# Working Paper No. 00-3/R Incomplete Markets, Borrowing Constraints, and the Foreign Exchange Risk Premium<sup>1</sup>

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#### Abstract

A large body of literature documents that returns from currency speculation are highly volatile and possess a predictable component, which is itself highly volatile and serially correlated. Explaining the returns from currency speculation through the presence of a risk premium has proven difficult, however. In particular, models with complete markets and time-separable preferences generate risk premia that are nearly constant. This paper solves a model consisting of two monetary economies with incomplete markets, in which agents are subject to borrowing constraints. The paper investigates if such a framework is able to account for the volatility and the size of the foreign exchange risk premium. The model succeeds in increasing substantially the volatility of the risk premium to about 30 percent of that in the data. However, this more volatile risk premium does not translate into sufficiently large predictable excess returns. It thus appears unlikely that excess returns from currency speculation can be uniquely explained by a time-varying risk premium in an incomplete-markets economy with exogenous borrowing constraints.

A well-known fact in international economics is that forward exchange rates are biased predictors of expected future spot rates and that there are consequently predictable excess returns from currency speculation. A large body of literature documents that these predictable expected returns, although small, are highly volatile and serially correlated. Two main approaches have been offered to explain this feature of the data, but, to date, no consensus has emerged. The first approach assumes that agents are risk-neutral and explains the bias by systematic forecast errors on the part of the traders (Lewis (1989); Frankel and Froot (1987); Tornell and Gourinchas (1996)). The second avenue retains the assumption of agents' rationality and explains the expected excess returns by the presence of a time-varying risk premium. This approach has had only limited success. Specifically, Arrow-Debreu economies composed of a moderately risk-averse representative agent with time-separable preferences generate risk premiums that have nearly no variance (see Macklem (1991); Engel (1992); Bekaert (1994)).<sup>1</sup> Their failure stems mainly from their inability to generate enough variability in an agent's intertemporal marginal rate of substitution (IMRS). In general, this lack of variability leads the models to fail the test proposed by Hansen and Jagannathan (1991), in which the ratio of the standard deviation of the IMRS to its expected value has to be greater than the estimated Sharpe ratio<sup>2</sup> of any zero net investment portfolio. For instance, Bekaert (1994) shows that a coefficient of relative risk aversion of at least 50 is necessary for his complete-markets framework to pass the Hansen-Jagannathan test.

This paper investigates whether the presence of undiversifiable risks, in a general equilibrium two-country monetary model in which markets are incomplete and agents face borrowing constraints, can generate foreign exchange risk premia that are consistent with the forward discount puzzle. The inability to insure fully against idiosyncratic risk implies that the agent's IMRS becomes more volatile. The model incorporates two endowment economies, composed of a continuum of (types of) infinitely lived agents facing both aggregate uncertainty, in the form of aggregate income and money growth rate shocks, and idiosyncratic income shocks.

<sup>&</sup>lt;sup>1</sup>Empirical tests of general equilibrium models with complete markets have also been unsuccessful in uncovering a time-varying risk premium (see Hodrick (1987) and Engel (1995) for surveys of the literature).

<sup>&</sup>lt;sup>2</sup>The Sharpe ratio is the ratio of the expected return to the standard deviation of the return,  $\frac{E(r)}{\sigma(r)}$ . The ratio of the standard deviation of the IMRS to its expected value,  $\frac{\sigma(IMRS)}{E(IMRS)}$ , is also called the market price of risk.

To provide an upper bound on the potential of the framework to account for the features of excess returns from currency speculation, I study the particular case in which agents cannot borrow to smooth their consumption. The paper shows that introducing uninsurable idiosyncratic risk drastically increases the market price of risk and the standard deviation of the risk premium, the latter to about 30 percent of that in the data. However, notwithstanding this significant increase in the volatility of the risk premium, the model is unable to account for the predictability of excess returns in the data. The introduction of uninsurable idiosyncratic risk is shown to also increase the covariance between the risk premium and the expected depreciation rate, cancelling out the effects of the higher volatility of the risk premium on the predictability of excess returns.

Recently, other papers have followed different approaches to resolve the puzzle through the presence of a risk premium. Compared with the standard framework, these papers generally have more volatile risk premiums, but they are unable to replicate the volatility that the risk premium shows in the data. This paper reaches a similar conclusion. In particular, Canova and Marrinan (1993) generate more volatile risk premiums by introducing exogenous shocks that follow GARCH processes in a Lucas (1982) model. However, the larger variation in the risk premium in their paper is due to an increase in the variance of the convexity term,<sup>3</sup> which is believed to be empirically small. In an attempt to increase the variability of the IMRS, some studies have introduced time-nonseparable preferences. Backus et al. (1993) show that habit persistence raises the standard deviation of the risk premium. However, their result comes at the cost of generating negatively autocorrelated forward premiums, which are highly positively autocorrelated in the data. Sibert (1996), using an overlapping-generations model, demonstrates that contrary to these previous studies, habit persistence has nearly no impact on the variance of the risk premium in her framework. Bekaert et al. (1997) show that although allowing for preferences that exhibit first-order risk aversion increases the standard deviation of the risk premium, their model still fails to produce empirically plausible risk premium volatility.

This paper differs from the above frameworks by departing from the completemarkets framework and assuming the presence of borrowing constraints. The main idea is that incomplete markets and borrowing constraints increase the variability

<sup>&</sup>lt;sup>3</sup>The convexity term is due to Jensen's inequality. Since expected profits can be measured in terms of both currencies, expected profits must exist, at least in terms of one of the two currencies, even if the agents are risk neutral.

of the IMRS and may lead to a more volatile risk premium.<sup>4</sup> Moreover, contrary to Backus et al. (1993) and Macklem (1991), the paper does not take prices to be a given random variable. In particular, prices are determined by the interaction of the agent's decisions in the home and the foreign country. The difficulty associated with an incomplete-markets framework is that the distribution of wealth matters for the determination of these prices. Typically, the cross-sectional distribution of agents' characteristics is a high dimensional object that is part of the set of state variables. In this regard, the paper provides an algorithm to solve international monetary models with incomplete markets. The algorithm adapts the work of Krusell and Smith (1997; 1998), in which the distribution of wealth is approximated by a function of the state variables, to an international monetary environment.

