

# WORKING PAPER NO. 02-19 OPTIMAL MONETARY POLICY

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# Abstract

Optimal monetary policy maximizes the welfare of a representative agent, given frictions in the economic environment. Constructing a model with two broad sets of frictions – costly price adjustment by imperfectly competitive firms and costly exchange of wealth for goods – we find optimal monetary policy is governed by two familar principles.

First, the average level of the nominal interest rate should be sufficiently low, as suggested by Milton Friedman, that there should be deflation on average. Yet, the Keynesian frictions imply that the optimal nominal interest rate is positive.

Second, as various shocks occur to the real and monetary sectors, the price level should be largely stabilized, as suggested by Irving Fisher, albeit around a deflationary trend path. (In modern language, there is only small "base drift" for the price level path as various shocks arise). Since expected inflation is roughly constant through time, the nominal interest rate must therefore vary with the Fisherian determinants of the real interest rate, i.e., with expected growth or contraction of real economic activity.

Although the monetary authority has substantial leverage over real activity in our model economy, it chooses real allocations that closely resemble those that would occur if prices were flexible. In our benchmark model, we also find some tendency for the monetary authority to smooth nominal and real interest rates.

# 1 Introduction

Three distinct intellectual traditions are relevant to the analysis of how optimal monetary policy can and should regulate the behavior of the nominal interest rate, output and the price level.

The Fisherian view: Early in this century, Irving Fisher [1923,1911] argued that the business cycle was "largely a dance of the dollar" and called for stabilization of the price level, which he regarded as the central task of the monetary authority. Coupled with his analysis of the determination of the real interest rate [1930] and the nominal interest rate [1896], the Fisherian prescription implied that the nominal interest rate would fluctuate with those variations in real activity which occur when the price level is stable.

The Keynesian view: Stressing that the market-generated level of output could be inefficient, Keynes [1936] called for stabilization of real economic activity by fiscal and monetary authorities. Such stabilization policy typically mandated substantial variation in the nominal interest rate when shocks, particularly those to aggregate demand, buffeted the economic system. Prices were viewed as relatively sticky and little importance was attached to the path of the price level.

The Friedman view: Evaluating monetary policy in a long-run context with fully flexible prices, Friedman [1969] found that an application of a standard microeconomic principle of policy analysis long used in public finance – that social and private cost should be equated – indicated that the nominal interest rate should be approximately zero. Later authors used the same reasoning to conclude that the nominal interest rate should not vary through time in response to real and nominal disturbances, working within flexible price models of business fluctuations.<sup>1</sup>

There are clear tensions between these three traditions if real forces produce expected changes in output growth that affect the real interest rate. If the price level is constant, then the nominal interest rate must mirror the real interest rate so that Friedman's rule must be violated. If the nominal interest rate is constant, as Friedman's rule suggests, then there must be expected inflation or deflation to accommodate the movement in the real rate so that Fisher's prescription cannot be maintained. The variation in both inflation and nominal interest rates generally implied by Keynesian stabilization conflicts with both the Friedman and Fisherian views.

We construct a model economy that honors each of these intellectual traditions and study the nature of optimal monetary policy. There are Keynesian features to the economy: output is inefficiently low because firms have market power and its fluctuations reflect the fact that all prices cannot be frictionlessly adjusted. However, as in the New Keynesian research on price stickiness that begins with Taylor [1980], firms are forward-looking in their price setting and this has dramatic implications for the design of optimal monetary policy. In our economy, there are also costs of converting wealth into consumption. These costs can be mitigated by the use of

<sup>&</sup>lt;sup>1</sup>See Chari and Kehoe [1999] for a survey.

money, so that there are social benefits to low nominal interest rates as in Friedman's analysis. The behavior of real and nominal interest rates in our economy is governed by Fisherian principles.

Following Ramsey [1927] and Lucas and Stokey [1983], we determine the allocation of resources that maximizes welfare (technically, it maximizes the expected, present discounted value of the utility of a representative agent) given the resource constraints of the economy and additional constraints that capture the fact that the resource allocation must be implemented in a decentralized private economy.<sup>2</sup> We assume that there is full commitment on the part of a social planner for the purpose of determining these allocations. We find that two familiar principles govern monetary policy in our economy:

The Friedman prescription for deflation: The average level of the nominal interest rate should be sufficiently low, as suggested by Milton Friedman, that there should be deflation on average. Yet, the Keynesian frictions generally imply that there should be a positive nominal interest rate.

The Fisherian prescription for eliminating price-level surprises: As shocks occur to the real and monetary sectors, the price level should be largely stabilized, as suggested by Irving Fisher, albeit around a deflationary trend path. (In modern language, there is only a small "base drift" for the price level path). Since expected inflation is relatively constant through time, the nominal interest rate must therefore vary with the Fisherian determinants of the real interest rate. However, there is some tendency for nominal and real interest rate smoothing relative to the outcomes in a flexible price economy.

By contrast, we find less support for Keynesian stabilization policy. Although the monetary authority has substantial leverage over real activity in our model economy, it chooses allocations that closely resemble those which would occur if prices were flexible. When departures from this flexible price benchmark occur under optimal policy, they are not always in the traditional direction: in one example, a monetary authority facing a high level of government demand chooses to contract private consumption relative to the flexible price outcome, rather than stimulating it.

The organization of the paper is as follows. In section 2, we outline the main features of our economic model and define a recursive imperfectly competitive equilibrium. In section 3, we describe the nature of the general optimal policy problem that we solve, which involves a number of forward-looking constraints. We outline how to treat this policy problem in an explicitly recursive form. Our analysis thus exemplifies a powerful recursive methodology for analyzing optimal monetary policy in richer models that could include capital formation, state dependent pricing and other frictions such as efficiency wages or search. In section 4, we identify four distortions present in our economic model, which are summary statistics for how its behavior can differ from a fully competitive, nonmonetary business cycle model. In section 5,

<sup>&</sup>lt;sup>2</sup>Our economy involves staggered prices. Ireland [1996], Goodfriend and King [2001], and Adao, Correia and Teles [2001] use a similar approach to study economies with pre-set prices.

we discuss calibration of a quantitative version of our model, including estimation of a money demand function.

In section 6, we discuss the results that lead to the first principle for monetary policy: the nominal interest rate should be set at an average level that implies deflation, but it should be positive. We show how this steady-state rate of deflation depends on various structural features of the economy, including the costs of transacting with credit – which give rise to money demand – and the degree of price-stickiness.<sup>3</sup> In our benchmark calibration, which is based on an estimated money demand function using post-1958 observations, the extent of this deflation is relatively small, about .75%. It is larger (about 2.3%) if we use estimates of money demand based on a longer sample beginning in 1948, which includes earlier observations when interest rates and velocity were both low.<sup>4</sup> In addition, a smaller degree of market power or less price stickiness make for a larger deflation under optimal policy.

In section 7, we describe the near-steady state dynamics of the model under optimal policy. Looking across a battery of specifications, we find that these dynamics display only minuscule variation in the price level. Thus, we document that there is a robustness to the Fisherian conclusion in King and Wolman [1999], which is that the price level should not vary greatly in response to a range of shocks under optimal policy. In fact, the greatest price level variation that we find involves less than a 0.5%change in the price level over 20 quarters, in response to a productivity shock which brings about a temporary but large deviation of output from trend, in the sense that the cumulative output deviation is more than 10% over the twenty quarters. Across the range of experiments, output under optimal policy closely resembles output that would occur if all prices were flexible and monetary distortions were absent. We refer to the flexible price, nonmonetary model as our underlying real business cycle framework. Although there are only small deviations of quantities under optimal policy from their real business cycle counterparts, because these deviations are temporary, they give rise to larger departures of real interest rates from those in the RBC solution. We relate the natures of these departures to the nature of constraints on the monetary authority's policy problem. Section 8 concludes.

# 2 The model

The macroeconomic model we study is designed to be representative of two recent strands of macroeconomic research. First, we view money as a means of economizing on the use of costly credit.<sup>5</sup> Second, we use a new Keynesian approach to price dynamics, viewing firms as imperfect competitors facing infrequent opportunities for

<sup>&</sup>lt;sup>3</sup>By the steady state, we mean the point to which the economy converges under optimal policy if there is no uncertainty.

<sup>&</sup>lt;sup>4</sup>Lucas [2000] highlights the importance of including intervals of low interest rates for estimation of the demand for money and the calculation of associated welfare cost measures.

<sup>&</sup>lt;sup>5</sup>As in Prescott [1987], Dotsey and Ireland [1996], and Lacker and Schreft [1996].

price adjustment.<sup>6</sup> To facilitate the presentation of these mechanisms, we view the private sector as divided into three groups of agents. First, there are households that buy final consumption goods and supply factors of production. These households also trade in financial markets for assets, including a credit market, and acquire cash balances which can be exchanged for goods. Second, there are retailers, which sell final consumption goods to households and buy intermediate products from firms. Retailers can costlessly adjust prices.<sup>7</sup> Third, there are producers, who create the intermediate products that retailers use to produce final consumption goods. These firms have market power and face only infrequent opportunities to adjust prices.

The two sources of uncertainty are the level of total factor productivity, a, and the level of real government purchases, g, which is assumed to be financed with lump sum taxes. These variables depend on an exogenous state variable  $\varsigma$ , which evolves over time as a Markov process, with the transition probability denoted  $\Upsilon(\varsigma, \cdot)$ . That is, if the current state is  $\underline{\varsigma}$  then the probability of the future state being in a given set of states B is  $\Upsilon(\underline{\varsigma}, B) = \Pr{\{\varsigma' \in B \mid \varsigma = \underline{\varsigma}\}}$ . We thus write total factor productivity as  $a(\varsigma)$  and real government spending as  $g(\varsigma)$ .

In this section, we describe a recursive equilibrium in this economy, with households and firms solving dynamic optimization problems given a fixed, but potentially very complicated, rule for monetary policy that allows it to respond to all of the relevant state variables of the economy, which are of three forms. Ignoring initially the behavior of the monetary authority, the model identifies two sets of state variables. First, there are the exogenous state variables just discussed. Second, since some prices are sticky, predetermined prices are part of the relevant history of the economy or, more generally, define a set of endogenous state variables. These endogenous state variables, s, evolve through time according to a multivalent function  $\Gamma$  where  $s' = \Gamma(s, p_0)$ , with  $p_0$  being an endogenous variable described further below. We allow the monetary authority to respond to  $\varsigma$  and s, but also to an additional vector of state variables  $\phi$ , which evolves according to  $\phi' = \Phi(\varsigma, s, \phi)$ , so this is a third set of states. In a recursive equilibrium,  $p_0$  is a function of the monetary rule, so that the states s evolve according to  $s' = \Gamma(s, p_0(\varsigma, s, \phi))$ ; we will sometimes write this as  $s' = \Gamma(s, \phi, \varsigma)$ . Hence, there is a vector of state variables  $\sigma = (s, \phi, \varsigma)$  that is relevant for agents, resulting from the stochastic nature of productivity and government spending; from the endogenous dynamics due to sticky prices; and, potentially, from the dynamic nature of the monetary rule.

# 2.1 Households

Households have preferences for consumption and leisure, represented by the timeseparable expected utility function,

<sup>&</sup>lt;sup>6</sup>Taylor [1980], Calvo [1983]

<sup>&</sup>lt;sup>7</sup>It is possible to eliminate the retail sector, but including it makes the presentation of the model easier.

$$\mathbf{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \right\}$$
(1)

The period utility function u(c, l) is assumed to be increasing in consumption and leisure, strictly concave and differentiable as needed. Households divide their time allocation – which we normalize to one unit – into leisure, market work n, and transactions time  $h_t$  so that  $n_t + l_t + h_t = 1$ .

Accumulation of wealth: Households begin each period with a portfolio of claims on the intermediate product firms, holding a previously determined share  $\theta_t$  of the per capita value of these firms. This portfolio generates current nominal dividends of  $\theta_t Z_t$  and has nominal market value  $\theta_t V_t$ .<sup>8</sup> They also begin each period with a stock of nominal bonds left over from last period which have matured and have market value  $B_t$ . Finally, they begin each period with nominal debt arising from consumption purchases last period, in the amount  $D_t$ . So, their nominal wealth is  $\theta_t V_t + \theta_t Z_t + B_t - D_t - T_t$ , where  $T_t$  is the amount of a lump sum tax paid to the government. With this nominal wealth and current nominal wage income  $W_t n_t$ , they may purchase money  $M_t$ , buy current period bonds in amount  $B_{t+1}$ , or buy more claims on the intermediate product firms. Thus, they face the constraint

$$M_t + \frac{1}{1 + R_t} B_{t+1} + \theta_{t+1} V_t \ge \theta_t V_t + \theta_t Z_t + B_t - D_t - T_t + W_t n_t.$$

We convert this nominal budget constraint into a real one, using a numeraire  $P_t$ . At present this is simply an abstract measure of nominal purchasing power but we are more specific later about its economic interpretation. Denoting the rate of inflation between period t-1 and period t as  $\pi_t = \frac{P_t}{P_{t-1}} - 1$ , the real flow budget constraint is

$$m_t + \frac{1}{1+R_t} b_{t+1} + \theta_{t+1} v_t \le \theta_t v_t + \theta_t z_t + \frac{b_t}{1+\pi_t} - \frac{d_t}{1+\pi_t} - \tau_t + w_t n_t,$$

with lower case letters representing real quantities when this does not produce notational confusion (real lump sum taxes are  $\tau_t = \frac{T_t}{P_t}$ ).<sup>9</sup>

Money and transactions: Although households have been described as purchasing a single aggregate consumption good, we now reinterpret this as involving many individual products – technically, a continuum of products on the unit interval – as in many studies following Lucas [1980]. Each of these products is purchased from a separate retail outlet at a price  $\overline{P}_t$ . Each customer buys a fraction  $\xi_t$  of goods with credit and the remainder with cash. Hence, the households' demand for nominal money satisfies  $M_t = (1 - \xi_t)\overline{P}_t c_t$ . The customer's nominal debt is correspondingly

 $<sup>{}^{8}</sup>Z_{t}$  and  $V_{t}$  are aggregates of the dividends and values of individual firms in a sense that we make more precise below.

<sup>&</sup>lt;sup>9</sup>For example  $m_t = \frac{M_t}{P_t}$  and  $v_t$ ,  $z_t$  and  $w_t$  are similarly defined. The two exceptions are the predetermined variables  $B_t$  and  $D_t$ , where  $b_t = \frac{B_t}{P_{t-1}}$  and  $d_t = \frac{D_t}{P_{t-1}}$ .

