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DO SUNK COSTS OF EXPORTING MATTER FOR
NET EXPORT DYNAMICS?

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Do Sunk Costs of Exporting Matter for Net Export Dynamics?*

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Abstract

Not all firms export every period. Firms enter and exit foreign markets. Previous research has suggested that these export participation decisions have significant aggregate implications. In particular, it has been argued that these export decisions are important for the comovements of net exports and the real exchange rate. In this paper, we evaluate these predictions in a general equilibrium environment. Specifically, assuming that firms face an up-front, sunk cost of entering foreign markets and a smaller period-by-period continuation cost, we derive the discrete entry and exit decisions yielding exporter dynamics in an otherwise standard equilibrium open economy business cycle model. We show that the export decisions of firms in the model are influenced by the business cycle in a manner consistent with evidence presented for U.S. exporters. However, in contrast to previous partial equilibrium analyses, model results reveal that the aggregate effects of these export decisions are negligible.

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1. Introduction

Recent studies have emphasized the importance of the entry and exit decisions of firms into foreign markets for net export and real exchange rate dynamics. This paper revisits this idea, extending the analysis to a general equilibrium environment. Specifically, we embed a model of establishment export dynamics into an equilibrium open economy business cycle model. Individual firms face a large, up-front sunk cost of entering a foreign market and a smaller, period-by-period cost of continuing in the foreign market. In the presence of idiosyncratic technology shocks, non-exporting firms start exporting only when the expected value of exporting covers the entry costs. Exporters continue to export as long as the value of doing so exceeds the continuation cost. Owing to heterogeneity in productivity, the value of entering the foreign market varies across nonexporters and the value of continuing in the foreign market varies across exporters. These values change over time so that the model generates a time-varying distribution of exporters and nonexporters.

The model is consistent with several empirical regularities documented in recent studies. First, most firms do not export. For example, among the U.S. manufacturing plants in the 1992 Census of Manufactures, Bernard, Eaton, Jensen, and Kortum (2003) report that only 21 percent of the plants actually export. Second, export decisions are quite persistent. For instance, in a balanced panel of manufacturing plants in the Annual Survey of Manufactures from the Longitudinal Research Database (LRD), over the period from 1984 to 1992, on average each year 87.4 percent of the exporters continued exporting in the following year and 86.1 percent of nonexporters did not export in the following year. Finally, exporters tend to be both bigger in terms of shipments, employees and capital, and more productive than nonexporters. Using the same data, Bernard and Jensen (1999a) find that U.S. exporters are 12 percent to 18 percent more productive, employ 77 percent to 95 percent more workers, use 13 percent to 20 percent more capital per worker, and produce 104 percent to 115 percent more output than nonexporters.[1]

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[1] The LRD is not a representative sample of manufacturing firms but is biased toward larger firms. Consequently,

A general theme running through the recent literature is that these export decisions have important implications for the dynamics of net exports. In particular, in a series of papers Baldwin (1988), Baldwin and Krugman (1989), and Dixit (1989a,b) develop partial equilibrium models of export decisions with sunk costs. They show that following a depreciation of the domestic real exchange rate, the sunk costs aspect of the export decisions lead foreign firms to continue serving the domestic market even though their goods may have become relatively more expensive. This idea, termed *exporter hysteresis*, is argued to have contributed to the dynamics of the U.S. net exports and real exchange rate in the mid to late 1980s. During this period, net exports declined as the real exchange rate depreciated and only started to increase with a lag of about two years. More generally, beyond this episode, these sunk costs are thought to contribute to the slow response of net exports to changes in the real exchange rate.

Our model of establishment dynamics contains the main feature leading to exporter hysteresis, sunk costs of exporting. The results contrast those in the previous literature. In particular, when business cycles are assumed to originate from exogenous shocks to aggregate productivity in each country, we find that export participation decisions do not noticeably alter the dynamics of the real exchange rate or net exports. Their properties are strikingly similar to those of the standard international business cycle model in Backus, Kehoe, and Kydland (1994). In that model, a positive innovation to productivity in one country leads its real exchange rate to depreciate and net exports to decline. Net exports then move into surplus with a delay of 5 quarters. These dynamics are governed by the familiar capital accumulation motive during an economic expansion in the standard model. We find that introducing export decisions does not noticeably increase this delay.

Our results are robust across a wide range of parameterizations that generate reasonable co-movements in economic activity across countries. The model generates business cycles close to the data only when goods from different countries are relatively poor substitutes compared to goods

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this database tends to understate the differences between exporters and non-exporters compared to the Census of Manufactures.

from the same country. When goods from the same country are fairly close substitutes, the total quantity imported depends very little on the number of different goods available. Demand for imports can be satisfied by purchasing many goods from a few different suppliers or a few goods from many different suppliers. Thus, the number of exporters, and hence foreign products, generally does not matter for aggregate trade dynamics for most reasonable parameterizations. That we find small aggregate implications of non-convexities in exporting is similar to the findings of Thomas (2002) and Veracierto (2003) regarding non-convexities in investment for aggregate dynamics.

We do find that the business cycle affects when firms start and stop exporting. In particular, exporters that would have stopped exporting in normal times delay doing so in an expansion. Similarly, an economic expansion will attract new exporters that in normal times would not have entered that market. However, because most firms are far from being at the margin of being indifferent to participating in foreign markets, the stock of exporters does not change much over the business cycle. Similarly, we find that an economic expansion at home leads to a slow and sustained expansion in the number of home firms that export. We show that these predictions of are consistent with evidence for U.S. firms.

The paper is organized as follows. The next section briefly reviews previous research related to the export decisions of firms and international business cycles. Section 3 develops a two-country dynamic general equilibrium model with export penetration and continuation costs. Section 4 discusses the quantitative implications of the model. Section 5 explores the sensitivity of the model to the costs of continuing to export, the characteristics of exporting firms, the substitutability between goods from the same and different countries, and the taste for variety. Section 6 presents new data on the timing of U.S. export decision over the business cycle and compares these moments to those generated by the model. Section 7 concludes.

2. Related Research

Researchers have developed dynamic partial equilibrium models of the discrete choice to export. The earliest models considered the export and pricing decisions of firms facing fixed costs of entering and continuing in foreign markets. (Examples of models of the export decisions with sunk costs include Baldwin (1988), Baldwin and Krugman (1989) and Dixit (1989a,b).) These models abstracted from most heterogeneity across firms. Instead, they focused on the export participation as well as industry trade and pricing dynamics in response to a largely exogenous process for exchange rates. As partial equilibrium models these papers are silent on aggregate trade and price dynamics

Recent, more empirically oriented work, has extended the original models of the export decision to allow for heterogeneity in the abilities and opportunities of production units. (Examples of models include Roberts and Tybout (1997), Aw, Chung and Roberts (1998), Clerides, Lach and Tybout (1998), Bernard and Jensen (1999a) and Das, Roberts and Tybout (2001)). These papers use the models to estimate the size of the sunk export costs and smaller continuation costs.[2] Using annual firm-level data on Colombian chemical producers from 1982 to 1991, Das, Roberts and Tybout (2001) estimate that export penetration costs account for between 18.4 percent and 41.2 percent of the annual value of a firm's exports. In 1999 U.S. dollars, these costs are estimated to be between $730,000 and $1.6 million, depending on plant size. They estimate continuation costs to be considerably smaller, on the order of 1 percent of the annual value of exports.

The dynamics of the trade balance and the terms of trade have been studied by Backus, Kehoe, and Kydland (1994), henceforth BKK, in an equilibrium international business cycle model. They find that a model with countries specialized in imperfectly substitutable goods and subject to exogenous aggregate productivity can generate the key features of the trade balance, namely, countercyclical net exports and a negative contemporaneous correlation with the terms of trade. A key prediction of this model is that the cross-correlation of the real exchange rate with the subsequent

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[2]These papers also focus on the extent to which firms become more productive by exporting. The evidence of this learning is less conclusive,and hence, we abstract from this channel.

net exports becomes positive within one quarter. This is counter to the common idea that there is a J-curve (Magee 1973), which suggests long and variable lags. BKK find that lags in shipment and capital accumulation can improve the fit of the model slightly. They also hypothesize that the type of fixed costs of exporting considered here could be important for generating greater delays. The frictions that give rise to export decisions have not been studied in an international business cycles framework.

The focus here on international trade costs is related to a number of papers that have focused on different economic questions. First, with respect to features of business cycles, Stockman and Tesar (1995) and Betts and Kehoe (2002) consider the effect of heterogeneity in trade costs across different goods.[3] Obstfeld and Rogoff (2000) consider trade costs that lead some goods to be traded only in some periods.[4] Second, the export decisions of firms introduces an extensive margin[5] to trade as the number of products available changes over time. Papers by Evenett and Venables (2002), Hummels and Klenow (2002), Kehoe and Ruhl (2002) and Ruhl (2003) study the growth in trade through the intensive and extensive margins. In our model, we find that the properties of the model are most sensitive to how consumers value additional varieties of foreign goods. It is this margin that generates the largest departures from the standard model of BKK. Finally, recent work by Bernard, Eaton, Jensen, and Kortum (2003) and Melitz (2003) also consider the role of firm heterogeneity in an international context. These papers focus on the pattern of trade and welfare gains from trade liberalization and do not consider aggregate fluctuations.

3. Model

We develop a two-country model with infinitely lived consumers and heterogeneous firms to study the international transmission of business cycles. The production side of the model is de-

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[3] Another approach has focused on frictions in international asset markets (see Baxter and Crucini (1995), Heathcoate and Perri (2002) and Kehoe and Perri (2002)).

[4] Ghironi and Meltiz (2004) develop a model of fixed trade costs to primarily study real exchange rate dynamics.

[5] Head (2002) and Cook (2002) study international business cycle models in which the number of firms varies over time due to fixed costs of entry. These models do not consider the export decisions of firms so that the available set of goods is the same across countries.

veloped to be consistent with certain characteristics and dynamics of exporters described in the previous section. This requires taking a stand on what determines a firm. We associate a firm with a unique variety of a differentiated good with a production process that is subject to idiosyncratic technology shocks.

There are two countries, *home* and *foreign*. Each country is populated by a large number of identical, infinitely lived consumers. In each period of time, the economy experiences an event s_t. Let $s^t = (s_0, \cdots, s_t)$ denote the history of events from period 0 up to and including period t. The probability of a history s^t, conditional on the information available at period 0, is defined as $\pi(s^t|s^0)$. The initial realization of an event at period 0, s_0, is given.

In each country there is a large number of monopolistically competitive firms each producing a differentiated intermediate good. The many intermediate good producers are normalized to a continuum with unit mass and are indexed $i \in [0, 1]$. An intermediate good producer uses capital and labor inputs to produce its variety of intermediate input. Firms differ in terms of total factor productivity, capital and the markets they serve. All firms sell their product in their own country but only some firms export their good abroad. When an intermediate good producer exports goods abroad, the producer incurs some international trading cost. The size of the cost depends on the producer's export status in the previous period. There is a (relatively) high up-front sunk cost τ_0 that must be borne to gain entry into the export market. In subsequent periods, to continue exporting, firms incur a lower but nonzero period-by-period fixed continuation cost τ_1. If a firm does not pay this continuation cost, then it ceases to export. In future periods, the firm can only begin exporting by incurring the entry cost τ_0 again. These costs are valued in units of labor in the destination market.

In each country, competitive final goods producers purchase intermediate inputs from those firms actively selling in that country.[6] The cost of exporting implies that the set of goods available

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[6] Final good production technology does not require capital or labor inputs. The final good production technology regulates a country's preferences over local and imported varieties.

to competitive final goods producers differs across countries. The entry and exit of exporting firms implies that the set of intermediate goods available in a country is changing over time. The final goods are used for both domestic consumption and investment.

We assume that there are no economies of scale to exporting. In particular, it is not possible for a single firm to incur the fixed cost of exporting and then export multiple different varieties of intermediate goods. We take the view that τ_0 is a per variety cost of starting to export. In practice, these fixed costs represent those costs associated with tailoring a product to the standards and taste of foreign consumers, establishing marketing and distribution networks, and learning about bureaucratic and administrative details in these new markets. For diverse goods, it is unlikely that exporting one good reduces the fixed costs of exporting a second good.

In this economy, there exists a complete set of one-period state-contingent nominal bonds denominated in the home currency. Let $B\left(s^{t+1}, s^t\right)$ denote the home consumer's holding of a bond purchased in state s^t with payoff in state s^{t+1}. Let $B^*\left(s^{t+1}, s^t\right)$ denote the foreign consumer's holding of this bond. The state-contingent bond $B(s^t)$ pays 1 unit of home currency if s^t occurs, and 0 otherwise. Let $Q(s^{t+1}|s^t)$ denote the nominal price of the state-contingent bond $B(s^{t+1})$ given s^t. All the intermediate and final good producers are owned by domestic consumers. It is assumed that these ownership claims cannot be traded.

A. Consumer's Problem

Home consumers choose consumption, labor and bond holdings to maximize their utility:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t|s_0\right) U\left[C\left(s^t\right), L\left(s^t\right)\right],$$

subject to the sequence of budget constraints,

$$P(s^t)C(s^t) + \sum_{s^{t+1}} Q(s^{t+1}|s^t)B(s^{t+1}) \leq P(s^t)W(s^t)L(s^t) + B(s^t) + \Pi(s^t) + P(s^t)T(s^t),$$

where $C(s^t)$ and $L\left(s^t\right)$ are the final good consumption and labor, respectively; $P(s^t)$ and $W\left(s^t\right)$ denote the price level and wage rate; $\Pi(s^t)$ is the sum of profits of the home country's intermediate good producers; and $T(s^t)$ is a lump sum transfer from the government. The discount factor is β.