By focusing on the effects of incomplete markets on the foreign exchange risk premium, this paper is complementary to previous research studying the implications of incomplete markets and asset-market frictions in open-economy models.<sup>5</sup> In a two-period model with production, Cole (1988) studied the implications of different international risk-sharing arrangements for business cycles. Building on this approach, Cole and Obstfeld (1991) show that the loss in welfare resulting from a ban in international asset trade is very small and could, therefore, help explain the apriori puzzling low amount of international intertemporal trade, as measured by the current-account balances. Mendoza (1991) also studies the properties of a small open economy subject to capital controls and shows, among others, that the agents' ability to smooth consumption are barely affected by the

<sup>&</sup>lt;sup>4</sup>The closed-economy literature on asset pricing and incomplete markets demonstrated that severe restrictions on borrowing and the persistence of idiosyncratic shocks have important effects on equilibrium asset prices. When these factors are taken into account, Lucas (1994), Heaton and Lucas (1996), Krusell and Smith (1997), and Storesletten et al. (1997) show that the models can generate significantly more suffering from idiosyncratic risks. In particular, this leads to a large increase in the market price of risk in the model, which is a key determinant of the equity premium.

<sup>&</sup>lt;sup>5</sup>This paper also complements work on the equity premium puzzle and the risk-free rate puzzle. The low variability of the IMRS is a central difficulty in explaining the behavior of both the stock and foreign exchange markets. However, the puzzle in international finance is not so much the low mean of the expected excess return from currency speculation but rather its high variance. A long list of papers relaxed the assumption of complete markets in closed-economy models (see Aiyagari (1994); Aiyagari and Gertler (1991); Constantinides and Duffie (1996); den Haan (1994); Heaton and Lucas (1996); Huggett (1993); Lucas (1994); Krusell and Smith (1997, 1998); Telmer (1993); Storesletten et al. (1997)).

presence of such controls and that, as a result, the effect on economic welfare are small. This earlier line of research led to more work on the impact of incomplete markets in business cycle models. The work of Baxter and Crucini (1995), for instance, showed that quantitative properties of a two-country general equilibrium model in which assets are restricted to a one-period noncontigent bond are similar to those when markets are complete when shocks are not very persistent or are transmitted rapidly across countries. They also found, as did Kollmann (1996), that the cross-country correlation of consumption is markedly lower and closer to the data under incomplete asset markets.

The remainder of the paper is organized as follows. Section 1 presents the model, while section 2 describes the numerical method used to solve it. I calibrate the model in section 3, while section 4 reports the results. Section 5 concludes.

### 1. Economic Environment

#### 1.1. Preferences and Forcing Processes

There are two countries, the home and the foreign, in which markets are incomplete: N (type of) ex-ante identical agents possess only a restricted set of assets to smooth their consumption. Only one good is perfectly traded on the world's market. In period t, residents of the home country are endowed with  $\xi$  units of the commodity.<sup>6</sup>  $\xi$  can take on two values,  $\xi^h$  and  $\xi^l$ , denoting a high- and a low- income level, respectively. It is assumed that the probability of receiving a good or bad income shock depends on the aggregate income shock, z, which is assumed to follow a two-state Markov process. The probability of receiving a low endowment, given the aggregate income shock, is also assumed to be dependent on the previous realization of the idiosyncratic shock. The large number of infinitely lived agents is risk-averse and cares about its holdings of real money balances. These agents wish to maximize:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t U_i(c_{i,t}, \frac{m_{i,t+1}}{p_t})\right\}, \ 0 < \beta < 1,$$
(1.1)

where  $c_{i,t}$  is the consumption at date t, by home agent i of the good,  $m_{i,t+1}$  is the amount of domestic currency held from period t to t+1 by agent i, and  $p_t$  is the

<sup>&</sup>lt;sup>6</sup>The structure of the foreign economy is identical to the home economy. Its full description in the text is thus omitted.

price level in terms of the good.<sup>7</sup> The home and foreign currencies are denoted by  $\overline{M}$  and  $\overline{N}$ , respectively. Monetary policies evolve according to the following processes:

$$\overline{M}' = (1+g_m)\overline{M} \tag{1.2}$$

$$\overline{N}' = (1 + g_n^*)\overline{N},\tag{1.3}$$

where  $g_m$  and  $g_n^*$  are the stochastic money growth rates, which are also assumed to follow two-state Markov processes. Monetary transfers,  $g_m \overline{M}$  and  $g_n^* \overline{N}$ , are distributed in a lump sum manner to national residents. Claims to future monetary transfers or claims to future output are not traded.

The variable e will denote the nominal exchange rate at date t, expressed in units of foreign currency. Since the consumption good is perfectly traded and there are no transport costs, the law of one price holds:

$$e = \frac{p}{p^*}.\tag{1.4}$$

#### 1.2. The Agent's Problem

Each agent in the home country faces the following budget constraint:

$$c + qb' + q^*b^{*'} + \frac{m'}{p} \le \omega$$
 (1.5)

where b',  $b^{*'}$ , m' are the new holdings, at time t, by a home resident, of home bonds, foreign bonds, and home currency, respectively.<sup>8</sup> The real prices of home and foreign bonds are given by  $q = p_b/p$  and  $q^* = ep_b^*/p$ , where  $p_b$  and  $p_b^*$  are the home and foreign bonds' nominal prices. The wealth of agent i at time t is represented by  $\omega$ , which follows the law of motion given by:

$$\omega' = \xi' + \frac{b'}{p'} + \frac{e'}{p'}b^{*'} + \frac{m'}{p'} + \frac{T}{p'}.$$
(1.6)

<sup>&</sup>lt;sup>7</sup>For the remainder of the paper, the time subscript t and the agent subscript i will be dropped. A superscript prime will denote a time t + 1 variable.

<sup>&</sup>lt;sup>8</sup>For expository purposes, I describe the model including foreign and domestic private bonds. However, since I will be interested in finding an upper bound to the model's potential to account for properties of risk premia, the reader should keep in mind that I will solve the model assuming that agents cannot borrow. Therefore, in equilibrium, agents will not be able to hold bonds.

An agent's wealth at time t is the sum of his endowment of the good, his return on his previous period bond and money holdings, and a lump-sum government transfer, T. The transfers are such that the government budget constraint is satisfied:

$$T = \overline{M}' - \overline{M} \tag{1.7}$$

Moreover, it is assumed that agents cannot borrow:

$$b' \ge 0, \tag{1.8}$$

$$b^{*'} \ge 0. \tag{1.9}$$

The aggregate state of the world is given by the aggregate income shocks, z and  $z^*$ , the monetary shocks,  $g_m$  and  $g_n^*$ , and by the measure, or distribution, of agents (across countries) over their individual wealth and employment status,  $\Gamma$ . This distribution is an endogenous variable in the model and no a priori restrictions are placed on it. In the section below describing the algorithm, more details are given concerning how I approximate this high-dimensional object. The agent's wealth and his current employment status represent his individual state variable. Let  $s = (z, z^*, g_m, g_n^*)$  represent the exogenous part of the aggregate state of the world. The aggregate laws of motion are given by the Markov chains for the aggregate income shocks and the money growth rate shocks and by an endogenous function H, which governs how  $\Gamma$  evolves through time:  $\Gamma' = H(\Gamma, s, s')$ . The section describing the algorithm also provides details regarding how to choose H.  $P(\Gamma, s)$ ,  $P^*(\Gamma, s)$ ,  $q(\Gamma, s)$ , and  $q^*(\Gamma, s)$  represent the equilibrium pricing functions for the good and for the bonds.