 $D_{t+1} = \xi_t \overline{P}_t c_t$ , which must be paid next period. Following our convention of using lower case letters to define real quantities, define  $\overline{p}_t \equiv \frac{\overline{P}_t}{P_t}$ . The real money demand of the household takes the form  $m_t = (1 - \xi_t)\overline{p}_t c_t$  and similarly  $d_{t+1} = \xi_t \overline{p}_t c_t$ .

We think of each final consumption goods purchase having a random fixed time cost – perhaps, the extent to which small children are clamoring for candy in the checkout queue – which must be borne if credit is used. This cost is known after the customer has decided to purchase a specific amount of the product, but before the customer has decided whether to use money or credit to finance the purchase. Let  $F(\cdot)$  be the cumulative distribution function for time costs. If credit is used for a particular good, then there are time costs  $\nu$  and the largest time cost is given by  $\bar{\nu}_t = F^{-1}(\xi_t)$ . Thus, total time costs are  $h_t = \int_0^{F^{-1}(\xi_t)} \nu \, dF(\nu)$ . The household uses credit when its time cost is below the critical level given by  $F^{-1}(\xi_t)$  and uses money when the cost is higher.

#### 2.1.1 Maximization Problem

Although the household's individual state vector can be written as its holdings of each asset  $(\theta, b, d)$ , it is convenient here – as in many other models – to aggregate these assets into a measure of wealth  $\varpi = v\theta + z\theta + \frac{b-d}{1+\pi} - \tau$ . We let U be the value function, i.e., the discounted expected lifetime utility of a household when it is behaving optimally. The recursive maximization problem is then

$$U(\varpi;\sigma) = \max_{\xi,c,l,n,m,\theta',b',d'} \{ u(c,l) + \beta E U(\varpi';\sigma') | \sigma \}$$
(2)

subject to

$$m + \frac{1}{1+R}b' + v\theta' \le v\theta + z\theta + \frac{b-d}{1+\pi} - \tau + wn = \varpi + wn$$
(3)

$$n = 1 - l - h \tag{4}$$

$$h = \int_{0}^{F^{-1}(\xi)} \nu \ dF(\nu)$$
 (5)

$$m = (1 - \xi)\overline{p}c \tag{6}$$

$$d' = \xi \overline{p}c \tag{7}$$

The household is assumed to view  $w, v, R, z, \overline{p}$  and  $\tau = T/P$  as functions of the state vector  $(\sigma)$ . The conditional expectation  $\beta EU(\varpi'; \varsigma', s', \phi') |\sigma\}$  $= \int U(\varpi'; \varsigma', s', \phi') \Upsilon(\varsigma, d\varsigma')$ , taking as given the laws of motion  $s' = \Gamma(\sigma)$  and  $\phi' = \Phi(\sigma)$  discussed above and the definition  $\varpi' = v'\theta' + z'\theta' + \frac{b'-d'}{1+\pi'} - \tau$ . We will return to discussion of the determinants and consequences of inflation later.

#### 2.1.2 Efficiency conditions

We consolidate the household's constraints (3) - (7) into a single constraint, by eliminating hours worked, as is conventional. We also substitute out for money, using  $m = (1 - \xi)\overline{p}c$ , and future debt, using  $d' = \xi \overline{p}c$  to simplify this constraint further. Let  $\lambda$ , which has the economic interpretation as the shadow value of wealth, represent the multiplier for this combined constraint. Then, we use the envelope theorem to derive  $D_1 U(\varpi,;\sigma) = \lambda$ .<sup>10</sup> (Our notation  $D_i$  means the first partial derivative of a function with respect to its  $i^{th}$  argument). We can then state the household's efficiency conditions as

$$c : D_1 u(c,l) = \lambda \left(1-\xi\right) \overline{p} + \beta E[\lambda' \frac{\overline{p}}{1+\pi'} \xi] |\sigma$$
(8)

$$\xi : \lambda \overline{p}c = \lambda w F^{-1}(\xi) + \beta E[\lambda' \frac{p}{1+\pi'}c] |\sigma$$
(9)

$$l : D_2 u(c,l) = w\lambda \tag{10}$$

$$b' : \frac{1}{1+R}\lambda = \beta E[\lambda' \frac{1}{1+\pi'}]|\sigma$$
(11)

$$\theta : v\lambda = \beta E \left[ \lambda' v' + \lambda' z' \right] |\sigma$$
(12)

as well as (3)-(7). Condition (8) states that the marginal utility of consumption must be equated to the full cost of consuming, which is a weighted average of the costs of purchasing goods with currency and credit. Condition (9) equates the marginal benefit of raising  $\xi$  – expanding its use of credit and decreasing its demand for money – to its net marginal cost, which is the sum of current time cost and future repayment cost. Condition (10) is the conventional requirement that the marginal utility of leisure is equated to the real wage rate times the shadow value of wealth. The last two conditions specify that holdings of stocks and bonds are efficient.

# 2.2 Retailers

We assume that retailers create units of the final good according to a constant elasticity of substitution aggregator of a continuum of intermediate products, indexed on the unit interval,  $i \in [0, 1]$ .<sup>11</sup> Retailers create q units of final consumption according to

$$q = \left[\int q(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}},\tag{13}$$

<sup>&</sup>lt;sup>10</sup>We use the phrase "envelope theorem" as short-hand for analyses following Benveniste and Scheinkman [1979], which supply derivatives of the value function under particular conditions that ensure its differentiability.

<sup>&</sup>lt;sup>11</sup>Note that this continuum of intermediate goods firms is distinct from the continuum of retail outlets at which consumers purchase final goods.

where  $\varepsilon$  is a parameter. In our economy, however, there will be groups of intermediate goods-producing firms which will all charge the same price for their good within a period and they can be aggregated easily. Let the *j*-th group have fraction  $\omega_j$  and charge a nominal price  $P_j$ . Then the retailer allocates its demands for intermediates across the *J* categories, solving the following problem.

$$\min_{q_j} (1+R) \sum_{j=0}^{J-1} \omega_j p_j q_j$$
(14)

subject to

$$\overline{q} = \left(\sum_{j=0}^{J-1} \omega_j q_j^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
(15)

where  $p_j = \frac{P_j}{P}$  is the relative price of the *j*-th set of intermediate inputs. Retailers view R and  $\{p_j\}_{j=0}^{J-1}$  as functions of  $\sigma$ . The nominal interest factor (1+R) affects the retailer's expenditures because, as is further explained below, the retailer must borrow to finance current production. This cost minimization problem leads to intermediate input demands of a constant elasticity form

$$q_j = \left(p_j^{-\varepsilon}\right)\bar{q}.\tag{16}$$

where  $\overline{q}$  is the retailer's supply of the composite good. Cost minimization also implies a nominal unit cost of production – an intermediate goods price level of sorts – given by

$$P = \left[\sum_{j=0}^{J-1} \omega_j P_j^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}.$$
(17)

This is the price index that we use as numeraire in the analysis above. Since the retail sector is competitive and all goods are produced according to the same technology, it follows that the final goods price must satisfy  $\overline{P} = (1 + R(\sigma))P$  and that the relative price of consumption goods is given by

$$\overline{p}(\sigma) = 1 + R(\sigma). \tag{18}$$

Since they have no market power or specialized factors, retailers earn no profits. Hence, their market value is zero and does not enter in the household budget constraint. At the same time, they are borrowers, making their expenditures at t and receiving their revenues at t+1. That is: for each unit of sales, the retail firm receives revenues in money or credit. Each of these are cash flows which are effectively in date t+1 dollars. If the firm receives money, then it must hold it "overnight." If the firm takes credit, then it is paid only at date t+1 with no explicit interest charges, as for example with "credit cards" in many countries.

# 2.3 Intermediate goods producers

The producers of intermediate products are assumed to be monopolistic competitors and face irregularly timed opportunities for price adjustment. For this purpose, we use a generalized stochastic price adjustment model due to Levin [1991], as recently exposited in Dotsey, King and Wolman's [1999] analysis of state dependent pricing. In this setup, a firm that has held its price fixed for j periods will be permitted to adjust with probability  $\alpha_j$ .<sup>12</sup> With a continuum of firms, the fractions  $\omega_j$  are determined by the recursions  $\omega_j = (1 - \alpha_j)\omega_{j-1}$  for j = 1, 2, ..., J-1 and the condition that  $\omega_0 = 1 - \sum_{j=1}^{J-1} \omega_j$ .

Each intermediate product i on the unit interval is produced according to the production function

$$y(i) = an(i) \tag{19}$$

with labor being paid a nominal wage rate of W and being flexibly reallocated across sectors. Nominal marginal cost for all firms is accordingly W/a. Let  $p(i) \equiv \frac{P(i)}{P}$  be the *i*-th intermediate goods producer's relative price and  $w = \frac{W}{P}$ , the real wage, so that real marginal cost is  $\psi = w/a$ .

Intermediate goods firms face a demand given by

$$y(i) = p(i)^{-\varepsilon}q(\sigma) \tag{20}$$

with the aggregate demand measure being  $q(\sigma) = c(\sigma) + g(\varsigma)$ , i.e., the sum of household and government demand.

#### 2.3.1 Maximization Problem

Intermediate goods firms maximize the present discounted value of their real monopoly profits given the demand structure and the stochastic structure of price adjustment. Using (19) and (20), current profits may be expressed as

$$z(p(i);\sigma) = p(i)y(i) - w(\sigma)n(i) = p(i)^{-\varepsilon}q(\sigma)\left[p(i) - \frac{w(\sigma)}{a(\varsigma)}\right].$$
 (21)

All firms that are adjusting at date t will choose the same nominal price, which we call  $P_0$ , which implies a relative price  $p_0 = \frac{P_0}{P}$ . The mechanical dynamics of relative prices are simple to determine. Given that a nominal price is set at a level  $P_j$ , then the current relative price is  $p_j = P_j/P$ . If no adjustment occurs in the next period, then the future relative price satisfies

$$p_{j+1}' = \frac{p_j}{1+\pi'}.$$
(22)

<sup>&</sup>lt;sup>12</sup>This stochastic adjustment model is flexible in that it contains the Taylor [1980] staggered price adjustment model as one special case (a four-quarter model would set  $\alpha_1 = \alpha_2 = \alpha_3 = 0$  and  $\alpha_4 = 1$ ), the Calvo [1983] model as another (this makes  $\alpha_j = \alpha$  for all j), and can be used to match microeconomic data on price adjustment.

A price-setting intermediate goods producer solves the following maximization problem:

$$v^{0}(\sigma) = \max_{p_{0}} \left[ z(p_{0};\sigma) + E\{\beta \frac{\lambda(\sigma')}{\lambda(\sigma)} \left[ \alpha_{1}v^{0}(\sigma') + (1-\alpha_{1})v^{1}(p_{1}',\sigma') \right] \} \right] |\sigma, \qquad (23)$$

with the maximization taking place subject to  $p'_1 = \frac{P'_1}{P'} = \frac{P_0}{P} \frac{P}{P'} = p_0/(1 + \pi')$ . A few comments about the form of this equation are in order. First, the discount factor used by firms equals households' shadow value of wealth in equilibrium, so we impose that requirement here. Second, as is implicit in our profit function, the firm is constrained by its production function and by its demand curve, which depends on aggregate consumption and government demand. Third, the firm knows that there are two possible situations at date t + 1. With probability  $\alpha_1$  it will adjust its price and the current pricing decision will be irrelevant to its market value  $(v^0)$ . With probability  $1 - \alpha_1$  it will not adjust its price and the current price will be maintained, resulting in a market value  $(v^1)$ , with the superscript j in  $v^j$  indicating the value of a firm which is maintaining its price fixed at the level set at date t - j, i.e.,  $P_{jt} = P_{0,t-j}$ . Thus, we have for  $j = 1, \ldots, J - 2$ ,

$$v^{j}(p_{j},\sigma) = z(p_{j};\sigma) + E\{\beta \frac{\lambda(\sigma')}{\lambda(\sigma)} [\alpha_{j+1}v^{0}(\sigma') + (1-\alpha_{j+1})v^{j+1}(p'_{j+1},\sigma')]\} |\sigma, \quad (24)$$

with  $p'_{j+1} = \frac{p_j}{1+\pi'}$ . Finally, in the last period of price fixity, all firms know that they will adjust for certain so that

$$v^{J-1}(p_{J-1},\sigma) = z(p_{J-1};\sigma) + E\{\beta \frac{\lambda(\sigma')}{\lambda(\sigma)} [v^0(\sigma')]\} | \sigma.^{13}$$
(25)

### 2.3.2 Efficiency conditions

In order to satisfy (23), the optimal pricing decision requires  $p_0$  to solve

$$0 = D_1 z (p_0; \sigma) + \beta E \left\{ \frac{\lambda'}{\lambda} (1 - \alpha_1) D_1 v^1 (p_1'; \sigma') \frac{1}{1 + \pi'} \right\} | \sigma.$$
 (26)

From (21), marginal profits are given by

$$D_1 z(p_j; \sigma) = q(\sigma) \left[ (1 - \varepsilon) p_j^{-\varepsilon} + \varepsilon \frac{w(\sigma)}{a(\sigma)} p_j^{-\varepsilon - 1} \right].$$
(27)

The optimal pricing condition (26) states that, at the optimum, a small change in price has no effect on the present discounted value. The presence of future inflation reflects the fact that  $p'_1 = p_0/(1 + \pi')$ , so that when the firm perturbs its relative

price by  $dp_0$ , it knows that it is also changing its one period ahead relative price by  $1/(1 + \pi')dp_0$ .<sup>14</sup> Equations (24) imply

$$D_1 v^j(p_j;\sigma) = D_1 z(p_j;\sigma) + \beta E \left\{ \frac{\lambda'}{\lambda} (1 - \alpha_{j+1}) D_1 v^{j+1}(p'_{j+1};\sigma') \frac{1}{1 + \pi'} \right\} |\sigma$$
(28)

for  $j = 1, \ldots, J - 2$ , while (25) implies

$$D_1 v^{J-1}(p_{J-1};\sigma) = D_1 z(p_{J-1};\sigma).$$
(29)

# 2.4 Defining the state vector s

We next consider the price component of the aggregate state vector. The natural state is the vector of previously determined nominal prices,  $[P_{1,t} \ P_{2,t} \ \dots \ P_{J-1,t}]^{15}$  Given these nominal prices and the current nominal price  $P_{0,t}$ , the price level is determined as  $P_t = [\sum_{j=0}^{J-1} \omega_j P_{j,t}^{(1-\varepsilon)}]^{\frac{1}{1-\varepsilon}}$ . However, our analysis concerns (i) households and firms that are concerned about real objectives as described above; and (ii) a monetary authority who seeks to maximize a real objective as described below. Accordingly, neither is concerned about the absolute level of prices in the initial period of our model (i.e., the time at which the monetary policy rule is implemented). For this reason, we define an alternative *real* state vector that captures the influence of predetermined nominal prices, but is compatible with any initial scale of nominal prices. In this section, we define this real state vector and describe some of its key properties. In appendix A, we provide a detailed derivation so that future analyses of richer economic models – containing capital, state dependent pricing and so forth – can make use of our approach.

