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IMPLICATIONS OF STATE-DEPENDENT PRICING FOR DYNAMIC MACROECONOMIC MODELS

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Abstract

State-dependent pricing (SDP) models treat the timing of price changes as a profit-maximizing choice, symmetrically with other decisions of firms. Using quantitative general equilibrium models that incorporate a "generalized (S,s) approach," we investigate the implications of SDP for topics in two major areas of macroeconomic research: the early 1990s SDP literature and more recent work on persistence mechanisms. First, we show that state-dependent pricing leads to unusual macroeconomic dynamics, which occur because of the timing of price adjustments chosen by firms as in the earlier literature. In particular, we display an example in which output responses peak at about a year, while inflation responses peak at about two years after the shock. Second, we examine whether the persistence-enhancing effects of two New Keynesian model features, namely, specific factor markets and variable elasticity demand curves, depend importantly on whether pricing is state dependent. In an SDP setting, we provide examples in which specific factor markets perversely work to lower persistence, while variable elasticity demand raises it.

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1 Introduction

State-dependent pricing models have long been viewed as a desirable vehicle for macroeconomic analysis because these models treat the timing of price changes as a profit-maximizing choice. SDP models make it possible to explore how the frequency of price changes responds to variations in model features, such as the form of the monetary policy rule, and to develop the implications of altered adjustment timing for the evolution of other macroeconomic variables. Yet, most macroeconomic investigations employ models with time-dependent pricing (TDP) for two reasons. First, until recently, it has not been possible to construct operational SDP models, frameworks in which the effects of alternative structural features could be explored or that could be readily taken to the data. By contrast, TDP models have proven to be a workhorse for both purposes. Second, macroeconomists have been unsure if incorporating state-dependent pricing behavior would have implications for the dynamics of economic models. Some have speculated that it would be relatively inconsequential in many contexts to adopt SDP rather than the more easily solved TDP setup. Others have expressed the view that incorporating state-dependence is unnecessary because the frequency of price changes does not vary much, at least in moderate inflations.

Using a battery of quantitative general equilibrium models developed along the lines of Dotsey, King, and Wolman [1999], we show that SDP modeling makes a difference in terms of model implications within two major areas of macroeconomic literature. First, as suggested by the 1990s literature – which made use of very different models – we show that there can be a quantitatively important effect of state-dependent pricing for economic outcomes under steady inflation and in response to monetary shocks. State-dependent pricing leads to novel macroeconomic dynamics, including a change in the lead-lag structure of output and inflation. In particular, we display an example in which output responses peak at about a year, while inflation peaks at about two years, in line with Friedman's [1992] summary of dynamic responses for the U.S. and other countries. Such dynamic responses have not previously been obtained in sticky price models, as stressed by Mankiw [2001], and the response depends critically on the price adjustment pattern endogenously chosen by firms. Second, it has been shown that specific factor markets and variable elasticity demand curves generate more persistent output effects of monetary shocks because they moderate the size of price changes that firms make. We investigate whether these results are sensitive to the incorporation of state-dependent pricing. We find that they can be: specific factor markets perversely work to lower persistence in the face of state-dependent pricing, while variable elasticity demand continues to raise it.

The organization of the remainder of the paper is as follows. Section 2 provides a little background on the literatures related to this paper. Section 3 describes the dynamic stochastic general equilibrium (DSGE) models that we employ in the paper. The next two sections of the paper provide our core findings. Section 4 evaluates whether modern quantitative state-dependent models have the four key implications highlighted by the early 1990s SDP literature. Section 5 evaluates the consequences of SDP for the two persistence-enhancing mechanisms stressed by some New Keynesian economists. The common finding of sections 4 and 5 is that state-dependent pricing has a rich set of implications for the dynamics of macroeconomic models, which differ substantially from those of time-dependent models.[1]

2 A little background

We begin by providing a quick overview of the two literatures on which we build.[2]

2.1 The early 1990s literature on state-dependent pricing

A decade ago, macroeconomists viewed dynamic models with state-dependent pricing as having very different implications from time-dependent models. For example, the influential textbook of Blanchard and Fischer [1989] reviewed a number of state-dependent pricing models and stressed how different the conclusions from SDP models were from TDP models, particularly in terms of the effects of monetary disturbances on real activity. Further contributions, published shortly after the textbook, increased the perceived discrepancy between time and state-dependent pricing models. Taken together, these developments through the early 1990s suggested the following ideas: (1) The steady-state pattern of price adjustment depends importantly on the nature of the demand and cost functions of the firm (Sheshinki and Weiss [1977,1983]); (2) The dynamic effect of money on output within state-dependent pricing models is dramatically different from that in time-dependent models, possibly involving complicated cyclical adjustment processes and nonlinear responses (Caplin and Leahy [1991]); (3) The evolution of the price level is substantially affected by the adjustment strategies of firms interacting with heterogeneous prices (Caballero and Engel [1993]); and (4) Multiple equilibria can readily arise in state-dependent pricing models because of complementarities in price-setting, even with the type of exogenous money stock rule

[1] We are pleased to have contributed this work to this volume and to have presented it at the April 2004 Carnegie-Rochester conference in honor of Alan Stockman, "The Economics of Exchange Rates" and the resulting conference volume. Comparing our title and that of the conference, a reader may plausibly wonder if there has been some mistake and our paper has accidentally fallen into the wrong collection. But we do not think that Alan will think so, since he has long argued in various conference discussions that it is important to incorporate state-dependent pricing into open economy modeling. In the last decade, research on "the new open economy macroeconomics" has explored the implications of sticky prices for the behavior of exchange rates. That literature has nearly exclusively concentrated on time-dependent pricing models. (The only exception we know is Landry [2003]). Our results suggest that the NOEM literature, by concentrating on time-dependent pricing models, may have missed some important dynamic implications of price-stickiness and reached inappropriate conclusions about the implications of structural features of models.

[2] The weblink at http://people.bu.edu/rking makes GAUSS and MATLAB code available to those interested in replicating and extending this research.

that nearly always guarantees a unique equilibrium in time-dependent models (Ball and Romer [1991]).

Accordingly, our first objective in this paper is to evaluate whether these core ideas remain as features in dynamic general equilibrium analysis, with specific emphasis on the effects of monetary shocks. Using a basic general equilibrium model, we find support for all of the core ideas from the early 1990s SDP literature.[3]

2.2 Recent work on output responses to monetary shocks

Within the last decade, there has been substantial research into the effects of monetary shocks and monetary policy rules within macroeconomic models that incorporate time-dependent pricing, most frequently along the lines of Taylor [1980] or Calvo [1983]. By contrast, there has been relatively little research on these topics within state-dependent pricing models. Initially, this was because state-dependent pricing models were not operational: it was difficult to solve them under general assumptions about the processes driving economic activity. But the Dotsey, King, and Wolman [1999] state-dependent pricing model provides one laboratory where these questions can be addressed.

One major focus of the recent literature on time-dependent models has been a "search for persistence mechanisms", in response to Chari, Kehoe, and McGrattan's [2000] provocative critique of Taylor-style pricing models. We look at two prominent ideas in the literature on New Keynesian macroeconomics: that there are factor markets specific to individual firms (Ball and Romer [1990], Kimball [1995] and Rotemberg [1996]) and that firms may face non constant elasticity demand curves that are of a "smoothed off kink" form (Ball and Romer [1990] and Kimball [1995]). The basic idea is that each of these mechanisms should moderate the magnitude of price adjustments that a firm would like to make, relative to those in a benchmark setting with flexible factors and a constant elasticity demand, thus making the price level response more sluggish and the nonneutrality of money more protracted.

In particular, we ask whether these New Keynesian mechanisms lead to increases in persistence that survive the introduction of state-dependent pricing. We find that there are very different conclusions for these two models. Within our state-dependent pricing framework, the introduction of local factor markets leads to more rapid price adjustment in the face of steady-state inflation and also more rapid adjustment in response to monetary shocks. Accordingly, time-dependent models that stress this mechanism are implicitly relying on very large costs of price adjustment in order to generate persistence. The variable demand elasticity specification works quite differently. First, in a steady state, this model produces more rapid adjustment – at given adjustment costs–than its constant elasticity counterpart. Second, in response to a monetary shock, this model produces slower adjustment initially than its

[3] There is one exception: our use of linear approximation methods makes it impossible for us to explore the implications of nonlinearities.

constant elasticity counterpart. Taking these two effects together, we find that the variable demand elasticity model enhances persistence in a state-dependent pricing environment.

3 DSGE models

We construct and study four models designed to be representative of much recent work in New Keynesian macroeconomics: production is linear in labor input; consumption and labor effort are separable in utility, and aggregate demand is governed by the quantity theory of money.[4] Thus, the only sophisticated element is the state-dependent pricing mechanism. While use of such simple models is limiting on some dimensions, it allows us to clearly illustrate the implications of state dependence for standard modeling. The four related models are as follows. Model I assumes that there is constant elasticity demand as in Dixit-Stiglitz [1977] and that there is a global labor market, two assumptions that allow for ready aggregation.[5] Model II allows for a variable demand elasticity, structured so that there is a "smoothed off" kink in the demand curve as suggested by Kimball [1995]. Models III and IV assume that there is a local labor market, a device used by authors such as Ball and Romer [1991].

3.1 The demand aggregator

Firms facing a declining demand elasticity will be less aggressive in pricing, as in the classic textbook discussion of a kinked demand curve. To develop a specific aggregator of the class suggested by Kimball [1995], we consider a general expenditure minimization problem facing households,

$$\min_{\{c(i)\}} \int_0^1 P(i)c(i)di \text{ subject to } \int_0^1 D(c(i)/c)di = 1, \tag{1}$$

where c is the total consumption aggregator implicitly defined by the demand aggregator D, which is an increasing concave function, and where $P(i)$ is the nominal price charged by the ith firm on the unit interval.

For any such aggregator, the aggregate price level, P, is implicitly defined by $\int_0^1 (\frac{P(i)}{P})(\frac{c(i)}{c})di = 1$. Expenditure minimization requires that $\frac{\Lambda}{P} D'\left(\frac{c(i)}{c}\right) = \frac{P(i)}{P}$,where Λ is the Lagrange multiplier on the constraint. For aggregators of the Kimball class, the first order condition can be solved to yield demand curves of the form

[4] Relative to our work in Dotsey and King (2001), we therefore abstract from investment and capital formation; from variable utilization; and features of household preferences and constraints that rationalize separate choices of hours and employment or provide motivations for simultaneously varying consumption and hours. We also abstract from the structural features that give rise to money demand.

[5] This is a standard set of assumptions in work on quantitative dynamic models beginning with King and Wolman [1996] and Yun [1996].

$c(i)/c = d(\frac{P(i)}{\Lambda})$, where Λ is determined by the condition $\int_0^1 D(d(\frac{P(i)}{\Lambda})di = 1$. Given the demand curve and the multiplier, the aggregate price level index is determined by $\int_0^1 (\frac{P(i)}{P})(\frac{c(i)}{c})di = 1$.

Our specific aggregator: We use a functional form for D that generates demand curves that are more elastic for firms that adjust their price than for firms whose relative price declines as a result of price fixity,

$$D(x) = \frac{1}{(1+\eta)\gamma}[(1+\eta)x - \eta]^{\gamma} - [1 + \frac{1}{(1+\eta)\gamma}].$$

One nice property of this specification is that the Dixit-Stiglitz aggregator is a special case when $\eta = 0$. The relative demand curves are given by

$$\frac{c(i)}{c} = \frac{1}{1+\eta}[((\frac{P(i)}{P})(\frac{P}{\Lambda}))^{1/(\gamma-1)} + \eta]. \tag{2}$$

i.e., they are the sum of a constant elasticity demand augmented by a constant. The Lagrange multiplier is given by $\frac{\Lambda}{P} = [\int_0^1 (P(i)/P)^{\gamma/(\gamma-1)} di]^{(\gamma-1)/\gamma}$. Conveniently, the aggregate price level index can be written as

$$P = \frac{1}{1+\eta}[\int_0^1 P(i)^{\gamma/(\gamma-1)} di]^{(\gamma-1)/\gamma} + \frac{\eta}{1+\eta}\int_0^1 P(i)di. \tag{3}$$

so that it is the sum of a DS and linear aggregator.

Figure 1 displays examples of the type of demand curves that can be generated with this aggregator. The benchmark case is a Dixit-Stiglitz specification with a demand elasticity of 10 (this involves choosing $\eta = 0$ and $\frac{1}{\gamma-1} = -10$, so that $\gamma = 0.9$. Over the range of demand plotted here, this curve appears nearly linear to the eye in panel A, but panel C confirms that the demand elasticity is constant. To study a variable elasticity demand curve, we choose the parameter η so that the demand curve has elasticity 10 at $c(i)/c = 1$, with γ then controlling the shape of the curve at other points.[6] In the figure, we use a value of $\gamma = 1.02$, which means that a 1.5 percent increase in price yields a 20 percent decrease in demand, which is intermediate between assumptions made by Kimball [1995] and Bergin and Feenstra [2000]. The marginal revenue schedules are plotted in panel B. The elasticity implications are shown in panel C: with $\gamma = 1.02$, a 20 percent decline in output means that the elasticity rises from 10 to 25, while a 10 percent rise in output means that the demand elasticity falls from 10 to 5. Finally, the profit implications at a marginal cost of 0.9 are shown in panel D.

3.2 Firms

We consider two labor market structures: one with global labor markets and the other where labor is tied to a specific firm. In the latter case, we assume that firms are

[6] As γ approaches 1 from above, the demand curve becomes increasingly concave.

small when it comes to assessing marginal cost but large when it comes to pricing.[7]

3.2.1 Factor demand and marginal cost

Production is linear in labor, $y(i) = an(i)$, where $y(i)$ is the output of an individual firm, a is the level of technology, and $n(i)$ is hours worked at a particular firm. Hence, real marginal cost, ψ_t, is given by $\psi_t = w_t/a$ in the case of global factor markets or by $\psi_t(i) = w_t(i)/a$ in the case of specific factor markets.

3.2.2 Price setting

Dotsey, King, and Wolman [1999] develop a model of dynamic pricing that can be readily integrated into a general equilibrium model. It also contains time and state-dependent pricing specifications as special cases. Basic features of our approach are: (i) firms are monopolistic competitors, facing demand for their product given by (2); (ii) within each period, some firms will adjust their price and all adjusting firms will choose the same nominal price P_t^*; (iii) the state of the economy includes a discrete distribution of firms, with firms of type j having last set their price j periods ago at the level P_{t-j}^*, so that we refer to j as the vintage of the price and denote the fractions of firms with this price as θ_{jt} $(j = 1, 2, .., J)$; and (iv) a fraction α_{jt} of vintage j firms decides to adjust its price and a fraction $1 - \alpha_{jt}$ decides not to adjust its price (all vintage J firms choose to adjust).[8]

The fraction of firms, after adjustment, that have a vintage j price is denoted ω_{jt} and these fractions play an important role in our analysis because they serve as weights in various aggregation contexts. The total fraction of adjusting firms (ω_{0t}) satisfies $\omega_{0t} = \sum_{j=1}^{J} \alpha_{jt}\theta_{jt}$ and fractions of firms $\omega_{jt} = (1 - \alpha_{jt}) \cdot \theta_{jt}$ maintain the price that they previously set in period $t - j$. Using these weights, for example, the perfect price level index is given by

$$P_t = \frac{1}{1+\eta}[\sum_{j=0}^{J-1} \omega_{jt} P_t(j)^{\gamma/(\gamma-1)}]^{(\gamma-1)/\gamma} + \frac{\eta}{1+\eta}\sum_{j=0}^{J-1} \omega_{jt} P_t(j).$$

Finally, the "beginning of period" fractions are mechanically related to the "end of period" fractions via $\theta_{j+1,t+1} = \omega_{jt}$ for $j = 0, 1, ..., J - 1$.

If the adjustment fractions α_j are treated as fixed through time, then the model collapses to Levin [1991], so that it contains Calvo [1983] and Taylor [1980] as special cases. In this interpretation, α_j plays two roles: it is the fraction of firms given the opportunity to adjust within a period, and it is also the probability of an individual firm being allowed to adjust after j periods, conditional on not having adjusted for $j - 1$ periods. Under state-dependent pricing we employ randomized fixed costs of

[7] The local labor market is not quite the "yeoman farmer" setting, as we allow individual workers to insure against the consumption risks associated with individual market conditions.

[8] Since all firms are in one of these situations, $\sum_{i=1}^{J} \theta_{it} = 1$.

adjustment to induce discrete adjustment by individual firms, while allowing for an adjustment rate that responds smoothly to the aggregate state of the economy.