The dynamic programming problem thus becomes:

$$V(\omega,\xi;\Gamma,s) = \max_{c,b',b^{*'},m'} \left\{ U(c,\frac{m'}{P(\Gamma,s)}) + \beta E[V(\omega',\xi';\Gamma',s')|\xi,s] \right\}$$
(1.10)

subject to 
$$c + q(\Gamma, s)b' + q^*(\Gamma, s)b^{*'} + \frac{m'}{P(\Gamma, s)} \le \omega$$
 (1.11)

$$\omega' = \xi' + \frac{b'}{P(\Gamma', s')} + \frac{e'(\Gamma', s')}{P(\Gamma', s')}b^{*'} + \frac{m'}{P(\Gamma', s')} + \frac{g'_m \overline{M}}{P(\Gamma', s')}$$
(1.12)

$$b' \ge 0 \tag{1.13}$$

$$b^{*'} \ge 0 \tag{1.14}$$

$$\Gamma' = H(\Gamma, s, s') \tag{1.15}$$

#### 1.3. Equilibrium

### 1.3.1. Definition

A recursive equilibrium consists of a set of individual decision rules and value functions,  $B(\omega,\xi;\Gamma,s)$ ,  $B^*(\omega,\xi;\Gamma,s)$ ,  $M(\omega,\xi;\Gamma,s)$ , and  $V(\omega,\xi;\Gamma,s)$ ; a set of pricing functions  $P(\Gamma,s)$ ,  $P^*(\Gamma,s)$ ,  $q(\Gamma,s)$  and  $q^*(\Gamma,s)$ ; and a law of motion H, such that (i)  $B(\omega,\xi;\Gamma,s)$ ,  $B^*(\omega,\xi;\Gamma,s)$ ,  $M(\omega,\xi;\Gamma,s)$ ,  $V(\omega,\xi;\Gamma,s)$  solve the dynamic programming problem above, (ii)  $B(\omega,\xi;\Gamma,s)$ ,  $B^*(\omega,\xi;\Gamma,s)$ ,  $M(\omega,\xi;\Gamma,s)$  are such that the bond and money markets clear:

$$\sum_{i=1}^{2N} b_i = 0, \tag{1.16}$$

$$\sum_{i=1}^{2N} b_i^* = 0, \tag{1.17}$$

$$\sum_{i}^{N} m_i' = \overline{M},\tag{1.18}$$

and

$$\sum_{i}^{N} n_{i}^{\prime} = \overline{N}, \qquad (1.19)$$

and the law of motion H is consistent with individual behavior.

### 1.3.2. Characterization

A solution to the agent's problem satisfies:

$$U_1(c, \frac{m'}{P(\Gamma, s)})q(\Gamma, s) > \beta E\left[U_1\left(c', \frac{m''}{P(\Gamma', s')}\right)\left(\frac{1}{P(\Gamma', s')}\right)|\xi, s\right], \quad (1.20)$$

$$U_1(c, \frac{m'}{P(\Gamma, s)})q^*(\Gamma, s) > \beta E\left[U_1\left(c', \frac{m''}{P(\Gamma', s')}\right)\left(\frac{e'(\Gamma', s')}{P(\Gamma', s')}\right)|\xi, s\right], \quad (1.21)$$

$$U_{1}\left(c,\frac{m'}{P(\Gamma,s)}\right)\left(\frac{1}{P(\Gamma,s)}\right) = U_{2}\left(c,\frac{m'}{P(\Gamma,s)}\right)$$

$$+\beta E\left[U_{1}\left(c',\frac{m''}{P(\Gamma',s')}\right)\left(\frac{1}{P(\Gamma',s')}\right)|\xi,s\right],$$
(1.22)

$$b' = 0 \,(\mu > 0), \tag{1.23}$$

$$b^{*'} = 0(\mu^* > 0), \tag{1.24}$$

where  $U_i$  denotes the derivative of the utility function with respect to the *ith* argument, and  $\mu$  and  $\mu^*$  are the multipliers of equations (1.13) and (1.14), respectively. The agent's optimal decision over money holding is determined by equation (1.22). Under this condition, the disutility of acquiring one unit of money should be equal to the expected discounted utility of next period's payoff, at the margin. Since agents cannot borrow and since bonds are supplied by the agents and, therefore, must be zero in net supply, conditions (1.23) and (1.24) determine the agent's allocation of domestic and foreign bonds. Since the borrowing constraints are strictly binding, equations (1.20) and (1.21) show that there is an upward pressure on the agents' subjective valuation of the bonds, as the marginal utility of receiving  $q(\Gamma, s)$  and  $q^*(\Gamma, s)$  units of currency is greater than the expected discounted disutility of repaying the debt. In other words, there is downward pressure on interest rates because agents, facing the borrowing constraints, cannot borrow as much as they would like to.

From equations (1.20) and (1.21) and the definition of q and  $q^*$ , we can derive each agent's valuation of domestic and foreign bonds:

$$p_b > \beta E\left[\frac{U_1(c', \frac{m''}{p'})}{U_1(c, \frac{m'}{p})}\left(\frac{p}{p'}\right)|\xi, s\right],\tag{1.25}$$

and

$$p_b^* > \beta E\left[\frac{U_1(c', \frac{m''}{p'})}{U_1(c, \frac{m'}{p})} \left(\frac{e'}{e}\right) \left(\frac{p}{p'}\right) |\xi, s\right].$$
(1.26)

The foreign exchange risk premium,  $E\left[\left(e'-f\right)|\xi,s\right]$ , can then be derived, once the forward exchange rate is computed using covered interest parity,  $\frac{f}{e} = \frac{p_b^*}{p_b}$ , and the equilibrium bond prices.

### 2. Solution Algorithm

This section describes the solution method. The reader interested only in the economic results can skip this section without any loss of continuity. The main problem associated with heterogeneous-agents models is that the wealth distribution matters for the determination of prices, i.e., it is part of the set of state variables. Since this distribution is a high-dimensional object, an approximation is needed to successfully solve these models numerically. Typically, authors have assumed either that there are only two types of agents in the economy or that there is a continuum of (types) of agents and no aggregate uncertainty. Without these assumptions, approximating the wealth distribution becomes more complicated. Basically, only two methods solve environments with a continuum of agents and aggregate uncertainty: parameterized expectations and the procedure proposed in Krusell and Smith (1997, 1998).

