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MONETARY POLICY IN A CHANNEL SYSTEM

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Abstract

Channel systems for conducting monetary policy are becoming increasingly popular. Despite its popularity, the consequences of implementing policy with a channel system are not well understood. We develop a general equilibrium framework of a channel system and study the optimal policy. A novel aspect of the channel system is that a central bank can "tighten" or "loosen" its policy without changing its policy rate. This policy instrument has so far been overlooked by a large body of the literature on the optimal design of interest-rate rules.

JEL: E40, E50, D83

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1 Introduction

Channel systems for conducting monetary policy are becoming increasingly popular.[1] Several central banks already use a channel system, and others are using at least some features of the channel system.[2] Despite its popularity, the consequences of implementing monetary policy with a channel system are not well understood. How does implementation of monetary policy in a channel system differ from plain-vanilla open market operations? Why do central banks choose different corridors? Most central banks choose an interest-corridor of 50 basis points (e.g., Australia, Canada, and New Zealand), while the European Central Bank (ECB) chooses one of 200 basis points. Why can some central banks control the overnight interest rate very tightly, while others cannot? For instance, the Euro repo rate fluctuates considerably around the minimum bid rate set by the ECB (Figure 1), and it tends to be above the minimum bid rate. In contrast, the overnight interbank cash rate in New Zealand is almost always equal to the policy rate (Figure 2).

There are several stylized facts that a reasonable theoretical model of channel systems has to explain. First, all central banks set a strictly positive interest-rate spread - defined as the difference between the lending and the deposit rates. Second, central banks typically react to changing economic conditions by increasing or decreasing their interest-rate corridor without changing its spread. Third, the money market

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[1]In a channel system, a central bank offers two facilities: a lending facility whereby it is ready to supply money overnight at a given lending rate against collateral and a deposit facility whereby banks can make overnight deposits to earn a deposit rate. The interest-rate corridor is chosen to keep the overnight interest rate in the money market close to the target rate. In a pure channel system, a change in policy is implemented by simply changing the corridor without any open market operations.

[2]For example, versions of a channel system are operated by the Bank of Canada, the Bank of England, the European Central Bank, the Reserve Bank of Australia, and the Reserve Bank of New Zealand. The US Federal Reserve System recently modified the operating procedures of its discount window facility in such a way that it now shares elements of a standing facility. Prior to 2003, the discount window rate was set below the target federal funds rate, but banks faced penalties when accessing the discount window. In 2003, the Federal Reserve decided to set the discount window rate 100 basis points above the target federal funds rate and eased access conditions to the discount window. The resulting framework is similar to a channel system, where the deposit rate is zero and the lending rate 100 basis points above the target rate.

rate tends to be in the middle or slightly above the middle of the corridor.

Figure 1: Interest rate channel of the European Central Bank.

EONIA (Euro Overnight Index Average) and Eurepo (reference rate for the GC repo market)
Source: European Banking Federation and ECB
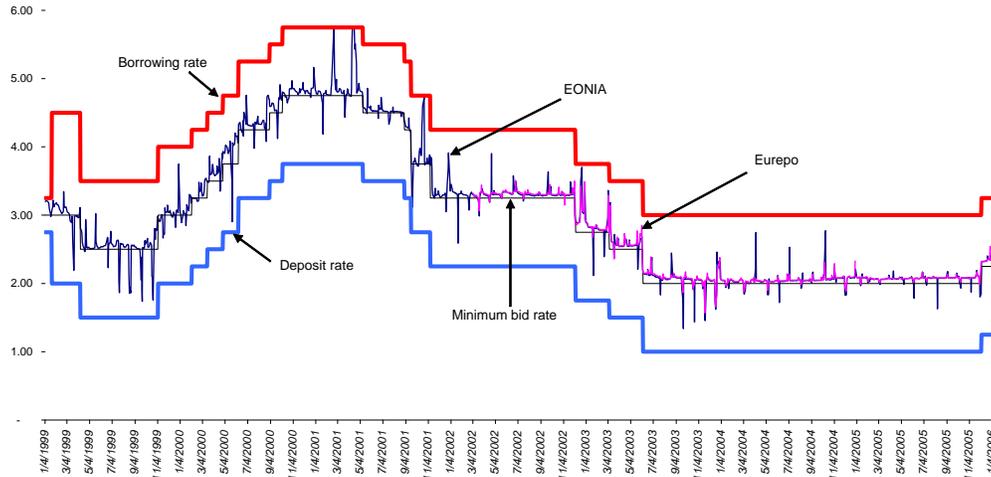
To study these stylized facts, we construct a dynamic general equilibrium model of a channel system with a money market and a welfare-optimizing central bank. Market participants are subject to idiosyncratic trading shocks that generate random liquidity needs. The shocks can be partially insured in a secured money market. To provide further insurance, the central bank operates facilities where market participants can borrow or deposit money at the specified rates. In accordance with central bank practice, there is no limit to the size of deposits on which interest is paid, and there is no limit to the size of a loan that a market participant can obtain provided that the loan is fully collateralized. Finally, the cost of pledging collateral is explicit and money is essential.[3]

Within this framework we answer the following three questions. First, what is the optimal interest-rate corridor? Second, what is the optimal collateral policy? Third, how does a change in the corridor affect the money market rate?

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[3]By 'essential,' we mean that the use of money expands the set of allocations (Kocherlakota 1998 and Wallace 2001).

Figure 2: Interest-rate channel of the Reserve Bank of New Zealand

Overnight Interbank Cash Rate
Source: Reserve Bank of New Zealand
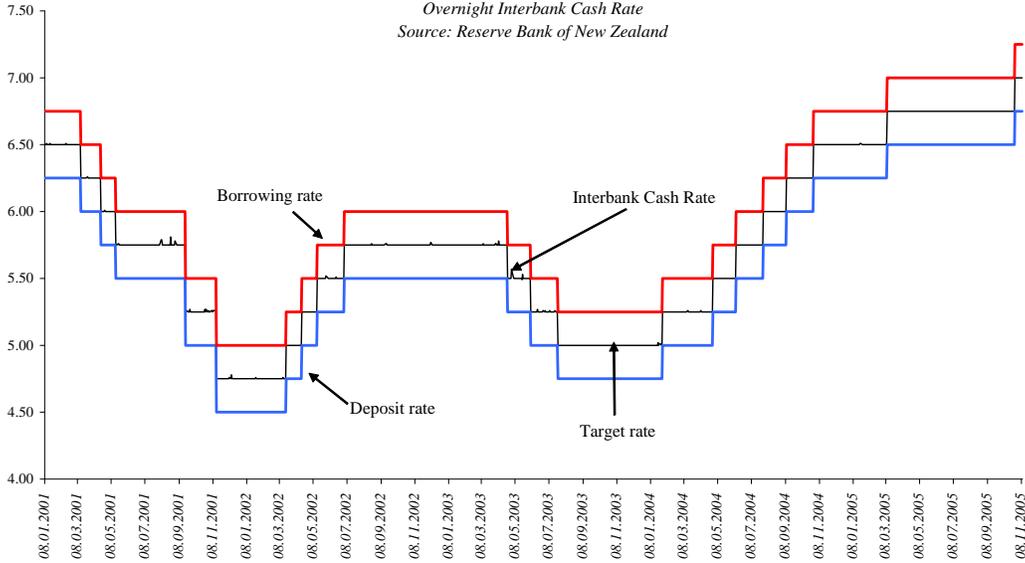
The following results emerge from our model. First, it is optimal to have a positive spread if the opportunity cost of holding collateral is positive, and the optimal spread is decreasing in the rate of return of the collateral.[4] Second, the money market rate is above the target rate if the opportunity cost of holding collateral is positive. This property of the model is consistent with the fact that the collateralized Eurepo rate tends to be above the minimum bid rate (Figure 1). Third, a central bank has two equivalent options for implementing a given policy: it can either shift the corridor while keeping the spread constant, or it can change the spread. For instance, to change its policy, it can keep the deposit rate constant and only change the borrowing rate, as done, for example, by the US Federal Reserve System, or it can shift the corridor without changing its spread as done by the European Central Bank.

An interesting aspect of the channel system is that a central bank can "tighten" or "loosen" its policy without changing its target rate. The reason is that by increasing the spread of the corridor symmetrically around the target rate, the central bank worsens the option for banks of accessing the standing facility. As a result, the policy regime is tighter.[5] This suggests that a characterization of policy through an

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[4]The rate of return of the collateral determines the opportunity costs for commercial banks of accessing the lending facility where a high rate of return implies a small or zero opportunity cost.

[5]This result suggests that the ECB with its 200-basis-point corridor implements a tighter mone-

interest-rate rule, as is commonly done in a large body of the literature, is incomplete. Rather, in a channel system, any policy must be characterized through an interest-rate *corridor* rule. We provide more discussion on this result in the literature section below.

Literature   There are very few theoretical studies of channel systems, and all of them are partial equilibrium models.[6] An early contribution is the model of reserves management under uncertainty by Poole (1968). Woodford (2000, 2001, 2003) discusses and analyzes the channel system to address the question of how to conduct monetary policy in a world with a vanishing stock of money. Whitesell (2006) evaluates reserves regimes versus channel systems. Elements of channel systems have been previously discussed in Gaspar, Quiros, and Mendizabal (2004) and Guthrie and Wright (2000).

It appears that there are two reasons why there is no other general equilibrium analysis of a channel system. First, money growth is endogenous in such a system. In contrast, most theoretical models of monetary policy characterize optimal policy in terms of a path for the money supply. In practice, however, monetary policy involves rules for setting nominal interest rates, and most central banks specify operating targets for overnight interest rates.[7] This paper, therefore, is a further attempt to break the apparent dichotomy (Goodhart, 1989) between theoretical analysis and central bank practices.

The second reason is related to the widespread belief that modeling the details of the framework used to implement a given interest-rate rule is unimportant when

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tary policy than the other central banks operating a channel system as mentioned before.

[6]There are general equilibrium models that study discount window loans. An early paper is Sargent and Wallace (1982). Williamson (2005) considers a general equilibrium model where the central bank provides one-period zero-nominal-interest loans. As in our model, these discount window loans are financed by the issue of outside money. He shows that this arrangement yields a Pareto optimal allocation and that this allocation can also be supported with unregulated interbank lending and without outside money.

[7]This fact is also emphasized in Woodford's (2003) book at the beginning of Chapter 2: "While virtually all central banks use short-term nominal interest rates (...) as their instrument (...), the theoretical literature in monetary economics has concerned itself almost entirely with the analysis of policies that are described by alternative (...) paths for the money supply."

analyzing optimal monetary policy. That is, it is taken for granted that the economic consequences of interest-rate rules do not hinge on the specific details of monetary policy implementation. However, our analysis reveals that a characterization of optimal policy and its implementation cannot be separated. To see this, consider any interest-rate rule in a system with zero deposit rate as operated, for example, by the US Federal Reserve System. Such an interest-rate rule uniquely determines how "tight" or "loose" policy is. In contrast, the same rule or any other interest-rate rule has no meaning in a channel system, since it does not determine whether a policy is "tight" or "loose." Consequently, in a channel system optimal policy must not only state an interest-rate rule, but it must also state an interest-rate *corridor* rule. This is a new insight, which goes beyond what we already know from the large and growing body of literature on the optimal design of interest-rate rules.