The problem of foreign consumers is analogous to this problem. Prices and allocations in the foreign country are represented with an asterisk. To be clear, money has no role in this economy. However, the local currency is used as a unit of account so that the foreign budget constraint is expressed as

$$P^*(s^t)C^*(s^t) + \sum_{s^{t+1}} \frac{Q(s^{t+1}|s^t)}{e(s^t)} B^*(s^{t+1}) \leq P^*(s^t)W^*(s^t)L^*(s^t) + \frac{B^*(s^t)}{e(s^t)} + \Pi^*(s^t) + P^*(s^t)T^*(s^t),$$

where * denotes the foreign variables and $e(s^t)$ is the nominal exchange rate.

The first order conditions for home consumers' utility maximization problems are

$$-\frac{U_L(s^t)}{U_C(s^t)} = W(s^t),$$

$$(1) \quad Q(s^{t+1}|s^t) = \beta\pi(s^{t+1}|s^t)\frac{U_C(s^{t+1})}{U_C(s^t)}\frac{P(s^t)}{P(s^{t+1})},$$

where $U_i(s^t)$ denotes the derivatives of the utility function with respect to its arguments. The price of the state-contingent bond is standard. With arbitrage, the complete asset markets assumption implies that the real exchange rate, $q(s^t)$, is proportional to the ratio of marginal utility of consumption across countries

$$(2) \quad q(s^t) \equiv \frac{e(s^t)P^*(s^t)}{P(s^t)} = \kappa\frac{U_C^*(s^t)}{U_C(s^t)},$$

where $\kappa = q(s^0)U_C(s^0)/U_C^*(s^0)$.[7]

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[7] In the simulation exercises, κ is normalized to be 1.

B. Final Good Producers

In the home country, final goods are produced using only home and foreign intermediate goods. A final good producer can purchase from any of the home intermediate good producers but can purchase only from those foreign intermediate good producers that are actively selling in the home market. The set of foreign firms actively selling in the home country is denoted by $\mathcal{E}^*(s^t)$, where $i \in \mathcal{E}^*(s^t)$ if the i_{th} firm is a foreign exporter in s^t.

The production technology of the firm is given by a constant elasticity of substitution (henceforth CES) function

$$(3) \quad D(s^t) = \left\{ a_1 \left[\int_0^1 y_h^d(i, s^t)^\theta di \right]^{\frac{\rho}{\theta}} + (1 - a_1) \left[N^* \left(s^t \right)^{-\lambda} \int_0^1 y_f^d(i, s^t)^\theta di \right]^{\frac{\rho}{\theta}} \right\}^{\frac{1}{\rho}},$$

where $D(s^t)$ is the output of final goods and $y_h^d(i, s^t)$ and $y_f^d(i, s^t)$ are inputs of intermediate goods purchased from home firm i and foreign firm i, respectively. The parameter a_1 determines the weight of home goods in final good consumption. The elasticity of substitution between intermediate goods that are produced in the same country is $1/(1 - \theta)$, and the elasticity of substitution between home and foreign aggregate inputs is $1/(1 - \rho)$.

With the export margin of the model, the measure of foreign varieties used in production of the composite foreign good changes over time. With a typical Dixit-Stiglitz aggregator there is a benefit to using smaller amounts of a greater number of varieties. To counteract the increasing returns to scale from this love-of-variety effect, we modify the aggregator of the foreign composite by introducing the additional term $N^{*-\lambda}$. This term allows us to separate the love-of-variety effect from the degree of market power, which is related to elasticity of substitution between individual varieties (Benassy 1996). We explore the role of these effects in our sensitivity analysis.

The final goods market is competitive. In each period t, given the final good price at home $P(s^t)$, the i_{th} home intermediate good price at home $P_h(i, s^t)$ for $i \in [0, 1]$, and the i_{th} foreign intermediate good price at home $P_f(i, s^t)$ for $i \in \mathcal{E}^*(s^t)$, a home final good producer chooses inputs

$y_h^d(i, s^t)$ for $i \in [0, 1]$, and $y_f^d(i, s^t)$ for $i \in \mathcal{E}^*(s^t)$ to maximize profits,

$$(4) \quad \max P\left(s^t\right) D\left(s^t\right) - \int_0^1 P_h\left(i, s^t\right) y_h^d\left(i, s^t\right) di - \int_0^1 P_f\left(i, s^t\right) y_f^d\left(i, s^t\right) di,$$

subject to the production technology (3) and the constraint that $y_f^d\left(i, s^t\right) = 0$ for $i \notin \mathcal{E}^*(s^t)$. Solving the problem in (4) gives the input demand functions,

$$(5) \quad y_h^d(i, s^t) = a_1^{\frac{1}{1-\rho}} \left[\frac{P_h(i, s^t)}{P_h(s^t)}\right]^{\frac{1}{\theta-1}} \left[\frac{P_h(s^t)}{P(s^t)}\right]^{\frac{1}{\rho-1}} D(s^t),$$

$$(6) \quad y_f^d(i, s^t) = (1 - a_1)^{\frac{1}{1-\rho}} \left[N^*\left(s^t\right)^\lambda \frac{P_f(i, s^t)}{P_f(s^t)}\right]^{\frac{1}{\theta-1}} \left[\frac{P_f(s^t)}{P(s^t)}\right]^{\frac{1}{\rho-1}} D(s^t), i \in \mathcal{E}^*(s^t)$$

where $P_h(s^t) = \left[\int_0^1 P_h(i, s^t)^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}}$, and $P_f(s^t) = N^*\left(s^t\right)^{\frac{\lambda}{\theta}} \left[\int_{i \in \mathcal{E}^*(s^t)} P_f(i, s^t)^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}}$. The zero-profit condition in the perfectly competitive market determines the price level of the final good as

$$(7) \quad P(s^t) = \left[a_1^{\frac{1}{1-\rho}} P_h(s^t)^{\frac{\rho}{\rho-1}} + (1 - a_1)^{\frac{1}{1-\rho}} P_f(s^t)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}.$$

C. International Trading Costs

An intermediate good producer can sell its product without frictions in its domestic market. However, it is costly to sell its product abroad. Producers that export to the foreign country face two sets of international trading costs. To enter the foreign market, an intermediate good producer has to pay a (relatively) high initial entry cost τ_0. This permits the firm to export in the current period. From the following period on, to continue exporting, the producer has to pay a lower but nonzero continuation costs τ_1 ($< \tau_0$). The export penetration cost (τ_0) and continuation cost (τ_1) are collected from foreign exporting firms by the domestic government, and distributed lump-sum to the domestic consumers. The government's budget constraint is given by

$$(8) \quad T(s^t) = \int_{i \in \mathcal{E}^*(s^t)} W\left(s^t\right) \left\{[1 - m^*(i, s^{t-1})]\tau_0 + m^*(i, s^{t-1})\tau_1\right\} di,$$

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where $m^*(i, s^t)$ is an indicator function denoting the export status of the i_{th} intermediate good producer in s^t. Let $m^*(i, s^t) = 1$ if the i_{th} foreign intermediate good producer is an exporter in s^t, 0 otherwise.[8] From (8), we see that the trade cost depends on the exporter status from the previous period. These trade costs imply that only a fraction

$$N^*(s^t) = \int_0^1 m^*(i, s^t) di$$

of foreign intermediate goods are available to home final good producers in state s^t.

D. Intermediate Goods Producers

In each country there is a large number of intermediate good producers normalized to a continuum with unit mass indexed $i \in [0, 1]$ who behave as monopolistic competitors. An intermediate good firm produces its differentiated good with a Cobb-Douglas production technology,

$$(9) \quad F(i, s^t) = A(i, s^t) k(i, s^{t-1})^\alpha l(i, s^t)^{1-\alpha} = y_h^d(i, s^t) + y_h^{*d}(i, s^t),$$

where $y_h(i, s^t)$ and $y_h^*(i, s^t)$ are the amounts of good i sold in the home and foreign intermediate goods markets, respectively, and $k(i, s^{t-1})$ and $l(i, s^t)$ are the capital and labor inputs of the firm i. Capital used in production is augmented by investment of final goods, $x(i, s^t)$. The law of motion for capital is given by

$$(10) \quad k(i, s^t) = (1 - \delta) k(i, s^{t-1}) + x(i, s^t),$$

where δ is the depreciation rate.

The term $A(i, s^t)$ denotes the productivity of the i_{th} firm and is composed of a country-wide

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[8] In reality, most of the costs are paid to agents that help exporting firms, not to the foreign government. However, no matter to whom the costs are paid initially, the payment goes to consumers ultimately. Additionally, in practice some of these costs are also paid to domestic agents. The results of the simulation exercises are not sensitive to the division of these costs across countries. To make matters simple, it is assumed that the costs are paid to the foreign government.

component $z(s^t)$, and a firm-specific component $\eta(i, s^t)$ such that

$$\ln A(i, s^t) = z(s^t) + \eta(i, s^t).$$

The country-wide component $z(s^t)$ may be correlated across countries and evolves according to a vector autoregressive process (VAR) with the foreign country-wide productivity, $z^*(s^t)$,

$$Z(s^t) = MZ(s^{t-1}) + \nu(s^t), \nu(s^t) \overset{iid}{\sim} N(0, \Omega),$$

where M is a coefficient matrix; $Z(s^t) = [z(s^t), z^*(s^t)]'$ and $\nu(s^t) = [\epsilon(s^t), \epsilon^*(s^t)]'$. The firm-specific productivity is independently, identically distributed across countries, firms, and time, $\eta(i, s^t) \overset{iid}{\sim} N(0, \sigma_\eta^2)$.

Consider the problem of an intermediate good producer from the home country in state s^t. The individual state of a firm is summarized by the triple (η, k, m), where we temporarily drop the firm index and aggregate state. The intermediate good producer chooses current prices P_h, P_h^*, inputs of labor l, investment x and the export decision m' to solve

$$V\left(\eta, k, m, s^t\right) = \max \Pi(h, i; s^t) + m'\Pi^*(h, i; s^t) + \sum_{s^{t+1}} \sum_{\eta'} Q\left(s^{t+1}|s^t\right) \Pr\left(\eta'\right) V\left(\eta', k', m', s^{t+1}\right),$$

$$\Pi(h, i; s^t) = P_h(i, s^t)y_h(i, s^t) - P(s^t)W(s^t)l(i, s^t) - P(s^t)x(i, s^t)$$

$$\Pi^*(h, i, s^t) = e(s^t)\left[P_h^*(i, s^t)y_h^*(i, s^t) - P^*(s^t)[m\tau_1 + (1-m)\tau_0]\right],$$

subject to the production technology (9), the law of motion for capital (10), and the constraints that supplies to home and foreign intermediate goods market $y_h(i, s^t)$ and $y_h^*(i, s^t)$ are equal to demands by home and foreign final good producers $y_h^d(i, s^t)$ and $y_h^{d*}(i, s^t)$ from (5) and its foreign analogue. Here, $\Pr\left(\eta'\right)$ denotes the probability of an idiosyncratic shock η'.

Let the value of the i_{th} producer if it exports in s^t be

$$V^1\left(\eta, k, m, s^t\right) = \max \Pi_h(i; s^t) + \Pi_h^*(i; s^t) + \sum_{s^{t+1}} \sum_{\eta'} Q\left(s^{t+1} | s^t\right) \Pr\left(\eta'\right) V\left(\eta', k', 1, s^{t+1}\right),$$

$$\Pi_h(i; s^t) = P_h(i, s^t) y_h(i, s^t) - P(s^t) w(s^t) l(i, s^t) - P(s^t) x(i, s^t)$$

$$\Pi_h^*(i, s^t) = e(s^t) P_h^*(i, s^t) y_h^*(i, s^t) - e(s^t) P^*(s^t) [m\tau_1 + (1-m)\tau_0],$$

and let the value of the i_{th} producer if it does not export in s^t be

$$V^0\left(\eta, k, m, s^t\right) = \max \Pi_h(i; s^t) + \sum_{s^{t+1}} \sum_{\eta'} Q\left(s^{t+1} | s^t\right) \Pr\left(\eta'\right) V\left(\eta', k', 0, s^{t+1}\right),$$

$$\Pi_h(i; s^t) = P_h(i, s^t) y_h(i, s^t) - P(s^t) w(s^t) l(i, s^t) - P(s^t) x(i, s^t)$$

Then, the actual value of i_{th} producer can be defined as

$$V\left(\eta, k, m, s^t\right) = \max\left\{V^1\left(\eta, k, m, s^t\right), V^0\left(\eta, k, m, s^t\right)\right\}.$$

Clearly the value of a producer depends on its export status and is monotonically increasing and continuous in η. Moreover V^1 intersects V^0 from below only once.[9] Hence, it is possible to solve for the firm-specific productivity at which a firm is indifferent between exporting or not exporting. This level of technology differs by the firms current export status. The critical level of technology for exporters and nonexporters, η_1 and η_0, satisfy

$$(11)\ V^1\left(\eta_1, k, 1, s^t\right) = V^0\left(\eta_1, k, 1, s^t\right),$$

$$(12)\ V^1\left(\eta_0, k, 0, s^t\right) = V^0\left(\eta_0, k, 0, s^t\right),$$

In general these critical technology levels will differ across firms based on their capital level.

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[9]If the difference between τ_0 and τ_1 is very large, $V^1 > V^0$ for all $\eta \in (-\infty, \infty)$ for some s^t. Since the data show that some of the previous exporters exit from foreign markets each period, it is assumed throughout that the shocks are small enough that this does not occur.

