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EXCHANGE RATES AND MONETARY POLICY REGIMES IN CANADA AND THE U.S.

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ABSTRACT This paper examines monetary regime switching in Canada and the United States and the implications of regime switching for exchange rates and key nominal and real macroeconomic aggregates for the two countries. Evidence of Markov regime switching in the process governing monetary base growth and in the bilateral exchange rate between the two countries is presented. Given the evidence, a two-country general equilibrium monetary model is constructed to account for observed properties of the U.S.-Canadian dollar exchange rate and for measured effects of monetary policy on key variables. Agents in the model face a monetary policy process with regime switching and form beliefs about regimes and money growth using observations and Bayesian learning. With the driving process for money growth rates parameterized using estimates from U.S. and Canadian data, quantitative implications of the model for behaviors of exchange rates and other key variables are examined. The findings are that inclusion of learning by agents contributes somewhat to the model's ability to account for persistence in effects of money shocks on variables, provided that the shocks themselves are persistent; inclusion of learning contributes little in accounting for business cycle fluctuations and exchange rate variability; inclusion of a nonlinear driving process for money growth rates is important for the model to account for long swings in exchanges rates; inclusion of learning adds only slightly to the ability of the model to account for long swings. The importance of nonlinearities in the driving process and the relative lack of importance of learning are consistent with other findings in the literature of learning effects in the face of regime switches.

1 Introduction

This paper examines monetary regime switching in Canada and the United States and the implications of regime switching for exchange rates and key nominal and real macroeconomic aggregates for the two countries. There is, of course, ample anecdotal evidence of monetary policy switches in these two countries. For example, following historically high inflation rates in the 1970s, the Bank of Canada announced a policy of gradualism in 1975: it would target M1 growth with gradually declining target ranges. By mid-1981, in the face of accelerating inflation, gradualism was dropped. It was followed by a policy with explicit inflation targets and increased emphasis on price stability as the appropriate target of monetary policy. In the U.S., an often cited shift in policy was the change in the operational instrument of policy, from 1979 through the early 1980s, from short-term interest rates to a narrow monetary reserve aggregate. While these and numerous other examples of changes in the objectives or instruments of policy in Canada and the U.S. can be listed, we consider measures of monetary regimes based on outcomes for growth rates of narrow monetary aggregates, rather than announced changes in operations or objectives. Using these measures, we test for the existence of regime switches in money growth rates in Canada and the U.S. Because of the strong trade links between the two countries, we also consider the behavior of exchange rates, as well as other key macroeconomic variables.

We find evidence of Markov regime switching in the process governing monetary base growth and in the bilateral exchange rate between the two countries. In particular, we estimate Markov regime-switching processes for monetary base growth rates and for exchange rates using Hamilton's (1989) maximum likelihood procedure applied to Canada and the U.S. We find that a money growth process with Markov regime switching between episodes of high and low average money growth rates is a better characterization than a simple single-mean linear driving process. The exchange rate between the two countries also displays evidence of regime switching, similar to the findings of Engel and Hamilton (1990) for bilateral exchange rates between the U.S. and Germany, France, and the U.K.

Given evidence of exchange rate and monetary regime switching, we construct a two-country general equilibrium monetary model to account for observed properties of the U.S.-Canadian dollar exchange rate and for measured effects of monetary policy on key nominal and real variables. The model incorporates a monetary policy process with regimes shifting according to a Markov transition law. Actual money growth in any period depends both on the regime of a country's monetary authority and a monetary control error. In any given period, agents cannot directly observe the regime from which money growth rates are drawn. Instead, beliefs are formed about the regime. These beliefs are rational expectations of the money growth process. They are formed from observed money growth rates and are updated using Bayesian learning. Our use of regime switching and Bayesian learning follows Andolfatto and Gomme (1997), who analyzed a closed economy model of the Canadian economy, and by Moran (1997, 1998), who traces out welfare costs of disinflation in models with Bayesian learning.

