have later shown that limiting the precision of information can improve incentives through this mechanism.)

In contrast, in our paper a strong reputation never has a negative effect on incentives, and we should emphasize that the reason forgetting may be beneficial is quite distinct from the above. In our analysis, forgetting helps the agents who have failed – those with the worst reputations — by giving them the chance for a fresh start (a central point in the policy debate surrounding this issue). Moreover, our characterization of forgetting seems to be closer than in Vercammen (1995) to the institutional details of credit bureau regulation in the United States (in which failures are erased, while successes may be reported forever), and allows us to capture its role in giving a fresh start to those who have failed, whose importance was stressed in the congressional debate discussed above. Finally, note that while in our paper we establish existence of an equilibrium and derive various properties of it as well as a series of conditions under which forgetting is optimal, Vercammen (1995) obtains very limited results for the model he describes, and his conclusions rely on an approximated solution of a numerical example (based on a few, rather ad-hoc simplifications).

The benefits of limiting the availability of information on borrowers' past histories have also been explored in a few other papers. Padilla and Pagano (2000) show, in the framework of a two-period model, that it may be optimal for the first-period lender not to share the private information he has acquired regarding the borrower with other lenders because this allows him to sustain a long-term contractual relationship with the borrower. Also, Crémer (1995) shows that using an inefficient monitoring technology can sometimes improve incentives when the principal cannot commit not to renegotiate because having less precise information limits the potential for renegotiation and hence allows for stronger punishments.[7]

Finally, while our paper and the ones mentioned above consider the effect of restricting credit histories on entrepreneurs' incentives and access to credit in a production economy, Chatterjee, Corbae, and Rios-Rull (2007) develop a model in which consumers borrow in order to insure themselves against income risk and weigh the benefits of defaulting against its reputational costs. They then compute an example and find that restricting credit histories is not beneficial.

II The Model

Consider an economy in which a continuum (of unit mass) of risk-neutral *entrepreneurs* is born in each period $z \in \mathbb{Z}$ (that is, time in the economy runs from $-\infty$ to $+\infty$). The entrepreneurs born at date z form generation z, and generations are all identical. Any entrepreneur of generation z has a constant probability $(1-\delta) \in [0,1)$ of dying at the end of each period, whatever the date of his birth. At the beginning of each period in which he is alive an entrepreneur is endowed with a new project, which requires one unit of financing in order to be undertaken. This project yields either R (success) or 0 (failure). Entrepreneurs have no resources of their own and output is nonstorable, so they must seek external financing in each period. Entrepreneurs also discount the future at the rate $\beta \leq 1$. Hence their "effective discount rate," which also takes into account the probability δ of survival, is $\tilde{\beta} = \delta \cdot \beta$.

We assume that there are two types of entrepreneurs. When any generation z is born, there is a set of measure $s_0 \in (0,1)$ of *safe* agents whose projects always succeed (i.e., their return is R with probability one), and a set of *risky* agents, with measure $1-s_0$, for whom the project may fail with some positive probability.[8] The returns on the risky agents' projects

[7] By contrast, in our model forgetting facilitates financing after failures, thus making punishments *weaker*.

[8] As discussed in Remarks 1 and 4, the property that the safe types' projects never fail is not essential, while a key role is played by the fact that the projects of the risky types always fail with a positive probability,

are independently and identically distributed among them. The success probability of a risky agent depends on his effort choice. If he chooses to exert high effort (h), incurring a utility cost $c > 0$, the success probability will be $\pi_h \in (0, 1)$. Hence his utility within a period, when his net revenue is x, is given by $x - c$. Alternatively, if he chooses to exert low effort (l), this is costless, but the success probability under low effort is only $\pi_l \in (0, \pi_h)$.

We assume:

Assumption 1. $\pi_h R - 1 > c$, $\pi_l R < 1$;

i.e., the project has a positive NPV if high effort is exerted (even when the cost of exerting high effort is taken into account), while it has a negative NPV under low effort.

In addition, we require the cost of effort c to be sufficiently high so that entrepreneurs face a nontrivial incentive problem. The following condition implies, as we will see, that when the entrepreneur is known for certain to be risky high effort cannot be implemented in a static framework.

Assumption 2. $\frac{c}{\pi_h - \pi_l} > R - 1/\pi_h$

Finally, we introduce one further parameter restriction, requiring that π_h and π_l not be too far apart. This condition is used to ensure the existence of an equilibrium.

Assumption 3. $\pi_h^2 \leq \pi_l$

In addition to entrepreneurs, there are lenders who provide external funding to entrepreneurs in the loan market. More specifically, we assume that in each period there are N risk-neutral lenders (where N is large) who compete among themselves on the terms of the contracts offered to borrowers. Each lender lives only a single period and is replaced by a new lender in the following period. Since lenders live only a single period, they cannot write long-term contracts. This is consistent with actual practice in U.S. unsecured credit markets where borrowers often switch between lenders. Furthermore, as we discuss below (see Remark 2), allowing long-term contracts would result in outcomes that are both more extreme (hence less realistic), and also yielding a lower level of total surplus, than those we obtain here.

A contract is then simply described by the interest rate r at which an entrepreneur is offered one unit of financing at the beginning of a period (if the entrepreneur is not offered — or does not accept — financing in this period we say $r = \emptyset$). If the project succeeds, the

whatever their effort level.

entrepreneur makes the required interest payment r to the lender. On the other hand, if the project fails, the entrepreneur is unable to make any payment and, therefore, defaults on the loan. We assume that there is limited liability, and so the debt is forgiven (i.e., discharged). So with no loss of generality, r can be taken to lie in $[0, R] \cup \emptyset$.

We assume that both an entrepreneur's type, as well as the effort he undertakes, are his private information. The loan market is thus characterized by the presence of both adverse selection and moral hazard. At the same time, in a dynamic framework such as the one we consider, the history of past financing decisions and past outcomes of the projects of an agent may convey some information regarding the agent's type and may, therefore, affect the contracts he receives in the future. Hence the agent cares for his reputation as determined by his past history, and this in turn may strengthen his incentives with respect to the static contracting problem. Since lenders do not live beyond the current period, we assume that there is a *credit bureau* that records this information in every period and makes it available to future lenders.

Let σ_t^i denote the *credit history* of agent $i \in [0, 1]$ who is known to have been funded for t periods. This credit history describes for each previous period $\tau < t$ in which the agent was funded, whether his project succeeded or failed. Hence, denoting by S a success and F a failure, σ_t^i is given by a sequence of elements out of $\{S, F\}$: $\sigma_t^i \in \Sigma_t \equiv \{S, F\}^t$. Observe that the bureau does not keep a record of periods in which the borrower is not financed (nor, as we explain below, of periods in which he is financed, the project fails, but this failure is forgotten). Similarly, a borrower's actual age is not observable by lenders.[9]

In addition to entrepreneurs' credit histories, lenders also have access to information on the set of contracts offered in the past. To be precise, let $\mathcal{C}_z(\sigma_t)$ denote the set of contracts offered at date z by the N lenders to entrepreneurs with credit history σ_t.

We assume that while the lenders present at an arbitrary date z know the set of contracts that were *offered* to borrowers in the past (i.e., they know $\mathcal{C}_{z'}(\sigma_t)$ for all $z' < z$ and all σ_t), they do not know the particular contracts that were *chosen* by an *individual* borrower. This in line with actual practice; while credit bureaus do not report the actual contracts adopted by individual borrowers, the set of contracts generally offered to borrowers with particular credit histories is available from databases such as "Comperemedia."

As discussed earlier, the focus of our paper is on the effect of restrictions on the information transmitted by credit bureau records. We model the *forgetting policy* in this economy as

[9]This is in in line with the provisions of the Equal Credit Opportunity Act (ECOA), which does not permit the use of age as a factor in granting credit.

follows. Consider an entrepreneur i, with credit history σ_t^i, whose project has failed. With probability q, the existence of this loan is suppressed (and naturally the failure of his project as well), and the borrower proceeds to the following period with an unchanged credit history σ_t^i.[10] Since his credit history is now shorter than that of agents with financing histories of the same length, but whose project did not fail, the borrower is thus pooled with the entrepreneurs with fewer periods of financing — for instance, with those belonging to the next generation. The parameter $q \in [0, 1]$ then describes the forgetting policy in the economy. Note that we take q as being fixed over time, which is in line with existing laws. As we will discuss, by pooling together entrepeneurs with different credit histories, the forgetting policy affects the terms of credit and hence the incentives of entrepreneurs and the lenders' policies. The main objective of our analysis is to study and evaluate these effects. Figure 2 illustrates the evolution of credit histories, under this model of forgetting.

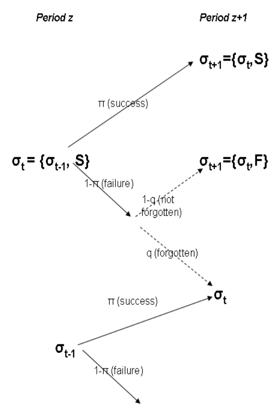

Figure 2: The Evolution of Credit Histories

Our representation of forgetting is clearly stylized, but we believe that it captures the essential feature of such policies as implemented in the United States. In particular, credit bureaus do indeed erase the entire record of a bad account when the statute dictates that such negative information can no longer be reported — exactly as in our paper. Also, just as in this paper, only negative information is erased in practice; positive information is reported indefinitely. The main difference between our formulation and actual practice is that, in the latter case, defaults are erased with the passage of time, rather than probabilistically. However, the consequences of higher values of the forgetting probability q are analogous to

[10] A similar, probabilistic approach to credit bureau regulation is also taken by Padilla and Pagano (2000).

those of allowing for a shorter period until negative information is forgotten.[11] Our main findings would in fact continue to hold if we considered the case in which negative information is instead kept on bureau records for a certain period of time; using q makes the analysis more tractable and the proofs cleaner, and provides us with a continuous characterization of the forgetting policy.

The timeline of a single period is then as follows. Each entrepreneur must obtain a loan of 1 unit in the market in order to undertake his project. Lenders simultaneously post the rate at which they are willing to lend 1 unit in this period to an entrepreneur with a given credit history, and do so for all possible credit histories at that date. At the same time, the risky entrepreneurs must choose their effort level — and incur the associated effort cost — basing their choice on the contracts they anticipate will be offered that period.[12] Next, each entrepreneur — both safe and risky — after observing the loans offered to him, chooses one of them (or none). If an entrepreneur is offered financing, and chooses one of the loans he is offered, he undertakes the project (funds lent cannot be diverted to consumption).

The outcome of the project is then realized at the end of the same period: if the project succeeds, the entrepreneur uses the revenue R to make the required payment r to the lender, while if the project fails, the entrepreneur defaults and makes no payment. The credit history of the entrepreneur is then updated. If the project was financed, a S is added to his history if the project succeeded in the period, and a F if it failed, and this failure was not forgotten. If the project was not financed, or it failed and that failure was forgotten (which occurs with probability q), then his credit history is left unchanged.

Next period, the same sequence is repeated: for each updated credit history, lenders choose the contracts they will offer and the risky entrepreneurs make their effort choice, and then each entrepreneur freely chooses which contract to accept among the ones he is offered, if any, and so on for every z.

To summarize, a lender's strategy consists in the choice of the contract to offer to en-

[11] This is exactly so for the polar cases of $q = 0$, which implies that all failures are kept in the record forever, and $q = 1$, where any failure is immediately forgotten.

[12] Having the risky entrepreneurs choose their effort before they see the contracts actually offered makes the analysis of lenders' deviations tractable in a situation like the one considered here, in which entrepreneurs with the same credit history may have been born at different dates and hence may have faced different offers of contracts in their lifetimes. If entrepreneurs could instead choose their effort level in any given period after observing the contracts offered by lenders in that period, the future updating of lenders' beliefs would be quite complicated following a deviation, as their effort strategies could not be stationary but depend on the different contracts offered to agents with the same credit history at different points in time. This case was examined in an earlier version of the paper (Elul and Gottardi, 2007) in an environment in which all entrepreneurs are born at the same time. In such an environment a different, less 'natural' specification of the forgetting rule is required; however, our analysis shows that the main qualitative results remain valid.

trepreneurs at any given date, for any possible credit history. The strategy of an entrepreneur specifies, in every period and for every possible credit history, the choice of a contract amongst the ones he is offered and, if the entrepreneur is risky, also his choice of effort. We will allow for mixed strategies with regard to effort; hence the effort level is given by a number $e \in [0, 1]$, denoting the probability with which the entrepreneur exerts high effort.[13] Thus $e = 1$ corresponds to a pure strategy of high (h) effort, and $e = 0$ to a pure strategy of low (l) effort. More generally, $\pi_e \equiv \pi_h e + \pi_l(1 - e)$ will denote the risky entrepreneurs' success probability when they exert effort e.

To evaluate the expected profit of a loan offered by a lender to an entrepreneur with credit history σ_t, at date z, an important role is played by the lender's belief, $p_z(\sigma_t, q)$, that the entrepreneur is a *safe* type. We term this the *credit score* of the entrepreneur. This belief is then updated over time on the basis of the entrepreneur's credit history σ_t, of the contracts $\mathcal{C}_{z'}(\cdot)$ offered in the past, and of the entrepreneurs' effort strategies, as we describe in detail below.

III Equilibrium

A Markov Perfect Equilibrium

In what follows we will focus on stationary *Markov Perfect Equilibria* (MPE) in which players' strategies depend on past events only through credit scores, and also do not depend on the date z. A key appeal of such equilibria is not only that players' strategies are simpler, but also that they resemble actual practice in consumer credit markets, where lending decisions are primarily conditioned on credit scores, most notably the "FICO score" developed by Fair Isaac and Company. In addition, we will discuss below the differences between MPE and other equilibria and argue that, in the latter, players' behavior is less plausible (see Remarks 2 and 3).

In particular, we will establish the existence and analyze the properties of *stationary, symmetric, sequential* MPE, where (i) all agents of a given type (i.e., all lenders, or all safe entrepreneurs, or all risky entrepreneurs with the same credit score) optimally choose the same strategy, at any date z, (ii) beliefs are determined by Bayes' Rule whenever possible and, when this is not possible, they must be consistent in the Sequential Perfect Equilibrium

[13] This is the only form of mixed strategies that we allow; we demonstrate below that mixing only occurs for at most a single period along the equilibrium path.

sense. We can now more formally describe the set of players' strategies for the Markov Perfect Equilibria that we consider.

