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**WHEN DO MORE PATENTS REDUCE R&D?**

Robert M. Hunt\*  
Federal Reserve Bank of Philadelphia

First Draft: September 2003

This Draft: January 2006

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA

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## **When Do More Patents Reduce R&D?**

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### **Abstract**

This paper develops a simple duopoly model in which investments in R&D *and* patents are inputs in the production of firm rents. Patents are necessary to appropriate the returns to the firm's own R&D, but patents also create potential claims against the rents of rival firms.

Analysis of the model reveals a general necessary condition for the existence of a positive correlation between the firm's R&D intensity and the number of patents it obtains. When that condition is violated, changes in exogenous parameters that induce an increase in firms' patenting can also induce a decline in R&D intensity. Such a negative relationship is more likely when (1) there is sufficient overlap in firms' technologies so that each firm's inventions are likely to infringe the patents of another firm, (2) firms are sufficiently R&D intensive, and (3) patents are cheap relative to both the cost of R&D and the value of final output.

Keywords: Patents, Patent Thickets

JEL Codes: 034

## **1. Introduction**

This paper presents a simple model that explores the relationship between the incentive to invent and the incentive to obtain patents. Unlike much of the previous literature, we do not treat these as a single decision. Instead we explore the factors required for investments in R&D and patents to be complementary in the manner typically assumed in most theoretical models and policy discussions.

We derive sufficient conditions for patents and R&D to be *substitute* inputs in the production of firm profits: there must be sufficient overlap between the firms' patented inventions, firms must be sufficiently R&D intensive, and patents must be relatively easy to obtain. The first requirement may be due to the nature of a technology, but it can also result from the manner in which patents are drafted and examined. The latter two depend both on the pecuniary costs of R&D and patents and the standards required for patentable inventions. In such environments, firms increase their patenting in order to tax the rents earned on a rival's inventions and to mitigate similar behavior by their rivals. Firms respond by reducing their R&D investments.

The remainder of the paper is organized as follows. Section 2 presents the model and Section 3 presents the main results. Section 4 investigates welfare implications. Section 5 places the results in the context of U.S. patent policy and empirical research on the use and effects of the patent system. All proofs are found in the appendix.

## **2. The Model**

A measure of consumers have a unit demand for the final output (inventions), which can be interpreted as improvements in product quality. There is a competitive fringe of firms that are

able to imitate and produce inventions at no cost, but they have no independent R&D capability. There are also two firms, sharing the same technology, that are capable of inventing and seeking property rights over their inventions. These two firms move simultaneously, deciding on the amount of R&D ( $x_i$ ) to perform and the number of patents ( $n_i$ ) to obtain. Both activities are subject to a constant marginal cost,  $R$  and  $C$ , respectively (final output is the numeraire). The required inputs are assumed to be purchased from competitive markets, so these prices also represent the social cost of performing R&D and obtaining patents.

An investment of  $x_i$  in R&D leads to a measure of the final goods invented

$f(x_i) = x_i^\alpha$ ,  $\alpha < 1$ . A firm that obtains a quantity of patents  $n_i$  is able to appropriate a share of rents associated with its own innovations. Let  $\theta(n_i) \in [0, 1]$  denote this share and assume it follows an exponential distribution; that is,  $\theta(n_i) = 1 - e^{-n_i}$ . Ignoring the other firm for a moment, firm  $i$  earns rents  $\theta(n_i)f(x_i)$ . The remainder,  $[1 - \theta(n_i)]f(x_i)$ , is unprotected, so it is supplied by the competitive fringe of imitators. Thus patents are essential to protecting the firm's return on R&D investments.

In addition, some of the rival's inventions may infringe one or more of  $i$ 's patents. This will depend on the number of patents firm  $i$  obtains, and the degree of overlap between the inventions produced by each firm. Let  $\beta \in [0, 1)$  denote this degree of overlap. Thus firm 1 may claim a share of the rents generated by firm 2 equivalent to  $\beta\theta(n_1)f(x_2)$ . This includes a share of what firm 2 would otherwise earn,  $\theta(n_1)f(x_2)$ , and a share,  $[1 - \theta(n_2)]f(x_2)$ , that would otherwise be supplied by the competitive fringe. Firm 2 engages in the same activity, which

extracts some rents from firm 1 and some output from the competitive fringe. The objective functions of the two firms are thus:

$$V^1 \equiv \theta(n_1)(1 - \beta\theta(n_2))f(x_1) + \beta\theta(n_1)f(x_2) - Rx_1 - Cn_1, \text{ and}$$

$$V^2 \equiv \theta(n_2)(1 - \beta\theta(n_1))f(x_2) + \beta\theta(n_2)f(x_1) - Rx_2 - Cn_2.$$

The parameter  $\beta$  is a parsimonious way of modeling the degree to which a firm's property rights depend on their inventions. When  $\beta = 0$ , each firm derives rents only from its own R&D investments. When  $1 > \beta > 0$ , a firm is able to lay claim to the inventions of others, but not as easily as it can claim inventions of its own making. Note that while the exact mechanism of the transfers is not specified in the model, holding R&D constant, the transfers impose no losses in the total potential rents that can be earned.