With driving processes for money growth rates parameterized from regime-switching estimates from the actual U.S. and Canadian economies, we examine quantitative implications of the model economy for behaviors of exchange rates and key nominal and real variables. In particular, we ask whether inclusion of learning by agents in the model in the face of monetary regime switching helps account for observed persistent effects of monetary innovations on exchange rates and other nominal and real variables; whether the model can account for regime switches, or "long swings," observed in the actual exchange rate series; and whether the model can account for business cycle fluctuations or second-moment properties of key variables.

We find that including learning by agents contributes somewhat to the model's ability to account for persistence in effects of money shocks on variables, provided that the shocks themselves are persistent, consistent with findings by Andolfatto and Gomme and by Moran. Including learning contributes little in accounting for business cycle fluctuations and exchange rate variability, however. We also find that the nature of the driving process for money growth rates is important for the model to account for long swings in exchange rates. When money growth rates display regime switching so, too, does the nominal exchange rate in the model. Absent regime switching in money growth, exchange rates do not display long swings. Learning, however, provides only a slight improvement in the ability of the model to generate exchange rate series characterized by long swings. The importance of nonlinearities in the driving process and the relative lack of importance of learning are consistent with recent findings of learning effects in the face of regime switches by Evans and Lewis (1995) and Kaminsky (1993).

The Bayesian learning mechanism in our model is employed to allow for inertia in updating expectations about money growth rates and inflation rates. This inertia can, in principle, be helpful in explaining observed persistence in the effects of monetary innovations on, among other variables, interest and exchange rates, and in explaining exchange rate variability. With uncertainty about monetary regimes, agents' inflation expectations and decisions that depend on those expectations take longer to adapt to transitory money shocks and actual regime shifts. In addition, the greater the uncertainty over which monetary regime is in place, the more expectations can fluctuate when subjected to sequences of money shocks.

Learning about monetary regimes has been used elsewhere to address features of exchange rate behavior other than persistence and variability, as the survey paper by Lewis (1995) and references therein clearly articulate. In contrast to existing work, we construct a general equilibrium model and quantitatively assess its performance under parameterizations consistent with empirical observations Also in contrast to existing international equilibrium models that allow monetary nonneutralities, inertia in the effects of monetary innovations arises endogenously from Bayesian learners rationally updating beliefs about monetary policy in the face of regime switches. While there is inertia, or a rigidity, in adjustments of expectations in the model, expectations are nonetheless rational. The endogenous rigidity of expectations that we analyze provides an alternative to the recent wave of open economy sticky price models, where nonneutrality stems from exogenous price rigidities, as in Obstfeld and Rogoff (1995, 1998), Chari, Kehoe, and McGrattan (1996), and Betts and Devereux (1998).

This paper proceeds as follows. Section 2 presents statistical properties of data drawn from the U.S. and Canadian economies, against which quantitative implications of the model that we construct can be compared. Section 3 presents an open economy general equilibrium model in which agents form beliefs about monetary policy. Quantitative properties of a parameterized version of the model are examined in section 4. Section 5 concludes.

2 Statistical Properties of International Data

2.1 Persistent Effects of Monetary Innovations and Volatilities of Variables

Table 1 presents some of the international data features against which we will evaluate the models developed in subsequent sections. Moments in the table are of logarithms of Hodrick-Prescott filtered, quarterly observations of exchange rates and outputs for Canada and the U.S. for various sample periods. The nominal exchange rate is the bilateral exchange rate for Canada vis-à-vis the U.S. dollar. Real exchange rates are measured using nominal exchange rates and the consumer price indexes of the two countries. The data are from the OECD's Quarterly National Accounts and the International Monetary Fund's

International Financial Statistics.

Table 1				
		St. Dev. (%)		
Sample Period	*Nom. FX*	Re*al FX*	*Can.* Re*al GDP*	*U.S.* Re*al GDP*
1962 : 2– 1996 : 2	2.520	2.696	1.526	1.670
1962 : 2– 1972 : 4	0.936	0.983	1.068	1.261
1973 : 1– 1979 : 4	2.629	2.583	0.636	1.825
1980 : 1– –1987 : 4	2.236	2.702	1.895	1.814
1988 : 1– 1996 : 2	2.948	3.371	1.349	0.976
		AR(1)		
1962 : 2– 1996 : 2	0.866	0.868	0.839	0.863
1962 : 2– 1972 : 4	0.722	0.612	0.542	0.756
1973 : 1– 1979 : 4	0.807	0.800	0.454	0.806
1980 : 1– –1987 : 4	0.737	0.815	0.833	0.781
1988 : 1– 1996 : 2	0.780	0.805	0.901	0.816