However, the assumption that firm-specific technology shocks are iid implies that each firm expects to draw the same level of technology tomorrow. Consequently, a firm's current capital stock is entirely determined by its export status in the previous period. As export status is a zero-one choice, the distribution of capital over firms is characterized by two mass points. This then implies that the critical technology level of an exporting firm also determines the technology of the marginal exporting firm, which we denote by $\eta_1\left(s^t\right)$. Among last period exporters, only those with a firm-specific productivity greater than $\eta_1(s^t)$ will continue to export in state s^t. Likewise, the critical technology of a nonexporter is denoted by $\eta_0\left(s^t\right)$.

From (11), (12), and the independence of the firm-specific productivity, the percentage of exporters in s^t among exporters and nonexporters in s^{t-1}, $n_1(s^t)$ and $n_0(s^t)$, respectively, can be defined as

$$n_1(s^t) = \Pr[\eta > \eta_1(s^t)],$$

$$n_0(s^t) = \Pr[\eta > \eta_0(s^t)].$$

Then, the law of motion for the export ratio among intermediate good producers, $N(s^t)$, is

(13) $N(s^t) = n_1(s^t)N(s^{t-1}) + n_0(s^t)[1 - N(s^{t-1})].$

Figure 1 illustrates the values of firms across firm-specific productivity depending on export status. In the absence of trade costs, the value of a firm that exports always exceeds the value of not exporting for all firm-specific productivity. This is true because, by exporting, the firm has a larger market for its goods. Without the fixed costs, all firms would export their good abroad. However, in the presence of international trade costs, it is not optimal for some firms to export goods abroad. The value of an exporting firm is reduced by the amount of the trade costs, τ_0 or τ_1 depending on the export status last period. Since the cost of being a new exporter exceeds the cost of continuing to export, $\tau_0 > \tau_1$, the value of being a new exporter is always lower than the value

of being a continuing exporter. This implies that $\eta_1(s^t) < \eta_0(s^t)$ for all s^t. Hence, the probability of being an exporter in s^t is always higher for last period exporters than last period nonexporters $(n_1(s^t) > n_0(s^t))$ and there is exporter hysteresis.

E. Equilibrium Definition

In an equilibrium, variables satisfy several resource constraints. The final goods market clearing conditions are given by $c(s^t) + \int x(i, s^t)di = D(s^t)$, and $c^*(s^t) + \int x^*(i, s^t)di = D^*(s^t)$. The intermediate goods market clearing conditions are $y_h^d(i, s^t) = y_h(i, s^t)$ for $i \in [0, 1]$, $y_f^d(i, s^t) = y_f(i, s^t)$ for $i \in \mathcal{E}^*(s^t)$, $y_f^{d*}(i, s^t) = y_f^*(i, s^t)$ for $i \in [0, 1]$, and $y_h^{d*}(i, s^t) = y_h^*(i, s^t)$ for $i \in \mathcal{E}(s^t)$. The labor market clearing conditions are $L(s^t) = \int_0^1 l(i, s^t)di$, and $L^*(s^t) = \int_0^1 l^*(i, s^t)di$. The profits of firms are distributed to the shareholders, $\Pi(s^t) = \int_0^1 \left[\Pi_h(i, s^t) + \Pi_h^*(i, s^t) \right] di$, and $\Pi^*(s^t) = \int_0^1 \left[\Pi_f(i, s^t) + \Pi_f^*(i, s^t) \right] di$. The government budget constraint is given by (8) and the foreign analogue. The international bond market clearing condition is given by $B(s^t) + B^*(s^t) = 0$. Finally, our decision to write the budget constraints in each country in units of the local currency permits us to normalize the price of consumption in each country as $P(s^t) = P^*(s^t) = 1$.

An equilibrium of the economy is a collection of allocations for home consumers $C(s^t)$, $L(s^t)$, $B(s^{t+1})$; allocations for foreign consumers $C^*(s^t)$, $L^*(s^t)$, $B^*(s^{t+1})$; allocations for home final goods producers $D(s^t)$, $y^d(h, i, s^t)$ for $i \in [0, 1]$, and $y^d(f, i, s^t)$ for $i \in \mathcal{E}^*(s^t)$; allocations for foreign final good producers $D^*(s^t)$, $y_f^{d*}(i, s^t)$ for $i \in [0, 1]$, and $y_h^{d*}(i, s^t)$ for $i \in \mathcal{E}(s^t)$; allocations and prices for home intermediate good producers $l(i, s^t)$, $x(i, s^t)$, $y_h(i, s^t)$, and $P_h(i, s^t)$ for $i \in [0, 1]$, $y_h^*(i, s^t)$ and $P_h^*(i, s^t)$ for $i \in \mathcal{E}(s^t)$; allocations and prices for foreign intermediate good producers $l^*(i, s^t)$, $x^*(i, s^t)$, $y_f(i, s^t)$ and $P_f(i, s^t)$ for $i \in \mathcal{E}^*(s^t)$, $y_f^*(i, s^t)$ and $P_f^*(i, s^t)$ for $i \in [0, 1]$; the export statuses of home and foreign intermediate good producers $m(i, s^t)$ and $m^*(i, s^t)$ for $i \in [0, 1]$; transfers $T\left(s^t\right), T^*\left(s^t\right)$ by home and foreign governments; real wages $W(s^t), W^*(s^t)$, real and nominal exchange rates $q\left(s^t\right)$ and $e\left(s^t\right)$; and bond prices $Q(s^{t+1}|s^t)$ that satisfy the following conditions: (i) the consumer allocations solve the consumer's problem; (ii) the final good producers' allocations

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solve their profit maximization problems; (iii) the intermediate good producers' allocations, prices, and export statuses solve their profit maximization problems; (iv) the market clearing conditions hold; and (v) the transfers satisfy the government budget constraint.

We focus on a stationary equilibrium. A stationary equilibrium consists of stationary decision rules and pricing rules that are functions of the state of the economy. The state of the economy is completely described by the distribution of the state variables (η, k, m) for all individual firms in both countries and the aggregate technology shocks. The state of the economy records the joint distribution of the capital stock, technology, and export status of firms in both countries. In general, keeping track of this distribution over time is computationally difficult. However, the assumption that firm-specific technology shocks are iid greatly simplifies the analysis, since it implies that last period's export status is sufficient to determine a firm's current capital stock. As firms are either exporters or nonexporters, at any point in time firms will have either a relatively low capital stock if they did not export yesterday or a relatively high capital stock if they did export yesterday. Consequently, the distribution of the capital stock in the economy is completely summarized by the aggregate shocks, Z and Z^*, the capital stock of exporters, K_1 and K_1^*, the capital stock of nonexporters, K_0 and K_0^*, and the share of exporters in each country, N and N^*.

F. Calibration

We now describe the functional forms and parameter values considered for our benchmark economy. The parameter values used in the simulation exercises are reported in Table 1. The instantaneous utility function is given as

$$U(C, L) = \frac{[C^\gamma (1 - L)^{1-\gamma}]^{1-\sigma}}{1 - \sigma},$$

where $1/\sigma$ is the intertemporal elasticity of substitution, and γ is the share parameter for consumption in the composite commodity.

In the steady state, the real interest rate is equal to $(1-\beta)/\beta$. The annual real return to capital

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is around 4 percent. This gives $\beta = 0.99$. The steady state constraint gives $Y = C + \delta K$. Dividing both sides by K, $\delta = \frac{Y}{K}\left(1 - \frac{C}{Y}\right)$. With the annual capital output ratio of 2.5 and consumption to output ratio of 0.75 as the average of the postwar U.S. data, $\delta = 0.025$. The curvature parameter, σ, determines the intertemporal elasticity of substitution and the relative risk aversion of consumers. We consider a value of $\sigma = 2$ as this is widely used in the international business cycle literature, e.g., Backus et al. (1994), Stockman and Tesar (1995), and Kehoe and Perri (2002).

The parameter θ determines an intermediate good producer's markup. Schmitt-Grohe (1997) summarizes the results of empirical studies estimating this markup. These estimates vary widely from 3 percent to 70 percent. Based on Basu and Fernald (1994), θ is set to be equal to 0.9 and yields an intermediate good producer's markup of about 11 percent. The parameter ρ determines the elasticity of substitution between home and foreign aggregates, $1/(1 - \rho)$. There is considerable disagreement over an appropriate value. Using the U.S. quarterly data of 163 industries at the 3-digit SIC level from 1980:1 to 1988:4, Gallaway et al. (2000) estimate that the elasticities range from 0.14 to 3.49. In the simulation exercises, ρ is set to $1/3$ so that the elasticity equals to 1.5 as in Backus et al. (1994) and Chari et al. (2001).

The parameter λ determines the love-of-variety. To our knowledge, there are no empirical estimates of this parameter. Consequently, we follow the literature, which implicitly assumes that the love-of-variety is tied to the elasticity of substitution across varieties, and set $\lambda = 0$. In this case, consumers have a preference for spreading consumption across more varieties. We examine three other cases for $\lambda \in \{\theta - 1, 1 - \theta, 1\}$. When $\lambda = \theta - 1$, we double the love-of-variety effect. When $\lambda = 1 - \theta$, we eliminate the love-of-variety effect so that consumers are indifferent between consuming n units of a single good or 1 unit of n identical goods. When $\lambda = 1$, consumers dislike variety and would rather concentrate all of their consumption in a single variety.[10]

In the model, we assume that profit income is attributed proportionally to labor and capital.

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[10]In this case, we assume that the dislike of variety is external to the consumer so that consumers will consume some of each variety but would prefer a world in which there were fewer choices.

We choose capital's share of income from postwar U.S. data to be $\alpha = 0.36$. The share parameter for consumption in the composite commodity, γ, is set to be equal to 0.294. This value is obtained from the observation that the average time devoted to work is 1/4 of the total available time, and the consumption-output ratio is about 0.75 in the postwar period.

We follow Kehoe and Perri (2002) in choosing the country specific productivity process. The model is simulated for 1000 times with 120 periods using the linearization methods suggested by King, et al. (1988a,b), and Klein (2000). The equations of the model are summarized in the appendix.

Exporter Characteristics and Hysteresis

The parameters τ_0, τ_1, a_1 and σ_η jointly determine the amount of trade, characteristics of exporters and nonexporters, and the dynamics of export status.[11] To pin these parameters down, we consider the following evidence. First, using annual data on U.S. firms in the LRD from 1984 to 1992, Bernard and Jensen (1999a) find that about 87.4 percent of exporters continue exporting in the next period, and among those that did not export last period, about 86.1 percent of firms remain in the nonexporter status. Consequently, we set $n_1 = n_0 = 3.5$ percent to match an average of the quarterly starter and stopper ratios. Second, Bernard and Jensen (1999a) find that exporters are 12 percent to 18 percent more productive than nonexporters. Finally, we note that for the U.S., the import to output ratio is approximately 15 percent. Choosing these parameters jointly to match these statistics yields values of $\tau_0 = 0.24897, \tau_1 = 0.05043, a_2 = 0.321$ and $\sigma_\eta = 0.5$.[12] The choice of $\sigma_\eta = 0.5$ is made as it leads exporters to be 15.5 percent more productive and to ship

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[11] An alternate approach to calibrate the firm shocks is to use previous estimate for the firm-specific productivity process. However, these studies tend to rely heavily on the sample of firms. With very small firms in the sample, the variance becomes very large. With only large firms, such as firms that can be found in S&P 500, the size of the variance becomes very small. Bernard, et al. (2003) estimate the distribution across plants of value added per workers using ASM 1992. They find that the sample standard deviation of the productivity across firms is about 0.76. However, their estimate differs due to the differences in production functions and the processes of technology shocks. For the robustness of the simulation results, various values of the standard deviation for the firm-specific productivity are considered.

[12] Under the zero export penetration costs, $\tau_0 = \tau_1 = 0$, a_2 set to be equal to 0.315 to match the exports to output ratio of 0.15.

90.2 percent more output (and hire 90.2 percent more workers). The characteristics of exporters in terms of employment, and output matches up well with the data as exporters produce 104 to 115 percent more output than nonexporters and hire 77 percent to 95 percent more workers. With these parameter values, on average, a nonexporter expects to pay about 16.5 percent of sales as entry costs, while an exporter expects to pay about 1.7 percent of sales to remain in the foreign market in the steady state. In total, these international trading costs represent 1.3 percent of GDP, or about 8 percent of exports.

Figure 2 shows how exporter characteristics (in a log scale) vary with the probability of exiting. The likelihood of exiting negatively affects the relative capital stock of the typical exporter but positively affects its productivity, employment, and output advantages. At one extreme, when export participation is essentially iid, there is no exporter hysteresis and it is the most productive firms that export each period regardless of their previous export decision. With export participation and technology independent across periods, all firms choose the same capital stock. At the other extreme, when export decisions are almost permanent, exporters and nonexporters are essentially the same in terms of productivity, but because exporters have a larger market for their goods, and expect to maintain this presence in future periods, they hire more workers and maintain a larger capital stock. Thus exporter hysteresis appears important in matching the observed exporter premia in the data.