In both the time-dependent and state-dependent settings, the firm's optimal pricing decision can be described using a dynamic programming approach. For example, a firm that last changed its price j periods ago must choose between continuing with a fixed nominal price, which implies a relative price of p_{jt}, ($p_{jt} = P^*_{t-j}/P_t$), and paying a fixed cost of adjusting its price (ξ). Each j-type firm has a value function of the form

$$v(p_t, \xi_t, s_t) = \max\{v_{jt}, v_{0t}\} \tag{4}$$

with

$$v_{jt} = z(p_{jt}, s_t) + \beta E_t \tfrac{\lambda_{t+1}}{\lambda_t} v(p_{j+1,t} \tfrac{P_t}{P_{t+1}}, \xi_{t+1}, s_{t+1})$$

$$v_{0t} = \max_{p^*_t}[z(p^*_t) + \beta E_t \tfrac{\lambda_{t+1}}{\lambda_t} v(p^*_t \tfrac{P_t}{P_{t+1}}, \xi_{t+1}, s_{t+1})] - w_t \xi_t$$

being, respectively, the values if the firm adjusts (v_{0t}) or does not adjust (v_{jt}). In these functions and below, s_t is a state vector that governs the evolution of the firm's demand and costs and $\frac{\lambda_{t+1}}{\lambda_t}$ is the ratio of future to current marginal utility, which is the appropriate discount factor. Real profits are given by $z(p_{jt}) = [p_{jt} - \psi_{jt}]c_{jt}$.

The dynamic program (4) implies that the optimal price satisfies an Euler equation that involves balancing pricing effects on current and expected future profits. That is, as part of an optimal plan, firms that reset their price will choose a price that satisfies

$$0 = \frac{\partial z(p^*_t, s_t)}{\partial p^*_t} + \beta E_t[\frac{\lambda_{t+1}}{\lambda_t} \frac{\partial v(p^*_t \frac{P_t}{P_{t+1}}, \xi_{t+1}, s_{t+1})}{\partial p^*_t}]. \tag{5}$$

Furthermore, for any given state of the economy, there is a unique cutoff value of the price-adjustment cost for each firm charging a relative price of p. All firms that draw an adjustment cost lower than this cutoff will optimally choose to adjust their price.[9] The endogenous adjustment fraction is determined by the menu cost of the marginal firm being just equal to the value gained, i.e.,

$$\xi(\alpha_{jt}) w_{0t} = v_{0t} - v_{jt}.$$

In the time dependent case, the fixed cost is either zero or infinite depending on when the firm last changed its price.

Iterating the Euler equation (5) forward, the optimal relative price, p^*_t, can be related to current and expected future variables:

$$p^*_t = \frac{\sum_{j=0}^{J-1} \beta^j E_t\{(\omega_{j,t+,j}/\omega_{0,t}) \cdot (\lambda_{t+j}/\lambda_t) \cdot \psi_{j,t+j} \cdot \epsilon_{j,t+j} \cdot c_{t+j}\}}{\sum_{j=0}^{J-1} \beta^j E_t\{(\omega_{j,t+j}/\omega_{0,t}) \cdot (\lambda_{t+j}/\lambda_t) \cdot (\epsilon_{j,t+j} - 1) \cdot (P_{t+j}/P_t) \cdot c_{t+j}\}}, \tag{6}$$

[9] As long as the inflation rate is nonzero and the maximum adjustment cost is finite, there will be a maximum number of periods during which any firm will leave its price unchanged. Thus, the state space for this problem is finite.

where $(\omega_{j,t+,j}/\omega_{0,t}) = (1-\alpha_{j,t+j}) \cdot (1-\alpha_{j-1,t+j-1}) \cdot ... \cdot (1-\alpha_{1,t+1})$ is the probability of nonadjustment from t through $t+j$, and $\epsilon_{j,t+j}$ is the elasticity of demand facing a firm with relative output of $c_{j,t+j}/c_{t+j}$.[10] According to (6), the optimal relative price is a fixed markup over real marginal cost ($p^* = \frac{\varepsilon}{\varepsilon-1}\psi$) if real marginal cost, the demand elasticity, and the price level are expected to be constant over time. More generally, (6) illustrates that the optimal price varies with current and expected future demands, aggregate price levels, real marginal costs, discount factors, the elasticity of demand, and adjustment probabilities. Intuitively, firms know that the price they set today may also apply in future periods, so the expected state of the economy in those future periods affects the price that they choose today. If, for example, marginal cost is expected to be high next period, a firm will set a high price in the current period, so as not to sell at a loss next period. Similarly, if the elasticity of demand is expected to be high next period, the firm will not raise its price as much in response to a nominal shock because it will lose a lot of business in the future. The conditional probability terms $(\omega_{j,t+,j}/\omega_{0,t})$ are present in time-dependent models, but they are not time-varying. In our setup, these conditional probability terms effectively modify the discount factor in a time-varying manner: a high probability of adjustment in some future period leads the firm to set a price that heavily discounts the effects on profits beyond that period.

3.3 The household

We want to have a household objective function that does not change radically when we consider local labor markets. Therefore, as in Dotsey and King (2001), we assume that idiosyncratic risks are pooled by households, so that they behave as if there is a super-household that chooses consumption and labor for each of its members. This avoids the potential complication of differential wealth among individuals that would arise when workers are tied to specific firms. The unified household approach conveniently provides full income insurance. Specifically, the household solves

$$\max_{c_{jt},n_{jt}} E_0\{\sum_t \beta^t \sum_j \omega_{jt}[\frac{1}{1-\sigma}c_{jt}^{1-\sigma} - \frac{\chi}{1+\phi}n_{jt}^{1+\phi}]\}$$
$$\text{subject to: } [\sum_j \omega_{jt}c_{jt}] \leq \sum_j \omega_{jt}[w_{jt}n_{jt} + z_{jt}],$$

where c_j and n_j are the consumption and labor effort of a household member working for a type j firm, and z_{jt} is the profits remitted to the household by a type j

[10] The pricing restriction (6) is a natural generalization of the type derived in time-dependent settings with exogenous adjustment probabilities that are constant through time as in Calvo [1983] (see, for example, King and Wolman [1996] and Yun [1996]). If the aggregator takes on a constant elasticity of substitution form, then the optimal pricing (in equation (6)) becomes the familiar expression found in Dotsey et al. (1999).

firm. In this setting–full insurance and utility that is separable in labor effort and consumption–all households consume the same amount, c_t. The first order condition determining labor supply is

$$w_{jt} = c_t^{\sigma} n_{jt}^{\phi},$$

and, hence, ϕ^{-1} is the Frisch labor supply elasticity.

We further impose the money demand relationship $M_t/P_t = c_t$. Ultimately, the level of nominal aggregate demand is governed by this relationship along with the central bank's supply of money.

3.4 Monetary policy and market clearing

The model is closed by assuming that nominal money supply growth follows an autoregressive process,

$$\Delta M_t = \rho \Delta M_{t-1} + m_t,$$

where m is i.i.d. and normally distributed. Depending on the structure of factor markets, equilibrium involves either a wage rate or a vector of wage rates that clear the labor market while simultaneously implying utility maximization and cost minimization. Further, the aggregate price level is such that the money demand equals money supply, and individual firms' prices are value maximizing.[11]

4 Evaluating predictions about SDP

We now evaluate whether the predictions of the early 1990s literature carry over to our dynamic general equilibrium setting. Throughout this section, we assume that the adjustment cost parameters are such that there is an approximately quadratic hazard function, in a sense made more specific below. In this section, we also restrict attention to the models in which there is a global labor market, so that there is a single real wage w_t.

We choose preference parameter values that produce a low elasticity of marginal cost with respect to real output, assuming that $\sigma = 0.25$ and $\phi = 0.05$ implying that the marginal cost elasticity is about 0.30.[12] Many studies in the early 1990s literature explicitly or implicitly assumed low elasticities of marginal cost with respect to output. For example, in their analyses of real rigidity [1990] and multiple equilibria

[11] There is no barrier to considering alternative monetary policy rules, such as interest rate rules, in our setting. However, we stick with the money supply rule for comparability with the results of other studies.

[12] Given that the household efficiency condition is $w_t = c_t^{\sigma} n_t^{\phi}$, given that consumption is equal to output, and given that labor is approximately equal to output, the elasticity is approximately $\sigma + \phi$..

[1991], Ball and Romer assumed explicitly that utility was linear in consumption ($\sigma = 0$) and that utility was close to linear in work (ϕ was small).[13]

4.1 Adjustment timing and the age distribution of prices

The first two predictions comes from Sheshinski-Weiss [1977, 1983]: (i) relatively small menu costs may lead firms to adopt lengthy periods of price inactivity within an inflationary steady state; and (ii) the shape of the firm's demand and cost conditions will be important for its frequency of price adjustment.

Figure 2 displays the hazard rates at an annual rate of 4 percent inflation for model I (Dixit-Stiglitz demand) and model II (Kimball kinked demand).[14] To begin, note that the chosen adjustment cost structure indeed leads to steady-state hazard functions that are roughly quadratic in the log relative price deviation, since log $(P_t^*) - \log(P_{t-j}^*) = j\log(\pi)$, where π is the steady-state inflation rate. The figure illustrates that the structure of demand and cost has a quantitatively important effect on hazard rates. Firms choose to adjust more frequently if there is a kinked demand curve. The average age of a price in the global DS model is 3.9 quarters and it is 2.2 quarters in the global K model. The expected duration of price fixity is about 8 quarters in the global DS model and it is 4.8 quarters in the global K model.[15]

The maximum adjustment cost is about 7.5 percent of production time in both of these economies, which is quite large (we call the fraction of total time devoted to price adjustment B and set it to $B = 0.015$: since the steady-state fraction of time that individuals devote to market work is $n = 0.20$, it follows that the adjustment cost is 7.5 percent of production work). However, because the highest adjustment cost is rarely paid, the average level of adjustment costs is only .42 percent of production time in model I (DS-global) and it is 0.86 percent of production time in model II (K-global), which are much smaller numbers. Another way of thinking about the magnitude of these costs is to measure the resources spent adjusting prices relative to sales, which is sometimes measured in the empirical literature on price adjustment costs: these are 0.37 percent and0 .78percent, respectively, for the two economies.

[13] Additional calibration information is as follows. First, we assume that there is a demand elasticity of 10 at the relative price of 1. With DS demand, this pins down $\varepsilon = -10$. With the K demand, we assume that $\gamma = 1.02$, leading to the demand specification displayed in Figure 1.

[14] Both of the demand models we study in this section satisfy a condition developed by Sheshinksi and Weiss, which is that $p\frac{\partial z}{\partial p}$ is decreasing in p. In their framework, this condition must be imposed if a higher rate of inflation is to increase the frequency of price adjustment.

[15] The two features are calculated as follows. First, the average age of price is just $\sum_{j=0}^{J-1} j * \omega_j$, where we adopt a "start of period" age convention. Second, the expected duration of price fixity is $\sum_{j=1}^{J-1} \frac{\omega_j}{\omega_0} = \frac{1}{\omega_0}$. In this expression, the probability of a price "surviving" until age j is $\frac{\omega_j}{\omega_0} = (1-\alpha_1)(1-\alpha_2)...(1-\alpha_j)$ so that the expression is the sum of the survival probabilities times the additional length of price fixity (1) that derives from each survival. But since the survival probabilities sum to one, there is a particularly simple form of this expression.

Figure 3 helps us understand why there is more rapid price adjustment in the economy with K-demand than with DS-demand: as a function of the firm's relative price, profits decline much more sharply when there are deviations of price from the $p = 1$ value that would be optimal in the absence of adjustment costs. The solid and dashed lines in each respective case are the value for all possible prices, while the stars and circles correspond to the prices actually chosen by the firm in the steady state.

4.2 Dynamic effects of monetary shocks

In the early 1990s literature on state-dependent pricing, Caplin and Leahy [1991] suggested that there would be strikingly different dynamics with endogenous timing of adjustment, in which the evolving distribution would play a critical role. In this section, we look at the dynamic response of output to an increase in the level of the money stock, which rises on impact by 1 percent and then gradually increases to 2 percent above its initial value.[16]

4.2.1 Model I: SDP dynamics with constant elasticity demand

The tendency for "front-loading" of price adjustments has been a much-discussed feature of sticky price models: if a firm expects the price level to increase in the future and if the firm expects to hold its nominal price fixed for a substantial time, then it will aggressively adjust its price in response to the expected future inflation. In Figure 4, it is clear that front-loading carries over to an SDP environment: the "reset price," which is the price set by adjusting firms, increases more than one-for-one with both the money stock and the price level.[17]

The SDP environment also involves dynamics that are very different from those in time-dependent models of the Taylor-Calvo-Levin form. Notably, there are complicated oscillatory dynamics in the price level, output, labor, marginal cost, and inflation. In fact, a fair reaction to these dynamics is that they are very far from any estimates that derive from vector autoregressions or other methods of tracing out empirical responses to monetary changes. But just as with the dynamic responses derived analytically by Caplin-Leahy [1991], which were also far from such empirical estimates, they illustrate that SDP models can deliver dramatically different dynamics for output and other variables than those in standard time-dependent models.

4.2.2 Model II: SDP dynamics with kinked demand

We next consider the effect of the same monetary shock in a setting with a "smoothed off kinked-demand curve" along the lines suggested by Kimball [1995]. There are

[16] That is, there is a value of $\rho = 0.5$ in the money supply specification.

[17] The reset price can increase by substantially more than the price level because only a fraction of the firms are adjusting prices.

very different dynamic responses displayed in Figure 5. Notably, in contrast to the DS model of the last subsection, the reset price is much less responsive under this specification: there is no "front-loading" of price adjustments. There are two very intriguing features. First, the stimulation of real activity lasts for about 10 quarters, but is now followed by a period of real contraction, which lasts for a substantial period but does not undo the effect of the initial stimulation. Second, while the real expansion of economic activity peaks after four quarters, the peak effect on inflation occurs much later.

These dynamics are not so evidentially at variance with various kinds of macroeconomic evidence.[18] First, in a critique of TDP sticky price models, Mankiw [2001] has argued that any macroeconomic model of the Phillips curve must produce a delayed surge in inflation that follows an initial real stimulation of economic activity. He uses this set of observations to critique standard New Keynesian sticky price models with Calvo price-setting. However, our simple state-dependent pricing model outcomes are reminiscent of Friedman's [1992] description of the dynamic effects of a change in money growth and they are also broadly consistent with Mankiw's description. Specifically, in response to a monetary shock, Friedman stressed that output responds before inflation. He also suggested that the output response is delayed by about six to nine months and is distributed over time.

In model II, the response of inflation is also distributed over time, but it occurs with more of a lag – up to 12 to 18 months. With respect to output, we do not produce the real activity delays that Friedman describes, although output in our model does take two to three quarters before achieving its maximal response. Significantly, however, the response of model inflation is delayed and does not peak until about six quarters.

4.2.3 Contrasting SDP with TDP in the kinked demand case

We now contrast the SDP model with a very specific TDP alternative, which we think is a natural benchmark: we use a TDP model that has exactly the same steady-state as the SDP model studied in Figure 5, but we freeze the adjustment rates at their steady state values. Differences between the dynamic responses, as reported in Figure 6, then are attributable to whether adjustment rates vary in the face of a monetary shock. First, the price level increases at about the same rate in the TDP (solid line) and SDP (dashed line) models during the first year, but then it increases more rapidly in the SDP model, leading to a surge in inflation during the second year. (The 'reset price' under TDP is marked with a '$\Diamond$', while that under SDP is marked with an '$\Box$'). Second, the oscillatory dynamics are attributable to changes in the rate of adjustment, since they are not present in the TDP variant.

[18] Anyone who has estimated vector autoregressions knows that there are many specifications that show monetary disturbances having an initial positive effect on real economic activity and then a negative one (although specification selection means that fewer of these are reported than estimated).

4.2.4 Understanding the incentives for adjustment

What are the incentives for price adjustment in the dynamic models? At one level, the answer is easy: there is a greater rate of adjustment if there is a greater value to adjusting. However, the determinants of $v_{0t} - v_{jt}$ are complicated, within and across the DS and K-demand models. Accordingly, we start here by focusing directly on a measure of one-period profit, which is revealing about the difference in adjustment incentives. We then discuss aspects of a dynamic decomposition for the K-demand case. Finally, we consider the evolution of the price level once again, displaying the importance of adjustment timing quantitatively.

Contrasting adjustment incentives: a static perspective To begin, we note that a rise in output and an associated increase in marginal cost are important features of Figure 5. We therefore start by looking at a measure of the static gain to price adjustment in the face of a 1 percent rise in real marginal cost, defined as

$$\frac{z(p^*, \psi) - z(p, \psi)}{z}$$

where $z(p, \psi) = pd(p) - \psi d(p)$; $z(p^*, \psi)$ is the level of profits at the statically optimal price; and $z = z(1, \psi = \frac{\varepsilon-1}{\varepsilon})$ is the level of profit under fully flexible prices.

We begin by graphing this measure as the dashed line in Figure 7 for the K-demand model. First, the price p^* is just above one because the firm faces the rapidly declining profit illustrated in Figure 3, and thus, a firm that is free to raise its price will not do so by very much in the face of the increase in marginal cost. Second, given the desirability of a small adjustment, it is intuitive that there is also a small loss of maintaining price $p = 1$, the price that would be optimal in the absence of the rise in marginal cost. Hence, in the kinked demand world, a firm with $p = 1$ also has only a small incentive to pay a fixed cost to adjust its price. However, should its price deviate significantly from $p = 1$, then the firm facing a kinked demand has a large incentive to adjust: this was the feature that led to more rapid steady-state adjustment under K-demand in Figure 2 above.

We also find the figure helpful in thinking about why there are larger incentives for price adjustment in response to a rise in marginal cost with DS-demand rather than K-demand: the DS model leads to larger desired price adjustments and therefore larger gains to adjustment near $p = 1$. But it leads to relatively smaller effects with large departures from $p = 1$, so that it is also compatible with more extended stickiness in the steady state.