To begin, recall that all adjusting firms choose a relative price  $p_{0,t}$ . Given the nominal state vector, this choice effectively determines the price level, i.e.,

$$P_t = \left[\omega_0 (p_{0,t} P_t)^{1-\varepsilon} + \sum_{j=1}^{J-1} \omega_j P_{j,t}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}} = \left[\frac{\sum_{j=1}^{J-1} \omega_j P_{j,t}^{(1-\varepsilon)}}{1-\omega_0 (p_{0,t})^{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}}$$

This suggests the value of defining an index of lagged nominal prices as

$$\widehat{P}_{t} = \left[\frac{1}{1-\omega_{0}}\sum_{j=1}^{J-1}\omega_{j}P_{j,t}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}.$$
(30)

<sup>&</sup>lt;sup>14</sup>There is a conceptual subtlety here that warrants some additional discussion. As described in the text, we view an individual firm as choosing  $p_0$  taking as given the actions of all other firms – including other adjusting firms – as these affect the price level, aggregate demand and so forth. Specifically, the firm views the actions of other adjusting firms as a function,  $\tilde{p}_0(\sigma)$ , with a law of motion for  $\sigma$  described earlier. In an equilibrium, there is a fixed point in that the decision rule of the individual firm  $p_0(\sigma)$  is equal to the function  $\tilde{p}_0(\sigma)$ . To avoid proliferation of notation, we simply use  $p_0(\sigma)$  to capture both concepts, with the hope that this does not produce confusion.

<sup>&</sup>lt;sup>15</sup>The state vector can alternatively be written as  $[P_{0,t-1} \ P_{0,t-2} \ \dots \ P_{0,t-(J-1)}]$ .

From above, we can see that variations in the price level relative to this index of lagged nominal prices arise solely due to  $p_{0,t}$  so that we define  $\gamma_1(p_{0,t}) = (1 - \omega_0)^{\frac{1}{1-\varepsilon}} [1 - \omega_0(p_{0,t})^{1-\varepsilon}]^{\frac{-1}{1-\varepsilon}} = P_t/\hat{P}_t.$ 

Using this indexed of lagged prices, we can express the real state of the economy as  $s = (s_1, \ldots, s_{J-2})$ . We choose to date this state vector as  $s_{t-1}$  to emphasize that it is predetermined in period t. These real states are relative prices – in terms of the index of lagged nominal prices – of the first J - 2 types of intermediate inputs,<sup>16</sup>

$$s_{j,t-1} = \frac{P_{j,t}}{\hat{P}_t} = \frac{P_{j,t}}{\left[\frac{1}{1-\omega_0}\sum_{n=1}^{J-1}\omega_n P_{n,t}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}}$$
(31)

for j = 1, ..., J - 2. Their evolution is straightforward to determine; we provide detailed derivations in the appendix. The first future state is given by

$$s_{1,t} = \frac{P_{1,t+1}}{\widehat{P}_{t+1}} = \frac{P_{0t}}{\widehat{P}_t} \frac{\widehat{P}_t}{\widehat{P}_{t+1}} = \frac{\gamma_0(p_{0,t})}{\gamma_2(p_{0,t}, s_{1,t-1}, \dots s_{J-2,t-1})}.$$
(32)

In (32),  $\gamma_0()$  is a function that describes the price set by adjusting firms relative to the index of predetermined prices and  $\gamma_2()$  describes inflation in the index of predetermined prices, with these functions being derived in appendix A.<sup>17</sup> Further, since  $P_{j,t} = P_{j+1,t+1}$ , the other future states satisfy

$$s_{j+1,t} = \frac{\widehat{P}_t}{\widehat{P}_{t+1}} s_{j,t-1} = \frac{s_{j,t-1}}{\gamma_2(p_{0,t}, s_{1,t-1}, \dots s_{J-2,t-1})}, \ j = 1, 2, \dots, J-3.$$
(33)

Taking all of these results together, it is clear that the real state vector evolves according to  $s' = \Gamma(s, p_0)$  as discussed above, which we can now write as  $s_t = \Gamma(s_{t-1}, p_{0,t})$ Accordingly, if  $p_{0,t}$  is a function simply of  $\sigma_t$ , this real state vector evolves according to  $s_t = \Gamma(s_{t-1}, p_0(\sigma_t))$ , which we write as  $s' = \Gamma(\sigma)$ .<sup>18</sup>

Given the real state vector, it is easy to calculate the relative prices that enter into the model, i.e.,

$$p_{j,t} = \frac{P_{j,t}}{P_t} = s_{j,t-1} \frac{\widehat{P}_t}{P_t} = s_{j,t-1} / \gamma_1(p_{0,t})$$
(34)

It is also easy to calculate the nominal variables that enter into the decision problems of individuals. For example, households and firms are concerned about future inflation

$$1 + \pi_{t+1} = \frac{P_{t+1}}{P_t} = \frac{P_{t+1}/\hat{P}_{t+1}}{P_t/\hat{P}_t}\frac{\hat{P}_{t+1}}{\hat{P}_t} = \frac{\gamma_1(p_{0,t+1})\gamma_2(p_{0,t}, s_{t-1})}{\gamma_1(p_{0,t})}.$$
(35)

<sup>16</sup>Note that we need only to include J - 2 such relative prices because the final relative price  $\frac{P_{J-1,t}}{\hat{P}_t}$  satisfies the identity  $1 = \left[\frac{1}{1-\omega_0}\sum_{j=1}^{J-1}\omega_j s_{j,t}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}$ .

 $\begin{bmatrix} r_t & r_{1-\omega_0} \not\simeq_{J^{-1}} & j_{J^{t-1}} & j_{J^{t-1}} \\ 1^7 \text{These functions are } \gamma_0(p_{0,t}) &= \left[ \frac{(1-\omega_0)(p_{0,t})^{1-\varepsilon}}{1-\omega_0(p_{0,t})^{1-\varepsilon}} \right]^{\frac{1}{1-\varepsilon}} \text{and} \quad \gamma_2\left(p_{0,t}, s_{1,t-1}, \dots s_{J-2,t-1}\right) \\ = \left[ \omega_1 \gamma_0(p_{0,t})^{1-\varepsilon} + \sum_{j=2}^{J-1} \omega_j s_{j-1,t-1}^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.$ 

<sup>&</sup>lt;sup>18</sup>Note that the household's endogenous state variables,  $\theta$ , b and d are not part of the aggregate state vector since, in equilibrium,  $\theta = 1$  and b - d = 0.

Therefore, we may write future inflation as  $1 + \pi(\sigma', \sigma)$ , under the working assumption that  $p_{0,t}$  is a function only of  $\sigma_t$ .

# 2.5 Monetary policy

Monetary policy determines the nominal quantity of money. However, just as we normalized other nominal variables by the index of predetermined prices, it is convenient to normalize the money stock by the index of predetermined prices, and thus to view the monetary authority as choosing the normalized money stock. With this normalization, we denote the policy rule by  $\mathfrak{M}(\sigma_t)$ , and the nominal money supply is given by

$$M_t = \mathfrak{M}(\sigma_t) \cdot \widehat{P}_t. \tag{36}$$

Real balances are given by  $m_t = \mathfrak{M}(\sigma_t) \cdot \frac{\hat{P}_t}{P_t} = \frac{\mathfrak{M}(\sigma_t)}{\gamma_1(p_{0,t})}$ .<sup>19</sup>

With the general function  $\mathfrak{M}(\sigma_t)$  we are not taking a stand on the targets or instruments of monetary policy. This notation makes clear, however, that the monetary authority's optimal decisions will depend on the same set of state variables as the decisions of the private sector.

# 2.6 Recursive equilibrium

We now define a recursive equilibrium in a manner that highlights the key elements of the above analysis.<sup>20</sup>

**Definition 1** For a given monetary policy function  $\mathfrak{M}(\sigma)$ , a Recursive Equilibrium is a set of relative price functions  $\lambda(\sigma)$ ,  $w(\sigma)$ ,  $\{p_j(\sigma)\}_{j=0}^{J-1}$ , and  $\overline{p}(\sigma)$ ; an interest rate function  $R(\sigma)$ ; a future inflation function  $\pi(\sigma', \sigma)$ ; aggregate production,  $q(\sigma)$ ; dividends,  $z(\sigma)$ ; intermediate goods producers' profits  $\{z_j(\sigma)\}_{j=0}^{J-1}$ ; value functions  $U(\cdot)$ and  $\{v^j(\cdot)\}_{j=0}^{J-1}$ ; household decision rules  $\{\xi(\sigma), c(\sigma), l(\sigma), n(\sigma), m(\sigma), \theta'(\sigma), b'(\sigma), d'(\sigma)\}_{j=0}^{S}$ ; retailers' relative quantities,  $\{q_j(\sigma)\}_{j=0}^{J-1}$ ; intermediate goods producers' relative prices,  $\{p_j(\sigma)\}_{j=0}^{J-1}$  and a law of motion for the aggregate state  $\sigma = (\varsigma, s, \phi)$ ,  $\varsigma' \sim \Upsilon(\varsigma, \cdot)$ ,  $s' = \Gamma(\sigma)$  and  $\phi' = \Phi(\sigma)$  such that: (i) households solve (2) - (7), (ii) retailers solve (14) - (15), (iii) price-setting intermediate goods producers solve (22) - (25), and (iv) markets clear.

<sup>&</sup>lt;sup>19</sup>It is clear from (36) that if the policy rule involves no response to the state, then this generally does not make the nominal money supply constant, because a constant  $\mathfrak{M}()$  implies  $M_t = \mathfrak{M} \cdot \hat{P}_t$ , meaning that the path of the money supply is proportional to the path of the index of predetermined prices. From (36), correspondingly, if the monetary authority makes the nominal money supply constant, it must make the index of predetermined prices part of the state vector, because a constant money supply M implies  $\mathfrak{M}(\sigma_t) = M/\hat{P}_t$ .

<sup>&</sup>lt;sup>20</sup>The household's real budget constraint (3) is not included in the equations that restrict equilibrium, as in many other models, since it is implied by market clearing and the government budget constraint. In equilibrium,  $\theta = 1$ , b - d = 0, and  $\tau = g$  so that  $\varpi = v + z - g$ . Thus, current inflation,  $\pi_t$ , does not enter into the household's decisions.

A more detailed description of equilibrium is contained in appendix B. While this definition details the elements of the discussion above that are important to equilibrium, it is useful to note that a positive analysis of this equilibrium can be carried out without determining the value functions  $U(\cdot)$  and  $\{v^j(\cdot)\}_{j=0}^{J-1}$ , but by simply relying on the first-order conditions. We exploit this feature in our analysis of optimal policy, which is the topic that we turn to next.

# **3** Optimal policy

Our analysis of optimal policy is in the tradition of Ramsey [1927] and draws heavily on the modern literature on optimal policy in dynamic economies which follows from Lucas and Stokey [1983]. In this paper, as in King and Wolman [1999], we adapt this approach to an economy which has real and nominal frictions. Here those frictions are monopolistic competition, price stickiness and the costly conversion of wealth into goods, with the cost affected by money holding. The outline of our multi-stage approach is as follows. First, we have already determined the efficiency conditions of households and firms that restrict dynamic equilibria, as well as the various budget and resource constraints. Second, we manipulate these equations to determine a smaller subset of restrictions that govern key variables, in particular eliminating  $M/\hat{P}$ so that it is clear that we are not taking a stand on the monetary instrument. Third, we maximize expected utility subject to these constraints, which yields constrained optimal allocations. Fourth, we find the absolute prices and monetary policy actions which lead these outcomes to be the result of dynamic equilibrium.<sup>21</sup>

For the purpose of this section, it is convenient to define a set of ratio variables,  $\kappa_{i,t} \equiv y_{i,t}/q_t$ . From the above analysis of demand, it is clear that these ratio variables are related to relative prices via  $\kappa_{i,t}^{-1/\varepsilon} = p_{i,t}$ . Using this definition, it is possible to describe a real policy problem restricted by production technology and implementation constraints. The staggered nature of pricing makes it a dynamic real policy problem, which contains restrictions on the motion of real state variables and forward-looking implementation constraints on states and controls.

# 3.1 Organizing the restrictions on dynamic equilibria

We begin by organizing the equations of section 2 so that they are a set of mainly real constraints on the policy maker. To aid in this process and in the statement of the optimal monetary policy problem as an infinite horizon dynamic optimization problem in the next subsection, it becomes useful to reintroduce time subscripts throughout this section.

 $<sup>^{21}</sup>$ We do not consider the possibility that optimal policy might involve randomization, as suggested by Bassetto [1999] and Dupor [2002].

#### 3.1.1 Restrictions implied by technology and relative demand

The first constraint is associated with production. Since  $n_t = \sum_{j=0}^{J-1} \omega_j n_{j,t}$ , (19) gives

$$a_t n_t = (\sum_{j=0}^{J-1} \omega_j \kappa_{j,t})(c_t + g_t).$$
(37)

The second constraint is associated with the aggregator (13), which applies to retailing of consumption and government goods, so that

$$1 = \left[\sum_{j=0}^{J-1} \omega_j \kappa_{j,t} \frac{\varepsilon_{-1}}{\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$
(38)

# 3.1.2 Restrictions implied by state dynamics

With staggered pricing, we previously showed that  $s = (s_1, \ldots, s_{J-2})$  evolved according to (32) and (33). Previously, we represented these J - 2 equations as  $s_t = \Gamma(s_{t-1}, p_{0,t})$ . Using the fact that  $\kappa_{0,t} = (p_{0,t})^{-\varepsilon}$ , there is a simple linkage between  $\kappa_{0,t}$  and the motion of real states.