The empirical regularities evident in Table 1 are that exchange rates, nominal and real, are more volatile than the output measures and display persistent movements. High persistence in exchange rate movements is indicated by the first-order autocorrelation coefficients of .87 for the full sample period 1962:2-1996:2. On the quantity side, for the full sample period, the first-order autocorrelations are .86 for U.S. GDP and .84 for Canadian GDP. For the full sample period, the standard deviations of nominal and real exchange rates are over one and a half times higher than the standard deviations of the output measures. In the 1980:1-1987:4 period, during which the U.S. dollar experienced large appreciations and subsequent depreciations against many currencies, there does not appear to be significantly greater variability in exchange rates between Canada and the U.S. than in other periods in the flexible exchange rate era. As in those other periods, the exchange rate is more variable than the output measures.

One quantitative issue to be addressed is whether movements in exchange rates drawn from simulations of our model are as persistent and highly volatile as in the actual data. A second issue is whether the dynamic responses of exchange rates, interest rates, and real variables to monetary shocks implied by our model correspond to impulse responses in actual data. Recent studies by Schlagenhauf and Wrase (1995a, 1995b) and Eichenbaum and Evans (1992) consider effects of monetary policy shocks on international variables using estimated vector autoregression (VAR) data representations. There are four key features of the empirical findings for the bilateral pairing of the U.S. and Canada: (i) A negative shock to U.S. monetary policy (positive shock to the federal funds rate) is associated with persistent nominal and real depreciations of the Canadian dollar vis-à-vis the U.S. dollar; (ii) Responses of Canadian and U.S. interest rates, outputs, and nominal and real exchange rates to a U.S. monetary policy shock all persist for many quarters beyond the period of the shock; (iii) A negative U.S. monetary shock is associated with impact increases in short-term U.S. interest rates and smaller increases in short-term Canadian interest rates, which implies a widening of the U.S.-Canada interest rate differentials; and (iv) A negative U.S. monetary shock is associated with an impact increase in U.S. output, increased U.S. output for a few quarters after the shock, and subsequent output decreases. Canadian output responses to U.S. money shocks are similar, although slightly smaller than the U.S. output responses. Consequently, when money growth falls in the U.S. relative to money growth in Canada, results (i) through (iv) arise in actual data.

We consider whether a parameterized version of the model we construct can account for contemporaneous nominal and real exchange rate, interest rate, and interest-rate-differential responses to shocks that alter the relative growth rates of money in Canada and the U.S. We also consider whether agents' beliefs about monetary policy can help account for effects of money shocks on exchange rates, interest rates, and output across the two countries that persist well beyond the period of the shock.[1]

Measuring effects of money shocks on variables in the model that we construct requires that we specify a process governing money growth rates and random shocks to money growth and that we assign values to parameters of that process. To impose discipline on the parameterization, we consider next empirical properties of the process governing the joint behavior of actual money growth rates in the Canadian and U.S. economies.

2.2 Characterization of the Money Growth Process

The data used to measure money growth are monetary base measures, adjusted for seasonals and for changes in reserve requirements, obtained from statistical releases of the U.S. Federal Reserve Board and of the Bank of Canada. Growth rates of the monetary base for each country are shown in Figure 1. To characterize the behavior of the process governing money growth in Canada and the U.S., begin by assuming that the data are best generated by a two-variable VAR representation.

$$\left(\begin{pmatrix} X_t^{U.S.} \\ X_t^{Can.} \end{pmatrix} - \begin{pmatrix} \bar{X}_t^{U.S.} \\ \bar{X}_t^{Can.} \end{pmatrix}\right) = \Psi \cdot \left(\begin{pmatrix} X_{t-1}^{U.S.} \\ X_{t-1}^{Can.} \end{pmatrix} - \begin{pmatrix} \bar{X}_{t-1}^{U.S.} \\ \bar{X}_{t-1}^{Can.} \end{pmatrix}\right) + \begin{pmatrix} e_t^{U.S.} \\ e_t^{Can.} \end{pmatrix}$$

where $\begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix} \in \left\{ \begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_L , \begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_H \right\}$ with regime switching,

or $\begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix} = \{\text{constant for all } t\}$ without switching.