The strategy of an entrepreneur consists in the choice, for every credit score p he may have and for any set of contracts $\mathcal{C}'$ he is offered, of accepting or not any of these contracts, and if so, which one. For the safe entrepreneurs, we denote this choice by $r^s(p, \mathcal{C}') \in \mathcal{C}' \cup \emptyset$, and for the risky ones by $r^r(p, \mathcal{C}')$. In addition, a risky entrepreneur has to choose the effort level he exerts.

An entrepreneur's choice depends not only on his immediate payoff, which depends on the contract chosen and the outcome of his project, but also on how this outcome will affect the contracts he is offered in the future. So we need to specify how lenders update their beliefs concerning the agent's type in light of the outcome of the current project. Let $p^S(p, \mathcal{C}')$ specify how lenders update their beliefs in case of success of the project of an entrepreneur who has credit score p and faced a set of contracts $\mathcal{C}'$. Analogously, $p^F(p, \mathcal{C}')$ denotes the updated belief in case of a failure (which is not forgotten), and $p^{\emptyset}(p, \mathcal{C}')$ the belief when the entrepreneur is not financed, or is financed, failed, and has this failure forgotten.

Observation 1. Since only risky agents can fail, $p^F(p, \mathcal{C}') = 0$ for any p and $\mathcal{C}' \neq \emptyset$.

The updating functions give the law of motion for the beliefs. In addition, we need to specify the initial credit score for agents with no credit history, denoted by p_0. Since the forgetting policy pools agents from different generations, p_0 will be endogenously determined in equilibrium, as discussed below.

Along the equilibrium path we denote by $\mathcal{C}(p)$ the set of contracts offered by lenders, and hence the posteriors by $p^S(p), p^F(p)$, and $p^{\emptyset}(p)$.

We are now ready to write the formal choice problem of the entrepreneurs. We begin with the risky entrepreneurs. It is convenient to separately consider their problem both on and off the equilibrium path. In the first case, they must choose whether to accept a contract from $\mathcal{C}(p)$ and which effort level to exert. Let $v^r(p)$ denote the maximal discounted expected utility that a risky entrepreneur with credit score p can obtain, along the equilibrium path. $v^r(\cdot)$ is recursively defined as the solution to the following problem:

$$v^r(p) = \max_{e\in[0,1], r\in\mathcal{C}(p)\cup\emptyset} \begin{cases} [e\pi_h + (1-e)\pi_l](R-r) - ec + \tilde{\beta}[e\pi_h + (1-e)\pi_l]v^r(p^S(p)) & \\ +\tilde{\beta}[e(1-\pi_h) + (1-e)(1-\pi_l)][qv^r(p) + (1-q)v^r(0)], & \text{if } r \neq \emptyset; \\ \tilde{\beta}v^r(p), & \text{if } r = \emptyset. \end{cases} \tag{1a}$$

When the agent chooses to accept a loan he is offered ($r \neq \emptyset$), the first line in (1a) represents the expected payoff from the current project plus the discounted continuation utility when the project succeeds, and the second line respresents the discounted continuation utility following a failure that is forgotten plus the one following a failure that is not forgotten. Note that in writing this expression we have used the fact that, by Observation 1, $p^F(p) = 0$. When the agent is not financed or rejects the contracts offered ($r = \emptyset$), by construction his credit history does not change. Thus, $p^{\emptyset}(p) = p$, by the Markov property. So, in this case his utility is simply the discounted utility of being financed next period, with his credit score unchanged. We denote the solution of problem (1a) by $e^r(p), r^r(p)$, which describes the risky entrepreneur's strategy for all possible values of p.

Off-the-equilibrium path, when the entrepreneur is faced with contracts $\mathcal{C}'$, he must choose which of the contracts offered to adopt, if any, with his effort choice given by $e^r(p)$, as determined in (1a) above. That is, the entrepreneur's maximal utility is given by:

$$v^r(p, \mathcal{C}') = \max_{r\in\mathcal{C}'\cup\emptyset} \begin{cases} [e^r(p)\pi_h + (1-e^r(p))\pi_l][(R-r) - e^r(p)c + \tilde{\beta}v^r(p^S(p, \mathcal{C}'))] & \\ +\tilde{\beta}[e^r(p)(1-\pi_h) + (1-e^r(p))(1-\pi_l)][qv^r(p^{\emptyset}(p, \mathcal{C}')) + (1-q)v^r(0)], & \text{if } r \neq \emptyset; \\ \tilde{\beta}v^r(p^{\emptyset}(p, \mathcal{C}')), & \text{if } r = \emptyset. \end{cases} \tag{1b}$$

Let $r^r(p, \mathcal{C}')$ denote the solution to this problem.

Similarly, letting $v^s(p)$ be the maximal discounted expected utility for a safe entrepreneur with credit score p, along the equilibrium path, we have:

$$v^s(p) = \max_{r\in\mathcal{C}(p)\cup\emptyset} \begin{cases} R - r + \tilde{\beta}v^s(p^S(p)), & \text{if } r \neq \emptyset; \\ \tilde{\beta}v^s(p), & \text{if } r = \emptyset. \end{cases} \tag{2a}$$

The solution to this problem is denoted by $r^s(p)$. The derivation of the off-the-equilibrium-path solution $r^s(p, C')$ is completely analogous. In particular, it solves:

$$v^s(p, \mathcal{C}') = \max_{r\in\mathcal{C}'\cup\emptyset} \begin{cases} R - r + \tilde{\beta}v^s(p^S(p, \mathcal{C}')), & \text{if } r \neq \emptyset; \\ \tilde{\beta}v^s(p^{\emptyset}(p, \mathcal{C}')), & \text{if } r = \emptyset. \end{cases} \tag{2b}$$

Since lenders cannot observe the specific contract chosen by an individual borrower in any given period, but only whether or not he was financed, we have:

Observation 2. Whenever an entrepreneur accepts financing, he will choose the contract with the lowest interest rate: i.e., for all p, C' we have $r^j(p, C') \in \{\min r \in C'\} \cup \emptyset$, for $j = s, r$. Also, entrepreneurs never refuse financing on the equilibrium path:[14] $r^j(p) \neq \emptyset$ for $j = s, r$, whenever $\mathcal{C}(p) \neq \emptyset$.

Observation 3. Observation 2 immediately implies that we cannot have separation in equilibrium. Since an entrepreneur with a given credit score always chooses the lowest rate offered (and never refuses financing), risky entrepreneurs are pooled with the safe ones until they fail.[15]

Next, we determine the expected net revenue for an arbitrary lender n from a loan with interest rate r offered to the entrepreneurs with credit score p, given the entrepreneurs' strategies, $r^s(\cdot), r^r(\cdot)$, and $e^r(\cdot)$, and the contracts $\mathcal{C}(p)$ offered by the other lenders. Given our focus on symmetric MPE, $\mathcal{C}(p)$ consists of a single contract $r(p)$.

The expression for lender n's profits will depend on which types of entrepreneurs accept his offer: this could be only the risky entrepreneurs, only the safe ones, both types, or none. In particular, when at least some of the entrepreneurs do not reject financing, if lender n's offer is lower than that of the other lenders ($r < r(p)$) he gains the entire market, and hence his profits are:

$$\Pi(r, p, r(p), r^s(\cdot), r^r(\cdot)) =$$
$$\begin{cases} pr, & \text{if } r < r(p), r^s(p, r(p) \cup r) \neq \emptyset, \text{ and } r^r(p, r(p) \cup r) = \emptyset; \\ (1-p)\pi_{e^r(p)} r, & \text{if } r < r(p), r^s(p, r(p) \cup r) = \emptyset, \text{ and } r^r(p, r(p) \cup r) \neq \emptyset; \\ \left[p + (1-p)\pi_{e^r(p)}\right] r, & \text{if } r < r(p), r^S(p, r(p) \cup r) \neq \emptyset, \text{ and } r^r(p, r(p) \cup r) \neq \emptyset. \end{cases} \tag{3a}$$

where recall that $\pi_e^r(p) \equiv e^r(p)\pi_h + (1 - e^r(p))\pi_l$. On the other hand, if he offers the same rate as all of the other lenders ($r = r(p)$), he shares the profits with the other $N - 1$ lenders:

$$\Pi(r, p, r(p), r^s(\cdot), r^r(\cdot)) =$$
$$\begin{cases} pr/N, & \text{if } r = r(p), r^s(p, r(p) \cup r) \neq \emptyset, \text{ and } r^r(p, r(p) \cup r) = \emptyset; \\ (1-p)\pi_{e^r(p)} r/N, & \text{if } r = r(p), r^s(p, r(p) \cup r) = \emptyset, \text{ and } r^r(p, r(p) \cup r) \neq \emptyset; \\ \left[p + (1-p)\pi_{e^r(p)}\right] r/N, & \text{if } r = r(p), r^S(p, r(p) \cup r) \neq \emptyset, \text{ and } r^r(p, r(p) \cup r) \neq \emptyset. \end{cases} \tag{3b}$$

[14] Since, as established above, $p^{\emptyset}(p) = p$.

[15] That is, experience a failure that is not forgotten.

Finally, if his offer is not accepted — either because it is higher than any other lenders' ($r > r(p)$), or because all entrepreneurs reject financing, or because he makes no offer — he receives zero:

$$\begin{aligned}&\Pi(r,p,r(p),r^s(\cdot),r^r(\cdot)) = \\ &\quad 0, \quad \text{if either } r > r(p), \text{ or } r^s(p,r(p)\cup r)=\emptyset \text{ and } r^r(p,r(p)\cup r)=\emptyset, \text{ or } r=\emptyset.\end{aligned} \tag{3c}$$

Since a lender lives only a single period, his objective is to choose r so as to maximize his expected profits given by (3a)-(3c). Given our focus on symmetric equilibria, $C(p)$ is then the contract r offered by lenders in equilibrium to entrepreneurs with credit score p.

We are now ready to give a formal definition of a MPE:

Definition 1. A symmetric, stationary sequential Markov Perfect Equilibrium is a collection of lenders' and borrowers' strategies $(r(\cdot), r^s(\cdot), r^r(\cdot), e^r(\cdot))$ and beliefs $p(\cdot)$, such that:

- Lenders maximize their total expected net revenue, given $r^s(\cdot), r^r(\cdot), e^r(p)$: for every p, $r = r(p)$ maximizes (3a)-(3c), when the other lenders offer $r(p)$;
- Entrepreneurs' strategies are sequentially rational. That is,
  - for all p, $(e^r(p), r^r(p))$ solve (1a), and for all p, C', $r^r(p, C')$ solves (1b)
  - for all p, $r^s(p)$ solves (2a), and for all p, C', $r^s(p, C')$ solves (2b).
- Beliefs are computed via Bayes' Rule whenever possible and are consistent otherwise.

The following notation will also be useful. Let $r_{zp}(p, e)$ denote the lowest interest rate consistent with lenders' expected profits being non-negative on a loan to entrepreneurs with credit score p, when all agents accept financing at this rate and risky entrepreneurs exert effort e. That is,

$$r_{zp}(p,e) \equiv \frac{1}{p + (1-p)(e\pi_h + (1-e)\pi_l)}. \tag{4}$$

Observe that $r_{zp}(p, e)$ is decreasing in both p and e and is larger than R when p and e are sufficiently close to zero (Assumption 1). Therefore, for $r_{zp}(p, e)$ to be an admissible interest rate that allows lenders to break even, either p or e must not be too low. In particular, let $p_{\text{NF}} \equiv \frac{1-\pi_l R}{(1-\pi_l)R}$ denote the lowest value of p for which this break-even rate is admissible when the risky entrepreneurs exert low effort, i.e., $r_{zp}(p_{\text{NF}}, 0) = R$. By contrast, when the risky entrepreneurs exert high effort ($e = 1$), $r_{zp}(p, 1) \leq R$ for all p: i.e., lenders can always break even.

B Existence and Characterization of Equilibrium

The following proposition establishes that a Markov Perfect Equilibrium exists, and characterizes its properties. The proof is constructive, and we show in Proposition 2 that this equilibrium is the MPE that maximizes total welfare.

Proposition 1. *Under assumptions 1-3, a (symmetric, stationary, sequential) Markov Perfect Equilibrium always exists with the following properties:*

i. Lenders make zero profits in equilibrium: either $r(p) = r_{zp}(p, e^r(p))$, or $r(p) = \emptyset$.

ii. Lenders never offer financing to entrepreneurs known to be risky with probability 1: $r(0) = \emptyset$, and so $v^r(0) = 0$.

iii. Furthermore, along the equilibrium path players' strategies are as follows:

a. When the cost of effort c is high, that is if $\frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)} \leq \frac{c}{\pi_h-\pi_l}$, an entrepreneur is financed if, and only if, $p \geq p_{NF}$ and if risky exerts low effort ($e^r(p) = 0$)

b. For intermediate values of the cost of effort, $\frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$, there exists $0 < p_l \leq p_m \leq p_h < 1$ (with $p_l \leq p_{NF}$) such that:

- *there is financing if and only if $p \geq p_l$*
- *risky entrepreneurs exert high effort if $p \geq p_h$, low effort if $p \in [p_l, p_m)$, and mix between high and low effort for $p \in [p_m, p_h)$ (with $e^r(p)$ strictly increasing for $p \in [p_m, p_h)$).*

c. When the cost of effort is low, $\frac{c}{\pi_h-\pi_l} \leq \frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$, there is financing for all $p > 0$, and risky entrepreneurs exert high effort ($e^r(p) = 1$).

Observe that, as discussed in Observation 3, the equilibrium described in Proposition 1 is a pooling equilibrium: all borrowers with the same credit score are offered the same contract.