The algorithm adapts the method of Krusell and Smith (1997) to a monetary economy context. The method concentrates on finding stationary equilibria. The idea is to assume that agents perceive prices as depending on a limited set of moments I of the wealth distribution. Krusell and Smith (1997,1998) find that, in a one-sector neoclassical growth model, the mean of the distribution alone suffices to generate accurate approximations. This result is due to the similarity in the marginal propensities to save across different agents.

The strategy will be to apply their result to the present framework. The idea is to start by approximating the distribution with its mean and verify whether, under some metric, the approximation is accurate. As I argue below, the mean turns out to be sufficient to bring about a good approximation. Here the mean of the distribution corresponds to the sum of the agents' endowment value and money supply in each country. Since there is no capital and since money is a veil in this economy, prices in this context are simply given as functions of the aggregate income shocks and the money growth rate shocks:  $\tilde{P}(s)$ , the home pricing function of the good, and  $\tilde{P}^*(s)$ , the foreign pricing function of the good, where  $s = (z, z^*, g_m, g_n^*)$ . The algorithm approximates the two functions by:

$$\widetilde{P}(s) = a_j \text{ if } s = s_j, j = 1, 2, ..., J$$
(2.1)

$$\widetilde{P^*}(s) = c_j \text{ if } s = s_j, j = 1, 2, ..., J$$
(2.2)

where J is the number of possible states.

Since agents cannot borrow, the following problem is solved:

$$V(\omega;s) = \max_{c,m'} \left\{ U(c, \frac{m'}{\tilde{P}(s)}) + \beta E\left[V(\omega';s')|\xi,s\right] \right\}$$
(2.3)

subject to 
$$c + \frac{m'}{\widetilde{P}(s)} \le \omega$$
 (2.4)

$$\omega' = \xi' + \frac{m'}{\widetilde{P}(s')} + \frac{g'_m \overline{M}}{\widetilde{P}(s')}$$
(2.5)

and to equations (2.1) and (2.2), and where  $e(s) = \frac{\widetilde{P}(s)}{\widetilde{P}^*(s)}$ . The algorithm involves the following steps:

- Generate random shocks for  $z, z^*, g_m, g_n^*, \xi$ , and  $\xi^*$ .
- Discretize the state space and restrict H to a finite set of moments  $H_I$  with chosen parameters. In the case here, the functions are given by equations (2.1)-(2.2).
- For each economy, solve (2.3) using value function iteration.
- Use the value function from the previous step to verify that the market clears and solve problem (2.6) below. That is, fix prices (to  $\hat{P}$  and  $\hat{P}^*$ ), derive the optimal decision rules for all agents, and iterate on prices until the markets clear. Thus:
  - Fix an initial wealth/employment distribution for a large number of agents and initial values for the aggregate shocks. Solve problem (2.6) and iterate on prices until all markets clear.

$$\widetilde{V}(\omega; s, \widehat{P}, \widehat{P^*}) = \max_{c,,m'} \left\{ U(c, \frac{m'}{\widehat{P}}) + \beta E\left[ V(\omega'; s') | \xi, s \right] \right\}$$
(2.6)

subject to 
$$c + \frac{m'}{\hat{P}} \le \omega$$
 (2.7)

$$\omega' = \xi' + \frac{m'}{\widetilde{P}(s')} + \frac{g'_m \overline{M}}{\widetilde{P}(s')}$$
(2.8)

and subject to (2.1), and (2.2).

 Use the decision rules from (2.6) to derive the new wealth/employment distribution. From the Markov processes, get new aggregate shocks.

- Repeat the procedure for a large number of periods, discarding the first part.
- For each state of the world, compute the mean and the variance of the simulated prices. If the mean of each simulated price series is close to the initial guess, given some convergence criteria, and the variance of the simulated series is small, then the algorithm has converged. Otherwise, update the initial guesses and go back to the third step.

One way to test the accuracy of the solution is to add another moment to approximate the distribution and assess if there are significant changes in the model's simulated data (for instance, in the risk premium statistical properties). However, that approach is computationally costly as it involves adding one more grid in the solution algorithm (for instance, a grid for the second moment of the distribution). In complicated models, a more practical approach is to look at the errors agents make when they use only the mean to approximate the distribution. In the current model, this would imply to look at the extent to which agents make mistakes when they approximate that future prices depend only on the mean of the wealth distribution. If agents make very small mistakes in their "forecasts" of future prices when they use the mean only, we can be more confident that the addition of more moments to the approximation would only have a very small quantitative impact on the results.

I employ a tight convergence criteria that dictates that a solution is found not only when, for every state of the world, the mean of the simulated prices series is close to the perceived prices,  $\tilde{P}(s)$ , that agents use to "forecast" future prices, but also when the standard deviation of the simulated price series is small. The convergence criteria used was that the percentage change between the mean of the simulated series and the guessed perceived prices, as well as the standard deviations of the simulated prices, be not greater than  $1 \times 10^{-6}$ . Given this strict convergence criteria, I find that the simulated price series, at every state of the world s, is a straight line at the level of the guessed perceived price,  $\tilde{P}(s)$ . Since the standard deviations of the price series are also very small, the agents only make very small mistakes when they follow the approximation rule given by the perceived prices. As a result, adding more moments would improve the agents' forecast of future prices, but the impact on the results would be quantitatively small, as Krusell and Smith (1997, 1998) found. Notice that the bond prices can be retrieved by computing the agents' subjective valuation of the bonds' payoffs, which are given by the agents' expected nominal intertemporal marginal rate of substitution. I look at an equilibrium in which the bonds' price is defined as the highest subjective bond valuation. At this price, all agents would like to borrow, except the one determining the bond price. This agent is therefore indifferent at zero bond holdings.

The simplex algorithm and the Newton-Raphson method (see Press et al. (1992)) are used to solve respectively the value function and the optimal decision rules in (2.3) and (2.6) above. Thirty grid points for individual wealth are used when solving (2.3). Cubic splines are used to interpolate between grid points. To solve for the market-clearing prices, the algorithm uses 2000 agents. Since the decision rules turn out to be linear, a simple bi-linear interpolation scheme computes the optimal decision off the grid. The two economies are simulated for 1100 periods, of which the first 100 periods are discarded.

## 3. Parametric Specifications

This section chooses the parameters' values, selects the transition probabilities for the idiosyncratic shock process, and estimates the processes for aggregate shocks.