Ψ is an autoregression matrix. Vectors $\begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_L$ and $\begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_H$ represent the long-term rate of money expansion in the two countries in each possible regime, which we denote by subscript L for low growth in the two-country system or H for high growth. The long-term money expansion rate depends on which monetary policy regime is in place. When switching is present, monetary policy regimes switch over time according to the transition law:

$$p_{ij} = Pr\left[\begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_t = \begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_j \mid \begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_{t-1} = \begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_i \right]$$
$$i, j \in \{L, H\}.$$

The sense in which $\begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_t$ is a long-term vector of money growth rates is captured by transition probabilities $p_{L,L}$ and $p_{H,H}$ being close to one. Innovations $e_t^{U.S.}$ and $e_t^{Can.}$ represent serially independent monetary control errors. The vector of control errors is drawn from normal distribution functions $N(0, \sigma_a^2)$ for regimes $a = L, H$ if the control error variabilities are regime dependent, and from distribution function $N(0, \sigma^2)$ if the variability is not regime dependent.

Given the possible representations above, we perform two tests. The first is a test to determine whether the data are best modeled by a VAR with a single mean money-growth process and with single-variance monetary control errors, or by one that has a Markov-switching mean vector along with single-variance control errors. The second test uses the preferred specification for mean money growth from the first test and goes on to test whether the data are best represented by a model with or without regime-dependent variabilities of the monetary control errors.

The first test is a likelihood ratio test of whether the driving process with regime switches in mean money growth rates, but not in monetary control error variances, represents the data better than the process without regime dependence. The log likelihood for the VAR without regime dependence is -326.99 and -317.32 for the VAR with regime switching in mean money growth. The likelihood ratio is therefore 19.36. Since the model without regime dependence places two restrictions on the specification relative to the mean-switching model, the chi-squared test is for two degrees of freedom. The null that the restrictions on mean money growth rates do not matter is easily rejected.

To further examine the possibilities of nonlinearities in the money growth process, we proceed next to maintain a Markov switching process in mean money growth rates and test whether the data are best represented with or without regime switching also in the covariance matrix of monetary control errors. The log likelihood for the model with a switching matrix is -314.92 and that for the model with a constant matrix is -317.32. The likelihood ratio is therefore 4.8. Since the model with a constant covariance matrix imposes three restrictions relative to the switching model, the chi-squared test is for three degrees of freedom. Given the chi-squared critical value for three degrees of freedom at a 5 percent confidence interval of 7.81, we cannot reject the null that the restrictions on the covariance matrix do not matter. The data therefore favor a constant covariance matrix model over a model with monetary control error variance that are state dependent.

Results of the two tests of the process governing Canadian and U.S. money growth point to a representation with switching in average, or long-term, growth rates in the two-country process, and with variabilities of monetary control errors that are not regime dependent. Estimates of the preferred model are in

Table 2.

Table 2	
Parameter	Estimates (standard errors in parentheses)
$\begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_H$	$\begin{pmatrix} 1.7656 \\ (.4943) \\ 2.6279 \\ (.4175 \end{pmatrix}$
$\begin{pmatrix} \bar{X}^{U.S.} \\ \bar{X}^{Can.} \end{pmatrix}_L$	$\begin{pmatrix} 1.1365 \\ (.3364) \\ 0.7918 \\ (.2454) \end{pmatrix}$
p_{HH}	.9687
p_{LL}	.9892
Ψ	$\begin{pmatrix} .7398 & .0246 \\ -.0269 & .2311 \end{pmatrix}$
V	$\begin{pmatrix} .2335 & -.0057 \\ -.0057 & 1.0188 \end{pmatrix}$

According to the estimates, the joint process governing Canadian and U.S. money growth rates is either in a "low" growth state or "high" growth state. The maximum likelihood estimates associate the low growth state with average quarterly monetary base growth of 1.14 percent in the U.S. and .79 percent in Canada, and the high growth state with average quarterly growth of 1.77 percent in the U.S. and 2.63 in Canada. The regimes that jointly characterize money growth evolution in the two countries are quite persistent. The estimates of p_{HH} and p_{LL} indicate that if the system is either in the high or low growth state, it is likely to remain in that state.