Since $r_{zp}(p, e)$ is decreasing in p, as observed above, the higher is p, the lower the interest rate offered to the entrepreneur, and hence the higher the current payment and future revenue he will receive in case of success. We will show that this implies that incentives become stronger as p increases, i.e., $e^r(p)$ is (weakly) increasing in p, even with forgetting.[16] We thus see that the cross-subsidization from the safe to the risky entrepreneurs that characterizes

[16] This is not simly an immediate consequence of the fact that the interest rate is decreasing in p, since with forgetting the utility following a failure is also increasing in p.

the pooling equilibrium has a beneficial effect on incentives; the higher p, the larger is this cross-subsidy to any single risky entrepreneur who is financed.

In particular, when c is high (region a.), incentives are weak, and risky entrepreneurs exert low effort whenever they are financed. Nevertheless, financing is still profitable for the lenders and occurs as long as p is not too low ($p > p_{\mathrm{NF}}$). By contrast, when c is low (region c.), incentives are strong enough that the risky entrepreneurs exert high effort for all $p > 0$, and as observed above, financing is profitable for all $p > 0$. The more interesting case occurs for intermediate values of c (region b.), where incentives depend on p. When p is sufficiently high ($p \geq p_m$), interest rates (both current and future) are low, which makes incentives strong enough that high effort can be sustained. By contrast, when $p < p_m$, interest rates are not sufficiently low to sustain high effort. Moreover, when p is particularly low ($p < p_l$), it is not feasible for lenders to break even, just as in region a.; therefore, there will be no financing.

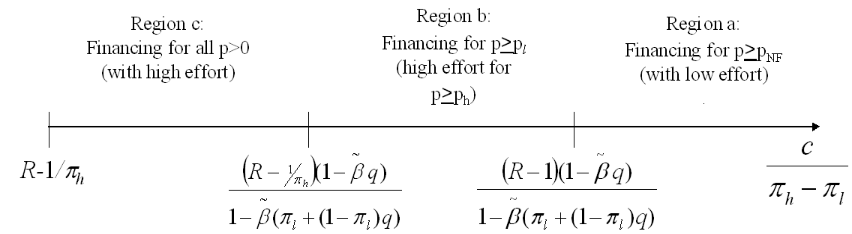

Figure 3: Equilibrium regions

In Figure 3 we plot where regions a., b., and c. lie in the space of possible values of the effort cost c (or more properly, $c/(\pi_h - \pi_l)$).[17] Figure 4 then illustrates the equilibrium outcomes obtained in region b., for different values of the credit score p. Recall that $0 < p_l \leq p_m \leq p_h < 1$, so the low-effort and mixing regions may be empty, while the high-effort and no-financing regions always exist.

Recall that a Markov Perfect Equilibrium requires that lenders use Bayes' Rule to update their beliefs whenever possible. We have already specified p^F and $p^\emptyset$, the updated beliefs in case of failure or no financing, respectively. We now show how p_0, the initial credit score, and p^S, the beliefs in case of success, are determined along the equilibrium path.

At an entrepreneur's date of birth (when he has no credit history), lenders' beliefs are given by the prior probability $p_0 \equiv p_0(s_0, q)$. To calculate this, recall that in each period, a

[17]The lower bound on $c/(\pi_h - \pi_l)$ in this figure is from Assumption 2.

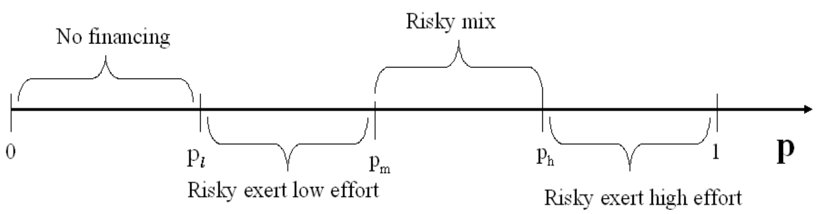

Figure 4: Financing pattern in region b.

unit mass of entrepreneurs is born; of them s_0 safe and $1-s_0$ risky. In addition, there are also entrepreneurs who were born in previous generations, but likewise have no credit history — these are risky agents who failed in all projects since they were born, and had each of these failures forgotten (and also survived). The total mass of such entrepreneurs is $(1-s_0)(1-\pi_{e^r(p_0)})\delta q + (1-s_0)(1-\pi_{e^r(p_0)})^2\delta^2 q^2 + \cdots$, where recall that $\pi_{e^r(p)} = e^r(p)\pi_h + (1-e^r(p))\pi_l$ denotes the probability of success with equilibrium effort strategy $e^r(p)$. Thus, at any point in time, the total mass of risky entrepreneurs with no credit history is $(1-s_0)\frac{1}{1-(1-\pi_{e^r(p_0)})\delta q}$, and so the initial credit score p_0 is given by:

$$p_0(s_0, q) = \frac{s_0}{s_0 + (1-s_0)\frac{1}{(1-(1-\pi_{e^r(p_0)})\delta q)}}$$

$$= s_0 \left[\frac{1-(1-\pi_{e^r(p_0)})\delta q}{1-s_0(1-\pi_{e^r(p_0)})\delta q} \right] \tag{5}$$

The initial credit score p_0 is clearly increasing in the measure of safe entrepreneurs s_0 born in each period. However, it is decreasing in the likelihood that a risky entrepreneur fails in the initial period and has this failure forgotten, as he then remains with credit score p_0. Hence p_0 is increasing in $\pi_{e^r(p_0)}$, the risky entrepreneurs' probability of success given their equilibrium effort strategy in the initial period, and decreasing in the probability of forgetting q (with $p_0(s,0) = s_0$).

The derivation of $p^S(p)$ is analogous. For a unit mass of entrepreneurs with credit score p, there is a measure p of safe entrepreneurs, and δp survive into the next period; these surviving entrepreneurs will have credit score $p^S(p)$. Similarly, there is a mass $1-p$ of risky entrepreneurs, and a measure $(1-p)\delta\pi_{e^r(p)}$ succeed at their projects, and survive into the next period. In addition, however, there are also risky entrepreneurs from previous generations, with credit score $p^S(p)$, whose projects failed, but whose failure was forgotten (and thus their score remains unchanged at $p^S(p)$). The total measure of these, per unit mass of entrepreneurs with score p, is $(1-p)\delta\pi_{e^r(p)}\left[(1-\pi_{e^r(p^S(p))})\delta q + (1-\pi_{e^r(p^S(p))})^2\delta^2 q^2 + \cdots\right]$.[18] Therefore:

$$p^S(p) = \frac{\delta p}{\delta p + (1-p)\delta\pi_{e^r(p)} + (1-p)\delta\pi_{e^r(p)}\left[(1-\pi_{e^r(p^S(p))})\delta q + (1-\pi_{e^r(p^S(p))})^2\delta^2 q^2 + \cdots\right]}$$

$$= \frac{p}{p + (1-p)\frac{\pi_{e^r(p)}}{1-\left(1-\pi_{e^r(p^S(p))}\right)\delta q}}. \tag{6}$$

We are now ready to prove Proposition 1. We first establish property ii. — that entrepreneurs who are known to be risky are never financed — and show that this is actually a general property of Markov equilibria. The basic intuition is that once an entrepreneur is known to be risky, his credit score remains the same regardless of the outcome of any future project. Hence in any Markov Perfect Equilibrium his continuation utility is also the same, which reduces his incentive problem to a static one, where we showed that financing cannot occur (Assumption 2).

Lemma 1. *Under Assumptions 1 and 2, any Markov Perfect Equilibrium is characterized by no financing when $p = 0$: i.e., $r(0) = \emptyset$ and hence $v^r(0) = 0$.*

This result implies that, in equilibrium, any entrepreneur who fails is excluded forever from financing (unless this failure is "forgotten").

The rest of the proof of Proposition 1 (in the Appendix) establishes the remaining general properties (i. and ii.) of the MPE, and the characterization of the parameter regions a., b., and c.

Next, we show that the equilibrium characterized in Proposition 1 is the MPE yielding the highest welfare. The welfare criterion we consider is the total surplus generated by entrepreneurs' projects that are financed; given agents' risk-neutrality, this is equivalent to the sum of the discounted expected utilities of all agents in the economy, including lenders.

Proposition 2. *The equilibrium constructed in Proposition 1 maximizes total surplus amongst all MPE.*

To prove the result, we first show that the construction of the equilibrium in Proposition 1 guarantees that the equilibrium implements the highest possible effort at any p. This is

[18] Since every such entrepreneur must have previously had a credit score of p, then succeeded and had his score updated to $p^S(p)$, before failing (and having this failure forgotten), one or more times.

clearly true for credit scores $p \geq p_h$, since high effort will be exerted in the current period, as well as in any future round of financing. The same is also true for $p < p_m$; as in the equilibrium of Proposition 1, the risky entrepreneurs exert low effort if financed, and this is the maximal effort level that can be sustained. The result is completed by showing this is true even when $p \in [p_m, p_h)$, i.e., in the mixing region of Proposition 1.

We conclude this section with several remarks concerning the robustness of our results to some of the assumptions and features of the model.

Remark 1. (Only Risky Agents Fail) In our setup, when an entrepreneur fails he is identified as risky and, in that case, can no longer obtain financing (since he would always exert low effort). It is a consequence of the assumption that only risky entrepreneurs can fail; this obviously simplifies the analysis. In section IV we consider an extension of our model in which the "safe" entrepreneurs can also fail; in this case, the posterior following a failure is no longer zero and hence may sometimes result in continued financing. Nevertheless, we present an example in which we show that the effect of forgetting is qualitatively similar to that obtained here — i.e., forgetting may still improve welfare because a sufficient number of failures will still result in exclusion.

Remark 2. (Long-term Contracts) It is also useful to compare the MPE we consider with the equilibria we would obtain if long term contracts were feasible; that is, if lenders lived forever, rather than a single period as assumed. In such a case lenders only need to break even over their entire lifetime, and not period-by-period and therefore, could use the time profile of their contracts to screen safe borrowers. This would lead to rather extreme and somewhat unrealistic contracts in equilibrium, where any net revenue to borrowers from the projects financed is postponed as far into the future as possible: that is, the interest payments would equal R in the initial periods, and subsequently be zero. Contracts that only entail a net revenue to borrowers after an uninterrupted string of successes of their projects are less attractive to the risky entrepreneurs, who face the risk of a failure in any period, and more attractive to the safe ones.

Nevertheless, it can be shown that in regions a. and b. a separating equilibrium with long-term contracts does not exist, as the risky entrepreneurs' effort cost is high enough that they cannot obtain financing on their own even with long term contracts; hence, in these regions the only equilibria exhibit pooling. Moreover, these pooling equilibria will generate less surplus than the one with short-term contracts (and forgetting) characterized in Proposition 1. The reason is that the postponement of payments that occurs with long-term contracts decreases the cross-subsidy from safe to risky entrepreneurs, and this will have a negative impact on the risky entrepreneurs' incentives; overall, there will be fewer periods of financing where high effort is exerted.

In region c., by contrast, the risky entrepreneurs will be able to obtain financing on their own and, therefore, may prefer to separate, rather than take a contract in which payments are postponed.[19] However, total welfare at such a separating equilibrium will also be lower than at the one with short-term contracts and forgetting, as the lack of any cross-subsidy means again that incentives are weaker and financing to risky entrepreneurs will be more limited.

Remark 3. (Non-Markov Equilibria) Observe that at the equilibrium we characterized the Markov property of players' strategies only binds at nodes where the entrepreneur is not financed, that is when $p = 0$ after a failure. This is because when an agent with $p > 0$ is financed, the updated belief in case of success will always be higher than the prior one, so p never hits the same value twice.

At non-Markov equilibria, by contrast, lenders' strategies may not be the same each time p equals zero. For example, the agent may be denied financing only temporarily after he fails. This threat of temporary exclusion could be enough to induce high effort even when the agent is known to be risky, and hence to make financing profitable for lenders.

Since these strategies imply that the entrepreneur is not treated identically at different nodes with $p = 0$, they require some coordination among lenders. Thus such non-Markov equilibria appear rather fragile, being open to the possibility of breakdowns in coordination, or to renegotiation (which is not the case for the MPE we consider).

Moreover, while these non-Markov equilibria without forgetting have some similarities with the MPE with forgetting, in that a risky entrepreneur who fails need not be permanently excluded, and thus will be given another chance, they exist only when c is low and lies in region c. of Proposition 1, so that incentives are sufficiently strong.[20] By contrast, in our MPE, efficient financing (i.e., with high effort) occurs after a failure is forgotten also for intermediate values of c (lying in region b.). This is because forgetting a failure in our setup entails pooling the risky types with the safe anew. So their reputation improves, which allows them to obtain a lower interest rate, and it is this gain in reputation (absent in the case of temporary exclusion considered in this remark) that further enhances incentives (see also Proposition 4 below).

[19] For instance, when $\tilde{\beta}$ is close to 1 it is not hard to show that a separating equilibrium exists.

[20] As it is only in region c. that the threat of exclusion alone is sufficient to sustain incentives (recall that financing obtains – with high effort — for all $p > 0$ in this case), without a cross-subsidy to the interest rate.

IV Optimal Forgetting

In this section we derive conditions under which forgetting entrepreneurs' failures is a socially optimal policy. That is when, in the equilibrium characterized in Proposition 1, total surplus is higher when $q > 0$ than with $q = 0$.

What are the effects of the forgetting policy on the equilibrium properties? A first effect is to make the exclusion process of the risky types slower; hence risky entrepreneurs with initial credit scores sufficiently high that they are financed in the first period of their life will be financed for more periods. This is welfare improving when the risky types exert high effort (in region c.), since the welfare generated from each period of financing of a risky entrepreneur is $G \equiv \pi_h R - 1 - c > 0$. By contrast, in region a. it is welfare decreasing since they exert low effort, and the welfare generated is $B \equiv \pi_l R - 1 < 0$.

A second effect of having forgetting is the weakening of incentives, since the punishment after default is lower. This is reflected in the fact that the boundaries of regions a.,b., and c. all shift to the left when q is increased. This has two negative effects. The first is that, for the same credit score p, a risky entrepreneur may switch from high to low effort. In addition, raising q may produce a shift from a region in which there is financing for all $p > 0$ (region c.), to one in which there is financing only for $p \geq p_{NF}$ (region a.) or $p \geq p_l$ (region b.); notice that this would reduce financing and hence decrease the surplus generated by the safe entrepreneurs.