#### 3.1. Preferences

The utility function is assumed to be of the following form:

$$\frac{\left[\left(ac^{\phi} + (1-a)\left(\frac{m'}{P}\right)^{\phi}\right)^{\frac{1}{\phi}}\right]^{1-\sigma}}{1-\sigma} \tag{3.1}$$

To determine a and  $\phi$ , I follow Chari, Kehoe and McGrattan (1998). They estimate a standard money-demand equation (which they derived from the first-order condition for a nominal bond in their model) and find a = 0.73 and  $\phi = -1.56$ .<sup>9</sup>

<sup>&</sup>lt;sup>9</sup>The utility function that Chari, Kehoe, and McGrattan use also includes leisure:  $\left[\left(ac^{\phi} + (1-a)\left(\frac{m^{0}}{P}\right)^{\phi}\right)^{\frac{\gamma}{\phi}}(1-l)^{1-\gamma}\right]^{1-\sigma}/(1-\sigma).$  However, since leisure enters multiplicatively, it does not affect the money-demand equation and, as a result, the estimates for a and  $\phi$ .

The discount rate is set to 0.9967, and I report the results for different coefficients of relative risk aversion.

#### 3.2. Shocks

The aggregate income shocks and the money growth rate shocks are assumed to follow a first-order vector autoregression. The VAR process is estimated using the growth rates of GDP,  $g_z$ , and M1,  $g_m$ , for Canada and the US. The data are quarterly and cover the period 1973:2 to 1999:3:

$$\begin{pmatrix} g'_z \\ g''_z \\ g'_m \\ g''_n \\ g''_n \end{pmatrix} = \begin{pmatrix} 0.41 & 0.06 & 0.14 & 0.06 \\ -0.31 & 0.70 & 0.20 & 0.10 \\ 0.32 & -0.03 & 0.33 & 0.42 \\ -0.002 & 0.07 & -0.04 & 0.86 \end{pmatrix} \begin{pmatrix} g_z \\ g'_z \\ g_m \\ g'_n \\ g'_n \end{pmatrix} + \begin{pmatrix} e'_z \\ e'_z \\ e'_m \\ e'_n \\ e'_n \end{pmatrix}.$$
(3.2)

The VAR process is then transformed to a two-state Markov process using the Tauchen and Hussey (1991) method.

Finally, as in Imrohoroglu (1989), the transition probabilities for the idiosyncratic shock process depend on the aggregate shock. Let  $\pi_{\xi^h\xi^l}^g$  be the probability that, when the economy is hit by a good aggregate shock, an agent receives a low endowment shock,  $\xi^l$ , given that the agent received a high endowment shock in the previous period,  $\xi^h$ . Therefore, depending on the realization of the aggregate income shock, the transition probabilities for the idiosyncratic shock are governed by the following two transition matrices:

$$\Pi^{g} = \begin{bmatrix} \pi^{g}_{\xi^{h}\xi^{h}} & \pi^{g}_{\xi^{h}\xi^{l}} \\ \pi^{g}_{\xi^{l}\xi^{h}} & \pi^{g}_{\xi^{l}\xi^{l}} \end{bmatrix}, \text{ and } \Pi^{b} = \begin{bmatrix} \pi^{b}_{\xi^{h}\xi^{h}} & \pi^{b}_{\xi^{h}\xi^{l}} \\ \pi^{b}_{\xi^{l}\xi^{h}} & \pi^{b}_{\xi^{l}\xi^{l}} \end{bmatrix}.$$
(3.3)

These probabilities are selected such that the percentage of agents receiving a bad idiosyncratic shock is 4 percent in good times and 8 percent in bad times. The average duration of a bad idiosyncratic shock is 1.5 model periods in good times (9 weeks) and 2.5 model periods in bad times (15 weeks).<sup>10</sup> These imply that the probabilities of governing the idiosyncratic process have the following features: (i)  $\pi_{\xi^h\xi^h}^g > \pi_{\xi^h\xi^h}^b$ , (ii)  $\pi_{\xi^l\xi^l}^g < \pi_{\xi^l\xi^l}^b$ , (iii)  $\pi_{\xi^l\xi^h}^g > \pi_{\xi^h\xi^h}^b$ , and (iv)  $\pi_{\xi^h\xi^l}^g < \pi_{\xi^h\xi^l}^b$ . Finally,

<sup>&</sup>lt;sup>10</sup>Receiving a bad idiosyncratic shock could be interpreted as the agent being unemployed and receiving only a fraction of its wage. The calibration of the transition probabilities is, consequently, similar to that of Imrohoroglu (1989) which set these parameters to match the

an agent receiving a bad idiosyncratic shock receives one-third of the endowment of an agent subject to a good shock.

The properties of the idiosyncratic shock's transition matrices will have important quantitative implications for the foreign exchange risk premium. For instance, Mankiw (1986) showed that the equity premium can be made arbitrarily large when a portion of the population is disproportionately affected by an economic downturn. This is, Mankiw's finding suggests that consumption volatility should be higher in bad times than in good times to raise the equity premium in model with idiosyncratic risk. Similarly, Constantinides and Duffie (1996) showed that, when individual income processes are nonstationary, the risk-free rate falls and the equity premium rises relative to the complete-markets case when the conditional variance of idiosyncratic shocks increases in bad times. These latter authors also demonstrated that, to defeat the possibility to self-insure, the persistence of idiosyncratic risks was very important in determining the size of the equity premium.

These previous findings mean that the correlation of idiosyncratic risk with aggregate risks and the persistence of individual shocks might be important in accounting for the volatility of the foreign exchange risk premium and the forward discount puzzle. Although agents cannot self-insure in the current framework, the duration of idiosyncratic risk can still affect the quantitative results in a non-trivial way. The reason is that the duration of a low endowment shock (i.e., D in the previous footnote), for instance, will raise  $\pi^i_{\xi^l\xi^l}$  and  $\pi^i_{\xi^h\xi^h}$ ,  $i = \{g, b\}$ . Therefore, I will conduct some robustness analysis, by varying that parameter in the calibration. Moreover, to verify the quantitative impact of the correlation between idiosyncratic shocks and economic downturn on the properties of risk premia, I will vary the percentage of agents receiving a high endowment shock when times are bad (i.e.,  $N^b$  in the previous footnote). In particular, a smaller  $N^b$  implies that the probability of receiving a low endowment is higher when the

average duration of unemployment, D, and the rate of employment, N, in good and bad times:

$$\pi^i_{\xi^l\xi^l} = 1 - \frac{1}{D^i}, \quad i = \{g, b\}$$

and

$$\pi^i_{\xi^h\xi^h} = \frac{N^i - (1 - \pi^i_{\xi^l\xi^h}) * (1 - N^i)}{N^i}, \quad i = \{g, b\}.$$

economy is hit by a bad aggregate shock, which raises consumption variability in economic downturns.