The upper portion of Figure 2 reproduces per capita growth rates of the monetary base for Canada and the U.S. over the sample period. The lower portion of the figure plots the smoothed probability that the process was in the high growth regime at each date in the sample. This inference, discussed in Hamilton (1994), uses the full sample of observations and the maximum-likelihood estimates of the regime-switching process (regime specific means, variability of the control errors, and transition probabilities) to draw an inference for each date t about the state of the process. The dates at which an econometrician would conclude that the process had switched between regimes (based on $pr(state = H \mid full\ sample\ of\ observations) \begin{smallmatrix}>\\<\end{smallmatrix} 0.5$) can be seen by tracing down from where the horizontal line at 0.5 probability crosses the smoothed inference probability. There are two clear regime switches in the figure. One, in the middle of 1971, occurs near the demise of the Bretton-Woods era. The other, in mid 1981, appears at the time that the Bank of Canada dropped its policy of gradualism, introduced in 1975, while the Federal Reserve was in the midst of a change in operating procedures.

2.3 Characterization of Exchange Rates

Given the evidence of regime switching in money growth rates in Canada and the U.S., it is natural to ask whether growth rates of other nominal variables in the two countries show evidence of regime switching. Because of our interest in the exchange rate effects of monetary policy, and in light of Engel and Hamilton's (1990) evidence of regime shifting for bilateral exchange rates between the U.S. and Germany, France, and the U.K., it is of interest to determine whether exchange rates exhibit evidence of regime switching. To explore this, we apply the two tests described above on the log first difference of the nominal exchange rate between Canada and the U.S. In particular, we first test to determine whether a process with regime switching in the mean appreciation of the nominal exchange rate is a better representation than a process without switching. Using the results from that test to characterize the mean, we then test whether the data-generating process is best characterized by a constant variance or by regime-dependent variances. The estimation results indicate that a process with regime switching in the mean and variance better describes the nominal exchange rate than a process without regime switches.

Estimates of the preferred model of the rate of nominal appreciation of the Canadian dollar relative to the U.S. dollar are shown in Table 3, where s_t denotes Canadian dollars per U.S. dollar in levels, and $R_1$and R_2 are used to denote the two regimes uncovered by the estimation procedure.

Table 3	
Parameter	Estimates (standard errors in parentheses)
$(\ln s_t - \ln s_{t-1})_{R_1}$	$-.0002$ (.0008)
$(\ln s_t - \ln s_{t-1})_{R_2}$	.0032 (.0019)
$p_{R_1R_1}$	.9576
$p_{R_2R_2}$	.9886
$AR(1)$	.2223 (.0748)
$Variance_{R_1}$	.00001
$Variance_{R_2}$	.00024

According to the estimates, the process governing the nominal exchange rate between the Canadian and U.S. dollar is either in a state of no change (very mild appreciation) of the Canadian dollar, state "R_1," or is in a state of depreciation of the Canadian dollar at a quarterly rate of about 0.3 percent, state "R_2." The upper portion of Figure 3 plots the nominal exchange rate, in Canadian dollars per U.S. dollar, over the sample period. The lower portion of the figure plots the smoothed probability that the process was in the state of mild Canadian dollar appreciation.

The estimates in the table show that the regimes that characterize the nominal exchange rate process are quite persistent. Inspection of the figure shows that by our estimates, switches between regimes are infrequent. In our sample period, the Canadian dollar began in a depreciation state, R_2, as we would expect on association with the sharp depreciation of the flexible rate associated with the Coyne affair. As we also expect, the Bayesian belief structure captures the fixed rate years between 1962 and 1970 as a period with little to no expected movements in the exchange rate. Moving into the flexible exchange rate era, our estimates identify five switches in expectations about the state of the nominal exchange rate, from little to no expected change where a high probability is assigned to state R_1, to an expected depreciation of the Canadian dollar where a low probability is assigned to state R_1. Periods during which the Canadian dollar is expected to depreciate are from late 1974 until late 1975; from late 1976 to early 1979; from late 1981 to the middle of 1982; from late 1983 to early 1986; and from late 1991 to the middle of 1994. These episodes, characterizing what Engel and Hamilton refer to as "long swings" in exchange rates, are what one might be led to believe from casual inspection of the actual Canadian-U.S. dollar exchange rate data in the upper portion of the figure.