The paper is structured as follows. Section 2 outlines the environment. The equilibrium is characterized in Section 3. Optimal monetary policy with an inactive money market is derived in Section 4, and Section 5 characterizes policy with an active money market. Finally, in Section 6, we discuss the policy implications that arise from the model, and Section 7 concludes. All proofs and a description of the Euro money markets and the ECB's operating procedures can be found in the Appendix.

2 Environment

We construct a dynamic general equilibrium model, populated by a [0,1]-continuum of infinitely lived market participants, where in each period three perfectly competitive markets open sequentially (Figure 3). The first market is a settlement market, where all claims from the previous period are settled. The second market is a money market, where lending and borrowing against collateral takes place. Finally, at the beginning of the third market, after the close of the money market, the standing facility and goods market open.

The sequence of markets and collateral requirements are motivated by the functioning of existing channel systems. For example, the key features of the ECB's implementation framework and of the Euro money market are the following.[8] First,

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[8] These key features are shared by all central banks that operate standing facilities.

any outstanding overnight loans at the ECB are settled at the beginning of the day. Second, most lending in the Euro money market and all credit obtained at the ECB's standing facility is collateralized. Third, the Euro money market operates between 7 am and 5 pm. Fourth, after the money market has closed, market participants can access the ECB's facilities for an additional 30 minutes. This means that after the close of the money market, the ECB's lending facility is the only possibility for obtaining overnight liquidity. Also, any late payments received can still be deposited at the deposit facility of the ECB.
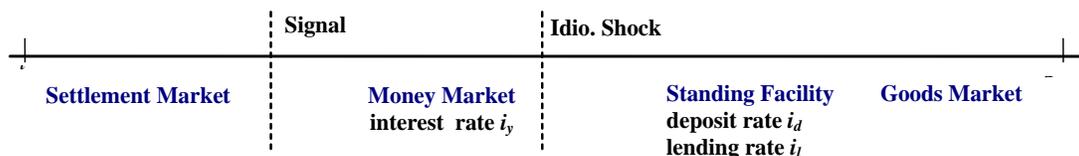
Figure 3: Sequence of markets.

We now discuss these markets in detail, starting with the goods market, which opens at the end of the period.

Goods market   At the beginning of the third market, agents receive idiosyncratic preference and technology shocks that determine whether they consume or produce in this market. With probability $1 - n$ an agent can consume and cannot produce: we refer to these agents as buyers. With probability n, an agent can produce and cannot consume: these are sellers. These trading shocks capture the liquidity shocks faced by commercial banks after the money market has closed.

A buyer gets utility $u(q)$ from q consumption in the third market, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$ and $u'(\infty) = 0$. Producers incur a utility cost $c(q) = q$ from producing q units of output. The discount factor is β where for technical reasons we assume that $\beta > n$.

A key friction is that market participants are anonymous. Since sellers require immediate compensation for their production effort, money is essential for trade.

8

Kocherlakota (1998), Wallace (2001), and Aliprantis et al. (2007) provide a detailed discussion of the features that generate an essential role for money. In addition to money we also have collateral, and we need to say why money plays this role and not collateral. We address this issue by assuming, as in Lester et al. (2007), that collateral cannot be physically brought into the goods market and market participants cannot recognize counterfeit claims to collateral, while they can always recognize currency.[9]

Money market   Participants in money markets face considerable uncertainty about their end-of-day liquidity position. This explains why they trade in the money market and *also* use the central bank's facilities after the money market has closed. To capture this obervation, we assume that at the beginning of the second market, participants receive a signal about the probability that they will become buyers or sellers in the goods market. With probability σ^k, an agent receives the information that he will be a seller with probability n^k, $k = H, L$, where $\varepsilon \equiv n^H - n^L \in [0, 1]$. We assume that $n = \sum_{k=H,L} \sigma^k n^k$ so that there is no aggregate uncertainty.

This modeling approach captures the idea that, when the money market is open, market participants receive information about their end-of-day cash holdings. Some market participants believe that they are likely to have excess cash at the end of the day, and others that they are likely to be short of cash. The difference in expected liquidity needs generates an incentive to trade in the money market. The imprecision of the signal captures the uncertainty that participants in money markets face about their end-of-day liquidity position.

There are three cases. If $\varepsilon = 0$, the signal contains no information, and so agents have no gains from trading in the money market. Consequently, no trade occurs in the money market. If $\varepsilon = 1$, there is no uncertainty about the liquidity shock in the goods market. Consequently, the portfolios are completely adjusted in the money market, and no agent accesses the facilities. Finally, if $\varepsilon \in (0, 1)$, the signal contains

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[9]Due to the essentiality of a medium of exchange, our paper is in the tradition of the search-theoretic approach to monetary economics, an approach initiated by the seminal paper of Kiyotaki and Wright (1989). Important contributions to this approach are Kiyotaki and Wright (1993), Trejos and Wright (1995), Shi (1995, 1997) and more recently Lagos and Wright (2005) and Rocheteau and Wright (2005). Wallace (2001) discusses why random matching and bargaining are not crucial for the essentiality of money.

some information about the future liquidity shock, but the information is not perfect. As a result, agents use both the money market and the standing facility to adjust their portfolio. For example, some agents will get the information that they will be sellers with high probability but then turn out to be buyers. These agents will first use the money market to trade away their cash and then use the lending facility to take out loans.

Settlement market In the first market, agents produce and consume general goods, repay loans, redeem deposits, and adjust their money balances. General goods are produced solely from inputs of labor according to a constant return to scale production technology where one unit of the consumption good is produced with one unit of labor generating one unit of disutility. Thus, producing h units of the general good implies disutility $-h$, while consuming h units gives utility h. The purpose of this market is that market participants can settle their overnight debt at the beginning of the period.[10]

2.1 Borrowing and lending facilities

At the beginning of the third market, after the idiosyncratic trading shocks are observed, the central bank offers a borrowing and a deposit facility. The central bank operates at zero cost and offers nominal loans ℓ at an interest rate i_ℓ and promises to pay interest rate i_d on nominal deposits d with $i_\ell \geq i_d$.

Since we focus on facilities provided by the central bank, we restrict financial contracts to overnight contracts. An agent who borrows ℓ units of money from the central bank in market 3 repays $(1 + i_\ell) \ell$ units of money in market 1 of the following period. Also, an agent who deposits d units of money at the central bank in market 3 of period t receives $(1 + i_d) d$ units of money in market 1 of the following period.

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[10]A convenient feature of these assumptions about preferences and technology is that they keep the distribution of money balances analytically tractable as in Lagos and Wright (2005). As we will see below, in equilibrium all households will hold the same amounts of money and collateral when they move on to the money market. Koeppl, Monnet, and Temzelides (2007) explain why the settlement market is necessary.

Collateral   As in current practice, we assume that the central bank provides credit only against collateral. In practice, collateral typically consists of low-risk and low-yield assets such as government securities. Here, we assume that general goods produced in market 1 can be stored and used as collateral. The storage technology has constant return to scales and yields $R \geq 1$ units of general goods in market 1 of the following period. We impose $\beta R \leq 1$, since when $\beta R > 1$, agents would store infinite amounts of goods, which is inconsistent with equilibrium.

The central bank operates the money market and keeps track of all financial arrangements and collateral holdings. In particular, collateral cannot be used to secure trade credit between a seller and a buyer in the goods market. Moreover, while the central bank keeps track of the financial history of agents, it has no knowledge of the agents' goods market transactions. Given this, money is still essential for trade in the goods market.[11]

Monetary policy   The central bank has three policy instruments: the deposit rate i_d, the lending rate i_ℓ, and lump-sum transfers $T = \tau M$, which we assume take place in market 1. Since central banks have no fiscal authority, we restrict these lump-sum transfers to be positive, that is, $\tau \geq 0$.[12] The lump-sum transfers are a substitute for open-market operations that we do not model here. Note that in a pure channel system central banks do not use open-market operations to affect the money market rate on a regular basis. Nevertheless, we don't rule this possibility out. Later, we will show that it is optimal to set $\tau = 0$.

Note that a central bank can change policy in two ways (even for $\tau = 0$). It can either increase or decrease $\delta \equiv i_\ell - i_d$, holding the policy rate (or target rate)

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[11]There is an inherent tension between money and credit, since the essentiality of money requires the absence of record keeping, whereas credit demands record keeping. Aiyagari and Williamson (2000) and Berentsen, Camera, and Waller (2007) show how credit can be introduced into a micro-founded model of money while keeping money essential.

[12]Andolfatto (2007) and Sanchez, Williamson and Wright (2007) investigate the informational structure in the Lagos-Wright framework that prevents the central bank from using lump-sum taxes and hence running the Friedman rule. We take a short-cut here by simply assuming that it cannot tax, which is consistent with central bank practice and the part of the literature which assumes that the central bank has no enforcement power and is therefore unable to tax (e.g., Kocherlakota 2003, Berentsen and Waller 2008).

$i_p = (i_\ell + i_d) / 2$ constant, or it can change i_p while holding δ constant. We will discuss the implications below.

In a channel system, the money stock evolves endogenously as follows

$$M_+ = M - i_\ell L + i_d D + \tau M \tag{1}$$

where M denotes the per capita stock of money at the beginning of period t. In the first market, total loans L are repaid. Since interest-rate payments by the agents are $i_\ell L$, the stock of money shrinks by this amount. Interest payments by the central bank on total deposits are $i_d D$. The central bank simply prints additional money to make these interest payments, causing the stock of money to increase by this amount. Finally, the central bank can also change the stock of money via lump-sum transfers $T = \tau M$ in market 1.

2.2 First-best allocation

In the Appendix, we show that at the beginning of a period the expected lifetime utility of the representative agent for a stationary allocation (q, b), where q is consumption and b collateral holdings, is given by

$$(1 - \beta) \mathcal{W} = (1 - n) [u(q) - q] + (\beta R - 1) b. \tag{2}$$

The first term on the right-hand side of the equation is the expected utility from consuming and producing the market 3 good. The second term is the utility of producing collateral and receiving the return in the following period.

It is obvious that the first-best allocation (q^*, b^*) satisfies $q = q^*$, where q^* is the value of q that solves $u'(q) = 1$. Moreover, $b^* = 0$ if $\beta R < 1$, and b^* is indeterminate if $\beta R = 1$. Thus, a social planner would never choose a positive amount of collateral when collateral is costly.