How should we interpret  $\beta$ ? For some industries it is a question of technology. Firms may draw from similar technical fields and arrive at similar solutions even when they apply them to different problems. This is particularly true for industries that advance through cumulative innovation and where firms may rely on a largely common set of building blocks derived from previous innovations. In addition, some products incorporate several, if not dozens, of potentially patentable innovations. Two examples might include semiconductors and computer software.

The size of  $\beta$  might also depend on the breadth of claims contained in patents.<sup>1</sup> If broad claims are regularly granted, it is more likely that firms will infringe each other's patents. Under this interpretation it is possible that patent breadth, and therefore  $\beta$ , can be influenced by policymakers or the courts.

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<sup>1</sup> The trade-off between patent breadth and length is studied in Gilbert and Shapiro (1990) and Klemperer (1990).

A third, and more controversial, interpretation is that  $\beta$  is a measure of a firm's effectiveness in obtaining property rights over things it has not really invented. While this is explicitly prohibited by U.S. patent law, it might nevertheless arise from mistakes in the examination of patent applications. This is a topic that has received considerable attention in recent years (FTC 2003, Jaffe and Lerner 2004, Merrill, Levin and Myers 2004).

This may be a particular problem for patents on computer programs, especially ones that implement methods of doing business, if the patent law's disclosure requirements are not adequately enforced.<sup>2</sup> In that case, one might not be certain what the applicant has invented and how far his or her claims should extend. In these areas, some researchers and practitioners worry that applicants can obtain relatively broad patents even though they have not yet started their R&D (Burk 2002, Burk and Lemley 2002, Flynn 2001, FTC 2003 (chapter 4)).

### 3. Equilibrium and Comparative Static Results

The first order conditions for firm 1 are

$$[1] \quad \theta_1 (1 - \beta \theta_2) f_1' - R = 0, \text{ and}$$

$$[2] \quad (1 - \theta_1) \{ (1 - \beta \theta_2) f_1 + \beta f_2 \} - C = 0,$$

where the subscripts refer to the variable for the appropriate firm. Note that in [1] the increase in revenue associated with additional R&D reflects the effect of firm 1's patenting and that of its rival, which the firm takes as given. In [2], the increase in revenue resulting from additional patenting includes the additional revenue firm 1 can extract from its rival.

The first order conditions imply the following relationship between R&D and patenting:

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<sup>2</sup> See 35 USC 112. An applicant must describe his or her invention in sufficient detail that a person skilled in the relevant art can replicate it without undue experimentation.

$$[3] \quad \frac{R}{C} = \frac{f_1' \left( \frac{\theta_1}{1-\theta_1} \right)}{f_1} \left[ \frac{(1-\beta\theta_2)f_1}{(1-\beta\theta_2)f_1 + \beta f_2} \right].$$

Let  $P \equiv R/C$  denote the relative cost of the inputs and  $\tau_1 \equiv (1-\beta\theta_2)f_1 / [(1-\beta\theta_2)f_1 + \beta f_2]$ .

Thus when R&D is significantly more expensive than patenting, the firm will obtain more patents for every increment of R&D it performs. The wedge  $\tau_1$  is decreasing in the rival's R&D investments and patenting, and in  $\beta$ , but is increasing in the firm's own R&D.

Substituting  $\theta(n) = 1 - e^{-n}$  in [3] and rearranging terms, we find that

$$[4] \quad \theta_i = \frac{Px_i}{\alpha\tau_i + Px_i}.$$

Thus, holding constant the rival's behavior, firm  $i$ 's patent portfolio is strictly increasing in the amount of R&D it performs.

**Proposition 1:** If the costs of obtaining patents ( $C$ ) and doing R&D ( $R$ ) are sufficiently small, there exists a unique, symmetric interior equilibrium,  $(x^*, n(x^*))$ , in pure strategies.