Given the high coherence between nominal and real exchange rates, we also find evidence of regime switching behavior in the real exchange rate, measured by the nominal Canadian-U.S. exchange rate multiplied by the ratio of the U.S. to Canadian consumer price indexes. In particular, we subjected the real exchange rate series to the same sequence of tests for regime switching in average appreciation and for regime-specific variability as performed on the nominal exchange rate series. The data support a specification of the real exchange rate process with regime switching, or long swings, in average real Canadian appreciation and with constant variance across regimes. Using re to denote the real exchange rate, in Canadian goods per unit of U.S. goods, the maximum likelihood estimates of the preferred specification are shown in Table 4.

Table 4	
Parameter	Estimates (standard errors)
$(\ln re_t - \ln re_{t-1})_{R_1}$	$-.0036$ (.0015)
$(\ln re_t - \ln re_{t-1})_{R_2}$	.0156 (.0029)
$p_{R_1 R_1}$	.9568
$p_{R_2 R_2}$	.8798
$AR(1)$	.0038 (.0895)
$Variance$	.00016

According to the estimates, the real exchange rate is in a state of no change (mild real appreciation) in the Canadian dollar, state "R_1," or is in a state of real depreciation at a quarterly rate of about 1.6 percent, state "R_2." Estimates of the transition probabilities also identify that the states are persistent.

3 An Open Economy Monetary Model with Learning

This section presents a simple open economy model and assesses its ability to account for features of the data from the Canadian and U.S. economies that we have just reviewed. To summarize, there are six important data features against which the model will be compared: (1) A temporary shock that lowers U.S. (Canadian) money growth relative to Canadian (U.S.) money growth is associated with persistent nominal and real depreciations (appreciations) of the Canadian dollar vis-à-vis the U.S. dollar; (2) Responses of U.S. and Canadian interest rates, outputs, and nominal and real exchange rates to a temporary monetary policy shock all persist for many quarters beyond the period of the shock; (3) A temporary shock that lowers U.S. (Canadian) relative to Canadian (U.S.) money growth is associated with impact increases in short-term interest rates and a widening of the U.S.-Canada (Canada-U.S.) interest rate differential; (4) Monetary shocks have real effects on Canadian and U.S. output that persist beyond the period of the shock; (5) Nominal and real exchange rates are more volatile than Canadian and U.S. real output levels; and (6) The exchange rate, nominal and real, between Canada and the U.S. displays long swings in the form of regime switching.

The model we construct assumes that agents potentially face a monetary policy that exhibits regime switching. Agents in the model form rational expectations about money growth innovations and about the monetary regime in place in any given period. Our interests in data drawn from simulations of the model economy are twofold. First, we are interested in whether the ability of the model to account for the data features listed above is enhanced by inclusion of a driving process for money growth, estimated from actual data, that exhibits regime switching. Second, we are interested in whether the model's ability to account for the data features is enhanced by inclusion of beliefs formed in the face of uncertainty about monetary regimes as opposed to expectations formed under certainty about the regime in place in every period.

An important modelling choice, especially when modelling effects of monetary innovations, is the degree of wage and price rigidity to assume. Because we wish to focus on effects in a model of allowing for adaptive expectations from rational agents who update beliefs, we assume that wages and prices are perfectly flexible. This helps focus on, for example, any extra persistence in effects of money shocks on key variables arising from the belief structure and regime uncertainty, as opposed to, say, overlapping wage or price contracts. Because we require monetary nonneutralities to account for the real effects of monetary innovations, however, some sort of rigidity must be assumed.