3 Symmetric stationary equilibrium

In period t, let $\phi \equiv 1/P$ be the real price of money in market 1, where P is the price of goods in market 1. We focus on symmetric and stationary equilibria, where all agents

follow identical strategies and where the real allocation is constant over time. In a stationary equilibrium, beginning-of-period real money balances are time-invariant

$$\phi M = \phi_+ M_+. \tag{3}$$

This implies that $\phi/\phi_+ = P_+/P = M_+/M$ is constant. Denote $\gamma \equiv M_+/M$ the time-invariant (endogenous) growth rate of the money supply.

$W(m, b, \ell, d, y)$ denotes the expected value of entering the first market with m units of money, b collateral, ℓ loans, d deposits and private credit y, where $y > 0$ means that the agent has borrowed money in the money market of the previous period. $Z(m, b)$ denotes the expected value from entering the money market with m units of money and b collateral. For notational simplicity, we suppress the dependence of the value function on the time index t. In what follows, we look at a representative period t.

3.1 Settlement

Denote the real gross interest rate on central bank loans and deposits, and private credit by $R_\ell \equiv \phi(1 + i_{\ell,-1})$, $R_d \equiv \phi(1 + i_{d,-1})$ and $R_y \equiv \phi(1 + i_{y,-1})$, respectively.[13] In the first market, the problem of the representative agent is:

$$W(m, b, \ell, d, y) = \max_{h, m_2, b_2} -h + Z(m_2, b_2)$$

$$s.t. \quad \phi m_2 + b_2 = h + \phi m + Rb + R_d d - R_\ell \ell - R_y y + \phi \tau M.$$

where h is hours worked in market 1, m_2 is the amount of money brought in to the second market, and b_2 is the amount of collateral brought in to the second market. Using the budget constraint to eliminate h in the objective function, one obtains the first-order conditions

$$Z_m \leq \phi \; (= \text{ if } m > 0) \tag{4}$$

$$Z_b \leq 1 \; (= \text{ if } b > 0) \tag{5}$$

$Z_m \equiv \frac{\partial Z(m_2, b_2)}{\partial m_2}$ is the marginal value of taking an additional unit of money into the second market in period t. Since the marginal disutility of working is one, $-\phi$ is the utility cost of acquiring one unit of money in the first market of period t.

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[13]The subscript -1 indicates that the interest rate has been agreed on in period $t - 1$.

$Z_b \equiv \frac{\partial V(m_2, b_2)}{\partial b_2}$ is the marginal value of taking additional collateral into the second market in period t. Since the marginal disutility of working is 1, -1 is the utility cost of acquiring one unit of collateral in the first market of period t. The implication of (4) and (5) is that all agents enter the following period with the same amount of money and the same quantity of collateral (which can be zero).

The envelope conditions are

$$W_m = \phi; W_b = R; W_\ell = -R_\ell; W_d = R_d; W_y = -R_y \tag{6}$$

where W_j is the partial derivative of $W(m, b, \ell, d, y)$ with respect to $j = m, b, \ell, d, y$.

3.2 Money market

Let y^k be the amount of money borrowed in the money market when signal $k = L, H$ is received. An agent who has m money and b collateral at the opening of market 2 has expected lifetime utility

$$Z(m, b) = \sum_{k=H,L} \sigma^k V^k(m + y^k, b, y^k)$$

where y^k solves

$$\max_{y^k} V^k(m + y^k, b, y^k) \text{ s.t. } y^k \le Rb/R_y \text{ and } m + y^k \ge 0.$$

The first inequality is the borrowing constraint. The second inequality implies that an agent cannot lend more money than he has. The first-order conditions are

$$V_m^k + V_y^k - \phi_+ \beta \lambda_{y\ell}^k + \phi_+ \beta \lambda_{yd}^k = 0 \tag{7}$$

where $\phi_+ \beta \lambda_{y\ell}^k$ is the multiplier on the borrowing constraint and $\phi_+ \beta \lambda_{yd}^k$ is the multiplier on the short-selling constraint. In any equilibrium, those agents who are likely to become sellers do not borrow money, and those who are likely to become buyers do not lend money. Consequently, we have $\lambda_{y\ell}^H = 0$ and $\lambda_{yd}^L = 0$, and so from (7) we have

$$V_m^H + V_y^H + \phi_+ \beta \lambda_{yd}^H = 0 \tag{8}$$
$$V_m^L + V_y^L - \phi_+ \beta \lambda_{y\ell}^L = 0. \tag{9}$$

The marginal value of collateral is $Z_b(m, b) = \sum_{k=H,L} \sigma^k \left[V_b^k + \beta\phi_+ \lambda_{y\ell}^k R/R_y \right]$. Then (9) gives us

$$Z_b(m, b) = \sum_{k=H,L} \sigma^k V_b^k + \sigma^L \left(R/R_y \right) \left(V_m^L + V_y^L \right) \tag{10}$$

since in any equilibrium $\lambda_{y\ell}^H = 0$.

The marginal value of money is $Z_m(m, b) = \sum_{k=H,L} \sigma^k \left(V_m^k + \beta\phi_+ \lambda_{yd}^k \right)$. Then from (8) we have

$$Z_m(m, b) = \sigma^L V_m^L - \sigma^H V_y^H. \tag{11}$$

Thus, the marginal value of money at the beginning of the money market is equal to the expected value of buying goods in market 3, $\sigma^L V_m^L$, plus the expected value of lending it in the money market, $-\sigma^H V_y^H$.

Finally, the market clearing condition is

$$\sum_{k=H,L} \sigma^k y^k = 0. \tag{12}$$

3.3   Liquidity shocks

At the beginning of market 3, an agent's state is revealed. Consider an agent who received signal k. Let q^k and q_s^k respectively denote the quantities he consumes or produces in market 3. Let ℓ_b^k (ℓ_s^k) and d_b^k (d_s^k) respectively denote the loan he obtains from the central bank and the amount of money he deposits in this market. If this agent holds m money, b collateral and private debt y at the opening of this market, he has expected lifetime utility

$$
\begin{aligned}
V^k(m, b, y) & = (1 - n^k)[u(q^k) + \beta W(m - pq^k + \ell_b^k - d_b^k, b, \ell_b^k, d_b^k, y)] \\
& \quad + n^k[-q_s^k + \beta W(m + pq_s^k - d_s^k + \ell_s^k, b, \ell_s^k, d_s^k, y)]
\end{aligned}
$$

where p is the price of goods in the third market and q^k, q_s^k, ℓ_s^k, ℓ_b^k, d_s^k and d_b^k are chosen optimally as follows.

It is obvious that buyers will never deposit funds in the central bank and sellers will never take out loans, and therefore, $d_b^k = 0$ and $\ell_s^k = 0$. For the rest of the paper, to simplify notation, we let $\ell^k \equiv \ell_b^k$ and $d^k \equiv d_s^k$. Accordingly, we obtain

$$
\begin{aligned}
V^k(m, b, y) & = (1 - n^k)[u(q^k) + \beta W(m - pq^k + \ell^k, b, \ell^k, 0, y)] \\
& \quad + n^k \left[-q_s^k + \beta W \left(m + pq_s^k - d^k, b, 0, d^k, y \right) \right]
\end{aligned}
$$

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where q_s^k, q^k, ℓ^k and d^k solve the following optimization problems.

A seller's problem is $\max_{q_s^k, d^k} \left[-q_s^k + \beta W \left(m + p q_s^k - d^k, b, 0, d^k, y \right) \right]$ s.t. $m + p q_s^k - d^k \geq 0$.[14] Using (6), the first-order condition reduces to

$$p \beta \phi_+ + p \beta \phi_+ \lambda_d = 1 \tag{13}$$

$$i_d = \lambda_d \tag{14}$$

where $\beta \phi_+ \lambda_d$ is the multiplier on the deposit constraint. The two conditions can be combined to give

$$p \beta \phi_+ (1 + i_d) = 1. \tag{15}$$

If an agent is a buyer, he solves the following maximization problem:

$$\max_{q^k, \ell^k} \quad u(q^k) + \beta W (m - p q^k + \ell^k, b, \ell^k, 0, y)$$

$$\text{s.t.} \quad p q^k \leq m + \ell^k \text{ and } \ell^k \leq \bar{\ell}$$

where the maximal amount that a buyer can borrow from the central bank $\bar{\ell}$ solves

$$R_\ell \bar{\ell} = bR - y R_y. \tag{16}$$

On the left-hand side is the total repayment $R_\ell \bar{\ell}$ when an agent borrows $\bar{\ell}$. On the right-hand side is the total disposable collateral at the time of repayment, since b units of collateral transform into Rb units of real goods. From this amount, the real debt obligation from the money market $y R_y$ must be deducted.

Using (6), the buyer's first-order conditions can be written as

$$u'(q^k) = p \beta \phi_+ (1 + \lambda_q^k) \tag{17}$$

$$\lambda_q^k = \lambda_\ell^k + i_\ell \tag{18}$$

where $\beta \phi_+ \lambda_q^k$ is the multiplier of the buyer's budget constraint and $\beta \phi_+ \lambda_\ell^k$ the multiplier of the borrowing constraint. Using (15) and combining (17) and (18) yields

$$u'(q^k) = \frac{1 + i_\ell + \lambda_\ell^k}{1 + i_d}. \tag{19}$$

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[14]Sellers can deposit their money holdings, including the proceeds from their latest transaction. This is in line with the institutional details described in the Introduction where banks can access the standing facility of the ECB 30 minutes after the close of the money market.

If the borrowing constraint is not binding, and the central bank sets $i_\ell = i_d$, trades are efficient. If the borrowing constraint is binding, then $u'(q) > 1$, and trades are inefficient even when $i_\ell = i_d$.

Using the envelope theorem and (17), the marginal value of money in market 3 is

$$V_m^k = (1 - n^k)u'(q^k)/p + n^k \beta R_d. \tag{20}$$

The marginal value of money has a straightforward interpretation. An agent with an additional unit of money becomes a buyer with probability $1 - n$, in which case he acquires $1/p$ units of goods yielding additional utility $u'(q)/p$. With probability n, he becomes a seller, in which case he deposits his money overnight, yielding the real return R_d. Note that the standing facility increases the marginal value of money because agents can earn interest on idle cash.

3.4 Liquidity premium

Since in equilibrium there is no default, the real return of collateral is βR. The real return, βR, is smaller than the marginal value, V_b, if $\lambda_\ell > 0$. To see this, use the envelope theorem to derive the marginal value of collateral in the third market

$$V_b^k = (1 - n^k)\beta \phi_+ \lambda_\ell^k R/R_\ell + \beta R. \tag{21}$$

Thus, the difference between the real return and the marginal value is $(1 - n^k)\beta \phi_+ \lambda_\ell^k R/R_\ell$. This quantity is positive if collateral relaxes the borrowing constraints of the buyers; i.e., if $\lambda_\ell^k > 0$. It is critical for the working of the model that $V_b^k > \beta R$. The reason is that, since $\beta R - 1$ is negative, agents are willing to hold collateral only if its liquidity value as expressed by the shadow price λ_ℓ^k is positive.