The proof is provided in the appendix. A sufficient condition for satisfying the second order condition and the participation constraint is that  $C \leq \tilde{C}$  where

$$[5] \quad \tilde{C} = \frac{\alpha(1-\alpha)}{-Ln(1-\alpha)} [1 + \beta(1-\alpha)] \left\{ \frac{\alpha^2}{R} [1 - \beta\alpha] \right\}^{\frac{\alpha}{1-\alpha}}.$$

Figure 1 plots this constraint in the space of input prices. Note that as we increase the output elasticity,  $\alpha$ , the constraint rotates clockwise. If we consider higher values of  $\beta$ , the combination of input prices where an interior equilibrium exists becomes slightly smaller.

Next, we derive some of the comparative static properties of the equilibrium. The proofs are found in the appendix.

**Proposition 2:** The equilibrium number of patents,  $n^*$ , is decreasing in both the cost of obtaining patents ( $C$ ) and the cost of doing R&D ( $R$ ). The equilibrium level of R&D,  $x^*$ , is decreasing in the cost of doing R&D. The equilibrium level of R&D,  $x^*$ , is decreasing (increasing) in the cost of obtaining patents ( $C$ ) when  $1 - 2\beta\theta(x^*) > 0$  ( $1 - 2\beta\theta(x^*) < 0$ ).

Thus the model generates the typical input price responses for patents, but not always for R&D. Ordinarily we expect that, where patents are essential to appropriating returns to innovation, reducing the cost of obtaining these property rights encourages more R&D. Proposition 2 shows this intuition holds only when the rival firm cannot extract a majority of the potential rents associated with firm  $i$ 's innovations. This is ensured if  $\beta < 1/2$ , but not otherwise. If  $\beta \geq 1/2$ , and firms are sufficiently active in their R&D and patenting, reductions in the cost of patenting will reduce R&D. In the appendix we show the following:

**Corollary 1:** Suppose  $\beta \geq 1/2$ . Then, if  $C \leq \tilde{C}$ ,  $1 - 2\beta\theta(x^*) \leq 0$  when  $C \leq \hat{C}$  where

$$\hat{C} = \left( \beta - \frac{1}{4\beta} \right) \left( \frac{\alpha}{4\beta R} \right)^{\frac{\alpha}{1-\alpha}}.$$

Figures 2 and 3 plot this constraint in the space of input prices, and we include the earlier constraint  $\tilde{C}$ . The only difference between them is that figure 3 assumes a higher output elasticity (0.85) than figure 2 (0.50). In figure 2  $\hat{C} < \tilde{C}$ , so the set of input prices that ensures existence of an interior equilibrium can be divided into a region where R&D is decreasing in the

cost of patenting and another where R&D is increasing in the cost of patents. In figure 3,  $\hat{C} \geq \tilde{C}$ , which implies there is no interior equilibrium where a decrease in the cost of patenting would increase R&D.

#### 4. Welfare Analysis

What would a social planner do? It is easy to show that the amount of innovation in the private equilibrium is always less than the first best outcome.<sup>3</sup> In the first best solution, patents are unnecessary. In a second best world, and where R&D subsidies cannot be funded from an external source, the social planner may “tax” patenting to stimulate private R&D investments.

To be concrete, suppose the planner can levy a tax  $\varepsilon$  such that the private cost of obtaining a patent is now  $c = C + \varepsilon$ , where  $C$  continues to denote the social cost of resources devoted to patent prosecution. The planner chooses  $\varepsilon$  to maximize  $f(x^*) - Rx^* - Cn^*$ , where  $(x^*, n^*)$  denote the outcomes of the symmetric private equilibrium with firms responding to the input prices  $R$  and  $c$ . In the appendix, we prove the following:

**Proposition 3:** A social planner limited to taxing patent activity always chooses a level of tax  $\varepsilon$  so that  $c \in (\hat{C}, \tilde{C}]$ .

In other words, the social planner will never permit the private cost of patents to be so low that the counterintuitive outcome defined in Proposition 2 would occur. The intuition follows immediately from the first order condition to the planner’s problem:

$$\frac{\partial x^*}{\partial C} [f'(x^*) - R] - \frac{dn^*}{dc} C = 0.$$

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<sup>3</sup> This is true even if we assume  $\beta$  is also a measure of the degree of overlap in the firms’ R&D programs.

So long as  $dn^*/dc < 0$  (see appendix) the first order condition can be satisfied only if the planner chooses  $\varepsilon$ , so that  $dx^*/dc < 0$ . At an optimum, the planner is trading off marginal reductions in the number of inventions that are worth  $f'(x^*) - R > 0$  against marginal reductions in resources devoted to patenting, which cost  $C$ .