A dynamic perspective on the adjustment with kinked demand The adjustment rate for a firm of vintage j is implicitly given by $\xi(\alpha_{jt})w_{0t} = v_{0t} - v_{jt}$. Accordingly, we can take a first order approximation to this expression and deduce that

$$\xi_\alpha(\alpha_j)(\alpha_{jt}-\alpha_j) = -[\frac{p_j\partial v_{jt}/\partial p_{jt}}{w_0}] * [\log(p_{jt}) - \log(p_j)] + \text{ other terms}$$

so that it is possible to explore the effects of the price level on adjustment incentives, holding fixed other factors. Specifically, we take the equilibrium solution for $\log(p_{jt})-\log(p_j)$ and then construct a synthetic series $\widetilde{\alpha}_{jt}-\alpha_j$ using the equation above. Given these synthetic series, we can also construct a synthetic series for the vintages, $\widetilde{\omega}_{jt}-\omega_j$, which is a dynamic simulation of sorts since it obeys the dynamic equations

$$\begin{aligned}
\widetilde{\omega}_{jt}-\omega_j &= (1-\alpha_j)(\widetilde{\omega}_{j-1,t-1}-\omega_j)-\omega_{j-1}(\widetilde{\alpha}_{jt}-\alpha_j) \\
\widetilde{\omega}_{0t}-\omega_0 &= \sum_{j=0}^{J-1}[\alpha_j(\widetilde{\omega}_{j-1,t-1}-\omega_j)+\omega_j(\widetilde{\alpha}_{jt}-\alpha_j)].
\end{aligned}$$

That is, the synthetic series for $\widetilde{\omega}_{jt}$ is constructed solely on the basis of variations in the synthetic adjustment rates $\{\widetilde{\alpha}_{jt}\}_{j,t}$, so that it too involves only the effects of p_{jt}. We have undertaken this decomposition and have found that effects of p_{jt} are dominant on α_{jt} – in the sense of high R^2 – except for those firms that just adjusted. The price effects capture variations in vintage fractions (ω_{jt}) virtually completely.[19]

The evolution of the price level once again Caballero and Engel [1993] emphasized that the behavior of the price level would be influenced by the interaction of the evolving distribution of prices and the evolving probability that individual price adjustments would take place. To explore this channel within our model, we consider the movement of a linear aggregate of the price level, $\overline{P}_t = \sum_{j=0}^{J-1}\omega_{jt}P_{jt}$. This price level can be decomposed directly into a part $\sum_{j=0}^{J-1}\omega_j P_{jt}$ that is the effect of price stickiness when steady-state weights are maintained and an additional component $\sum_{j=0}^{J-1}(\omega_{jt}-\omega_j)P_{jt}$ that derives from the interaction of evolving adjustment rates and

[19] See Appendix D, for these simulations.The adjustment rate for a firm of vintage j is implicitly given by $\xi(\alpha_{jt})w_{0t} = v_{0t}-v_{jt}$, which we can write as

$$\begin{aligned}
w_{0t}\xi(\alpha_{jt}) &= -z_{jt}+(v_{0,t}-\beta E_t[\frac{\lambda_{t+1}}{\lambda_t}v_{0,t+1}]) \\
&\quad +\beta E_t[\frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_{j+1,t+1})(w_{0,t+1}\xi(\alpha_{j+1,t+1})+w_{0,t+1}\Xi(\alpha_{j+1,t+1})]
\end{aligned}$$

Accordingly, it is more generally possible to link variations in adjustment rates to three factors: profits (z_{jt}); a measure of the urgency of adjustment ($v_{0,t}-\beta E_t[\frac{\lambda_{t+1}}{\lambda_t}v_{0,t+1}]$); and an "option value" of adjustment term that involves future adjustment costs. Further, the effects of profitability can be decomposed into consequences of relative price variations; marginal cost variations; and aggregate demand variations. We have undertaken some exploration of the analytics and quantitative performance of such measures in our model, but these experiments are not reported because of the dominance of the effect of the price level on relative prices.

past prices. That is, a useful decomposition of the price level suggested by this model is

$$\overline{P}_t = \sum_{j=0}^{J-1} \omega_{jt} P_{jt} = \sum_{j=0}^{J-1} \omega_j P_{jt} + \sum_{j=0}^{J-1} (\omega_{jt} - \omega_j) P_{jt} \tag{7}$$

In our framework, we want to calculate a linear decomposition that captures the elements highlighted by (7). To develop such a linear decomposition, we begin by noting that the linear aggregate is related to the perfect (exact, nonlinear) price index (3) according to $\frac{\overline{P}_t}{P_t} = [\sum_{j=0}^{J-1} \omega_{jt} \frac{P_{jt}}{P_t}]$,. Differentiating this expression, we find that we can express the motion of the linear aggregate as follows,

$$\begin{aligned} \frac{d\overline{P}_t}{\overline{P}_t} &= \frac{dP_t}{P_t} + \frac{1}{\sum_{j=0}^{J-1} \omega_j p_j} \{ [\sum_{j=0}^{J-1} \omega_j p_j \frac{dp_{jt}}{p_{jt}}] + [\sum_{j=0}^{J-1} p_j d\omega_{jt}] \} \\ &= \frac{1}{\sum_{j=0}^{J-1} \omega_j p_j} \{ [\sum_{j=0}^{J-1} \omega_j p_j (\frac{dp_{jt}}{p_{jt}} + \frac{dP_t}{P_t})] + [\sum_{j=0}^{J-1} p_j d\omega_{jt}] \}. \end{aligned}$$

i.e., as the sum of a term that captures the effect of nominal price adjustments at fixed weights and a term that captures the effects of changes in adjustment probabilities. Applying this decomposition to the K-demand model, we produce Figure 8.

The top panel of this figure shows that the exact price level (3) and the linear aggregator are indistinguishable to the eye in this economy, but that there is an important difference between these and the fixed hazard part of the price level, which is $\frac{1}{\sum_{j=0}^{J-1} \omega_j p_j} \{ [\sum_{j=0}^{J-1} \omega_j p_j (\frac{dp_{jt}}{p_{jt}} + \frac{dP_t}{P_t})] \}$. Interestingly, this difference is minuscule during the first few quarters after the monetary shock hits, but it becomes important later on, rising with inflation, as the second panel shows. The background to this panel is Figure 2, which shows that 22 percent of the firms in the economy are adjusting each period in steady-state. The second panel of the figure shows that this fraction rises by about 3 percent during the second year after the shock, which is when inflation peaks (adjustment rates here are measured as a deviation from the steady-state level).

On the basis of this analysis, we conclude that this case has strong effects of the type identified by Caballero and Engel [1993]: the interaction of the nearly quadratic hazard, sticky nominal prices, and the price level is at the heart of understanding the dynamics of inflation. Concretely, the delayed response in inflation shown in Figure 5 arises because there is initially little movement in the price level, so that firms have little incentive to pay to adjust prices. However, as the price level continues to rise, more firms have this incentive and their collective action produces a further rise in the price level, which additionally reinforces the extent of adjustment.

4.3 Multiplicity and nonexistence

Ball and Romer [1991] highlight the possibility of multiple equilibria in basic SDP models, stressing that changes in the price level can alter the privately optimal pattern of price adjustment for firms. In our DSGE model, we have found that there apparently is a substantial part of the parameter space in which there are both multiplicity and nonexistence according to the criteria of Blanchard and Kahn [1980]. That we find these regions in the K-demand case is perhaps not too surprising, given the central role that the price level played in triggering adjustment in the prior section.

In Figure 9, for the K-demand model, we calculate the number of stable eigenvalues of the dynamic model at each point in a grid of the adjustment cost parameter B – which is the largest value the adjustment cost can be – and the labor utility parameter ϕ. If there is a '*' in the figure, it means that there is a unique, stable rational expectations solution: the number of stable eigenvalues is equal to the number of predetermined variables. Since we are studying $B = 0.015$ and $\phi = 0.05$ in the figures above, we start by noting that there is a '*' in that location. We also note that there is a region around this point in which there is uniqueness, but that it is close to the border with a region of nonexistence. At other points in the figure, there are fewer stable eigenvalues than predetermined variables, which implies nonexistence according to Blanchard-Khan, so that we put an 'o' in that location. Finally, there are points in which there are more stable eigenvalues than predetermined variables, which implies multiplicity (nonuniqueness) according to Blanchard-Khan, so that we put a '$\diamondsuit$' in that location. John and Wolman [2004] have begun the important work of exploring the conditions under which dynamic multiple equilibria occur in SDP models, together with providing economic interpretation about these findings. Their analysis suggests that this is a complex and subtle topic.

It is important to stress that nonexistence and nonuniqueness do not always arise. For one example, if we were to produce a version of this figure for the comparable TDP model, then all points would be a unique equilibrium: this buttresses the Ball-Romer idea that multiplicity is related to state dependence. For another, a version of this figure for the DS model examined above (demand elasticity =10) would also lead to uniqueness for all parameter values in this grid. Finally, in exploring both K and DS models with a global labor market, a higher elasticity of marginal cost to output (over one) and the adjustment cost distribution similar to that used in DKW [1999], we also did not find nonexistence or nonuniqueness. But in some investigations, nonexistence and nonuniqueness can arise for precisely the parameter values that interest a researcher. For example, we would like to look at adjustment cost specifications with a B smaller than 0.015, so as to reduce the extent of steady-state stickiness. But we cannot because this moves us out of the region of solvability. Moreover, in exploring SDP model dynamics in the current investigation, we have encountered – particularly in models with local factor markets – many cases in which there are apparently multiple equilibria or there is nonexistence. In our experience, Figure 9 is representative in that it suggests that there is indeed a complicated relationship, since the relevant regions are discontinuous.

5 Specific factors and persistence

An important line of macroeconomic research has explored the implications of the various sticky-price model features for the time paths of real and nominal variables, with one particular topic being the persistence of real effects in the wake of Chari, Kehoe and McGrattan [2000]. Work by Kimball [1995] and Rotemberg [1996] viewed each firm as having a pool of workers to draw from as opposed to buying labor in a competitive market. Hence, even if the firm purchases labor competitively, it knows that an increase (decrease) in its demand will raise (lower) the wage rate and it takes this into account in pricing its product. The intuition behind this result is as follows. An increase in the current price cuts demand, which lowers marginal cost when factors are specific. In turn, the lower marginal cost makes it efficient to price less aggressively. For this reason, Kimball [1995] and Rotemberg [1996] suggested there would be increased price sluggishness and persistence if one switches from a global to local view of factor markets. They also discuss the fact that in setting a low price, the firm must balance the fact that there will be high demand in the future and that this output must be produced at high cost, but they conclude that the overall effect is to make firms price less aggressively and to increase price level sluggishness.

We use different parameter values to explore this idea. First, we assume that there is a higher elasticity of marginal cost with respect to output and in particular that it is about 1.5 (we do this by assuming $\sigma = 1$ and $\phi = 0.5$). Second, since Kimball and Rotemberg both used Calvo-like models, we assume that there is an adjustment cost structure that makes the DS-global version into an "approximate Calvo" model within the steady-state, having an adjustment hazard of about 0.2 for eight quarters before complete adjustment occurs.[20]

5.1 The promise

We begin by illustrating the promise of the specific factors mechanism, calculating the output impulse responses for an approximate Calvo model and displaying it in Figure 10.[21] The dramatic returns to the introduction specific factors appears in the output responses of models III and IV, with specific factors alone (model III) producing virtually the same persistence as the variable elasticity of demand specification (II).[22]

[20] We also choose these parameter values – more in line with values used in the real business cycle literature such as King, Plosser, and Rebelo [1988] – because there is no endogenous persistence in this case, as emphasized by Chari, Kehoe, and McGrattan [2000]. Hence, any increase in persistence will be attributable to specific factors. The choice also allows us to evaluate whether, as is sometimes suggested, local factor markets substitute for a low marginal cost elasticity in generating persistence.

[21] The weights are those from the global DS model developed in the next section.

[22] In his conference comments, Susanto Basu stressed the symmetry of specific factors and variable demand elasticity under Calvo pricing. Thus, our parametric specification – although not designed

The combination of the two New Keynesian mechanisms, as originally suggested by Kimball, yields a great deal of persistence.

5.2 Approximate Calvo

We want to explore the effects of state dependence within a battery of models that have an approximate Calvo form, i.e., a steady-state hazard that is roughly constant for a number of periods. Accordingly, we select the parameters of our cost function so that there is a flat hazard for the DS-global setting for eight quarters, which is a "truncated Calvo" steady state. The necessary cost function, $\xi(\alpha)$, is one that is fairly flat until $\alpha = .2$ then rises very sharply to close to the maximum cost. Faced with this adjustment cost, firms with a range of different values of $(v_0 - v_j)/w_0$ will all choose $\alpha = 0.2$. When $(v_0 - v_j)/w_0 \geq B = 0.015$, then all firms will choose to adjust ($\alpha = 1$).

With this cost structure in hand, we can explore the effect of changing the structure of demand and the effect of localizing factors on hazard rates and vintage fractions, as we did previously for the alternative cost specification. Figure 11 displays the results, revealing some worth highlighting. First, as suggested above, the figure displays an "approximate Calvo" form of adjustment: the optimal hazard is about 0.2 until full adjustment occurs. Second, in the global factor market setting, as above, the shift from DS-demand to K-demand lowers the number of periods over which there is incomplete adjustment by firms, cutting it from eight in the DS case to four in the K-demand case. Third, for both of the local market cases, the results are dramatic: moving from global to local markets cuts the interval of partial adjustment to just one period. To understand this, we return to the original intuition from Kimball [1995] and Rotemberg [1996]: with a fixed hazard, a firm sets its price relatively less aggressively than under global markets because it wants to take advantage of low current marginal cost, which occurs when price is raised above the benchmark value of one. In doing so, as discussed above, it must balance the fact that there will be high demand in the future and that this output must be produced at high cost. But these future periods of low profits – resulting from high demand and high cost occurring together – can be avoided through payment of an adjustment cost, so that the firm makes aggressive use of this option in both local market settings. In fact, under the current parameterization, it keeps prices fixed for only two periods (including the initial period of price adjustment).

This dramatic implication of very short intervals of price fixity for the DS-local and K-local models *could* be altered by assuming larger values of the maximum price adjustment cost (which is here set equal to 0.015). But, then, the conclusion would be that models with local factor markets require substantially higher adjustment costs to obtain a specified pattern of "near Calvo" adjustment. In fact, in order to produce price fixity of four periods in the DS case, we must ramp up adjustment costs so that

for this purpose – corresponds to essentially equivalent strength of these two mechanisms.

5.85 percent of labor effort is devoted to price changes at a cost of 5.5 percent of sales. Thus, a TDP pricing model with local labor markets and four vintages of firms is ignoring tremendous incentives that firms have for adjusting their price. This level of menu costs strikes us as implausible.

5.3 Consequences of endogenous adjustment

With exogenous adjustment timing, there were large gains in persistence from moving from global to local markets. We now explore the implications of moving from global to local factor markets with endogenous adjustment timing, using the same cost of adjustment structure discussed above. Figure 12 displays the effect of moving from a global to local market under state-dependent pricing with the DS-demand structure and Figure 13 displays the effect with the K-demand structure: each indicates that the persistence gain suggested by Figure 10 also turns into a persistence loss under state-dependent pricing.

Looking across this pair of figures, it is clear that there is more persistence with model IV (K-local) than with model III (DS-local). However, more importantly, this pair of figures illustrates a principle: economic mechanisms that have one set of consequences under time-dependent pricing (as in Figure 10) can have a very different set of consequences under state-dependent pricing (as in Figure 12 and 13) because the mechanisms alter the incentives agents have to adjust the timing of their price changes.

5.4 The effect of K-demand on dynamics once again

It is important to stress that persistence is not necessarily reduced when a model feature lowers the number of periods of price-fixity in the steady-state, which we illustrate by considering the global market case under "approximate Calvo" cost structure of this section. As background, Figure 11 shows that the number of periods of price-fixity is roughly halved when DS-demand is replaced by K-demand. Figure 14 shows the effects of moving from DS-demand to K-demand on the dynamic response to a monetary shock (in this diagram, a solid line refers to the K-demand model and a dashed line refers to the DS model). Despite the smaller number of price vintages, the K-demand model continues to have the important implication discussed above: the K-demand makes firms less aggressive on the pricing front, converting the more than 2 percent change in the reset price on impact to about a 0.8 percent change in the reset price on impact. That is, even though the current framework is one with a higher elasticity of marginal cost to output and a different structure of adjustment costs, the price level still is initially more sluggish than under DS-demand, which brings about both a larger real output response and a more persistent one.

In terms of the dynamics of the inflation rate, the K-demand model also leads to a peak inflation rate that lags the output peak, although it does so only by one or two quarters in this case. However, the change in the adjustment cost function from one involving a nearly quadratic hazard to one involving a nearly constant hazard does mean that there is a quite different decomposition of the sources of variations in the price level. If we were to reproduce Figure 8 for the current adjustment cost structure, then we would find that there was only a minuscule difference between the various price level measures and a very small change in the fraction of firms altering the timing of their price adjustment in the face of the monetary shock.

6 Summary and conclusions

What are the implications of state-dependent pricing models for dynamic macroeconomic modeling? In this paper, we showed that these are rich and varied, working within a battery of quantitative dynamic general equilibrium models.

We began by investigating whether some of the results of the 1990s literature on state-dependent pricing carried over to our models, which are constructed along the lines proposed by Dotsey, King, and Wolman [1999]. This earlier literature reached the general conclusion that SDP models were very different from the more commonly employed time-dependent pricing models (TDP models). More specifically, it suggested the following ideas: (1) the steady-state pattern of price adjustment depends importantly on the nature of the demand and cost functions of the firm; (2) the dynamic effect of money on output within state-dependent pricing models is dramatically different from that in time-dependent models, possibly involving complicated cyclical adjustment processes and nonlinear responses; (3) the evolution of the price level is substantially affected by the adjustment strategies of firms interacting with heterogeneous prices; and (4) multiple equilibria can readily arise in state-dependent pricing models, because of complementarities in price-setting, even with the type of exogenous money stock rule that nearly always guarantees a unique equilibrium in time-dependent models. Working with assumptions characteristic of that literature, specifically that there is a low elasticity of marginal cost with respect to output and that there is a hazard function that rises quadratically in a measure of price gaps, we found support for all of these ideas, except that our use of linear approximation methods precluded studying nonlinear dynamics. Exploring the dynamic response of output and inflation to monetary shocks in a model with a "smoothed off kinked demand curve," we unexpectedly found a pattern of output and inflation dynamics that has been suggested to be inconsistent with sticky price models: output peaking after four quarters, and inflation peaking nearly a year later.

In evaluating the implications of state-dependent pricing for dynamic macroeconomic models, we also considered issues related to ongoing research into model features that can lead to larger persistence of output responses to monetary shocks. Working with an adjustment cost structure that was designed to produce a relatively flat hazard function over eight quarters in the reference case of a constant elasticity demand curve and a global labor market, we found that two model modifications – a variable demand elasticity and a local labor market – led to sharply reduced intervals of stickiness. The kinked demand curve model had a flat hazard over four quarters rather than eight; the local labor market models had only one period of incomplete price adjustment. For this reason, it turned out that the local labor market friction lowered persistence under SDP rather than raising persistence as it does under TDP. However, the result for the kinked demand curve was that there was larger persistence (relative to constant elasticity demand) even though the steady-state duration of price fixity was smaller under kinked rather than constant elasticity demand. Taken together, these examples show that state-dependent pricing may alter the conclusions that a researcher would draw about the effect of structural elements of a model.

In closing their 1989 discussion of state-dependent pricing and time-dependent pricing, Blanchard and Fischer considered the types of economic exchanges that might be best modeled using either approach, but they could only summarize a few empirical studies about price adjustment dynamics (notably Cecchetti [1986] and Kashyap [1995]). [23, 24] Recent work by Bils and Klenow [2004] and Klenow and Kryvtsov [2004] is providing valuable new information about the behavior of consumer goods prices in the U.S., both in terms of the timing and magnitude of adjustments, and many studies are underway for other countries.[25] It is clear from this ongoing work that the average duration of price fixity differs substantially across industries and that there are important period-to-period changes in the fractions of goods whose prices are changed. It is also clear that aspects of this work raise challenges for existing models of price adjustment, both time-dependent and state-dependent. Learning further about the general implications of these pricing models for macroeconomic dynamics, as we have here, will be a central component of the important project of taking SDP models to data.

[23] See Willis [2001] for an interesting modern reworking of Cecchetti's analysis of magazine prices.

[24] See Wolman [2003] for a comprehensive survey and critical appraisal of a variety of evidence on duration of price fixity and the magnitude of price adjustment costs.

[25] Notably, the cooperative project being sponsored by the European Central Bank on Inflation Persistence.

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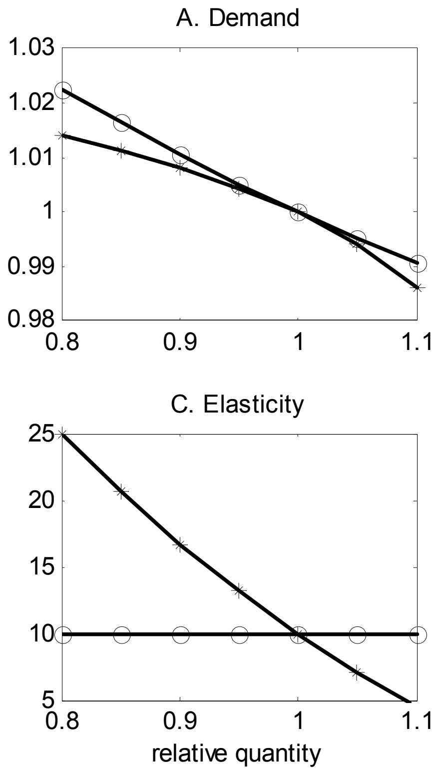

Figure 1: Alternative Demands: Dixit-Stiglitz (o) and Kimball Kink (*)

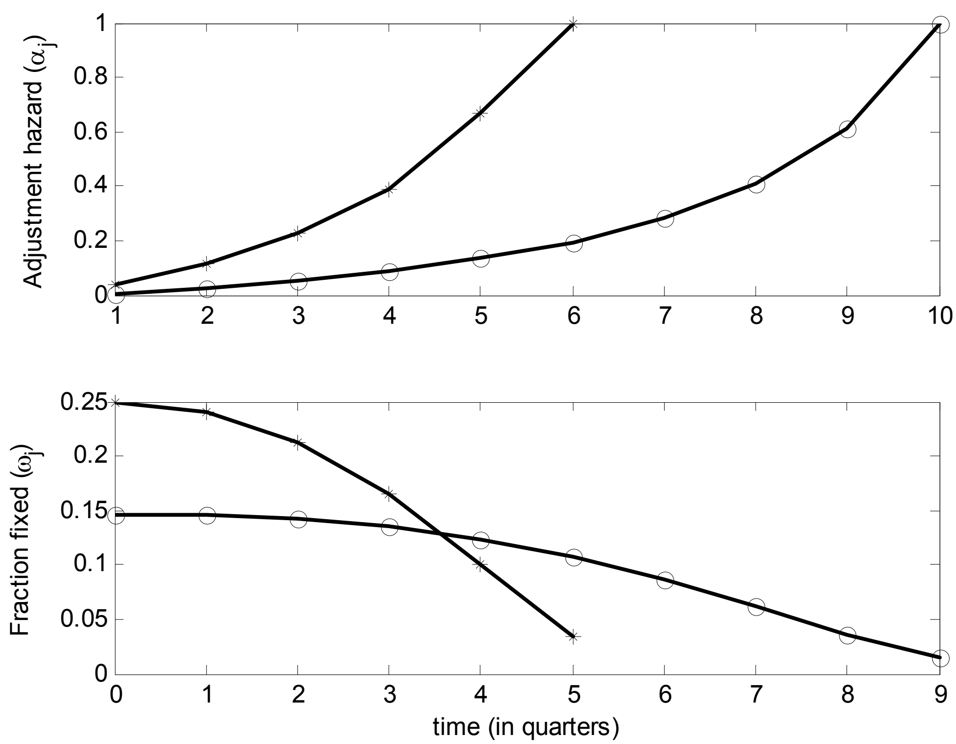

Figure 2: Adjustment rates (α) and vintage fractions (ω) for global labor market models with DS demand (o) and K demand (*)

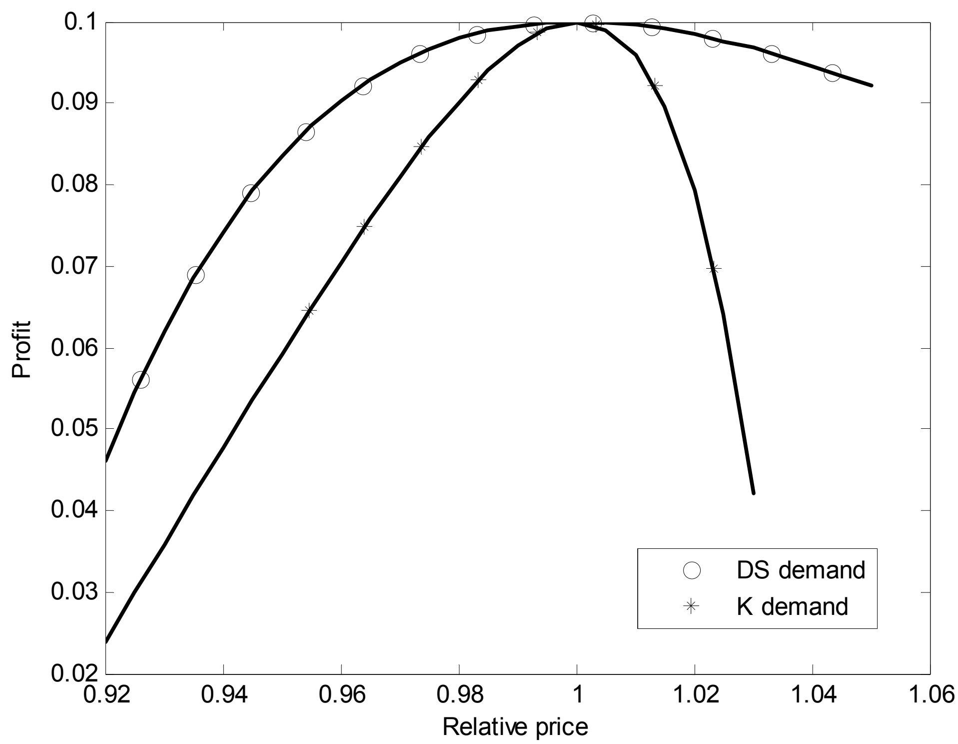

Figure 3: Profit for DS and K demand at $\psi = \frac{\varepsilon - 1}{\varepsilon} = .9$

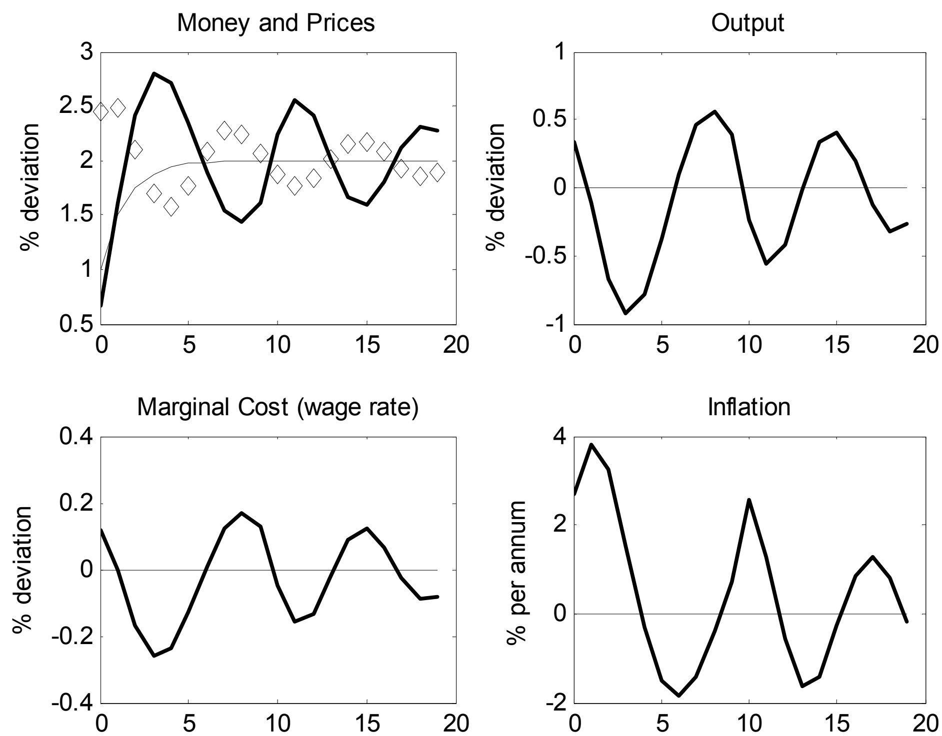

Figure 4: Dynamic Responses to a Monetary Shock with DS demand. The three responses in the first panel are the money stock (light line); the price level (dark line); and the optimal price set by adjusting firms ($\Diamond$).

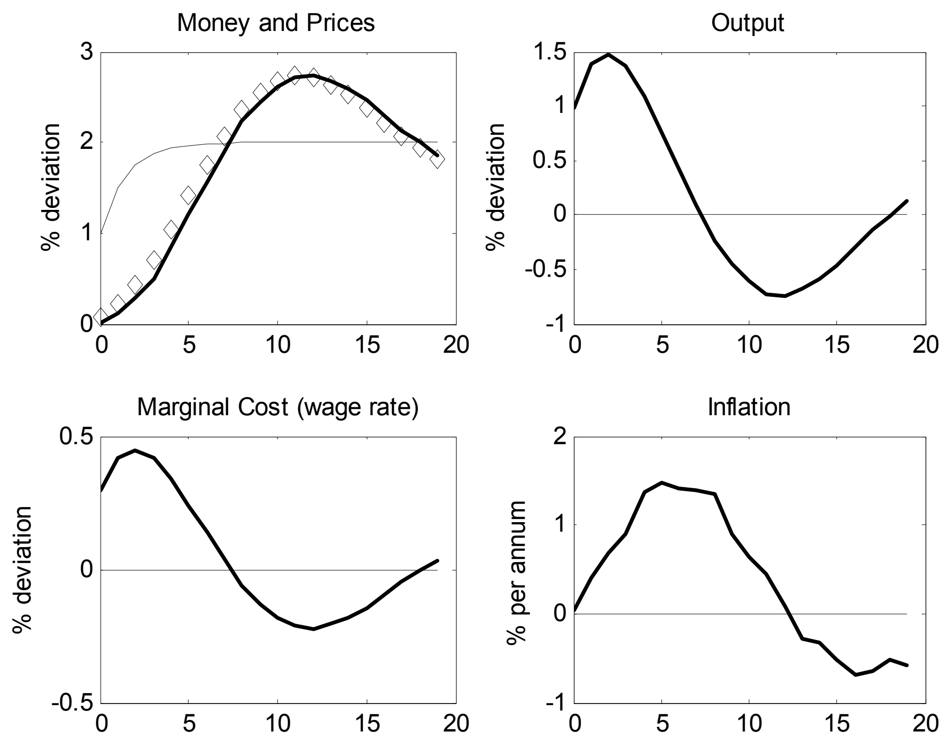

Figure 5: Dynamic Responses to a Monetary Shock with K demand. The three responses in the first panel are the money stock (light line); the price level (dark line); and the optimal price set by adjusting firms ($\Diamond$).

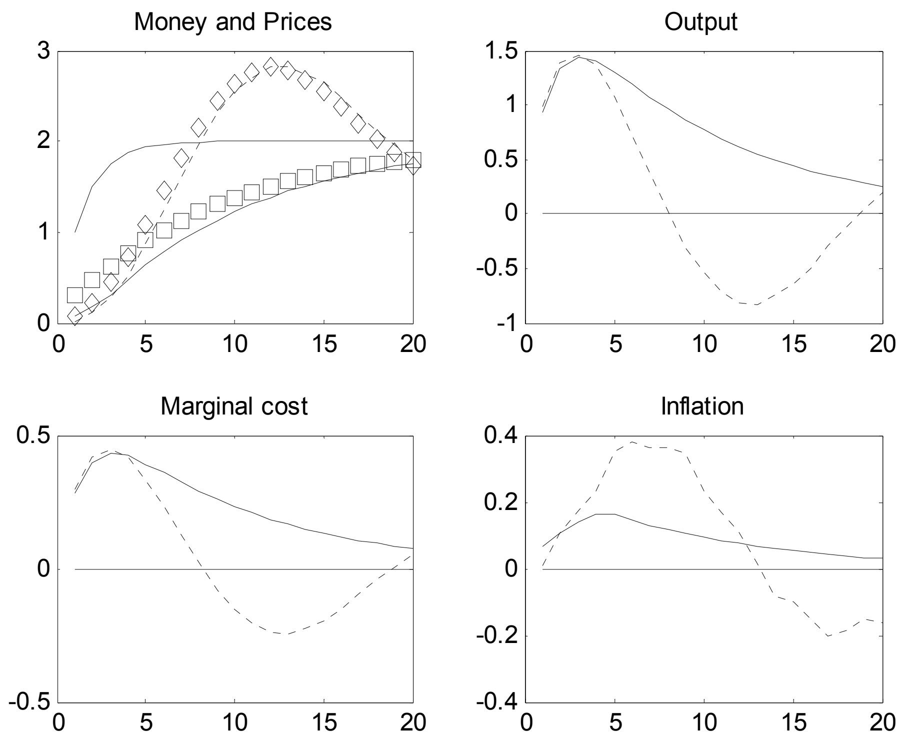

Figure 6: Comparison of TDP (—) and SDP(- -) responses with the K demand specification. In the first panel, in addition to the price levels and money stock, the ◊ is the re-set price in the SDP model and the □ is the re-set price in the TDP model

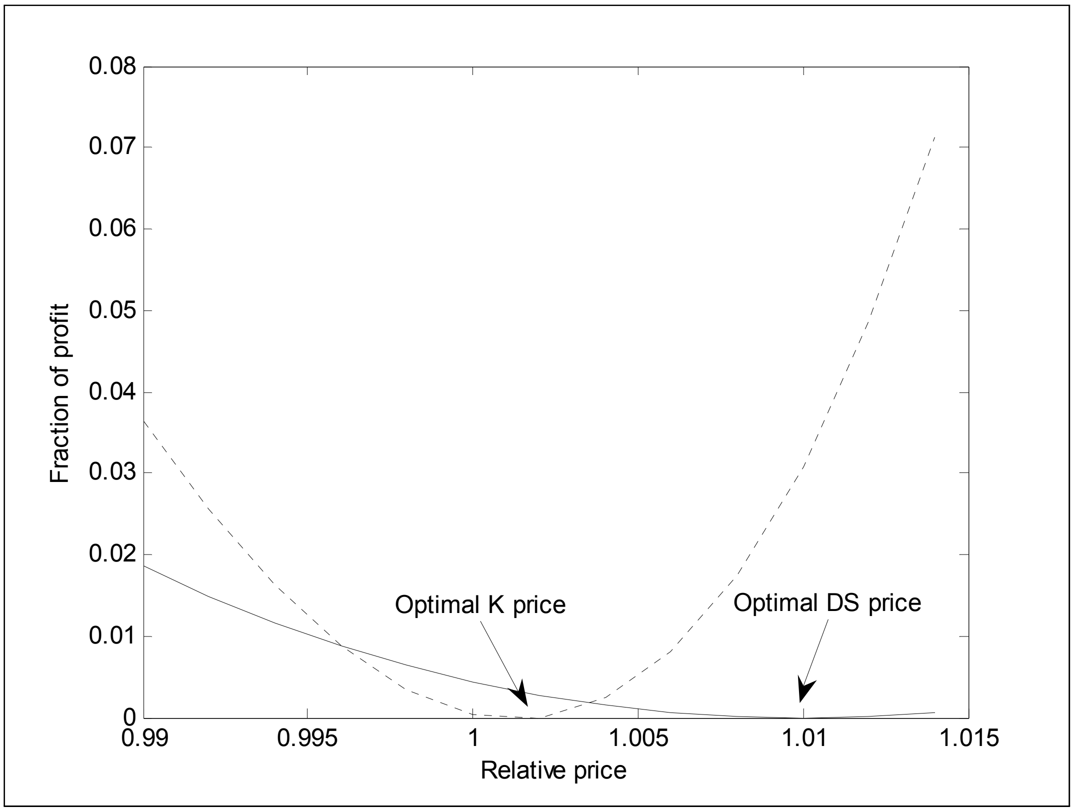

Figure 7: Static profits

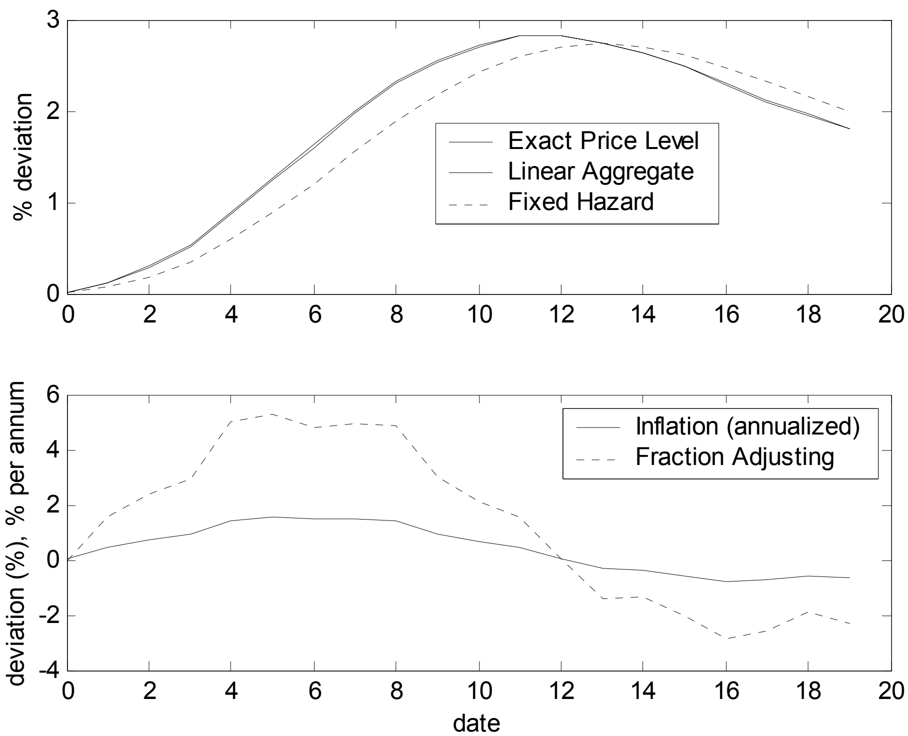

Figure 8: Sources of variation in the price level

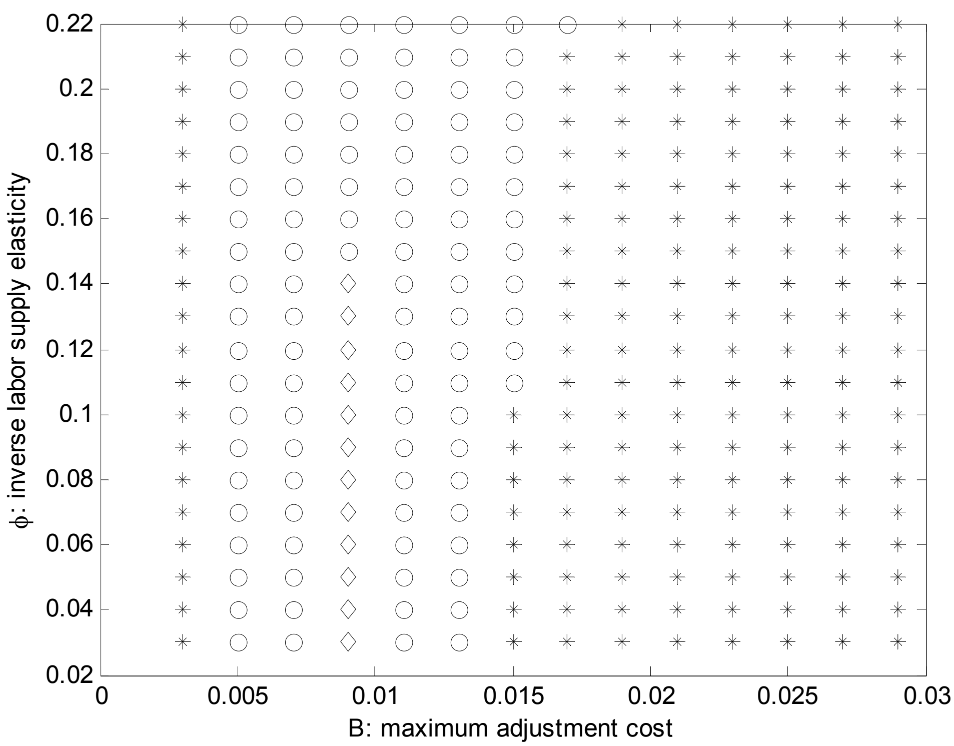

Figure 9: Equilibrium: Uniqueness (*), Nonexistence (o) or Multiplicity (◊)

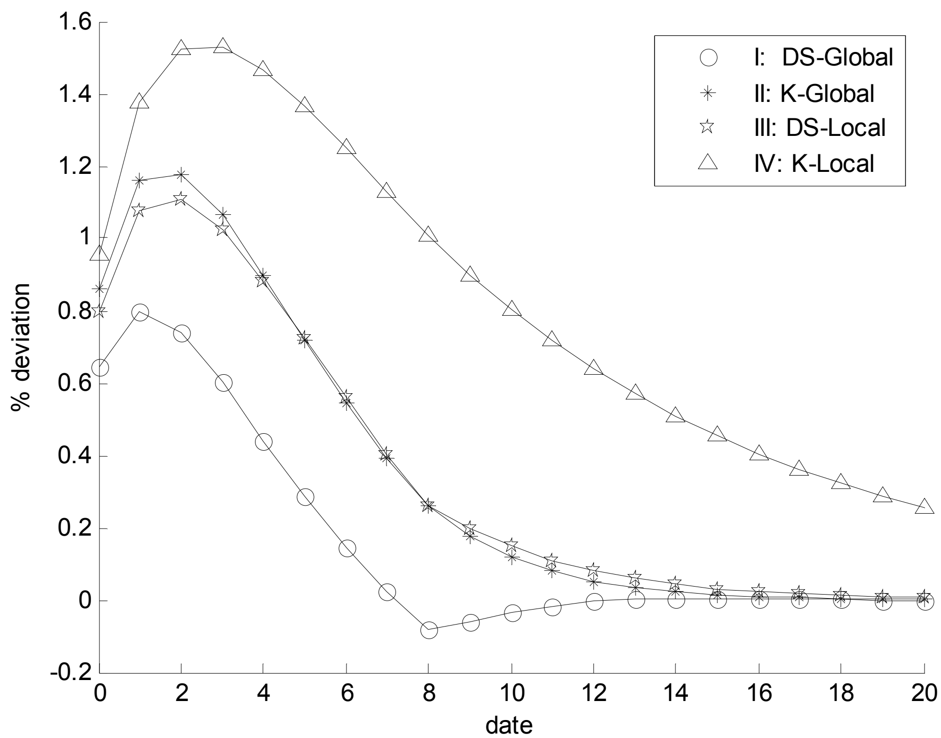

Figure 10: Introduction of local labor markets or kinked demand (or both) increases output persistence under time-dependent pricing.

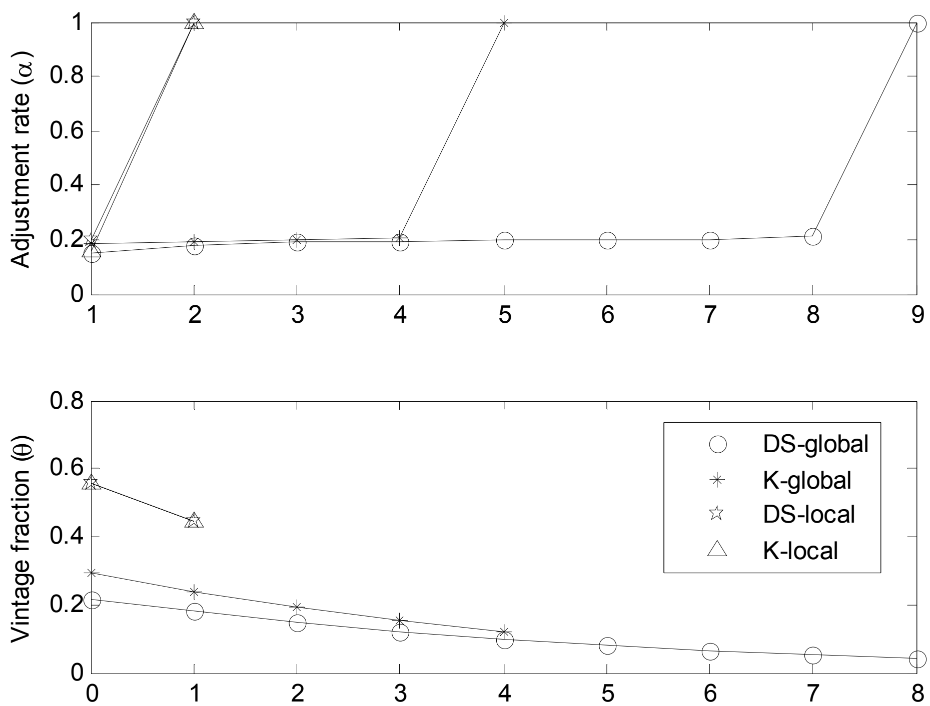

Figure 11: Adjustment hazards and vintage fractions under cost distribution leading to near-constant hazard

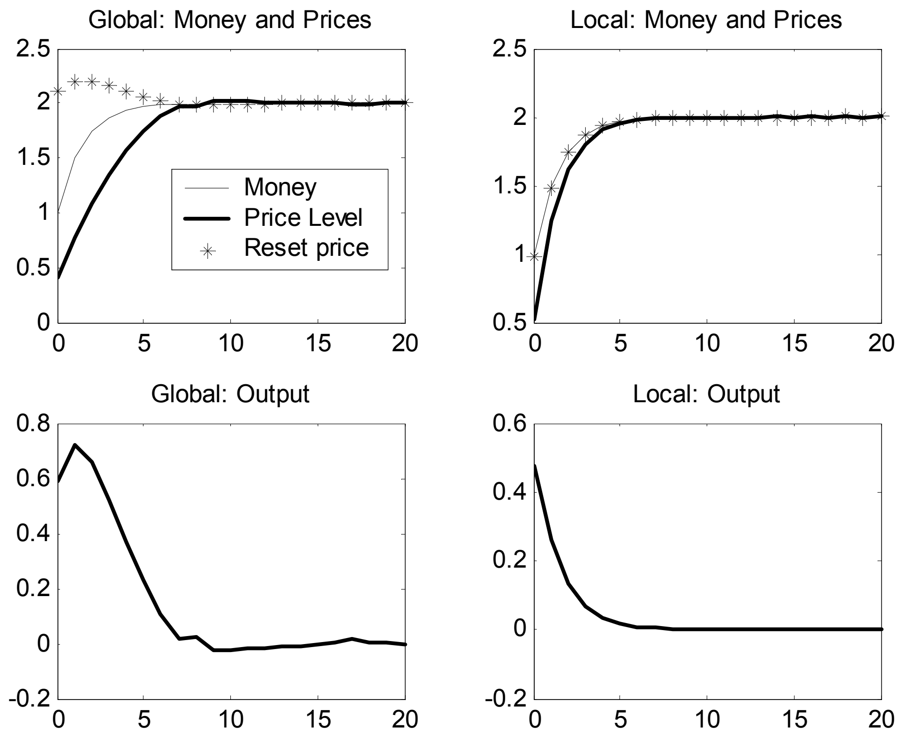

Figure 12: Consequences of localizing labor market under DS demand

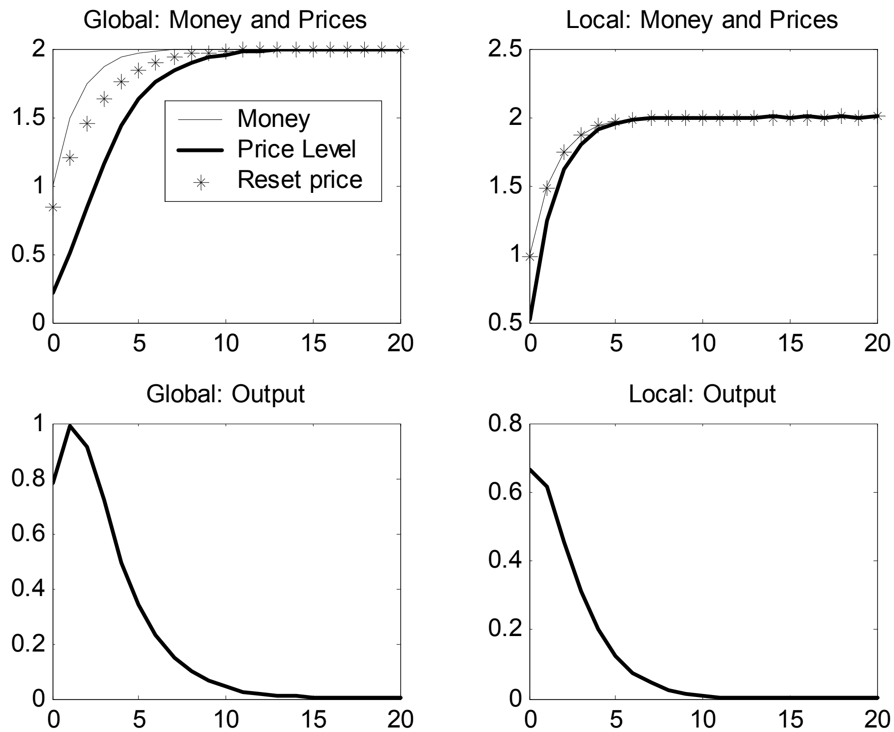

Figure 13: Consequences of localizing labor market under K demand

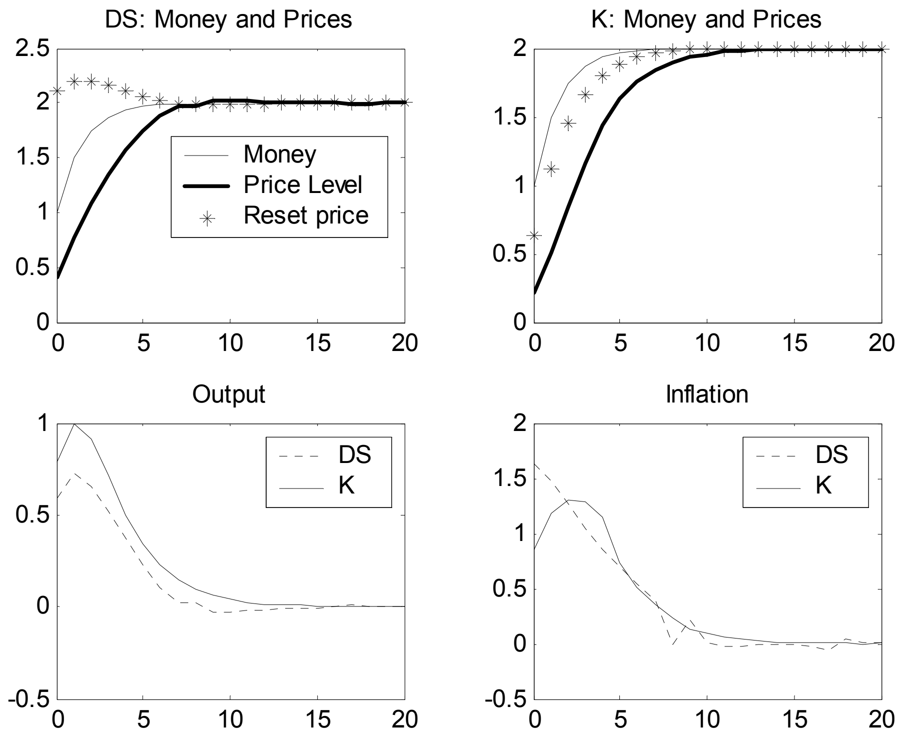

Figure 14: Persistence implications of DS and K demand with endogenous timing of price adjustment

Appendices

A State-dependent pricing models

State-dependent pricing models view the timing of adjustment as endogenous, rather than as a parametric feature of a model economy. Following the early work of Barro [1972], state-dependent pricing models attracted the attention of a number of researchers through the early 1990s. But since the models were mathematically challenging, individual studies chose simplifying assumptions to make the problem tractable.

A.1 Steady-state adjustment

A pair of influential studies by Sheshinski and Weiss [1977,1983] provided a detailed partial equilibrium analysis of the effect of inflation on price adjustment in the presence of fixed costs, working within an economic environment that was otherwise stationary. Their analyses drew on prior work on optimal discrete adjustment policies in the presence of fixed costs that began with Scarf's [1959] work on inventory adjustment and is sometimes described as the S,s adjustment literature.

In their analyses, a key reference point was the constant relative price – which we will call p^*– that the firm would set in the absence of fixed costs of adjustment. Analyzing the effects of certain inflation within a continuous time framework, SW [1977] showed that a firm would adopt a strategy of adjusting its price periodically, starting at a relative price $\overline{p} > p^*$ and adjusting when inflation eroded the relative price to a value $\underline{p} < p^*$. That is: it would set a nominal price that implied a path of the real price that was initially high in comparison to the frictionless price and then declined through time as the real value of the nominal price was eroded by inflation. Their analysis showed that an increase in the rate of inflation would unambiguously raise the size of nominal price adjustments (the ratio $\overline{p}/\underline{p}$) and that larger adjustment costs would also raise $\overline{p}$ and lower $\underline{p}$. However, somewhat surprisingly, their analysis showed that there must be a restriction on the shape of the profit function for higher inflation to increase the frequency of price adjustment. If $z(p, ...)$ is the profit function, they showed that $p\frac{\partial z(p,...)}{\partial p}$ must be decreasing in price for higher inflation to have the expected effect.[26] Finally, they reported simulation analyses indicating that – with a quadratic profit function – small fixed costs can lead to price fixity on the part of firms of one to two years, a finding that they attributed to the profit effects of $p = p^*$ being small in the neighborhood of p^*.

Working within a setting in which inflation was either zero or increased by a random amount, SW [1983] explored the effect of uncertain inflation on the optimal

[26] The demand specifications used in our analysis satisfy these conditions.

pricing problem, showing that it again took the (S,s) form. They then looked at the implications of inflation uncertainty for the timing and magnitude of price adjustment. The main finding of the paper was developed by a clever certainty equivalence argument: the optimal policy under uncertain inflation would be of the same form as under certain inflation. All that was necessary to study the effects of uncertain inflation was to determine the particular "certainty equivalent" inflation rate, which they showed depended on the parameters of the stochastic process for inflation and the real interest rate, but not on the shape of the firm's profit function. Using this apparatus they explored the effects of increasing the mean and variance of the inflation rate on the values of $\overline{p}$ and lower $\underline{p}$, as well as the expected duration of price fixity.

A.2 State-dependent pricing and dynamics

While the SW studies focused on the behavior of an individual firm, the analyses of Caplin and Spulber [1987] and Caplin and Leahy [1991] focused on the real effects of money on output within basic general equilibrium models. Delicately balancing rigor and tractability, CS built a continuous time general equilibrium model that would maintain the optimality of the (S,s) policies as developed by SW and yet allow for analysis of aggregates. The CS model involved (i) a demand curve for the firm's output that depended on its relative price and aggregate real balances; (ii) a cost function for the firm that depended on the volume of its output; (iii) a price level that was an aggregate of the prices of individual firm prices; (iv) a money supply rule that specified that the money supply could not decline, but could increase by a stochastic amount; and (v) an initial uniform distribution of prices relative to the money stock. Taken together, these ingredients led to a striking result: increases in the money stock were neutral even though prices were sticky. In essence, this result occurred because an increase in money would lead the firms with the lowest relative prices to re-set their prices at the highest level, causing the price level to rise one-for-one with the money stock and hence leaving real aggregate demand unaffected. This finding was dramatically different from those arising in time-dependent models.

Caplin and Leahy [1991] reexamined the interaction of money, output, and the price level within a model that made two important modifications in the prior analysis of Caplin and Spulber [1987]. CL assumed that the (log) money supply was a driftless random walk, so that it could either rise or fall. CL also assumed that firms followed two-sided adjustment strategies, adjusting their price – normalized by the money stock– upward if it fell sufficiently and reducing it if it became too high. Assuming that there was a uniform distribution of initial prices, they were able to develop a simple relationship between money, prices, and output, which again differed substantially from the TDP case and also preserved the uniform distribution. In essence, there were three regimes. First, if the lowest normalized price was at the level that triggered an adjustment, then a money supply increase would trigger an increase in the price level and be neutral, just as in CS. But if there was a decrease in the money supply, then all nominal prices would remain unchanged and there would be a negative effect on real output. Second, if the highest normalized price was at the level that triggered an adjustment, then a money supply decrease would trigger a decrease in the price level and be neutral, just as in CS. But if there was a increase in the money supply, then all nominal prices would remain unchanged and there would be a positive effect on real output. Third, if neither of the above conditions were satisfied, then no nominal price adjustments would occur and there would be a direct effect of money on output. The Caplin and Leahy [1991] analysis implied that the effects of money would depend strongly on the state of the economy, again suggesting dramatically different linkages than in time-dependent pricing models. They stressed that the evolving distribution of relative prices was a key determinant of the effect of monetary disturbances and they concluded that it was important to systematically investigate how such evolving distributions contributed to macroeconomic phenomena. Finally, they also constructed some suggestive sample paths of money, output, and prices that suggested that SDP dynamic responses would look quite different from the standard responses in TDP models.

A.3 The dynamics of the price level

Caballero and Engel [1993] focused attention on the implications of state-dependent pricing for the behavior of the price level, working within a simple and yet empirically rich framework that they applied to U.S. inflation data. The upshot of the CE analysis was two-fold. First, they suggested that price adjustment hazards should be best viewed as an approximately quadratic function of price gaps.[27] Second, they suggested that the dynamics of the price level would be materially affected by tracking aspects of the distribution of prices through time owing to the incentives for price adjustment that this distribution implied for individual firms. At an annual data frequency, their estimates suggested that the fraction of firms adjusting ranged between 49 percent and 59 percent during the 1960-1990 interval, increasing substantially during the high inflation period in the middle of the sample and particularly in response to changes in oil prices.

A.4 Multiple Equilibria

Ball and Romer [1991] demonstrated that state-dependent pricing models could differ significantly from time-dependent models in terms of uniqueness of equilibria. In particular, there was a new possibility of multiple equilibria due to a basic complementarity in the price-adjustment decisions of firms: an increase in a given firm's price raises the price level (perhaps only by a very small amount if it acts alone)

[27] A view that has been implemented in some later work on the evolution of inflation in Spanish sectoral data (Estrada and Hernando [1999]).

which in turn makes it desirable for other firms to raise their price because their relative prices have fallen. Considering a monetary shock, they showed it could be individually rational in the presence of a fixed menu cost for a firm not to adjust price if all others did not adjust and the price level therefore remained constant. They also showed it could be individually rational for a firm to change price one-for-one with the monetary shock if all others did as well. Because such a multiplicity depends critically on a firm choosing whether or not to adjust price, it was precluded in time-dependent models.

B Stochastic adjustment costs

Costs of price adjustment play an important role in our analysis, so that this appendix discusses aspects of the DKW modeling of these costs and the relationship to conventional modeling of nonstochastic adjustment costs in models of investment, labor demand, and so on. It is convenient to start by thinking about the adjustment process as though there were just one vintage, with a firm considering whether to stay with its preset price and earn v or to adjust and earn v_0.

B.1 A direct adjustment cost interpretation

Panel A of Figure 15 displays one vision of the adjustment cost structure: it displays the costs to the owner of a portfolio of firms, under the assumption that a fraction of firms α adjust. As is conventional, there are positive, increasing, and convex labor costs of adjustment, which we call $\Xi(\alpha)$.

If the portfolio owner is equating the marginal cost of increasing the rate of adjustment α by a small amount, faces a wage rate of w, and has a gain of $v_0 - v$, then

$$w\xi(\alpha) = [v_0 - v] \tag{8}$$

is the relevant efficiency condition, where $\xi(\alpha) = \Xi_\alpha$ is the *marginal* labor cost of adjustment. This marginal adjustment cost is shown in panel B of Figure 15.

Linear approximation of the condition $\xi(\alpha) = \frac{v_0 - v}{w}$ then indicates that the slope of the marginal adjustment cost curve is relevant for the response of the adjustment rate, since

$$\xi_\alpha[d\alpha] = d[\frac{v_0 - v}{w}]$$

Thus, as is conventional in adjustment cost settings, it is $\Xi_{\alpha\alpha} = \xi_\alpha$, which is relevant for the local behavior of adjustment rate response, i.e., the second derivative of costs is important for adjustment because the adjustment cost function is locally quadratic.

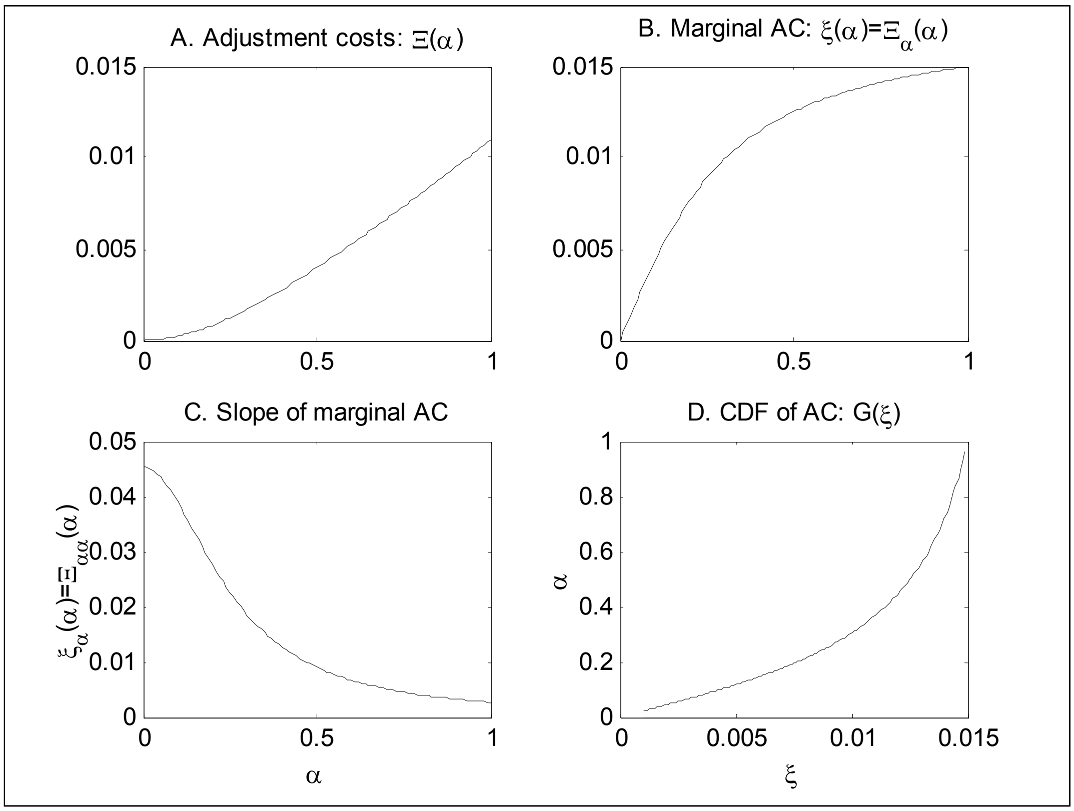

Figure 15: Adjustment costs leading to a nearly quadratic hazard

B.2 Stochastic adjustment costs based on a cdf

Next, suppose that adjustment costs are stochastic and idiosyncratic across firms, being governed by a cumulative distribution function $G(x)$ on the interval $0 \leq x \leq B$ and suppose also that the density function is $g(x)$. The truncated mean level of adjustment costs plays a central role in our analysis. It is defined as

$$\Xi(\xi) = \int_0^{\xi} x g(x) dx$$

This measure has two interpretations. First, as in the previous section, it can be interpreted as the adjustment costs paid by a holder of a portfolio of firms if the largest adjustment cost is ξ. Second, it gives the expected value of an individual firm's adjustment costs if there is an adjustment rule that specifies that ξ is the highest adjustment cost paid by any adjusting firm. This is an unconditional expectation, in the sense that it does not take into account information about whether the firm adjusts or not, but simply recognizes that adjustment costs are paid only in some situations (i.e., the form of the adjustment rule that truncates the distribution).

Under this adjustment rule, a firm's probability of adjustment is

$$\alpha(\xi) = G(\xi) = \int_0^{\xi} g(x) dx$$

so that the expectation of any individual firm's costs *conditional on adjustment* is given by

$$\frac{\Xi(\xi)}{\alpha(\xi)}$$

From the standpoint of the adjustment cost distribution, efficient adjustment requires that

$$\alpha = G(\xi) = G(\frac{v_0 - v}{w})$$

Further, the sensitivity of the response of the adjustment rate to $\frac{v_0 - v}{w}$ is determined by density of adjustment costs, i.e.,

$$d\alpha = g(\xi) * d(\frac{v_0 - v}{w})$$

This is the vision of adjustment costs developed in Caballero and Engel [1999] and Dotsey, King, and Wolman [1999]. Both of these analyses specify a distribution of "fixed costs of adjustment" and develop the implications for that assumed distribution. However, there are considerable differences in the assumed distributions, with Caballero and Engel [1999] using a distribution close to that shown in Figure 15 and DKW using one that is closer to Figure 16 below.

B.3 Stochastic adjustment via the cost function

We now consider the relationship between these two approaches. In line with Figure 15, we could specify an *inverse* cdf

$$\xi = F(\alpha)$$

and associated function $f(\alpha) = F_\alpha(\alpha)$. Since the inverse cdf (or marginal adjustment cost function) is defined by the requirement that

$$\xi = F(G(\xi))$$

and its derivative thus satisfies $1 = F_\alpha G_\xi = f(\alpha)g(\xi)$.

Above, we discussed the total adjustment cost function in Figure 15. We also said that the truncated mean was

$$\Xi(\xi) = \int_0^\xi xg(x)dx.$$

We now show that these two alternative definitions of total cost Ξ are the same as a function of α. To begin, use the change of variables $x = F(a); g(x) = \frac{1}{F'}$; and $dx = F'(a)da$ so that

$$\Xi(\alpha) = \int_0^\alpha F(a)da$$

That is: the truncated mean is just the area under the inverse cdf function and the truncated mean is also the relevant measure of total adjustment costs.

Further, as above, efficient adjustment implies that $\xi(\alpha) = \frac{v_0 - v}{w}$ so that the slope of the marginal adjustment cost curve relevant for the dynamic response of the adjustment rate is $\xi(\alpha) = f(\alpha)$, the reciprocal of the density function (since $1 = F_\alpha G_\xi = f(\alpha)g(\xi)$).

$$\xi(\alpha)[d\alpha] = d[\frac{v_0 - v}{w}]$$

Again, the interpretation is that the adjustment rate responds most strongly when the density of adjustment costs is largest.

Given the above, we look again at Figure 15, emphasizing four aspects of the figure. First, panel A shows the level of adjustment costs, the truncated mean $\Xi(\alpha)$. Second, the rate of adjustment is determined by the inverse cdf, $F(\alpha)$. Hence, since efficient adjustment involves $\xi(\alpha) = \frac{v_0 - v}{w}$, this involves variations along the inverse cdf. Third, the extent of response to variations in $\frac{v_0 - v}{w}$ is governed by the slope of the inverse cdf (or cost function). Fourth, the cdf itself is shown in panel D.[28]

[28] For any cdf, the associated "adjustment cost function" $\Xi(\alpha)$ is positive, increasing and convex for $\alpha > 0$. That is: $\Xi > 0$, $\Xi_\alpha > 0$, and $\Xi_{\alpha\alpha} > 0$, with the third condition being guaranteed by the fact that $\Xi_{\alpha\alpha} = 1/g > 0$.

B.4 Extension to multiple vintages

Now, as in our main analysis, we consider the extension to multiple periods of price fixity. The results above enter our analysis in two ways. First, the expected value of a firm depends on its expected future adjustment costs,

$$v_j = \pi_j + \beta E\{\frac{\lambda'}{\lambda}[\alpha'_{j+1}v'_0 - w'\Xi'_{j+1}\} + (1 - \alpha'_{j+1})v'_{j+1}]\}. \tag{9}$$

Second, the labor devoted to price adjustment in a particular bin is given by

$$\Xi_j\theta_j$$

and total adjustment costs are $n^p = \sum_{j=1}^{J} \Xi_j\theta$. Third, the possibility that there are multiple bins now raises a new set of issues concerning the interaction between the steady-state and the nature of adjustment. In a stationary state, there must be increasing adjustment rates, such as illustrated in Figure 15, which are consistent with the requirements $\xi(\alpha_j) = \frac{v_0 - v_j}{w}$ and the value function recursions (9) above. Accordingly, the variation in adjustment rates is related to the slope of the cost function,

$$[d\alpha_j] = \frac{1}{\xi_\alpha(\alpha_j)}d[\frac{v_0 - v_j}{w}] = g(\xi_j)d[\frac{v_0 - v_j}{w}]$$

which is also the density of adjustment costs, as the final equality stresses. In a multiple vintage approach, it is accordingly the case that the properties of the cdf matter at various points: it is $g(\xi_j)$ that enters in the response above rather than $g(\xi)$.

B.5 Approximate Calvo

In the test, we want to explore the effects of state dependence within a battery of models that have an approximate Calvo form, i.e., a steady-state hazard that is roughly constant for a number of periods. Accordingly, we select the parameters of our cost function so that there is a flat hazard for the DS-global setting for eight quarters, which is a "truncated Calvo" steady-state. Figure 16 displays the nature of the adjustment costs necessary for this result. In panel B, we see that the necessary cost function, $\xi(\alpha)$, is one that is fairly flat until $\alpha = o.2$ then rises very sharply to close to the maximum cost. Faced with this adjustment cost, firms with a range of different values of $(v_0 - v_j)/w_0$ will all choose $\alpha = o.2$. When $(v_0 - v_j)/w_0 \geq B = 0.015$, all firms will choose to adjust ($\alpha = 1$).

B.6 Functional form

The inverse tangent (or arctangent) is a monotonically increasing function that maps the real line into the interval $(-\pi, \pi)$. It has concave, convex, nearly linear, and s-

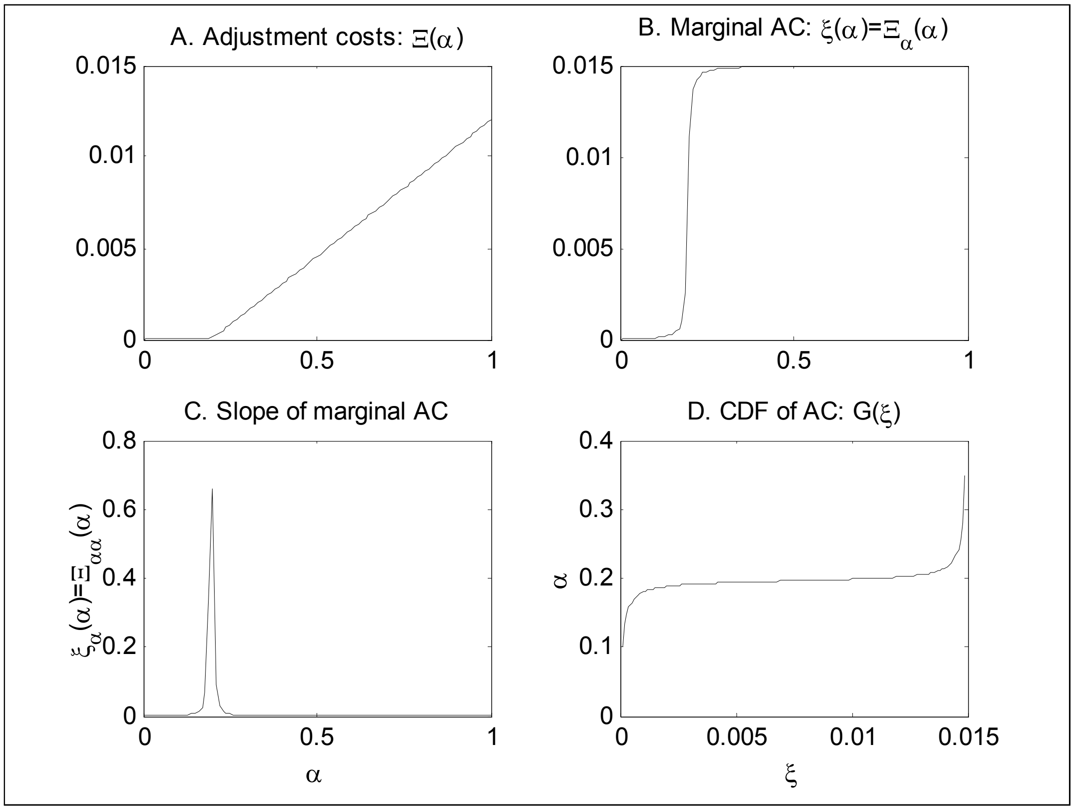

Figure 16: Adjustment costs leading to near constant hazard ($\alpha \approx 0.2$).

shaped pieces. Hence, an inverse cdf that selects part of this function can be used to explore a variety of different assumptions about the cost function within a common functional form.

We proceed as follows. First, we select a part of the standardized arctangent $s(x)$– shown in Figure A1–that we would like to use, i.e., an interval ($\underline{x}$,$\overline{x}$). Then, we assume that

$$x(\alpha) = (\overline{x} - \underline{x}) * \alpha + \underline{x}$$

where α is restricted to range $0 \leq \alpha \leq 1$. Finally, we assume that the inverse CDF takes the form

$$\xi(\alpha) = K_1 s(x(\alpha)) + K_2$$

We further assume that the inverse CDF takes on a zero value at $\alpha = 0$ and a value of B at $\alpha = 1$. That is:

$$\begin{aligned} 0 &= K_1 s(\underline{x}) + K_2 \\ B &= K_1 s(\overline{x}) + K_2 \end{aligned}$$

so that the values of the parameters are given by

$$\begin{aligned} K_1 &= \frac{B}{s(\overline{x}) - s(\underline{x})} \\ K_2 &= -\frac{Bs(\underline{x})}{s(\overline{x}) - s(\underline{x})} \end{aligned}$$

B.6.1 Evaluating the conditional mean

To evaluate Ξ, we proceed as follows. First, the fact that $\int \tan^{-1}(z)dz = z * \tan^{-1}(z) - \frac{1}{2}\ln(1+z^2)$ implies that

$$\int s(z)dz = z * s(z) - \frac{1}{2\pi}\ln(1+z^2)$$

Second, using the change of variables $y = (\overline{x} - \underline{x}) * a + \underline{x} = ba + \underline{x}$, we can determine that

$$\begin{aligned} & \int_0^\alpha [K_1 s(a * b + \underline{x}) + K_2]da \\ = & \frac{K_1}{b}\int_{\underline{x}}^{\alpha b + \underline{x}} s(y)dy + K_2 \int_0^\alpha da \\ = & \frac{K_1}{b}[(\alpha b + \underline{x})s(\alpha b + \underline{x}) - \underline{x}s(\underline{x})] \\ & -\frac{K_1}{2\pi b}[\ln(1 + (\alpha b + \underline{x})^2) - \ln(1 + (\underline{x})^2)] + K_2\alpha \end{aligned}$$

We use this to compute the extent of adjustment costs Ξ.

B.6.2 Marginal adjustment costs

The linearizations require the derivatives of the adjustment cost functions above. The derivative of the conditional mean is

$$\frac{d}{d\alpha}\Xi(\alpha) = \frac{d}{d\alpha}\int_0^{\alpha} F(a)da = \xi(\alpha)$$

for all cost functions. The derivative of the specific inverse CDF is

$$\frac{d}{d\alpha}\xi(\alpha) = \frac{d}{d\alpha}[\frac{K_1}{\pi}\tan^{-1}(\alpha * b + \underline{x}) + K_2] = b\frac{K_1}{\pi}\frac{1}{1 + (\alpha * b + \underline{x})^2}$$

using the fact that $\frac{d}{dx}\tan^{-1}(x) = 1/[1 + x^2]$.

B.6.3 Calculating the CDF

The form of the inverse CDF or cost function makes it easy to determine the form of the CDF. That is:

$$\alpha = G(\xi) = \frac{1}{b}\tan(\frac{\xi - K_2}{K_1/\pi}) - \frac{\underline{x}}{b}$$

The density can be readily calculated using $\frac{d}{dx}\tan(x) = (\cos(x))^{-2}$, so that it takes the form

$$g(\xi) = \frac{1}{bK_1/\pi}[\cos(\frac{\xi - K_2}{K_1/\pi})]^{-2}$$

B.6.4 Some selections

By appropriate choices of the range of the parameters above, we can replicate a variety of cdfs or cost functions. Figure 17 displays three selections (the parameter choices for "nearly Calvo" fall outside of the plotted range, with $\underline{x} = -50$ and $\overline{x} = 205$).

The implied adjustment cost functions (with B=0.15) are plotted next: the horizontal axis is α and the vertical axis is $\xi(\alpha)$.

For additional details on adjustment cost selection, see the replication materials for this project, which are available at **http://people.bu.edu/rking/**, specifically the documentation on adjustment cost utilities.

C Dynamics of adjustment rates

The adjustment rate for a firm of vintage j is implicitly given by $\xi(\alpha_{jt})w_{0t} = v_{0t} - v_{jt}$. Accordingly, we can take a first order approximation to this expression and deduce that

$$\xi_{\alpha}(\alpha_j)(\alpha_{jt} - \alpha_j) = -[\frac{p_j \partial v_{jt}/\partial p_{jt}}{w_0}] * [\log(p_{jt}) - \log(p_j)] + \text{other terms}$$

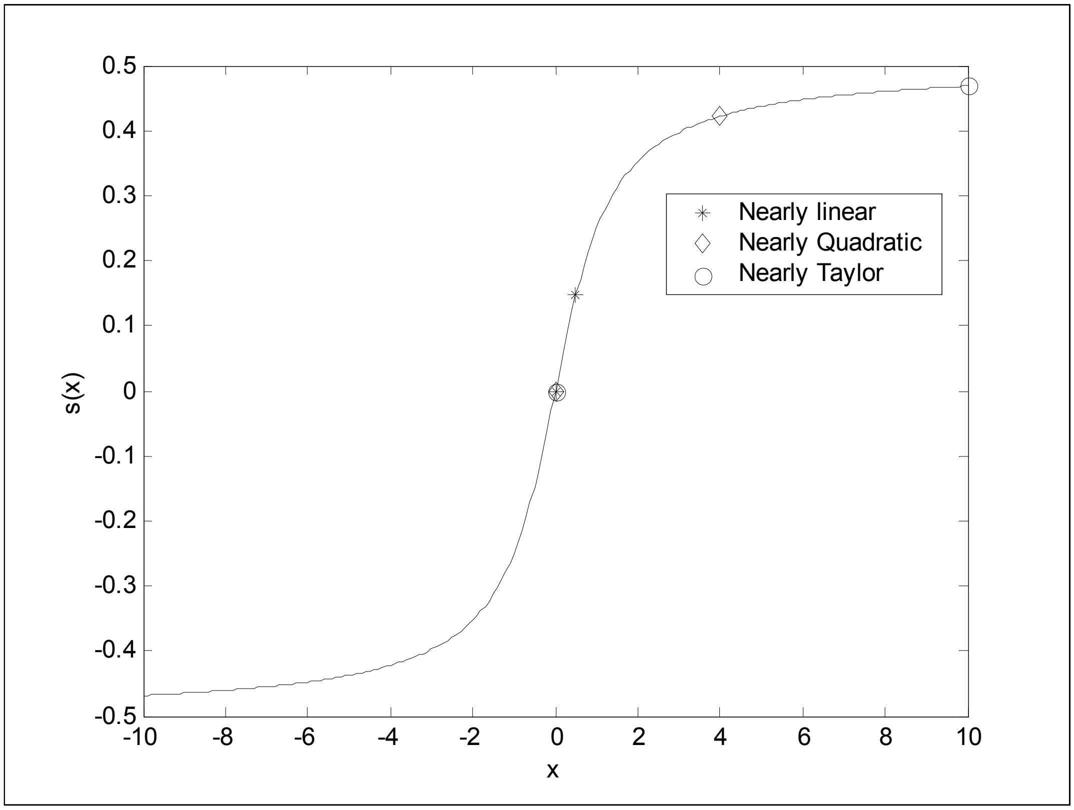

Figure 17: Standardized arc tangent function and some segments

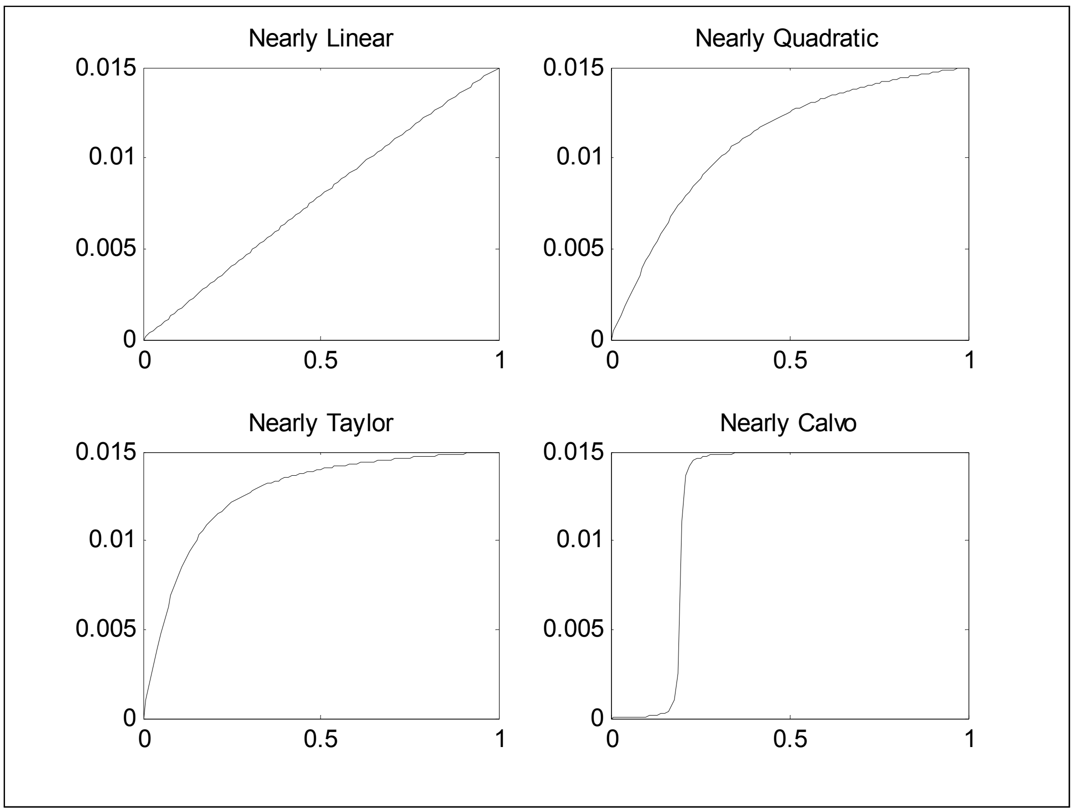

Figure 18: Alternative adjustment cost functions (inverse CDFs)

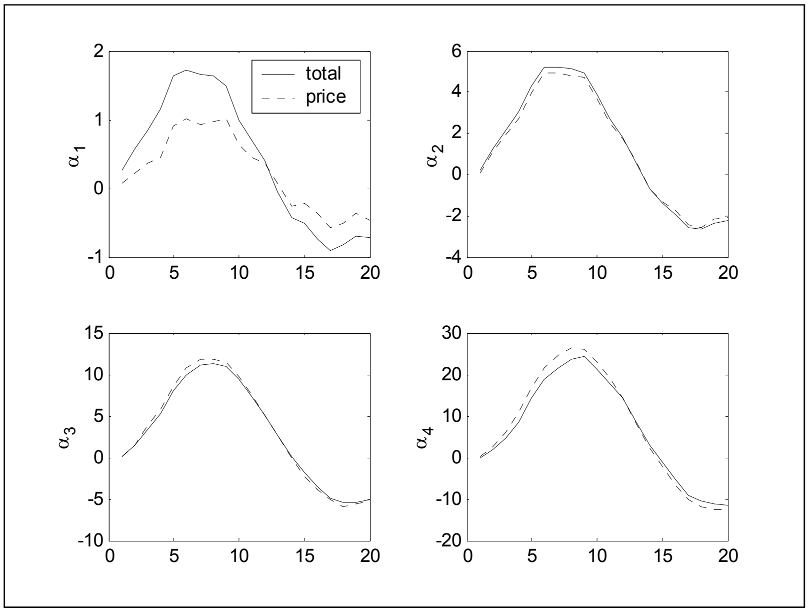

Figure 19: Effects of relative price on adjustment rates

so that it is possible to explore the effects of the price level on adjustment incentives, holding fixed other factors. Specifically, we take the equilibrium solution for $\log(p_{jt}) - \log(p_j)$ and then construct a synthetic series $\widetilde{\alpha}_{jt} - \alpha_j$ using the equation above. Given these synthetic series, we can also construct a synthetic series for the vintages, $\widetilde{\omega}_{jt} - \omega_j$, which is a dynamic simulation of sorts, since it obeys the dynamic equations

$$\begin{aligned}
\widetilde{\omega}_{jt} - \omega_j &= (1 - \alpha_j)(\widetilde{\omega}_{j-1,t-1} - \omega_j) - \omega_{j-1}(\widetilde{\alpha}_{jt} - \alpha_j) \\
\widetilde{\omega}_{0t} - \omega_0 &= \sum_{j=0}^{J-1} [\alpha_j(\widetilde{\omega}_{j-1,t-1} - \omega_j) + \omega_j(\widetilde{\alpha}_{jt} - \alpha_j)].
\end{aligned}$$

That is, the synthetic series for $\widetilde{\omega}_{jt}$ is constructed solely on the basis of variations in the synthetic adjustment rates $\{\widetilde{\alpha}_{jt}\}_{j,t}$, so that it too involves only the effects of p_{jt}.

The striking results of this analysis are reported in Figures 19 and 20. Figure 19 shows that the effects of p_{jt} are dominant on α_{jt} except for those firms that just adjusted, with this exception seeming plausible on the basis of our prior analysis of static profit gain in the main text. The price effects capture variations in vintage fractions (ω_{jt}) virtually completely.

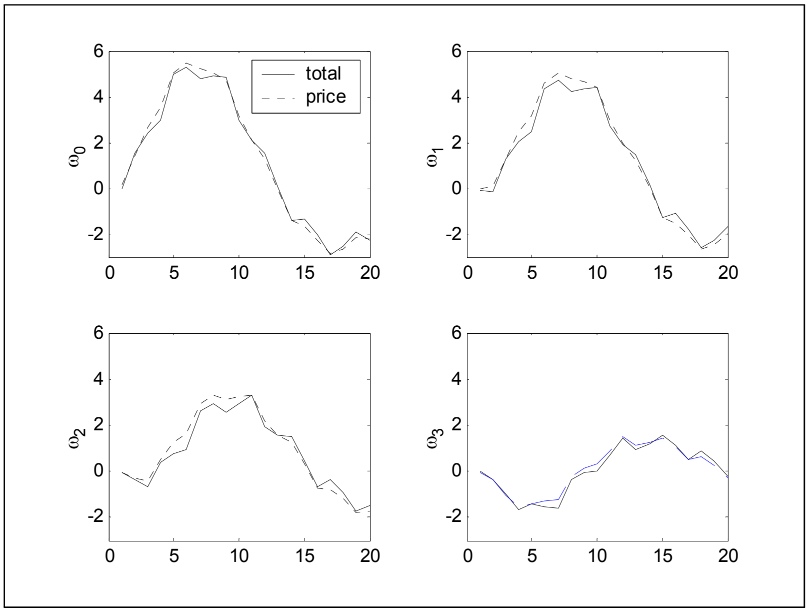

Figure 20: Effects of relative prices on vintage fractions

D Computation

There are two parts to the computation of the model. Consider that the model is a set of equations of the form

$$E_t F(Y_{t+1}, Y_t, X_{t+1}, X_t) = 0$$

where Y_t is a vector of endogenous variables and X_t is a vector of exogenous variables.

D.1 Computation of stationary point

The stationary point of the model involves finding a vector Y, given a vector X, such that

$$F(Y, Y, X, X) = 0$$

Additional discussion to be added.

D.2 Computation of local approximation

The local approximation takes the form

$$F_1 * E_t(Y_{t+1} - Y) + F_2(Y_t - Y) + F_3 E_t(X_{t+1} - X) + F_4(X_t - X) = 0$$

where the F_i are matrices of partial derivatives. We compute those matrices analytically and numerically, offering a cross-check on the dynamic system's approximation.

E Model equations

This appendix spells out the complete set of equations of our model. The discussion is divided into seven blocks of equations governing: (1) the dynamics of the price distribution; (2) the behavior of relative prices, relative demand and the price level; (3) cost, profit, and labor demand; (4) consumption demand and labor supply; (5) firm value, efficient pricing, and efficient adjustment; (6) definitions and aggregate consistency conditions; (7) decomposition of price adjustment dynamics.

E.1 Dynamics of the price distribution

The first block of model equations describes the evolution of the price distribution, including the dynamics of lagged relative prices; the dynamics of the "end of period" distribution (ω); and (iii) the start of period distribution (θ).

1A. Evolution of lagged relative prices

$$p_{j+1,t+1} = p_{j,t} \tag{1.A}$$

1B. Restrictions on omegas

$$1 = \sum_{0}^{J-1} \omega_{jt} \tag{1.B}$$

1C. Relate omegas to thetas and alphas

$$\omega_{jt} = (1 - \alpha_{jt}) \cdot \theta_{jt} \text{ for } j = 1 \text{ to } J - 1 \tag{1.C}$$

1D. Next period thetas in terms of current omegas

$$\theta_{j+1,t+1} = \omega_{jt} \qquad j = 1 \text{ to } J - 1 \tag{1.D}$$

E.2 Relative demand, relative prices, and the price level

The second block determines the current relative prices, current relative demands, the price level, and a related demand multiplier. Given the lagged relative prices, many current relative prices evolve based on the extent of inflation. Relative demands depend on these current relative prices in ways that are governed by the aggregator specification, which also imposes restrictions on the price level and the related multiplier.

2A. Current relative prices

$$p_{jt} = \frac{P^*_{t-j}}{P_t} = \frac{p_{j-1,t-1}}{\pi_t} \qquad j = 1 \text{ to } J-1 \tag{2.A}$$

2B. Define relative demand

$$y_t(i) = x_t(i) \cdot y_t \qquad j = 0 \text{ to } J-1 \tag{2.B}$$

2C. Determinants of demand fractions

The demand structure stems from the specification of the aggregator

$$1 = \int_0^1 \phi\left(x_i\right) di$$

where $\phi(x_i)$ takes the specific functional form

$$\phi(x(i)) = \frac{1}{(1+\eta)\,\gamma}\left[(1+\eta)\,x\,(i) - \eta\right]^{\gamma} + 1$$

If we minimize the nominal cost

$$P = \int_0^1 P(i)x(i)di$$

of a package of $x(i)$ consistent with the constraint, then we have a first order condition of the form

$$-P(i) + Z\phi'(x(i))$$

where Z is the multiplier on the constraint.

Using the efficiency condition, the demand function is

$$x_t(i) = \frac{1}{1-\eta}\left[\left(\left(\frac{P_t\left(i\right)}{P_t}\right)\left(\frac{P_t}{Z_t}\right)\right)^{\frac{1}{\gamma-1}} + \eta\right] = \frac{1}{1-\eta}\left[\left(\frac{p_t(i)}{\zeta_t}\right)^{\frac{1}{\gamma-1}} + \eta\right]$$

We then apply this specification to each of the bins,

$$x_{jt} = \frac{1}{1-\eta}\left[\left(\frac{p_{jt}}{\zeta_t}\right)^{\frac{1}{\gamma-1}} + \eta\right] \tag{2.C}$$

2D. Price level restriction

The price level is defined as

$$\begin{aligned} P &= \int_0^1 P(i)x(i)di \\ P_t &= \sum_0^{J-1} \omega_{jt}P_{jt}x_{jt} \end{aligned}$$

Hence, it implies a restriction on relative prices and demands, which is followed by its loglinear counterpart,

$$1 = \sum_0^{J-1} \omega_{jt}p_{jt}x_{jt} \tag{2.D}$$

2E. Aggregate multiplier restriction

From the aggregator constraint

$$1 = \sum_0^{J-1} \omega_{jt}\phi\left(x_{jt}\right) \tag{2.E}$$

E.3 Cost, profit, and labor demand

The third block involves production, cost, profit and labor demand specifications

3A. Production fuction/labor demand

The prodution function is a simple linear expression, which also governs labor demand when prices are sticky,

$$y(i) = an(i) \qquad \text{for } j = 0 \text{ to } J-1 \tag{3.A}$$

3B. Marginal cost

Given the production function, real marginal cost is the wage rate divided by productivity,

$$\psi_t(i) = \frac{w_t(i)}{a_t} \qquad \text{for } j = 0 \text{ to } J-1 \tag{3.B}$$

3C. Profits

Profits can be expressed in a number of different ways,

$$z_{jt} = \left(p_{jt} - \psi_{jt}\right) y_{jt} = \left(p_{jt} - \psi_{jt}\right) x_{jt} y_t = p_{jt} y_{jt} - w_{jt} n_{jt} \tag{3.C}$$

for $j = 0$ to $J-1$

3D. Marginal profit equations

Marginal profits enter in pricing efficiency conditions and these take the form

$$mz_{jt} = y_{jt} + \left(p_{jt} - \psi_{jt}\right) s_{jt} y_t \tag{3.D}$$

where s_{jt} is the slope of the demand curve. This expression involves only one marginal cost term if the factor market is global, since $\psi_{jt} = \psi_t$ in this case.

3E. Demand slopes

One element of marginal profits is the demand slope,

$$s_{jt} = \frac{1}{1-\eta} \frac{1}{\gamma - 1} \left[(p_{jt})^{\frac{\gamma-2}{\gamma-1}} \zeta_t^{\frac{1}{\gamma-1}} \right] \tag{3.E}$$

E.4 Consumption demand and labor supply

The fourth block involves consumption demand, labor supply, and various labor aggregations.

The household solves the following problem:

$$\max \left\{ \sum_t \beta^t \sum_j \omega_{jt} \left[\frac{1}{1-\sigma} c_{jt}^{1-\sigma} - \frac{\chi}{1+\phi} n_{jt}^{1+\phi} \right] \right\}$$

$$\text{subject to} \quad : \quad \left[\sum_j \omega_{jt} c_{jt} \right] \leq \sum_j \omega_{jt} \left[w_{jt} n_{jt} + z_{jt} \right]$$

4A. Marginal utility of consumption

$$c_{jt}^{-\sigma} = \lambda_t = c_t^{-\sigma} \tag{4.A}$$

4B. Total labor

$$n_t = n_t^P + n_t^y \tag{4.B}$$

4C. Labor supply

If the market is global, then

$$w_t = \frac{1}{\lambda_t} n_t^{\phi} \tag{4.C global}$$

where labor supply is the sum of labor in production and labor in price adjustment.

If there is a local labor market, then there is a labor supply equation that is pertinent for each "bin" $j = 1, 1., , , J - 1$,

$$w_{jt} = \frac{1}{\lambda_t} n_{jt}^{\phi} \tag{4.C local}$$

Adjusting firms must hire labor to adjust prices, so that the labor supply specification is a little different for the residents of "bin 0," who are paying the adjustment costs.

$$w_t = \frac{1}{\lambda_t} (n_{0t} + \frac{n_t^p}{\omega_{0t}})^{\phi} \tag{4.C local, bin 0}$$

4D. Labor in price adjustment

$$n_t^p = \sum_{j=1}^{J} \theta_{jt} \Xi_{jt} = \Xi_{Jt} + \sum_{j=1}^{J-1} \theta_{jt} \left(\Xi_{jt} - \Xi_{Jt} \right) \tag{4.D}$$

By way of backgound, the derivation of (4.D) is

$$\begin{aligned}
\text{using } 1 &= \sum_{j=1}^{J} \theta_{jt} \\
\text{then } \theta_{Jt} &= 1 - \sum_{j=1}^{J-1} \theta_{jt} \\
\text{and consequently } n_t^p &= \sum_{j=1}^{J-1} \theta_{jt} \Xi_{jt} + \theta_{Jt} \Xi_{Jt} \\
n_t^p &= \sum_{j=1}^{J-1} \theta_{jt} \Xi_{jt} + \left(1 - \sum_{j=1}^{J-1} \theta_{jt} \right) \Xi_{Jt} \\
n_t^p &= \Xi_{Jt} + \sum_{j=1}^{J-1} \theta_{jt} \left(\Xi_{jt} - \Xi_{Jt} \right)
\end{aligned}$$

4E. Labor in production

$$n_t^y = \sum_{j=0}^{J-1} \omega_{jt} n_{jt}^y \tag{4.E}$$

E.5 Firm value, efficient pricing, and efficient adjustment

The fifth block involves firm value functions, marginal conditions for pricing and efficient adjustment, and a defintion of the marginal value of having a higher price.

5A. Value function recursions

The value function recursions are defined in marginal utility units to make them easier to approximate.

$$v_j = \lambda z_j + \beta E \left\{ \left[\alpha'_{j+1} v'_0 - w' \Xi'_{j+1} \lambda' + \left(1 - \alpha'_{j+1} \right) v'_{j+1} \right] \right\} \tag{5.A standard}$$

$$v_{J-1} = \lambda z_{J-1} + \beta E \left\{ [v_0' - w' \Xi_J' \lambda'] \right\} \tag{5.A last}$$

5B. Marginal value recursions

The marginal values are also denominated in marginal utility units, so that pricing efficiency requires

$$0 = \lambda_t \frac{\partial z\left(p_{0t}, s_t\right)}{\partial p_{0t}} + \beta E_t \left[(1 - \alpha_{1,t+1}) \frac{\partial v_1 \left(p_{0t} \frac{P_t}{P_{t+1}}, s_{t+1} \right)}{\partial p_{0t}} \right] \tag{5.B first}$$

$$\frac{\partial v_j \left(p_{jt}, s_{t+1}\right)}{\partial p_{jt}} = \lambda_t \frac{\partial z\left(p_{jt}, s_t\right)}{\partial p_{jt}} + \beta E_t \left[(1 - \alpha_{j+1,t+1}) \frac{\partial v_{j+1} \left(p_{j,t} \frac{P_t}{P_{t+1}}, s_{t+1} \right)}{\partial p_{jt}} \right] \tag{5.B}$$

$$\frac{\partial v_{J-1} \left(p_{J-1,t}, s_{t+1}\right)}{\partial p_{J-1,t}} = \lambda_t \frac{\partial z\left(p_{J-1,t}, s_t\right)}{\partial p_{J-1,t}} \tag{5.B last}$$

5C. Optimal adjustment

The efficiency condition for optimal adjustment is

$$\xi\left(\alpha\right) = \frac{v_0 - v_j}{\lambda w} \tag{5.C }$$

5D. Expected adjustment costs

As discussed in appendix B of the working paper,

$$\Xi(\alpha) = \int_0^{\alpha} F(a) da \tag{5.D.}$$

E.6 Aggregate definitions and consistency conditions

6A. Aggregate demand

$$c_t = y_t \tag{6.A }$$

6B. Monetary equilibrium

$$\frac{M_t}{P_t} = c_t \tag{6.B}$$

6C. Monetary rule

$$\Delta \ln M_t = \rho \ln M_{t-1} + m_t \tag{6.C}$$

6D. Inflation

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{6.D}$$

6E. Lagged price level
The accounting relationship is

$$P_{t+1}^L = P_t \tag{6.E}$$

6F. Linear Aggregate Output

$$y_t^l = \sum_0^{J-1} \theta_{jt} y_{jt} = \sum_0^{J-1} \theta_{jt} x_{jt} y_t \tag{6.F}$$

E.7 Decomposition of adjustment dynamics

The equations in this section are based on various analytical approximations described in appendix D.

7A. Linear price level

As its name suggests, the linear price level is a linear aggregation of the existing prices in the economy

$$\overline{P}_t = \sum_{j=0}^{J-1} \omega_{jt} P_{jt} \tag{7.A}$$

7B. Price level without changing adjustment rates

$$\overline{P}_t = \sum_{j=0}^{J-1} \omega_j P_{jt} \tag{7.B}$$

7C. The effects of relative price on adjustment rates

$$\xi_\alpha(\alpha_j)(\widetilde{\alpha}_{jt} - \alpha_j) = -[\frac{p_j \partial v_{jt} / \partial p_{jt}}{w_0}] * [\log(p_{jt}) - \log(p_j)] + \text{other terms} \tag{7.C}$$

7D. Restriction on the synthetic ex post fractions

$$\sum_{j=0}^{J-1} (\widetilde{\omega}_{j,t} - \omega_j) = 0 \tag{7.D}$$

7E. Evolution of the synthetic ex post fractions

$$\widetilde{\omega}_{jt} - \omega_j = (1 - \alpha_j)(\widetilde{\theta}_{j-1,t-1} - \theta_{j-1}) - \theta_{j-1}(\widetilde{\alpha}_{jt} - \alpha_j) \tag{7.E}$$

7F. Evolution of the synthetic ex ante fractions

$$\widetilde{\theta}_{jt} - \theta_j = \widetilde{\omega}_{j-1,t-1} - \omega_{j-1} \tag{7.F}$$