To derive the liquidity premium on the collateral, assume, for simplicity, that $\varepsilon = 0$, and then use the first-order conditions (5) and (19) to write (21) as follows:

$$1 - \beta R = (1 - n)\left[u'(q)\beta R/\Delta - \beta R\right] \tag{22}$$

The term $\beta R/\Delta$, where $\Delta \equiv R_\ell/R_d$, is the price of goods in terms of collateral in market 3. A buyer can use one unit of collateral to borrow R/R_ℓ units of money, which allows him to acquire $(R/R_\ell)/p = \beta R (R_d/R_\ell) = \beta R/\Delta$ units of goods.

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The right-hand side of equation (22) is the collateral's liquidity premium. While collateral costs 1 util to produce, its return is $\beta R \leq 1$. Hence, if $\beta R < 1$, agents need an incentive to hold collateral. This is provided by making collateral liquid.[15]

4   No trade in the money market

In this section, to focus on the central bank's facilities, we consider the case where the signal contains no information, i.e., $\varepsilon = 0$. In this case, agents have no gains from trading in the money market. Consequently, they use the lending and deposit facilities only to adjust their money holdings. We will consider the case $\varepsilon \in (0, 1)$ in Section 5.

To define a symmetric stationary equilibrium, use the first-order condition (5) and (22) to get

$$\frac{1 - \beta R}{\beta R} \geq (1 - n)\left[u'(q)/\Delta - 1\right] \ (= \ \text{if } b > 0). \tag{23}$$

Then (4), (15), (20), and taking into account that in a stationary equilibrium $M_+/M = \phi/\phi_+ = \gamma$, yield

$$\frac{\gamma - \beta\left(1 + i_d\right)}{\beta\left(1 + i_d\right)} = (1 - n)\left[u'(q) - 1\right]. \tag{24}$$

Also, from (1), since $L = (1 - n)\ell$ and $D = nd$, we get

$$\gamma = 1 + i_d - (1 - n)(i_\ell - i_d)\frac{z_\ell}{z_m} + \tau \tag{25}$$

where $z_m \equiv m/p$ and $z_\ell \equiv \ell/p$. To derive this equation, we use $d = m + pq_s$, market clearing $nq_s = (1 - n)q$, and we take into account that in symmetric equilibrium all agents hold identical amounts of money when they enter market 3. Then, from the budget constraint of the buyer, we have

$$q = z_m + z_\ell. \tag{26}$$

Finally, since $\beta R < 1$ in any equilibrium where agents hold collateral, it must be the

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[15] In microfounded models of money, liquidity premia arise naturally. See, for example, Lagos (2005), Lester, Postlewaite and Wright (2007), or Telyukova and Wright (2006).

case that the borrowing constraint is binding and so from (15) and (16) we get[16]

$$z_\ell = \beta R b / \Delta. \tag{27}$$

We can use these five equations to define a symmetric stationary equilibrium. They determine the endogenous variables $(\gamma, q, z_\ell, z_m, b)$. Note that all other endogenous variables can be derived from these equilibrium values.

Definition 1 *A symmetric stationary equilibrium with $\varepsilon = 0$ is a policy (i_d, i_ℓ, τ) and a time-invariant list $(\gamma, q, z_\ell, z_m, b)$ satisfying (23)-(27) with $z_\ell \geq 0$ and $z_m \geq 0$.*

Let

$$\tilde{\Delta} \equiv \frac{1 - \beta n + \tau/(1 + i_d)}{1/R - n\beta}. \tag{28}$$

Then we have the following:

Proposition 1 *For any (i_d, i_ℓ, τ) with $i_\ell \geq i_d \geq 0$, there exists a unique symmetric stationary equilibrium such that*

$$z_\ell > 0 \ \ and \ z_m = 0 \ \ \ if \ and \ only \ if \ \ \Delta = 1$$
$$z_\ell > 0 \ \ and \ z_m > 0 \ \ \ if \ and \ only \ if \ \ 1 < \Delta < \tilde{\Delta}$$
$$z_\ell = 0 \ \ and \ z_m > 0 \ \ \ if \ and \ only \ if \ \ \Delta \geq \tilde{\Delta}.$$

Several points are worth mentioning. First, market participants are willing to acquire collateral if the borrowing rate relative to the deposit rate is not too high; i.e., if $\Delta < \tilde{\Delta}$. Second, the critical value $\tilde{\Delta}$ is increasing in R and τ, and so is b. Furthermore, agents increase their collateral holdings and hence finance a larger share of their consumption by borrowing if R or τ is increased. Third, if $\Delta = 1$, agents are not willing to hold money across periods. They just use collateral to borrow money to finance their consumption. This, however, does not mean that money is not used, since it still plays the role of a medium of exchange in market 3. It means only that agents do not want to hold it across periods.

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[16]If the borrowing constraint is non-binding ($\lambda_\ell = 0$), equation (21) reduces to $V_b = \beta R$, implying from (5) that $b = 0$ since we have $\beta R < 1$. Consequently, in any equilibrium where agents hold collateral, it must be the case that the constraint is binding.

How should we interpret the result that we can have $z_\ell > 0$ and $z_m = 0$ for large values of i_ℓ *and* i_d if $\Delta = 1$? The reason is that the cost of transforming collateral into consumption depends not only on i_ℓ *but also on* i_d. To see this, note that with one unit of collateral a buyer can borrow R/R_ℓ units of money, which allows him to acquire $(R/R_\ell)/p$ units of goods. From the first order conditions of the sellers, the price of goods is $p = 1/[\beta R_d]$. Hence, he can aquire $\beta R (R_d/R_\ell) = \beta R/\Delta$ units of goods. The crucial point is that even though the buyer pays a high nominal interest rate, the price of goods adjusts to reflect the fact that the seller can deposit the money and earn the high interest i_d. The same explanation holds for $z_\ell > 0$ and $z_m > 0$ if $1 < \Delta < \tilde{\Delta}$.

Given a real allocation $\{q(\Delta), b(\Delta)\}$ any pair (i_ℓ, i_d) satisfying $\Delta = \frac{1+i_\ell}{1+i_d}$ is consistent with this allocation. Thus, there are many ways to implement a given policy Δ. The allocations differ only in the rate of inflation.[17] This can be seen from (25) which can be written as follows

$$\frac{\gamma - \tau}{1 + i_d} = 1 - (1-n)(\Delta - 1)\frac{z_\ell}{z_m}.$$

Since the right-hand side of the equation is a constant for a given Δ, the inflation rate $\gamma - 1$ is increasing in i_d. To keep Δ constant when increasing i_d, one needs to increase i_ℓ accordingly.

4.1 Optimal policy

We now derive the optimal policy. The central bank's objective is to maximize the expected lifetime utility of the representative agent. It does so by choosing lump-sum transfers τ, consumption q and collateral holding b to maximize (2) subject to the constraint that its choice is consistent with the allocation given by (23)-(26). Given τ, the policy is implemented by choosing Δ.

Assume first that it is optimal to set $\Delta \geq \tilde{\Delta}$. In this case, no agent is using the lending facility which implies that $b = 0$. Moreover, from (24) and (25) q satisfies

$$\tilde{q}(\tau) = u'^{-1}\left(\frac{1 - \beta n + \tau/(1 + i_d)}{\beta(1 - n)}\right).$$

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[17]One can show that this is not the case when sellers can't access the deposit facility after the goods market.

Note that $\tilde{q}$ is independent of Δ when $\Delta \geq \tilde{\Delta}$, and so any $\Delta \geq \tilde{\Delta}$, implements the same real allocation $(b, q) = (0, \tilde{q})$. Now consider the largest q that the central bank can implement. From (23) the largest q is attained when $\Delta = 1$. It satisfies

$$\hat{q} = u'^{-1} \left[\frac{1/(\beta R) - n}{1 - n} \right].$$

Thus, the policy $\Delta = 1$ attains the allocation $(b, q) = (\hat{q}/(\beta R), \hat{q})$, since no agent is holding money across the period when $\Delta = 1$. Accordingly, the central bank is constrained to choose quantities q such that $\hat{q} \geq q \geq \tilde{q}(\tau)$.

Finally, it can be shown (see the proof of Proposition 1) that when $1 \leq \Delta < \tilde{\Delta}$, b and q solve

$$\frac{1 - \beta R}{\beta R} = (1 - n) [u'(q)/\Delta - 1] \tag{29}$$

$$q = \beta R b F(\Delta; \tau) \tag{30}$$

where

$$F(\Delta; \tau) = \frac{1}{\Delta} \left[1 + \frac{(1 - n)(\Delta - 1)}{1 + \beta n(\Delta - 1) - \Delta/R + \tau/(1 + i_d)} \right].$$

Thus, the central bank is constrained to choose an allocation that satisfies (29) and (30), and so the central bank's maximization problem is

$$\max_{q, b, \tau} \ (1 - n) [u(q) - q] + (\beta R - 1) b$$

$$\text{s.t.} \quad q = \beta R b F \left(\frac{\beta R(1 - n) u'(q)}{1 - n\beta R}; \tau \right) \tag{31}$$

$$\text{and } \hat{q} \geq q \geq \tilde{q}(\tau)$$

where to derive (31), we use (29) to replace Δ in (30).

Proposition 2 $\tau = 0$ *is optimal. Also, there exists a critical value $\overline{R}$ such that if $R < \overline{R}$, then the optimal policy is $\Delta \geq \tilde{\Delta}$. Otherwise, the optimal policy is $\Delta \in \left(1, \tilde{\Delta}\right)$.*

The striking result of Proposition 2 is that it is never optimal to set a zero interest-rate spread $\delta = i_\ell - i_d$. The reason is that, for society, the use of collateral is costly, since $\beta R - 1$ is negative. The benefit of collateral is that it increases consumption above $q = \tilde{q}$. The central bank thus faces a trade-off. It can encourage the use

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of costly collateral to increase consumption. The optimal policy simply equates the marginal benefit of additional consumption to the marginal cost of holding collateral. It is interesting to note that, in contrast to collateral, the use of fiat money is not costly for society since money can be produced without cost.

If R is small ($R < \overline{R}$), it is optimal for the central bank to discourage the use of collateral.[18] It does so by implementing an interest-rate policy that satisfies $\Delta \geq \tilde{\Delta}$. In contrast, if the rate of return is sufficiently high, it sets $\Delta \in \left(1, \tilde{\Delta}\right)$ so that agents finance some of their consumption using the lending facility. An increase in R reduces the optimal Δ. In the limit, as $R \to 1/\beta$, the holding of collateral becomes costless. We consider the optimal policy in this limiting case below.