G. Measurement

Prior to evaluating the model, we consider some important measurement issues in comparing the model to the data. First, consider the notion of import prices. The number of varieties imported influences the ideal price index but are not included in the price indices of statistical agencies (see Feenstra 1994). For consistency then, we measure the price of imports as the weighted average price

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without the scale effect.

$$P_{IM,t} = \left[\int_{i \in \mathcal{E}_t^*} \frac{1}{N_t^*} p_{Fit}^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}} = N_t^{*\frac{1-\theta}{\theta}} P_{F,t},$$

$$P_{EX,t} = e_t P_{IM,t}^*.$$

This also alters the way that our CPI is defined

$$P_{C,t} = \left[a_1^{\frac{1}{1-\rho}} P_{Ht}^{\frac{\rho}{\rho-1}} + a_2^{\frac{1}{1-\rho}} N_t^{*\frac{\rho(\theta-1)}{(\rho-1)\theta}} P_{IM,t}^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho}{\rho-1}}.$$

The number of foreign varieties is included in the CPI as changes in the extensive margin affect the relative weight on the composite imported and domestic goods. In the subsequent analysis, we note how these changes from the ideal prices affect the results.

4. Results

In this section, we consider the dynamic behavior of the export participation model versus the standard model of BKK. These models' distinct export participation decisions suggest that substantial differences should exist in the aggregate trade dynamics. In particular, the export decision model includes the key feature emphasized by previous authors as important for explaining net export dynamics: sunk costs of exporting. When shocks to productivity change the relative cost of producing foreign goods, foreign exporters may be slow to exit the home market and home exporters may be slow to enter the foreign market. These changes in export participation influence trade flows and thus net export and real exchange rate dynamics. In contrast, the standard model does not have this channel.

For comparison sake, we modify the standard model of BKK to include heterogenous, monopolistically competitive firms. We denote this as the *No Costs* model. We also compare our model of endogenous entry and exit to a third, *Fixed exit*, model in which the cost of continuing exporting is stochastic. Typically an exporter can continue exporting for free but periodically receives a shock

that requires repaying the start-up cost in order to export. In this model, the firms that stops exporting are nearly identical to those that continue exporting, so that even the least productive exporters continue exporting. We calibrate the model to match the trade share, entry and exit rates.

Figure 3 depicts the first 25 periods of each economy's response to a persistent aggregate productivity shock driven by a 1 standard deviation productivity shock. The figure reveals similar net export and real exchange rate dynamics across all three models.

The three models generate very similar real exchange rate behavior, although the model with exogenous exit has the smallest change in the real exchange rate and our model of endogenous exit has the largest change. In all three models, net exports decline on impact and then increase, moving into surplus with a lag. In the no cost model, net exports move into surplus in the eighth quarter following the shock, while with sunk costs of exporting, net exports go into surplus in the seventh quarter.

Next we consider the cross-correlation function of net exports and the real exchange rate for these three models. Figure 4 plots the correlation between q_t and nx_{t+k} with twelve quarters of leads and lags. The dynamics of the no cost model have been discussed extensively in BKK. Our model of sunk costs generates nearly identical dynamics. This is not surprising given the similarities of the impulse responses.

BKK show that lags in the time to trade or build capital can shift the cross-correlation function between net exports and the real exchange rate to the right. We find that fixed costs of trade do not have any noticeable impact on these comovements. In our model, exporters can begin exporting in the same period in which they incur the cost of entering the market. This suggests that focusing on the delays firms face in expanding their foreign sales may be what matters most for understanding the dynamics between the real exchange rate and net exports.

We now consider the properties of the model by examining the simulated model's moments. We report the Hodrick-Prescott filtered statistics for the data, the benchmark economy, and some variations on that economy in Table 2. We discuss the variations in the next section. The data are

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for the U.S. economy from 1975:1 through 2004:3. Since our focus is on trade dynamics between industrialized economies, we remove the effect of petroleum from our measures of net exports and relative prices. Net exports are measured as the nominal trade balance net of petroleum imports. The terms of trade is measured as the ratio of the price of non-oil imports to the price of exports.

Casual inspection of these tables indicates that there are a number of dimensions on which the standard model does not match certain features of international business cycles.[13] For the most part, the inclusion of sunk costs of exporting does not appear to noticeably alter model performance along these dimensions.

Table 2 reveals that the standard deviations of output, investment, employment, consumption, net exports, and the real exchange rate are essentially identical for the standard no cost model and the model with fixed costs of exporting.

The model with sunk costs differs slightly from the no cost model in two dimensions. First, the sunk cost model generates slightly less comovement in economic activity, since investment, output, and employment are less correlated across countries than in the no cost model. The difference is small though, less than 0.01 percentage points. Second, net exports are slightly more persistent with sunk costs of exporting. Again, this difference is minor, only 0.02 percentage points.

The discussion above raises two issues. First, given that exit is largely exogenous in the fixed exit model and endogenous in the sunk cost model, so that the exiting exporters exiting in the fixed exit model are on average much more productive than those exiting with endogenous exit, why are the aggregate dynamics so similar? Second, how can the presence of costs which lead firms to change their participation in export markets have so little impact on aggregate dynamics? To resolve these questions, we consider the differences across different parameterizations of our economies.

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[13] The model exhibits low volatility of relative prices, too much consumption risk sharing and not enough comovement in economic activity. These puzzles are discussed in Backus, Kehoe, and Kydland (1995).

5. Sensitivity

We consider some alternate specifications. All parameters for these specifications are described in Table 1.

Exporter Persistence

We begin by examining the sensitivity of our results to the amount of hysteresis in the economy, measured by the probability that a current exporter stops exporting in the following period. We consider two extremes. First, we consider the case in which there is a 0.5 percent probability of exiting so that exporters are almost permanently exporting. Next, we consider the case where there is a 50-50 chance of continuing to export in the following period. The results are reported in the columns *High Persistence* and *IID Exporters*, respectively. Surprisingly, the properties of these calibrations are nearly identical to our baseline case.

As we have already seen, the persistence of exporter status affects the exporter premium. That exporter characteristics do not affect the model's properties suggests that it doesn't matter whether there are a few productive exporters selling a lot or many productive exporters selling a little each. That these models perform so similarly suggests that the distribution of firms' characteristics does not matter[14] and explains why the fixed exit and endogenous exit models are nearly identical.

Taste for Variety

To further identify the source of the model's invariance to export decisions, we now explicitly consider how consumers value changes in the number of varieties available when exporters enter and exit. The parameter λ controls the taste for variety. We consider two cases. At one extreme, we double the love-of-variety in the standard model, $\lambda = \theta - 1$. At the other extreme, we consider the case where consumers strongly dislike variety and would like to concentrate consumption in a single good[15], $\lambda = 1$. The results are reported, respectively, in the columns *Love Variety* and *Hate*

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[14]This is a statement about exporter decisions and business cycles. This is not saying that these considerations are unimportant for welfare.

[15]We assume the taste for variety is external to the consumer so that consumers will choose some of each variety available.

Variety.

Changing the taste for variety primarily alters the international comovement of activity. In particular, we see that international risk sharing, measured by consumption correlations, is increasing in the love-of-variety, while business cycle synchronization, measured by comovements in economic activity, is decreasing in the love-of-variety.[16]

When consumers dislike variety, an expansion in the number of imported goods lowers the marginal utility of an additional imported good. This acts as both a negative shock to the marginal utility of consumption and a shift in taste toward locally produced goods. This implies that an expansion at home that leads more home firms to export will lead to an expansion in production in foreign and a much smaller expansion in consumption. When consumers love variety, these effects operate in reverse.

High Markups

We now consider the effect of making goods from the same country less substitutable. For the sake of comparison, we include the results for the no cost model. Making goods less substitutable has two effects. First, it raises the market power of individual producers. Second, it increases the love-of-variety. To identify the role of each channel we also consider the case in which there is no love-of-variety effect, reported in the column *CRS*.

The model with no love-of-variety channel is nearly identical to the no cost model, while the sunk cost model differs noticeably. Based on this, we conclude that understanding how variety is valued is critical to evaluating the role of export participation for both business cycle dynamics and welfare considerations.

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[16]When there is strong love or hate of variety effect, whether the fixed costs are paid in home or foreign goods or labor matters for international business cycles. Alessandria and Choi (2002) show that with fixed costs paid in units of the home final good, when consumers hate variety there is no consumption-output anomaly as comovements in output and consumption are approximately equal.

Elasticity of Substitution

Continuing with high markups, we now consider the effect of making home and foreign varieties equally substitutable ($\theta = 1/(1 - 1/\rho)$). As we've noted, there is a large range of Armington elasticities and $\rho = 3$ fits in this range. The results for the no cost model and the sunk cost model with no love-of-variety effect are reported in the last two columns. We find that the models differ in that the model with sunk costs generates net exports that are substantially more volatile (0.29 vs. 0.20) and generates less comovement between the real exchange rate and net exports (0.38 and 0.23). More important, this calibration generates very little business cycle synchronization so that international comovements are far from what we see in the data. This is a larger problem in the sunk cost model, since we find that economic activity is even less synchronized than in the standard model.

Our sensitivity analysis indicates that the reason the benchmark model does not differ much from the no cost model is that home and foreign composites are not very substitutable and goods from the same country are very close substitutes. With a low markup and very little love-of-variety, it essentially doesn't matter whether consumers have a lot of a few goods or a little of many goods.

6. Exporters and Business Cycles

We now study the export participation decisions of firms over the business cycle in the model and the data. First, we discuss the export participation decisions in the model and then we turn to the data. Figure **5** depicts the first 25 periods of each economy's response to a persistent aggregate productivity shock driven by a 1 standard deviation productivity shock in the home country.

The export sector in both countries expands on impact. The number of home exporters expands gradually and persistently, while the number of foreign exporters expands only for the first two periods and then begins contracting. The difference in the number of home and foreign exporters largely mirrors the dynamics of net exporters. The sudden and large expansion of the foreign export sector is driven by the large increase in investment at home, temporarily raising the demand for

foreign goods. The sustained increase in the home export sector is a result of the persistent cost advantage of home firms and the resulting net export surpluses.

We now compare the predictions of the model to the data. The ideal data for such an analysis are a panel of firm-level exports by destination markets for multiple source countries. To our knowledge, such data does not exist. We do have data on the total number of U.S. exporters by certain destination markets from 1995 to 2003, as reported in the Census Department's annual Profile of Exporting Firms. We focus on export participation by U.S. exporters to a limited set of OECD countries.

For each destination i we have data on the number of exporters, N_t^i, and the total value of exports, EX_t^i, deflated by the U.S. GDP deflator. For each country we collect data on real GDP, Y_t^i, and the bilateral real exchange rate with the U.S., q_t^i. We also include data on U.S. GDP.

The model is annualized to match the data. The parameters are reported in Table 1. All data are Hodrick-Prescott (1997) filtered with a smoothing parameter of 6.25 (as suggested by Ravn and Uhlig (2002)). Figure 6 plots the comovement of exporters with each of these four variables for the data and the model. For each variable, there is substantial heterogeneity in comovements across countries; however, the pattern of dynamics are consistent across countries so that we report the average comovement. In these plots, we also include the moments predicted by our model of endogenous entry and exit. The model fits the data surprisingly well. We describe each panel separately.

First, the data show that the number of exporters is highly correlated with the value of exports. There is almost no relationship between exports and exporters at leads and lags. In the model, we find a stronger contemporaneous correlation between exports and exporters than in the data. The model also predicts that exporters tend to lag trade flows. This is a feature of the sunk cost aspect of trade. In particular, if trade is high today, then exporters delay exiting so that the stock of exporters will remain high in the following period. In theory, with the fixed costs, high future trade flows will also lead to high current exporters. That we don't find much evidence of this may derive from the

persistent exogenous productivity shocks. The largest effect of these shocks is upon impulse, which is unanticipated.

The second feature of the data is that U.S. exporters tend to expand to foreign markets once the U.S. economy is already booming, so that exporters tend to lag domestic GDP. This relationship is predicted by the model. That domestic business cycles are strongly correlated with export decisions seems to result from the impulse being a productivity shock that persistently lowers the costs of producing goods for the export market.

The third feature of interest of the data concerns the dynamics of the real exchange rate and number of exporters. Here we find that the data and the model match up quite well. There is virtually no contemporaneous relationship between the real exchange rate and exporters. This can be understood in the following way. The real exchange rate is the relative price of the two baskets of goods. There is substantial home bias in these baskets so that the real exchange rate is essentially determined by the ratio of productivity across countries. On the other hand, from the impulse responses, we see that a positive productivity shock in one country leads to an expansion in the export sector in both countries. Given an increase in productivity that leads to an expansion of the export sector can occur in either country, there is no relationship between the real exchange rate and exporters.

There is some evidence, though, that following a depreciation of the real exchange rate, U.S. exporters tend to enter foreign markets. Moreover, we see that after U.S. exporters expand into a market the real exchange rate tends to appreciate. This dynamic pattern can be understood from the impulse responses. Following a positive productivity shock, the real exchange rate depreciates and the export sector expands. This expansion is slow and sustained. During this expansion, the real exchange rate begins to depreciate as it returns to steady state.

Finally, we consider the relationship between destination GDP and the number of U.S. exporters. The data show that U.S. exporters enter foreign markets in anticipation of a foreign expansion. In contrast, the model predicts that entry will lag the foreign expansion. One possibility

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for this discrepancy with the data is that some foreign expansions were triggered in part by predictable trade liberalizations. If this is the case, then exporters have an incentive to stay in foreign markets longer, even if current sales are fairly low. This feature of the model would also generate lower comovements with trade and the number of exporters.

7. Conclusions

It has long been argued that there are trading frictions in place that are important for understanding net export dynamics. Among these frictions, sunk costs of exporting, which lead firms to slowly exit foreign markets, are believed to have large consequences. In this paper, we embed a model of these costs in an equilibrium business cycle model. We find that the dynamics of net exports and the real exchange rate do not differ much from those in a model without these sunk costs. This is robust across many specifications for which the business cycle properties of the model are close to the data. We interpret this to imply that lags in expanding trade flows are potentially more important for net export dynamics than the costs of entering and continuing exporting.