To minimize the length of time over which the rigidity is assumed to bind on agents, we consider a simple open economy "liquidity," or differential-participation model. In the model, innovations in monetary policy are nonneutral because of differential participation by different agent types in absorbing monetary injections. The limited participation feature arises given that households make portfolio decisions for a period prior to observing money shocks in the period. With portfolio decisions that are determined prior to a period's money shock, a monetary injection in financial markets leaves financial intermediaries in the model flush with liquidity. To induce firms that borrow to finance input acquisitions to absorb funds, downward "liquidity effect" pressure is placed on the nominal interest rate. If this liquidity effect pressure is sufficiently strong to outweigh the standard Fisherian anticipated inflation effect, a positive monetary innovation gives rise to lower nominal interest rates. Since lower nominal rates reduce firms' borrowing costs, more inputs are acquired and real activity expands. After the period of a money shock, however, agents are able to rearrange their portfolios.

There are two countries in the model, Canada and the U.S., which we generically label domestic and foreign, linked by trade in goods and currencies. A multi-member household inhabits each country. Households consist of shoppers, firm managers, workers, and financial intermediaries. Each household member has distinct tasks to perform during each period in markets for goods, labor, and currencies. At the end of each period, all household members reunite to pool resources, and all of a country's per-household wealth consequently resides with a representative household.

3.1 Trading Opportunities

The trading opportunities, objectives, and constraints of households are isomorphic across countries. For brevity, we provide details for the representative domestic household's decisions and opportunities only. The foreign analogs are straightforward and involve obvious notation alterations. The representative domestic household begins period t with K_t^D units of capital and A_t^D units of domestic currency carried forward from the previous period.[2]

At the beginning of period t, the household divides nominal wealth A_t^D by sending a deposit of N_t^D domestic currency units with its financial intermediary member to the domestic financial market. The remaining $A_t^D - N_t^D$ is allocated to trade in the currency exchange market. In the exchange market, domestic and foreign households trade currencies to arrange balances for use in purchasing consumption goods.

Domestic currency available to the domestic household in the exchange market consists of $A_t^D - N_t^D$, from the initial wealth allocation, along with wage receipts of the household worker. The worker supplies $\tilde{H}_t^D$ labor units in the domestic labor market at nominal wage W_t^D. In the foreign exchange market, $A_t^D - N_t^D + \alpha_1 W_t^D \tilde{H}_t^D$ units of domestic currency are divided into a domestic currency balance, $M_{D,t}^D$, and a foreign currency balance, $M_{F,t}^D$, at nominal exchange rate e_t (expressed in domestic per foreign currency units). Note that we allow for some of the household worker's wage receipts, $\alpha_1 W_t^D \tilde{H}_t^D$, with $0 \leq \alpha_1 \leq 1$, to be used in currency trades in the foreign exchange market. The household's nominal allocation in the foreign exchange market is:

$$A_t^D - N_t^D + \alpha_1 W_t^D \tilde{H}_t^D = M_{D,t}^D + e_t M_{F,t}^D \tag{1}$$

The household shopper purchases $C_{D,t}^D$ units of home-produced goods at price P_t^D, and $C_{F,t}^D$ units of foreign goods at price P_t^F, subject to the cash constraints:

$$P_t^D C_{D,t}^D \leq M_{D,t}^D \tag{2}$$

$$P_t^F C_{F,t}^D \leq M_{F,t}^D \tag{3}$$

When the constraints bind as equalities, the shopper returns home at the end of the period with goods, but no cash.[3]

The financial intermediary receives a monetary injection X_t^D in the financial market, which is deposited on behalf of the household. The intermediary then holds $N_t^D + X_t^D$ units of cash, which it lends to domestic firms. Loanable cash supplied by the intermediary is[4]:

$$\tilde{L}_t^D = N_t^D + X_t^D \tag{4}$$

The firm manager hires workers, undertakes investment, and holds the household's capital stock K_t^D. Prior to producing output, the firm borrows L_t^D domestic currency units from an intermediary to finance acquisition of H_t^D units of labor at wage W_t^D per unit and to finance capital accumulation, in the face of a cash constraint:

$$W_t^D H_t^D + \alpha_2 P_t^D I_t^D \leq L_t^D \tag{5}$$

The firm purchases $I_t^D = K_{t+1}^D - (1-\delta)K_t^D$ units of home-produced goods to add to the household's capital stock and finances fraction $\alpha_2 P_t^D I_t^D$, with $0 \leq \alpha_2 \leq 1$, using cash obtained by borrowing from an intermediary. Capital and consumption goods are indistinguishable in the domestic goods market and sell at common price P_t^D.