According to Proposition 2, it is optimal to set $\tau = 0$. To see why, note that $\hat{q}$ is independent of τ, and $\tilde{q}$ is decreasing in τ. Therefore, increasing τ increases the set of attainable allocations, but only by decreasing the lower bound of the feasible q's. Then, since F is decreasing in τ, an increase in τ either increases b, decreases q or both. This reduces welfare unambiguously. Hence, it is optimal to set τ to zero. The intuition is that an increase in τ is equivalent to an increase in inflation. The inflation tax reduces the sellers' willingness to produce for money and so agents substitute bonds for money.

We now consider the case when holding collateral is costless, i.e., when $R = 1/\beta$. To avoid indeterminacies of the equilibrium allocation, we consider the limiting allocation when the rate of return of the collateral satisfies $R \to 1/\beta$.[19] In this limiting case, the critical value is $\tilde{\Delta} = \frac{1-\beta n}{\beta - \beta n} > 1$ and Proposition 1 continues to hold. With costless collateral, the optimal policy is $i_\ell \to i_d$. This policy implements the first-best allocation q^*. Interestingly, under this policy when $i_\ell \to i_d > 0$ the price level approaches infinity. The reason is that agents are now unwilling to hold money, since it is strictly dominated in return by collateral.

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[18]This is similar to Lagos and Rocheteau (2004), albeit in a very different context. They construct a model where capital competes with fiat money as a medium of exchange. They show that when the socially efficient stock of capital is low (which is the case when the rate of return is low), a monetary equilibrium exists that dominates the non-monetary one in terms of welfare. So in this case, it would be optimal to discourage the use of capital as a medium of exchange.

[19]At $R = 1/\beta$ agents are indifferent to how much collateral they acquire.

Policy implications   We conclude this section with a summary of the policy implications of the model. First, market participants are willing to borrow if the borrowing rate relative to the deposit rate is not too high. Second, if $i_\ell = i_d$ so that the spread is zero, agents are not willing to carry money across periods. Third, if holding collateral is costly, it is never optimal to set a zero interest-rate corridor because of the trade-off between costly collateral and extra consumption. Fourth, lump-sum injections of money are never optimal. Fifth, an efficient allocation can be attained only if holding collateral is costless, i.e., if $R \to 1/\beta$. In this case, the optimal policy is to set a zero interest-rate corridor.

5   Trade in the money market

We now assume that $\varepsilon > 0$. Recall that at the beginning of the money market, agents receive a signal about the probability that they will become a consumer or a producer in the third market. With probability σ^k, an agent receives the information that he will be a seller with probability n^k, $k = H, L$.

We focus on the case where $\varepsilon = n^H - n^L$ is small. This case captures the situation where agents' liquidity needs in the money market are not too different from each other and not too different from their initial beliefs. In this case, they are reluctant to pledge all their collateral or to sell all their money holdings in the money market. Consequently, the short-selling constraints in the money market are non-binding. This essentially means that the money market rate remains strictly within the interest-rate corridor, which is consistent with the experience of central banks that operate a channel system (see Figures 1 and 2). We again focus on symmetric and stationary equilibria, where all agents follow identical strategies and where the real allocation is constant over time.

In what follows, we assume that the central bank does not make lump-sum transfers ($\tau = 0$), since we have shown that this is optimal for the case when there is no trade in the money market. With an active money market, central bank loans satisfy $L = \sigma^H \left(1 - n^H\right) \ell^H + \sigma^L \left(1 - n^L\right) \ell^L$ and deposits $D = \sigma^H n^H d^H + \sigma^L n^L d^L$. Accordingly, (1) can be written as

$$M_+ = M - \left[\sigma^H \left(1 - n^H\right) \ell^H + \sigma^L \left(1 - n^L\right) \ell^L\right] i_\ell + \left(\sigma^H n^H d^H + \sigma^L n^L d^L\right) i_d \quad (32)$$

Using the market clearing conditions in the goods and money markets, we can write this equation as follows

$$M_+/M = 1 + i_d - (i_\ell - i_d) \left[\sigma^L (1 - n^L) \ell^L/M + \sigma^H (1 - n^H) \ell^H/M \right]. \qquad (33)$$

It is interesting to compare (33) with (25) (when $\tau = 0$). As before, the entire stock of money earns interest i_d. The only difference is the amount of loans that the central bank provides. Without a money market, the amount is $(1 - n) \ell/M$; with a money market, it is $\sigma^L (1 - n^L) \ell^L/M + \sigma^H (1 - n^H) \ell^H/M$.

Let $\hat{\Delta} \equiv R_\ell/R_y$. In the Appendix we prove:

Lemma 3 *A symmetric stationary equilibrium where no short-selling constraint is binding in the money market is a time-invariant list $\left(\hat{\Delta}, q^L, q^H, z^L, z^H, z_m, b, \gamma \right)$ and a policy (i_d, i_ℓ) satisfying*

$$R\gamma = 1 + i_y \qquad (34)$$

$$\frac{\beta R b}{\Delta} = \sigma^H q^H + \sigma^L q^L - z_m \qquad (35)$$

$$z^H = -\sigma^L \left(q^L - q^H \right) \left(\frac{\hat{\Delta}}{\hat{\Delta}-1} \right) = -\frac{\sigma^L}{\sigma^H} z^L \qquad (36)$$

$$z_m = \left(\frac{\hat{\Delta}}{\hat{\Delta}-1} \right) \frac{\left\{ (\hat{\Delta}-1) \left[\sigma^L (1-n^L) q^L + \sigma^H (1-n^H) q^H \right] - \varepsilon \sigma^L \sigma^H \left(q^L - q^H \right) \hat{\Delta} \right\} R(\Delta-1)}{R\hat{\Delta} - \Delta + (1-n) R\hat{\Delta}(\Delta-1)} \qquad (37)$$

$$\hat{\Delta} = \frac{\Delta}{n\beta R(1-\Delta)+\Delta} \qquad (38)$$

$$u'(q^k) = \frac{n^k}{1-n^k} \Delta \frac{1-n\beta R}{n\beta R}, \quad k = H, L, \qquad (39)$$

with $b \geq 0$, $z^L < \beta R b \hat{\Delta}/\Delta$ and $z^H > -z_m$.

We discuss the policy implications of Lemma 3 in Section 6.

Proposition 4 *For any $1 < \Delta < \tilde{\Delta}$, there exists a critical value $\varepsilon_1 > 0$, defined in the proof, such that, if $\varepsilon < \varepsilon_1$, a symmetric monetary equilibrium exists where no short-selling constraint in the money market binds.*

Note first that the system of equations (34) - (39) can be solved recursively. Equations (38) and (39) yield $\hat{\Delta}$, q^L and q^H. Using these values, we can then derive $\left(z^L, z^H, z_m, b, \gamma \right)$ from the remaining equations. One then has to check that the required inequalities hold. The inequality $b \geq 0$ simply requires that policy is such that agents have an incentive to acquire collateral, which is satisfied whenever $\Delta < \tilde{\Delta}$

(defined by (28)). The inequality $z^L < \beta Rb\hat{\Delta}/\Delta$ requires that those agents who are likely to become buyers are not pledging all their collateral to acquire money in the money market, and the inequality $z^H > -z_m$ requires that those agents who are likely to become sellers are not lending all their money.

6 Discussion of the policy implications

We now discuss the key implications of our model for the behavior of the money market rate, inflation, liquidity, collateral requirement, and the use of interest-rate rules. These results can be found by inspecting equations (34) and (38). For this discussion, let us define the policy interest rate $i_p \equiv (i_\ell + i_d)/2$.

Money market rate In the introduction, we have seen that the money market rate tends to be in the middle or above the target rate and changes one-to-one with a shift in the corridor (see also Figures 1 and 2). Our model replicates these facts. To see this, we can write (38) as follows

$$i_y = i_\ell - n\beta R\delta. \tag{40}$$

where $\delta = i_\ell - i_d$. Inspection of (40) reveals the following result: First, if the spread δ is kept constant, i_y changes in i_ℓ one-to-one. Second, if $n\beta R = 1/2$, then $i_y = i_p$.[20] Our model suggests that the money market interest rate lies exactly on the policy rate if, for example, $n = 1/2$ and $\beta R \to 1$. It is reasonable to assume that $n = 1/2$, since it means that on average a bank is equally likely to borrow or to provide cash in the money market and also equally likely to be either short of money or have excess cash at the end of the day. The second assumption means that holding collateral has no cost. Third, as mentioned in the introduction, the Euro money market rate tends to be above the minimum bid rate i_p. Our model has a simple explanation for this observation. With $n = 1/2$ and $\beta R < 1$, we have $i_y = \frac{i_\ell(2-\beta R)+i_d\beta R}{2} > i_p$. Thus, costly collateral generates a money market rate that tends to be above the target rate.

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[20] The first two results exactly match the behavior of the overnight money market rate of the channel system operated by the Reserve Bank of New Zealand. See Figure 2 in the Introduction.

Inflation To see the implications of our model for inflation, $1 + \pi \equiv \gamma$,[21] we can rewrite (34) as follows

$$1 + \pi = (1 + i_y) / R.$$

By defining $1 + r \equiv R$, we get the standard expression for the Fisher equation $(1 + r)(1 + \pi) = 1 + i_y$. It is interesting to note that the nominal interest rate of the Fisher equation is the money market rate i_y and not i_ℓ or i_d. Using (40), we can rewrite the Fisher equation as follows:

$$1 + \pi = (1 + i_\ell) / R - n\beta\delta. \tag{41}$$

From this expression, it is clear that inflation is increasing in i_ℓ and i_d. If we keep the spread δ constant and shift the corridor up, inflation is also increasing. Finally, inflation is also increasing if we increase the spread δ symmetrically around the policy rate when $n < 1/(2\beta R)$. As we have argued above, this condition is likely to be fulfilled, since on average $n = 1/2$, which implies that the inequality reduces to $\beta R < 1$.

Liquidity We can interpret n as a measure for liquidity in the money market. If $n = 1/2$, as mentioned above, banks are equally likely to have excess money or too little money at the end of the day. If $n < 1/2$, a bank is more likely to be short of money at the end of the day. The implications of changes in n for the money market rate can again be explored by considering (40). From this equation it is clear that an increase in liquidity (i.e., an increase in n) reduces the money market rate. Furthermore, n affects how close the money market rate is to the policy rate.[22]

Collateral requirement What is the optimal collateral requirement? Inspection of (40) reveals that a higher return on collateral, R, reduces the money market rate. From (41), one can also see that an increase in R reduces inflation and, as discussed above, gets the money market rate closer to the target rate. But the most important

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[21]Since we study steady state allocations, money growth and inflation are perfectly correlated. Then, through the Fisher equation, long-run money growth and interest rates are positively correlated, as confirmed by the data (see McCandless and Weber 1995).