Thus far we have treated the overlap parameter  $\beta$  as an exogenous aspect of the technological environment. Suppose the social planner has some control over the magnitude of  $\beta$ , perhaps through legal doctrines that determine the breadth of patent claims. What would she do? In the appendix, we show:

**Proposition 4:**  $n^*$  is decreasing in  $\beta$ . A sufficient condition for  $\partial x^*/\partial \beta < 0$  is  $\alpha \geq 1/2$ .

Thus the social planner would prefer less overlap and therefore more narrow patents. Firms would respond, however, by increasing their patent activity. These results, however, depend on some stark attributes of the model. If it were generalized to consider the possibility of differential effects of patent breadth for one's own inventions compared to the rival's inventions, it is likely the comparative static calculation for  $\partial x^*/\partial \beta$  would be more ambiguous.

There is, of course, another obvious remedy—merge the two firms. This would internalize the problem created by the overlap and, in the absence of imitation by the competitive fringe, attain the first best outcome. But once the model is generalized to include racing, a single firm may do less R&D than two firms (Lee and Wilde 1980), and this would be compounded in a dynamic model where the monopolist is concerned about replacing its own profits (Reinganum 1985).

## 5. Discussion

This paper develops a simple model that illustrates the relationship between firms' R&D and patenting decisions. It is typically the case that these two activities are complementary—firms that do a lot of R&D also tend to patent more. And ordinarily, reducing the cost of R&D, or of patenting, will stimulate additional investments in R&D.

But as the model illustrates, this intuition does not always hold. Each firm cares about the patent strategies of its rival, which affects the rents it earns on its own discoveries, as well as the rents earned when the rival infringes its own patents. A necessary condition is a significant overlap between the rights granted to each firm ( $\beta$  must be at least  $\frac{1}{2}$ ). Then, if firms are sufficiently active in their R&D and patenting, incremental reductions in the cost of obtaining patents result in less, rather than more R&D. This does not imply the elimination of R&D investments, but rather less innovation than would otherwise occur.

Thus there may be instances where raising patent costs can actually induce more R&D. This might be achieved via a patent tax or by increasing the requirements that must be satisfied in order to obtain a patent. For example, policymakers could increase the *inventive step* (the standard of nonobviousness in U.S. law) required to obtain a patent so that the most trivial advances over the prior art do not qualify for patent protection (Hunt 1999).

The model suggests that the perverse outcome is more likely to occur in high-tech industries that advance rapidly via cumulative innovation (and which also obtain many patents) than in industries that are not research intensive and which do not build up large patent portfolios.<sup>4</sup> Previous empirical work has identified a number of industries with such

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<sup>4</sup> A similar intuition obtains from dynamic models of innovation. Hunt (2004) shows that a low inventive step, making patents easy to obtain, can reduce the rate of innovation in industries where R&D is the most productive.

characteristics, including electronics, computers, and semiconductors (Levin et al., 1987; Cohen Nelson, and Walsh 2000; Hall and Ziedonis 2001). These industries account for most of the rapid growth in U.S. patenting in recent years (Hall 2003). They are also the industries where researchers identify what is sometimes called “strategic patent” behavior, including the assembly of large portfolios for wholesale cross-licensing and possibly deceptive patent prosecution (Grindley and Teece 1997, Graham and Mowery 2004).

Bessen and Hunt (2003) present empirical results consistent with the phenomenon modeled here in the context of patenting computer software. Obtaining such patents was difficult, but not impossible, during the 1970s and early 1980s. Over time, however, courts have become more receptive to such patents and their numbers have grown rapidly, especially among firms in the industries described above (and much less so among firms in the software industry). All else equal, firms that concentrated on obtaining software patents experienced a statistically and economically significant decline in their R&D intensity relative to other firms.

Of course, this is a highly stylized model that omits a number of important considerations from the analysis. For example, the deleterious effects of excessive patenting might be mitigated via cross-licensing or patent pools, assuming these can be negotiated without excessive transactions costs.<sup>5</sup> Both of these solutions raise a new set of questions, particularly in the context of antitrust policy, which are not addressed here.<sup>6</sup>

Nor is it immediately obvious that successful licensing arrangements improve innovation incentives in an environment of “cheap” patents. Bessen (2003) develops a model of “patent

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<sup>5</sup> FTC (2003), Chapter 3, provides a detailed discussion of reasons why these transactions costs may be large for some industries. For an empirical analysis of the effects of fragmented property rights in the semiconductor industry, see Ziedonis (2003).

<sup>6</sup> See, for example, the papers by Choi (2003), Farrell and Shapiro (2005), and Shapiro (2001, 2003).

thickets” and examines the efficacy of licensing in a regime of either high or low patent standards. He finds that under low patent standards firms are able to assemble large patent portfolios, which they use to make aggressive licensing demands. This encourages more patenting, but less R&D. Under higher standards, such a strategy is not cost effective.

There are a number of interesting extensions to consider. For example, one could make the cost of obtaining a certain measure of patents depend on the amount of R&D the firm does. All else equal, this would tend to reduce the space of parameter values where the counterintuitive outcome described in Proposition 2 would occur. Another generalization is to analyze an asymmetric environment in which an established firm, with a preexisting patent portfolio and capital stock, competes with one or more potential entrants. That would allow us to examine issues such as hold-up and entry deterrence. This is a topic for future research.

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## Appendix

### Proof of Proposition 1:

We begin by checking the second order condition to the firm problem. Substituting [4] into  $V_n^1 = 0$  and rearranging terms, we have

$$\Psi \equiv \alpha[1 - \beta\theta_2]f_1 - Rx_1 - \alpha\tau C = 0.$$

The first derivative is

$$\left. \frac{\partial \Psi}{\partial x_1} \right|_{x_1^*} \equiv [1 - \beta\theta_2] \{ \alpha - \theta_1 - \alpha[1 - \theta_1](1 - \tau) \}.$$

In order to ensure the first order conditions are associated with a local maximum when  $\beta = 0$  (which implies  $\tau = 1$ ), we need  $\theta_1(x_1^*) > \alpha$ . This will ensure we have an interior equilibrium when firm 1's profits are generated only from the inventions it invents. Let  $\tilde{x}$  denote the value of  $x$  s.t.  $\theta_1(x_1^*) = \alpha$  when firm 2 is playing the same strategy (i.e., in the symmetric case). Since  $\theta$  is strictly increasing in  $x$ ,  $\partial \psi / \partial x_1 < 0 \forall x > \tilde{x}$ . Thus for the symmetric case, if there is a local maximum, it will be unique. Using [4], we can derive this critical R&D intensity:

$$\tilde{x} = \frac{\alpha^2 \tau}{1 - \alpha P}.$$

Substituting for  $f(x) = x^\alpha$  in  $V_x^1 = 0$ , again assuming symmetric behavior, yields

$$x^* = \left( \frac{\alpha\theta[1 - \beta\theta]}{R} \right)^{\frac{1}{1-\alpha}}.$$

Substituting  $\alpha$  for  $\theta$  in the previous expression and equating it to  $\tilde{x}$  allows us to solve for the critical cost of obtaining patents in terms of  $R$ ,  $\beta$ , and  $\alpha$ :

$$C \leq \bar{C} \equiv (1 - \alpha)[1 + \beta(1 - \alpha)] \left\{ \frac{\alpha^2}{R} (1 - \beta\alpha) \right\}^{\frac{\alpha}{1-\alpha}}.$$

For values of  $C < \bar{C}$ , the second order condition is satisfied even when  $\beta = 0$ .

Next, we need to verify that  $V(x_1^*, n(x_1^*)) \geq 0$ . Substituting for  $x_1^*$  and  $n_1^*$  from the first order conditions,

$$V(x_1^*, n(x_1^*)) = ([1 - \beta\theta_2]f_1 + \beta f_2) \{ \theta_1[1 - \alpha\tau] + (1 - \theta_1)Ln(1 - \theta_1) \}.$$

For symmetric case when  $x = \tilde{x}$  the expression in brackets reduces to  $\{ \alpha[1 - \alpha\tau] + (1 - \alpha)Ln(1 - \alpha) \}$ . This is positive for sufficiently small values of  $\tau$ , but is negative for  $\tau = 1$ . Thus, in order to ensure the participation constraint is satisfied when  $\beta = 0$ , we require a value of  $\bar{C} = \lambda \bar{C}$  s.t.

$$V(\tilde{x}, n(\tilde{x})) = f(\tilde{x})[1 + \beta(1 - \alpha)] \{ \alpha[1 - \alpha] + \lambda \bar{C} Ln(1 - \alpha) \} = 0,$$

which implies that  $-\lambda \bar{C} Ln(1 - \alpha) = \alpha(1 - \alpha)f(\tilde{x})$ . When  $\beta = 0$ ,

$$\bar{C} = (1 - \alpha) \left\{ \frac{\alpha^2}{R} \right\}^{\frac{\alpha}{1-\alpha}} \quad \text{and} \quad f(\tilde{x}) = \left\{ \frac{\alpha^2 \bar{C}}{1 - \alpha R} \right\}^\alpha = \left\{ \frac{\alpha^2}{R} \right\}^{\frac{\alpha}{1-\alpha}}.$$