We do find that export decisions can potentially alter trade dynamics, but for a new reason. Export decisions change the number of different varieties of goods available in a country. If consumers value or dislike variety, then export decisions have the potential to alter the international transmission of business cycles. There is little independent evidence of the taste for variety, but we find that it is the key source of any differences between the benchmark model and a model with export decisions. That we find comovements are closer to the data when consumers dislike variety suggests that the gains to variety may be small or even negative. If this is the case, then the welfare gains from increased variety may be overstated.

Our model of export decisions and the business cycle sheds some light on the timing of these export decisions. We find that U.S. exporters tend to expand into foreign markets when U.S. GDP is high and in anticipation of future foreign GDP growth. We also find some evidence that the current real exchange rate is unrelated with current export participation, but that a depreciation leads to

more entry in subsequent periods. More empirical work is necessary to determine the robustness of these results.

The current model has a number of shortcomings. On the micro side, we have concentrated on a limited set of facts about exporters. In particular, we have focused on the differences between the average exporters and average nonexporters with little concern about the difference among firms within these sectors. Clearly, there are large differences between major exporters, like GM, Ford, and Boeing, and the rest of the export sector which may matter. Also, we have not focused on the pattern of export growth of plants. On the macro side, there are aspects of international business cycles that our benchmark model cannot explain. In particular, the model predicts too little comovement in economic activity across countries and relative prices that are much too smooth compared to the data. Perhaps, in an environment in which these puzzles are less pronounced, export decisions may have a greater impact on net export dynamics.

A. Data Appendix

Aggregate Data: Aggregate moments are computed for the U.S. over the period 1975:1 through 2004:3. The following data are from the BEA: Consumption, GDP, Fixed Investment, Imports, and Exports; nominal GDP, exports and non-oil imports are used to construct the ratio of net exports to GDP; and the terms of trade measured as the ratio of the price of non-oil imports to the price of exports. The Real Exchange Rate series is the Federal Reserve's trade weighted dollar index. Labor is measured as total non-farm employees (CES0000000001).

Exporter Data: The number of exporters is collected from annual reports of the U.S. Census Bureau: Profile of U.S. Exporting Companies. Trade is from U.S. Census Bureau: U.S. Trade (Imports, Exports and Balance) by country database. These are deflated using the U.S. GDP deflator from the BEA. Real exchange rates are constructed using bilateral exchange rate from Haver Analytics and annual inflation rates from the IFS World tables. Real GDP is also from the IFS World tables and for the U.S. from the BEA. The exporter data are from 1995 to 2003. The other variables are from 1994 to 2004. The destination countries are Australia, Belgium, Canada, France, Germany, Italy, Japan, Korea, Mexico, Netherlands, Spain, Switzerland, and the United Kingdom.

B. Appendix

The following equations describe the complete economy.

Consumer's Problem: The first order conditions for the home consumer are:

$$(14) \quad -\frac{U_L(s^t)}{U_C(s^t)} \;=\; W(s^t),$$

$$(15) \quad Q(s^{t+1}|s^t) \;=\; \beta Pr(s^{t+1}|s^t)\frac{U_C(s^{t+1})P(s^t)}{U_C(s^t)P(s^{t+1})}.$$

Similarly, the first order conditions for the foreign consumer are:

$$(16) \quad -\frac{U_L^*(s^t)}{U_C^*(s^t)} \;=\; W^*(s^t),$$

$$(17) \quad Q^*(s^{t+1}|s^t) \;=\; \beta Pr(s^{t+1}|s^t)\frac{U_C^*(s^{t+1})P^*(s^t)}{U_C^*(s^t)P^*(s^{t+1})},$$

$$(18) \quad Q(s^{t+1}|s^t) \;=\; \beta Pr(s^{t+1}|s^t)\frac{U_C^*(s^{t+1})e(s^t)P^*(s^t)}{U_C^*(s^t)e(s^{t+1})P^*(s^{t+1})}.$$

From the state contingent bond equations (15) and (18), we get

$$(19) \quad \frac{U_C(s^{t+1})P(s^t)}{U_C(s^t)P(s^{t+1})} = \frac{U_C^*(s^{t+1})e(s^t)P^*(s^t)}{U_C^*(s^t)e(s^{t+1})P^*(s^{t+1})}.$$

The real exchange rate is defined as $q(s^t) = \frac{e(s^t)P^*(s^t)}{P(s^t)}$. Iterating on (19) yields

$$(20) \quad q(s^t) = \kappa\frac{U_C^*(s^t)}{U_C(s^t)},$$

where $\kappa = q(s^0)U_C(s^0)/U_C^*(s^0)$. For the simulations κ is normalized to be 1.

Final Good Producer's Problem: The focs for the home final good producer give the input demand functions

$$(21)\ y^d(h,i,s^t) = a_1^{\frac{1}{1-\rho}}\left[\frac{P(h,i,s^t)}{P(h,s^t)}\right]^{\frac{1}{\theta-1}}\left[\frac{P(h,s^t)}{P(s^t)}\right]^{\frac{1}{\rho-1}}D(s^t),$$

$$(22)\ y^d(f,i,s^t) = (1-a_1)^{\frac{1}{1-\rho}}\left[\frac{P(f,i,s^t)}{P(f,s^t)}\right]^{\frac{1}{\theta-1}}\left[\frac{P(f,s^t)}{P(s^t)}\right]^{\frac{1}{\rho-1}}D(s^t),$$

where $P(h,s^t) = \left[\int_0^1 P(h,i,s^t)^{\frac{\theta}{\theta-1}}di\right]^{\frac{\theta-1}{\theta}}$, and $P(f,s^t) = \left[\int_{i\in\mathcal{E}^*(s^t)} P(f,i,s^t)^{\frac{\theta}{\theta-1}}di\right]^{\frac{\theta-1}{\theta}}$. The zero-profit condition in final goods implies that

$$(23)\ \ P(s^t) = \left[a_1^{\frac{1}{1-\rho}}P(h,s^t)^{\frac{\rho}{\rho-1}} + (1-a_1)^{\frac{1}{1-\rho}}P(f,s^t)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}.$$

The resource constraint for the final goods gives

$$(24)\ \ D(s^t) = C(s^t) + I(s^t).$$

Intermediate Good Producer's Problem: The first order conditions for the i_{th} home intermediate good producer give

$$(25)\ \frac{P(h,i,s^t)}{P(s^t)} = q(s^t)\frac{P^*(h,i,s^t)}{P^*(s^t)} = \frac{W(s^t)}{\theta F_L(i,s^t)},$$

$$(26)\ \ \ \ P(s^t) = \sum_{s^{t+1}}\sum_{\eta(i,s_i^{t+1})} Q(s^{t+1}|s^t)Pr[\eta(i,s^{t+1})]\left\{\left(\frac{\alpha}{1-\alpha}\right)\right.$$
$$\left. \cdot \frac{P(s^{t+1})W(s^{t+1})L(i,s^{t+1})}{K(i,s^t)} + P(s^{t+1})(1-\delta)\right\}.$$

The marginal cost of production is equal to $W(s^t)/F_L(i,s^t)$ and prices are a constant mark-up over marginal cost.

The resource constraint is defined as for good $i \in [0,1]$

$$(27)\ \ y(h,i,s^t) + m(i,s^t)y^*(h,i,s^t) = A(i,s^t)K(i,s^{t-1})^\alpha L(i,s^t)^{1-\alpha}.$$

From the demand functions for intermediate goods (21) and (22), and the price decisions (25), the labor demand function can be obtained from (27).

$$(28)\ L(i,s^t) = \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu}{\theta-1}}A(i,s^t)^{\frac{1-\nu}{\alpha}}K(i,s^{t-1})^{1-\nu}\left\{a_1^{\frac{1}{1-\rho}}\left[\frac{P(h,s^t)}{P(s^t)}\right]^{\mu}D(s^t)\right.$$
$$\left. + m(i,s^t)(1-a_1)^{\frac{1}{1-\rho}}q(s^t)^{\frac{1}{1-\theta}}\left[\frac{P^*(h,s^t)}{P^*(s^t)}\right]^{\mu}D^*(s^t)\right\}^{\nu},$$

where $\nu = \frac{1-\theta}{1-\theta(1-\alpha)}$, and $\mu = \frac{1}{1-\theta} - \frac{1}{1-\rho}$. Since $\eta(i,s^t)$ follows an iid. normal distribution, equation (26) implies that $K(i,s^t)$ is independent of $\eta(i,s^t)$ but depends on the firm's export status, $m(i,s^t)$,

and the state of the world, s^t.

$$(29) \quad K(i, s^t) = \begin{cases} K_0(s^t) & \text{if } m(i, s^t) = 0, \\ K_1(s^t) & \text{if } m(i, s^t) = 1. \end{cases}$$

Hence, the sufficient statistics for the distribution of the capital among home intermediate good producers are $K_0(s^t)$, $K_1(s^t)$, and $N(s^t)$.

Marginal Exporters: Let $L_{m,m'}(i, s^t)$ and $I_{m,m'}(i, s^t)$ be the potentially sub-optimal levels of labor inputs and investment for the i_{th} firm when $m(i, s^{t-1}) = m$ and $m(i, s^t) = m'$, respectively. Clearly $I_{m,m'}(i, s^t) = I_{m,m'}(s^t) = K_{m'}(s^t) - (1-\delta)K_m(s^t)$. The problem of firm i with state (η, k, m) in aggregate state s^t is to solve the following problem

$$V\left(i, \eta, k, m; s^t\right) = \max\left\{V^0\left(i, \eta, k, m; s^t\right), V^1\left(i, \eta, k, m; s^t\right)\right\}$$

where V^0 is the maximal value of not exporting in the current period and V^1 is equal to the maximal value of exporting this period. From the mark-up pricing (25), the value of the i_{th} firm can be rewritten as

$$(30) \ V^{m'}\left(\eta, k, m, s^t\right) = \left[\frac{1 - \theta(1-\alpha)}{\theta(1-\alpha)}\right] P(s^t)W(s^t)L_{m,m'}(i, s^t) - P(s^t)I_{m,m'}(s^t)$$
$$+ \sum_{s^{t+1}}\sum_{\eta'} Pr(\eta')Q(s^{t+1}|s^t)V\left(\eta', k', m', s^{t+1}\right)$$

where m, $m' \in \{0, 1\}$. The firm-specific productivity of marginal exporters among last period exporters and non-exporters, $\eta_1(s^t)$ and $\eta_0(s^t)$, satisfy

$$(31) \quad V^1\left(\eta_j(s^t), K_j(s^{t-1}), m; s^t\right) = V^0\left(\eta_j(s^t), K_j(s^{t-1}), m; s^t\right),$$

where $m, j = \{0, 1\}$. Let $\zeta_j \in [0, 1]$ denote the identity of the firm with a shock such that $\eta\left(\zeta_j, s^t\right) = \eta_j\left(s^t\right)$, then the marginal exporter conditions (31) can be rewritten as

$$(32) \ 0 = \left[\frac{1 - \theta(1-\alpha)}{\theta(1-\alpha)}\right] P(s^t)W(s^t)\left[L_{m,1}(\zeta_j, s^t) - L_{m,0}(\zeta_j, s^t)\right]$$
$$- P(s^t)[K_1(s^t) - K_0(s^t)] - e(s^t)P^*(s^t)\tau_m$$
$$+ \sum_{s^{t+1}}\sum_{\eta'} Pr(\eta')Q(s^{t+1}|s^t)\left\{V\left[\eta', K_1(s^t), 1; s^{t+1}\right] - V\left[\eta', K_0(s^t), 0; , s^{t+1}\right]\right\}$$

$$(33) \ L_{m,m'}(i, s^t) = \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu}{\theta-1}} e^{\frac{1-\nu}{\alpha}[z(s^t)+\eta(i,s^t)]} K_m(s^{t-1})^{1-\nu}\left\{a_1^{\frac{1}{1-\rho}}\left(\frac{P(h, s^t)}{P(s^t)}\right)^\mu D(s^t)\right.$$
$$\left. + m'(1-a_1)^{\frac{1}{1-\rho}} q(s^t)^{\frac{1}{1-\theta}}\left(\frac{P^*(h, s^t)}{P^*(s^t)}\right)^\mu D^*(s^t)\right\}^\nu.$$

Among last period exporters, if the firm-specific productivity $\eta(i, s^t)$ is greater (less) than $\eta_1(s^t)$, the producer will (will not) export goods abroad in s^t. Among last period non-exporters, if the firm-specific productivity $\eta(j, s^t)$ is greater (less) than $\eta_0(s^t)$, the producer will (not) export goods abroad in s^t. Thus, the percentage of exporters in s^t among non-exporters and exporters in s^{t-1}, $n_0(s^t)$ and $n_1(s^t)$, respectively, can be defined as

$$(34) \quad n_m(s^t) = 1 - \Phi[\eta_m(s^t)],$$

where $m = \{0, 1\}$. $\Phi(\eta)$ is the *cdf*. of $\eta(i, s^t)$. $N(s^t)$ is the percentage of exporters in s^t among all intermediate good producers. $N(s^t)$ evolves as

$$(35) \quad N(s^t) = n_1(s^t)N(s^{t-1}) + n_0(s^t)[1 - N(s^{t-1})].$$

Aggregate Variables
Capital and Investment: The aggregate capital at home in s^t is defined as

$$
\begin{aligned}
K(s^t) &= \int_{i \in \mathcal{E}(s^t)} K_1(s^t)di + \int_{i \notin \mathcal{E}(s^t)} K_0(s^t)di \\
(36) \qquad &= [1 - N(s^t)]K_0(s^t) + N(s^t)K_1(s^t).
\end{aligned}
$$