## 4. Results

Before turning to the simulated results, I will first briefly report some well-known empirical features of foreign exchange markets. First, we can construct an empirical measure of the risk premium by using the fitted values of a regression of the realized returns from currency speculation, e' - f, on a constant and the forward premium at time t, f - e. Table 1 reports the results for five countries for the period covering 1974:1 to 1999:12. As most empirical studies found, estimates of  $\beta_1$  are for the most part smaller than -1, except for the Italian line, which has an estimate of -0.9973.<sup>11</sup> Nonzero estimates of  $\beta_1$  imply that returns from currency speculation have a predictable component. The table shows that, for all currencies, the mean of the risk premium is close to zero. On the other hand, the risk premium is highly volatile: from 0.59 percent per month for the Canadian dollar to about 1 percent per month for the yen. Table 1 also summarizes some interesting properties of forward premia. In particular, it shows that forward premia are highly autocorrelated, with the coefficients ranging from 0.876 to 0.952, and that the risk premia are more volatile than forward premia. This will turn out to be important in understanding the results form the simulations below.

<sup>&</sup>lt;sup>11</sup>Surveying the results over 75 published articles, Froot (1990) finds an average  $\widehat{\beta_1}$  of -1.88.

	Canada	France	Italy	Japan	UK			
OLS Regression								
$\widehat{\alpha_1}$	-0.0023	-0.0016	-0.0035	0.0066	-0.0044			
	(0.0007)	(0.0018)	(0.0022)	(0.0022)	(0.0019)			
$\widehat{\beta_1}$	-1.3819	-1.1545	-0.9973	-1.4746	-1.5512			
	(0.1354)	(0.1787)	(0.1190)	(0.1896)	(0.1991)			
$R^2$	0.2521	0.1190	0.1852	0.1636	0.1642			
Risk Premium								
Mean	0.0013	0.0042	0.0105	-0.0036	0.0052			
Std	0.0059	0.0101	0.0122	0.0139	0.0113			
Forward Premium								
Mean	-0.0026	-0.0050	-0.0140	0.0069	-0.0062			
Std	0.0043	0.0088	0.0123	0.0094	0.0073			
Autocorr.	0.944	0.897	0.897	0.911	0.938			

 Table 1: Summary Statistics

The exchange rates are US dollars per unit of foreign currency.

To put the results in perspective, I first solve the model assuming that markets are complete. Table 2 reports the results concerning the mean and the standard deviation of the risk and the forward premiums, as well as the autocorrelation of the latter. It also shows the predictability of excess returns by reporting the estimated coefficient,  $\hat{\beta}_1$ , of a regression of the realized return, e' - f, on a constant and the forward premium, f - e. Finally, the table reports the model's market price of risk,  $\frac{\sigma(IMRS)}{E(IMRS)}$ . The first column of the table describes the Canadian data that I use to assess the model's performance; in the second one, the model's results, under a coefficient of relative risk aversion equal to 2, are reported.

	E(rp)	$\sigma(rp)$	E(fp)	$\sigma(fp)$	$\rho(fp)$	$\widehat{\beta_1}$	$\frac{\sigma(IMRS)}{E(IMRS)}$
Data							
	0.0013	0.0059	-0.0026	0.0043	0.944	-1.3819	
Model							
$\sigma = 2$	$1 \times 10^{-6}$	$3 \times 10^{-7}$	-0.0004	0.0067	0.4522	$-3 \times 10^{-6}$	0.0003

 Table 2: Sample and Implied Moments Under Complete Markets

**rp**=risk premium, **fp**=forward premium,  $\widehat{\beta_1}$  is the estimated slope coefficient from  $e' - f = \alpha_1 + \beta_1(f - e) + u'$ .

It is obvious that, as was previously pointed out in the literature, an environment with complete markets cannot generate either a high market price of risk or a volatile risk premium. In fact, the risk premium, under complete markets, is nearly constant. As has been pointed out in the literature, the potential of a model to account for the excess returns in the data critically depends on the extent to which the IMRS varies. The lack of variation of the IMRS implies excess returns that vary very little also. As a result, excess returns from currency speculation are unpredictable: the slope coefficient in a regression of the excess returns from currency speculation on the forward premium is zero for all cases. Note also that, even though forward premiums are autocorrelated in the model, the serial correlation is still insufficiently large to match the data. Overall, the results are similar to the ones of complete-markets frameworks of Macklem (1991) and Bekaert (1994).

	E(rp)	$\sigma(rp)$	E(fp)	$\sigma(fp)$	$\rho(fp)$	$\widehat{\beta_1}$	$\frac{\sigma(IMRS)}{E(IMRS)}$	
	Data							
	0.0013	0.0059	-0.0026	0.0043	0.944	-1.3819		
	Model							
$\sigma = 2$	0.0002	0.0008	0.0009	0.0096	0.3972	0.0495	0.0528	
$\sigma = 3$	0.0002	0.0013	0.0009	0.0095	0.3771	0.0595	0.1028	
$\sigma = 4$	0.0002	0.0017	0.0008	0.0093	0.3615	0.0694	0.1314	

 Table 3: Sample Moments and Implied Moments

rp=risk premium,  $\overline{\mathsf{fp}}$ =forward premium,  $\widehat{\beta_1}$  is the estimated slope coefficient from  $e' - f = \alpha_1 + \beta_1(f - e) + u'$ .

The introduction of undiversifiable idiosyncratic risks and borrowing constraints drastically alters the properties of excess returns and the market price of risk generated by the model. Table 3 summarizes the results for different levels of risk aversion. Notice that the standard deviation of the risk premium increases significantly. With a coefficient of relative risk aversion of three, the standard deviation of the risk premium is now 14 percent of that in the data. And when  $\sigma$ is raised to 4, that percentage increases to 30 percent. Backus et al. (1993) obtain similar results by introducing time-nonseparable preferences: with moderate habit persistence their risk premium is approximately 15 percent of the estimated risk premium volatility in the data. However, contrary to a framework with habit persistence, the increase in the variance of the risk premium does not come at the cost of generating negatively autocorrelated forward premiums. In all cases, the forward premium has an autocorrelation of approximately 0.4, which is, however, still lower than the serial correlation of 0.9 observed in the data. On the other hand, the model generates forward premia that are too volatile compared to the data.