### 3.1.3 Restrictions implied by household behavior

The household's decision rules are implicitly restricted by the equations (3) - (7) and (8) - (12). A planner must respect all of these conditions, but it is convenient for us to use some of them to reduce the number of choice variables, while retaining others. In particular, combining (8), (11) and (18), we find that the household requires that the marginal utility of consumption is equated to a measure of the full price of consumption, which depends on  $\lambda_t$  as is conventional, but also on  $R_t$  and  $\xi_t$  because money or credit must be used to obtain consumption.

$$D_1 u(c_t, l_t) = \lambda_t \left[ 1 + R_t \left( 1 - \xi_t \right) \right]$$
(39)

Combining (9), (11) and (18), the efficient choice between money and credit as a means of payment is restricted by

$$Rc = wF^{-1}(\xi) = \frac{D_2 u(c,l)}{\lambda} F^{-1}(\xi)$$
(40)

which indicates how credit use is related to market prices and quantities.<sup>22</sup>

The nominal interest rate enters into each of these equations but, since it is an intertemporal price, it also enters in the bond efficiency condition (11), which takes the form  $\lambda_t \frac{1}{1+R_t} = \beta E_t [\lambda_{t+1} \frac{1}{1+\pi_{t+1}}]$ . We manipulate this equation to make more

<sup>&</sup>lt;sup>22</sup>Since  $\xi = 1 - \frac{M}{P_c}$ , this is also restriction that implicitly defines the demand for money,  $\frac{M}{P_c}$ , as a function of a small number of variables, i.e.,  $\frac{M}{P_c} = 1 - F(\frac{R_c}{w})$ . We exploit this in our analysis below.

transparent the constraints that it places on real variables. In particular, multiplying through by  $P_{0t} = P_{1,t+1}$ ; using the definition of relative prices; and using  $p_{jt} = \kappa_{jt}^{-1/\varepsilon}$ , we arrive at

$$\kappa_{0t}^{-1/\varepsilon} \lambda_t \frac{1}{1+R_t} = \beta E_t [\kappa_{1,t+1}^{-1/\varepsilon} \lambda_{t+1}]$$
(41)

which is a forward-looking constraint, reflecting the intertemporal nature of (11).

Combining equations (4) and (5) to eliminate transactions time, we can write

$$n_t = 1 - l_t - \int_0^{F^{-1}(\xi_t)} \nu dF(\nu) = n(l_t, \xi_t).$$
(42)

so that only  $l_t$  and  $\xi_t$  are choices for the optimal policy problem.

We do not drop the other household conditions, but rather use them to construct variables which do not enter directly in the optimal policy problem, but are relevant for the decentralization, such as real money demand as  $m_t = (1 - \xi_t)\overline{p}_t c_t = m(c_t, l_t, \xi_t)$ and real transactions debt as  $d_{t+1} = \xi_t \overline{p}_t c_t = d(c_t, l_t, \xi_t)$ .

### 3.1.4 Restrictions implied by firm behavior

Price-setting behavior of intermediate good producers is captured by the form of marginal value recursions (26) - (29), with (28) reproduced here for the reader's convenience,<sup>23</sup>

$$D_1 v^j(p_j; \sigma) = D_1 z(p_j; \sigma) + \beta E \left\{ \frac{\lambda'}{\lambda} (1 - \alpha_{j+1}) D_1 v^{j+1}(p'_{j+1}; \sigma') \frac{1}{1 + \pi'} \right\} | \sigma.$$

We rewrite this expression by multiplying both sides by  $\lambda_t p_{j,t}$ , transforming (26) - (29) to expressions of the form

$$0 = x(\kappa_{0t}, c_t, l_t, \lambda_t, g_t, a_t) + \beta E_t \left[ \chi_{1,t+1} \right],$$
(43)

$$\frac{\chi_{j,t}}{1-\alpha_j} = x(\kappa_{jt}, c_t, l_t, \lambda_t, g_t, a_t) + \beta E_t \left[\chi_{j+1,t+1}\right]$$
(44)

$$\frac{1}{1 - \alpha_{J-1}} \chi_{J-1,t} = x(\kappa_{J-1,t}, c_t, l_t, \lambda_t, g_t, a_t).$$
(45)

where (44) holds for j = 1, 2, ..., J - 2, where

$$x(\kappa_{jt}, c_t, l_t, \lambda_t, g_t, a_t) = (c_t + g_t) \left( \lambda_t \left( 1 - \varepsilon \right) \kappa_{j,t}^{\frac{\varepsilon - 1}{\varepsilon}} + \varepsilon \frac{D_2 u\left( c_t, l_t \right)}{a_t} \kappa_{j,t} \right)$$
(46)

<sup>&</sup>lt;sup>23</sup>The expressions (26) and (29) are essentially special cases of this expression, with  $D_1 v^0(p_0; \sigma) = D_1 v^{J-1+1}(p_j; \sigma) = 0.$ 

and where

$$\chi_{j,t} = \left[ (1 - \alpha_j) \lambda_t p_{jt} D_1 v^j(p_{j,t}) \right].$$

Note that the function  $x(\kappa_{jt}, c_t, l_t, \lambda_t, g_t, a_t)$  is simply shorthand that makes the expressions look neater. By contrast, the variables  $\chi_{j,t}$  actually replace the expression  $(1 - \alpha_j)\lambda_t p_{jt} D_1 v^j(p_{j,t})$ .

# 3.2 The optimal policy problem

The monetary policy authority maximizes (1) subject to the constraints just derived, including a number of constraints which introduce expectations of future variables into the time t constraint set. One way to proceed is to define a Lagrangian for the dynamic optimization problem, with the result being displayed in Table 1. In this Lagrangian,  $\mathbf{d}_t$  is a vector of decisions that includes real quantities, some other elements and the nominal interest rate  $R_t$ . Similarly,  $\mathbf{\Lambda}_t$  is a vector of Lagrange multipliers chosen at t. This problem also takes the initial exogenous ( $\varsigma_0$ ) and endogenous states  $s_{-1} = (s_{j,-1})_{j=1}^{J-2}$  as given. Finally, it embeds the various definitions above, including  $x(\kappa_{j,t}, c_t, l_t, \lambda_t, g_t, a_t)$  etc.

In Table 1, there are two types of constraints to which we attach multipliers. The first three lines correspond to the forward-looking constraints: (41), which is a kind of Fisher equation, and (43) - (45), which are the implementation constraints arising from dynamic monopoly pricing. We stress these constraints by listing them first in Table 1 and in other tables below. The remainder are conventional constraints which either describe point-in-time restrictions on the planner's choices or the evolution of the real state variables that the planner controls.

One can then find the first order conditions to this complicated dynamic optimization problem. Because the problem is dynamic and has fairly large dimension at each date, there are many such conditions. Further, as is well-known since the work of Kydland and Prescott [1977], this problem is inherently nonstationary. As an example of this aspect of the policy problem, consider the first order condition with respect to  $\chi_{jt}$  for some j satisfying 0 < j < J-1 which would arise if uncertainty is momentarily assumed absent. At date 0, this condition takes the form

$$0 = -\frac{\phi_{j,0}}{1 - \alpha_j}$$

but for later periods, it takes the form

$$0 = \{\phi_{j-1,t-1} - \frac{\phi_{j,t}}{1 - \alpha_j}\}.$$

Notice that the difference between these two expressions is the presence of a lagged multiplier, so that they would be identical if  $\phi_{j-1,-1}$  were added to the right-hand side of the former.

#### 3.2.1 Augmenting the optimal policy problem

We now augment the policy problem with a full set of lagged multipliers, corresponding to the forward-looking constraints. In doing so, we generalize the Lagrangian to that displayed in Table 2, effectively making the problem stationary.

The Fisher equation (41): For each date s,  $\lambda_s$  appears in period s-1 via the expression  $-E_{s-1}\varphi_{s-1}\kappa_{1,s}^{-\frac{1}{\varepsilon}}\lambda_s$  and then in period s as  $\beta E_s \varphi_s \frac{1}{1+R_s}\kappa_{0,s}^{-\frac{1}{\varepsilon}}\lambda_s$ . By contrast, the  $\lambda_0$  does not have the first term. To make the first order conditions time invariant, we therefore add  $-\beta \varphi_{-1} \kappa_{1,0}^{-\frac{1}{\varepsilon}}\lambda_0$ , which introduces the lagged multiplier  $\varphi_{-1}$  into our problem.

Implementation constraints arising from intermediate goods pricing (43 - 45): There are a number of implications of the constraints involving optimal price-setting by the intermediate goods firms.

First,  $\chi_{1,s}$  typically appears in period s-1 as  $\beta E_{s-1}\phi_{0,s-1}\chi_{1,s}$  and in period s as  $E_s \frac{1}{1-\alpha_1}\phi_{1,s}\chi_{1,s}$ . The exception is  $\chi_{1,0}$  which does not have the first term. We therefore append the term,  $\beta \phi_{0,-1}\chi_{1,0}$  to the optimization problem, which introduces another lagged multiplier,  $\phi_{0,-1}$ .

Second, for each  $j = 2, ..., J-2, \chi_{j,s}$  enters the problem twice, in  $\beta E_{s-1}\phi_{j-1,s-1}\chi_{j,s}$ and in  $-E_s \frac{1}{1-\alpha_j}\phi_{j,s}\chi_{j,s}$ . Again, an exception is  $\chi_{j,0}$  which does not have the first term. We add these terms,  $\beta \phi_{j-1,-1}\chi_{j,0}$  for j = 2, ..., J-2. This introduces the lagged multipliers  $\phi_{1,-1}, \ldots, \phi_{J-3,-1}$ 

Finally,  $\chi_{J-1,s}$  usually enters the problem twice, in  $\beta E_{s-1}\phi_{J-2,s-1}\chi_{J-1,s}$  and in  $-E_s\phi_{J-1,s}\chi_{J-1,s}$ . As above, an exception is  $\chi_{J-1,0}$  which does not have the first term. We add the term  $\beta\phi_{J-2,-1}\chi_{J-1,0}$  to our problem, and hence introduce the lagged multiplier  $\phi_{J-2,-1}$ .

If we set the lagged multipliers  $[\varphi_{-1}, (\phi_{j,-1})_{j=0}^{J-2}]$  all to 0, then we have exactly the same problem as before. Accordingly, we can always find the solution to the Table 1 problem from the Table 2 problem. However, the explicit introduction of these variables allows us to now examine a fully recursive formulation of the problem, as we explain next.

Before turning to this topic, we note that in Table 2 we define  $U^*(s_{-1}, \phi_{-1}, \varsigma_0)$ as the value of the Lagrangian evaluated at the optimal decisions, where  $\phi_{-1} = [\varphi_{-1}, (\phi_{j,-1})_{j=0}^{J-2}]$ . As is familiar from the static context of static optimization, this value function for the optimal policy problem has two important properties. First, it depends on the parameters of the problem, which here are  $s_{-1}, \phi_{-1}, \varsigma_0$ . Second, it is the solution to the problem of maximizing the objective (1) subject to the constraints discussed above, so we use the notation  $U^*$  to denote the planner's value function.

# 3.2.2 The recursive form of the policy problem

Working on optimal capital taxation under commitment, Kydland and Prescott [1980] began the analysis of how to solve such problems using recursive methods. They

proposed augmenting the traditional state vector with a lagged multiplier as above and then described a dynamic programming approach. Important recent work by Marcet and Marimon [1999] formally develops the general theory necessary for a recursive approach to such problems.

In our context, the fully recursive form of the policy problem is displayed in Table 3. There are a number of features to point out. First, the state vector for the policy problem is given by  $\varsigma_t$ ,  $s_{t-1}$  and  $\phi_{t-1} \equiv [\varphi_{t-1}, (\phi_{j,t-1})_{j=0}^{J-2}]$ . That is: we have now determined the extra state variables to which the monetary authority was viewed as responding in section 2 above. Second, we can write the optimal policy problem in a recursive form similar to a Bellman equation; Marcet and Marimon [1999] describe such a recursive form as a saddlepoint functional equation. Third, as  $E_t U^*(s_t, \phi_t, \varphi_{t+1})$  summarizes the future effects of current choices, there is a dramatic simplification of the problem, with future constraints eliminated, as is a conventional benefit of employing dynamic programming.

# 3.3 FOCs, Steady States, and Linearization

Given this particular recursive form, it is a straightforward activity-if a somewhat lengthy one-to determine the first order conditions that circumscribe optimal policy. As in conventional dynamic programs, these first order conditions can involve the derivatives of the future value function (i.e., the derivatives of  $U^*(s_t, \phi_t, \varphi_{t+1})$ ) with respect to elements of  $s_t$  or  $\phi_t$ . Application of the conventional envelope theorem method supplies these necessary derivatives. As with other dynamic programs, the first order conditions may be represented as a system of equations of the form

$$0 = E_t \{ \mathbf{F}(\underline{Y}_{t+1}, \underline{Y}_t, \underline{X}_{t+1}, \underline{X}_t) \}$$

where  $\underline{Y}_t$  is the vector of all endogenous states, multipliers, and decisions and  $\underline{X}_t$  is a vector of exogenous variables. In our context,  $\underline{Y}_t = [\eta_t, \varsigma_t, , c_t, l_t, \xi_t, (\kappa_{j,t})_{j=1}^{J-1}, (\chi_{j,t})_{j=0}^{J-1}, s_{t-1}, \phi_{t-1}]'$  and  $\underline{X}_t = [a_t, g_t]'$ . Our computational approach involves two steps. First, we calculate a stationary

Our computational approach involves two steps. First, we calculate a stationary point defined by  $\mathbf{F}(\underline{Y}, \underline{Y}, \underline{X}, \underline{X}) = 0$ . Second, we then (log)linearize the above system and calculate the local dynamic behavior of quantities and prices given a specified law of motion for the exogenous states  $\varsigma$ , which is also taken to be (log)linear.

# **3.4** Real and nominal aspects of the policy problem

The approach of Lucas and Stokey [1983] is to formulate the optimal policy problem entirely in terms of real quantities, but our analysis above stops short of fully utilizing this approach. There are two elements that are incomplete in this regard. First, in our formulation of the policy problem, the initial real state  $s_{t-1}$  was described as a vector of relative prices. We also showed how the evolution of the state was determined by the ratio of real quantities  $\kappa_{0,t}$ . Alternatively one can interpret the initial state as a vector involving relative quantities<sup>24</sup>:

$$s_{j,t-1} = \frac{\kappa_{j-1,t-1}^{-\varepsilon}}{\left[\frac{1}{1-\omega_0}\sum_{n=1}^{J-1}\omega_n\kappa_{n-1,t-1}^{(\varepsilon-1)/\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}, \ j = 1, ..., J-2.$$

While this interpretation helps make it possible to express the policy problem in our model entirely in terms of real quantities, it seems more natural in the staggered pricing environment to view the initial state as involving relative prices rather than relative quantities.<sup>25</sup>

Second, we have left the nominal interest rate  $R_t$  and the marginal utility of wealth  $\lambda_t$  in the our formulation of the optimal policy problem, although these variables can be eliminated to produce an entirely real problem.<sup>26</sup> However, we have chosen not do so in order to let us more readily analyze the consequences of variations in nominal interest rates on economic activity and welfare in this work and in future research.