The aggregate investment at home in s^t is defined as

$$(37) \quad I(s^t) = K(s^t) - (1 - \delta)K(s^{t-1}).$$

Labor Demand: The average labor demand in s^t from last period non-exporters and exporters, $L_0(s^t)$ and $L_1(s^t)$, can be defined as

$$L_0(s^t) = \frac{\int_{i \notin \mathcal{E}(s^{t-1})} L(i, s^t)di}{1 - N(s^{t-1})}, L_1(s^t) = \frac{\int_{i \in \mathcal{E}(s^{t-1})} L(i, s^t)di}{N(s^{t-1})}.$$

As η is iid from (33)

$$
\begin{aligned}
(38)\ L_m(s^t) &= \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu}{\theta-1}} e^{\frac{1-\nu}{\alpha}z(s^t)} K_m(s^{t-1})^{1-\nu} \left\{ a_1^{\frac{1}{1-\rho}} \left[\left(\frac{P(h, s^t)}{P(s^t)}\right)^{\mu} D(s^t)\right]^{\nu} \right. \\
&\quad \cdot \int_{-\infty}^{\eta_m(s^t)} e^{\frac{1-\nu}{\alpha}\eta} \phi(\eta)d\eta + \left[a_1^{\frac{1}{1-\rho}} \left(\frac{P(h, s^t)}{P(s^t)}\right)^{\mu} D(s^t) + (1-a_1)^{\frac{1}{1-\rho}} q(s^t)^{\frac{1}{1-\theta}} \right. \\
&\quad \left. \cdot \left(\frac{P^*(h, s^t)}{P^*(s^t)}\right)^{\mu} D^*(s^t)\right]^{\nu} \int_{\eta_m(s^t)}^{\infty} e^{\left(\frac{1-\nu}{\alpha}\right)\eta} \phi(\eta)d\eta \left. \right\},
\end{aligned}
$$

where $m = \{0,1\}$. $\phi(\eta)$ is the *pdf*. of η. The home aggregate labor demand is defined as

$$(39) \quad L(s^t) = [1 - N(s^{t-1})]L_0(s^t) + N(s^{t-1})L_1(s^t).$$

Capital Decision Rules: The capital decision rules (26) are defined as

$$(40) \quad 1 = \sum_{s^{t+1}} Q(s^{t+1}|s^t)\frac{P(s^{t+1})}{P(s^t)}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{W(s^{t+1})L_m(s^{t+1})}{K_m(s^t)} + (1-\delta)\right].$$

Price Indices: From the mark-up pricing (25), and the labor demand function for the i_{th} firm, the price of the i_{th} firm can be rewritten as

$$
\begin{aligned}
(41)\ \left[\frac{P(h, i, s^t)}{P(s^t)}\right]^{\frac{\theta}{\theta-1}} &= \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu+\theta-1}{\theta-1}} K_{m(i,s^{t-1})}^{1-\nu} e^{\left(\frac{1-\nu}{\alpha}\right)[z(s^t)+\eta(i,s^t)]} \left\{ a_1^{\frac{1}{1-\rho}} \left[\frac{P(h, s^t)}{P(s^t)}\right]^{\mu} D(s^t) \right. \\
&\quad \left. + m(i, s^t)(1-a_1)^{\frac{1}{1-\rho}} q(s^t)^{\frac{1}{1-\theta}} \left[\frac{P^*(h, s^t)}{P^*(s^t)}\right]^{\mu} D^*(s^t) \right\}^{\mu-1},
\end{aligned}
$$

$$(42) \quad \frac{P^*(h, i, s^t)}{P^*(s^t)} = \frac{P(h, i, s^t)}{q(s^t)P(s^t)}.$$

34

Then, the aggregate export price $P^*(h, s^t)$ can be expressed as

$$(43) \quad \left[\frac{P^*(h, s^t)}{P^*(s^t)}\right]^{\frac{\theta}{\theta-1}} = \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu+\theta-1}{\theta-1}} q(s^t)^{\frac{\theta}{1-\theta}} e^{\left(\frac{1-\nu}{\alpha}\right)z(s^t)} \left\{a_1^{\frac{1}{1-\rho}} \left[\frac{P(h, s^t)}{P(s^t)}\right]^{\mu} D(s^t)\right.$$

$$+ (1-a_1)^{\frac{1}{1-\rho}} q(s^t)^{\frac{1}{1-\theta}} \left[\frac{P^*(h, s^t)}{P^*(s^t)}\right]^{\mu} \left. D^*(s^t)\right\}^{\nu-1}$$

$$\cdot \left\{[1-N(s^t)]K_0(s^{t-1})^{1-\nu} \int_{\eta_0(s^t)}^{\infty} e^{\left(\frac{1-\nu}{\alpha}\right)\eta} \phi(\eta) d\eta\right.$$

$$+ N(s^t)K_1(s^{t-1})^{1-\nu} \int_{\eta_1(s^t)}^{\infty} e^{\left(\frac{1-\nu}{\alpha}\right)\eta} \phi(\eta) d\eta\left.\right\}.$$

Similarly, the aggregate home price $P(h, s^t)$ can be expressed as

$$(44) \quad \left[\frac{P(h, s^t)}{P(s^t)}\right]^{\frac{\theta}{\theta-1}} = q(s^t)^{\frac{\theta}{\theta-1}} \left[\frac{P^*(h, s^t)}{P^*(s^t)}\right]^{\frac{\theta}{\theta-1}} + \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu+\theta-1}{\theta-1}}$$

$$\cdot e^{\left(\frac{1-\nu}{\alpha}\right)z(s^t)} \left\{a_1^{\frac{1}{1-\rho}} \left[\frac{P(h, s^t)}{P(s^t)}\right]^{\mu} D(s^t)\right\}^{\nu-1}$$

$$\cdot \left\{[1-N(s^t)]K_0(s^{t-1})^{1-\nu} \int_{-\infty}^{\eta_0(s^t)} e^{\left(\frac{1-\nu}{\alpha}\right)\eta} \phi(\eta) d\eta\right.$$

$$+ N(s^t)K_1(s^{t-1})^{1-\nu} \int_{-\infty}^{\eta_1(s^t)} e^{\left(\frac{1-\nu}{\alpha}\right)\eta} \phi(\eta) d\eta\left.\right\}.$$

From the aggregate price index (23),

$$(45) \quad 1 = a_1^{\frac{1}{1-\rho}} \left[\frac{P(h, s^t)}{P(s^t)}\right]^{\frac{\rho}{\rho-1}} + (1-a_1)^{\frac{1}{1-\rho}} \left[\frac{P(f, s^t)}{P(s^t)}\right]^{\frac{\rho}{\rho-1}}.$$

Values of Firms: Let $V_m(s^t), m = \{0, 1\}$, be the average values of firms among the firms that have the same export status, m, in s^{t-1}. Clearly

$$V_0(s^t) = \frac{1}{1-N(s^{t-1})} \int V\left[\eta, K_0(s^{t-1}), 0, s^t\right] \phi(\eta) d\eta,$$

$$V_1(s^t) = \frac{1}{N(s^{t-1})} \int V\left[\eta, K_1(s^{t-1}), 1, s^t\right] \phi(\eta) d\eta.$$

These average values of firms can be rewritten as

$$(46) \quad V_m(s^t) = \left[\frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}\right] P(s^t)W(s^t)L_m(s^t) - P(s^t)\left\{[1-n_m(s^t)]K_0(s^t)\right.$$

$$+n_m(s^t)K_1(s^t)\} + (1-\delta)P(s^t)K_m(s^{t-1}) - n_m(s^t)e(s^t)P^*(s^t)\tau_m$$

$$+ \sum_{s^{t+1}} Q(s^{t+1}|s^t)\left\{[1-n_m(s^t)]V_0(s^{t+1}) + n_m(s^t)V_1(s^{t+1})\right\},$$

and the difference between $V_1(s^t)$ and $V_0(s^t)$ gives

$$(47) \quad V_1(s^t) - V_0(s^t) = \left[\frac{1 - \theta(1-\alpha)}{\theta(1-\alpha)}\right] P(s^t)W(s^t)[L_1(s^t) - L_0(s^t)]$$
$$-P(s^t)\left\{[n_0(s^t) - n_1(s^t)][K_0(s^t) - K_1(s^t)]\right\}$$
$$+(1-\delta)P(s^t)[K_1(s^{t-1}) - K_0(s^{t-1})]$$
$$-e(s^t)P^*(s^t)[n_1(s^t)\tau_1 - n_0(s^t)\tau_0]$$
$$+[n_1(s^t) - n_0(s^t)]\sum_{s^{t+1}} Q(s^{t+1}|s^t)[V_1(s^{t+1}) - V_0(s^{t+1})].$$

The conditions for marginal exporters can be rewritten as

$$(48) \quad 0 = [1 - \theta(1-\alpha)] P(s^t) \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu+\theta-1}{\theta-1}} e^{\left(\frac{1-\nu}{\alpha}\right)[z(s^t)+\eta_m(s^t)]} K_m(s^{t-1})^{1-\nu}$$
$$\cdot \left\{\left[a_1^{\frac{1}{1-\rho}}\left(\frac{P(h,s^t)}{P(s^t)}\right)^\mu D(s^t) + (1-a_1)^{\frac{1}{1-\rho}} q(s^t)^{\frac{1}{1-\theta}}\left(\frac{P^*(h,s^t)}{P^*(s^t)}\right)^\mu D^*(s^t)\right]^\nu\right.$$
$$\left. - \left[a_1^{\frac{1}{1-\rho}}\left(\frac{P(h,s^t)}{P(s^t)}\right)^\mu D(s^t)\right]^\nu\right\} - P(s^t)[K_1(s^t) - K_0(s^t)] - e(s^t)P^*(s^t)\tau_m$$
$$+ \sum_{s^{t+1}} Q(s^{t+1}|s^t)[V_1(s^{t+1}) - V_0(s^{t+1})].$$

Notice that by substituting $[V_1(s^{t+1}) - V_0(s^{t+1})]$ with (47), (48) becomes a static equation.

Exports and Imports: The real imports are defined as

$$(49) \quad IM(s^t) = \int_{i \in \mathcal{E}^*(s^t)} \frac{P(f,i,s_i^t)y(f,i,s^t)}{P(f,s^t)} di = (1-a_1)^{\frac{1}{1-\rho}}\left[\frac{P(f,s^t)}{P(s^t)}\right]^{\frac{1}{\rho-1}} D(s^t).$$

Similarly, the real exports are defined as

$$(50) \quad EX(s^t) = \int_{i \in \mathcal{E}(s^t)} \frac{e(s^t)P^*(h,i,s^t)y^*(h,i,s^t)}{e(s^t)P^*(h,s^t)} di = (1-a_1)^{\frac{1}{1-\rho}} q(s^t)\left[\frac{P^*(h,s^t)}{P^*(s^t)}\right]^{\frac{1}{\rho-1}} D^*(s^t).$$

The gross domestic product, $Y(s^t)$ is defined as

$$(51) \quad Y(s^t) = \frac{\int_0^1 \left[P(h,i,s^t)y(h,i,s^t) + e\left(s^t\right) P^*(h,i,s^t)y^*(h,i,s^t)\right] di}{P(h,s^t)}.$$

Equilibrium: Under the normalization of price indices, $P(s^t) = P^*(s^t) = 1$, we have 6 dynamic equations:[17]
  (40) and foreign analogue for $K_0(s^t)$ and $K_0^*(s^t)$, and $K_1(s^t)$ and $K_1^*(s^t)$;
  (47) and foreign analogue for $V_1(s^t) - V_0(s^t)$ and $V_1^*(s^t) - V_0^*(s^t)$; and, 37 static equations:
  (14) and foreign analogue for $W(s^t)$ and $W^*(s^t)$;
  (20) for $q(s^t)$;
  (24) and foreign analogue for $D(s^t)$ and $D^*(s^t)$;
  (34) and foreign analogue for $n_0(s^t)$ and $n_0^*(s^t)$, and $n_1(s^t)$ and $n_1^*(s^t)$;

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[17]$Q(s^{t+1}|s^t)$ is substituted by other variables using (15) and (18).

(35) and foreign analogue for $N(s^t)$ and $N^*(s^t)$;
(36) and foreign analogue for $K(s^t)$ and $K^*(s^t)$;
(37) and foreign analogue for $I(s^t)$ and $I^*(s^t)$;
(38) and foreign analogue for $L_0(s^t)$ and $L_0^*(s^t)$, and $L_1(s^t)$ and $L_1^*(s^t)$;
(39) and foreign analogue for $L(s^t)$ and $L^*(s^t)$;
(43) and foreign analogue for $P^*(h, s^t)$ and $P(f, s^t)$;
(44) and foreign analogue for $P(h, s^t)$ and $P^*(f, s^t)$;
(45) and foreign analogue for $P(s^t)$ and $P^*(s^t)$;
(48) and foreign analogue for $\eta_0(s^t)$, $\eta_0^*(s^t)$, $\eta_1(s^t)$, and $\eta_1^*(s^t)$[18];
(49) and foreign analogue for $IM(s^t)$ and $IM^*(s^t)$;
(50) and foreign analogue for $EX(s^t)$ and $EX^*(s^t)$;
(51) and foreign analogue for $Y(s^t)$ and $Y^*(s^t)$.