The test proposed in Hansen and Jagannathan (1991) provides a means to shed some light on the results. The test imposes a lower bound on the standard deviation of the model's IMRS. Consider an unconstrained agent in the home country. The Euler equation must satisfy:

$$E\left[\lambda'(e'-f)\right] = 0.$$
 (4.1)

When no borrowing is allowed, the agent determining the bonds' price is instead considered. Hansen and Jagannathan (1991) show that the Sharpe ratio imposes a lower bound on the standard deviation of the IMRS to its mean (i.e., the market price of risk) that the model must satisfy. In the present case, the lower bound can be derived as:

$$\frac{E(e'-f)}{\sigma(e'-f)} \le \frac{\sigma(\lambda)}{E(\lambda)}.$$
(4.2)

Backus et al. (1993) estimate the Sharpe ratio for currency speculation for the Canadian dollar to be 0.293 per month. Bekaert and Hodrick (1992) estimate bounds as high as 0.776 when US equity investments and German, Japanese, and UK equity and foreign exchange investments are jointly considered.

When  $\sigma$  equals 4, the model is able to substantially increase the market price of risk to about 45 percent of that estimated in the data. Storesletten et al. (1997) obtain comparable results, although Krusell and Smith (1997) can generate significantly higher market price of risk. Table 5 shows that as the market price of risk increases, so does the standard deviation of the risk premium. To increase the volatility of the risk premium to empirically plausible values, the economies would need to be even riskier. In a similar fashion, Backus et al. (1993) show that, with habit persistence, the standard deviation of the risk premium increases as the market price of risk rises. To generate a standard deviation of expected returns from currency speculation that is half its estimated value in the data, they need preferences that exhibit strong habit persistence, so that agents are very sensitive to small changes in consumption.

Compared to a complete-markets framework, increases in the standard deviation of the risk premium materialize in larger deviations of  $\widehat{\beta}_1$  from zero. However, it is clear that the model still fails to match the slope coefficient,  $\widehat{\beta}_1$ , estimated from the data. Compared to a coefficient of -1.3819 the largest estimated slope coefficient is 0.0694. As Bekaert et al. (1997) note, one explanation is that, while necessary, it is not sufficient to raise the standard deviation of the risk premium to explain the predictability of excess returns from currency speculation: the slope coefficient is also a function of the forward premium and the expected rate of depreciation, which are endogenous variables. These variables will also be affected by changes in the underlying structure of the economy. This can be seen from the definition of  $\beta_1$  and noticing that the forward premium can be decomposed into a risk premium and the expected rate of currency depreciation:

$$\beta_1 = \frac{cov(e'-f;f-e)}{var(f-e)} = \frac{cov(rp;E(\Delta e')) - var(rp)}{var(E(\Delta e')) + var(rp) - 2cov(rp;E(\Delta e'))}, \quad (4.3)$$

where  $E(\Delta e')$  denotes the expected change in the exchange rate. Therefore, estimates of  $\beta_1$  that are less than -1 imply that:

$$var(rp) > cov(rp; E(\Delta e')) > var(E(\Delta e'))$$
(4.4)

and

$$var(rp) > var(fp). \tag{4.5}$$

Table 4 reports the components of  $\beta_1$  for the different levels of relative risk aversion in Table 3:

	$\widehat{\beta_1}$	$\sigma(rp)$	$\sigma(E(\Delta e'))$	$cov(rp; E(\Delta e'))$
$\sigma = 2$	0.0495	0.0008	0.0101	0.0012
$\sigma = 3$	0.0595	0.0013	0.0101	0.0018
$\sigma = 4$	0.0694	0.0017	0.0101	0.0022

Table 4. Decomposition of the Slope Coefficient

Obviously, since the estimated  $\beta_1$ s from the model are close to 0, the variance of the risk premium is less than that of the forward premium, and the inequality in equation (4.5) is reversed. The last column of the table shows, however, that the introduction of incomplete markets and borrowing constraints correctly generates a positive covariance between the risk premium and the expected rate of depreciation. However, these features increase that covariance to such an extent that it neutralizes the effects on  $\hat{\beta}_1$ . In fact, the model generates the following inequalities:

$$var(E(\Delta e')) > cov(rp; E(\Delta e')) > var(rp),$$

which is the opposite ordering of (4.4). Thus, the impact that idiosyncratic risk has on the covariance of the risk premium and the expected depreciation rate in part underlies the failure of the model in explaining the predictability of excess returns.

#### 4.1. Sensitivity Analysis

#### 4.1.1. Persistent Endowment Shocks

In this section, I first document the effects of varying the duration of a low endowment spell on the results when the coefficient of relative risk aversion is set to three. As section 3.2 previously discussed, Constantinides and Duffie (1996) showed that the persistence of idiosyncratic risks had an important impact on asset prices. Table 5 reports the results of changing the duration of a low endowment shock when times are bad (i.e.,  $D^b$ ).<sup>12</sup> The table shows that increasing the persistence of idiosyncratic risk slightly increases the standard deviation of the risk premium and raises the market price of risk. However, these effects are relatively small, considering that when  $D^b$  equals 20, the duration of a low endowment shock has been increased approximately tenfold, compared to the benchmark calibration Such a number implies that  $\pi^b_{\xi^l\xi^l}$ , the probability of receiving a low endowment shock given that the agent received one in the previous period. is 0.95, compared to 0.6 under the benchmark calibration.<sup>13</sup> It would also imply that, empirically, the duration of a bad endowment spell would be more than two years. The effects on  $\hat{\beta}_1$ , however, are relatively bigger. Yet, the coefficient estimate remains small and of the wrong sign.

	E(rp)	$\sigma(rp)$	E(fp)	$\sigma(fp)$	$\rho(fp)$	$\widehat{\beta_1}$	$\frac{\sigma(IMRS)}{E(IMRS)}$
Data							
	0.0013	0.0059	-0.0026	0.0043	0.944	-1.3819	
Model ( $\sigma = 3$ )							
$D^b = 5$	0.0002	0.0021	0.0008	0.0091	0.3513	0.0882	0.1603
$D^b = 10$	-0.0004	0.0022	0.0014	0.0089	0.3704	0.1195	0.176
$D^b = 20$	-0.0005	0.0022	0.0016	0.0088	0.404	0.1282	0.18

Table 5. Persistence of Idiosyncratic Shocks

#### 4.1.2. Correlation Between Idiosyncratic and Aggregate Risks

To verify the quantitative impact of the correlation between idiosyncratic shocks and economic downturns on the properties of risk premia, I vary the percentage of agents receiving a high endowment shock when times are bad (i.e.,  $N^b$ ). I simulate the models when the number of agents receiving a high endowment shocks in bad

<sup>&</sup>lt;sup>12</sup>Since we know from the works of Mankiw (1986) and Constantinides and Duffie (1996) that the correlation between aggregate and idiosyncratic risk is important in determining the size of the risk premium, I concentrate the increase of the duration of low endowment shocks in bad times to get an upper bound on the effects of more persistent low endowment shocks on the results.