# 4 Four distortions

Our macroeconomic model has the property that there are four readily identifiable routes by which nominal factors can affect real economic activity.

# 4.1 Defining the distortions

We discuss these four distortions in turn, using general ideas that carry over to a wider class of macroeconomic models.

*Relative price distortions:* In any model with asynchronized adjustment of nominal prices, there are distortions that arise when the price level is not constant. In our model, the natural measure of these distortions is

$$\delta_t = \frac{a_t n_t}{(c_t + g_t)} = \left[\sum_{j=0}^J \omega_j (P_{jt}/P_t)^{-\varepsilon}\right].$$
 (47)

<sup>24</sup>The definition of the real states implies that  $s_{j,t-1} = [P_{j-1,t-1}/P_{t-1}]/[\frac{1}{1-\omega_0}\sum_{n=1}^{J-1}\omega_n(P_{n-1,t-1}/P_{t-1})^{(1-\varepsilon)}]^{\frac{1}{1-\varepsilon}}$  since (i)  $P_{jt} = P_{j-1,t-1}$ ; and (ii) the index of lagged prices is homogeneous of degree one. The expression in the text then follows directly from the definition of  $\kappa_{j,t-1}$ .

<sup>25</sup>However, the results are insensitive to which interpretation one prefers.

<sup>26</sup>Using (39) and (40), one finds that  $\lambda(c_t, l_t, \xi_t) = [D_1 u(c_t, l_t) - (1 - \xi_t) D_2 u(c_t, l_t) F^{-1}(\xi_t) / c_t]$ and  $R(c_t, l_t, \xi_t) = \frac{D_2 u(c_t, l_t) F^{-1}(\xi_t)}{[D_1 u(c_t, l_t) - (1 - \xi_t) D_2 u(c_t, l_t) F^{-1}(\xi_t) / c_t]}]$ . These functions then can be imposed on the planning problem, with  $\lambda$  and R eliminated as choice variables and the last two terms in Tables 1 and 2 eliminated. If all relative prices are unity, then  $\delta$  takes on a value of one. If relative prices deviate from unity, which is the unconstrained efficient level given the technology, then  $\delta_t$ measures the extent of lost aggregate output which arises for this reason.

The markup distortion: If all firms have the same marginal cost functions, then we can write  $W_t = \Psi_t a_t$ . Here W is the nominal wage,  $\Psi_t$  is nominal marginal cost and  $a_t$  is the common marginal product of labor. If we divide by the perfect (intermediate good) price index, then this expression can be stated in real terms as

$$w_t = \psi_t a_t = \frac{1}{\mu_t} a_t \tag{48}$$

so that real marginal cost  $\psi_t$  acts like a sales tax shifter.

Some recent literature has described this second source of distortions in terms of the average markup  $\mu_t \equiv P_t/\Psi_t$ , which is the reciprocal of real marginal cost  $\psi_t$ , stressing that the monetary authority has temporary control over this markup tax because prices are sticky, enabling it to erode (or enhance) the markups of firms with sticky prices.<sup>27</sup> According to this convention, which we follow here, a higher value of the markup lowers real marginal cost and works like a tax on productive activity.

Since movements in  $\delta_t$  and  $\mu_t$  (or  $\psi_t$ ) are not necessarily related closely together, it is best to think about these two factors from the standpoint of fiscal analysis – in which there can be separate shocks to the level of the production function and its marginal products – rather than reasoning from the effects of productivity shocks which traditionally shift both in RBC analysis.

Inefficient shopping time: The next distortion is sometimes referred to as "shoe leather costs." But in our model, it is really "shopping time costs," as in McCallum and Goodfriend [1988], since it is in time rather than goods units. In (42) above, it is  $h_t = \int_0^{F^{-1}(\xi_t)} \nu dF(\nu)$ . Variations in  $h_t$  work like a shock to the economy's time endowment. Pursuing the fiscal analogy discussed above, this is similar to a conscription (lump sum labor tax).

The wedge of monetary inefficiency: In transactions-based monetary models, there is also an effect of monetary policy on the full cost of consumption. In (39) above, it is  $D_1 u(c_t, l_t) = \lambda_t [1 + R_t (1 - \xi_t)]$ . This equation highlights as a wedge of monetary inefficiency the product of the nominal interest rate and the extent of monetization of exchange  $(1 - \xi_t)$ . Pursuing the fiscal policy analogy discussed above, it is like a consumption tax relative to the non-monetary model.

# 4.2 Selectively eliminating one or more distortions

Since the four distortions all enter into our model, it can be difficult to determine which distortion is giving rise to a particular result. In our analysis below, we selectively eliminate one or more distortions. In doing so, we are imagining that there is a fiscal authority which can offset the distortions in the following ways.

<sup>&</sup>lt;sup>27</sup>See Woodford [1995], King and Wolman [1996] and Goodfriend and King [1997].

Eliminating variations in relative price distortions. This modification involves resolving the model with  $\delta(c_t + g_t) = a_t n_t$  replacing  $\delta_t(c_t + g_t) = a_t n_t$ . Since relative price distortions affect the constraint  $\delta_t(c_t + g_t) = a_t n_t$  but do not affect the marginal costs of firms or the wages of workers, they can be interpreted as an additive productivity shock-relative to a benchmark level of  $\delta$ -with an effect of  $(1/\delta_t - 1/\delta)a_t n_t$ . Accordingly, the elimination of relative price distortions can be understood as involving a fiscal authority which decreases its spending by an amount  $\tilde{g}_t = (\delta^{-1} - \delta_t^{-1})a_t n_t$ , where  $\delta$  is a benchmark level of distortions with  $\delta = 1$  corresponding to no distortions. Total government spending would then be  $g_t - \tilde{g}_t$ .

Eliminating variation in the markup distortion. This involves re-solving the model with  $w_t = \psi a_t$  replacing  $w_t = \psi_t a_t = \frac{1}{\mu_t} a_t$ . Using the idea that the markup is like a sales tax, we can think of this as involving a fiscal authority which adjusts an explicit sales/subsidy tax on intermediate goods producers so that  $(1 + \tau_t^i) \frac{1}{\mu_t} = (1 + \tau^i)$ , where  $(1 + \tau^i) = \psi$  is a benchmark level of the net tax on intermediate goods producers from the two sources.

Eliminating variations in inefficient shopping time. Eliminating variations in the resources used by credit involves holding the right hand side of  $l_t + n_t = 1 - h_t$  fixed. A fiscal interpretation of this is that a fiscal authority varies the amount of its lump sum confiscation of time similarly to the changes in lump sum confiscation of goods discussed for relative price distortions.

Eliminating variations in the wedge of monetary inefficiency. This modification involves holding  $(1 + (1 - \xi_t)R_t)$  fixed at a specified level. A fiscal interpretation is that there is a consumption tax rate which is varied so that  $(1 + (1 - \xi_t)R_t)(1 + \tau_t^c)$ is held constant at a specified level.

# 4.3 Distortions under "neutral" policy

One possible choice for the monetary authority of real outcomes is sometimes described as neutral policy, as in Goodfriend and King [1997]. It involves making the path of the price level constant through time, thus minimizing relative price distortions but leaving the markup at  $\mu = \frac{\varepsilon}{\varepsilon-1}$  and allowing variations in the two monetary distortions as the real economy fluctuates over time in response to variations in the real conditions  $g_t$  and  $a_t$ . Under this regime, real activity fluctuates in a manner which is identical to how it would behave if prices were flexible and if the monetary authority stabilized the price level. In its essence, this is the Fisherian proposal for eliminating business fluctuations via price stabilization.

At least after a brief startup period associated with working off an inherited distribution of relative prices, such an outcome is always feasible for the monetary authority in our economy. To the extent that the monetary authority chooses to depart from these neutral outcomes, it is because it is responding to the distortions identified in this section. For one example, a monetary authority might choose a lower average rate of inflation, to reduce time costs, as suggested by Friedman. For another example, a monetary authority might choose to stabilize the fluctuations in real economic activity that would occur under neutral policy, changing the extent to which the markup distortion is present in booms and contractions. Such stabilization policy would be of the general form advocated by Keynes.

# 5 Choice of parameters

Given the limited amount of existing research on optimal monetary policy using the approach of this paper and given the starkness of our model economy, we have chosen the parameters with two objectives in mind. First, we want our economy to be as realistic as possible, so we calibrate certain parameters to match certain features of the U.S. economy as discussed below. Second, we want our economy to be familiar to economists who have worked with related models of business cycles, fiscal policy, money demand, and sticky prices. Our benchmark parametric model is as follows, with the time unit taken to be one quarter of a year.

# 5.1 Preferences

We assume the utility function is logarithmic,  $u(c, l) = \ln c + 3.3 \ln (l)$ , with the parameter set so that agents work approximately .20 of available time. We assume also that the discount factor is such that the annual interest rate would be slightly less than three percent ( $\beta = 0.9928$ ). This choice of the discount factor is governed by data on one year T-bill rates and the GDP deflator.

# 5.2 Monopoly power

We assume that the demand elasticity,  $\varepsilon$ , is 10. This means that the markup would be 11.11% over marginal cost if prices were flexible. Hall [1988] argues for much higher markups, whereas Basu and Fernald [1997] argue for somewhat lower markups. Our choice of  $\varepsilon = 10$  is representative of other recent work on monopolistically competitive macroeconomic models; e.g., Rotemberg and Woodford [1999] use  $\varepsilon = 7.88$ . We also explore the implications of a lower elasticity of demand (higher markup).

# 5.3 Distribution of price-setters

A key aspect of our economy is the extent of exogenously imposed price stickiness. We use a distribution suggested by Wolman [1999], which has the following features. First, it implies that firms expected a newly set price to remain in effect for five quarters. That is: the expected duration of a price chosen at t, which is  $\alpha_1 1 + (1 - \alpha_1)\alpha_2 2 + (1 - \alpha_1)(1 - \alpha_2)\alpha_3 3 + ...$  is equal to 5. Second, this estimate is consistent with the recent empirical work on aggregate price adjustment dynamics by Gali and Gertler [1999] and Sbordone [2002]. Third, rather than assuming a constant hazard  $\alpha_i = \alpha$  as in the Calvo [1983] model, our weights involve an increasing hazard, which is consistent with available empirical evidence and recent work on models of state dependent pricing. The particular adjustment probabilities  $\alpha_i$  and the associated distribution are given in Table 4; the average age of prices is  $\sum_{j=0}^{J-1} j\omega_j = 2.3$  for the benchmark parameterization. We explore some implications of assuming greater price flexibility below

# 5.4 Credit costs and money demand

Our model establishes a direct link between the distribution of credit costs and the demand for money, which was highlighted above in (40). Our money demand function,

$$\frac{M_t}{\overline{P}_t c_t} = 1 - \xi_t = 1 - F(\frac{R_t c_t}{w_t})$$
(49)

embodies the negative effect of the interest rate and the positive effect of a scale variable – consumption expenditure – stressed in the transactions models of Baumol [1952] and Tobin [1956] as well as the positive effect of the wage rate stressed by Dutton and Gramm [1973]. That is, the fraction of goods purchased with credit is higher when the interest cost Rc is greater or when the wage rate w is lower: the ratio Rc/w is the time value of interest foregone by holding money to buy consumption.

#### 5.4.1 Estimating the demand for money

We use the following procedure to estimate the demand for money. First, we posit that the distribution of credit costs is of the following "generalized beta" form:

$$F(x) = \underline{\xi} + \overline{\xi}B(\frac{x}{\kappa}; b_1, b_2) \tag{50}$$

for  $0 < x \leq \kappa$ . The basic building block of this distribution is the beta distribution,  $y = B(z; b_1, b_2)$ , which maps from the unit interval for z into the unit interval for y. It is a flexible functional form in that the parameters  $b_1, b_2$  can be used to approximate a wide range of distributions.<sup>28</sup> In the general expression (50), we allow for the standard beta-distribution's independent variable to be replaced by  $x/\kappa$ , which essentially changes the support of the distribution of costs to  $(0, \kappa)$ . In addition, we make it possible for some goods to be pure cash or pure credit goods:  $\underline{\xi}$  is a mass point at zero credit costs, allowing for the possibility that there are some goods that will always be purchased with credit;  $\overline{\xi} \leq 1 - \underline{\xi}$  similarly allows for goods for which money will always be used.

<sup>&</sup>lt;sup>28</sup>See, for example, Casella and Berger [1990], pages 107-108, for a discussion of the beta distribution. The beta cdf takes the form  $\left[\int_{0}^{x} (z)^{b_{1}-1}(1-z)^{b_{2}-1}dz\right]/\beta(b_{1},b_{2})$ , where  $\beta(b_{1},b_{2}) = \Gamma(b_{1})\Gamma(b_{2})/[\Gamma(b_{1}+b_{2})]$  is the  $\beta$  function, which is in turn based on the  $\Gamma$  function as shown.

We use quarterly economic data to construct empirical analogues to our model's variables: a measure of the nominal stock of currency; a measure of nominal consumption expenditures per capita; a measure of the nominal interest rate; and a measure of the hourly nominal wage rate.<sup>29</sup> The ratios  $\frac{M_t}{P_tc_t}$  and  $(\frac{R_tc_t}{w_t})$  are shown in Figure 1.<sup>30</sup> Since there is not too much low frequency variation in  $(\frac{c_t}{w_t})$ , the Figure mainly reflects the fact that the velocity of money and the nominal interest rate move together. Figure 1 highlights the fact that we explore two sample periods. First, we look at the sample 1948.1 through 1989.4. Our choice of the endpoint of this "long sample" is based on evidence that an increasing portion of currency was held outside of the U.S. during the 1990s.<sup>31</sup> The key feature of this longer sample period is that there is an initial interval of low nominal interest rates which makes the opportunity cost of money holding (Rc/w) quite low. Second, we look at 1959.1-1989.4 since some analysts have argued that the earlier period is no longer relevant for U.S. money demand behavior.

Two estimated money demand functions are displayed in Figure 1, one for the shorter sample and one for the longer sample. Each money demand function is estimated by selecting the parameters  $[\underline{\xi}, \overline{\xi}, \kappa, b_1, b_2]$  so as to minimize the sum of squared deviations between the model and the data.<sup>32</sup>

#### 5.4.2 Implications of the money demand estimates.

We stress three implications of the money demand estimates.

The estimated cost distribution: The parameter estimates over the two sample periods also imply distributions of credit costs, which are displayed in panel A of Figure 2. The first point to note is that the two costs cdfs are very similar for opportunity cost measures exceeding .002, as were the money demand functions in

<sup>&</sup>lt;sup>29</sup>The basic data used is a three month treasury bill rate; the FRB St. Louis's currency series; real personal consumption expenditures (billions of chained 1996 dollars); the personal consumption expenditures series chain-type price index (1996=100); civilian noninstitutional population and average hourly earnings of production workers in manufacturing. The ratio m/c is formed by taking the ratio of currency to nominal consumption expenditures, which is itself a product of real expenditures and the data. The ratio Rc/w is formed by multiplying the quarterly nominal treasury bill rate by nominal per capita consumption expenditures and then dividing by nominal average hourly earnings.

 $<sup>^{30}</sup>$  The wage rate in the model is a wage per quarter, with the quantity of time normalized to one. The wage rate in the data is an hourly wage rate. Assuming that the time endowment per quarter is 16 hours per day, 7 days per week and 13 weeks per quarter, there are then 1456 hours per quarter. We therefore divide the data series Rc/w by this number of hours to get a measure that conforms to the theory.

<sup>&</sup>lt;sup>31</sup>See Porter and Judson [1996].

<sup>&</sup>lt;sup>32</sup>The nonlinear regression chooses the five parameters to minimize the sum of squared errors,  $\frac{1}{T}\sum_{t=1}^{T} \left[\frac{M_t}{P_t c_t} - (1 - F(x_t))\right]^2$  with  $x_t = \left(\frac{R_t c_t}{w_t}\right)$  and  $F(x_t) = \underline{\xi} + \overline{\xi}B(\frac{x_t}{\kappa}; b_1, b_2)$ . The point estimates for the short sample are  $[\underline{\xi} = .6394, \overline{\xi} = .1155, \kappa = .0127, b_1 = 2.8058, b_2 = 10.4455]$  and those for the long sample are  $[\underline{\xi} = .0658, \overline{\xi} = 0.6859, \kappa = .0126, b_1 = 0.4824, b_2 = 7.1304]$ .

Figure 1. Below this point, the two functions differ substantially. The short sample period suggests that there are many goods (about two-thirds) that have zero credit costs. The longer sample period suggests that there are many more goods with small, but non-neglible transactions costs.

This figure anticipates the results presented below, by indicating not only the lowest interest rate data point as 'o' but also the optimal level of the nominal interest rate as '\*'. For the short sample, the optimal nominal interest rate happens to be virtually identical to the minimum value in the sample, while for the longer sample the optimum is slightly above the minimum value.

The money demand elasticities: Given the cost distribution (50), there is not a single "money demand elasticity." But we can still compute the relevant elasticity at each point, producing panel B of Figure 2. For the long sample period, the money demand elasticity is less (in absolute value) than one-half, and for the short sample period, it is less than one-third. The triangle in panel B indicates the money demand elasticity at the mean interest rate for the sample in question.

Bailey-Friedman calculations. Positive nominal interest rates lead individuals in this model to spend time in credit transactions activity that could be avoided if the nominal interest rate were zero. Given the estimated money demand function, with its associated distribution of credit costs, we can calculate this time cost as  $h = \int_0^{(Rc/w)} \nu dF(\nu)$ , which is the area under the inverse money demand function.<sup>33</sup> If all goods were purchased with credit, the short (long) sample money demand estimates imply that individuals would spend approximately 0.03% (0.05%) of their time endowment in credit transactions.<sup>34</sup>

# 6 Optimal policy in the long run

There are two natural reference points for thinking about optimal policy in the long run. The first reference point is Friedman's [1969] celebrated conclusion that the nominal interest rate should be sufficiently close to zero so that the private and social

$$\left[\int_{0}^{y} z(z)^{b_{1}-1} (1-z)^{b_{2}-1} dz\right] / \beta(b_{1},b_{2}) = \frac{\Gamma(b_{1}+1)\Gamma(b_{1}+b_{2})}{\Gamma(b_{1})\Gamma(b_{1}+b_{2}+1)} B(y;b_{1}+1,b_{2})$$

so  $h = \kappa \overline{\xi} \frac{\Gamma(b_1+1)\Gamma(b_1+b_2)}{\Gamma(b_1)\Gamma(b_1+b_2+1)} B(\frac{(Rc/w)}{\kappa}; b_1+1, b_2).$ 

 $<sup>^{33}</sup>$ The "generalized beta" distribution makes this a particularly simple calculation because the truncated mean of a beta distribution is.

<sup>&</sup>lt;sup>34</sup>While this number may seem implausibly small to some readers, reference to Figures 1 and 2 helps understand why it is not given our transactions demand for money. As seen in Figure 1, the largest amount of credit use – implying a rate of money to consumption of about .25 – begins to take place when the opportunity cost is about .005, which translates to annualized interest rate of just under 10% as seen in Figure 2. With the estimated money demand over the short sample, the money demand curve cuts the axis at less than m/c = .4, implying an increase in m/c of .15 = .4 - .25. Using a triangle to approximate the integral, we find that the approximate cost saving is  $\frac{1}{2}(.005) * .15 = .000375$  or .0375%.

costs of money-holding coincide. At this point, the economy minimizes the costs of decentralized exchange. The second reference point is an average rate of inflation of zero, which minimizes relative price distortions in steady state. In this section, we document the intuitive conclusion that the long-run inflation rate should be negative – but not as negative as suggested by Friedman's analysis – when both sticky price and exchange frictions are present.

# 6.1 The four distortions at zero inflation

If there is zero inflation in the benchmark economy–which uses the credit cost technology with parameters set from the short sample estimates–then it is relatively easy to determine the levels of the four distortions. With zero inflation, the nominal and real interest rates are each equal to 2.93 percent per annum. The parameters of the credit cost technology imply that 65.6 percent of transactions are financed with credit ( $\xi = .656$ ) and that the ratio of real money to consumption is about 34 percent.

The markup is equal to that which prevails in the static monopoly problem,  $\mu = \frac{\varepsilon}{\varepsilon - 1} = 1.\overline{11}$ , so that price is roughly eleven percent higher than real marginal cost in the steady-state.

There are no relative price distortions – all firms are charging the same, unchanging price – so that  $\delta = 1$ . Further, marginal relative price distortions are also small.

The wedge of monetary inefficiency is positive, but relatively small in this steady state. It is calculated from the above discussion as

$$(1 + (1 - \xi) * R) = (1 + (1 - .656) * .0072) = 1.0025$$

where the calculation of the wedge uses the quarterly nominal interest rate .0072.

Time costs associated with use of credit are quite small, approximately .004% of the time endowment. Recall that the maximal time costs - associated with using credit for all purchases - are about 0.03%. At zero inflation, time spent on credit transactions involves only 14% of the maximum time that could be spent on credit transactions.

# 6.2 The benchmark result on long-run inflation

Even though the distortions associated with money demand are small at zero inflation, a monetary authority maximizing steady-state welfare would nonetheless choose a lower rate of inflation, for the reasons stressed by Friedman [1969]. When we solve the optimal policy problem for the benchmark model using the short-sample estimates displayed in Figure 1 above, we find that the asymptotic rate of inflation – the steady state under the optimal policy – is negative 76 basis points (-0.76% at an annual rate). Given that we assume a steady-state real interest rate of 2.93% percent (as determined by time preference), the long-run rate of nominal interest is 2.17%. This result raises two sets of questions. First, how do the four distortions isolated earlier in the paper contribute to this finding? Second, how do variations away from the benchmark parameter values affect the optimal long-run inflation rate? Each of these questions is addressed in Table 5 and in the discussion below.

# 6.3 Optimal inflation with fewer distortions

We now alter the monetary authority's problem – relative to the benchmark case – by selectively eliminating one or more distortions. Table 5 shows the effect of various modifications of the mix of distortions.<sup>35</sup>

Why is disinflation desirable? Starting with the zero inflation steady-state rate of inflation, the Table shows that both the wedge of monetary inefficiency and time costs play a role in reducing the inflation rate from zero to the benchmark level of -.76%. Table 5 shows that the wedge of monetary inefficiency has a moderate influence on the optimal long-run rate of inflation. If it is eliminated by itself, then the inflation rate rises from -.76% to -.54%, so that the wedge accounts for almost 30% of the deviation from zero inflation. It also shows that if we only eliminate time costs, then the inflation rate rises further, from -.76% to -.28%, so that time costs alone account for almost 65% of the deviation from the zero inflation position.<sup>36</sup>

Why is there less deflation than at the Friedman Rule? If prices are flexible, then the Friedman rule is optimal even though there is imperfect competition. In fact, Goodfriend [1997] notes that a positive markup makes the case stronger in a sense because the additional labor supply induced by declines in the wedge and time costs yield a social marginal product of labor which exceeds the real wage.

To evaluate why there is a benchmark rate of inflation of -.76% per annum – as opposed to a Friedman rule level of -2.93% per annum – it is necessary to eliminate variations in either the relative price distortion or the markup distortion. We suppose that the markup distortion is fixed at the zero inflation level, i.e.,  $\mu = \frac{\varepsilon - 1}{\varepsilon} = 1.11$ . In this case, Table 2 shows that there is a slightly more negative rate of inflation than with a variable markup, a finding which is consistent with the facts that in this model, the average markup (i) is decreasing in the inflation rate near zero inflation; and (ii) does not respond importantly to variations in the inflation rate near zero inflation. The first fact explains why eliminating the distortion makes the optimal inflation rate more negative, since the monetary authority does not encounter an increasing markup in the modified problem as it lowers the inflation rate from a starting point of zero. The second fact explains why the effect is a small one quantitatively.

<sup>&</sup>lt;sup>35</sup>The table also presents results of the sensitivity analysis to be discussed below.

<sup>&</sup>lt;sup>36</sup>Time costs and the wedge interact nonlinearly in determining the long run inflation rate. Therefore, adding up the contributions of the two effects in isolation does not yield the long run inflation rate from the benchmark case with both effects present.

# 6.4 Sensitivity Analysis

We now explore the sensitivity of the steady-state rate of inflation to two aspects of the model. First, holding the parameters of money demand fixed at the benchmark levels, we explore the consequences of various structural features of the model. These results are presented in panel A of Table 5. Second, we discuss the long-run rate of inflation using the parameter estimates from the long sample. These results are presented in panel B of Table 5.

#### 6.4.1 Changing features of the model

We explore the consequences of changing the degree of monopoly power and the extent of price stickiness.

Monopoly power: Decreasing the demand elasticity ( $\varepsilon$ ) to 6 leads to a larger deflation, 1.34% per year, because this lowers the costs of relative price distortions. The money demand distortions become relatively more important, pushing the optimum closer to the Friedman rule.

Price stickiness: we change the distribution of prices ( $\omega$ ) to [0.3, 0.28, 0.25, 0.2, 0.1]. With this distribution, the expected duration of a newly adjusted price is 3.8 quarters. The inflation rate in the long run under optimal policy is -1.2%. Optimal policy comes closer to the Friedman rule in this case because the relative price distortions associated with deviations from zero inflation are smaller the more flexible are prices.

# 6.4.2 Credit costs based on the long sample

If we solve the optimal policy problem with the longer sample estimates, Panel B shows that there is much more deflation, reflecting the increased gains from substitution away from costly credit at low interest rates. The asymptotic rate of deflation is -2.30%, implying a nominal interest rate of only 0.63\%. The other structural features continue to affect the long-run inflation rate in the manner described above.

# 7 Dynamics under optimal policy

We now discuss the nature of the dynamic response of the macroeconomy under optimal policy. The reference point for this discussion is the response of real quantities if prices are flexible and there are no money demand distortions. After discussing this case, we begin by studying optimal policy response in a situation in which there are distortions from imperfect competition and sticky prices, but there are no money demand distortions. We contrast the effects of shocks to productivity and demand. We then turn to analyzing the effects of these same shocks when the monetary authority is confronted with money demand distortions as well.

# 7.1 The real business cycle solution

If intermediate goods firms have market power but can flexibly adjust their prices and if there are no money demand distortions, then the loglinear approximation dynamics of consumption and leisure are

$$\log(c_t/c) = \frac{a}{a-g} \log(a_t/a) - \frac{g}{a-g} \log(g_t/g)$$
  
$$\log(l_t/l) = \frac{g}{a-g} [\log(a_t/a) - \log(g_t/g)]$$

with the approximate dynamics of the real interest rate given by  $r_t - r = E_t [\log(c_{t+1}/c) - \log(c_t/c)]$ , where  $r = \beta^{-1} - 1.^{37}$  The consumption dynamics then imply that

$$r_t - r = \frac{a}{a - g} E_t(\log(a_{t+1}/a) - \log(a_t/a)) - \frac{g}{a - g} E_t(\log(g_{t+1}/g) - \log(g_t/g)).$$

This real business cycle (RBC) solution is the benchmark for our subsequent analysis. We study impulse responses to productivity and government purchase shocks, under the assumption that each is first order autoregressive with a parameter  $\rho$ . Under this assumption, all of the macro variables in the RBC solution have simple solutions. For example, assuming that  $\log(a_t/a) = \rho \log(a_{t-1}/a) + e_{at}$ , the impulse response of the level of consumption to a productivity shock is just  $\log(c_{t+j}/c) = \frac{a}{a-g}\rho^j e_{at}$  and that of the real interest rate is just  $r_t - r = \frac{a}{a-g}(\rho - 1)\rho^j e_{at}$ . Since  $\rho < 1$ , the real interest rate is low when the level of consumption is high, because consumption is expected to fall back to its stationary level.

# 7.2 Optimal policy without money demand distortions

In this section, we explore dynamic responses to productivity and government demand shocks in variants of our model with the money demand distortions eliminated, which is the case previously studied in King and Wolman [1999]. Our procedure is to make two uses of the first order conditions from the optimal policy problem.<sup>38</sup> First, we solve these conditions for a stationary point, which is the long run limit that will occur under optimal policy. Second, we study the response to shocks near this stationary point, working also under the assumption that these shocks occur in the stationary distribution that obtains under optimal policy. <sup>39</sup>

<sup>&</sup>lt;sup>37</sup>Derivation of approximate dynamics is facilitated by recognizing that without money demand or relative price distortions, our model is governed by  $c_t + g_t = a_t (1 - l_t)$ ,  $w_t = \psi a_t$  with  $\psi = \frac{\varepsilon - 1}{\varepsilon}$  and  $w_t D_1 u(c_t, l_t) = D_2 u(c_t, l_t)$ . With  $u(c, l) = log(c) + \theta \log(l)$ , there is an exact closed form solution  $c_t = \frac{\psi}{\psi + \theta} (a_t - g_t)$  and  $l_t = \frac{\theta}{\psi + \theta} (\frac{a_t - g_t}{a_t})$ . <sup>38</sup>Above, we wrote the planner's first order conditions as  $0 = E_t \{\mathbf{F}(\underline{Y}_{t+1}, \underline{Y}_t, \underline{X}_{t+1}, \underline{X}_t)\}$ . The

<sup>&</sup>lt;sup>38</sup>Above, we wrote the planner's first order conditions as  $0 = E_t \{ \mathbf{F}(\underline{Y}_{t+1}, \underline{Y}_t, \underline{X}_{t+1}, \underline{X}_t) \}$ . The first step involves finding  $0 = \mathbf{F}(\underline{Y}, \underline{Y}, \underline{X}, \underline{X})$ . The second step involves solving the linear rational expectations model near this stationary point and computing the response of the policy authority.

<sup>&</sup>lt;sup>39</sup>Technically, we compute the optimal policy response with the initial vector of lagged multipliers taking on its steady-state value.

Without money demand distortions, the long run limit involves a zero inflation steady state. One focal point of our discussion, here and below, in on the response of the price level to our two shocks under optimal policy.

#### 7.2.1 Productivity shocks

Figure 3 displays the response of economic activity under optimal policy when there are persistent variations in productivity (the autoregressive coefficient is set equal to .95). For the purpose of discussing this figure and the others below, we use the RBC solution as the reference point. Optimal policy here is to exactly replicate the RBC solution for quantities and this involves holding the path of the price level exactly constant through time.

Turning to the details of the graph, it is constructed under the assumption that there are no government purchases in the steady state, so that consumption moves one-for-one with the productivity shock and labor is predicted to be constant. The level of the productivity shock is 1.0% and the expected growth rate of consumption at date 0 is then  $(\rho - 1) = -.05$ . We state the real interest rate in annualized terms, so that the impact effect on the real and nominal interest rate is -.20 or a decline of 20 basis points relative to the steady-state level of the rate.

In this setting, then, there is no Keynesian stabilization policy: the government does not choose to smooth out the fluctuations that would occur if prices were flexible, even though there are monopoly distortions present in the economy which make output inefficiently low. At the same time, in order to bring about this flexible price solution, it is necessary for policy to be activist. For example, if the interest rate is the policy instrument, then it must move with the underlying determinants of the real interest rate.

#### 7.2.2 Government purchase shocks

Figure 4 displays the response of economic activity under optimal policy when there are persistent variations in government purchases (the autoregressive coefficient is again set equal to .95). In this setting, the response of economic activity deviates from the flexible price solution, in a manner that is particularly evident in the path of interest rates.

Under the RBC solution, the basic mechanism is that there is a persistent, but ultimately temporary, drain on the economy's resources. In response to this drain, the representative agent consumes fewer market goods and takes less leisure, so that work effort rises. The real interest rate again reflects the response of consumption growth: it rises because consumption is expected to grow back toward the steady state as the government purchase shock disappears.

Under optimal policy, this basic picture is overlaid with an initial interval during which labor input and consumption are reduced relative to the levels that would prevail if prices were flexible. There is an important sense in which this is counterintuitive from a traditional perspective on stabilization policy: the monetary authority works to *increase* the variability of consumption stemming from a real shock rather than mitigate it. Working with pre-set pricing model of the sort developed by Ireland [1996] and Adao, Correia and Teles [2001], Goodfriend and King [2001] argue that the key to understanding the effects of government purchases is to recognize that optimal policy selects a state contingent pattern of consumption taking into account its influence on the contingent claims price  $\lambda(c, l) = D_1 u(c, l)$ .<sup>40</sup> Relative to the RBC solution, the government will want to have less consumption when government purchases are high because this increases the contingent claims value of g, making it easier to satisfy the implementation constraint. Our staggered pricing model displays a similar incentive, but a dynamic one: the monetary authority wants to depress the consumption path to an extent while there are predetermined prices. In line with this, Figure 4 shows that the optimal plan involves consumption which is transitorily low relative to the RBC solution. Because consumption is expected to grow toward the RBC path in these periods, the real interest rate – which continues to be described by  $r_t - r = E_t [\log(c_{t+1}/c) - \log(c_t/c)]$ -is high relative to the RBC path. The magnitude of this interest rate variation is substantial relative to the RBC component, because there is a temporary initial consumption shortfall, which implies rapid growth.

In our setting, then, it is not desirable for the government to stabilize consumption in the face of government purchase shocks, even though it is feasible for it to do so. Rather, the optimal policy is to somewhat reinforce the negative effects that g has on consumption, thus attenuating the effects on employment and output. But, since the implied movements in real marginal cost are temporary, they have little consequence for the path of the price level.

# 7.3 Optimal policy in the benchmark model

We now calculate the response of the economy to productivity and government demand shocks in the benchmark model, in which we restore the two monetary distortions discussed in section 6. In each case, we find that the solutions involve some interest rate smoothing, in both real and nominal terms.

#### 7.3.1 Productivity shocks

Figure 5 shows the response of the economy to a productivity shock. On impact, consumption is slightly lower than the RBC response and then subsequently exceeds this level very slightly. But small differences in consumption paths translate into larger differences in growth rates and interest rates: rather than falling by 20 basis

<sup>&</sup>lt;sup>40</sup>To draw this conclusion, Goodfriend and King contrast a small open economy facing exogenous contingent claims prices with a closed economy setup with endogenous contingent claims prices.

points on impact, the nominal and real interest rates decline by a good bit less (the nominal rate falls by 7 basis points and the real rate by 8 basis points).

The dynamic behavior of real and nominal interest rates is of some interest. The real interest rate is smoothed relative to the RBC solution, but only during the first few quarters, presumably because this is the interval when the effects of pre-existing prices are important for the trade-offs that the monetary authority faces. Afterwards, the real interest rate closely tracks the underlying real interest rate associated with the RBC response. There is a small amount of expected inflation, which makes the nominal interest rate even less responsive to the productivity shock than the real rate.

Yet the total effect on the price level is very small: it is about 0.25% over fifteen quarters, while productivity is inducing a cumulative rise in consumption of about 11%.<sup>41</sup> Even though they are not exactly those of the flexible price solutions, the real responses are quite close in form, indicating that the monetary authority does not make much use of the leverage that it has over real activity to undertake stabilization policy.

The motivation for interest rate smoothing in this economy involves the money demand distortions, as a comparison of the results of this section with those of (7.2.1) above makes clear. More specifically, we have found that it is the time cost distortion, as opposed to the wedge of monetary inefficiency, which accounts for most of the interest rate smoothing. It is interesting to note that maximal time costs which seem to be quite small can motivate the monetary authority to deliver significant smoothing of nominal interest rates. On the other hand, this smoothing results in only small variations in the price level, so the costs in terms of relative price distortions are small.<sup>42</sup>

#### 7.3.2 Government purchases

Figure 6 shows the response of economic activity to a change in government purchases in the benchmark model. In contrast to the analysis of section (7.2.2), the response of the economy under optimal policy now much more closely resembles that in the RBC benchmark. That previous analysis indicated that optimal policy sought to increase the variability of real and nominal interest rates in response to a government purchase shock, but this incentive is now curtailed by the effect of such interest rate changes on the monetary distortions, especially the time cost. More specifically, the interest rate smoothing motivation approximately cancels out the earlier effects, leading to outcomes that closely resemble the flexible price solution.

<sup>&</sup>lt;sup>41</sup>That is: the total effect on productivity over fifteen quarters is given by  $\frac{1-(.95)^{16}}{1-.95} = 11.2$  and over the infinite horizon it is given by  $\frac{1}{1-.95} = 20$ .

<sup>&</sup>lt;sup>42</sup>In ongoing research, we are exploring the determinants of interest rate smoothing using a dynamic version of the method of eliminating selective distortions. Woodford [1999] discusses optimal interest rate smoothing in a related model.

# 7.4 Robustness

In Figure 7, we summarize the interest rate and price level responses to productivity and demand shocks in the benchmark model in the left hand column; we record these same responses for a version of the model using the long-sample money demand estimates in the right hand column. While there are differences across shocks and money demand specifications, the Figure illustrates that the optimal policy responses involve very small variations in the price level. While real interest rate behavior under optimal policy can deviate somewhat from the RBC solution, significant deviations are transitory, lasting only a few periods.

# 8 Summary and conclusions

Optimal monetary policy depends on the nature of frictions present in the economy. In this analysis, we have described a modern monetary model which there are a range of frictions – imperfect competition, sticky prices and the costly exchange of wealth for consumption – and explored the consequences for economic activity under optimal monetary policy. More specifically, we initially developed a recursive equilibrium for a model economy with these three frictions. We then described how to calculate optimal allocations using the approach pioneered by Ramsey [1927], but also placed this analysis in recursive form. To derive quantitative results, we estimated a model of money demand, which determined the extent of transactions cost-savings, and we calibrated other aspects of the model in ways consistent with much recent research on imperfect competition and sticky prices.

As suggested by Friedman [1969], we found that deflation was one feature of an optimal monetary policy regime. The extent of this deflation was small (about 0.75%) if we used estimates of money demand based on a sample that focused on post 1950 observations. It was larger (about 2.3%) if we used estimates of money demand based on a longer sample that included earlier observations when interest rates and velocity were both low.

We studied the dynamic responses of economic activity under optimal policy to productivity and government purchase shocks, using three different assumptions about money demand. These dynamic responses are anchored by the dynamics of the underlying real business cycle model. Depending on the nature of the shocks and the details of money demand, there can be interesting departures of real interest rates and real activity from their counterparts in the real business cycle model. However, in all cases optimal monetary policy involves very little "base drift" in the path of the price level, relative to the deflationary steady state path.

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Table 1:Standard Lagrangian for optimal policy problem43

$$\begin{split} \min_{\{\Lambda_t\}_{t=0}^{\infty}} \max_{\{d_t\}_{t=0}^{\infty}} \mathbf{E}_0 \{ \sum_{t=0}^{\infty} \beta^t \mathbf{E}_t [ u\left(c_t, l_t\right) \\ &+ \varphi_t (\kappa_{0,t}^{-1/\varepsilon} \lambda_t \frac{1}{1+R_t} - \beta E_t (\kappa_{1,t+1}^{-1/\varepsilon} \lambda_{t+1})) \\ &+ \varphi_{0,t} (x(\kappa_{0,t}, c_t, l_t, \lambda_t, g_t, a_t) + \beta E_t \chi_{1,t+1}) \\ &+ \sum_{j=1}^{J-2} \phi_{j,t} (x(\kappa_{j,t}, c_t, l_t, \lambda_t, g_t, a_t) + \beta E_t \chi_{j+1,t+1} - \frac{\chi_{jt}}{1-\alpha_j}) \\ &+ \phi_{J-1,t} (x(\kappa_{J-1,t}, c_t, l_t, \lambda_t, g_t, a_t) - \frac{\chi_{J-1,t}}{1-\alpha_{J-1}}) \\ &+ \eta_{1,t} (a_t n(l_t, \xi_t) - (\sum_{j=0}^{J-1} \omega_j \kappa_{j,t})(c_t + g_t)) \\ &+ \eta_{2,t} (1 - (\sum_{j=0}^{J-1} \omega_j \kappa_{j,t} \frac{\varepsilon^{-1}}{\varepsilon}) \frac{\varepsilon}{\varepsilon^{-1}}) \\ &+ \vartheta_t (D_1 u(c_t, l_t) - \lambda_t (1 + R_t (1 - \xi_t))) \\ &+ \vartheta_t [\lambda_t R_t c_t - D_2 u(c_t, l_t) F^{-1} (\xi_t)] \} \end{split}$$

<sup>43</sup>In this table,  $\mathbf{d}_t = \left\{ c_t, l_t, \xi_t, \lambda_t, (\kappa_{j,t})_{j=0}^{J-1}, (\chi_{j,t})_{j=1}^{J-1}, (s_{j,t})_{j=1}^{J-2}, R_t \right\}$  is a vector of decisions that includes real quantities, some other elements  $(\lambda_t, (\chi_{j,t})_{j=1}^{J-1})$  and the nominal interest rate  $R_t$ . Further,  $\mathbf{\Lambda}_t = \left\{ \varphi_{t,} (\phi_{j,t})_{j=0}^{J-1}, \eta_{1t}, \eta_{2t}, (\zeta_{j,t})_{j=1}^{J-2}, \vartheta_t, \varrho_t \right\}$  is a vector of Lagrange multipliers chosen at t.

 Table 2:

 An augmented Lagrangian for optimal policy problem<sup>44</sup>

$$\begin{split} U^*(s_{-1}, \phi_{-1}, \varsigma_0) &= \\ \min_{\{\Lambda_t\}_{t=0}^{\infty}} \max_{\{\mathbf{d}_t\}_{t=0}^{\infty}} \mathbf{E}_0 \{\sum_{t=0}^{\infty} \beta^t \mathbf{E}_t [u(c_t, l_t) \\ &+ \varphi_t \kappa_{0,t}^{-1/\varepsilon} \lambda_t \frac{1}{1+R_t} - \varphi_{t-1} \kappa_{1,t}^{-1/\varepsilon} \lambda_t \\ &+ \sum_{j=0}^{J-1} \phi_{j,t} x(\kappa_{j,t}, c_t, l_t, \lambda_t, g_t, a_t) \\ &+ \sum_{j=1}^{J-1} (\phi_{j-1,t-1} - \frac{\phi_{j,t}}{1-\alpha_j}) \chi_{j,t} \\ &+ \eta_{1,t} (a_t n(l_t, \xi_t) - (\sum_{j=0}^{J-1} \omega_j \kappa_{j,t})(c_t + g_t)) \\ &+ \eta_{2,t} (1 - (\sum_{j=0}^{J-1} \omega_j \kappa_{j,t} \frac{\varepsilon_{-1}}{\varepsilon}) \frac{\varepsilon}{\varepsilon_{-1}}) \\ &+ \frac{1}{2} \sum_{j=1}^{J-2} \zeta_{j,t} (\Gamma_j (s_{t-1}, \kappa_{0,t}^{-\varepsilon}) - s_{j,t}) \\ &+ \vartheta_t (D_1 u(c_t, l_t) - \lambda_t (1 + R_t (1 - \xi_t))) \\ &+ \varrho_t (\lambda_t R_t c_t - D_2 u(c_t, l_t) F^{-1} (\xi_t))] \} \end{split}$$

 $<sup>\</sup>frac{44}{4} \text{In this table, } \mathbf{d}_{t} = \left\{ c_{t}, l_{t}, \xi_{t}, \lambda_{t}, (\kappa_{j,t})_{j=0}^{J-1}, (\chi_{j,t})_{j=1}^{J-1}, (s_{j,t})_{j=1}^{J-2}, R_{t} \right\} \text{ is a vector of decisions that includes real variables, some other elements } (\lambda_{t}, (\chi_{j,t})_{j=1}^{J-1}) \text{ and the nominal interest rate } R_{t}. \text{ Further, } \mathbf{\Lambda}_{t} = \left\{ \varphi_{t}, (\phi_{j,t})_{j=0}^{J-1}, \eta_{1t}, \eta_{2t}, (\zeta_{j,t})_{j=1}^{J-2}, \varsigma_{t}, \vartheta_{t}, \varrho_{t} \right\} \text{ is a vector of Lagrange multipliers chosen at } t.$ Finally, note that  $[s_{-1}, \phi_{-1}, \varsigma_{0}] = [(s_{j,-1})_{j=1}^{J-2}, (\omega_{-1}, (\phi_{j,-1})_{j=0}^{J-2}), \varsigma_{0}]$ 

# Table 3:Fully recursive form of optimal policy problem45

$$U^*\left(s_{t-1}, \phi_{t-1}, \varsigma_t\right) = \\ \min_{\{\Lambda_t\}} \max_{\{\mathbf{d}_t\}} \{u\left(c_t, l_t\right) + \beta E_t U^* s_t, \phi_t, \varsigma_{t+1}\}$$

$$+ \varphi_t \kappa_{0,t}^{-1/\varepsilon} \lambda_t \frac{1}{1+R_t} - \varphi_{t-1} \kappa_{1,t}^{-1/\varepsilon} \lambda_t \\ + \sum_{j=0}^{J-1} \phi_{j,t} x(\kappa_{j,t}, c_t, l_t, \lambda_t, g_t, a_t) \\ + \sum_{j=1}^{J-1} [\phi_{j-1,t-1} - \frac{\phi_{j,t}}{1-\alpha_j}] \chi_{j,t}$$

$$+ \eta_{1,t} [a_t n(l_t, \xi_t) - (\sum_{j=0}^{J-1} \omega_j \kappa_{j,t})(c_t + g_t)] \\ + \eta_{2,t} [1 - (\sum_{j=0}^{J-1} \omega_j \kappa_{j,t} \frac{\varepsilon - 1}{\varepsilon})^{\frac{\varepsilon}{\varepsilon - 1}}] \\ + \sum_{j=1}^{J-2} \zeta_{j,t} [\Gamma_j(s_{t-1}, \kappa_{0,t}^{-\varepsilon}) - s_{j,t}] \\ + \vartheta_t [D_1 u(c_t, l_t) - \lambda_t (1 + R_t (1 - \xi_t))] \\ + \varrho_t [\lambda_t R_t c_t - D_2 u(c_t, l_t) F^{-1} (\xi_t)] \}$$

<sup>&</sup>lt;sup>45</sup>In this table,  $\mathbf{d}_t = \left\{ c_t, l_t, \xi_t, \lambda_t, (\kappa_{j,t})_{j=1}^{J-1}, (\chi_{j,t})_{j=0}^{J-1}, (s_{j,t})_{j=1}^{J-2}, R_t \right\}$  is a vector of decisions that includes real quantities, some other elements  $(\lambda_t, (\chi_{j,t})_{j=0}^{J-1})$  and the nominal interest rate  $R_t$ . Further,  $\mathbf{\Lambda}_t = \left\{ \varphi_{t,} (\phi_{j,t})_{j=1}^{J-2}, \eta_{1t}, \eta_{2t}, (\zeta_{j,t})_{j=1}^{J-2}, \vartheta_t, \varrho_t \right\}$  is a vector of Lagrange multipliers chosen at t. Finally, note that  $[s_{t-1}, \phi_{t-1}, \varsigma_t] = [(s_{j,t-1})_{j=1}^{J-2}, (\omega_{t-1}, (\phi_{j,t-1})_{j=0}^{J-2}), \varsigma_t]$ 

$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$	$\alpha_8$	$\alpha_9$
0.014	0.056	0.126	0.224	0.350	0.504	0.686	0.897	1
$\omega_0$	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$	$\omega_5$	$\omega_6$	$\omega_7$	$\omega_8$
0.198	0.195	0.184	0.161	0.125	0.081	0.040	0.012	0.001

Table 4:Price adjustment probabilitiesand the associated distribution weights

# Table 5:Effect of eliminating various distortionson the long-run optimal inflation rate46A. Short-sample money demand specification

		Benchmark	Sensitivity Analysis	
			Increase	Increase
			Demand	Price
	Eliminate		Elasticity	Flexibility
1		-0.76	-1.34	-1.21
2	Wedge	-0.54	-0.78	-0.84
3	Time Costs	-0.28	-0.86	-0.59
4	Wedge, Time Costs	0	0	0
5	Markup	-0.81	-1.48	-1.27

# B. Long-sample money demand specification

		Benchmark	Sensitivity Analysis	
			Increase	Increase
			Demand	Price
	Eliminate		Elasticity	Flexibility
1		-2.30	-2.84	-2.80
2	Wedge	-2.03	-2.53	-2.61
3	Time Cost	-0.21	-0.62	-0.42
4	Wedge, time cost	0	0	0
5	Markup	-2.41	-2.93	-2.82

<sup>&</sup>lt;sup>46</sup>The benchmark model is in row 1, i.e., all distortions are present; the wedge of monetary inefficiency is eliminated in row 2; shopping time costs are eliminated in row 3; and both forms of monetary distortion are eliminated in row 4. In row 5, the markup is fixed at the zero inflation level  $\varepsilon/(\varepsilon - 1)$ . The columns are as follows: benchmark calibration discussed in section 5; (b) demand elasticity for the differentiated products set to 6 instead of 10;.(c) the distribution of firms ( $\omega$ ) is modified from that in table 1 to  $\omega = 0.3, 0.28, 0.25, 0.2, 0.10$ . In this case, no firm goes more than five periods with the same price, and the expected duration of a price is 3.8 quarters instead of 5.0 quarters as in the benchmark case.





Figure 2A: Implied cost cdf











Figure 7. Real Interest Rate and Price-level Behavior

# A Derivation of the real state vector

As discussed in section (2.4), the natural price component of the state vector suggested by the model is the vector of previously determined nominal prices  $[P_{1,t} P_{2,t} \dots P_{J-1,t}]$ . This appendix develops the alternative real state vector employed in our work. Since draws entirely on the definition of the price level, it could therefore be used in more complicated environments that included additional dynamic features such as capital formation or the distribution of price setters that arises in models of state dependent pricing.

Using the definitions of the price level,  $P_t = \left[\frac{1}{1-\omega_0}\sum_{j=1}^{J-1}\omega_j P_{j,t}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}$ , and the index of lagged prices,  $\hat{P}_t = \left[\frac{1}{1-\omega_0}\sum_{j=1}^{J-1}\omega_j P_{j,t}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}$ , we can write

$$\frac{P_t}{\hat{P}_t} = [\frac{1 - \omega_0}{1 - \omega_0 (p_{0,t})^{1 - \varepsilon}}]^{\frac{1}{1 - \varepsilon}} = \gamma_1(p_{0,t})$$

where

$$\gamma_1(p_{0,t}) \equiv [\frac{1-\omega_0}{1-\omega_0(p_{0,t})^{1-\varepsilon}}]^{\frac{1}{1-\varepsilon}}$$

We define the real state of the economy as  $s = (s_1, \ldots, s_{J-2})$  via

$$s_{j,t-1} = \frac{P_{j,t,}}{\hat{P}_t} = \frac{P_{j,t}}{\left[\frac{1}{1-\omega_0}\sum_{n=1}^{J-1}\omega_j P_{n,t}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}}$$

The lagged deflator evolves through time according to

$$\frac{\widehat{P}_{t+1}}{\widehat{P}_{t}} = \left[\frac{\left[\sum_{j=1}^{J-1}\omega_{j}P_{j,t+1}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}}{\widehat{P}_{t}}\right] = \left[\frac{\left[\sum_{j=1}^{J-1}\omega_{j}P_{j-1,t}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}}{\widehat{P}_{t}}\right]$$

$$= \left[\omega_{1}\left(\frac{P_{0,t}}{\widehat{P}_{t}}\right)^{1-\varepsilon} + \sum_{j=2}^{J-1}\omega_{j}s_{j-1,t-1}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}$$

Hence, all of the future states depend only on the  $s_{t-1}$  and on  $\frac{P_{0,t}}{\hat{P}_t}$ . This latter expression depends only on  $p_{0,t}$ , as follows.

$$\frac{P_{0,t}}{\widehat{P}_t} = \frac{P_{0,t}}{P_t} \frac{P_t}{\widehat{P}_t} = \left[\frac{(1-\omega_0)(p_{0,t})^{1-\varepsilon}}{1-\omega_0(p_{0,t})^{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}} = \gamma_0(p_{0,t})$$

Hence, we can write the dynamics of the deflator as

$$\frac{\widehat{P}_{t+1}}{\widehat{P}_t} = \gamma_2(p_{0,t}, s_{1,t-1}, \dots s_{J-2,t-1}).$$

where  $\gamma_2(p_{0,t}, s_{1,t-1}, \dots s_{J-2,t-1}) = \left[\omega_1 \gamma_0(p_{0,t})^{1-\varepsilon} + \sum_{j=2}^{J-1} \omega_j s_{j-1,t-1}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}$ .

# **B** Recursive Equilibrium

For a given monetary policy function  $\mathfrak{M}(\sigma)$ , a *Recursive Equilibrium* is a set of relative price functions  $\lambda(\sigma)$ ,  $w(\sigma)$ ,  $\{p_j(\sigma)\}_{j=0}^{J-1}$ , and  $\overline{p}(\sigma)$ ; an interest rate function  $R(\sigma)$ ; a future inflation function  $\pi(\sigma', \sigma)$ ; aggregate production,  $q(\sigma)$ ; dividends,  $z(\sigma)$ ; intermediate goods producers' profits  $\{z_j(\sigma)\}_{j=0}^{J-1}$ ; value functions  $U(\cdot)$  and  $\{v^j(\cdot)\}_{j=0}^{J-1}$ ; household decision rules  $\{\xi(\sigma), c(\sigma), l(\sigma), n(\sigma), m(\sigma), \theta'(\sigma), b'(\sigma), d'(\sigma)\}$ ; retailers decision rules  $\{q_j(\sigma)\}_{j=0}^{J-1}$ ; intermediate goods producers' decision rules  $\{p_j(\sigma)\}_{j=0}^{J-1}$ ; intermediate goods producers' decision rules  $\{p_j(\sigma)\}_{j=0}^{J-1}$ ; intermediate solve  $(\varsigma, s, \phi)$ ,  $s' = \Gamma(\sigma)$  and  $\phi' = \Phi(\sigma)$  such that: (i) households solve (2) - (7), (ii) retailers solve (14) - (15), (iii) price-setting intermediate goods producers solve (22) - (25), and (iv) markets clear. This requires (1) - (7), below, be satisfied.

- 1. Households solve (2) subject to (3) (7) taking as given  $(w(\sigma), v(\sigma), R(\sigma), z(\sigma), \tau(\sigma), \overline{p}(\sigma))$ . The solution to this problem, U, is attained by  $\{\xi(\sigma), c(\sigma), l(\sigma), n(\sigma), m(\sigma), \theta'(\sigma), b'(\sigma), d'(\sigma)\}$  and  $\lambda(\sigma)$  satisfies  $D_2u(c(\sigma), l(\sigma)) = w(\sigma)\lambda(\sigma)$ .
- 2. Retailers solve (14) subject to (15), taking  $R(\sigma)$ ,  $\{p_j(\sigma)\}_{j=0}^{J-1}$  and  $\overline{q} = q(\sigma)$  as given. The solution is described by  $\overline{p}(\sigma)$  and  $q_j(\sigma)_{j=0}^{J-1}$ .
- 3. Price-setting intermediate goods producers solve (23) (25) subject to (22) and (21), taking as given  $a(\varsigma)$ ,  $w(\sigma)$ ,  $\pi(\sigma', \sigma)$  and  $q(\sigma)$  as given. The solution to this problem,  $\{v^j(\cdot)\}_{j=0}^{J-1}$ , is attained by  $\{p_j(\sigma)\}_{j=0}^{J-1}$  and profits are given by  $z_j(\sigma) \equiv z(p_j(\sigma); \sigma), j = 0, \ldots, J-1$ .
- 4. Intermediate and retail goods market equilibrium. Output of intermediate goods is  $q_j(\sigma) = p_j(\sigma)^{-\varepsilon} q(\sigma)$ . Given the price of the retail good,  $\overline{p}(\sigma) = 1 + R(\sigma)$ , production of retail goods is then given by  $q(\sigma) = c(\sigma) + g(\varsigma)$ .
- 5. Equilibrium in the labor markets. The demand for labor is given by  $n_j(\sigma) = \frac{q_j(\sigma)}{a(\varsigma)}$ , for  $j = 0, \ldots, J 1$ , implying total employment  $n(\sigma) = \sum_{j=0}^{J-1} \omega_j n_j(\sigma)$ . In equilibrium,  $n(\sigma) = 1 l(\sigma) h(\sigma)$ .
- 6. Equilibrium in the money and asset markets. The demand for real balances by households must equal its supply, the demand for bonds by retail firms must equal their supply by households and, finally, households must hold the portfolio of intermediate goods producers who return all profits as dividends.<sup>47</sup>

(a) 
$$m = (1 - \xi(\sigma))\overline{p}(\sigma) c(\sigma) = \mathfrak{M}(\sigma) / \gamma_1(p_0(\sigma))$$

<sup>&</sup>lt;sup>47</sup>The bond market condition reflects the fact that retail goods firms in our model are left holding money and credit claims at the end of each period and cannot use this revenue to repay their liabilities until the beginning of the next period. As a result, they must borrow to finance their purchases of intermediate inputs.

- (b)  $b'(\sigma) = (1 + R(\sigma)) \overline{p}(\sigma) c(\sigma)$ (c)  $\theta(\sigma) = 1$  and  $z(\sigma) = \sum_{j=0}^{J-1} \omega_j z_j(\sigma)$
- 7. <u>Rational Expectations</u>. The future inflation function satisfies  $\pi(s', \varsigma', \sigma) = \frac{\gamma_1(p_0(\sigma'))\gamma_2(p_0(\sigma),s)}{\gamma_1(p_0(\sigma))}$  where  $\sigma = (\varsigma, s, \phi), \ \sigma' = (\varsigma', s', \phi')$  and  $s' = \Gamma(\sigma)$  with  $s'_1 = \frac{\gamma_0(p_0(\sigma'))}{\gamma_2(p_0(\sigma),s)}$  and  $s'_{j+1} = \frac{s_j}{\gamma_2(p_0(\sigma),s)}$ , for  $j = 0, \ldots, J-3$ .