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Table 1: Parameter Values

Benchmark Model	
Preferences	$\beta = 0.99, \quad \sigma = 2, \quad \theta = 0.9, \quad \rho = 1/3, \quad \gamma = 0.308$
Production	$\alpha = 0.36, \quad \delta = 0.025, \quad a_1 = 1, \quad a_2 = 0.321$
Productivity	$M_{11} = M_{22} = 0.95,\ M_{12} = M_{12} = 0,$ $Var(\epsilon) = Var(\epsilon^*) = \sigma_\epsilon^2 = 0.007^2$ $Corr(\epsilon, \epsilon^*) = 0.25$ $\sigma_\eta = 0.50$
Trade costs	$\tau_0 = 0.0592, \quad \tau_1 = 0.01$
Variations	
No Cost	$a_2 = 0.315$
Fixed Exit	$\tau_0 = 0.2527, \quad \tau_1 = 0, \text{Pr (exit shock)} = 0.0367$
Permanent Exporters	$a_2 = 0.322, \tau_0 = 0.110,\ \tau_1 = 0.012$
IID Exporters	$a_2 = 0.317, \tau_0 = \tau_1 = 0.0047$
Love Variety	$\lambda = -0.1, a_2 = 0.328, \tau_0 = 0.049,\ \tau_1 = 0.01$
Hate Variety	$\lambda = 1, a_2 = 0.255, \tau_0 = 0.049,\ \tau_1 = 0.01$
High Markup	$\theta = 2/3, \gamma = 0.393$
No Cost	$a_2 = 0.315$
Sunk	$a_2 = 0.348\ , \tau_0 = 0.16,\ \tau_1 = 0.052$
CRS	$\lambda = 1/3, a_2 = 0.322,\ \tau_0 = 0.16,\ \tau_1 = 0.052$
High Armington	$\rho = \theta = 2/3$
No Cost	$a_2 = 0.561$
Sunk-CRS	$\lambda = 1/3, a_2 = 0.589, \tau_0 = 0.16, \tau_1 = 0.052$
Annual	$M_{11} = 0.95^4,\ \sigma_\epsilon^2 = 0.0242^2,\ \sigma_\eta = 0.25,\ \beta = 0.96,$ $\delta = 0.10,\ \tau_0 = 0.021, \quad \tau_1 = 0.01$

Table 2: Business Cycle Statistics

| | | | | | | | | | High Markup | | | $\rho = \theta$ | |
	Data	No Costs	Sunk Cost	Fixed Exit	High Persistence	IID Exporters	Love Variety	Hate Variety	No Costs	Sunk Cost	CRS	No Costs	Sunk-CRS
Standard deviation (in percent)													
Y	1.42	1.29	1.31	1.30	1.31	1.30	1.32	1.21	1.24	1.31	1.25	1.32	1.35
NX/Y	0.46	0.16	0.17	0.16	0.16	0.16	0.19	0.09	0.10	0.16	0.11	0.20	0.29
Standard deviation (relative to output)													
C	0.79	0.36	0.36	0.35	0.35	0.36	0.35	0.37	0.41	0.40	0.41	0.37	0.36
I	3.25	3.36	3.39	3.36	3.37	3.37	3.44	3.19	3.92	4.12	3.94	4.33	4.67
L	0.85	0.47	0.47	0.47	0.47	0.47	0.47	0.46	0.42	0.43	0.42	0.49	0.49
RER	2.81	0.33	0.32	0.32	0.32	0.32	0.31	0.37	0.41	0.37	0.40	0.23	0.21
Domestic correlation with output													
C	0.83	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.95	0.96	0.91	0.90
I	0.93	0.98	0.98	0.98	0.98	0.98	0.97	0.99	0.98	0.96	0.98	0.95	0.93
L	0.85	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.98	0.98
NX/Y	-0.38	-0.52	-0.52	-0.52	-0.52	-0.52	-0.53	-0.48	-0.39	-0.44	-0.40	-0.10	-0.22
RER	0.16	0.56	0.56	0.57	0.57	0.56	0.57	0.55	0.60	0.61	0.60	0.58	0.59
$Q, NX/Y$	0.07	-0.49	-0.50	-0.49	-0.50	-0.49	-0.50	-0.49	-0.31	-0.40	-0.33	0.38	0.23
Persistence													
Y	0.87	0.70	0.70	0.70	0.70	0.70	0.70	0.69	0.70	0.71	0.71	0.71	0.72
C	0.91	0.74	0.74	0.74	0.74	0.74	0.74	0.73	0.73	0.74	0.74	0.74	0.74
I	0.84	0.69	0.69	0.69	0.69	0.69	0.70	0.67	0.69	0.71	0.69	0.68	0.70
L	0.95	0.69	0.69	0.69	0.69	0.69	0.70	0.67	0.69	0.71	0.69	0.69	0.70
NX/Y	0.90	0.69	0.71	0.70	0.72	0.69	0.73	0.63	0.75	0.81	0.78	0.87	0.88
RER	0.81	0.79	0.79	0.79	0.79	0.79	0.80	0.77	0.77	0.78	0.77	0.85	0.87
International correlation*													
Y	0.51	0.21	0.20	0.20	0.20	0.21	0.19	0.27	0.19	0.15	0.19	0.06	0.02
C	0.32	0.52	0.52	0.53	0.52	0.52	0.53	0.47	0.58	0.60	0.59	0.72	0.74
I	0.29	0.01	-0.01	0.00	-0.01	0.00	-0.05	0.15	0.03	-0.10	0.02	-0.24	-0.37
L	0.43	0.18	0.17	0.18	0.18	0.18	0.15	0.26	0.12	0.05	0.11	-0.24	-0.29

*International correlations are from Kehoe and Perri (2000).

Figure 1: Value of Firms

Values

Exporting Firm
(after Costs τ_1)

Exporting Firm
(before Costs)

Non-exporting Firm

Exporting Firm
(after Costs τ_0)
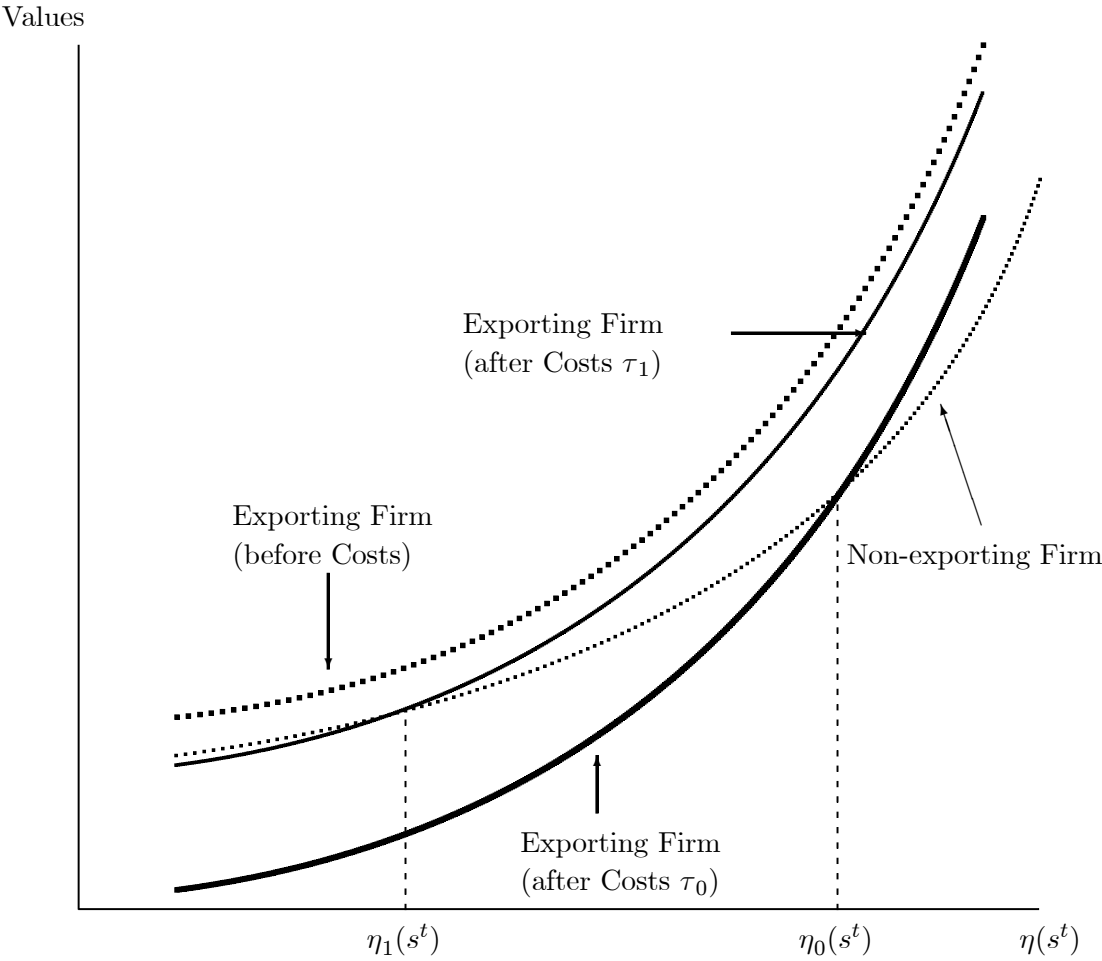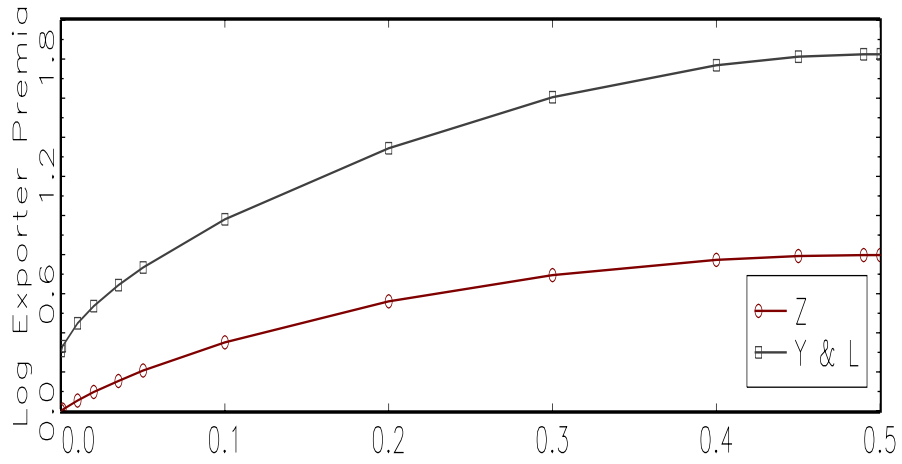
$\eta_1(s^t)$ $\eta_0(s^t)$ $\eta(s^t)$

Figure 2 Exporter Characteristics and Hysteresis
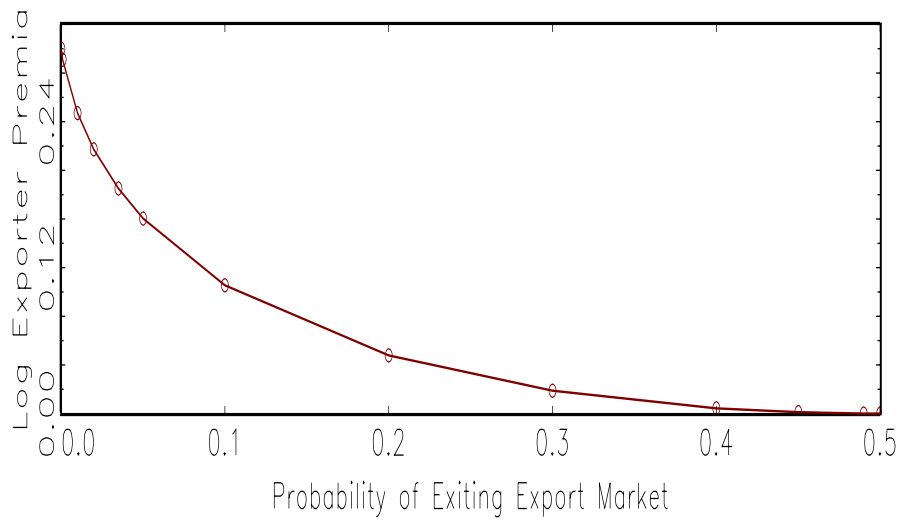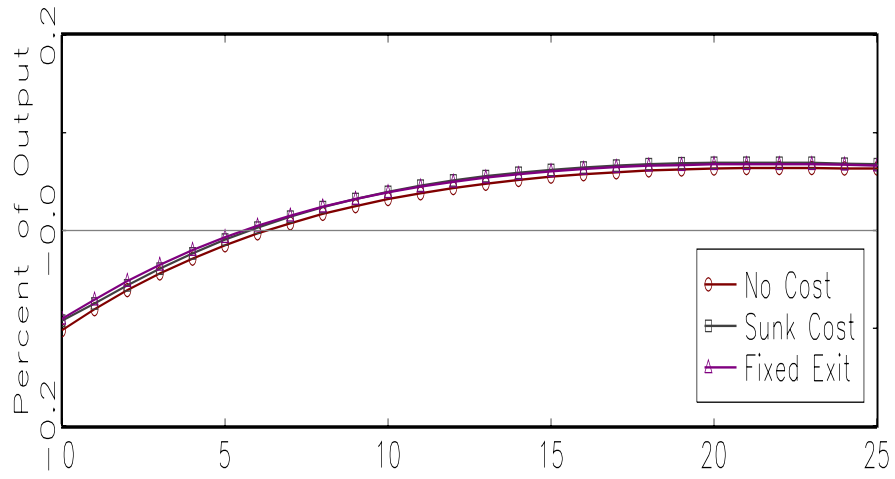a. Technology, Output and Labor

b. Capital

Figure 2:

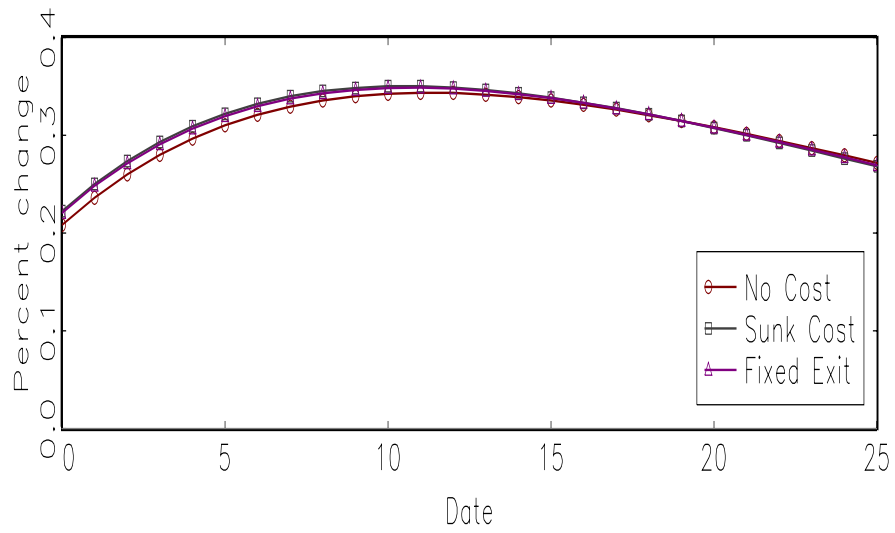# Figure 3
a. Net Exports

b. Real Exchange Rate

Figure 3:
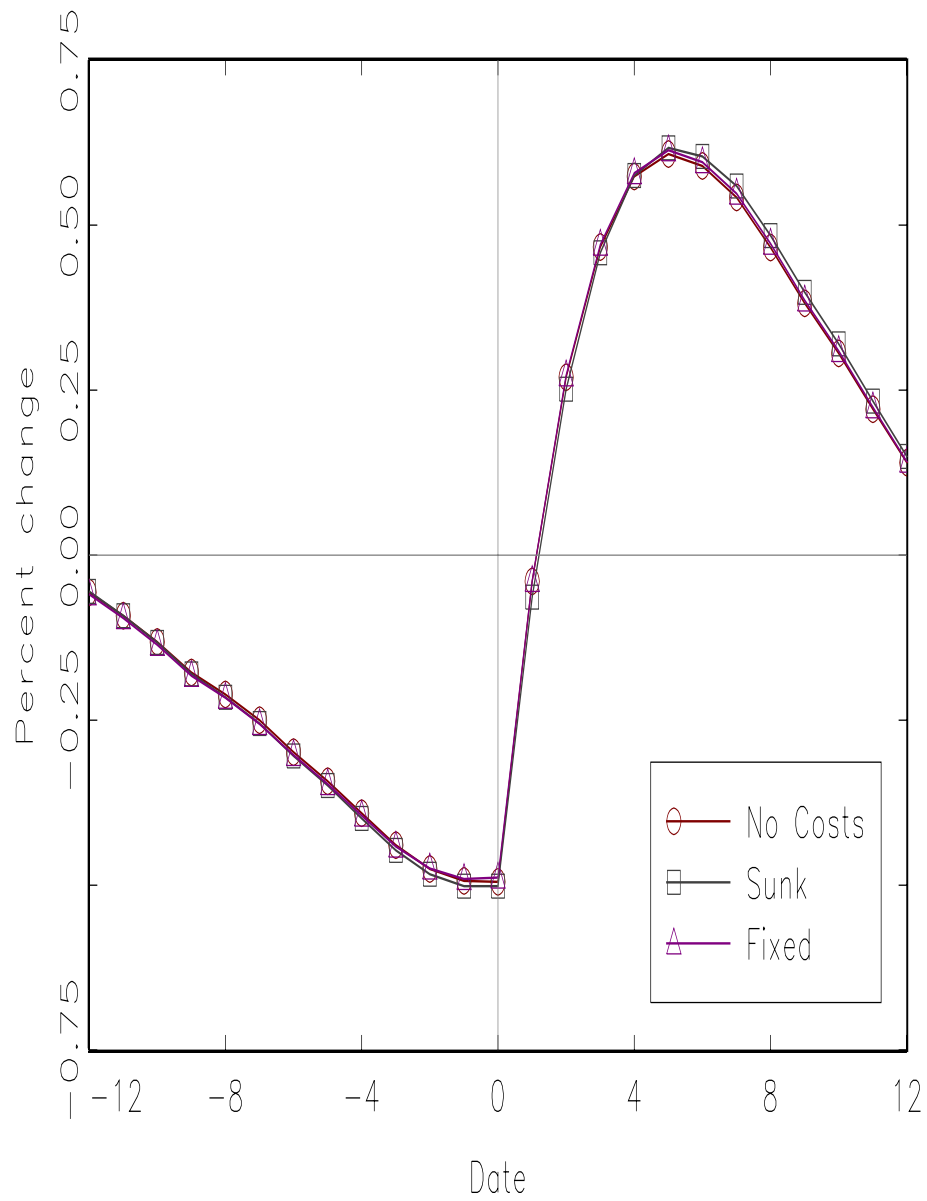
Figure 4
Correlation of NX_{t+k} and RER_t

Figure 4:

Figure 5: Exporter Dynamics − Impulse Response

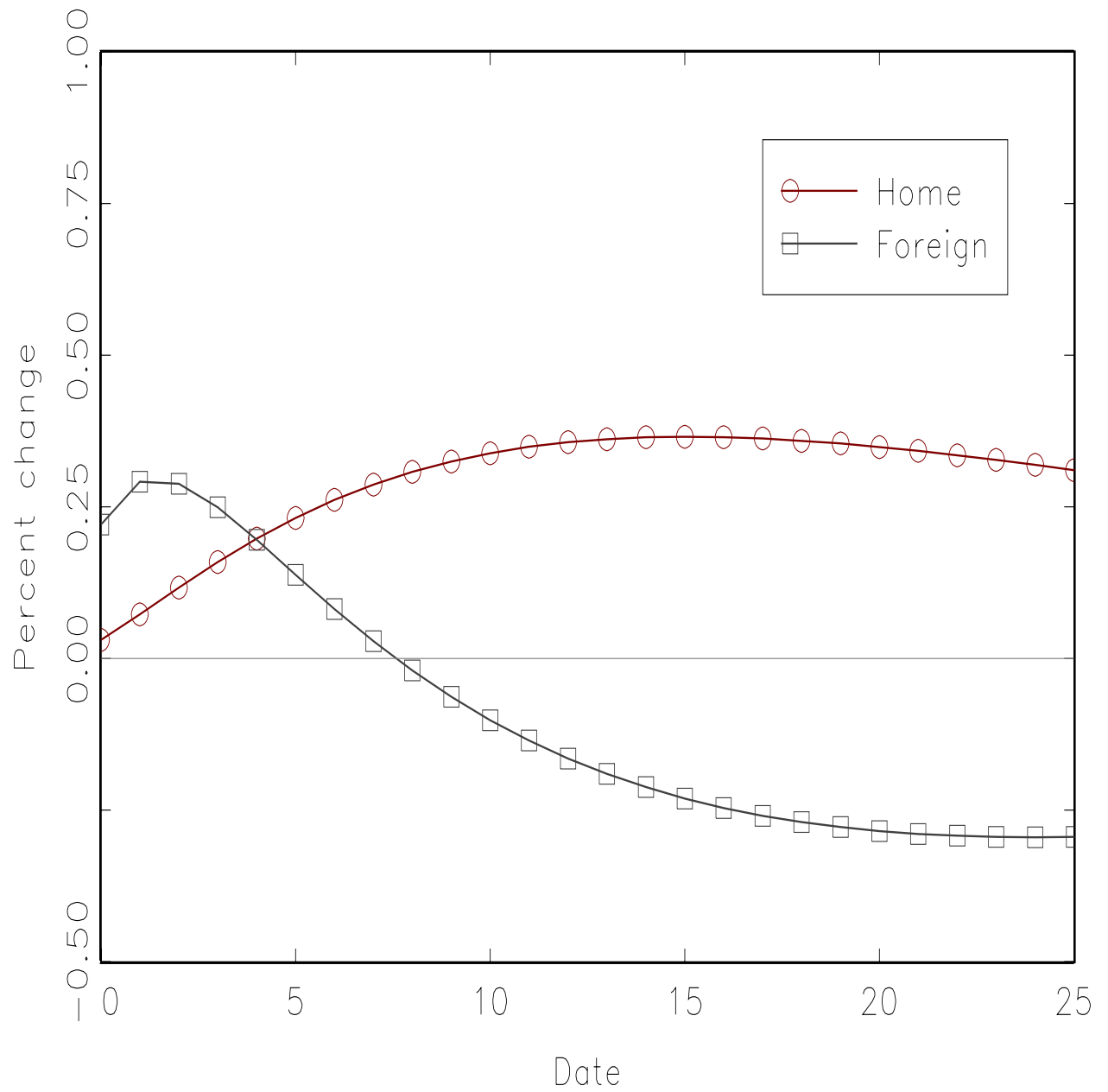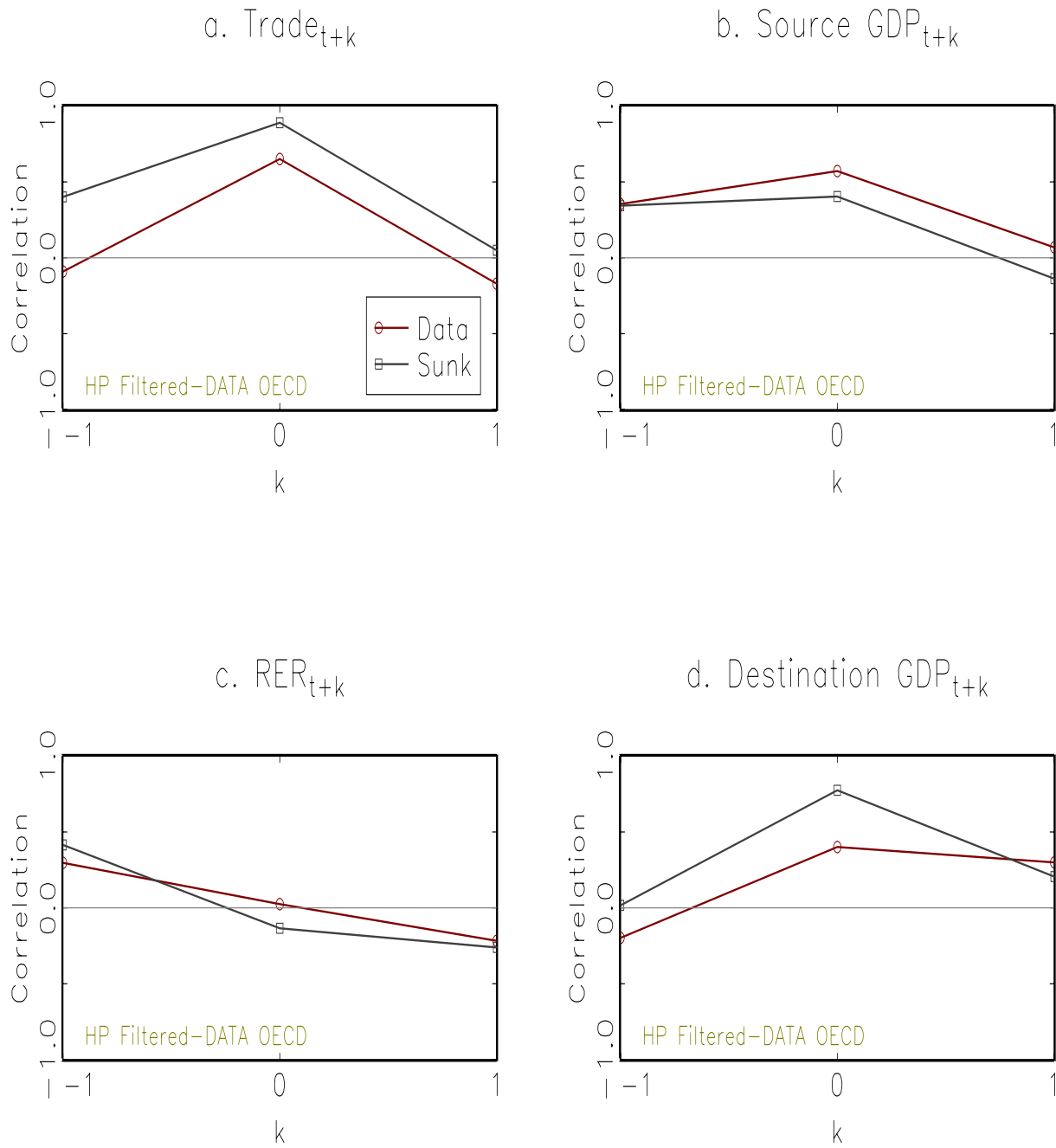Figure 5:

Figure 6 Exporter Dynamics – Data and Model

a. Trade$_{t+k}$

b. Source GDP$_{t+k}$

c. RER$_{t+k}$

d. Destination GDP$_{t+k}$

Figure 6: