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Monetary Policy Implementation Frameworks: A Comparative Analysis*

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Abstract

We compare two stylized frameworks for the implementation of monetary policy. The first framework relies only on standing facilities, while the second framework relies only on open market operations. We show that the Friedman rule cannot be implemented when the central bank uses standing facilities, while it can be implemented with open market operations. For a given rate of inflation, we show that standing facilities unambiguously achieve higher welfare than just conducting open market operations. We conclude that elements of both frameworks should be combined. Also, our results suggest that any monetary policy implementation framework should remunerate both required and excess reserves.

1 Introduction

In this paper, we compare two frameworks for the implementation of monetary policy. In the first framework, the central bank (CB) operates a channel system. In a channel system, the CB offers two standing facilities: One lending facility where it stands ready to lend funds

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against collateral and one deposit facility where it accepts and remunerates deposits. The CB does not intervene in any other way. In a second framework, the CB conducts open market operations (OMOs), and does not provide access to any standing facilities. Our analysis allows to clearly identify the cost and benefits of these two polar scenarios and it suggests ways in which these systems can be combined.

In practice, CBs often adopt a mix of these two approaches but with different emphasis. Some systems rely primarily on the use of standing facilities, while other systems rely on OMOs and may not provide standing facilities. For example, the Federal Reserve uses OMOs to implement its monetary policy but does not operate standing facilities. While the discount window is a source of credit for banks, it is not considered a regular source of funding.[1] At the other end of the spectrum are CBs that run "narrow corridors," such as the Bank of Canada or the Reserve Bank of Australia. In such systems, the credit and deposit facilities operated by the CB play a preeminent role. Our goal is to shed light on some of the welfare costs and benefits of different approaches to implement monetary policy. We are also interested in monetary policy implementation because the Federal Reserve has received authority to pay interest on reserves as of October 2008. This authority gives the Federal Reserve the opportunity to modify the way it implements monetary policy in important ways.[2]

We base our analysis on a variant of the model of Berentsen and Monnet (2008), hereafter BM. They use a general equilibrium model that is tractable, as well as provides a rationale for the use of CB reserves to study the implementation of monetary policy using only standing facilities (a pure channel system). Modeling the reasons for which agents (or banks) hold reserves is important because the monetary policy implementation framework modifies the incentives for holding reserves. We depart from BM's assumption that agents pledge goods as collateral. Instead, we introduce Treasury securities, or government bonds, as eligible collateral. Agents can exchange reserves for securities, with each other and with the CB, in a Treasury market.[3] This market allows agents to adjust their reserves holdings after they observe a signal about whether or not they are likely to need reserves. As in BM, we consider monetary policy implementation systems with a daily reserve maintenance period, in which the level of required reserves is normalized to zero.

[1] In December 2007, the Federal Reserve started to provide reserves through a term auction facility to alleviate some stress in financial markets. Whether this facility will become a regular tool is not yet clear. More details are available at: http://www.federalreserve.gov/monetarypolicy/taf.htm.

[2] See, for example, Keister, Martin, and McAndrews (2008).

[3] Note that the money supply, in our model, consists only of CB reserves.

We obtain two main results. First, a pure channel system is unable to achieve the efficient allocation. The intuition is simple: In our model with a microfounded role for money, the lower the inflation rate is, the higher welfare is. However, the rate of inflation is directly linked to the growth rate of the stock of reserves. A CB that uses a pure channel system can only control the stock of reserves via the rate it charges at the two standing facilities. If the lending rate is high, agents have to repay a large amount of interests to the CB. Therefore, the stock of available reserves shrinks, everything else constant. Naturally, this depends on agents still borrowing at the lending facility: If agents do not borrow, agents do not repay interests to the CB and the stock of reserves cannot shrink. As the rate of inflation becomes sufficiently low, the opportunity cost of holding reserves decreases and agents prefer to hold reserves than using the CB's borrowing facility. This effect imposes a lower bound on the growth rate of the supply of reserves, which is necessarily higher than the efficient growth rate, i.e. the Friedman rule.

Our second result is that the channel system achieves a higher welfare than a system relying on open market operations, for rates of inflation that can be achieved by both systems. In our model, agents learn whether they may need reserves in the Treasury market, and an agent's real value for reserves depends on this information. A property of the efficient allocation (the first best if there is no inflation above the Friedman rule and the second best otherwise) is that agents value reserves the same, independently of their information. That is, it is efficient that they are insured against their information shock in the Treasury market. This means that agents must be indifferent between holding bonds and reserves in the Treasury market.

With inflation, those agents with no need for reserves, always prefer to hold bonds. This preference is strict when the central bank conducts open market operations, as there is no possible insurance mechanism against inflation. Therefore, open market operations cannot achieve the second best. Surprisingly, this is not the case with the channel system: By paying an interest on deposits, the CB can actually compensate the cost of inflation for agents who do not need reserves. Then, these agents can be indifferent between holding bonds and reserves. However, an interest on deposits is itself inflationary, so that agents who actually need reserves are hurt. To reduce inflationary pressure, the CB can set the lending rate above the deposit rate. While agents who use the lending facility pay more, they indirectly benefit from this policy: Since inflation is lower, each dollar they borrow has more value. This policy is feasible if the lending rate is not too high. Indeed, if it is too high, no agents would use the lending facility, as they would instead sell all their bonds on the Treasury market. Therefore, when the interest rate channel is narrow, i.e., the lending rate is not too high relative to the deposit rate, agents can be indifferent between holding bonds and reserves in the Treasury market.

To summarize, we obtain our second result because the CB that uses a channel system can charge different interest rates to borrowers and depositors. This essentially create a beneficial transfer from agents who borrow at the CB to depositors. Such a transfer does not exist when the CB conducts OMOs as it cannot (price) discriminate between borrowers and lenders.

The recent literature on the implementation of monetary policy, as well as the literature on banks' reserves management problem initiated by Poole (1968), is mainly confined to analyzing the issue of monetary policy implementation within a given framework and does not contrast the performance of different systems in welfare terms. For example, the literature has been concerned with the behavior of the fed funds rate (see for instance Hamilton 1996, Furfine 1999), or reducing the volatility of the interbank market rates (Whitesell, 2006a,b, and Holthausen, Monnet and Wuerz, 2007). Woodford (2000) argues that the CB can implement monetary policy even if it does not have direct control of the money supply. Goodfriend (2002) proposes a monetary implementation framework in which the CB pays interest on reserves at the policy rate and expands the supply of reserves considerably. Ennis and Weinberg (2007) also consider the benefits of paying interest on reserves and the impact it has on daylight credit in a simple model.

The remainder of the analysis proceeds as follows. In Section 2 we describe the model. In Section 3 we study an implementation framework that relies on OMOs. In Section 4 we study an implementation framework that relies on a channel. Section 5 compares the two frameworks, and Section 6 discusses our results and concludes.

2 The Environment

There is a [0,1] continuum of infinitively-lived agents, which we associate with banks, a government, and a CB. Time is discrete and three perfectly competitive markets open sequentially in each period. These markets – the Treasury market, the goods market, and the settlement market – are described below. The discount factor is β.

The government issues a fixed number $\bar{B}$ of consols (bonds) with nominal return $\tilde{R}$ per unit in each period. We assume that these securities are book entry at the CB so that they are illiquid. The government finances these bonds using lump-sum tax. Since we consider a stationary environment, we assume that the government adjusts the nominal return to inflation, so that the real return is constant.[4]

In the Treasury market, agents trade securities with each other and, if it is active, with the CB. At the beginning of this market, each agent receives information regarding their use of cash. We model this as a preference shock $\varepsilon \in \{0,1\}$. $\varepsilon = 1$ with probability $\mu \in [0,1]$. Agents with $\varepsilon = 0$ know they will have no need for cash, while agents with $\varepsilon = 1$ are likely to need some. This difference in expected need for cash generates an incentive for trading in the Treasury market.

In the goods market, agents produce or consume a perishable good but cannot trade securities with the CB or with each other.[5] With probability $1-n$, a bank can consume but cannot produce in the goods market; we refer to these agents as consumers. With probability n, an agent can produce but cannot consume; we call these agents producers. Consuming q units of goods in the second market generates utility $\varepsilon u(q)$, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$ and $u'(\infty) = 0$. Producing q units of output has a utility cost $c(q) = q$. The first-best allocation in the goods market is denoted q^* and satisfies $u'(q^*) = 1$. All trades are anonymous and agents' trading histories are private information. Since producers require immediate compensation for their production effort, reserves are essential for trade.[6]

In the goods market, the CB can operate standing facilities. After agents observe their idiosyncratic shock, they can either borrow reserves from, or deposit reserves with, the CB. The CB operates the standing facilities at zero cost. It offers nominal loans at an interest rate i_ℓ and promises to pay interest rate i_d on nominal deposits, with $i_\ell \geq i_d$. This condition eliminates the possibility for arbitrage in which agents borrow and subsequently make a deposit at interest $i_d > i_\ell$, increasing their reserves holdings at no cost. We restrict financial contracts to overnight contracts. An agent who borrows ℓ units of reserves from the CB repays $(1+i_\ell)\,\ell$ units at the settlement stage. Similarly, an agent who deposits d units of reserves at the CB receives $(1+i_d)\,d$ units at the settlement stage. All loans must be secured with Treasury securities.

In the settlement stage, agents can produce and consume a general good, settle their claims with the CB, and trade securities with one another or the CB. In addition, agents and the CB can buy securities from the government. General goods are produced solely from

[4] See Kocherlakota (2003) or Shi (2005) for a reason why securities should or could be illiquid, respectively.

[5] We modify this assumption in the Appendix and show that our results are basically unchanged.

[6] By essential we mean that the use of reserves expands the set of allocations (Kocherlakota, 1998 and Wallace, 2001).

labor according to a production technology with constant return to scale. Producing one unit of the consumption good generates one unit of disutility while consuming one unit of it gives one unit of utility.[7]

The government levies a (nominal) lump-sum tax during the settlement stage to finance interest payment on its debt. The CB does not have the authority to tax agents but can make lump-sum transfers of reserves to agents during the settlement stage. We denote these (nominal) transfers by $\pi \geq 0$.

In the remainder of the paper, we assume that the CB operates only a subset of its monetary policy implementation tools. In the next section, the CB is active only in trading securities and shuts down access to the standing facilities. In the following section, the CB is inactive in the Treasury market but provides access to the standing facilities.

2.1 Modeling Choice

Our assumptions on the timing of events are motivated by the CBs' practice. CBs that rely primarily on OMOs often intervene in markets early in the morning, before most of the banks' payment activity takes place. For instance, in the United States, the Desk at the Federal Reserve Bank of New York conducts its intervention around 10:30 a.m.[8] To the contrary, banks can access standing facilities at any time of the day and even after the money market is closed. Therefore, banks that are short of reserves but still need to make an unexpected payment can do so by accessing the lending facility. For instance, Hartmann, Manna, and Manzanares (2001) report that the euro area money market opens at around 8:00 a.m. and closes at around 5:45 p.m. Still, banks in the euro area can access the standing facilities at their National CB until 6:15 p.m., or 15 minutes after TARGET (the euro area Real-Time Gross Settlement system) closes.

To get to this structure, we assume that, conditional on receiving the signal $\varepsilon = 1$, banks have to make a payment with probability $1 - n$, at the end of the day. If they think it is likely that they will have to make a payment at the end of the day (i.e., they received the signal $\varepsilon = 1$), banks can borrow funds on the Treasury market. Our Treasury market is the equivalent of a secured interbank market. As is the case in reality, banks can access standing

[7] The linear preferences in market 1, first introduced by Lagos and Wright (2005) to get a degenerate distribution of reserves holdings at the beginning of a period, allow us to interpret transactions that are taking place in the first market as settlement transactions, as in Koeppl, Monnet and Temzelides (2008).

[8] See Edwards (1997) for details.

facilities even after payments are last made, i.e., even after the goods market closes, while OMOs are conducted before banks know whether they will have to make a payment, i.e., before the final schock is realized. Still, we show in Section 7.2 in the Appendix, that our results are robust to a change in the timing of OMOs. We do so by allowing the CB to intervene in markets after agents have observed their shock. While the equilibrium conditions are naturally somewhat affected, our results are unchanged.[9]

To match some of the important details of a channel system, we assume that banks have to pledge collateral when they access the lending facility. CBs around the world only extend collateralized loans, but we keep the reasons for this requirement out of the model. Collateral requirements impose costs on banks, and in our model with no default risk, it would be optimal for the CB to make uncollateralized loans. Although we keep default out of the model, it is relatively easy to integrate it back in: One can think of the discount factor as integrating a default probability δ, such that the effective discount factor is $\beta = (1 - \delta)\,\tilde{\beta}$ and $\tilde{\beta}$ is the time discount factor.[10]

It is also worth discussing why we associate banks with agents in our economy. The basic idea is that each bank can serve exactly one agent and competition implies that a bank is maximizing this agent's payoff. When agents are in need of cash, the bank accesses the Treasury market or the standing facilities on behalf of the agent. In this sense, there is no distinction between what a bank does and what an agent would do. Also, since agents themselves could access the Treasury market or the standing facilities, there is no essential role for a "bank" in our model, such as the provision of liquidity insurance as in Diamond and Dybvig (1983) for example. Our model is, therefore, not a model of banking, although it still is a model of monetary policy implementation.[11] Introducing the necessary frictions giving rise to a nontrivial role for banks would add an additional layer of complexity, as banks would then have to manage an aggregate portfolio of deposits and loans. Monetary policy could only have a role if the liquidity insurance within the bank is imperfect, that is, if the bank itself is subject to an aggregate liquidity shock (for instance, withdrawals could be correlated). In our model, we bypass this difficulty as the agent's shock (being a consumer or a producer) is

[9] One should note that conducting an OMO after the goods market closes is equivalent to opening a deposit facility. Indeed, after the goods market closes, no bank is interested in obtaining reserves from the CB. Banks with excess reserves are willing to sell those reserves to the CB at any price corresponding to a gross interest rate greater than or equal to 1.

[10] See Chapman, Chiu and Molico (2009) for a model of the channel system with default.

[11] See Mattesini, Monnet, and Wright (2009) for a model in which banks are essential.

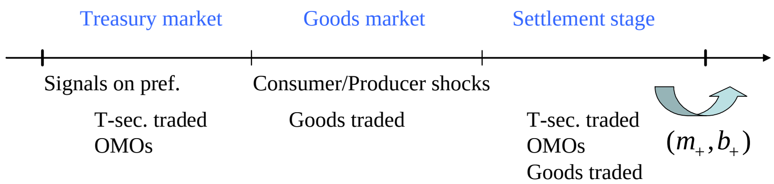

Figure 1: Time line with OMOs.

itself the bank's aggregate shock. Needless to say, it would be very interesting to extend the model to include a nontrivial role for banks, and this is left for future research.

3 Open Market Operations

In this section we study our economy when the CB engages in OMOs in the Treasury market. Since the CB can print reserves at no cost, it does not face a budget constraint when it buys securities. However, it cannot sell more securities than what it acquired in the previous settlement stage. In this section, we assume that the CB does not operate standing facilities. The time line is, therefore, as in Figure 1.

We use B and b to denote the stock of Treasuries held by the CB and by agents (the private sector), respectively, at the beginning of the securities market so that $\bar{B} \equiv B + b$. During the Treasury market, the CB can buy Y securities from the private sector (the usual convention applies that a negative purchase is a sale). The stock of securities held by the private sector becomes $b' \equiv b - Y$, while the stock of securities held by the CB becomes $B' = B + Y = \bar{B} - b'$. At the settlement stage, agents get the interest rate on their securities and the CB makes a lump-sum transfer.

Let m denote an agent's holding of reserves at the beginning of the Treasury market. We will show that agents choose the same holdings of reserves and securities at the settlement stage so m and b are identical for all agents. Let M denote the stock of reserves at the beginning of the Treasury market. The stock of reserves increases by ηY if the CB buys Y securities at a price η. Therefore, the stock of reserves at the end of the Treasury market is given by

$$M' = M + \eta Y.$$

Following the settlement stage, the stock of money is

$$M_+ = M' + \rho Y' - \tilde{R}\,(B+Y) + \pi,$$

where $\pi = \tilde{R}\,(B+Y) + \tau$ denotes lump-sum transfers of reserves made by the CB to the agents and the government. The CB transfers the interests on its bond holdings to the government, and we assume that the government redistributes it lump-sum to the agents. In addition, the CB can make an additional transfer of τ. Y' is the amount of securities the CB buys and ρ the price. Hence, the evolution of the supply of reserves is given by

$$M_+ = M + \eta Y + \rho Y' + \tau. \tag{1}$$

We assume that the CB may not force people to buy the securities it is trying to sell at a given price. Instead, it determines the amount of securities it wants to buy or sell, taking the demand schedule for prices η and ρ into account.

3.1 Equilibrium

We solve the equilibrium backward, first considering the settlement stage, then the goods market trades, and finally the Treasury market. In period t, let ϕ_t be the real price of reserves in the settlement stage. We focus on symmetric and stationary equilibriums in which all agents follow identical strategies and where real allocations are constant over time. For notational simplicity we suppress the time index t and consider a representative period t. In a stationary equilibrium end-of-period real reserves balances are time invariant

$$\phi M = \phi_+ M_+. \tag{2}$$

Define the growth rate of the money supply, i.e., inflation, as $\gamma \equiv M_+/M = \phi/\phi_+$. We let $V(m,b)$ denote the expected value of entering the settlement stage with m units of reserves and b securities. $Z(m,b)$ denotes the expected value of entering the Treasury market with m units of reserves and b securities. During the Treasury market, agents reallocate their portfolio based on the information they have about their type in the goods market. $W\,(m,b)$ denotes the expected value of entering the goods market with a portfolio (m,b).

3.1.1 Settlement Stage

In the settlement stage, the problem of an agent with portfolio (m, b) is:

$$V(m,b) = \max_{h,m_+,b_+} -h + \beta Z(m_+, b_+)$$
$$s.t. \quad \phi m_+ + \phi\rho b_+ = h + \phi(m - T) + \phi\left(\rho + \tilde{R}\right) b + \phi\pi.$$

where h is hours worked in the settlement stage. Using the budget constraint to eliminate h in the objective function, one obtains the first-order conditions

$$\beta Z_{m_+} = \phi, \tag{3}$$
$$\beta Z_{b_+} \leq \phi\rho (= \text{ if } b_+ > 0), \tag{4}$$

where $Z_{m_+} \equiv \frac{\partial Z(m_+,b_+)}{\partial m_+}$, and $Z_{b_+} \equiv \frac{\partial Z(m_+,b_+)}{\partial b_+}$, are the marginal value of taking an additional unit of reserves, and securities, into the Treasury market, respectively. Since the marginal disutility of working is 1, $-\phi$ is the utility cost of acquiring one unit of reserves in the settlement stage and $-\phi\rho$ is the utility cost of acquiring one unit of securities in the settlement market. The implication of (3) and (4) is that all agents enter the following period with the same amount of reserves and the same quantity of securities (which can be zero). The envelope conditions are

$$V_m = \phi \quad \text{and} \quad V_b = \phi\left(\tilde{R} + \rho\right). \tag{5}$$

where V_j is the partial derivative of $V(m, b)$ with respect to $j = m, b$.

Market clearing also requires that

$$B + Y + Y' = \bar{B} - b_+. \tag{6}$$

3.1.2 Treasury Market

In the Treasury market, agents receive their idiosyncratic shock ε, indicating whether they will need cash in the next market. Those with a shock $\varepsilon = 0$ know that they will have no use for cash, and they may wish to buy bonds instead of holding onto their cash until the next settlement stage. Those with a shock $\varepsilon = 1$ know that there is some chance that they will need cash to consume. Therefore, they may opt for selling bonds. η denotes the market clearing price. An agent's expected lifetime utility when entering the Treasury market with a portfolio (m, b) is

$$Z(m,b) = \mu W^1(m - \eta y^1, b + y^1) + (1-\mu)\, W^0(m - \eta y^0, b + y^0),$$

where y^{ε}, the quantity of Treasuries bought in the market depending on ε, is chosen optimally as indicated below. Agents with shock ε solve

$$\begin{aligned} max_{y^{\varepsilon}} \quad & W^{\varepsilon}(m - \eta y^{\varepsilon}, b + y^{\varepsilon}) \\ \text{s.t.} \quad & -b \leq y^{\varepsilon} \leq \frac{m}{\eta}. \end{aligned}$$

For convenience, we use *real* Lagrange multipliers and denote the ones for the first and second constraints by $\phi\lambda_b^{\varepsilon}$ and $\phi\lambda_m^{\varepsilon}$, respectively. The first-order condition for this problem is

$$W_b^{\varepsilon} - \eta W_m^{\varepsilon} - \phi\lambda_m^{\varepsilon} + \phi\lambda_b^{\varepsilon} = 0. \tag{7}$$

Since $u'(0) = +\infty$, no agent with $\varepsilon = 1$ will leave the Treasury market without reserves, because there is no other opportunity to get reserves in the goods market in this system. Therefore, $\lambda_m^1 = 0$. To the contrary, agents with $\varepsilon = 0$ derive no utility from cash purchase and will, therefore, never acquire cash, so that $\lambda_b^0 = 0$.

Given $\lambda_b^0 = 0$, we can use (7) to find an expression for the marginal value of securities when entering the Treasury market

$$\begin{aligned} Z_b(m,b) &= \mu\left(W_b^1 + \lambda_b^1\right) + (1-\mu) W_b^0 & (8) \\ &= \mu\eta W_m^1 + (1-\mu) W_b^0. & (9) \end{aligned}$$

An additional unit of securities in the Treasury market allows agents to acquire additional reserves at a price η. Also, since $\lambda_m^1 = 0$, the marginal value of reserves when entering the Treasury market is

$$Z_m(m,b) = \mu W_m^1 + \frac{(1-\mu)}{\eta} W_b^0. \tag{10}$$

Clearly, by combining (9) and (10), we obtain an arbitrage condition that must hold for agents to be willing to hold both cash and securities, $\eta Z_m = Z_b$. Market clearing requires that

$$(1-\mu) y^0 + \mu y^1 + Y = 0. \tag{11}$$

3.1.3 Goods Market

After the Treasury market, agents receive idiosyncratic shocks that determine whether they are consumers or producers (whether they have a high or a low need for reserves) as they enter the goods market. An agent who received shock ε is a consumer with probability $1-n$ and a producer with probability n. The expected payoff of an agent ε and portfolio (m,b) is

$$W^{\varepsilon}(m,b) = (1-n) W^{\varepsilon,c}(m,b) + n W^{\varepsilon,p}(m,b). \tag{12}$$

Let q and q_p^ε denote the quantities consumed and produced in the goods market by agents who received shock ε, respectively (only those agents with $\varepsilon = 1$ consume and q denotes their consumption level). Producers solve the following problem:

$$W^{\varepsilon,p}(m,b) = \max_{q_p^\varepsilon} \left[-q_p^\varepsilon + V(m + pq_p^\varepsilon, b) \right].$$

Using (5), the first-order condition reduces to

$$p\phi = 1. \tag{13}$$

Hence, those agents are indifferent as to the amount they produce and we will just assume that $q_p^1 = q_p^0 = q_p$, where market clearing requires $q_p = (1-n)\,\mu q/n$. The marginal value of reserves and securities for producers in the goods market are respectively,

$$W_m^{\varepsilon,p} = V_m = \phi \quad \textit{and} \quad W_b^{\varepsilon,p} = V_b = \phi\left(\tilde{R} + \rho\right). \tag{14}$$

A producer will only be able to use his reserves in the settlement stage, hence the value of reserves is the same in the goods market as it is in the next settlement stage. This is also true of the marginal value of securities. An agent with $\varepsilon = 0$ will never consume so that $W^{0,c}(m,b) = V(m,b)$. However a consumer solves the following problem:

$$\begin{aligned} W^{1,c}(m,b) = \max_{q} \quad & u(q) + V(m - pq, b) \\ \text{s.t.} \quad & pq \leq m. \end{aligned}$$

Let $\phi\bar{\lambda}_m^1$ denote the real Lagrange multiplier of this type's budget constraint. Using (5) and (13), the first-order conditions can be written as

$$u'(q) = 1 + \bar{\lambda}_m^1. \tag{15}$$

If the budget constraint is binding, then $u'(q) > 1$, which means trades are inefficient. Otherwise, trades are efficient. Using the envelope theorem and (15), the marginal value of reserves in the goods market for agents with $\varepsilon = 1$ is

$$W_m^{1,c} = \frac{1}{p}u'(q) = \phi u'(q). \tag{16}$$

The marginal value of reserves has a straightforward interpretation. A consumer with an additional unit of reserves acquires $1/p$ units of goods yielding additional utility $u'(q)/p$.

However, the value of bringing an additional security is the same as the value of this security in the next settlement stage, since securities are illiquid in the goods market:

$$W_b^1 = W_b^0 = \phi \left(\tilde{R} + \rho \right). \tag{17}$$

Combining (12), (14), and (16), the marginal values of reserves in the goods market for an agent with shock $\varepsilon \in \{0, 1\}$ are,

$$W_m^1 = (1-n)\phi u'(q) + n\phi, \tag{18}$$

$$W_m^0 = \phi. \tag{19}$$

3.1.4 Symmetric Stationary Equilibrium

Combining (10) and (18)-(19) we obtain

$$Z_m(m,b) = \mu\left[(1-n)\phi u'(q) + n\phi\right] + (1-\mu)\,\phi\frac{\left(\tilde{R}+\rho\right)}{\eta}. \tag{20}$$

The marginal value of cash in the Treasury market has two parts. First, an additional unit of cash can be used to purchase $1/\eta$ units of bonds promising a real return $\phi\left(\tilde{R}+\rho\right)$ in the next settlement stage. Agents with $\varepsilon = 0$ prefer this option. Second, an additional unit of cash can be used in the goods market by consumers to purchase $1/p = \phi$ units of goods with a marginal value $u'(q)$, or it can be held onto the next settlement stage by producers with a real return of ϕ. Agents with $\varepsilon = 1$ will prefer this option. Equation (3) can be rewritten as $Z_m = \phi_{-1}/\beta$. Also, in a stationary equilibrium $\gamma = \phi_{-1}/\phi$. Therefore, using (20) we obtain

$$\frac{\gamma}{\beta} = \mu\left[(1-n)u'(q) + n\right] + (1-\mu)\,\frac{\left(\tilde{R}+\rho\right)}{\eta}. \tag{21}$$

On the left-hand side of (21) is the real marginal cost of holding cash across periods, i.e., the inflation rate. On the right-hand side is the real marginal benefit from an additional unit of cash. Equations (4) and (8) give a no arbitrage condition

$$\eta \leq \rho_-.$$

In other words, it must cost more to acquire bonds in the previous period than the price at which they can be sold in the Treasury market. Since we consider a stationary equilibrium and the amount of bonds available in the economy is constant, we have $\phi\rho = \phi_{-}\rho_{-}$, so that we must have

$$\gamma\eta \leq \rho.$$

We now want to solve for the quantities traded on the Treasury market. From (7) we have,

$$\mu\lambda_b^1 = \frac{\rho}{\beta} - \left(\tilde{R} + \rho\right),$$

so that $\lambda_b^1 \geq 0$, if and only if $\rho \geq \beta\tilde{R}/(1-\beta)$. In other words, if consumers are constrained, bonds carry a liquidity premium, as they are worth more than their intrinsic value, i.e., the discounted stream of their payments $\beta\tilde{R}/(1-\beta)$. Similarly, we have

$$\lambda_m^0 = \tilde{R} + \rho - \eta,$$

and $\lambda_m^0 > 0$ if and only if $\tilde{R} + \rho > \eta$. That is, producers will naturally choose to spend all their cash acquiring bonds if they are cheap relative to their return. Therefore, we obtain

$$\begin{aligned} y^0 &= m/\eta, \text{ if } \tilde{R} + \rho > \eta, \\ y^1 &= -b, \text{ if } \rho > \beta\tilde{R}/(1-\beta), \\ y^1 &= -\frac{(1-\mu)}{\mu}y^0 - \frac{1}{\mu}Y \text{ otherwise.} \end{aligned}$$

Finally, the market clearing condition for bonds on the settlement market is

$$Y + Y' = \bar{B} - B - b_+ = b - b_+.$$

and so

$$b_+ = b - (Y + Y').$$

Therefore we can define an equilibrium with OMOs as follows.

Definition 1 *Given the CB policy $(\tau, B, Y/M, Y'/M)$, a symmetric stationary equilibrium is a list (γ, q, η, ρ) that solves*

$$\gamma = 1 + \eta\frac{Y}{M} + \rho\frac{Y'}{M} + \frac{\tau}{M}, \tag{22}$$

$$\frac{\gamma}{\beta} = \mu\left[(1-n)u'(q) + n\right] + (1-\mu)\frac{\left(\tilde{R} + \rho\right)}{\eta}, \tag{23}$$

$$\rho = \gamma\eta, \tag{24}$$

where

$$\begin{aligned} y^0 &= m/\eta, \text{ if } \tilde{R}+\rho > \eta, \\ y^1 &= -b, \text{ if } \rho > \beta\tilde{R}/(1-\beta), \\ y^1 &= -\frac{(1-\mu)}{\mu}y^0 - \frac{1}{\mu}Y \text{ otherwise.} \end{aligned}$$

In the Appendix, we characterize four types of equilibrium, which depend on the agents' desire to hold bonds. More precisely, let υ^{ε} be the real return on cash for agents with a shock ε and let ζ be the real return on collateral, as evaluated in the Treasury market. Intuitively, in any equilibrium cash has relatively more value for those agents with $\varepsilon = 1$, or $\upsilon^1 \geq \upsilon^0$. Also in any equilibrium, there must be a positive demand and a positive supply of bonds in the Treasury market. This implies that cash must have a higher value than bonds for those agents with $\varepsilon = 1$ and inversely for agents with $\varepsilon = 0$, or $\upsilon^1 \geq \zeta \geq \upsilon^0$. Finally, we can rewrite (23) as $\gamma/\beta = \mu\upsilon^1 + (1-\mu)\zeta$.[12]

3.2 Welfare with Open Market Operations

Given a real allocation q and z_b, we show in the Appendix that welfare is given by

$$(1-\beta)\mathcal{W} = (1-n)[u(q)-q].$$

The problem of the CB is to choose Y and Y' so as to maximize the welfare function, given the implied allocation is an equilibrium. q^* denotes the efficient allocation. In the Appendix, we show the following result.

Proposition 2 *Suppose $M_0 \leq B_0\beta\tilde{R}/(1-\beta)$, then the efficient allocation q^* is an equilibrium allocation with OMOs.*

The equilibrium implementing the efficient allocation q^* requires a sufficiently large initial stock of bonds relative to the money supply, or $M_0 \leq B_0\beta\tilde{R}/(1-\beta)$. In this equilibrium, bonds are as liquid as cash, and agents are indifferent as to which asset to hold in the Treasury market, or $\upsilon^1 = \zeta = \upsilon^0$. Not surprisingly, this is the case if the Friedman rule hold $\gamma = \beta$. The CB implements this equilibrium by selling bonds in the Treasury market to pump money

[12] The four types of equilibrium are defined by the four cases $\upsilon^1 = \zeta = \upsilon^0$, $\upsilon^1 = \zeta > \upsilon^0$, $\upsilon^1 > \zeta = \upsilon^0$, and $\upsilon^1 > \zeta > \upsilon^0$.

out of the system at the rate γ. The condition $M_0 \leq B_0\beta\tilde{R}/(1-\beta)$ can be restated as $M_0 \leq \rho B_0$ where $\rho = \beta\tilde{R}/(1-\beta)$. It says that the cash value of outstanding bonds must be higher than the stock of cash, when bonds are fairly priced. This condition ensures that the CB can implement the Friedman rule, $\gamma = \beta$, by selling enough bonds in the Treasury market. In this equilibrium, $\upsilon^1 = \zeta = \upsilon^0$ and bonds do not carry a liquidity premium.

We supposed so far that the CB could retain the profit from selling its securities. Suppose that this is no longer the case. Then it must be that $\gamma \geq 1$. In this case what is the best feasible policy? We show the following result in the Appendix.

Proposition 3 *Suppose the CB cannot retain profit, then $\gamma = 1$ and the best equilibrium allocation with OMOs satisfies*

$$\beta\left[(1-n)\,u'(q)+n\right]=1. \tag{25}$$

In all possible equilibria, welfare is decreasing in γ. As inflation rises, cash loses its value and producers are unwilling to produce as much. Therefore, when the CB cannot retain profit, it can only reach a second best equilibrium, one in which the CB implements the lowest possible level of inflation, here $\gamma = 1$. In this equilibrium, since there is some inflation (relative to the Friedman rule) those agents with $\varepsilon = 0$ prefer to hold bonds over cash so that $\zeta > \upsilon^0$. Given this, the best equilibrium is the one that minimizes distortion for agents with $\varepsilon = 1$. Inflation's distortions in the goods market cannot be eliminated with monetary policy, since there is no possibility for OMOs there.[13] Hence, the budget constraint will bind for consumers. However distortions in the Treasury market can be eliminated for agents with $\varepsilon = 1$. In particular, $\varepsilon = 1$ agents can be indifferent between holding bonds and cash, or $\upsilon^1 = \zeta$, so that their short-selling constraint is not binding. In this case, we obtain, using $\gamma/\beta = \mu\upsilon^1 + (1-\mu)\,\zeta$, that $\upsilon^1 = 1/\beta$, which is condition (25). Hence, in the Treasury market, the discounted marginal value of cash for $\varepsilon = 1$ agents equals their marginal cost (all in real terms). An example of policy that implements $\gamma = 1$ is one of pure repos, or $Y = -Y'$.

4 Channel System

In this section, we study an implementation framework that relies solely on standing facilities. We assume that the CB does not conduct OMOs. When $\gamma = 1$, the result above shows that

[13] We relax this assumption in Section 7.2 in the Appendix. There, we show that a CB that conducts OMOs can provide some insurance against inflation, as the best achievable allocation satisfies $u'(q) = 1/\beta$.

this is without loss of generality.

This framework is very similar to the model studied in BM's 2008 paper. However, to make a legitimate comparison with OMOs, we modify BM's model in one important dimension. They assume that agents can produce at a cost an asset bearing a real and exogenous return, similar to a Lucas tree. In this paper, instead, we study the channel system under the assumption that agents must pledge bonds as collateral in order to borrow from the CB.

In the channel system, agents can access the CB's lending and deposit facilities after they observe their idiosyncratic shock on the goods market. The CB operates the standing facilities at zero cost, offers nominal loans ℓ at an interest rate i_ℓ, and promises to pay interest rate i_d on nominal deposits d with $i_\ell \geq i_d$. This condition eliminates the possibility for arbitrage where agents borrow and subsequently make a deposit at interest $i_d > i_\ell$, thus increasing their reserves holdings at no cost. We restrict financial contracts to overnight contracts. An agent who borrows ℓ units of reserves from the CB repays $(1 + i_\ell)\,\ell$ units of reserves at the settlement stage. Similarly, an agent who deposits d units of reserves at the CB receives $(1 + i_d)\,d$ units of reserves at the settlement stage. We assume that all loans must be secured with Treasury securities. In a channel system, the stock of reserves evolves endogenously as follows

$$M_{+1} = M - i_\ell L + i_d D + \pi, \tag{26}$$

where M denotes the per capita stock of reserves at the beginning of period t, and π is a lump-sum transfer of reserves to agents. In the settlement stage, total loans L are repaid. Since interest rate payments by the agents are $i_\ell L$, the stock of reserves shrinks by this amount. Interest payments by the CB on total deposits are $i_d D$. The CB simply prints additional reserves to make these interest payments; therefore the stock of reserves increases by this amount. Note that since the CB does not hold any Treasury securities, and as the return on Treasury securities is financed via lump-sum taxes on agents, the supply of reserves is not affected by the outstanding amount or the rate of return on securities. The time line then is as in Figure 2.

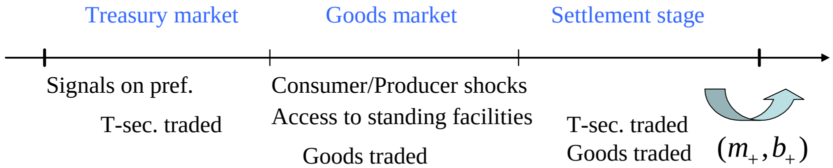

Figure 2: Time line with standing facilities.

4.1 Settlement Stage

In the first market, the problem of a representative agent is:

$$V(m, b, \ell, d) = \max_{h, m_+, b_+} -h + \beta Z(m_+, b_+)$$
$$s.t. \quad \phi m_+ + \phi\rho b_+ = h + \phi(m - T) + \phi\left(\tilde{R} + \rho\right) b + \phi(1 + i_d)d - \phi(1 + i_\ell)\ell + \phi\pi.$$

The first-order conditions are

$$\beta Z_{m_+} \leq \phi \ (= \text{ if } m_+ > 0\ ), \tag{27}$$
$$\beta Z_{b_+} \leq \phi\rho \ (= \text{ if } b_+ > 0\ ). \tag{28}$$

Once again, we focus on equilibriums in which $m > 0$. The implication of (27) and (28) is that all agents exit the settlement stage with the same portfolio of reserves and securities (which can be zero). The envelope conditions are

$$V_m = \phi; V_b = \phi\left(\tilde{R} + \rho\right); V_\ell = -\phi(1 + i_\ell); V_d = \phi(1 + i_d), \tag{29}$$

where V_j is the partial derivative of $V(m, b, \ell, d)$ with respect to $j = m, b, \ell, d$.

4.2 Treasury Market

The Treasury market functions as in the previous section. In particular, the first-order condition from an $\varepsilon-$agent's problem is still

$$W_b^\varepsilon - \eta W_m^\varepsilon - \phi\lambda_m^\varepsilon + \phi\lambda_b^\varepsilon = 0. \tag{30}$$

Since agents can obtain reserves from the standing facility, an agent with $\varepsilon = 1$ may choose to leave this market with no cash. It can also be the case that agents with $\varepsilon = 0$ leave the market with only cash, since they can deposit it at the CB. Therefore, we cannot conclude yet that $\lambda_m^1 = \lambda_b^0 = 0$. However, we will later make the argument that in equilibrium we must have $\lambda_m^1 = \lambda_b^0 = 0$. So that we still obtain

$$Z_b(m,b) = \mu\eta W_m^1 + (1-\mu) W_b^0 \tag{31}$$

and

$$Z_m(m,b) = \mu W_m^1 + \frac{(1-\mu)}{\eta} W_b^0. \tag{32}$$

Market clearing requires that

$$(1-\mu) y^0 + \mu y^1 = 0. \tag{33}$$

4.3 Goods Market

As in the previous section, at the beginning of the goods market, agents receive idiosyncratic shocks that determine whether they have a high or a low need for reserves. We let q^ε and q_s^ε, respectively, denote the quantities consumed by a buyer and produced by a seller in the goods market. Let ℓ_b^ε (ℓ_s^ε) and d_b^ε (d_s^ε), respectively, denote the loan obtained and the amount of reserves deposited by a buyer (seller) in the goods market. An agent who has m reserves and b securities at the opening of the goods market has an expected lifetime utility

$$\begin{aligned} W^\varepsilon(m,b) &= (1-n)[\varepsilon u(q^\varepsilon) + V(m - pq^\varepsilon - d_b^\varepsilon + \ell_b^\varepsilon, b, \ell_b^\varepsilon, d_b^\varepsilon)] \\ &\quad + n[-q_s^\varepsilon + V(m + pq_s^\varepsilon - d_s^\varepsilon + \ell_s^\varepsilon, b, \ell_s^\varepsilon, d_s^\varepsilon)], \end{aligned}$$

where $q^\varepsilon, q_s^\varepsilon$, ℓ_s^ε, $\ell_b^\varepsilon, d_s^\varepsilon$, and d_b^ε are chosen optimally as follows.

Agents with $\varepsilon = 0$ never consume so that $q^0 = 0$. Then let $q^1 = q$. Agents with a high need for reserves never deposit reserves with the CB and those with a low need for reserves never take out loans but deposit all their reserves. Therefore $d_b^1 = 0$ and $\ell_b^1 = \ell_s^\varepsilon = 0$ for $\varepsilon = 0, 1$. To simplify the notation, let $\ell \equiv \ell_b^1$ and $d^\varepsilon \equiv d_s^\varepsilon$. Accordingly, we get

$$\begin{aligned} W^1(m,b) &= (1-n)[u(q) + V(m - pq + \ell, b, \ell, 0)] \\ &\quad + n\left[-q_s + V\left(m + pq_s - d^1, b, 0, d^1\right)\right], \\ W^0(m,b) &= (1-n)V(m - d_b^0, b, 0, d_b^0) + n\left[-q_s^\varepsilon + V\left(m + pq_s^\varepsilon - d^0, b, 0, d^0\right)\right]. \end{aligned}$$

where q_s^ε, q, ℓ, and d_b, d^ε solve the following optimization problems.

The problem of producers is $\max_{q_s,d}\left[-q_s+V\left(m+pq_s-d,b,0,d\right)\right]$ s.t. $m+pq_s-d\geq 0$. Using (29), the first-order condition reduces to

$$p\phi+p\phi\lambda_d = 1, \tag{34}$$

$$i_d = \lambda_d, \tag{35}$$

where $\phi\lambda_d$ is the multiplier on the deposit constraint. The two conditions can be combined to get

$$p\phi\left(1+i_d\right)=1. \tag{36}$$

Comparing (13) and (36) we can already observe one major difference between OMOs and a system relying on standing facilities. With standing facilities, the deposit facility makes money more valuable as depositor earn interest on their deposit. Therefore, ceteris paribus, the price of goods p is lower with a lending facility than with OMOs.[14] Consumers solve the following maximization problem:

$$\max_{q,\ell} \quad u(q)+V(m-pq+\ell,b,\ell,0)$$
$$\text{s.t.} \quad pq\leq m+\ell \text{ and } \ell\leq\bar{\ell},$$

where

$$\bar{\ell}\equiv(\tilde{R}+\rho)b/\left(1+i_\ell\right) \tag{37}$$

is the maximal amount that a buyer can borrow from the CB, as b units of Treasury securities become $(\tilde{R}+\rho)b$ units of reserves at the beginning of the settlement market. Finally, the securities must also cover the interest payment. Using (29), consumers' first-order conditions can be written as

$$u'(q)=p\phi(1+\lambda_q), \tag{38}$$

$$\lambda_q=\lambda_\ell+i_\ell, \tag{39}$$

where $\phi\lambda_q$ is the multiplier on a consumer's budget constraint and $\phi\lambda_\ell$ is the one on the borrowing constraint. Using (36) and combining (38) and (39) yields

$$u'(q)=\frac{1+i_\ell+\lambda_\ell}{1+i_d}. \tag{40}$$

[14] This result is similar to the one in Berentsen, Camera and Waller (2007).

If the borrowing constraint is not binding and the CB sets $i_\ell = i_d$, trades are efficient. If the borrowing constraint is binding, then $u'(q) > 1$, which means trades are inefficient even when $i_\ell = i_d$. Using the envelope theorem and (38), the marginal value of reserves in the goods market is

$$W_m^1 = (1-n)u'(q)/p + n\phi(1+i_d), \text{ and} \tag{41}$$
$$W_m^0 = \phi(1+i_d). \tag{42}$$

Note that the standing facility increases these marginal values because agents can earn interest on idle reserves. Now we can derive the marginal value of securities for each type of agents using (40),

$$W_b^1 = \left[(1-n)\frac{(1+i_d)}{(1+i_\ell)}u'(q) + n\right]\phi\left(\tilde{R}+\rho\right), \text{ and} \tag{43}$$
$$W_b^0 = \phi\left(\tilde{R}+\rho\right). \tag{44}$$

4.4 Symmetric Stationary Equilibrium

First we show that the equilibrium price $1/\eta$ in the Treasury market is bounded above by $(1+i_\ell)/(\tilde{R}+\rho)$ and below by $(1+i_d)/(\tilde{R}+\rho)$.

Lemma 4 *In any equilibrium*

$$1 + i_\ell \geq \frac{\tilde{R}+\rho}{\eta} \geq 1 + i_d. \tag{45}$$

Proof. Suppose $(\tilde{R}+\rho)/[\eta(1+i_\ell)] > 1$. This means that with one unit of reserves, agents with $\varepsilon = 1$ are able to purchase $1/\eta$ bonds, which allows them to borrow $(\tilde{R}+\rho)/[\eta(1+i_\ell)] > 1$ units of reserves from the CB. In addition, those agents who do not need to borrow get $(\tilde{R}+\rho)/\eta \geq 1$ next period and are, therefore, willing to purchase bonds. But this implies that all agents want to buy bonds, and this cannot be an equilibrium. Suppose now $(\tilde{R}+\rho)/[\eta(1+i_d)] < 1$, then all agents would sell bonds to acquire reserves, since they would obtain at least $\eta(1+i_d)$ amount of reserves by using the deposit facility, higher than the bond return $\tilde{R}+\rho$. This again cannot constitute an equilibrium. ■

Notice that $(\tilde{R}+\rho)/\eta \geq 1 + i_d$ implies that $\varepsilon = 0$-agents will bring some bonds over to the goods market, so that their short-selling constraint on securities cannot possibly bind in the Treasury market, i.e., $\lambda_b^0 = 0$. Similarly, $(\tilde{R}+\rho)/\eta \leq 1+i_\ell$ implies that agents with $\varepsilon = 1$ will bring cash to the goods market, so that their cash short-selling constraint cannot bind, i.e., $\lambda_m^1 = 0$. Therefore, the above argument validates (32) and (31) as equilibrium equations.

We now solve for the quantities traded in the Treasury market. Using (30), we obtain that $\phi\lambda_b^1 = \eta W_m^1 - W_b^1$. Using equations (41) and (43), we obtain the expression for λ_b^1,

$$\mu\lambda_b^1 = \frac{\rho}{\beta} - (\tilde{R}+\rho)\left\{1+\mu(1-n)\left[u'(q)\frac{(1+i_d)}{(1+i_\ell)} - 1\right]\right\},$$

so that $\lambda_b^1 \geq 0$, if and only if $\rho \geq (\tilde{R}+\rho)\beta\left\{1+\mu(1-n)\left[u'(q)\frac{(1+i_d)}{(1+i_\ell)} - 1\right]\right\}$. Notice that bonds can be used to borrow from the CB. This gives bonds an additional value that equals $\mu(1-n)\left[u'(q)\frac{(1+i_d)}{(1+i_\ell)} - 1\right]$. This liquidity premium depends on whether agents are constrained in the goods market. If agents are severely constrained, then the liquidity premium is high by (40). If agents are not constrained, then bonds do not carry any liquidity premium. Therefore, agents with $\varepsilon = 1$ are indifferent between holding bonds or cash whenever the bond price perfectly reflects the intrinsic value plus the liquidity premium. In this case, $\lambda_b^1 = 0$, i.e., their constraint on the Treasury market does not bind.

We now solve for λ_m^0 to analyze cases in which $\varepsilon = 0$-agents are constrained on the Treasury market. From (30) we obtain that $\phi\lambda_m^0 = W_b^0 - \eta W_m^0$. Using (42) and (44), we obtain the expression for λ_m^0

$$\lambda_m^0 = \tilde{R}+\rho-\eta(1+i_d),$$

so that $\lambda_m^0 \geq 0$, if and only if $\left(\tilde{R}+\rho\right)/\eta \geq 1+i_d$. That is, $\varepsilon = 0$-agents are cash constrained if bonds return more than depositing cash with the CB. Therefore, we have the following equilibrium relations

$$y^0 = m/\eta, \text{ if } \tilde{R}+\rho > \eta(1+i_d), \quad (46)$$

$$y^1 = -b, \text{ if } \rho > (\tilde{R}+\rho)\beta\left\{1+\mu(1-n)\left[u'(q)\frac{(1+i_d)}{(1+i_\ell)} - 1\right]\right\}, \quad (47)$$

$$0 = (1-\mu)y^0 + \mu y^1. \quad (48)$$

The money supply evolves according to (26), which can be simplified to[15]

$$\gamma = 1+i_d+(i_d-i_\ell)\mu(1-n)\frac{\ell}{M}+\frac{\pi}{M}, \quad (49)$$

[15] See the Appendix for details.

where

$$\ell = pq - \left(M - \eta y^1\right) \leq b(\tilde{R} + \rho)/(1 + i_\ell). \tag{50}$$

Also, using (28), (31), (41), and (44), we obtain the following equilibrium condition for money holding,

$$\frac{\gamma}{\beta} = \mu\left[(1-n)\,u'(q) + n\right](1 + i_d) + (1-\mu)\,\frac{\tilde{R} + \rho}{\eta}. \tag{51}$$

Finally, (31) and (32) give us the no arbitrage condition

$$\gamma\eta = \rho. \tag{52}$$

Definition 5 *A symmetric stationary equilibrium is a policy* (π, i_d, i_ℓ) *and a time-invariant list* $(\gamma, q, \ell/M, \rho, \eta)$ *satisfying (45)-(52).*

In the Appendix, we show the following result.

Proposition 6 *The first best* q^* *is not an equilibrium allocation of any channel system.*

This result has a simple intuition: at the efficient allocation, cash has a lower value than bonds on the Treasury market and, therefore, all agents would demand bonds. To understand why this is the case, notice that q^* is an equilibrium allocation if consumers are unconstrained and $i_\ell = i_d$. When $i_\ell = i_d$, the cost of borrowing at the CB for consumers is exactly compensated by the extra quantity that sellers are willing to produce, since they themselves earn interests on their deposits. This implies that $\gamma = 1 + i_d \geq 1$, as deflation otherwise requires $i_\ell > i_d$. As consumers are unconstrained in the goods market, it must be that agents with $\varepsilon = 1$ have enough bonds on the Treasury market to acquire the cash they may need. Therefore, bonds are priced at their fair value in the settlement stage, i.e., $\rho = \beta\tilde{R}/(1-\beta)$. However, this means that the return from bonds in the Treasury market $\left(\tilde{R} + \rho\right)/\eta = (1 + i_d)/\beta$ is higher than the return from cash in the goods market, which is $1 + i_d$, given the equilibrium allocation is q^*. This obviously cannot be an equilibrium, because all agents would prefer to buy bonds than to hold onto cash.

4.5 No Lending Facility

To understand the equilibrium with lending facility, it is first instructive to study the case in which the CB does not offer any lending facility but only a deposit facility and set $\pi = 0$. Then (46)-(48) become

$$y^0 = m/\eta, \text{ if } \tilde{R} + \rho > \eta(1 + i_d), \quad (53)$$

$$y^1 = -b, \text{ if } \rho > \beta(\tilde{R} + \rho), \quad (54)$$

$$y^1 = -\frac{(1-\mu)}{\mu} y^0 \text{ otherwise.} \quad (55)$$

Notice that the absence of a lending facility reduces the value of holding collateral for agents with $\varepsilon = 1$. Therefore, their short-selling constraint binds for lower values of ρ than in the case with a lending facility. Since $\ell = 0$, (26) becomes

$$\gamma = 1 + i_d. \quad (56)$$

The equilibrium conditions (51) and (52) remain unchanged.

We have the following result, the proof of which is relegated to the Appendix.

Proposition 7 *With no lending facility, the best feasible equilibrium allocation satisfies*

$$\beta\left[(1-n)\, u'(q) + n\right] = 1. \quad (57)$$

In an economy with no lending facility, a CB has no possibility to reduce the money supply. However, since inflation is perfectly correlated with the interest rate on deposits, $\gamma = 1 + i_d$, (36) implies that the deposit rate exactly offsets the effects of inflation in the goods market. Hence, whether the CB chooses $i_d > 0$ or sets $i_d = 0$ does not modify the goods market allocation q. However, as there is inflation, agents with $\varepsilon = 0$, prefer to hold bonds to cash, so that $\zeta > v^0$. When bonds are priced at their fair value, the short-selling constraint on the Treasury market is not binding as bonds and cash for $\varepsilon = 1$ agents bear the same rate of return, i.e., $v^1 = \zeta$. We can show that such an equilibrium exists and the consumption allocation is given by (57). Notice that this allocation is the same as the one defined by (25). Hence, a channel system with no lending facility can achieve the same allocations as a system with only OMOs. Also, absent any lending facility, the CB cannot insure agents against the shock they receive in the goods market: Had they known in the Treasury market, agents who do not need cash in the goods market, would have preferred to sell all their bonds, as $\zeta > v^0$.

4.6 Lending Facility

In the previous sections, we showed that 1) a channel system cannot implement the first best allocations and 2) a channel system cannot do better than OMOs when the CB uses only a deposit facility. In this section, we consider the case in which the CB offers a lending and a deposit facility but has to redistribute any profit lump-sum. In other words, we require $\gamma \geq 1$. We show the following result.

Proposition 8 *Suppose $\bar{B}_0/M_0$ is high enough, then there is an equilibrium in a channel system with $i_\ell > i_d$ and where q is such that*

$$(1-n)\, u'(q) + n < 1/\beta.$$

The equilibrium that we consider is one in which $\varepsilon = 1$-agents are indifferent between holding bonds and cash in the Treasury market, consumers are unconstrained when they borrow at the lending facility, and inflation is lower than $1 + i_d$. Indeed, when agents borrow at the CB, $\gamma < 1 + i_d$, as interests on loans $i_\ell L$ now reduce the stock of money in circulation. However, there is borrowing at the CB only if agents hold collateral. Hence, agents with $\varepsilon = 1$ access the lending facility, only if they are unconstrained on the Treasury market. This implies that $v^1 = \zeta$, where $v^1 = [(1-n)\, u'(q) + n]\,(1 + i_d)$ and $\zeta = (\tilde{R} + \rho)/\eta$.

Since agents are unconstrained at the lending facility, bonds do not carry any liquidity premium. Therefore, bonds must be priced at fair value: $\rho = \beta\tilde{R}/(1-\beta)$ which implies $(\tilde{R} + \rho)/\eta = \gamma/\beta$. In other words, the bonds return equals the cash return. The marginal value of cash must, however, be equal to its marginal cost on the settlement stage, so that (51) applies. Since $v^1 = \zeta$, we obtain that

$$\frac{\gamma}{\beta} = [(1-n)\, u'(q) + n]\,(1 + i_d)$$

and since the lending facility reduce inflation below $1 + i_d$, we obtain that consumption can increase above the level that we obtain when there is no lending facility. In other words, paying interest on deposits makes consumers better off if inflation is lower than the deposit rate: Then inflation does not erode all the benefits of using the deposit facility. Inflation is reduced whenever consumers access the lending facility at a higher rate than the deposit rate.

Why can agents consume the amount q such that $u'(q) = (1 + i_\ell)/(1 + i_d) > 1$? When agents pledge one unit of collateral at the lending facility, (37) implies that they obtain $(\tilde{R} + \rho)/(1 + i_\ell)$ units of cash. With it, (36) implies that they can purchase $1/p = \phi\,(1 + i_d)$ units of goods. Hence, one additional unit of collateral at the lending facility has a value $u'(q)\,\phi\,(1 + i_d)\left[(\tilde{R} + \rho)/(1 + i_\ell)\right]$. If agents do not borrow, then they just get the return from the collateral $\phi(\tilde{R} + \rho)$. When consumers are unconstrained, they are indifferent between the two options (accessing or not accessing the lending facility), so that $u'(q)(1+i_d)/(1+i_\ell) = 1$.

In the Appendix, we also show that this equilibrium exists if the stock of bonds is high enough (justifying why the collateral constraint does not bind). Notice that we need that bonds are priced at their fair value and do not carry any liquidity premium. This is only the case when consumers are unconstrained on the goods market, while still borrowing at the CB. This is crucial, since this implies that there is lower inflation in this environment than in the case with no lending facility. Indeed, absent any lending facility we had $v^1 = \zeta > v^0$. However, introducing lending facilities allows a reduction in inflation relative to the deposit rate, which increases v^0. By chosing the correct combination of interest rate, the CB can fully insures agents against their shocks, in the sense that $v^1 = \zeta = v^0$. This is achieved by charging borrowers a larger interest than the deposit rates. While the borrowers are a-priori worse-off, they actually benefit from this policy as it lowers inflation, which increases the value of cash. The fact that the CB can increase v^0 by increasing the lending rate resembles a transfer from consumers to those agents who do not need cash.

Finally, notice that the equilibrium allocation is pinned down by $u'(q) = (1+i_\ell)/(1+i_d)$. Therefore,

$$1 < \frac{(1+i_\ell)}{(1+i_d)} < \frac{1-\beta n}{\beta(1-n)},$$

and the lending rate cannot be too high, i.e., it is optimal that the CB implements a narrow channel.

5 Discussion

In this section, we compare the two implementation frameworks studied in the paper. We are interested in these systems' effectiveness in implementing the Friedman rule and in the welfare they yield for an exogenously fixed rate of growth of the supply of reserves.

5.1 Implementation of the Friedman Rule

The welfare maximizing allocation, q^*, can be achieved when monetary policy is implemented with OMOs and the CB sells bonds so as to contract the money supply according to the Friedman rule. However, this allocation cannot be implemented in a channel system. Hence, ideally implementing monetary policy using OMOs is better than using a channel system.

It is interesting to understand why the Friedman rule cannot be implemented in a channel system. Some insight can be gained by looking at the equations governing the evolution of the supply of reserves in both systems. When the CB uses OMOs, the supply of reserves evolves according to

$$M_{+} = M - \rho Y - \eta Y'.$$

Hence, if $Y' = 0$, the supply of reserves can shrink when Y is set appropriately. This is not the case however when the CB uses a channel system. In this case, the supply of reserves evolves according to

$$M_{+} = M - i_{\ell} L + i_{d} D,$$

where L and D are total loans and deposits respectively. Notice that the only way to implement the Friedman rule is to set $i_{\ell} > i_{d}$ high enough so as to shrink the supply of reserves. This obviously implies a cost on borrowers. However, reserves are costless to hold when the Friedman rule is in place. Therefore, if the Friedman rule is implemented, there will be no borrowing at the standing facility if the CB charges a positive interest rate and, as a consequence, the supply of reserves cannot shrink. This limits the ability of the CB to increase the rate of return on money, thereby making it impossible to implement the Friedman rule.

5.2 Welfare with a Given Inflation Rate

In this section, we consider which framework yields higher welfare for an exogenously given level of inflation. First, we define a feasible level of inflation under each system: γ is a feasible level of inflation under a given monetary policy implementation framework if there exists a monetary policy under that framework that can achieve γ. We offer the following result.

Proposition 9 *If $\gamma < \bar{\gamma}$, then γ is OMO-feasible but not channel-feasible. For all $\gamma \geq \bar{\gamma}$, welfare is higher when γ is implemented using a channel system.*

If a channel system can implement a given level of inflation, then this system is preferable to OMOs. The intuition is rather simple. Producers are willing to produce more goods for a given amount of cash since they can earn interest on their profit by using the deposit facility. However, as we showed in sections 4.5 and 4.6, this is not enough for standing facilities to be better than OMOs. In addition to the deposit facility, agents should have access to a lending facility that imposes a higher interest rate $i_{\ell} > i_{d}$. By treating borrowers and depositors asymmetrically, a lending facility can perform a transfer from banks that need cash to those that use the deposit facility. This transfer improves welfare, since it allows lower inflation and additional production (since inflation is lower). If there is no lending facility, or if the standing facilities treat agents symmetrically $i_\ell = i_d$, the interest rate paid on a deposit facility is equal to inflation and standing facilities cannot do better than OMOs.

5.3 Policy Implications

The results above suggest that an optimal system of implementation of monetary policy should include some elements of both of the pure systems we have studied in this paper. If it wants to achieve inflation rates that are sufficiently low, a CB operating a channel system may need to hold a portfolio of securities and make OMO to affect the evolution of the supply of reserves. Conversely, a CB implementing monetary policy using OMO may want to pay interest on reserves to moderate some of the distortions that arise away from the Friedman rule.

This result is relevant for an important policy question in the United States. In October 2008, the Federal Reserve received the authority to pay interest on reserves. This was motivated in part by the financial crisis that started in 2007, and which has greatly influenced monetary policy and its implementation. Once the crisis subsides, the Federal Reserve will have a new tool at its disposal. In the implementation framework of the Federal Reserve prior to the authorization to pay interest on reserves, banks were required to hold reserves against a fraction of their deposits and neither required nor excess reserves were remunerated. This led to a couple of potential distortions. On the one hand, banks did expend resources in an effort to minimize their reserves requirement. One manifestation of such efforts was the creation of sweep accounts.[16] On the other hand, taking the requirement as given, banks tried to minimize the amount of reserves they held above their requirement since such reserves were costly at the margin.

As noted by Vice Chairman Kohn in a testimony to Congress before the new law was passed, "The Board has long supported legislation that would authorize the Federal Reserve to pay depository institutions interest on the balances they hold at Reserve Banks. As we previously have testified, paying interest on required reserve balances would remove a substantial portion of the incentive for depositories to engage in reserve avoidance measures,

[16] A sweep account transfers funds from deposit accounts, against which the banks would have to hold reserves, into another account against which no reserves need to be held at the end of each day. This allows the bank to minimize the amount of reserves it must hold.

and the resulting improvements in efficiency should eventually be passed through to bank borrowers and depositors. Having the authority also to pay interest on contractual clearing and excess reserve balances as well as required reserves would enhance the Federal Reserve's ability to efficiently conduct monetary policy."[17]

In our model, there are no reserve requirements, since all reserves held are excess reserves. An agent who is a producer is in a similar position to a bank holding excess reserves. Our analysis implies that this is suboptimal as it may distort the agent's incentives. In particular, our analysis provides support for the argument that it may be optimal for the Federal Reserve System to pay interest on both required and *excess* reserves.

6 Conclusion

This paper studies two stylized implementation frameworks for monetary policy. In one case, the CB only relies on OMOs, while in the other case, the CB operates standing facilities. In our environment, holding reserves is costly if the CB does not implement the Friedman rule. The implementation frameworks can reduce this cost in different ways.

If the CB can keep its profits, then OMOs can achieve the Friedman rule and thus the efficient allocation. However, this is not the case if the CB must rebate all its profits, for example to a fiscal authority. To the contrary, standing facilities cannot implement the Friedman rule. To reach the Friedman rule, the CB must be able to shrink the money supply sufficiently. With only standing facilities, this can be achieved only if banks use the lending facility and the CB's lending rate is higher than the deposit rate. However, if the rate of growth of the money supply is low enough, agents will prefer to hold money, rather than the bonds. This is because the opportunity cost of holding money, in terms of foregone interest, becomes small compared with the cost of accessing the CB's lending facility. However, this implies that agents cannot pledge collateral at the CB, and, therefore, cannot borrow. When we compare the two frameworks at rates of inflation that both can implement the Friedman rule, we find that the framework using standing facilities achieves higher welfare. When $i_\ell > i_d$, standing facilities create a transfer from banks that access the lending facility to banks that use the deposit facility. Such a transfer is absent when the CB conducts monetary policy through OMOs, since the CB cannot discriminate between borrowers and depositors. Finally,

[17] The transcript of the testimony can be found at
http://www.federalreserve.gov/newsevents/testimony/kohn20060301a.htm

our results highlight the benefits of using both OMOs and standing facilities to implement monetary policy. They also suggest that CBs should pay interest on both required and excess reserves.

7 Appendix

7.1 Derivation of Equation (49)

We know that

$$\begin{aligned} L &= \mu(1-n)\left[pq-\left(M-\eta y^1\right)\right] \\ D &= \mu n\left(M-\eta y^1+pq_s\right)+(1-\mu)n\left(M-\eta y^0+pq_s\right)+(1-\mu)(1-n)\left(M-\eta y^0\right) \\ &= npq_s+nM+(1-\mu)(1-n)M-\mu n\eta y^1-(1-\mu)\eta y^0 \\ &= \mu(1-n)pq+(1-\mu(1-n))M+\mu(1-n)\eta y^1 \\ &= M+\mu(1-n)\left[pq-\left(M-\eta y^1\right)\right] \end{aligned}$$

$$\begin{aligned} M_{+1} &= M-i_\ell(L)+i_d D+\pi, && (58)\\ &= M-i_\ell(\mu(1-n)\left[pq-\left(M-\eta y^1\right)\right]) && (59)\\ &\quad +i_d\left[M+\mu(1-n)\left[pq-\left(M-\eta y^1\right)\right]\right]+\pi, && (60)\\ &= M(1+i_d)+(i_d-i_\ell)\mu(1-n)\left[pq-\left(M-\eta y^1\right)\right]+\pi, && (61)\\ &= M(1+i_d)+(i_d-i_\ell)\mu(1-n)\ell+\pi, && (62) \end{aligned}$$

Therefore,

$$\gamma = 1+i_d+(i_d-i_\ell)\mu(1-n)\frac{\ell}{M}+\frac{\pi}{M}.$$

where $\ell = pq-(M-\eta y^1)$.

7.1.1 Equilibrium with OMOs

Equilibrium and Proof of Proposition 2. Case in which $\lambda_b^1=0$, $\lambda_m^0=0$ (or $\upsilon^1=\zeta=\upsilon^0$).

In this case,

$$\rho = \frac{\beta}{1-\beta}\tilde{R},$$

and since $\lambda_m^0 = 0$, we also have

$$\gamma = \beta,$$

so that

$$q = q^*$$

Since this equilibrium has the first best allocation as its ouctome, let us analyze whether it exists. This equilibrium exists if there is a policy Y, Y' such that $\gamma = \beta$. Hence, using (22) with $\tau = 0$, we must have

$$\beta = 1 + \frac{\tilde{R}}{1-\beta} \frac{(Y + \beta Y')}{M}$$

so that

$$\frac{Y}{M} + \beta \frac{Y'}{M} = -\frac{(1-\beta)^2}{\tilde{R}}$$

Set $Y' < 0$ and $Y' = -Y - \theta$ (i.e., the CB buys bonds in the Treasury market but less than what it sells in the settlement stage, where θ is how much more it has to sell than it just bought). Then,

$$\begin{aligned} \frac{Y}{M} + \beta \frac{(-\theta - Y)}{M} &= -\frac{(1-\beta)^2}{\tilde{R}} \\ \beta \frac{\theta}{M} &= \frac{(1-\beta)^2}{\tilde{R}} + \frac{Y}{M}(1-\beta) \end{aligned}$$

since we consider a stationary equilibrium, $Y/M = Y_0/M_0$. Setting $Y_0/M_0 = 0$, we get that $Y' = -\theta$, where

$$\frac{\theta}{M} = \frac{(1-\beta)^2}{\beta \tilde{R}}$$

This policy is feasible if and only if the CB has enough bonds, i.e., if

$$\begin{aligned} \sum_{t=0}^{\infty} \theta_t &\leq B_0 \\ \frac{(1-\beta)^2}{\beta \tilde{R}} \sum_{t=0}^{\infty} M_t &\leq B_0 \end{aligned}$$

Since $M_{t+1} = \gamma M_t = \beta M_t$, this policy is feasible if and only if

$$\begin{aligned} \frac{(1-\beta)}{\beta \tilde{R}} &\leq \frac{B_0}{M_0} \\ M_0 &\leq \frac{\beta \tilde{R}}{(1-\beta)} B_0 \end{aligned}$$

the money stock is less than the lifetime discounted value of the CB's stock of bonds.

Case in which $\lambda_b^1 > 0$, $\lambda_m^0 = 0$ (or $\upsilon^1 > \zeta = \upsilon^0$) In this case, we have $\eta = \tilde{R} + \rho$, which we can write

$$\rho = \frac{\gamma \tilde{R}}{1-\gamma}$$

Therefore, the return on bonds is as good as cash for an agent with $\varepsilon = 0$ in the Treasury market. By replacing the value for η and ρ in (22) and (23), we obtain

$$\gamma = 1 + \frac{\tilde{R}}{(1-\gamma)} \frac{(Y + \gamma Y')}{M} \tag{63}$$

$$\frac{\gamma}{\beta} = \mu \left[(1-n)u'(q) + n\right] + (1-\mu) \tag{64}$$

We also need to check that $\lambda_b^1 > 0$, which gives us the restriction (if $\gamma < 1$)

$$\gamma > \beta.$$

In this case, notice that the first best is only attainable for $\gamma \to \beta$. Since then the equilibrium equations boil down to the one for the case in which $\lambda_b^1 = 0$ and $\lambda_m^0 = 0$.

Case in which $\lambda_b^1 = 0$, $\lambda_m^0 > 0$ (or $\upsilon^1 = \zeta > \upsilon^0$) In this case,

$$\rho = \frac{\beta}{1-\beta} \tilde{R}$$

Since $\lambda_m^0 > 0$, we have $\eta = \rho/\gamma < \tilde{R} + \rho$ so that holding bonds yield a higher return than holding cash for those agents with $\varepsilon = 0$. Replacing the above expression for ρ, we obtain that this equilibrium exists only if

$$\gamma > \beta.$$

Then replacing the value for η and ρ in (22) and (23), we obtain

$$\gamma = 1 + \frac{\beta}{1-\beta} \tilde{R} \frac{(Y + \gamma Y')}{\gamma M}$$

$$\frac{\gamma}{\beta} = \left[(1-n)u'(q) + n\right]$$

Comparing (23) with the equation above, observe that $\left(\tilde{R} + \rho\right)/\eta = (1-n)u'(q) + n$, so that the bond return equals the yield on cash for agents with $\varepsilon = 1$.

Given Y'/M and Y/M, the first equation gives us γ, and then we get q from the second. An equilibrium with $\lambda_b^1 = 0$ exists for feasible Y'/M and Y/M such that the above equation holds if $\gamma > \beta$. The first best is clearly not attainable in this case, and the best equilibrium allocation is achieved when $\gamma \to \beta$.

Case in which $\lambda_b^1 > 0$, $\lambda_m^0 > 0$ (or $\upsilon^1 > \zeta > \upsilon^0$). In this case, both borrowing short-selling constraints are binding such that $y^1 = -b$ and $y^0 = M/\eta$. By replacing these values in the market clearing condition in the Treasury market, we obtain

$$\eta = \frac{(1-\mu)\,M}{(\mu b - Y)}$$

Notice that this implies $\mu > Y/b$. By replacing the value for η and ρ in (22) and (23) we obtain

$$\begin{aligned} \gamma &= 1 + \frac{(1-\mu)\,M}{(\mu b - Y)} \frac{(Y + \gamma Y')}{M} \\ \gamma &= \frac{\mu\,(b - Y)}{\mu\,(b + Y') - Y - Y'} \\ \gamma &= \frac{\mu\,(1 - Y_0/b_0)}{\mu\,(1 + Y_0'/b_0) - (Y_0 + Y_0')\,/b_0}. \end{aligned}$$

Also,

$$\frac{\gamma}{\beta}\left[1 - \beta\,(1-\mu)\right] = \mu\left[(1-n)u'(q) + n\right] + \frac{\tilde{R}\,(\mu b - Y)}{M} \tag{65}$$

Given the initial conditions M_0, B_0 and policies, these two equations define an equilibrium. We also need to check that $\lambda_m^0 > 0$, which gives us the restriction

$$\gamma > 1 - \frac{\tilde{R}\,(\mu b - Y)}{(1-\mu)\,M}$$

Finally, we need to check that $\lambda_b^1 > 0$, which gives us the restriction

$$\gamma > \frac{\beta}{(1-\beta)} \frac{\tilde{R}\,(\mu b - Y)}{(1-\mu)\,M}.$$

Notice that q^* is not attainable here. At q^*, we must have from (65) $\frac{\beta}{(1-\beta)} \frac{\tilde{R}(\mu b - Y)}{M} = 1$. Then the above restriction implies $\gamma > 1/\,(1-\mu)$. Since $\upsilon^1 > \zeta$ and $\gamma/\beta = \mu\upsilon^1 + (1-\mu)\,\zeta$, it must be that $\upsilon^1 > 1$, which is a contradiction.

Proof of Proposition 3.

Note that welfare is decreasing in γ in all four equilibriums. Therefore, the CB should seek to implement $\gamma = 1$. It is easy to see that $\gamma = 1$ cannot be an equilibrium in the case in which $\upsilon^1 = \zeta = \upsilon^0$ and $\upsilon^1 > \zeta = \upsilon^0$. Let us now consider the case in which $\upsilon^1 > \zeta > \upsilon^0$.

In this case, we have $\gamma/\beta = 1/\beta = \mu \upsilon^1 + (1-\mu)\,\zeta$. So that $\upsilon^1 > 1/\beta$. Finally, in the case in which $\upsilon^1 = \zeta > \upsilon^0$, we have $\upsilon^1 = 1/\beta$. Therefore, this is the case the CB should aim for. It is easy to see that the policy $Y = -Y'$ (pure repos) implements $\gamma = 1$. The rest of the proposition follows from the definition of the equilibrium.

7.1.2 Channel System with No Lending Facility: Equilibrium

Proof of Proposition 6.

Suppose there is a channel system that achieves the first best. Then since $u'(q^*) = 1$, (40) implies that $\lambda_\ell = 0$, and $i_\ell = i_d$. In turn, this implies that $\gamma = 1 + i_d$. From (47) we have

$$\frac{1}{\beta} \geq \frac{\tilde{R} + \rho}{\rho}, \tag{66}$$

as the term in square bracket equals 1. Then replacing $\gamma = 1 + i_d$ and $u'(q^*) = 1$ in (51) we get

$$\frac{1}{\beta} = \mu + (1-\mu)\,\frac{\tilde{R} + \rho}{\rho},$$

which contradicts (66).

Proof of Proposition 7.

It is easy to see that only $\lambda_m^0 > 0$ is an equilibrium, so that $y^0 = M/\eta$. Indeed, since $\gamma\eta = \rho$, we obtain $\eta\,(1 + i_d) = \rho$, since $\gamma = 1 + i_d$. Hence, we always get that $\tilde{R} + \rho > \eta\,(1 + i_d) = \rho$. Therefore we need to consider only two types of equilibrium, one where $\rho > \beta\tilde{R}/\,(1-\beta)$ and the other in which $\rho = \beta\tilde{R}/\,(1-\beta)$.

Case in which $\lambda_m^0 > 0$, $\lambda_b^1 = 0$ $(\upsilon^1 = \zeta > \upsilon^0)$ In this case,

$$\rho = \frac{\beta}{1-\beta}\tilde{R},$$

and replacing this value and the expression for η and $\gamma = 1 + i_d$ in (51), we obtain

$$\frac{1}{\beta} = (1-n)\,u'(q) + n. \tag{67}$$

This is an equilibrium only if $y^1 > -b$, since $\lambda_b^1 = 0$. Using the Treasury market clearing condition together with $y^0 = M/\eta$, we obtain

$$b > \frac{(1-\mu)}{\mu}\frac{M}{\eta}.$$

Notice that $b = \bar{B}$, since the CB does not hold any bonds. Replacing the value for η, we obtain that $\lambda_m^0 > 0$, $\lambda_b^1 = 0$ is an equilibrium if and only if

$$\frac{\beta \tilde{R}}{(1-\beta)} \bar{B} = \rho \bar{B} > \frac{1-\mu}{\mu} M (1+i_d).$$

(Multiplying both sides by ϕ, we have an expression linking the real value of bonds on the left side to the real value of cash multiplied by a constant on the right side.) Hence, for this to be an equilibrium $1 + i_d$ should be low enough.

Case in which $\lambda_m^0 > 0$, $\lambda_b^1 > 0$ ($v^1 > \zeta > v^0$) In this case, $y^0 = M/\eta$ and $y^1 = -b = -\bar{B}$, so that η is given by the market clearing condition on the Treasury market

$$\eta = \frac{1-\mu}{\mu} \frac{M}{\bar{B}}.$$

Hence,

$$\rho = \frac{1-\mu}{\mu} \frac{M}{\bar{B}} (1+i_d).$$

Replacing these values in (51), we obtain

$$\frac{1}{\beta} = \mu \left[(1-n) u'(q) + n\right] + (1-\mu) \frac{\tilde{R} + \rho}{\rho}. \tag{68}$$

This is an equilibrium if and only if $\rho > \beta \tilde{R}/(1-\beta)$ and $(\tilde{R} + \rho)/\eta > 1 + i_d$. We know the second condition is always satisfied. Turning to the first condition, this requires

$$\frac{1-\mu}{\mu} \frac{M}{\bar{B}} (1+i_d) > \frac{\beta}{1-\beta} \tilde{R}.$$

Hence, this equilibrium exists if $1 + i_d$ is large enough.

Notice that

$$\frac{\tilde{R} + \rho}{\rho} = \frac{\tilde{R}}{\rho} + 1 = \tilde{R} \frac{\mu}{(1-\mu)} \frac{\bar{B}}{M (1+i_d)} + 1 < \frac{1}{\beta}.$$

Therefore using (68), we have

$$\frac{1}{\beta} < \left[(1-n) u'(q) + n\right]$$

Comparing this with (67), we find that welfare is highest in the first equilibrium (where i_d is relatively small). When i_d is too large, bonds are relatively unattractive and agents cannot get many reserves for their bonds on the Treasury market.

7.1.3 Channel System with a Lending Facility ($\lambda_\ell > 0$): Equilibrium

Suppose now $\lambda_\ell > 0$. In words, agents are constrained when they borrow from the CB. Then

$$\ell = \frac{(\tilde{R}+\rho)}{(1+i_\ell)}(\bar{B}+y^1).$$

Since agents borrow from the CB, it must be that $\lambda_b^1 = 0$, so that they do not sell all their bonds on the Treasury market, and hence (47) gives

$$\frac{1}{\beta} = \frac{(\tilde{R}+\rho)}{\rho}\left\{1+\mu(1-n)\left[u'(q)\frac{(1+i_d)}{(1+i_\ell)}-1\right]\right\}. \tag{69}$$

Consider now the case in which $\lambda_m^0 > 0$, then (48) implies that

$$y^1 = -\frac{1-\mu}{\mu}\frac{M}{\eta}$$

The budget constraint on the goods market gets us

$$\begin{aligned} pq &= M - \eta y^1 + \ell, \text{ or} \\ \phi\ell &= q\frac{1}{(1+i_d)} - \frac{1}{\mu}\phi M \end{aligned}$$

replacing y^1 in the expression for ℓ above, we obtain

$$\phi\ell = \frac{(\tilde{R}+\rho)}{\rho(1+i_\ell)}(\phi\rho\bar{B} - \frac{1-\mu}{\mu}\phi M\gamma)$$

Hence, combining the last two equations, we get

$$q\mu\frac{(1+i_\ell)}{(1+i_d)} - \phi M(1+i_\ell) = \frac{(\tilde{R}+\rho)}{\rho}(\mu\phi\rho\bar{B} - (1-\mu)\phi M\gamma)$$

Replacing this value in (49), we have an expression for γ (with $\pi = 0$).

$$\gamma = (1-n)(1+i_\ell) + n(1+i_d) - (i_\ell - i_d)\mu(1-n)q\frac{1}{\phi M(1+i_d)}$$

Also, we have from (51)

$$\frac{1}{\beta} = \mu(1-n)u'(q)\frac{1}{\gamma} + (1-\mu)\frac{\tilde{R}+\rho}{\rho} + \mu n\frac{(1+i_d)}{\gamma}. \tag{70}$$

and arranging (69),

$$\frac{1}{\beta} = \mu(1-n)u'(q)\frac{(1+i_d)}{(1+i_\ell)}\frac{(\tilde{R}+\rho)}{\rho} + (1-\mu)\frac{(\tilde{R}+\rho)}{\rho} + \mu n\frac{(\tilde{R}+\rho)}{\rho}.$$

Combining both equations above, we obtain the equilibrium expression for the market rate,

$$\begin{aligned}(1-n)u'(q)\frac{1}{\gamma} + n\frac{(1+i_d)}{\gamma} &= (1-n)u'(q)\frac{(1+i_d)}{(1+i_\ell)}\frac{(\tilde{R}+\rho)}{\rho} + n\frac{(\tilde{R}+\rho)}{\rho} \\ \frac{(\tilde{R}+\rho)}{\eta} &= (1+i_\ell)\frac{(1-n)u'(q)+n(1+i_d)}{(1-n)u'(q)(1+i_d)+n(1+i_\ell)}\end{aligned}$$

Notice that $\frac{(\tilde{R}+\rho)}{\eta} < 1+i_\ell$, as required in equilibrium also, $\frac{(\tilde{R}+\rho)}{\eta} > 1+i_d$ if and only if $1+i_\ell > (1+i_d)^2$. Then the equilibrium in this case is defined by ϕ_0, ρ, q, and γ that satisfies,

$$\begin{aligned}\frac{(\tilde{R}+\rho)}{\rho}\gamma &= (1+i_\ell)\frac{(1-n)u'(q)+n(1+i_d)}{(1-n)u'(q)(1+i_d)+n(1+i_\ell)} \\ \gamma &= (1-n)(1+i_\ell)+n(1+i_d) - \frac{(i_\ell-i_d)}{(1+i_\ell)}(1-n)\frac{q}{\phi M}\mu\frac{(1+i_\ell)}{(1+i_d)} \\ \frac{\gamma}{\beta} &= \mu(1-n)u'(q)+(1-\mu)\frac{\tilde{R}+\rho}{\rho}\gamma + \mu n(1+i_d) \\ \frac{q}{\phi M}\mu\frac{(1+i_\ell)}{(1+i_d)} - (1+i_\ell) &= \mu\frac{\phi\rho\bar{B}}{\phi M}\frac{(\tilde{R}+\rho)}{\rho} - (1-\mu)\frac{(\tilde{R}+\rho)}{\rho}\gamma\end{aligned}$$

Combining the second and the fourth equations, we get:

$$\gamma = \frac{1+i_d - \frac{(i_\ell-i_d)}{(1+i_\ell)}(1-n)\mu\frac{\phi\rho\bar{B}}{\phi M}\frac{(\tilde{R}+\rho)}{\rho}}{1-\frac{(i_\ell-i_d)}{(1+i_\ell)}(1-n)(1-\mu)\frac{(\tilde{R}+\rho)}{\rho}}$$

Consider now the case in which $\lambda_m^0 = 0$. Then

$$\frac{(\tilde{R}+\rho)}{\eta} = 1+i_d.$$

and since $\lambda_b^1 = 0$, we have

$$\frac{\gamma}{\beta} = (1+i_d)\left\{1+\mu(1-n)\left[u'(q)\frac{(1+i_d)}{(1+i_\ell)} - 1\right]\right\}. \tag{71}$$

$$\gamma = (1-n)(1+i_\ell)+n(1+i_d) - (i_\ell-i_d)\mu(1-n)q\frac{1}{\phi M(1+i_d)}$$

Proof of Proposition 8:

Suppose $i_\ell > i_d$, and consider the following candidate allocation,

$$u'(q) = \frac{1+i_\ell}{1+i_d}, \tag{72}$$

such that from (40) $\lambda_\ell = 0$. In other words, agents are not constrained when they borrow from the CB. Also, consider an equilibrium in which agents borrow from the CB. Then it must be that they still hold bonds in the goods market, so that $y^1 > -b$. Therefore, any equilibrium in which agents borrow from the CB has $\lambda_b^1 = 0$. Combining (72) and (47), $\lambda_b^1 = 0$, if and only if $\rho = \beta\tilde{R}/(1-\beta)$, or

$$\frac{\tilde{R}+\rho}{\rho} = \frac{1}{\beta} \tag{73}$$

Now consider the case in which $\lambda_m^0 > 0$. (46) implies that

$$\frac{\tilde{R}+\rho}{\eta} = \frac{\tilde{R}+\rho}{\rho}\gamma > 1+i_d$$

Combining both equations above, we obtain

$$1+i_d > \gamma > \beta(1+i_d). \tag{74}$$

where the first inequality follows from the fact that $i_\ell > i_d$ and $\ell > 0$. Now replacing (73) in (51), we get

$$\frac{\gamma}{\beta} = [(1-n)u'(q)+n](1+i_d) \tag{75}$$

Notice that (74) and (75) give us the desired result.

We now check that this equilibrium exists. Replacing $u'(q)$ using (72),

$$\frac{\gamma}{\beta} = (1-n)(1+i_\ell)+n(1+i_d) \tag{76}$$

Using (49) and (76) we get

$$\begin{aligned} 1+i_d+(i_d-i_\ell)\mu(1-n)\frac{\ell}{M} &= \beta(1+i_d)+(1-n)\beta(i_\ell-i_d) \\ (1+i_d)(1-\beta)-(1-n)\beta(i_\ell-i_d) &= \beta(i_\ell-i_d)\mu(1-n)\frac{\ell}{M} \\ 1+\mu\frac{\ell}{M} &= \frac{(1-\beta)}{\beta}\frac{(1+i_d)}{(i_\ell-i_d)(1-n)} \end{aligned}$$

Finally to check that this is indeed an equilibrium, we need to verify that (using the budget constraint on the goods market)

$$0 < \frac{\ell}{M} \leq \frac{\left(\bar{B} + y^1\right)}{M} \frac{\left(\tilde{R} + \rho\right)}{\left(1 + i_\ell\right)} \tag{77}$$

where $y^1 = -\frac{(1-\mu)}{\mu} y^0 = -\frac{(1-\mu)}{\mu} M/\eta$. Focusing on the first inequality and replacing the value for y^1 and p, this requires,

$$\begin{aligned} \frac{(1-\beta)}{\beta} \frac{(1+i_d)}{(i_\ell - i_d)(1-n)} &> 1, \text{ or} \\ \frac{1-\beta n}{\beta(1-n)} &> \frac{1+i_\ell}{1+i_d} \end{aligned}$$

and focusing on the second inequality, this requires,

$$\frac{(1-\beta)}{\beta} \frac{(1+i_d)}{\mu(i_\ell - i_d)(1-n)} - \frac{1}{\mu} \leq \left(\frac{\phi\rho\bar{B}}{\phi M} - \frac{(1-\mu)}{\mu}\gamma \right) \frac{1}{\beta(1+i_\ell)} \tag{78}$$

where $\phi\rho\bar{B}$ is the real price of bonds (a constant) and ϕM is the real price of reserves, also a constant. Notice that for all $\bar{B}$ and M, this inequality puts a limit as to how close i_ℓ can be to i_d, i.e., on the size of the interest rate channel. In other words, if the lending rate is too close to the deposit rate, borowing is relatively cheap and consumers will want to borrow too much from the CB. To reduce borrowing, the CB must increase i_ℓ.

Replacing ℓ in the budget constraint on the goods market and $p\phi(1+i_d) = 1$ gives us ϕM, as

$$\begin{aligned} pq &= M - y^1\eta + \ell, \text{ or} \\ \phi M &= \frac{\beta}{(1-\beta)} \mu (1-n) \frac{(i_\ell - i_d)}{(1+i_d)^2} q \end{aligned}$$

Setting $M_0 = \frac{\beta}{(1-\beta)} \mu (1-n) \frac{(i_\ell - i_d)}{(1+i_d)^2} q$, we know that $\phi_0 = 1$, and since $\rho_0 = \beta\tilde{R}_0/(1-\beta)$, we can pick $\bar{B}_0$ high enough, so that the above (78) is satisfied.

7.2 Open Market Operations in the Goods Market

In this section, we consider the model in which the Treasury market is open in the goods market, once preference shocks are realized and before trades actually take place.

In the settlement stage, the problem of an agent with portfolio (m, b) is now:

$$V(m,b) = \max_{h,m_+,b_+} -h + \beta W(m_+, b_+)$$
$$s.t. \quad \phi m_+ + \phi \rho b_+ = h + \phi(m-T) + \phi\left(\rho + \tilde{R}\right) b + \phi\pi.$$

Notice that the value of the portfolio (m_+, b_+) is now directly evaluated in the goods market: Since the Treasury market is open in the goods market, we can simply assume that agents wait for the realization of their shock and ignore their signals before trading their Treasury securities. Using the budget constraint to eliminate h in the objective function, one obtains the first-order conditions

$$\beta W_{m_+} = \phi, \tag{79}$$
$$\beta W_{b_+} \leq \phi\rho(= \text{ if } b_+ > 0), \tag{80}$$

The envelope conditions are

$$V_m = \phi \quad \text{and} \quad V_b = \phi\left(\tilde{R} + \rho\right). \tag{81}$$

7.2.1 Goods Market

When they enter the goods market, agents can be either consumers or producers. An agent who received shock $\varepsilon = 1$ need reserves with probability $1 - n$ and does not need any with probability n. An agent who received shock $\varepsilon = 0$ has no need for reserves. In general, the expected payoff of an agent with shock ε and portfolio (m, b) is

$$W^{\varepsilon}(m,b) = (1-n)W^{\varepsilon,c}(m,b) + nW^{\varepsilon,p}(m,b). \tag{82}$$

Let q and q_p^{ε} denote the quantities consumed and produced in the goods market by agents who received shock ε, respectively (only those agents with $\varepsilon = 1$ consume, and q denotes their consumption level). Producers (whether they get $\varepsilon = 0$ or $\varepsilon = 1$) solve the following problem:

$$W^{\varepsilon,p}(m,b) = \max_{q_p^{\varepsilon}, y_p^{\varepsilon}} \left[-q_p^{\varepsilon} + V(m - \eta y_p^{\varepsilon} + pq_p^{\varepsilon}, b + y_p^{\varepsilon})\right]$$
$$\text{s.t.} \quad -b \leq y_p^{\varepsilon} \leq \frac{m}{\eta}$$

Since securities have to be exchanged before goods are traded, the sales pq_p^ε do not enter the short-selling constraints. Since producers have no need for cash, their securities short-selling constraint does not bind, $\phi\lambda_b = 0$, and the first-order conditions are

$$\begin{aligned} p\phi &= 1 \\ -\eta V_m + V_b &= \phi\lambda_m^\varepsilon \end{aligned}$$

where $\phi\lambda_m$ is the real multiplier on the cash short-selling constraint. Using the envelope conditions (81), we obtain

$$\left(\tilde{R} + \rho\right) - \eta = \lambda_m^\varepsilon$$

Therefore, $\lambda_m^\varepsilon > 0$ so that $y_p^\varepsilon = \frac{m}{\eta}$ if $\tilde{R} + \rho > \eta$. In other words, agents who do not need reserves buy as many Treasury bills as possible if they are cheap relative to their return. The envelope conditions are then

$$\begin{aligned} W_m^{\varepsilon,p} &= V_m + \phi\lambda_m^\varepsilon/\eta = \phi\left(\tilde{R} + \rho\right)/\eta \\ W_b^{\varepsilon,p} &= V_b = \phi\left(\tilde{R} + \rho\right) \end{aligned}$$

For those agents who do not need reserves, the incremental value of cash is the value of purchasing Treasury bills, while the incremental value of Treasury bills is just their real return on the next settlement stage.

Agents with $\varepsilon = 0$ who cannot produce solve

$$\begin{aligned} W^{0,c}(m,b) = \max_{y_c^0} \quad & V(m - \eta y_c^0, b + y_c^0) \\ \text{s.t.} \quad & -b \le y_c^0 \le \frac{m}{\eta} \end{aligned}$$

As these agents have no need for cash, the short-selling constraint on treasuries will not bind. The first order and envelope conditions then give

$$\begin{aligned} \left(\tilde{R} + \rho\right) - \eta &= \lambda_m^{0,c} \\ W_m^{0,c} &= \phi\left(\tilde{R} + \rho\right)/\eta \\ W_b^{0,c} &= \phi\left(\tilde{R} + \rho\right) \end{aligned}$$

Finally, agents with $\varepsilon = 1$ who want to consume solve

$$\begin{aligned} W^{1,c}(m,b) = \max_{q, y_c^1} \quad & u(q) + V(m - \eta y_c^1 - pq, b + y_c^1) \\ \text{s.t.} \quad & -b \leq y_c^1 \leq \frac{m}{\eta} \\ & pq \leq m - \eta y_c^1 \end{aligned}$$

Since these agents need reserves, the short-selling constraint on reserves will never bind and the first-order conditions are

$$\begin{aligned} u'(q) - pV_m - \phi p \lambda &= 0 \\ -\eta V_m + V_b + \phi \lambda_b^1 - \eta \phi \lambda &= 0 \end{aligned}$$

Using the envelope conditions on the settlement stage and $p\phi = 1$, we obtain

$$\begin{aligned} u'(q) &= \left(\tilde{R} + \rho\right)/\eta + \lambda_b^1/\eta \\ \lambda &= \left(\tilde{R} + \rho\right)/\eta - 1 + \lambda_b^1/\eta \end{aligned}$$

Consumers equalize the marginal utility of reserves to the marginal value of a Treasury security. Whenever their short-selling constraint is binding, reserves have more value than a security alone. This is the case (i.e., $y_c^1 = -b$) if $u'(q) > \left(\tilde{R} + \rho\right)/\eta$. The envelope conditions for these agents are

$$\begin{aligned} W_m^{1,c} &= V_m + \phi\lambda = \phi\left(\tilde{R} + \rho\right)/\eta + \phi\lambda_b^1/\eta \\ W_b^{1,c} &= V_b + \phi\lambda_b^1 = \phi\left(\tilde{R} + \rho\right) + \phi\lambda_b^1 \end{aligned}$$

The incremental value of reserves is again the value of purchasing Treasury bills. However, Treasury bills are more valuable for $\varepsilon = 1$ agents, since they can be sold to relax their cash constraint.

Independently of the shock received, the value of a portfolio (m,b) for an agent that enters the goods market is therefore

$$\begin{aligned} W(m,b) &= \mu\left[nW^{1,p}(m,b) + (1-n)W^{1,c}(m,b)\right] \\ &\quad + (1-\mu)\left[nW^{0,p}(m,b) + (1-n)W^{0,c}(m,b)\right] \end{aligned}$$

so that we obtain the envelope conditions

$$
\begin{aligned}
W_m &= \phi \frac{\left(\tilde{R}+\rho\right)}{\eta} + \mu(1-n)\phi\left[u'(q) - \frac{\left(\tilde{R}+\rho\right)}{\eta}\right] \\
W_b &= \phi\left(\tilde{R}+\rho\right) + \mu(1-n)\phi\eta\left[u'(q) - \frac{\left(\tilde{R}+\rho\right)}{\eta}\right]
\end{aligned}
$$

Combining them with (79) and (80), we get

$$
\frac{\gamma}{\beta} - \frac{\left(\tilde{R}+\rho\right)}{\eta} = \mu(1-n)\left[u'(q) - \frac{\left(\tilde{R}+\rho\right)}{\eta}\right]
$$

and the no arbitrage condition $\rho \geq \eta_+$ with equality if $b > 0$. Using our stationarity assumptions $\phi\rho = \phi_-\rho_-$, we get

$$
\rho \geq \gamma\eta \text{ with } = \text{ if } b > 0.
$$

We will still assume that $q_p^1 = q_p^0 = q_p$, where market clearing requires $q_p = (1-n)\mu q/n$. The other equilibrium condition (22) (obtained from combining (1) and (6)) are still valid and are determined as before. Therefore, we can define an equilibrium with OMOs in the goods stage as follows.

Definition 10 *Given the CB policy* $(\tau, B, Y/M, Y'/M)$*, a symmetric stationary equilibrium is a list* (γ, q, η, ρ) *that solves*

$$
\begin{aligned}
\gamma &= 1 + \eta\frac{Y}{M} + \rho\frac{Y'}{M} + \frac{\tau}{M} \\
\frac{\gamma}{\beta} &= \frac{\left(\tilde{R}+\rho\right)}{\eta} + \mu(1-n)\left[u'(q) - \frac{\left(\tilde{R}+\rho\right)}{\eta}\right] \\
\rho &= \gamma\eta
\end{aligned}
$$

and

$$
\begin{aligned}
y_p^0 &= y_p^1 = y_p = \frac{m}{\eta} \quad \textit{if} \ \ \tilde{R}+\rho > \eta \\
y_c^1 &= -b \ \ \textit{if} \ \ u'(q) > \left(\tilde{R}+\rho\right)/\eta \\
y_c^1 &= -\frac{(1-\mu)+\mu n}{\mu(1-n)} y_p - \frac{1}{\mu(1-n)} Y \quad \textit{otherwise}
\end{aligned}
$$

Let us suppose once again that the CB has to redistribute its profit from OMOs, or $\gamma \geq 1$. Then the best-case scenario is that consumers are not constrained on the goods market, so that $u'(q) = \left(\tilde{R} + \rho\right)/\eta$. In this case, the best equilibrium is $\gamma = 1$ so that $\rho = \eta$. Then bonds are priced at their fair value since we have

$$\begin{aligned}\left(\tilde{R} + \rho\right)/\rho &= 1/\beta \\ \rho &= \beta\tilde{R}/(1-\beta)\end{aligned}$$

Notice that, in this case $u'(q) = 1/\beta$. This is the best allocation that can be achieved when the CB conducts OMOs in the goods market.

Proposition 11 *Suppose the CB makes no profit, then $\gamma = 1$ and the best equilibrium allocation with OMOs in the goods market satisfies*

$$u'(q) = 1/\beta$$

In this equilibrium, notice that OMOs can perfectly insure agents against their liquidity shock since they all value cash at $1/\beta$. However, OMOs has little control over the inflation rate which remains at $\gamma = 1$.

7.2.2 Standing Facilities

We now derive the equilibrium conditions when the CB does not conduct OMOs but offers standing facilities instead. The Treasury market is still open in the goods market.

When they enter the goods market, agents can be either short or long of reserves. An agent who received shock $\varepsilon = 1$ has a high need for reserves with probability $1-n$ and a low need for reserves with probability n. An agent who received shock $\varepsilon = 0$ has no need for reserves. The expected payoff of an agent ε and portfolio (m, b) is

$$W^{\varepsilon}(m,b) = (1-n)W^{\varepsilon,c}(m,b) + nW^{\varepsilon,p}(m,b).$$

Let q and q_p^{ε} denote the quantities consumed and produced in the goods market by agents who received shock ε, respectively (only those agents with $\varepsilon = 1$ consume, and q denotes their consumption level). Producers (whether they get $\varepsilon = 0$ or $\varepsilon = 1$) solve the following problem:

$$\begin{aligned} W^{\varepsilon,p}(m,b) = \max_{q_p^{\varepsilon}, y_p^{\varepsilon}, d} \quad & \left[-q_p^{\varepsilon} + V(m - \eta y_p^{\varepsilon} + p q_p^{\varepsilon} - d, b + y_p^{\varepsilon}, 0, d)\right] \\ \text{s.t.} \quad & -b \leq y_p^{\varepsilon} \leq \frac{m}{\eta} \\ & d \leq m - \eta y_p^{\varepsilon} + p q_p^{\varepsilon} \end{aligned}$$

where we have already taken into account that producers do not borrow at the CB. As producers have no need for cash, their short-selling constraint on securities does not bind, $\phi\lambda_b = 0$, and the first-order conditions are

$$\begin{aligned} -1 + pV_m + p\phi\lambda_d^\varepsilon &= 0 \\ -\eta V_m + V_b - \eta\phi\lambda_d^\varepsilon &= \phi\lambda_m^\varepsilon \\ -V_m + V_d - \phi\lambda_d^\varepsilon &= 0 \end{aligned}$$

where $\phi\lambda_m^\varepsilon$ is the multiplier on the cash short-selling constraint and $\phi\lambda_d^\varepsilon$ is the multiplier on the deposit constraint. Using the envelope conditions on the settlement stage, $V_d = \phi(1 + i_d)$ we obtain

$$\begin{aligned} p\phi(1 + i_d) &= 1 \\ \left(\tilde{R} + \rho\right) - \eta(1 + i_d) &= \lambda_m^\varepsilon \\ \lambda_d^\varepsilon &= i_d \end{aligned}$$

Therefore, $\lambda_m^\varepsilon > 0$ so that $y_p^\varepsilon = \frac{m}{\eta}$ if $\tilde{R} + \rho > \eta(1 + i_d)$. In other words, agents who are long in reserves buy as many bonds as possible if the bond return is higher than the deposit rates on reserves. Also, the shadow price of reserves λ_d^ε is naturally the deposit rate i_d. The envelope conditions are then

$$\begin{aligned} W_m^{\varepsilon,p} &= V_m + \phi\lambda_m^\varepsilon/\eta + \phi\lambda_d^\varepsilon = \phi\left(\tilde{R} + \rho\right)/\eta \\ W_b^{\varepsilon,p} &= V_b = \phi\left(\tilde{R} + \rho\right) \end{aligned}$$

Agents with $\varepsilon = 0$ who cannot produce solve

$$\begin{aligned} W^{0,c}(m, b) = \max_{y_c^0} \quad & V(m - \eta y_c^0 - d, b + y_c^0, 0, d) \\ \text{s.t.} \quad & -b \leq y_c^0 \leq \frac{m}{\eta} \\ & d \leq m - \eta y_c^0 \end{aligned}$$

As these agents are long in reserves, the short-selling constraint on securities will not bind. The first order and envelope conditions then give

$$\begin{aligned} \left(\tilde{R} + \rho\right) - \eta(1 + i_d) &= \lambda_m^{0,c} \\ \lambda_d^0 &= i_d \\ W_m^{0,c} &= \phi\left(\tilde{R} + \rho\right)/\eta \\ W_b^{0,c} &= \phi\left(\tilde{R} + \rho\right) \end{aligned}$$

Hence, once again $y_c^0 = m/\eta$ if $\tilde{R} + \rho > \eta(1 + i_d)$. Finally, agents with $\varepsilon = 1$ who are short in reserves solve

$$
\begin{aligned}
W^{1,c}(m, b) = \max_{q, y_c^1, \ell} \quad & u(q) + V(m - \eta y_c^1 + \ell - pq, b + y_c^1, \ell, 0) \\
\text{s.t.} \quad & -b \leq y_c^1 \leq \frac{m}{\eta} \\
& pq \leq m + \ell - \eta y_c^1 \\
& \ell \leq (b + y_c^1)\left(\tilde{R} + \rho\right)/(1 + i_\ell)
\end{aligned}
$$

where we have taken into account that in equilibrium, these agents do not use the deposit facility. Since these agents need reserves, their short-selling constraint will never bind and the first order conditions are

$$
\begin{aligned}
u'(q) - pV_m - \phi p \lambda &= 0 \\
-\eta V_m + V_b + \phi \lambda_b^1 - \eta \phi \lambda + \phi \lambda_\ell \left(\tilde{R} + \rho\right)/(1 + i_\ell) &= 0 \\
V_m + V_\ell + \phi \lambda - \phi \lambda_\ell &= 0
\end{aligned}
$$

Using the envelope conditions on the settlement stage and $p\phi = 1/(1 + i_d)$, we obtain

$$
\begin{aligned}
u'(q) &= \frac{1 + i_\ell}{1 + i_d} + \frac{\lambda_\ell}{1 + i_d} \\
\phi \lambda_b^1 &= \phi u'(q)(1 + i_d)\left[\eta - \frac{\left(\tilde{R} + \rho\right)}{(1 + i_\ell)}\right] \\
\lambda &= \lambda_\ell + i_\ell
\end{aligned}
$$

Agents who need reserves equalize the marginal utility of reserves to the marginal value of a Treasury security. Whenever their short-selling constraint is binding ($\lambda_b^1 > 0$), they will have no more bonds in the goods stage, and, therefore, they will not be able to borrow at the CB. In this case $\lambda_\ell > 0$. This is the case ($y_c^1 = -b$) if $1 + i_\ell > \left(\tilde{R} + \rho\right)/\eta$, i.e., borrowing at the CB is more expensive than "borrowing" on the Treasury market. The envelope conditions for these agents are

$$
\begin{aligned}
W_m^{1,c} &= V_m + \phi \lambda = \phi\left(\tilde{R} + \rho\right)/\eta + \phi \lambda_b^1/\eta \\
W_b^{1,c} &= V_b + \phi \lambda_b^1 + \phi \lambda_\ell \left(\tilde{R} + \rho\right)/(1 + i_\ell) \\
&= \eta V_m + \eta \phi \lambda = \phi\left(\tilde{R} + \rho\right) + \phi \lambda_b^1
\end{aligned}
$$

The value of a portfolio (m, b) for an agent that enters the goods market is, therefore,

$$\begin{aligned} W(m,b) &= \mu\left[nW^{1,p}(m,b) + (1-n)W^{1,c}(m,b)\right] \\ &\quad + (1-\mu)\left[nW^{0,p}(m,b) + (1-n)W^{0,c}(m,b)\right] \end{aligned}$$

so that we obtain the envelope conditions

$$\begin{aligned} W_m &= \phi\frac{\left(\tilde{R}+\rho\right)}{\eta} + \mu(1-n)\phi u'(q)\frac{(1+i_d)}{(1+i_\ell)}\left[1+i_\ell - \frac{\left(\tilde{R}+\rho\right)}{\eta}\right] \\ W_b &= \phi\left(\tilde{R}+\rho\right) + \mu(1-n)\phi u'(q)\frac{(1+i_d)}{(1+i_\ell)}\left[\eta(1+i_\ell) - \left(\tilde{R}+\rho\right)\right] \end{aligned}$$

Combining them with (79) and (80), we get

$$\frac{\gamma}{\beta} - \frac{\left(\tilde{R}+\rho\right)}{\eta} = \mu(1-n)u'(q)\frac{(1+i_d)}{(1+i_\ell)}\left[1+i_\ell - \frac{\left(\tilde{R}+\rho\right)}{\eta}\right]$$

as well as the no arbitrage condition $\rho \geq \eta_+$ with equality if $b > 0$. Using the stationarity assumptions $\phi\rho = \phi_-\rho_-$, we get

$$\rho \geq \gamma\eta \text{ with } = \text{ if } b > 0$$

We will still assume that $q_p^1 = q_p^0 = q_p$, where market clearing requires $q_p = (1-n)\mu q/n$.

The money supply still evolves according to

$$\gamma = 1 + i_d + (i_d - i_\ell)\mu(1-n)\ell/M$$

An equilibrium with a standing facility is, therefore, characterized by the following equations

$$1 + i_\ell \geq \left(\tilde{R}+\rho\right)/\eta \geq 1 + i_d \tag{83}$$

$$\frac{\gamma}{\beta} - \frac{\left(\tilde{R}+\rho\right)}{\eta} = \mu(1-n)u'(q)\frac{(1+i_d)}{(1+i_\ell)}\left[1+i_\ell - \frac{\left(\tilde{R}+\rho\right)}{\eta}\right] \tag{84}$$

$$\gamma = 1 + i_d + (i_d - i_\ell)\mu(1-n)\ell/M \tag{85}$$

$$\rho = \gamma\eta \tag{86}$$

$$\phi\ell = q/(1+i_d) - \phi\left(M - \eta y_c^1\right) \tag{87}$$

and

$$\begin{aligned} y_p^{\varepsilon} &= y_c^0 = y^0 = \frac{m}{\eta} \text{ if } \left(\tilde{R}+\rho\right)/\eta > 1+i_d \\ y_c^1 &= -b \text{ and } \ell = 0 \text{ if } 1+i_\ell > \left(\tilde{R}+\rho\right)/\eta \\ \mu\left(1-n\right) y_c^1 &= -\left[(1-\mu)+\mu n\right] y^0 \end{aligned}$$

Proposition 12 *There is an equilibrium with a standing facility such that the allocation q^s satisfies $u'(q^s) < 1/\beta$.*

Proof. The best equilibrium with a standing facility is when consumers are not constrained at the lending facility, i.e., $\lambda_\ell = 0$ while still borrowing at the CB. This requires that they still have bonds to pledge as collateral, so that $y_c^1 > -b$, or $\lambda_b^1 = 0$. These two requirements impose

$$u'(q) = \frac{1+i_\ell}{1+i_d}, \tag{88}$$

and

$$\frac{\left(\tilde{R}+\rho\right)}{\eta} = 1+i_\ell.$$

Then (84) implies that for money to be valued, its return has to be equal to the one on securities or,

$$\frac{\gamma}{\beta} = \frac{\left(\tilde{R}+\rho\right)}{\eta} = 1+i_\ell.$$

The inflation rate is, therefore, $\gamma = \beta(1+i_\ell)$. We obtain from (85) that $\gamma \le 1+i_d$ (with strict inequality whenever $\ell > 0$) so that in this equilibrium

$$\frac{1+i_\ell}{1+i_d} \le \frac{1}{\beta}, \tag{89}$$

with strict inequality if there is borrowing at the CB. Combining (88) and (89) we obtain the result whenever $\ell > 0$ and $i_\ell > i_d$. We now show that such an equilibrium exists.

Since $\left(\tilde{R}+\rho\right)/\eta > 1+i_d$, we get $y_p^{\varepsilon} = y_c^0 = y^0 = m/\eta$, so that

$$y_c^1 = -\frac{\left[(1-\mu)+\mu n\right]}{\mu(1-n)} M/\eta$$

Since all the cash in the economy is in the hands of consumers, this implies that they borrow at the CB an amount

$$\phi\ell = \frac{q}{(1+i_d)} - \frac{\phi M}{\mu(1-n)} \tag{90}$$

Therefore, we need

$$\begin{aligned}
\phi\ell &= \frac{q}{(1+i_d)} - \frac{\phi M}{\mu(1-n)} \leq \phi\left(b + y_c^1\right)\frac{\left(\tilde{R}+\rho\right)}{(1+i_\ell)} \\
&\quad \frac{q}{(1+i_d)} - \frac{\phi M}{\mu(1-n)} \leq b\phi\eta - \frac{[(1-\mu)+\mu n]}{\mu(1-n)}\phi M
\end{aligned}$$

where we have used $\left(\tilde{R}+\rho\right)/(1+i_\ell) = \eta$. This implies that agents are unconstrained at the standing facility if

$$\frac{q}{(1+i_d)} - \phi M \leq \bar{B}\phi\eta$$

which implies a high enough deposit rates. ϕM is given by (85) once we replace for $\phi\ell$

$$\begin{aligned}
\gamma &= 1 + i_d + (i_d - i_\ell)\,\mu(1-n)\,\frac{\frac{q}{(1+i_d)} - \frac{\phi M}{\mu(1-n)}}{\phi M} \\
\frac{\beta\frac{1+i_\ell}{1+i_d} - 1}{1 - \frac{1+i_\ell}{1+i_d}}\phi M &= \frac{\mu(1-n)\,q}{(1+i_d)} - \phi M \\
\phi M\Delta\left[\frac{1-\beta}{\Delta - 1}\right] &= \frac{\mu(1-n)\,q}{(1+i_d)} \qquad (91)
\end{aligned}$$

where $\Delta = (1+i_\ell)/(1+i_d)$ and where q is given by (88). Since $\Delta > 1$, we get that $\phi M > 0$. Finally, we need to check that $\ell > 0$. Using (90), this imposes

$$\frac{q}{(1+i_d)} > \frac{\phi M}{\mu(1-n)} \qquad (92)$$

and replacing ϕM using (91), this requires

$$\frac{1}{\beta} > \frac{1+i_\ell}{1+i_d}$$

which is satisfied. This completes the proof. ■

Notice that, in the equilibrium in the proof, all agents value cash in the same way so that once again the channel policy insures agents against their liquidity shock. Since consumers are not constraint, $\lambda_b^1 = 0$ and from the envelope condition, all agents cash valuation is $\left(\tilde{R}+\rho\right)/\eta$. And as before, the difference between the deposit and the lending rates explain why the CB can achieve the allocation q^s. If we now combine (11) and (12), we obtain that even if OMOs take place once the shocks are realized, it is better to use the channel system than OMOs, given a level of inflation that is implementable using both systems. Therefore, the result in the main body of the paper is robust to a change in the timing of OMOs.

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