Taken together, the above observations imply that in region a. forgetting is always welfare decreasing, since both of these effects are negative. As for region c. as long as the level of q is not too large (so that it does not induce a shift out of this region), forgetting will be welfare increasing because the first effect is positive while the second, negative one, is not present.

Let $q(s_0)$ denote the welfare maximizing level of q (which clearly depends on the proportion s_0 of safe types born in each period, as the equilibrium depends on it). Formally, we obtain:

Proposition 3. *The welfare maximizing forgetting policy for high and low values of c, respectively, is as follows:*

1. *If* $\frac{c}{\pi_h - \pi_l} \geq \frac{R-1}{1-\tilde{\beta}\pi_l}$*, no forgetting is optimal for all* s_0*:* $q(s_0) = 0$.
2. *If* $\frac{c}{\pi_h - \pi_l} < \frac{R-1/\pi_h}{1-\tilde{\beta}\pi_l}$*, for all* $s_0 > 0$*, some degree of forgetting is strictly optimal:* $q(s_0) > 0$.

Thus in region c., where incentives are strong and high effort is implemented everywhere, some positive level of forgetting is optimal.

We now turn our attention to region b., that is, to intermediate values of c. An important feature of region b. is that the level of effort varies along the equilibrium path (switching at some point, after a sequence of successes, from low to high). The weakening of incentives due to forgetting now manifests itself not only in the change of the boundaries of this region, which again shift to the left as q increases, but also possibly in the change of the value of p along the equilibrium path where the switch from low to high effort takes place, i.e., $p_m(q)$ and $p_h(q)$.[21] These switching points are key to the analysis of the welfare impact of raising q, since an extra period of financing with high effort makes a positive contribution to the social surplus, while one with low effort makes a negative contribution.

Notice first that when $p_0(s_0, 0) = s_0 > p_h(0)$, high effort is always exerted by a risky entrepreneur when financed. Hence an analogous argument to that used to prove part 2. of Proposition 3 establishes that the socially optimal level of q is above 0 in this case.

On the other hand, when $s_0 \in [p_{\text{NF}}, p_h(0)]$ raising q above 0, while leading to a lower probability of exclusion, does not necessarily increase welfare.[22] There is in fact a trade-off between the positive effect when high effort is exerted (which happens after a sufficiently long string of successes allows the borrower's credit score p to exceed $p_h(q)$), and the negative effect when low effort may be exerted (when the string of successes is so short that $p < p_h(q)$). There are two facets to the negative effect just mentioned when $p < p_h(q)$. As discussed above, an agent whose failure is forgotten has an opportunity to exert low effort once again, which lowers the social surplus. In addition, a longer string of successes will be required until a risky entrepreneur switches to high effort, for several reasons. First, "the updating will be slower,"that is, $p^S(p)$ will be closer to p. Also, $p_0(s_0, q)$ will be lower because there will be more risky entrepreneurs among those with no credit history, namely those who failed in their first period and had their failure forgotten. One last effect to be considered is the change in $p_h(q)$; since incentives are weaker due to the fact that failures are less costly, $p_h(q)$ may also increase[23] with q, thus generating an additional welfare loss.

In the second part of the next Proposition, we will show that in region b. the positive effect of raising q prevails over the negative one when $s_0 \in [p_{\text{NF}}, p_h(0)]$, provided (i) agents

[21] To highlight the dependence of these points (introduced in Proposition 1) on q, they are now written as functions of q.

[22] Obviously, when $s_0 < p_{\text{NF}}$ the borrower is excluded from financing and raising q will not affect welfare.

[23] This is indeed typical, although not always the case, because a higher value of q also increases the continuation utility upon success.

are sufficiently patient ($\tilde{\beta}$ close to 1), (ii) s_0 is sufficiently high, (iii) $|B|$ is sufficiently small relative to G, (iv) $p_h(0)$ is not too high, and (v) $p_0(s_0, 1) \geq p_{NF}$. The first three conditions, in particular, are needed because the positive effect follows the negative one along the equilibrium path. The fourth condition limits how much $p_h(q)$ can increase as we raise q, and the final condition ensures that raising q never pushes the initial credit score into the no-financing region, which would clearly be suboptimal.
We then obtain:

Proposition 4. *For intermediate values of* c, $\frac{R-1/\pi_h}{1-\tilde{\beta}\pi_l} \leq \frac{c}{\pi_h - \pi_l} < \frac{R-1}{1-\tilde{\beta}\pi_l}$, *the optimal policy may also exhibit forgetting. More precisely:*

1. *If* $s_0 > p_h(0)$, *welfare is maximized at* $q(s_0) > 0$.

2. *If* $s_0 \in [p_{NF}, p_h(0)]$, *when* $p_0(s_0, 1) \geq p_{NF}$, $\frac{p_h(0)(1-\pi_h^2)+\pi_h^2}{[1-\pi_h(1-\pi_h)(1-p_h(0))]} < \frac{\pi_l s_0((1-\pi_l)-(1-\pi_h)B/G)}{\pi_l s_0(1-\pi_l)-(1-\pi_h)(1-(1-\pi_l)s_0)B/G}$ *and* $\tilde{\beta}$ *is sufficiently close to 1, we also have* $q(s_0) > 0$.

This Proposition reflects the trade-offs between the positive and negative effects of forgetting. In particular, the second inequality appearing in part 2. incorporates conditions (ii) - (iv) stated above.[24] Figure 5 illustrates the welfare-maximizing forgetting policy, as derived in Propositions 3 and 4, as a function of the cost of effort c.

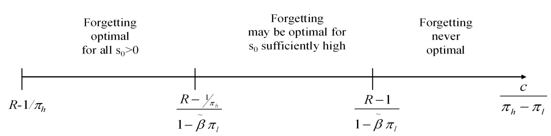

Figure 5: Welfare-maximizing forgetting policy, as a function of c

[24] While the second inequality in part 2. is stated in terms of $p_h(0)$, an endogenous variable, it is possible to show that it is not vacuous, that it is satisfied for an open set of parameter values (see also the example below in the text). In particular, let $\pi_l \to 1/R$, so that both $B \to 0$ and $p_{\mathrm{NF}} \to 0$. Then the term on the right-hand side of the inequality approaches π_l. If, in addition, $\pi_l > \frac{\pi_h^2}{1-\pi_h+\pi_h^2}$, the inequality will hold when $p_h(0)$ is sufficiently small, which occurs when we are close to the boundary between region c. and region b., i.e., for c close to (but above) $\frac{(\pi_h-\pi_l)(R-1/\pi_h)}{1-\tilde{\beta}\pi_l}$.

While the previous results give conditions under which some $q > 0$ maximizes total welfare, we can also determine when $q(p_0) = 1$, i.e., when welfare is maximized by keeping absolutely no record of any failure. This will be the case when the risky entrepreneurs exert high effort when financed. Now, from Proposition 1 and Assumption 2 it is easy to see that region c. becomes empty as $q \to 1$, and so a sufficient set of conditions for $q = 1$ to be optimal will be that we remain in region b. even when $q = 1$ ($p_h(1) < 1$), and that we are in the high-effort portion of this region ($p_0(s_0, 1) \geq p_h(1)$).[25] More precisely:

Proposition 5. *$q = 1$ maximizes total welfare when*[26] $p_0(s_0, 1) \geq \frac{1-\pi_h\left(R-\frac{c}{\pi_h-\pi_l}\right)}{(1-\pi_h)\left(R-\frac{c}{\pi_h-\pi_l}\right)}$. *When $\frac{c}{\pi_h-\pi_l} \leq R-1$, this condition will be satisfied for s_0 sufficiently close to* 1.

Remark 4. (Risky Entrepreneurs Can Fail Even Under High Effort) As we discussed above, forgetting failures provides a social benefit through the additional periods of financing under high effort which it permits. In the light of this, we can also understand the importance of our assumption that the risky entrepreneur can fail even when he exerts high effort, i.e., that $\pi_h < 1$. When this is not the case and we have $\pi_h = 1$ (as, for example, in Diamond, 1989) high effort ensures success, and there is no benefit from forgetting a failure, since such failures only result from low effort, which is inefficient.

An Example Here we consider a numerical example to illustrate the results obtained. Let $R = 3, \pi_h = 0.43$, $\pi_l = 0.3333$, $c = 0.15$, $\tilde{\beta} = 0.95$, and $\delta = 0.999$. For these values, Assumptions 1 and 2 are satisfied and we are in region b. of Proposition 1, for which high effort is implemented in equilibrium when $p \geq p_h(q)$. The threshold $p_h(0)$ above which high effort is exerted when $q = 0$ can be computed from equation (12) in the Appendix, which yields: $p_h(0) = 0.084$.

When s_0 is above this threshold ($s_0 > 0.084$), from Proposition 4 we know that $q(s_0) > 0$ is optimal, because forgetting failures increases the rounds of financing to risky entrepreneurs, and in these new rounds they always exert high effort. As we see in Figure 7 below, the optimal forgetting policy $q(s_0)$ in this region is given by high values of q (close to 1).

When $s_0 \in [p_{\text{NF}}, p_h(0)) = [0.00005, 0.084)$ low effort is exerted with $q = 0$ in the initial

[25] When $\delta = 1$, these two conditions are also necessary, as $p^S(p) = p$ when $q = 1$.

[26] The inequality $p_0(s_0, 1) \geq \frac{1-\pi_h\left(R-\frac{c}{\pi_h-\pi_l}\right)}{(1-\pi_h)\left(R-\frac{c}{\pi_h-\pi_l}\right)}$ is equivalent to $p_0(s_0, 1) \geq p_h(1)$. And $\frac{c}{\pi_h-\pi_l} \leq R-1$ ensures that $p_h(1) < 1$, i.e., we remain in region b. as $q \to 1$.

round(s) of financing.[27] For the parameters of this example, the inequality stated in part 2. of Proposition 4 is satisfied if $s_0 > 0.002$, since the net surplus $G = \pi_h R - 1 - c = 0.14$ generated by projects undertaken with high effort is sufficiently high, relative to the negative surplus $B = -0.0001$ generated by projects undertaken with low effort. Notice also that for $s_0 \geq 0.002$ $p_0(s_0, 1)$ is always larger than p_{NF}. Hence some degree of forgetting will be optimal for $\tilde{\beta}$ sufficiently close to 1, since the additional periods of high effort provided by forgetting outweigh the cost of the extra periods of low effort at the start of an agent's lifetime. In particular, we find that this is indeed the case when $\tilde{\beta} = 0.95$.

Consider $s_0 = 0.05$. When $q = 0$, we have $p^S(p_0) = 0.136 > p_h(0)$, and so low effort is exerted for the first round of financing along the equilibrium path, and high effort forever after the first success, as long as the project succeeds. On the other hand, when $q > 0$, more rounds of financing with low effort may be needed before risky entrepreneurs begin to exert high effort because $p_0(q)$ is lower than s_0, the updating is slower and, finally, $p_h(q)$ may also be higher. In Figure 6 we plot the number of successes that are required under low effort, starting from $p_0(q)$, until $p_h(q)$ is reached. Figure 6 also plots the welfare associated with these different specifications of the forgetting policy, allowing us to determine that when $s_0 = 0.05$, the optimum is $q = 0.635$; for such value of q two successes under low effort are required, starting from $p_0(0.635) = 0.029$, until reaching $p_h(0.635) = 0.091$. Figure 7 then plots the optimal policy for all values of $s_0 \in (0, 1)$.[28]

Next, we examine the consequences of relaxing, in the context of this example, the assumption that the safe entrepreneurs' projects always succeed. Let their success probability be $\pi = 0.95$, while all other parameters are unchanged. In this case, the posterior of an entrepreneur whose project fails will be above zero, and he may still receive additional rounds of financing. In particular, entrepreneurs who fail will continue to be financed as long as their credit score remains above $p_{\text{NF}} = 0.00005$. For $q = 0$ and $s_0 = 0.05$, this means that, starting from an initial credit score $p_0 = 0.05$, an entrepreneur can experience two consecutive failures before being excluded from further financing.[29] The optimal forgetting policy is now $q = 0.69$; that is, forgetting is still optimal and the optimal q is actually higher than when the safe entrepreneurs never fail (it was $q = 0.635$ when $\pi = 1$). The reason is that forgetting now also benefits the safe entrepreneurs, as they may be excluded from

[27]In this example, the risky entrepreneurs never randomize in their effort choice along the equilibrium path.

[28]Although the condition in 2. of Proposition 4 is violated for $s_0 \leq 0.0009$, we can nevertheless still have $q(s_0) > 0$, since the condition is only sufficient, not necessary.

[29]For higher scores, more failures are possible.

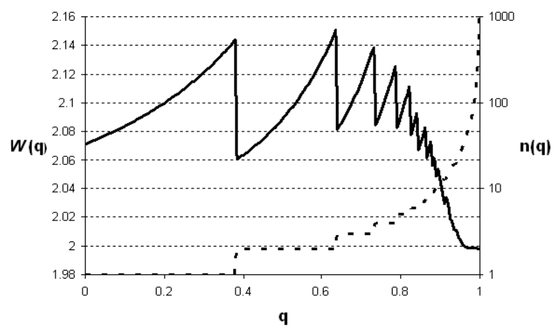

Figure 6: Total welfare (solid line), and number of successes required to reach high-effort region (dashed line), as a function of q; when $s_0 = 0.05$

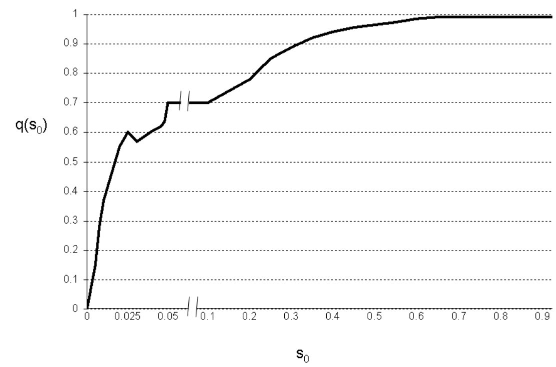

Figure 7: Welfare-maximizing value of q, as a function of s_0

financing after experiencing sufficiently many failures.

Discussion — Empirical Evidence and Policy Implications

Our model captures many of the key points made in the Congressional debate surrounding the adoption of the FCRA, which we summarized in the Introduction. As such, it allows us to determine conditions under which the positive arguments prevail over the negative ones.

Notice first that the main argument put forward in favor of forgetting — that it allows individuals to obtain a true fresh start and hence to continue being productive members of society — is echoed in our model, where the positive effect on welfare of forgetting is that it gives risky entrepreneurs who fail access to new financing.[30] By improving their reputation, this may induce them to exert high effort and hence increase aggregate surplus. Furthermore, all of the arguments made against forgetting also operate in our model: (i) forgetting weakens incentives by reducing the penalty for failure — in our set-up, as we raise q, region c. shrinks, and region a. increases in size; (ii) by erasing the records of those who defaulted in the past, there is an increased risk that frauds will be committed in the future — the analog in our model is that forgetting "slows down" the weeding out of risky entrepreneurs, hence the average quality of borrowers is lower; and (iii) forgetting can lead

[30]Two other arguments were also made in favor of forgetting — that old information may be less relevant, and that there is limited storage space; these do not have a role in our model. Furthermore, even if old information were less relevant (as would be the case if the type of an entrepreneur could change over time), lenders would take this into account and give less weight to past information anyway.

to tighter lending standards — in our model this may be seen most sharply in the fact that raising q can shift us from region c., where there is financing for all $p > 0$, to region b., where financing may not occur (for $p < p_l$),[31] or even if it does, it is at a higher interest rate (for $p \in (p_l, p_h)$).

In addition, while the policy debate suggested that (iv) another negative effect of forgetting is that it forces safe agents to subsidize the risky ones, this is in fact socially optimal in our environment because it thereby improves the risky entrepreneurs' incentives.[32]

Our results are also consistent with the empirical evidence in Musto (2004). Forgetting clearly leads to higher credit scores for those who fail, and thus to more credit — in our model without forgetting they would have $p = 0$, and no credit. Moreover, Musto's second point — that those who have their failure forgotten are more likely to fail in the future than those who are observationally equivalent (i.e., with the same score) is also an implication of the model, since only the risky agents ever have their failure forgotten. However, in contrast to Musto's suggestion that these laws are inefficient, Propositions 3 and 4 show that forgetting may be optimal.

Our model can also help us understand the international evidence, and in particular the relationship between forgetting clauses and the provision of credit. An implication of our model is that, if the forgetting clause is optimally determined and economies only differ with regard to the strength of the incentive problems in them (as captured by c), there will be a positive relationship between credit volume and the degree of forgetting (as measured by q). The first reason is that forgetting is optimal when incentives are strong, i.e., for low values of c. Also, in this case, the introduction of a forgetting policy further increases the volume of credit, since it gives entrepreneurs who fail another chance at financing. This relationship is consistent with the empirical evidence reported in Figure 1. Those countries in which information is only reported for a limited period of time have higher provision of credit than those that never forget defaults.

Finally, while we have shown that forgetting past defaults can be welfare improving, this would never arise in equilibrium as the outcome of the choice of lenders. As shown in Lemma 1, there cannot exist any Markov Perfect Equilibrium in which agents who are known to be risky (as is the case for those who failed) obtain financing. Thus forgetting can only occur through government regulation of the credit bureau's disclosure policies.

[31] See Proposition 1. Just as suggested in the policy debate, the cohorts who are excluded from financing as a result of the introduction of such a policy are those with a low credit score p_0 — the worst risks.

[32] Since only they face a moral hazard problem.

V Conclusion

In this paper, we have investigated the effects of restrictions on the information available to lenders on borrowers' past performance. These restrictions may facilitate a "fresh start" for borrowers in distress, but also affect their incentives. To analyze them, we have considered an environment where borrowers need to seek funds repeatedly, and the borrower-lender relationship is characterized by the presence of both moral hazard and adverse selection. In such a framework, we have determined the effects of these restrictions on borrowers' incentives as well as on lenders' behavior, and hence on access to credit and overall welfare. We found that imposing limits on the information available to lenders is desirable when either borrowers' incentives are sufficiently strong or the average quality of borrowers in the market is not too low. Even if neither of the conditions is satisfied, the result still holds, provided the cost of bad incentives is not too high and agents are sufficiently patient. In these cases imposing such limits is welfare-improving and increases credit volume, otherwise the reverse may obtain. We also show that these findings may help explain some features of the empirical evidence.

VI Appendix — Proofs

Proof of Lemma 1 — No financing when known to be risky

If $p = 0$, we must have $p^S(p, C') = 0 = p^F(p, C')$ whatever C', i.e., the agent will be known to be risky in the future as well. Furthermore, under assumption 1, if the agent is known to be the risky type, he can only be financed in a given period if he exerts high effort with some probability, as otherwise lenders cannot break even. But for high effort (or mixing) to be incentive compatible, the utility from high effort must be no less than that from low effort, i.e., the interest rate r offered must be such that:

$$\pi_h(R-r) - c + \pi_h\tilde{\beta}v^r(p^S(0)) + (1-\pi_h)q\tilde{\beta}v^r(0) + (1-\pi_h)(1-q)\tilde{\beta}v^r(p^F(0)) \geq$$

$$\pi_l(R-r) + \pi_l\beta v^r(p^S(0)) + (1-\pi_l)q\tilde{\beta}v^r(0) + (1-\pi_l)(1-q)\tilde{\beta}v^r(p^F(0)),$$

which simplifies to the static incentive compatibility condition:

$$\frac{c}{\pi_h - \pi_l} \leq R - r, \tag{7}$$

since when $p = 0$ we have $p^S = p^F = 0$.

By Assumption 2, this can be satisfied only if $r < 1/\pi_h$, in which case lenders cannot break even. Thus the agent cannot be financed in equilibrium if he is known to be risky. Finally, since this agent is never financed, it is immediate that $v^r(0) = 0$.■

Proof of Proposition 1 — Characterization of the Equilibrium
To complete the proof of Proposition 1, we establish the remaining properties of the MPE and the specific features of this equilibrium for parameter regions a., b., and c.

We now verify the characterization of the equilibrium strategies provided for each region and show that there are no profitable deviations by lenders.

a. To show that the strategies specified in the Proposition constitute an MPE when $\frac{c}{\pi_h-\pi_l} \geq \frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$, we need to demonstrate that (a-i) low effort is incentive compatible for $p \geq p_{\text{NF}}$; (a-ii) $r(p) = r_{zp}(p,0) \leq R$ for $p \geq p_{\text{NF}}$, i.e., it is admissible; and (a-iii) there are no profitable deviations by lenders.

a-i. Given the above strategies and implied beliefs, from (1a) we get:

$$v^r(p) = \pi_l(R - r_{zp}(p,0)) + \pi_l\tilde{\beta}v^r(p^S(p)) + (1-\pi_l)q\tilde{\beta}v^r(p), \tag{8}$$

since from Lemma 1, $v^r(p^F(p)) = v^r(0) = 0$.

By the same argument used to derive (7) above, for low effort to be IC we need:

$$\frac{c}{\pi_h - \pi_l} \geq R - r_{zp}(p,0) + \tilde{\beta}[v^r(p^S(p)) - qv^r(p)].$$

Solving (8) for $v^r(p^S(p))$ and substituting above, we obtain the low-effort incentive compatibility constraint:

$$\frac{c\pi_l}{\pi_h - \pi_l} \geq v^r(p)(1 - \tilde{\beta}q), \tag{9}$$

But since $r_{zp}(p,0) > r_{zp}(1,0) = 1$ for all $p < 1$, we have

$$v^r(p) < \frac{\pi_l(R-1)}{1-\tilde{\beta}(\pi_l + (1-\pi_l)q)},$$

where the term on the right-hand side is the present discounted utility of a risky entrepreneur who is financed in every period (until he has a failure that is not forgotten), exerting low effort, and at the rate $r = 1$.

This immediately verifies (9) for c in this region.

a-ii. $r_{zp}(p,0) \leq R$ if and only if $\frac{1}{p+(1-p)\pi_l} \leq R$, or equivalently $p \geq p_{\text{NF}}$.

a-iii. Consider a deviation by a lender. First note that lenders make zero profits in equilibrium, so refusing to offer a contract would never be profitable.

Nor can a lender profit by offering a different interest rate for $p \geq p_{\text{NF}}$. To see this, first note that a higher rate than $r(p)$ would not be accepted by any borrower. But what if a lender offers a lower rate r', so that the set of contracts offered is $\mathcal{C}'$?

We show first that a sequentially rational strategy for entrepreneurs is to never refuse financing when it is offered (just as on the equilibrium path). In this case, with all entrepreneurs accepting the offer, we have $p^S(p,\mathcal{C}') = p^S(p)$, since borrowers choose their effort before they observe the lender's deviation. By contrast, if a (single) entrepreneur were to refuse financing, he would have credit score $p^{\emptyset}(p,\mathcal{C}') = p$, and hence lower utility. That is, the posteriors following the lender deviation are the same as on the equilibrium path, and hence it is optimal for all entrepreneurs to accept financing (at the lowest rate offered, by Observation 2).

This then implies that the lender would earn a negative profit from this deviation, since he is offering a rate below $r_{zp}(p,e(p))$.

A similar argument shows that a lender cannot profit by offering financing at $p < p_{\text{NF}}$.

b. Next, we show that for intermediate values of c, $\frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$, an MPE exists characterized by $0 < p_l \leq p_m \leq p_h < 1$ such that: for $p \geq p_l$ entrepreneurs are always financed, $e^r(p) = 1$ for $p \geq p_h$, $e^r(p) \in (0,1)$ and is (strictly) increasing in p for $p \in [p_m, p_h)$, $e^r(p) = 0$ for $p \in [p_l, p_m)$ and $r(p) = r_{zp}(p, e^r(p))$.

We begin by characterizing the values of (b-i) p_h, (b-ii) p_m and (b-iii) p_l, showing that the effort choices specified above for the risky entrepreneurs are optimal. In (b-iv) we demonstrate that there are no profitable deviations for lenders.

b-i. Let $\tilde{p}^S(p) = \frac{p}{p+(1-p)\frac{\pi_h e+\pi_l(1-e)}{1-(1-(\pi_h e+\pi_l(1-e)))q\delta}}$; this is the posterior belief, following a success, that an entrepreneur is risky, when the prior belief is $p \in (0,1)$ and the effort undertaken if risky is e, calculated using Bayes' Rule. Also, let $\tilde{v}^r(p,1)$ denote the discounted expected utility for a risky entrepreneur with credit score p when he is financed in every period until experiencing a failure that is not forgotten, he exerts high effort ($e=1$), beliefs are updated according to $\tilde{p}^S(p,1)$, and the interest rate is $r_{zp}(p',1)$ for all $p' \geq p$. Then

$\tilde{v}^r(p,1)$ satisfies the following equation:[33]

$$\tilde{v}^r(p,1) = \pi_h(R - r_{zp}(p,1)) - c + \tilde{\beta}\pi_h \tilde{v}^r(\tilde{p}^S(p,1),1) + \tilde{\beta}(1-\pi_h)q\tilde{v}^r(p,1). \tag{10}$$

We then define p_h as the value of p that satisfies the following equality:

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p_h,1) + \tilde{\beta}\tilde{v}^r(\tilde{p}^S(p_h,1),1) - \tilde{\beta}q\tilde{v}^r(p_h,1) \tag{11}$$

Or, using (10) to simplify this:

$$\frac{c\pi_l}{\pi_h - \pi_l} = \tilde{v}^r(p_h,1)(1-\tilde{\beta}q) \tag{12}$$

Observe that, since $\tilde{p}^S(p,1)$ is strictly increasing in p, and $r_{zp}(p,1)$ is strictly decreasing, $\tilde{v}^r(p,1)$ is strictly increasing in p. Thus the term on the right-hand side of (12) is increasing in p, and so (12) has at most one solution.

By a continuity argument, it can then be verified that:

Claim 1. *A solution $p_h \in (0,1)$ to (12) always exists.*[34]

Given the monotonicity of the term on the right-hand side of (12), it is immediate that the incentive compatibility constraint for high effort is satisfied for all $p \geq p_h$.

b-ii. Next, we find p_m, the lower bound of the region where risky agents mix over effort.

For mixing to be an equilibrium strategy at p, risky entrepreneurs must be indifferent between high and low effort, i.e.,

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p,e) + \tilde{\beta}v^r(\tilde{p}^S(p,e)) - \tilde{\beta}qv^r(p) \tag{13}$$

for some $e \in [0,1]$. Note that effort e is exerted at p in equilibrium, so $v^r(p) = \pi_e(R - r(p)) - c \cdot e + \tilde{\beta}\pi_e v^r(\tilde{p}^S(p,e)) + \tilde{\beta}q(1-\pi_e)v^r(p)$. Thus (13) can also be rewritten as

$$\frac{c\pi_l}{\pi_h - \pi_l} = v^r(p)(1-\tilde{\beta}q) \tag{14}$$

Now, let $\left(\tilde{p}^S\right)^{-1}(p_h,1)$ denote the preimage of p_h according to the map $\tilde{p}^S(p,1)$, i.e.,

[33]Note that while $\tilde{v}^r(p,1)$ and $\tilde{p}^S(p,e)$ are well defined for all $p \in (0,1)$, they only coincide with the equilibrium values $v^r(p)$ and $p^S(p)$ when both $p \geq p_h$ and $e = e^r(p) = 1$.

[34]The proofs of claims 1-6 can be found in Appendix B.

$\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h, 1), 1\right) = p_h$.[35] We then define p_m to be the lowest value of $p \geq \left(\tilde{p}^S\right)^{-1}(p_h, 1)$ for which a solution of (14) can be found for some e. By the construction of p_h, $e = 1$ is a solution to (14) when $p = p_h$, and so $p_m \leq p_h$. It can be shown that:

Claim 2. *A lowest value p_m always exists and, moreover, $p_m > \left(\tilde{p}^S\right)^{-1}(p_h, 1)$.*

This implies that there is at most a single period of mixing along the equilibrium path. It can also be shown that:

Claim 3. *For all $p \in [p_m, p_h]$, there exists a solution $e^r(p)$ to (14), with $e^r(p)$ strictly increasing in p.*

If there is more than one solution to (14) at p, we choose the highest.

b-iii. We determine p_l, the lower bound on the financing region, and demonstrate that low effort is incentive compatible in $[p_l, p_m)$.

♦ If $p_m \geq p_{\mathrm{NF}}$, set $p_l = p_{\mathrm{NF}}$. By construction, $r_{zp}(p, 0) \leq R$ for all $p \geq p_{\mathrm{NF}}$; hence the contract $r_{zp}(p, e^r(p))$ is admissible for all $p \geq p_{\mathrm{NF}}$.

Alternatively, if $p_m < p_{\mathrm{NF}}$ set p_l to be the lowest value of $p \geq p_m$ such that the contract $r_{zp}(p, e^r(p))$ is admissible (i.e., not greater than R). Since $r_{zp}(p, e)$ is decreasing in e, $r_{zp}(p, e^r(p)) \leq r_{zp}(p, 0)$ for all $p \in [p_m, p_{\mathrm{NF}}]$, so $p_l \leq p_{\mathrm{NF}}$. In this case we also redefine p_m, with some abuse of notation, to be equal to p_l; following this redefinition the low effort region $[p_l, p_m)$ is then empty in this case.

Observe that in either case we have $p_l > 0$. Furthermore, $p_l \leq p_{\mathrm{NF}}$, which implies that $r_{zp}(p, 0) > R$ for $p < p_l$. Finally, $p_l \leq p_m$, with p_m as defined above.

♦ To conclude, we show that low effort is incentive compatible for $p \in [p_l, p_m)$. It suffices to consider the case $p_l = p_{\mathrm{NF}}$ since when $p_l < p_{\mathrm{NF}}$, we showed immediately above that $p_l = p_m$, in which case there is no low-effort region.

Claim 4. *The contract $r_{zp}(p, 0)$ satisfies the low-effort IC constraint for $p \in [p_{NF}, p_m)$.*

The argument is a little lengthier in this case and proceeds by induction on p, iterating the map $\left(\tilde{p}^S\right)^{-1}(p, 1)$.

b-iv. By the same argument as in a-iii. above, there can be no profitable lender deviations.

[35] That is, when the prior belief is $\left(\tilde{p}^S\right)^{-1}(p_h, 1)$ and the entrepreneur exerts high effort if risky, the posterior belief of lenders after observing a success is equal to p_h.

c. Finally, consider the low values of c: $\frac{c}{\pi_h-\pi_l} \leq \frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$. Note first that, by Assumption 1, $r_{zp}(p,1) \leq R$ for all $p > 0$, so $r(p) = r_{zp}(p,1)$ is always admissible. Also, the argument that there are no profitable deviations for lenders is again the same as the one in a-iii. So it only remains to verify that risky entrepreneurs indeed prefer to exert high rather than low effort for all $p > 0$.

For high effort to be incentive compatible for all $p > 0$, we need to show that

$$\frac{c\pi_l}{\pi_h - \pi_l} \leq \tilde{v}^r(p,1)(1-\tilde{\beta}q), \tag{15}$$

since in this region $v^r(p) \equiv \tilde{v}^r(p,1)$, for $\tilde{v}^r(p,1)$ defined in (10) above.

Notice that, for any $p > 0$, a lower bound for $\tilde{v}^r(p)$ is given by $\frac{\pi_h(R-1/\pi_h)-c}{1-\tilde{\beta}(\pi_h+q(1-\pi_h))}$, which is the present discounted utility for a risky entrepreneur who is financed in every period (until a failure that is not forgotten) at $r = 1/\pi_h$ and exerts high effort.[36] But then the upper bound on c that defines region c. immediately implies (15). This completes the proof of Proposition 1. ■

Proof of Proposition 2 — Efficiency of Equilibrium

We begin by showing that the equilibrium constructed in Proposition 1 maximizes $e^r(p)$, the effort exerted by the risky entrepreneurs, for any p; this will play an important role in the proof of the proposition. This result is intuitive, as the equilibrium of Proposition 1 was constructed recursively, with effort chosen to be maximal at each stage.

Claim 5. *The equilibrium constructed in Proposition 1 maximizes the risky entrepreneurs' effort $e^r(p)$ across all symmetric sequential MPE.*

The following corollary is immediate, since for lenders to break even when $p < p_l$ a higher level of effort is needed than in the equilibrium of Proposition 1, contradicting Claim 5.

Corollary 1. *No MPE can implement financing when $p < p_l$.*

Recall that welfare is given by the total surplus accruing from the agents' projects that are financed. Let $W(s_0, q)$ denote the total surplus at the MPE of Proposition 1 when the measure of safe entrepreneurs born into every generation is s_0, and let $\overline{\mathcal{W}}(s_0, q)$ denote the total surplus at a different MPE. We then conclude by showing that:

Claim 6. $W(s_0, q) \geq \overline{\mathcal{W}}(s_0, q)$

[36] This follows immediately from the fact that $\tilde{v}^r(p,1)$ is the present discounted utility under the same circumstances except that the interest rate is $r(p) = r_{zp}(p,1) < 1/\pi_h$ for all $p > 0$.

The proof of this claim is by induction on p, relying at each stage on the fact that surplus will be higher whenever effort is higher. The result then follows from Claim 5 above.■

Proof of Proposition 3 – Optimal Forgetting (regions a. and c.)
1. When $\frac{c}{\pi_h-\pi_l} \geq \frac{R-1}{1-\pi_l\tilde{\beta}}$, since $\frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$ is decreasing in q, the condition defining region a. in Proposition 1 is satisfied for all q. At the MPE, there is financing only when $p_0 \geq p_{\text{NF}}$ and risky entrepreneurs never exert high effort, regardless of the value of q. Hence if $p_0 \geq p_{\text{NF}}$, the total surplus generated in equilibrium by the loans to risky entrepreneurs is $\frac{B}{1-(\pi_l+(1-\pi_l)q)\tilde{\beta}}$, which is strictly decreasing in q since $B < 0$. Thus $q = 0$ is optimal. If on the other hand $p_0 < p_{\text{NF}}$, such surplus is zero for all q, and so $q = 0$ is also (weakly) optimal.
2. Again notice that $\frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$ is decreasing in q. Thus when $\frac{c}{\pi_h-\pi_l} < \frac{R-1/\pi_h}{1-\tilde{\beta}\pi_l}$, the condition defining region c. of Proposition 1 is satisfied for all $q \in [0, q^*]$, where $q^* = \frac{(R-1/\pi_h)-\frac{c}{\pi_h-\pi_l}(1-\tilde{\beta}\pi_l)}{\tilde{\beta}\left((R-1/\pi_h)-\frac{c}{\pi_h-\pi_l}(1-\pi_l)\right)} > 0$. Hence at the MPE there is always financing whatever p_0 is, and for all $q \in [0, q^*]$, and risky entrepreneurs always exert high effort. That is, for $q \in [0, q^*]$, the total surplus generated in equilibrium by the loans to risky entrepreneurs is

$$\frac{G}{1-(\pi_h+(1-\pi_h)q)\tilde{\beta}}.$$

Now this is increasing in q since $G > 0$. Thus any $q \in (0, q^*]$ dominates $q = 0$ and the optimal value will be $q(p_0) \geq q^*$.[37]■

Proof of Proposition 4 – Optimal Forgetting (region b.)
1. When $s_0 > p_h(0)$ the proof is an immediate corollary of part 2. of Proposition 3.
2. Since $p_0(s_0, 1) \geq p_{\text{NF}}$, the agents will always be financed at the initial date, irrespective of q. Thus, by the argument used to prove Proposition 3, it suffices to show that we can increase the surplus generated by the risky entrepreneurs' projects. Recall that $W^r(s_0, q)$ denotes the surplus from the risky agents' projects, when the forgetting policy is q and the measure of safe types born into each generation is s_0. We will show that under the conditions stated in the proposition, we can find some $\bar{q} > 0$ such that $\mathcal{W}^r(s_0, \bar{q}) > \mathcal{W}^r(s_0, 0)$.

We proceed as follows. For any $q > 0$ we first find a threshold $\tilde{p}_h(q)$ for $p_h(q)$, relative to $p_h(0)$, such that if $p_h(q) < \tilde{p}_h(q)$ then the surplus from risky entrepreneurs' projects is higher at q than at 0. We then show that the parameter restrictions stated in the Proposition indeed ensure the existence of $\bar{q} > 0$ such that $p_h(\bar{q}) \leq \tilde{p}_h(q)$.

Let $n(s_0, q)$ denote the number of successes (or forgotten failures), starting from the

[37]The optimal value of q could be higher than q^*, which would push us out of region c. and into region b.

prior $p_0(s_0, q)$, until the risky entrepreneurs first exert high effort with probability 1, when the forgetting policy is q. In the simple case in which there is no mixing in equilibrium, the surplus $\mathcal{W}^r(n(s_0, q), q)$ from the risky entrepreneur's projects can be defined recursively as follows:

$$\begin{aligned} \mathcal{W}^r(n, q) &= (\pi_l R - 1) + \pi_l \tilde{\beta} \mathcal{W}^r(n-1, q) + (1-\pi_l) q \tilde{\beta} \mathcal{W}^r(n, q) \\ \mathcal{W}^r(0, q) &= \tfrac{\pi_h R - 1 - c}{1-(\pi_h + (1-\pi_h) q)\tilde{\beta}}, \end{aligned} \tag{16}$$

where $\mathcal{W}^r(0, q)$ is the surplus generated by the risky entrepreneur's projects once he is in the high-effort region. When there is mixing in equilibrium, the exact expression for $\mathcal{W}^r$ depends on the equilibrium level of effort exerted in the mixing region. However, since there can only be a single period of mixing (in the period before high effort is exerted with probability 1), we can bound the surplus generated by lending to the risky entrepreneurs.

In particular, an upper bound on the surplus $\mathcal{W}^r(n(s_0, 0), 0)$ generated with the forgetting policy $q = 0$ can be obtained by assuming that high effort in the mixing region with probability 1.[38] In this case we have:

$$\mathcal{W}^r(s_0, 0) \leq \frac{B(1-(\pi_l\tilde{\beta})^{n(s_0,0)-1})}{1-\pi_l\tilde{\beta}} + \frac{G(\pi_l\tilde{\beta})^{n(s_0,0)-1}}{1-\pi_h\tilde{\beta}}. \tag{17}$$

Similarly, considering instead the case in which low effort is exerted with probability 1 in the mixing region, we obtain a lower bound on $\mathcal{W}^r(n(s_0, q), q)$:

$$\mathcal{W}^r(s_0, q) \geq B \frac{\frac{1-\left(\frac{\pi_l\tilde{\beta}}{1-(1-\pi_l)q\tilde{\beta}}\right)^{n(s_0,q)}}{1-\left(\frac{\pi_l\tilde{\beta}}{1-(1-\pi_l)q\tilde{\beta}}\right)}}{1-(1-\pi_l)q\tilde{\beta}} + G \frac{\left(\frac{\pi_l\tilde{\beta}}{1-(1-\pi_l)q\tilde{\beta}}\right)^{n(s_0,q)}}{1-(\pi_h+(1-\pi_h)q)\tilde{\beta}}. \tag{18}$$

So to show that $\mathcal{W}^r(s_0, q) > \mathcal{W}^r(s_0, 0)$, it suffices to show that we can find $q > 0$ such that:

$$\frac{B(1-(\pi_l\tilde{\beta})^{n(s_0,0)-1})}{1-\pi_l\tilde{\beta}} + \frac{G(\pi_l\tilde{\beta})^{n(s_0,0)-1}}{1-\pi_h\tilde{\beta}} < B \frac{\frac{1-\left(\frac{\pi_l\tilde{\beta}}{1-(1-\pi_l)q\tilde{\beta}}\right)^{n(s_0,q)}}{1-\left(\frac{\pi_l\tilde{\beta}}{1-(1-\pi_l)q\tilde{\beta}}\right)}}{1-(1-\pi_l)q\tilde{\beta}} + G \frac{\left(\frac{\pi_l\tilde{\beta}}{1-(1-\pi_l)q\tilde{\beta}}\right)^{n(s_0,q)}}{1-(\pi_h+(1-\pi_h)q)\tilde{\beta}}.$$

[38] That is, so there are only $n_0(s_0, 0) - 1$ periods in which low effort is exerted with positive probability.

Letting $\tilde{\beta} \to 1$ and simplifying, the above expression reduces to:

$$\frac{B}{G}\frac{1-\pi_l^{n(s_0,0)-1}}{1-\pi_l}+\frac{\pi_l^{n(s_0,0)-1}}{1-\pi_h}<\frac{B}{G}\frac{1-\left(\frac{\pi_l}{1-(1-\pi_l)q}\right)^{n(s_0,q)}}{(1-\pi_l)(1-q)}+\frac{\left(\frac{\pi_l}{1-(1-\pi_l)q}\right)^{n(s_0,q)}}{(1-\pi_h)(1-q)},$$

since $1-(\pi_l+(1-\pi_l)q)=(1-\pi_l)(1-q)$ and $1-(\pi_h+(1-\pi_h)q)=(1-\pi_h)(1-q)$. This is equivalent to

$$\frac{B/G}{1-\pi_l}+\pi_l^{n(s_0,0)-1}\left(\frac{1}{1-\pi_h}-\frac{B/G}{1-\pi_l}\right)<\frac{B/G}{(1-\pi_l)(1-q)}+\frac{1}{1-q}\left(\frac{\pi_l}{1-(1-\pi_l)q}\right)^{n(s_0,q)}\left(\frac{1}{1-\pi_h}-\frac{B/G}{1-\pi_l}\right) \tag{19}$$

It will be useful to rewrite (19) in terms of a condition on $p_h(q)$ and $p_h(0)$. To this end, notice that $p_h(0)$ and $n(s_0,0)$ are related by the following expression: $n(s_0,0)$ is the smallest integer for which[39]

$$\frac{p_0(s_0,0)}{p_0(s_0,0)+(1-p_0(s_0,0))\pi_l^{n(s_0,0)}}\geq p_h(0), \tag{20}$$

so that when $\delta=1$ we have $\pi_l^{n(s_0,0)-1}\leq\frac{1}{\pi_l}\frac{s_0}{1-s_0}\left(\frac{1-p_h(0)}{p_h(0)}\right)$.[40]

Similarly, as $\delta\to 1$, an upper bound for $n(s_0,q)$[41] is given by the lowest integer that satisfies

$$\frac{p_0(s_0,q)}{p_0(s_0,q)+(1-p_0(s_0,q))\left(\frac{\pi_l}{1-(1-\pi_l)q}\right)^{n(s_0,q)}}\geq p_h(q). \tag{21}$$

This implies that $\left(\frac{\pi_l}{1-(1-\pi_l)q}\right)^{n(s_0,q)-1}\geq\frac{p_0(s_0,q)}{1-p_0(s_0,q)}\left(\frac{1-p_h(q)}{p_h(q)}\right)$, or

$$\left(\frac{\pi_l}{1-(1-\pi_l)q}\right)^{n(s_0,q)}\geq\left(\frac{\pi_l}{1-(1-\pi_l)q}\right)\frac{p_0(s_0,q)}{1-p_0(s_0,q)}\left(\frac{1-p_h(q)}{p_h(q)}\right).$$

[39]When there is no mixing in equilibrium, i.e., $p_m(0)=p_h(0)$, the validity of (20) follows immediately from the definition of $p_h(0)$ and $n(s_0,0)$. The fact that it also holds with mixing can be seen by noticing that in such case the probability of success is greater or equal than when low effort is exerted, and so the posterior is $\tilde{p}^S(p,e^r(p))\leq\tilde{p}^S(p,0)$. Hence $n(s_0,0)$ will be greater or equal than the term satisfying (20). But $n(s_0,0)$ cannot be strictly greater, as this would imply that we mix for more than a single period, which we have shown (in the proof of Proposition 1) cannot happen.