<sup>&</sup>lt;sup>13</sup>Note that since the algorithm is tailored to solve stationary models, I cannot study the case where  $\pi^b_{\ell^l \ell^l} = 1$ .

times is 90%, 88%, and 86%. Remember that under the benchmark calibration, 92% of the agents receive a high endowment shocks in bad times. As  $N^b$  falls, the probability of receiving a low endowment increases when the economy is hit by a bad aggregate shock.<sup>14</sup> Therefore, as  $N^b$  falls, consumption variability increases in economic downturns, which may increase the volatility of the risk premium. The results are reported in Table 6.

	E(rp)	$\sigma(rp)$	E(fp)	$\sigma(fp)$	$\rho(fp)$	$\widehat{\beta_1}$	$\frac{\sigma(IMRS)}{E(IMRS)}$	
Data								
	0.0013	0.0059	-0.0026	0.0043	0.944	-1.3819		
	Model ( $\sigma = 3$ )							
$N^b = 0.9$	0.0003	0.0022	0.0013	0.0142	0.3725	0.066	0.107	
$N^b = 0.88$	0.0003	0.0033	0.0019	0.0188	0.3674	0.0746	0.109	
$N^b = 0.86$	0.0002	0.0044	0.0024	0.024	0.3556	0.065	0.1113	

Table 6. Correlation Between Idiosyncratic and Aggregate Risks

The table shows that as  $N^b$  falls and the probability of receiving a low endowment increases, the market price of risk and the volatility of the foreign exchange risk premium increase. When the employment rate in bad times falls to 86%, the standard deviation of the risk premium reaches 0.44%, more than three times its level under the benchmark calibration. Notice also that this increase in the volatility of the risk premium does not come at the cost of generating less persistent forward premia. In fact, the autocorrelation of the forward premium is not sensitive to changes in  $N^b$ . However, as in the previous exercise, varying the percentage of agents employed in bad times does not significantly affect the estimated slope coefficient in a regression of the excess returns from currency speculation on the forward premium. The simulated slope coefficient remains no larger than 7.5% and has the wrong sign.

<sup>14</sup>Remember that, that probability is equal to:

$$\pi^{b}_{\xi^{h}\xi^{h}} = \frac{N^{b} - (1 - \pi^{b}_{\xi^{l}\xi^{h}}) * (1 - N^{b})}{N^{b}},$$

which is a function of the employment rate in bad times,  $N^b$ .

## 4.1.3. Decomposing $\widehat{\beta}_1$

To understand why relatively volatile risk premia do not translate into more biased coefficient estimates in the previous exercises, in this section I again decompose  $\hat{\beta}_1$  into the volatility of the risk premium and that of the expected depreciation rate, as well as the covariance between these two series.

	$\widehat{\beta_1}$	$\sigma(rp)$	$\sigma(E(\Delta e'))$	$cov(rp; E(\Delta e'))$
$D^b = 5$	0.0882	0.0021	0.0101	0.0027
$D^b = 10$	0.1195	0.0022	0.0101	0.0029
$D^b = 20$	0.1282	0.0022	0.0101	0.003
$N^b = 0.9$	0.066	0.0022	0.0153	0.003
$N^b = 0.88$	0.0746	0.0033	0.0206	0.0044
$N^b = 0.86$	0.065	0.0044	0.0259	0.0056

Table 7. Decomposition of the Slope Coefficient ( $\sigma = 3$ )

Table 7 reports the results of this decomposition. Again, in both exercises, the model generates the following ordering:

$$var(E(\Delta e')) > cov(rp; E(\Delta e')) > var(rp).$$

Therefore, the ordering is the reverse of what it should be to explain the sign and the magnitude of the estimated slope coefficient. The model generates a larger coefficient estimate when the duration of idiosyncratic risks rises than when  $N^b$ increases because more persistent idiosyncratic shocks lead to a higher covariance term without significantly affecting either the volatility of the risk premium of that of the expected rate of depreciation. In contrast, these latter statistics increase as the number of agents receiving a low endowment shock in bad times rises, leaving the estimated slope coefficient relatively unchanged.

### 5. Conclusion

This paper investigated the effects of incomplete markets and borrowing constraints on the foreign exchange risk premium. It provided an upper bound on the potential of uninsurable idiosyncratic risk to account for the volatility of the risk premium and on the predictability of excess returns in the data. With respect to the properties of the risk premium, these features were shown to be important: the variability of the risk premium increases drastically compared to that in a complete-markets framework. In particular, the model can generate relatively high-risk premium volatility, to about 30 percent of that in the data. Another interesting aspect of the model is that, contrary to models incorporating habit persistence, the increase in the variance of the risk premium does not come at the cost of generating negatively autocorrelated forward premiums. This is important since forward premiums display high serial correlation. The paper, moreover, showed that the cases in which the risk premium is more volatile also display markedly higher market price of risk. However, the model still failed the test proposed by Hansen and Jagannathan (1991).

Notwithstanding this significant increase in the volatility of the risk premium, the model is unable to account for the predictability of excess returns in the data. The introduction of uninsurable idiosyncratic risk was shown to also increase the covariance between the risk premium and the expected depreciation rate, cancelling out the effects of the higher volatility of the risk premium on the predictability of excess returns. Successful models of the risk premium will need mechanisms that can simultaneously raise the volatility of the risk premium while keeping the covariance relatively constant.

One limitation of the current approach is that the no-borrowing constraint was not endogenously derived in the model. This is a simplifying assumption which nevertheless remains at odds with how foreign exchange markets actually work. An alternative approach would be to study the properties of the foreign exchange risk premium in models that endogenously derive the credit constraints. Some recent works on the subject showed that deriving the constraints endogenously has important effects on the properties of dynamic general equilibrium models and can help improve the match between the models and the data. For instance, Alvarez and Jermann (2000) showed that, in a closed-economy context calibrated to US data, a model in which credit constraints are endogenously determined to deter agents from defaulting can generate empirically plausible equity premia and Sharpe ratios. In the open-economy literature, Kehoe and Perri (2000) present a relatively similar framework in which countries face potential exclusions from future access to capital markets if they decide to default on their debt. They show that when the credit constraints are so determined, an otherwise standard open-economy business cycle model generates more plausible correlation between macroeconomic variables across countries. The borrowing constraints could also be modeled following the work of Bernanke and Gertler (1989), Kiyotaki and More (1997) or that of Aiyagari and Gertler (1999). For instance, Mendoza (2000) follows this route to study the implications of dollarization for an economy facing credit-market imperfections. Studying the properties of risk premia and the forward discount puzzle in such frameworks offers a promising avenue for future research.

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