This implies that  $\lambda = -\alpha / Ln(1 - \alpha) \in [0, 1]$ . Thus the second order condition and the participation constraint are satisfied when

## Appendix

$$C \leq \tilde{C} \equiv \frac{\alpha[1+\beta(1-\alpha)]}{-\ln(1-\alpha)} \left\{ \frac{\alpha^2}{R}(1-\beta\alpha) \right\}^{\frac{\alpha}{1-\alpha}}.$$

### Proof of Propositions 2 and 4:

We return to the first order conditions for the symmetric equilibrium:

$$V_x = \theta(1-\beta\theta)f'(x) - R = 0$$

$$V_n = (1-\theta)f(x)[1+\beta(1-\theta)] - C = 0$$

The associated Jacobian is

$$|J| = \begin{vmatrix} \theta(1-\beta\theta)f'' & (1-\theta)f'(1-2\beta\theta) \\ (1-\theta)f'[1+\beta(1-\theta)] & -(1-\theta)f[1+2\beta(1-\theta)] \end{vmatrix},$$

which reduces to

$$|J| = ((1-\theta)f')^2 \left\{ \left( \frac{\theta}{1-\theta} \right) \left( \frac{1-\alpha}{\alpha} \right) [1+2\beta(1-\theta)] - (1-2\beta\theta)[1+\beta(1-\theta)] \right\}.$$

$|J| > 0$  for values of  $C \leq \tilde{C}$ . The comparative static results follow:

$$\frac{\partial n^*}{\partial R} = \frac{-1}{|J|} \cdot \begin{vmatrix} V_{xx} & -1 \\ V_{nx} & 0 \end{vmatrix} = \frac{-(1-\theta)f'}{|J|} [1+\beta(1-\theta)] < 0$$

$$\frac{\partial n^*}{\partial C} = \frac{-1}{|J|} \cdot \begin{vmatrix} V_{xx} & 0 \\ V_{nx} & -1 \end{vmatrix} = \frac{\theta f''}{|J|} (1-\beta\theta) < 0$$

$$\frac{\partial n^*}{\partial \beta} = \frac{-1}{|J|} \cdot \begin{vmatrix} V_{xx} & -\theta^2 f' \\ V_{nx} & (1-\theta)^2 f \end{vmatrix} = \frac{\theta(1-\theta)(f')^2}{|J|} \left\{ \left( \frac{1-\alpha}{\alpha} \right) [1-\beta\theta] - \left( \frac{\theta}{1-\theta} \right) [1+\beta(1-\theta)] \right\} < 0$$

$$\frac{\partial x^*}{\partial R} = \frac{-1}{|J|} \cdot \begin{vmatrix} -1 & V_{xn} \\ 0 & V_{mn} \end{vmatrix} = \frac{-(1-\theta)f}{|J|} [1+2\beta(1-\theta)] < 0$$

$$\frac{\partial x^*}{\partial C} = \frac{-1}{|J|} \cdot \begin{vmatrix} 0 & V_{xn} \\ -1 & V_{mn} \end{vmatrix} = \frac{-(1-\theta)f'}{|J|} (1-2\beta\theta)$$

$$\frac{\partial x^*}{\partial \beta} = \frac{-1}{|J|} \cdot \begin{vmatrix} -\theta^2 f' & V_{xn} \\ (1-\theta)^2 f & V_{mn} \end{vmatrix} = \frac{(1-\theta)^3 f f''}{|J|} \left\{ [1-2\beta\theta] - \left( \frac{\theta}{1-\theta} \right)^2 [1+2\beta(1-\theta)] \right\}$$

A sufficient condition for  $\partial x^*/\partial \beta \leq 0$  is  $\theta(x^*) \geq 1/2$ , which is assured if  $\alpha \geq 1/2$ .

### Proof of Corollary 1:

Suppose  $\beta \geq 1/2$  and let  $\hat{\theta} = 1/2\beta$ . Substituting for  $\theta$  in [2] and solving for  $x$  yields the following:

$$\hat{x} = \left( \frac{2\beta C}{4\beta^2 - 1} \right)^{\frac{1}{\alpha}}.$$

To determine when  $x^* \geq \hat{x}$ , we insert  $\hat{x}$  into [1] and solve for values of  $C$  that satisfy:

## Appendix

$$C \leq \hat{C} \equiv \left( \beta - \frac{1}{4\beta} \right) \left( \frac{\alpha}{4\beta R} \right)^{\frac{\alpha}{1-\alpha}}.$$

### Proof of Proposition 3:

The social planner imposes a tax,  $\varepsilon$  so that firms pay  $c = C + \varepsilon$  in order to obtain patents. The planner chooses  $\varepsilon$  to maximize  $W(x^*) = f(x^*) - Rx^* - Cn(x^*)$  where  $(x^*, n(x^*))$  represent the firms' choice of R&D and patents in a symmetric equilibrium with input costs  $R$  and  $c$ . The associated first order condition is given in the text. Substituting for  $R$  and  $C$  using, [1] and [2], we find that

$$W(x^*) = f(x^*)[1 + \beta(1-\theta)] \left\{ \frac{1-\alpha\theta[1-\beta\theta]}{1+\beta(1-\theta)} + (1-\theta)Ln(1-\theta) \right\} \geq V(x^*),$$

where the inequality is strict as long as  $x^* < \infty$ . This implies there is a region of input costs just above the set defined by  $\tilde{C}$  where the planner would be willing to subsidize R&D (from some outside source) to make an interior equilibrium feasible. But we have ruled this out in the proposition.

The proof requires that  $dn^*/dC \leq 0$ . The derivative itself is

$$\frac{dn^*}{dC} = \frac{\partial n^*}{\partial C} + \frac{\partial n^*}{\partial x} \cdot \frac{\partial x^*}{\partial C} = \frac{1}{|J|} \left\{ \theta f''(1-\beta\theta) - \frac{dn^*}{dx} (1-\theta) f'(1-2\beta\theta) \right\}.$$

Substituting  $c$  for  $C$  in [2] implies  $n^* = -Ln(c/M)$  where  $M \equiv f(x)[1 + \beta(1-\theta)] = f(x)[1 + \beta(c/M)] \equiv f(x)m$ . This allows us to solve a quadratic equation for  $m$ :  $m = (1 + \sqrt{1 + 4\beta c/f})/2$ . We can then calculate

$$\frac{\partial n^*}{\partial x} = \frac{f'}{f} \left( 1 - \frac{4\beta c}{4fm\sqrt{1+4\beta c/f}} \right) = \frac{f'}{f} \cdot \frac{fm + \beta c}{fm\sqrt{1+4\beta c/f}} \equiv \frac{f'}{f} \eta.$$

Returning to our derivative:

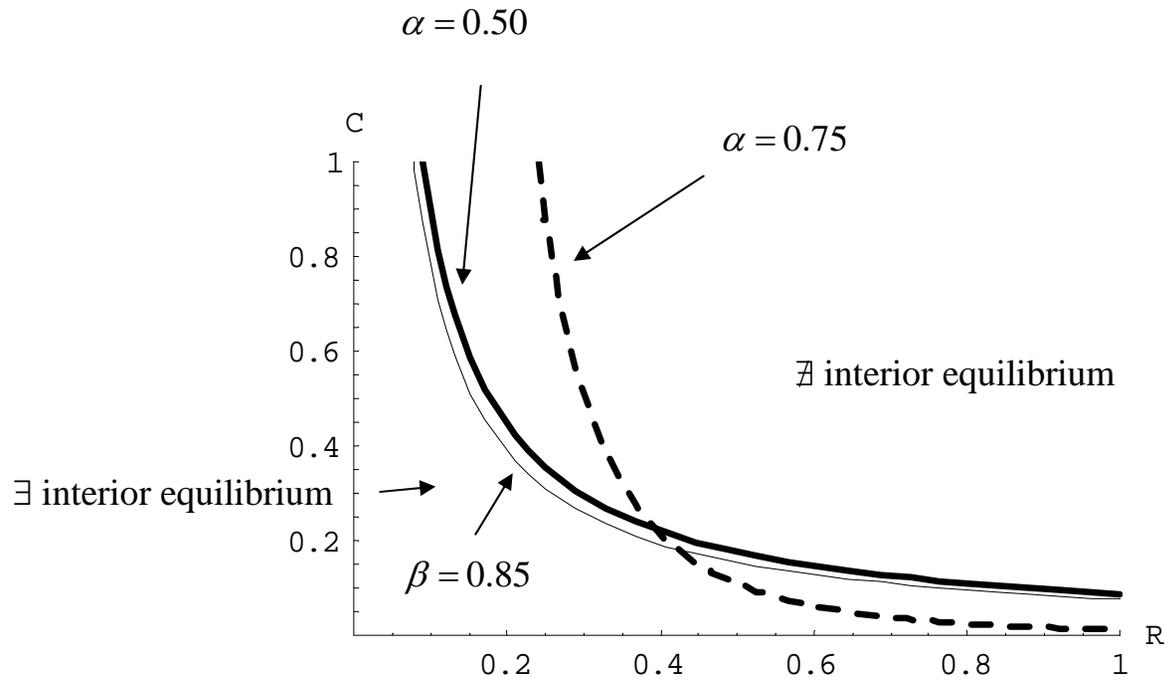
$$\frac{dn^*}{dC} = \frac{-(1-\theta)(f')^2(1-\beta\theta)}{f|J|} \left\{ \left( \frac{\theta}{1-\theta} \right) \left( \frac{1-\alpha}{\alpha} \right) + \eta \left( \frac{1-2\beta\theta}{1-\beta\theta} \right) \right\} < 0.$$

If it is feasible (i.e.,  $\hat{C} < \tilde{C}$ ), the social planner sets  $\varepsilon$  so that

$$\frac{-dx^*}{dC} (f' - R) = \frac{(1-\theta)(f')^2(1-\beta\theta)}{f|J|} \left\{ \left( \frac{\theta}{1-\theta} \right) \left( \frac{1-\alpha}{\alpha} \right) + \eta \left( \frac{1-2\beta\theta}{1-\beta\theta} \right) \right\} C.$$

Otherwise, it sets  $\varepsilon$  so that  $C + \varepsilon = \tilde{C}$ .

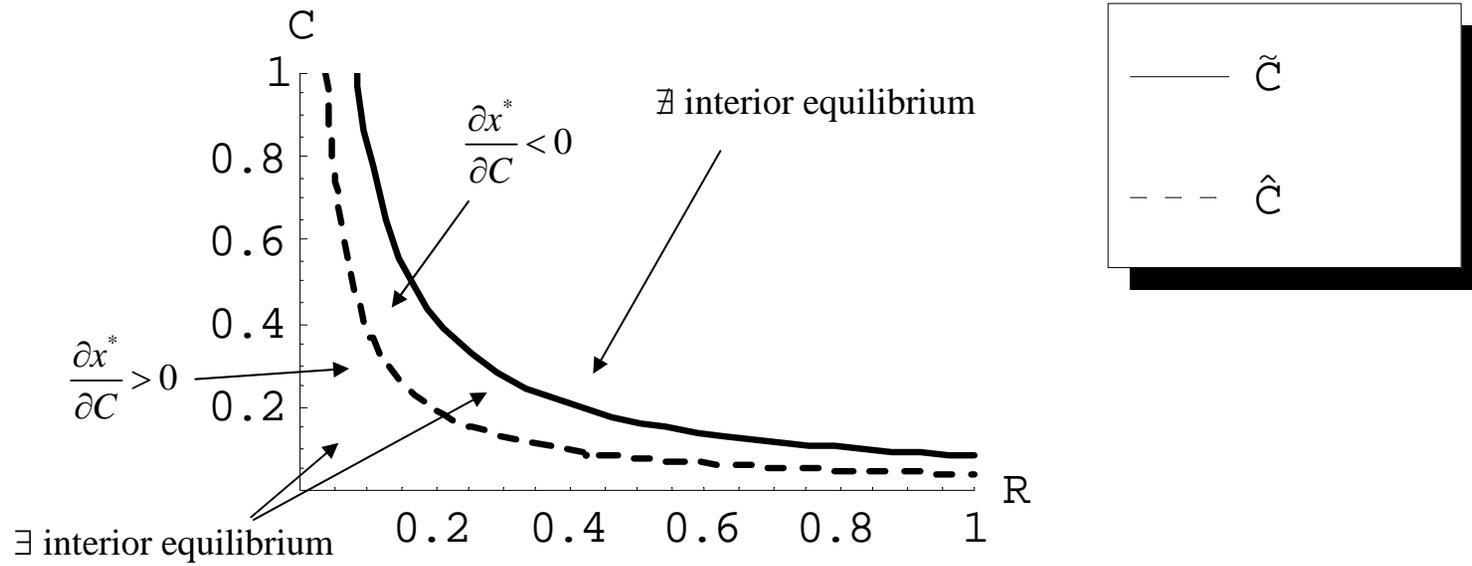
# Figure 1



Note: Boundaries are defined by  $\tilde{C}$  (see text).

The default values used in this figure are  $\beta = 0.25$  and  $\alpha = 0.50$ .

Figure 2



Note:  $\beta = 0.60$  and  $\alpha = 0.5$ .

Figure 3