[40]We use equations (5) and (6) for $q=0$.

[41]Since from equation (6) we know that $p^S(p)$ is increasing in $e^r(p^S(p))$, the upper bound is obtained by assuming that low effort is exerted even in the final period when $p_h(q)$ is reached.

If we now substitute the expression for $p_0(s_0, q)$ from (5) as $\delta \to 1$, we then obtain:

$$\left(\frac{\pi_l}{1-(1-\pi_l)q}\right)^{n(s_0,q)} \geq \pi_l \frac{s_0}{1-s_0}\left(\frac{1-p_h(q)}{p_h(q)}\right).$$

Thus to satisfy (19) it suffices to show that:

$$\begin{array}{l} \frac{B/G}{1-\pi_l} + \frac{1}{\pi_l}\frac{s_0}{1-s_0}\left(\frac{1-p_h(0)}{p_h(0)}\right)\left(\frac{1}{1-\pi_h} - \frac{B/G}{1-\pi_l}\right) < \\ \frac{1}{1-q}\left[\frac{B/G}{1-\pi_l} + \pi_l \frac{s_0}{1-s_0}\left(\frac{1-p_h(q)}{p_h(q)}\right)\left(\frac{1}{1-\pi_h} - \frac{B/G}{1-\pi_l}\right)\right] \end{array} \tag{22}$$

We will obtain a welfare improvement by taking $q \to 1$. In order for us to satisfy (22) as $q \to 1$, however, the right-hand side of the inequality above must be positive. This will be the case if:

$$p_h(q) < \tilde{p}_h(q) \equiv \frac{\pi_l s_0\left((1-\pi_l) - (1-\pi_h)B/G\right)}{\pi_l s_0(1-\pi_l) - (1-\pi_h)(1-(1-\pi_l)s_0)B/G}. \tag{23}$$

We now show that the condition on B/G stated in the proposition ensures that we can find $\bar{q}$ close to 1 such that $p_h(\bar{q})$ satisfies (23) and so we can achieve a welfare improvement. We begin by providing a convenient upper bound for $p_h(q)$.

For intermediate values of c, lying in the region where type b. equilibria obtain when $q = 0$, $p_h(0)$ belongs to $(0, 1)$ and satisfies equation (12) above. It is then easy to see from the definition of this region in Proposition 1 that, when $\tilde{\beta}$ is sufficiently close to 1, c will remain in the same region for any $q > 0$.[42] So for $\tilde{\beta}$ close to 1, $p_h(q)$ also lies in $(0, 1)$ and so equation (12) relates $p_h(0)$ and $p_h(q)$:

$$\tilde{v}^r(p_h(0), 1; 0) = \tilde{v}^r(p_h(q), 1; q)(1 - \tilde{\beta}q). \tag{24}$$

Recall that $\tilde{v}^r(p, 1; q)$, derived in (10), denotes the discounted expected utility of a risky entrepreneur with credit score p, when he exerts high effort for all $p' > p$ and the contracts offered are $r_{zp}(p, 1)$, highlighting the dependence of the utility on the forgetting policy q.

By a similar argument to that in the proof of parts a. and c. of Proposition 1, a (strict) upper bound for $\tilde{v}^r(p_h(0), 1; 0)$ is given by the utility of being financed in the current period at the rate $r_{zp}(p_h(0), 1)$, and in future periods at the rate $r = 1$, until a failure occurs, all the while exerting high effort, i.e., by $\pi_h(1 - r_{zp}(p_h(0), 1)) + \frac{\pi_h(R-1)-c}{1-\tilde{\beta}\pi_h}$. Conversely, when the forgetting policy is q, a (strict) lower bound for $\tilde{v}^r(p_h(q), 1; q)$ is given by $\frac{\pi_h(R - r_{zp}(p_h(q),1))-c}{1-\tilde{\beta}(\pi_h+(1-\pi_h)q)}$,

[42] For $\tilde{\beta}$ close to 1, the boundaries of the region are approximately equal to $\frac{(R-1/\pi_h)}{1-\pi_l}$ and $\frac{(R-1)}{1-\pi_l}$, both independent of q.

that is, the utility of a risky agent when financed at the constant rate $r_{zp}(p_h(q),1)$ until he experiences a failure that is not forgotten, still exerting high effort. Together with (24) this implies that:

$$\pi_h(1-r_{zp}(p_h(0),1))+\frac{\pi_h(R-1)-c}{1-\tilde{\beta}\pi_h}>(1-\tilde{\beta}q)\frac{\pi_h(R-r_{zp}(p_h(q),1))-c}{1-\tilde{\beta}(\pi_h+(1-\pi_h)q)}.$$

When $\tilde{\beta}\to 1$, the above inequality becomes

$$\pi_h(1-r_{zp}(p_h(0),1))+\frac{\pi_h(R-1)-c}{1-\pi_h}>\frac{\pi_h(R-r_{zp}(p_h(q),1))-c}{1-\pi_h},$$

or, simplifying: $r_{zp}(p_h(q),1)>(1-\pi_h)r_{zp}(p_h(0),1)+\pi_h$.
Using the definition of $r_{zp}(\cdot,\cdot)$ in (4), the previous expression can be rewritten as follows:

$$\frac{1}{p_h(q)+(1-p_h(q))\pi_h}>(1-\pi_h)\frac{1}{p_h(0)+(1-p_h(0))\pi_h}+\pi_h,$$

or

$$\begin{aligned}&p_h(0)+(1-p_h(0))\pi_h>(1-\pi_h)[p_h(q)+(1-p_h(q))\pi_h]+\pi_h[p_h(q)+(1-p_h(q))\pi_h][p_h(0)+(1-p_h(0))\pi_h]\\&=[p_h(q)+(1-p_h(q))\pi_h]\left[1-\pi_h(1-\pi_h)(1-p_h(0)\right]],\end{aligned}\tag{25}$$

which is in turn equivalent to:

$$p_h(0)(1-\pi_h)+\pi_h>[p_h(q)(1-\pi_h)+\pi_h]\left[1-\pi_h(1-\pi_h)(1-p_h(0))\right],$$

i.e.,

$$\frac{p_h(0)(1-\pi_h)+\pi_h}{1-\pi_h(1-\pi_h)(1-p_h(0))}>p_h(q)(1-\pi_h)+\pi_h.$$

The above inequality implies that when $\tilde{\beta}$ is close to 1 the following upper bound on the level of $p_h(q)$ must hold, for all q:

$$p_h(q)<\bar{p}_h\equiv\frac{p_h(0)(1-\pi_h^2)+\pi_h^2}{1-\pi_h(1-\pi_h)(1-p_h(0))}.\tag{26}$$

Finally, recall that for q close to 1, a sufficient condition for q to implement a welfare improvement offer $q=0$ is that $p_h(q)<\tilde{p}_h(q)$, which is given in (23 by $\tilde{p}_h(q)=p_h(q)<\tilde{p}_h(q)\equiv\frac{\pi_l s_0((1-\pi_l)-(1-\pi_h)B/G)}{\pi_l s_0(1-\pi_l)-(1-\pi_h)(1-(1-\pi_l)s_0)B/G}$.

Hence, the condition in the proposition implies that for q close to 1 we have $\bar{p}_h<\tilde{p}_h(q)$.

Thus, on the basis of the previous discussion, we can conclude that there exists $\bar{q}$ yielding a welfare improvement over $q = 0$. ■

Proof of Proposition 5 — $q = 1$ optimal

Note that when $q = 1$ a lower bound for $\tilde{v}^r(p, 1)$ is given by $\frac{\pi_h(R - r_{zp}(p,1)) - c}{1 - \tilde{\beta}}$; this bound holds with equality when $\delta = 1$. That is, $\tilde{v}^r(p, 1)(1 - \tilde{\beta}) \geq \pi_h(R - r_{zp}(p, 1)) - c$. So high effort is incentive compatible at p if $\pi_h(R - r_{zp}(p, 1)) - c \geq \frac{c\pi_l}{\pi_h - \pi_l}$, or $R - r_{zp}(p, 1) \geq \frac{c}{\pi_h - \pi_l}$. Substituting $r_{zp}(p, 1) = \frac{1}{p + (1-p)\pi_h}$, we obtain the following upper bound for $p_h(1)$:

$$p_h(1) \leq \frac{1 - \pi_h(R - \frac{c}{\pi_h - \pi_l})}{(1 - \pi_h)(R - \frac{c}{\pi_h - \pi_l})}. \tag{27}$$

Now for $q = 1$ to be optimal, it suffices that $p_0 \geq p_h(1)$ and also that $p_h(1) < 1$. The conditions in the proposition then follow from (27), using (5) above. As discussed above, these conditions are also necessary when $\delta = 1$.

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Appendix B — Proofs of Claims 1-6

Claim 1: *A solution $p_h \in (0,1)$ to (12) always exists.*

Since $\tilde{p}^S(p,1)$ and $r_{zp}(p,1)$ are both continuous for all $p \in (0,1)$, $\tilde{v}^r(p,1)$ is also continuous. As $p \to 1^-$, $r_{zp}(p,1) \to 1$ and $\tilde{p}^S(p,1) \to 1$, and so $\tilde{v}^r(p,1) \to \frac{\pi_h(R-1)-c}{1-\tilde{\beta}(\pi_h+(1-\pi_h)q)}$. And since $c < \frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$ in this region, for p close to 1 we have

$$\frac{c\pi_l}{\pi_h - \pi_l} < \tilde{v}^r(p,1)(1-\tilde{\beta}q)$$

Conversely, as $p \to 0^+$ it is immediate to see that (since $\tilde{p}^S(p,1) \to 0$ and $r_{zp}(p,1) \to 1/\pi_h$), $\tilde{v}^r(p,1) \to \frac{\pi_h(R-1/\pi_h)-c}{1-\tilde{\beta}(\pi_h+(1-\pi_h)q)}$. Then since $c > \frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$ in this region, for p close to 0 we have

$$\frac{c\pi_l}{\pi_h - \pi_l} > \tilde{v}^r(p,1)(1-\tilde{\beta}q)$$

Thus by the continuity $\tilde{v}^r(\cdot,1)$, there must be a solution $p_h \in (0,1)$ to (12); moreover, by the monotonicity of $\tilde{v}^r(\cdot,1)$ this solution is unique. ■

Claim 2: *A lowest value p_m always exists and, moreover, $p_m > \left(\tilde{p}^S\right)^{-1}(p_h,1)$.*

We first establish that there cannot be a solution to (14) at $p = \left(\tilde{p}^S\right)^{-1}(p_h,1)$. To see this, note first that for any $p' \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$ we have $\tilde{p}^S(p',e) \geq p_h$ for all e. Suppose that $e \in [0,1]$ does in fact solve (14) at $p = \left(\tilde{p}^S\right)^{-1}(p_h,1)$. Then we must have $v^r(p) = v^r(p_h)$ (since $e=1$ also solves (14) at p_h). But on the other hand,

$$\begin{aligned} v^r(p) &\equiv \pi_e(R - r_{zp}(p,e)) - c\cdot e + \pi_e\tilde{\beta}\tilde{v}^r(\tilde{p}^S(p,e),1) + (1-\pi_e)\tilde{\beta}qv^r(p) \\ &= \pi_h(R - r_{zp}(p,e)) - c + \pi_h\tilde{\beta}\tilde{v}^r(\tilde{p}^S(p,e),1) + (1-\pi_h)\tilde{\beta}qv^r(p) \\ &< \pi_h(R - r_{zp}(p_h,1)) - c + \pi_h\tilde{\beta}\tilde{v}^r(\tilde{p}^S(p_h,1),1) + (1-\pi_h)\tilde{\beta}qv^r(p_h) \equiv v^r(p_h). \end{aligned}$$

Here the second line follows from the first because the risky agent is indifferent between high and low effort at p, and the inequality in the third line follows because (i) by Assumption 3, we have $\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h,1),e\right) \leq \tilde{p}^S(p_h,1)$ for any e and (ii) we have $v^r(p) = v^r(p_h)$ by assumption. But this contradicts our assertion that $v^r(p) = v^r(p_h)$, and so we conclude that (14) cannot be satisfied at $p = \left(\tilde{p}^S\right)^{-1}(p_h,1)$. Moreover, note that this argument also implies that low effort is in fact incentive compatible at $p = \left(\tilde{p}^S\right)^{-1}(p_h,1)$ (this will be used below).

We now take p_m to be the lowest $p > \left(\tilde{p}^S\right)^{-1}(p_h,1)$ for which there is a solution to (14). By the continuity of $\tilde{v}^r(p,1)$ and $r_{zp}(p,e)$, such minimum value p_m must exist. ■

Claim 3: *For all $p \in [p_m, p_h]$, there exists a solution $e^r(p)$ to (14), with $e^r(p)$ strictly increasing in p.*

Suppose a solution to (14) with respect to e exists for some $p \in [p_m, p_h]$; since we can take $p = p_m$, this is always possible. Let $e^r(p)$ denote this solution (if there is more than one solution, we pick the highest one).

To prove the claim, it suffices to show that for all $p' \in (p, p_h)$ a solution $e^r(p')$ of (14) also exists, and $e^r(p') > e^r(p)$.

Since $p \geq p_m > \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, we have $\tilde{p}^S(p, e^r(p)) > p_h$ and, for all $p' > p$, $\tilde{p}^S(p', e') > p_h$ for all e'.

Now, by construction we have $\frac{c\pi_l}{\pi_h - \pi_l} = v^r(p)$, where

$$v^r(p) = \pi_{e^r(p)}(R - r_{zp}(p, e^r(p))) - c \cdot e^r(p) + \pi_{e^r(p)}\tilde{\beta}\tilde{v}^r(\tilde{p}^S(p, e^r(p))) + (1 - \pi_{e^r(p)})\tilde{\beta}qv^r(p),$$

or

$$v^r(p) = \frac{\pi_{e^r(p)}(R - r_{zp}(p, e^r(p))) - c \cdot e^r(p) + \pi_{e^r(p)}\tilde{\beta}\tilde{v}^r(\tilde{p}^S(p, e^r(p)))}{1 - (1 - \pi_{e^r(p)})\tilde{\beta}q}$$

More generally, for any $p' \geq \left(\tilde{p}^S\right)^{-1}(p_h, 1)$ and $e \in [0, 1]$ we can define

$$\tilde{v}^r(p', e) = \frac{\pi_e(R - r_{zp}(p', e)) - c \cdot e + \pi_e\tilde{\beta}\tilde{v}^r(\tilde{p}^S(p', e))}{1 - (1 - \pi_e)\tilde{\beta}q}$$

as the utility function the risky agent would receive if the equilibrium implemented effort e at p.

Now, by the construction of p_h, we know that we must have $\tilde{v}^r(p', e)(1 - \tilde{\beta}q) < \frac{c\pi_l}{\pi_h - \pi_l}$ for $p' \in (\left(\tilde{p}^S\right)^{-1}(p_h, 1), p_h)$. Conversely, since $p' > p$, from the monotonicity of $\tilde{v}^r(\cdot, 1)$ and $\tilde{p}^S(\cdot, 1)$ we can also conclude that $\tilde{v}^r(p'^r(p)) > v^r(p)(1 - \tilde{\beta}q) = \frac{c\pi_l}{\pi_h - \pi_l}$.

So by the continuity of $\tilde{v}^r(\cdot, 1)$ and $r_{zp}(\cdot, \cdot)$, there must be a solution $e' \in (e^r(p), 1)$ to (14) at p'.

In addition, this argument also implies that there can be no solution to (14) for $p \in \left(\left(\tilde{p}^S\right)^{-1}(p_h, 1), p_h), p_m\right)$. ■

Claim 4: *The contract $r_{zp}(p, 0)$ satisfies the low-effort IC constraint for $p \in [p_{NF}, p_m)$.*

We proceed by induction.

- First consider $p \in \left[\max[p_{NF}, \left(\tilde{p}^S\right)^{-1}(p_h, 1)], p_m\right)$.

  Recall that in proving Claim 3 we defined $\tilde{v}^r(p, e)$ to be the utility the risky agent would receive if the equilibrium implemented effort e at p, where $p \in \left[\left(\tilde{p}^S\right)^{-1}(p_h, 1)], p_h\right)$.

Intuitively, low effort was not incentive compatible in this region, that would contradict the construction of p_m as minimal. To see this, suppose it were not true, i.e., $\tilde{v}^r(p, 0)(1 - \tilde{\beta}q) > \frac{c\pi_l}{\pi_h - \pi_l}$. Now, from the construction of p_h as minimal, we also know that we have $\tilde{v}^r(p, 1)(1 - \tilde{\beta}q) < \frac{c\pi_l}{\pi_h - \pi_l}$. Then the continuity of $\tilde{v}^r(p, e)$ in e would imply that there must be a solution e' to (14) at p, which contradicts the construction of p_m as minimal.

So we conclude that $e^r(p) = 0$ for $p \in \left[p_{\text{NF}}, \left(\tilde{p}^S\right)^{-1}(p_h, 1)], p_m\right)$. By the monotonicity of $r_{zp}(p, 0)$ and $\tilde{v}^r(p, 1)$, we can also conclude that $v(p)$ is monotonic for $p \geq \max[p_{\text{NF}}, \left(\tilde{p}^S\right)^{-1}(p_h, 1)]$.

- If $\max[p_{\text{NF}}, \left(\tilde{p}^S\right)^{-1}(p_h, 1)] = p_{\text{NF}}$, then we are done. Otherwise, we need to iterate the argument. Recall that we have already demonstrated that low effort is incentive compatible at $p' = \left(\tilde{p}^S\right)^{-1}(p_h, 1)$ and that $v^r(p)$ is monotonic for $p \geq p'$.

  We conclude the proof by showing that if low effort is incentive compatible at some p' and that $v^r(p)$ is monotonic for $p \geq p'$, then it is incentive compatible for $p \geq \left(\tilde{p}^S\right)^{-1}(p', 0)$ and also $v^r(p)$ is monotonic for $p \geq \left(\tilde{p}^S\right)^{-1}(p', 0)$.

  Now, $p^S(p) = \tilde{p}^S(p, 0)$ is increasing in p, and also $p^S(p) \geq p'$ when $p \geq \left(\tilde{p}^S\right)^{-1}(p', 0)$, thus from the monotonicity of $v^r(\cdot)$ we have $v(p^S(p)) < v(p^S(p'))$. Also, $r_{zp}(p, 0)$ is monotonic in p. These then imply that $v^r(p) < v^r(p')$ and so low effort is also incentive compatible at p. The desired monotonicity of $v(p)$ for $p \geq \left(\tilde{p}^S\right)^{-1}(p', 0)$ follows immediately.

  We can then iterate the same argument as above, and continue doing so until reaching p_{NF}. ■

Claim 5: *The equilibrium constructed in Proposition 1 maximizes the risky entrepreneurs' effort $e^r(p)$, across all symmetric sequential MPE.*

First note that this is immediate for region c., since the equilibrium of Proposition 1 implements high effort for all $p > 0$. As far as region a. for the values of c in this region, it is not hard to show that only low effort can be incentive compatible.

So it suffices to consider region b., the intermediate values of c (where high effort is implemented with probability 1 if and only if $p \geq p_h$). Suppose that this is not the case and that there exists some other equilibrium that implements higher effort at some p'. Without loss of generality we can take $p' < p_h$. Let $e^r(p), p^S(p), r(p)$, and $v^r(p)$ denote the effort, updating function, interest rate, and risky-entrepreneur utility, respectively, for the equilibrium of Proposition 1, and let $\overline{e}^r(p), \overline{p}^S(p), \overline{r}(p)$, and $\overline{v}^r(p)$ denote the same for this other equilibrium, where $\bar{e}^r(p'^r(p')$ for some $p' < p_h$.

We begin by noting that $v^r(p) \geq \overline{v}^r(p)$ for all $p \geq p_h$.[43] This then implies that the equilibrium of Proposition 1 maximizes effort for $p \in \left[\left(\tilde{p}^S\right)^{-1}(p', 0)\right)$, i.e., $e^r(p) \geq \bar{e}^r(p)$. If this were not the case, then the choice of p_m as minimal and $e^r(p)$ as maximal would be violated. We can then also conclude that $v^r(p) \geq \overline{v}^r(p)$ for $p \geq \left(\tilde{p}^S\right)^{-1}(p', 0)$.

To establish this result for the remaining values of p, recall that we showed in the proof of Proposition 1 that low effort is the only incentive compatible effort choice at $p \geq \left(\tilde{p}^S\right)^{-1}(p', 0)$. Since $v^r(p) \geq \overline{v}^r(p)$ for $p \geq \left(\tilde{p}^S\right)^{-1}(p', 0)$, this must also be the case for any other equilibrium, since this inequality implies that any future continuation utility would be (weakly) higher in the equilibrium of Proposition 1. It is then straightforward to extend this argument to lower values of p. ■

Claim 6: $W(s_0, q) \geq \overline{\mathcal{W}}(s_0, q)$.

First consider the case $p_0(s_0, q) < p_l$. From (5) it is clear that since the other equilibrium implements lower effort at any p, we must have $\bar{p}_0(s_0, q) \leq p_0(s_0, q)$. Thus from Corollary 1 there can be no financing in either equilibrium and so $W(s_0, q) = \overline{\mathcal{W}}(s_0, q)$.

Next, when $p_0(s_0, q) \geq p_l$, total surplus can be defined as follows:

$$\mathcal{W}(s_0, q) = \mathcal{W}^s(s_0) + \mathcal{W}^r(s_0),$$

where $W^s(s_0) = \frac{s_0(R-1)}{1-\tilde{\beta}}$ and $W^r(s_0) = (1 - s_0)w^r(p_0(s_0, q))$, for $w^r(\cdot)$ defined recursively: $w^r(p) \equiv \left[\pi_{e(p)}R - 1 - c \cdot e^r(p)\right] + \pi_{e(p)}\tilde{\beta}w^r(p^S(p)) + (1 - \pi_{e(p)})q\tilde{\beta}w^r(p)$.[44] We can similarly define $\overline{\mathcal{W}}(s_0)$, $\overline{\mathcal{W}}^s(s_0), \overline{\mathcal{W}}^r(s_0)$, and $\overline{w}^r(p)$ for the other equilibrium.

It is immediate that $W^s(s_0) \geq \overline{\mathcal{W}}^s(s_0)$. So we will restrict attention in what follows to establishing the same result for $W^r(s_0)$.

Suppose first that there is financing for all $p > p_0(s_0, q)$ in both equilibria. Now, it follows that since the equilibrium of Proposition 1 implements high effort for $p \geq p_h$, we have $w^r(p) \geq \bar{w}^r(p)$. Also note that $w^r(p)$ is constant above p_h. So if $p_0(s_0, q) \geq p_h$, it is

[43] This is immediate if the other equilibrium also implements high effort at every successor node. Suppose, however, that this is not the case and that high effort is *not* implemented at some successor node $\bar{p}^S$ (without loss of generality consider the closest such successor node). Then we must have $\bar{v}^r(\bar{p}^S) \leq v^r(\bar{p}^S)$. The result then follows, since both equilibria implement the same effort level between p and $\bar{p}^S$, while interest rates for the equilibrium of Proposition 1 are minimal.

[44] Here $\pi_{e(p)} \equiv \pi_h e^r(p) + \pi_l(1 - e^r(p))$ is the risky entrepreneurs' success probability given the equilibrium effort level at p, and similarly $\pi_{\overline{e}(p)}$ for the other equilibrium.

then immediate that $W^r(s_0) \geq \overline{\mathcal{W}}^r(s_0)$, since from (5) we know that $p_0(s,0) \geq \bar{p}_0(s_0,q)$.[45] Otherwise, we proceed by induction.

Consider next $p \in [p_m, p_h)$. We know from Claim 5 that $e^r(p) \geq \bar{e}^r(p)$, which also implies that $\bar{p}^S(p) \geq p^S(p) \geq p_h$, and thus that $w^r(\bar{p}^S(p)) \geq \overline{w}^r(\bar{p}^S(p))$. Also note that $w^r(p') > 0$ for $p' \geq p_h$. So

$$\begin{aligned}
\overline{w}^r(p) &= \frac{\big(\pi_{\bar{e}(p)}R - 1 - c\bar{e}^r(p)\big) + \pi_{\bar{e}(p)}\tilde{\beta}\overline{w}^r(\bar{p}^S(p))}{1-(1-\pi_{\bar{e}(p)})\tilde{\beta}q} \\
&\leq \frac{\big(\pi_{\bar{e}(p)}R - 1 - c\bar{e}^r(p)\big) + \pi_{\bar{e}(p)}\tilde{\beta}w^r(\bar{p}^S(p))}{1-(1-\pi_{\bar{e}(p)})\tilde{\beta}q} \\
&\leq \frac{\big(\pi_{e(p)}R - 1 - ce^r(p)\big) + \pi_{e(p)}\tilde{\beta}w^r(\bar{p}^S(p))}{1-(1-\pi_{e(p)})\tilde{\beta}q},
\end{aligned}$$

where the final inequality follows because $\bar{e}^r(p) \leq e^r(p) \leq 1$.[46] If we now replace $w^r(\bar{p}^S(p))$ with $w^r(p^S(p))$ in the right-hand side of the inequality, this cannot decrease its value, since we showed that $w^r(p')$ is constant for $p' \geq p_h$ (and $\bar{p}^S(p) \geq p^S(p)$). This then demonstrates that $w^r(p) \geq \bar{w}^r(p)$ for $p \in [p_m, p_h)$. Another consequence of this is that $w^r(p)$ is weakly increasing for $p \geq p_m$.

Next, observe that for lower values of p; $p \in [p_l, p_m)$, from Claim 5 we know that both equilibria implement low effort in this region, and so $w^r(p) \geq \bar{w}^r(p)$. This again also implies that $w^r(p)$ is weakly increasing. Then since $p_0(s_0,q) \geq \bar{p}(s_0,q)$, we can conclude that $W^r(s_0) \geq \overline{\mathcal{W}}^r(s_0)$.

If the other equilibrium implements exclusion for $p \geq p_l$, it is not hard to extend the argument above, once we note that financing generates positive social surplus in every period (since the lenders make zero profits, on average). ■

[45] Since $p_0(s,0)$ is decreasing in the effort exerted in the inital period, and this effort cannot be higher in the other equilibrium.

[46] The reason is that increasing from $\bar{e}^r(p)$ to $e^r(p)$ raises the probability of success (and hence continuing rather than staying at the same score). Since the agent exerts high effort at $\bar{p}^S(p)$, this then increases welfare generated by the risky agents.