[22]In practice, central banks attempt to influence n through open market operations. We leave this aspect of the model to future research.

aspect of the collateral requirement is that it affects the real allocation. Inspection of (39) reveals that an increase in R yields higher consumption and, consequently, higher welfare. Therefore, in practice, optimal collateral requirements do not distort market participants' portfolio choices. That is, market participants should hold assets eligible as collateral because they yield high returns and not because of the liquidity premium implied by the borrowing facility.

Interest-rate rules   Finally, a central bank can tighten its policy without changing its policy rate by simply increasing the corridor symmetrically around the policy rate. This can be seen by rewriting Δ as follows

$$\Delta = \frac{1 + i_\ell}{1 + i_d} = \frac{1 + i_\ell}{2i_p - i_\ell}.$$

It is evident that $\partial \Delta / \partial i_\ell > 0$. Hence, from (39), a symmetric increase of the spread around the policy rate decreases consumption.

Therefore, in a channel system, interest-rate rules (i.e., rules that specify a path for the policy rate i_p) are incomplete. The reason is that such a rule does not determine whether a policy is "tight" or "loose." Rather, in a channel system, any policy must be characterized through an interest-rate *corridor* rule.

7   Conclusion

We have analyzed the theoretical properties of a channel system of interest-rate control in a dynamic general equilibrium model with infinitely lived agents and a central bank. With this model, we could match several stylized facts regarding the use of channel systems by central banks. Moreover, we could derive several policy implications that we have summarized in Section 6. Perhaps the most important result is that interest-rate rules are meaningless in a channel system. In a channel system, any policy must be characterized through an interest-rate *corridor* rule. This is a new insight, which goes beyond what we already know from the large and growing body of literature on the optimal design of interest-rate rules.

While our paper is a first step toward analyzing a channel system in a general equilibrium model, many aspects have remained unexplored. For example, why is

there so little volatility in New Zealand's money-market interest rate (see Figure 2) and so much in the case of the European Central Bank (Figure 1)? Moreover, we know little about optimal monetary policy in a channel system under stress due to aggregate shocks. These are some of the issues left for future research.

Finally, a complementary modeling approach addresses monetary policy as a mechanism design problem (e.g., Wallace, 2005). Such an approach could potentially explain why central banks increasingly use channel systems to implement monetary policy. Our paper could not answer this question, since we have taken the channel mechanism as given to study its properties.

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8   APPENDIX

8.1   Background[23]

To understand some of the features of our environment, it is useful to have some information on how the money market functions and on monetary policy procedures at central banks that operate a standing facility. This section does not aim at being general, and we will, therefore, concentrate on the case of the euro money markets and the ECB's operating procedures.

Operating procedures of the ECB   The ECB has two main instruments for the implementation of its monetary policy. First, it conducts weekly main refinancing operations that are collateralized loans with a one-week maturity. Main refinancing operations are implemented using a liquidity auction, where banks bid for liquidity. Bids consist of an amount of liquidity and an interest rate. The total amount to be allocated is announced before the auction. Following the auction, the ECB allocates liquidity according to the bidded rates, in a descending order. The minimum bid rate is the main policy rate used by the ECB to implement monetary policy.

Second, the ECB offers a lending facility with a lending rate that is 100 basis points higher than its minimum bid rate, and a deposit facility, with a deposit rate that is 100 basis points below its minimum bid rate. At the lending facility, liquidity is provided either in the form of overnight repurchase agreements or as overnight collateralized loans, whereby the ownership of the asset is retained by the debtor. In both cases, banks have to resort to safe, eligible assets as defined by the ECB. Eligible banks can access the standing facilities at any time of the day. The use of the standing facility largely depends on banks' activities on the euro money markets during the day.

The euro money markets   There are two segments for the euro money market. The first segment is the unsecured money market, where banks borrow and lend cash to each other without resorting to collateral. The reference interest rate on the

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[23]This section draws heavily on materials from ECB (2005), ECB (2004), BIS (2003), and Hartmann, Manna and Manzanares (2001).

unsecured money market is the Euro OverNight Index Average (EONIA), calculated by the ECB. The second segment is the secured money market where agents lend at different maturities against collateral. This is the largest money market segment. There are several reference interest rates, depending on maturities (Euro Interbank Offered Rates, or Euribors) and on whether the collateral pledged belongs to a general collateral pool (Euripo).

Transactions on both segments of the money market are settled using the two large-value payment systems operating in the euro area, the Trans-European Automated Real-time Gross settlement Express Transfer system (TARGET) and Euro1. These large-value payment systems are essential in finalizing the transfer of funds for transactions taking place in money markets. Therefore, the opening and closing hours of money markets are closely related to the operating hours of these payment systems.

TARGET settles payments with immediate finality in central bank money and operates between 7 a.m. and 6 p.m. C.E.T. with a cut-off time of 5 p.m. for customer payments.[24] Eligible institutions hold accounts at TARGET, which are debited or credited depending on market participants' orders. Intraday credit is provided free of charge as long as it is fully collateralized. Banks may also access the deposit or lending facilities after making a request at the latest 30 minutes after the actual closing time of TARGET.[25] After the close of TARGET, an overdraft position on a bank's TARGET account is automatically transformed into an overnight loan via a recourse to the lending facility, again against eligible assets.

Euro1 is a private, large-value payment system offered by the Euro Banking Association (EBA). Euro1 functions as a sort of netting system, whereby on each settlement day, at any given time, each participant will have only one single payment obligation or claim with respect to the community of other participants as joint creditors/debtors. In particular, there are no bilateral payments, claims or obligations between participants. Euro1 settles in central bank money at the ECB at the end of the day. After the cut-off time of 4 p.m. C.E.T., clearing banks with debit positions will pay their single obligations into the EBA settlement account at the ECB through

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[24]The unsecured segment opens around 8 a.m. and closes around 5:45 p.m.

[25]On the last Eurosystem business day of a minimum reserve maintenance period, the deposit facility can be accessed for 60 minutes after the actual closing of TARGET.

TARGET. After all amounts due have been received, the ECB will pay the clearing banks with credit positions also using TARGET.

In this paper, we model two specific features of the description above. First, banks cannot carry overnight overdrafts on their TARGET accounts, and they have to borrow either on the money markets or at the lending facility in order to cover their TARGET positions. When TARGET closes, euro money markets are also closed. As a consequence, the central bank standing facility is, at the end of the day, the only recourse to overnight liquidity. Also, since participants can access the standing facility 30 minutes after the close of TARGET, any late payments received on a TARGET account can be deposited at the standing facility of the ECB. In the first part of the paper, we model this aspect of the liquidity management problem. Second, banks can predict when a payment is due or incoming so that, with a well-functioning money market, the likelihood of resorting to the standing facilities should be small. However, there may be unexpected payments to be made that can force banks to hold an overdraft on their TARGET account. In the second part of the paper, we add a money market to the model. There, banks are able to trade their liquidity when they are confident that they will end up the day with a credit on their central bank account. Given it is the most important segment of the money market, we concentrate on the secured interbank money market.

8.2   Channel system of the Bank of England

Here, we discuss the channel system operated by the Bank of England. As shown in Figure 4, the Sterling Overnight Interbank Average rate (SONIA) was very volatile until the first quarter of 2006. Before this date the bank's implementation framework consisted of a 100-basis-point corridor, non-remunerated daily reserves requirements and a somewhat restricted access to the borrowing facility.[26] From January 2000 to May 2006, the SONIA was on average 5 basis points *below* the Bank of England target rate, while the daily gilt repo rate with two-week maturity was on average 11 basis points *below* the target rate over the same period. Furthermore, the bank decreased its target rate from 4% to 3.75% in February 2003. However, the SONIA

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[26]For details on the Bank of England implementation framework, we refer the reader to Clews (2005).

averaged 3.95% over the period when the bank rate was 4%, and averaged 3.76% after its easing. Hence, while monetary policy targeted a 25-basis-point easing, the Bank effectively implemented a 19-basis-point easing.

Figure 4: The channel system of the Bank of England

Source: European Banking Federation and ECB
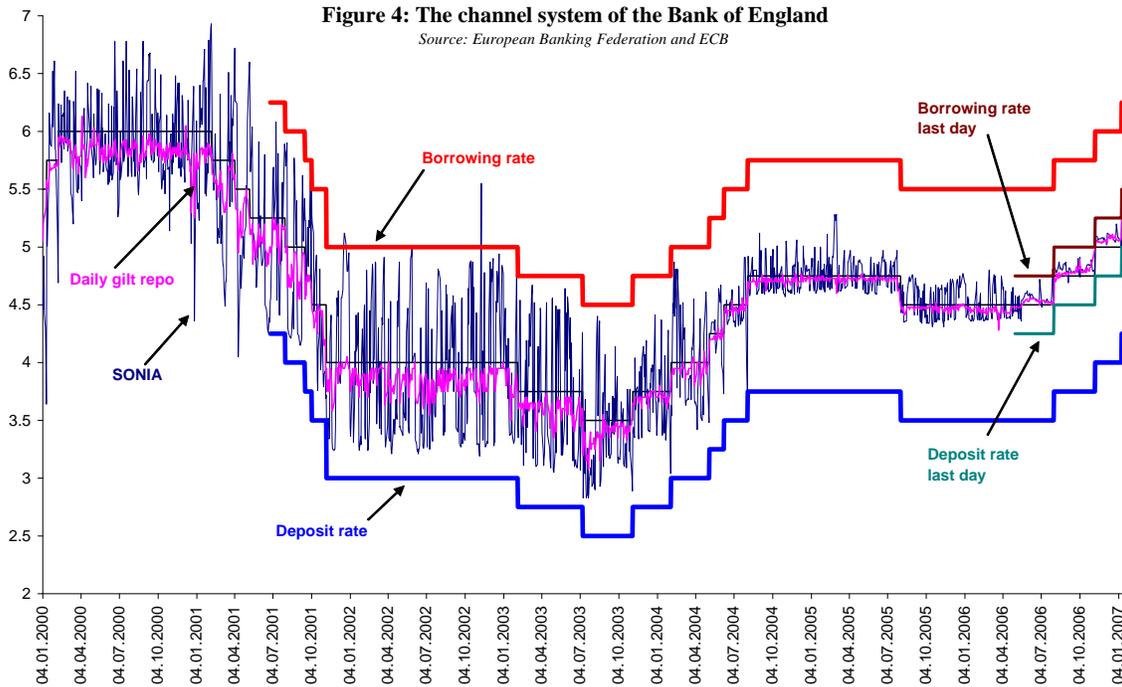
Hence, the implementation framework was not very efficient in implementing monetary policy. As a result of this inefficiency, the Bank of England reformed its implementation framework in 2006. It introduced 1) a 25-basis-point corridor on the last day of the maintenance period, 2) remuneration on reserves within limits at the official bank rate and 3) open market operations to ensure that there is an equal (and small) chance of using either facility. As Figure 4 illustrates, this reform resulted in an immediate decrease in the variability of the SONIA and repo rates. Furthermore, the SONIA is now on average 5 basis points *above* the bank's target rate, and more surprisingly, the repo rate is also on average 5 basis points *above* the target rate. Therefore, the reform of the monetary implementation framework increased the average difference between the bank's target rate and the SONIA by 10 basis points and the difference between the repo rate and the bank's target rate by 16 basis points.

8.3 Welfare

In this Appendix, we show that if the central bank's objective is to maximize the expected discounted utility of the representative agent, the central bank's objective is to maximize (2). To derive (2), we must first calculate hours worked in market 1. The money holdings at the opening of the first market are $\tilde{m} = 0$, having bought, and $\tilde{m} = m + pq_s$, having sold. Hence, hours worked are

$$
\begin{aligned}
h_b &= \phi[m_+ + (1+i_\ell)\ell] - (R-1)b - \phi\tau M \\
h_s &= \phi[m_+ - (1+i_d)(m+pq_s)] - (R-1)b - \phi\tau M.
\end{aligned}
$$

Since $h = nh_s + (1-n)h_b$, by using (1) and rearranging we get $h = -(R-1)b$. Then, welfare is given by

$$
\begin{aligned}
\mathcal{W} &= -b + (1-n)\left[u(q) - q\right] + \sum_{j=1}^{\infty} \beta^j \left\{(1-n)\left[u(q) - q\right] + (R-1)b\right\} \\
&= \frac{(1-n)[u(q) - q] + (\beta R - 1)b}{1-\beta}.
\end{aligned}
$$

8.4 Proofs

Proof of Proposition 1. For ease of exposition, we assume $\tau = 0$. The proof can be easily replicated when $\tau > 0$. We first prove the *only if* part. Assume first $z_\ell = 0$ and $z_m > 0$. Then from (24) and (25) we get

$$
\frac{1-\beta}{\beta} = (1-n)\left[u'(q) - 1\right] \tag{42}
$$

and from (23) we have

$$
\frac{1-R\beta}{R\beta} \geq (1-n)\left[u'(q)/\Delta - 1\right]. \tag{43}
$$

Use (43) to replace $u'(q)$ in (42) and rearrange to get $\Delta \geq \tilde{\Delta}$.

Assume now that $z_\ell > 0$ and $z_m > 0$. Then from (25) $z_\ell > 0$ implies that $1+i_d > \gamma$. Use (24) to replace γ and rearrange to get $\Delta < \tilde{\Delta}$. Next divide (25) by $1 + i_d$ and solve for Δ to get

$$
\Delta = 1 + \frac{z_m}{z_\ell} \frac{1 + i_d - \gamma}{(1-n)(1+i_d)} > 1
$$

since $1 + i_d > \gamma$. Hence, we have $1 < \Delta < \tilde{\Delta}$, if $z_\ell > 0$ and $z_m > 0$.

Finally, assume that $z_\ell > 0$ and $z_m = 0$. Then, the previous equation immediately implies that $\Delta = 1$.

We now prove the *if* part. From (24) and (25) we get

$$1 - n\beta - \beta(1-n)u'(q) = (1-n)(\Delta - 1)\frac{z_\ell}{z_m} \tag{44}$$

and from (23) we have

$$\Delta\left(\frac{1}{R} - n\beta\right) \geq \beta(1-n)u'(q) \tag{45}$$

Assume first that $1 < \Delta < \tilde{\Delta}$. Use (44) to rewrite (45) as follows

$$1 - n\beta - \Delta\left(\frac{1}{R} - n\beta\right) \leq (1-n)(\Delta - 1)\frac{z_\ell}{z_m}.$$

Rearrange to get

$$0 < \tilde{\Delta} - \Delta \leq \frac{(1-n)(\Delta - 1)}{(1/R - n\beta)}\frac{z_\ell}{z_m}.$$

Hence, $1 < \Delta < \tilde{\Delta}$ implies $\frac{z_\ell}{z_m} > 0$.

Assume next that $\Delta \geq \tilde{\Delta}$. Then from (44) we have

$$1 - n\beta - \beta(1-n)u'(q) \geq (1-n)\left(\tilde{\Delta} - 1\right)\frac{z_\ell}{z_m}.$$

Then $z_\ell > 0$ immediately implies that

$$0 > \tilde{\Delta} - \Delta \geq \frac{(1-n)\left(\tilde{\Delta} - 1\right)}{(1/R - n\beta)}\frac{z_\ell}{z_m}$$

which is a contradiction. Hence $\Delta \geq \tilde{\Delta}$ implies $z_\ell = 0$.

Existence and uniqueness when $\tilde{\Delta} \leq \Delta$: In this case $z_\ell = b = 0$ and from (25) $\gamma = 1 + i_d$. Then, from (24) and (25) we get (42). Since the right-hand side of (42) is strictly decreasing in q, there exists a unique q that solves (42). Finally, from (26) we have $z_m = q$.

Existence and uniqueness when $1 < \Delta < \tilde{\Delta}$: The system of equations (23)-(26) with $z_\ell = \beta Rb/\Delta$ can be reduced as follows. Equations (26) and $z_\ell = \beta Rb/\Delta$ imply $z_m = q - \beta Rb/\Delta$. Then, multiply both sides of (25) by z_m and replace z_m to get

$$(q - \beta Rb/\Delta)[\gamma - (1 + i_d)] = -(1-n)z_\ell(i_\ell - i_d).$$

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Use (24) to eliminate γ and rearrange to get

$$(q - \beta Rb/\Delta)\{1 - (1-n)\beta[u'(q) - 1] - \beta\} = (1-n)\frac{\beta Rb}{(1+i_\ell)}(i_\ell - i_d).$$

Hence, an equilibrium is defined by the following two equations:

$$\frac{1}{R\beta} = (1-n)u'(q)/\Delta + n$$

$$(q - \beta Rb/\Delta)\{1 - (1-n)\beta[u'(q) - 1] - \beta\} = (1-n)\frac{\beta Rb}{(1+i_\ell)}(i_\ell - i_d)$$

We can use the first equation to replace for $u'(q)$ in the second to get

$$\frac{1}{R\beta} = (1-n)u'(q)/\Delta + n$$
$$q = \beta RbF(\Delta).$$

If we substitute q in the first expression, we get

$$\frac{1}{R\beta} = (1-n)u'\left[\beta RbF(\Delta)\right]/\Delta + n \equiv RHS. \tag{46}$$

The left-hand side of (46) is constant while the right-hand side is decreasing in b for a given $1 \leq \Delta < \tilde{\Delta}$. Moreover, we have $\lim_{b\to 0} RHS = +\infty$ and $\lim_{b\to\infty} RHS = n < \frac{1}{R\beta}$. Hence, for any policy Δ with $1 \leq \Delta < \tilde{\Delta}$, a unique $b > 0$ exists. Then, from (30) a unique value for q exists. Accordingly, a unique symmetric stationary equilibrium exists.

Finally, we have $\lim_{\Delta \to \tilde{\Delta}} F(\Delta) = +\infty$ and so $b \to 0$. ∎

Proof of Proposition 2. We first show that $\tau = 0$ is optimal. Note that $\hat{q}$ is independent of τ, and $\tilde{q}$ is decreasing in τ. Therefore, increasing τ does only decrease the lower bound of the set of attainable allocations. Second, $F(\Delta; \tau)$ is decreasing in τ, so that an increase in τ either increases b, decreases q, or both. This reduces welfare. Hence, since $\tau \geq 0$, it is optimal to set τ to zero.

We now assume $\tau = 0$. Substituting (31) into the objective function, the problem becomes

$$\max_q \ (1-n)\left[u(q) - q\right] + (\beta R - 1)\frac{q}{\beta RF\left(\frac{R\beta(1-n)u'(q)}{1-nR\beta}\right)}$$

$$\text{s.t.} \ \ \hat{q} \geq q \geq \tilde{q}.$$

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After rearranging, the first-order condition is

$$(1-n)\left[u'(q)-1\right]+\frac{1-\beta R}{\beta RF\left(\Delta\right)}\left[\frac{F'\left(\Delta\right)\Delta}{F\left(\Delta\right)}\frac{u''\left(q\right)q}{u'\left(q\right)}-1\right]=\hat{\lambda}-\tilde{\lambda}$$

where $\hat{\lambda}$ is the multiplier of the first inequality, and $\tilde{\lambda}$ is the multiplier of the second inequality. Consider the first-order condition and note that

$$\Delta\left(q\right)=\frac{R\beta\left(1-n\right)u'\left(q\right)}{1-nR\beta}.$$

Suppose that the optimal q is such that $\Delta=1$; i.e., $q=\hat{q}$. In this case, $\tilde{\lambda}=0$ and $\hat{\lambda}>0$, implying that $\Theta\left(\hat{q},R\right)>0$. Then we have $F\left(1\right)=1$, $F'\left(1\right)=\frac{1-nR}{R-1}$ and so

$$\Theta\left(\hat{q},R\right)=\frac{1-\beta R}{\beta R}\frac{1-nR}{R-1}\frac{u''\left(\hat{q}\right)\hat{q}}{u'\left(\hat{q}\right)}<0$$

which is a contradiction. Thus, in any equilibrium $q<\hat{q}$, implying $\Delta>1$.

Now suppose that the optimal q is such that $\Delta=\tilde{\Delta}$; i.e., $q=\tilde{q}$. In this case, $\tilde{\lambda}>0$ and $\hat{\lambda}=0$, implying that $\Theta\left(\tilde{q},R\right)<0$. One can show that $\lim_{\Delta\to\tilde{\Delta}}F\left(\Delta\right)=\infty$, $\lim_{\Delta\to\tilde{\Delta}}F'\left(\Delta\right)=\infty$, $\lim_{\Delta\to\tilde{\Delta}}\frac{F'(\Delta)\Delta}{F(\Delta)}=\infty$ and $\lim_{\Delta\to\tilde{\Delta}}\frac{F'(\Delta)\Delta}{F(\Delta)F(\Delta)}=\frac{(1-1/R)}{(1/\Delta)^2(1-n)(\Delta-1)^2}$. Moreover, $(1-n)\left[u'(q)-1\right]=1/\beta-1$. Accordingly, we get

$$\Theta\left(\tilde{q},R\right)=1/\beta-1+\frac{1-\beta R}{\beta R}\frac{R\left(1-\beta n\right)^2}{\left(R-1\right)\left(1-n\right)}\frac{u''\left(\tilde{q}\right)\tilde{q}}{u'\left(\tilde{q}\right)}$$

Consider first $R\to1$. Then we have $\lim_{R\to1}\Theta\left(\tilde{q},R\right)=-\infty$. Now consider $R\to1/\beta$. Then we have $\lim_{R\to1/\beta}\Theta\left(\tilde{q},R\right)=1/\beta-1>0$. Since $\frac{1-\beta R}{\beta}\frac{(1-\beta n)^2}{(R-1)(1-n)}$ is monotonically decreasing in R, we have a unique critical value $\overline{R}$ such that $\Theta\left(\tilde{q},\overline{R}\right)=0$. Thus, if $R<\overline{R}$, $q=\tilde{q}$ and if $R>\overline{R}$, q solves $\Theta\left(q,R\right)=0$. ∎

Proof of Lemma 3. A stationary equilibrium requires that $M_+/M=\phi/\phi_+=\gamma$. To prove Lemma 3, note first that using the fact that $\lambda_\ell^k=u'(q^k)\left(1+i_d\right)-\left(1+i_\ell\right)$, $\lambda_q^k=\lambda_\ell^k+i_\ell$ and $\lambda_d^k=i_d$, the marginal value of money, the marginal value of collateral and the marginal value of private debt in market 3 can be written as follows

$$V_m^k = \beta\phi_+\left(1+i_d\right)\left\{1+\left(1-n^k\right)\left[u'(q^k)-1\right]\right\} \tag{47}$$

$$V_b^k = \beta R\left\{1+\left(1-n^k\right)\left[u'(q^k)/\Delta-1\right]\right\} \tag{48}$$

$$V_y^k = -\beta\phi_+\left(1+i_y\right)\left\{1+\left(1-n^k\right)\left[u'(q^k)/\Delta-1\right]\right\}. \tag{49}$$

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To derive (34) rewrite the first-order condition (5) by using equations (15), (10), and (47)-(49) to get

$$\frac{1-\beta R}{\beta R} = \sigma^H \left(1 - n^H\right) \left[\frac{u'(q^H)}{\Delta} - 1\right] + \sigma^L \frac{\Delta}{\hat{\Delta}} \left\{\frac{\Delta-\hat{\Delta}}{\Delta} + \left(1 - n^L\right)\left[u'(q^L) - 1\right]\right\} \qquad (50)$$

Then, rewrite the first-order condition (4) by using equations (15), (11), (47)-(49) to get

$$\frac{\gamma-\beta(1+i_d)}{\beta(1+i_d)} = \sigma^L(1 - n^L)\left[u'(q^L) - 1\right] + \sigma^H \frac{\Delta}{\hat{\Delta}} \left\{\frac{\Delta-\hat{\Delta}}{\Delta} + \left(1 - n^H\right)\left[\frac{u'(q^H)}{\Delta} - 1\right]\right\}. \qquad (51)$$

Finally, combine (50) with (51) to get (34).

To derive (35), note that in any equilibrium, the budget constraints in the goods market hold with equality and so

$$pq^k = m^k + \ell^k = M + y^k + \ell^k, \qquad k = H, L. \qquad (52)$$

Then, use (52) to substitute y^H and y^L in the money market's market clearing condition (12) and rearrange to get (35).

To derive (36) combine (12) and (52).

To derive (37), use (34) to write (33) as follows

$$\frac{R\hat{\Delta} - \Delta}{R\hat{\Delta}(\Delta - 1)} = \sigma^L(1 - n^L)\ell^L/M + \sigma^H(1 - n^H)\ell^H/M.$$

Then, use (52) to substitute ℓ^H and ℓ^L and rearrange to get

$$\frac{R\hat{\Delta}-\Delta}{R\hat{\Delta}(\Delta-1)} = -(1 - n) + \frac{1}{z_m}\left[\sigma^L(1 - n^L)q^L + \sigma^H(1 - n^H)q^H - \sigma^L(n^H - n^L)z^L\right].$$

Finally use (36) and solve for z_m to get (37).

Note that equations (50) - (37) must hold in any monetary equilibrium, where agents hold collateral. We now consider the case where no short-selling constraint is binding in the money market to derive (38) and (39).

When no short-selling constraint is binding in the money market, $\lambda^H_{md} = \lambda^L_{m\ell} = 0$, and so from (7) $V^L_m + V^L_y = V^H_m + V^H_y = 0$. Then, (47) and (49) imply

$$u'(q^k) = \frac{n^k}{1 - n^k} \frac{(\Delta - \hat{\Delta})}{(\hat{\Delta} - 1)}, \qquad k = H, L.$$

Using these expressions to replace $u'(q^H)$ and $u'(q^L)$ in (50), and solving for $\hat{\Delta}$ yields (38). Finally, to derive (39) use (38) to replace $\hat{\Delta}$ in the above equations. ∎

Proof of Proposition 4. The first thing to note is that the system of equations (34) - (39) can be solved recursively as described in the text. It remains to be shown under which conditions the short-selling constraints in the money market are non-binding. Thus, we need to verify that $y^k < Rb/\left[\phi_+ \left(1 + i_y\right)\right]$ and that $m + y^k > 0$. Using the seller's first-order condition and dividing by p, we can write these conditions as follows

$$z^k < \beta Rb \hat{\Delta}/\Delta \text{ and } z_m + z^k > 0.$$

Since $z^L > z^H$, it is sufficient to check that $z^L < \beta Rb \hat{\Delta}/\Delta$. Along the same lines, since $z^L > z^H$, it is sufficient to check that $z^H > -z_m$.

Let us first consider whether $z^H > -z_m$. From (36) and (37) $z^H > -z_m$ if

$$\sigma^L \left(q^L - q^H\right) < \frac{(\hat{\Delta}-1)\left[\sigma^L(1-n^L)q^L+\sigma^H(1-n^H)q^H\right]-\sigma^L\sigma^H\left(q^L-q^H\right)(n^H-n^L)\hat{\Delta}}{\Phi}$$

where $\Phi = \left(R\hat{\Delta} - \Delta\right)/\left[R\left(\Delta - 1\right)\right] + (1 - n)\hat{\Delta}$. Note that $\Phi > (1 - n)\hat{\Delta}$ since $R\hat{\Delta} > \Delta$.

Then $n^H - n^L = \varepsilon$ and $\sigma^L n^L + \sigma^H n^H = n$ yield $n^H = n + \sigma^L \varepsilon$ and $n^L = n - \sigma^H \varepsilon$. Using these relations and rearranging yields

$$q^L - q^H < \frac{(\hat{\Delta}-1)(1-n)\left(\frac{\sigma^H}{\sigma^L}q^H+q^L\right)-\varepsilon\sigma^H\left(q^L-q^H\right)}{\Phi}.$$

Divide by q^H and rearrange to get

$$\frac{q^L}{q^H}\left[\Phi - \left(\hat{\Delta} - 1\right)(1 - n) + \varepsilon\sigma^H\right] < \Phi + \frac{\sigma^H}{\sigma^L}\left(\hat{\Delta} - 1\right)(1 - n) + \varepsilon\sigma^H.$$

The left-hand side is larger than zero since $\Phi > (1 - n)\hat{\Delta}$. Moreover, it is strictly smaller than the right-hand side at $\varepsilon = 0$ (since $q^L = q^H$ at $\varepsilon = 0$). Then, divide the inequality by $\left[\Phi - (1 - n)\left(\hat{\Delta} - 1\right) + \sigma^H \varepsilon\right]$ to get

$$\frac{q^L}{q^H} < \frac{\Phi+\frac{\sigma^H}{\sigma^L}\left(\hat{\Delta}-1\right)(1-n)+\sigma^H\varepsilon}{\Phi-\left(\hat{\Delta}-1\right)(1-n)+\sigma^H\varepsilon}.$$

The left-hand side is increasing in ε and the right-hand side is decreasing. Therefore, there is a unique $\tilde{\varepsilon}_1$, such that $z_H > -z_m$ when $\varepsilon < \tilde{\varepsilon}_1$.

We next check $\beta Rb\hat{\Delta}/\Delta > z^L$. From $\sigma^H q^H + \sigma^L q^L = z_m + \frac{\beta Rb}{\Delta}$, we need $\sigma^H q^H + \sigma^L q^L > z_m + z^L/\hat{\Delta}$, or replacing for z_m and z^L, and rearranging we need

$$\sigma^H q^H + \sigma^L q^L > \frac{\hat{\Delta}\left\{(\hat{\Delta}-1)\left[\sigma^L(1-n^L)q^L+\sigma^H(1-n^H)q^H\right]-(n^H-n^L)\sigma^L\sigma^H\left(q^L-q^H\right)\hat{\Delta}\right\}}{(\hat{\Delta}-1)\Phi} + \frac{\sigma^H\left(q^L-q^H\right)}{(\hat{\Delta}-1)}.$$

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Multiply through by $\left(\hat{\Delta} - 1\right)$ and arrange to obtain

$$\left(\sigma^H q^H + \sigma^L q^L\right)\hat{\Delta} - q^L > \frac{\hat{\Delta}\left\{\left(\hat{\Delta}-1\right)\left[\sigma^L(1-n^L)q^L+\sigma^H(1-n^H)q^H\right]-(n^H-n^L)\sigma^L\sigma^H\left(q^L-q^H\right)\hat{\Delta}\right\}}{\Phi}.$$

Use $n^H = n + \sigma^L \varepsilon$ and $n^L = n - \sigma^H \varepsilon$ to substitute n^H and n^L and rearrange to get

$$\left(\sigma^H q^H + \sigma^L q^L - \frac{q^L}{\hat{\Delta}}\right)\Phi > (1-n)\left(\sigma^L q^L + \sigma^H q^H\right)\left(\hat{\Delta} - 1\right) - \sigma^L \sigma^H\left(q^L - q^H\right)\varepsilon.$$

This expression is satisfied at $\varepsilon = 0$, since we have $\Phi > (1-n)\hat{\Delta}$. Dividing both sides by $\sigma^H q^H + \sigma^L q^L$, and rearranging gives

$$\frac{\Phi}{\hat{\Delta}\left(\sigma^H \frac{q^H}{q^L}+\sigma^L\right)} < \Phi - \left(\hat{\Delta} - 1\right)(1-n) + \frac{\sigma^L\sigma^H\varepsilon\left(1-\frac{q^H}{q^L}\right)}{\sigma^H \frac{q^H}{q^L}+\sigma^L}.$$

Since $\frac{q^H}{q^L}$ is decreasing in ε, the left-hand side is increasing in ε, and the right-hand side is also increasing in ε. Therefore, given this constraint does not bind at $\varepsilon = 0$, either it never binds or it binds for some $\varepsilon > \acute{\varepsilon}_1$. Thus, if $\varepsilon < \varepsilon_1 = \min\{\tilde{\varepsilon}_1, \acute{\varepsilon}_1\}$, a unique equilibrium exists where no short-selling constraint binds. $\blacksquare$