Determining the household's nominal wealth evolution requires accounting for currency brought home at the end of the period by household members. From (1) - (3), the shopper brings home goods but no cash when the constraints bind as equalities. The household worker brings home any wage receipts not allocated to the foreign exchange market, $(1-\alpha_1)W_t^D \tilde{H}_t^D$. The firm manager, after the close of trading in goods markets, pays loan obligation $R_{L,t}^D L_t^D$, where $R_{L,t}^D$ is the gross domestic loan rate. The manager brings home capital and cash profits of:

$$P_t^D Y_t^D - R_{L,t}^D L_t^D - (1-\alpha_2) P_t^D I_t^D \tag{6}$$

where Y_t^D is real output per domestic household.

The intermediary receives loan repayments $R_{L,t}^D \tilde{L}_t^D = R_{L,t}^D (N_t^D + X_t^D)$ and pays a gross deposit return $R_{D,t}^D (N_t^D + X_t^D)$. The intermediary returns home at the end of the period with its household's own deposit return, $R_{D,t}^D (N_t^D + X_t^D)$, plus cash derived from intermediation $R_{L,t}^D (N_t^D + X_t^D) - R_{D,t}^D (N_t^D + X_t^D)$. Thus, the intermediary brings home a cash balance of:

$$R_{L,t}^D (N_t^D + X_t^D) \tag{7}$$

Combining cash brought home by the household firm manager in (6) and the intermediary in (7) and cash that the household worker did not send to the foreign exchange market gives the household's end-of-period nominal wealth:

$$\begin{aligned} A_{t+1}^D &= P_t^D Y_t^D - R_{L,t}^D L_t^D - (1-\alpha_2) P_t^D I_t^D + \\ & R_{L,t}^D (N_t^D + X_t^D) + (1-\alpha_1) W^D \tilde{H}_t^D \end{aligned} \tag{8}$$

3.2 Preferences, Technology, and Shocks

The household maximizes utility measure:

$$U = E_t \sum_{j=0}^{\infty} (\beta_D)^{t+j} \mu(C_{t+j}^D, l_{t+j}^D) \tag{9}$$

with $0 < \beta_D < 1$. Domestic consumption of home-produced goods, $C_{D,t}^D$, and foreign-produced goods, $C_{F,t}^D$, is aggregated according to a simple Cobb-Douglas aggregator:

$$C_t^D = (C_{D,t}^D)^{\upsilon} (C_{F,t}^D)^{1-\nu} \tag{10}$$

and momentary utility takes the form:

$$\mu(C_t^D, 1 - \tilde{H}_t^D) = \frac{1}{\rho^D} \left\{ (C_t^D)^{\gamma} (l_t^D)^{1-\gamma} \right\}^{\rho^D} \tag{11}$$

Leisure is $l_t^D = 1 - \tilde{H}_t^D$, with the time endowment normalized to unity. Foreign utility is the same as (9)-(11) except for obvious notation alterations.

For output production, each domestic firm possesses the technology:

$$Y_t^D = f^D(K_t^D, H_t^D, T^D) = (K_t^D)^{\alpha^D} (T^D H_t^D)^{1-\alpha^D} \ , T^D = \exp(\theta_t^D), \tag{12}$$

where $0 < \alpha^D < 1$, and T^D represents a random productivity innovation. Foreign firms' technologies are the same as above except for notation.

Shocks to labor productivities evolve according to the bivariate autoregression:

$$\begin{bmatrix} \theta_t^D \\ \theta_t^F \end{bmatrix} = \begin{bmatrix} T_{DD} & T_{DF} \\ T_{FD} & T_{FF} \end{bmatrix} \begin{bmatrix} \theta_{t-1}^D \\ \theta_{t-1}^F \end{bmatrix} + \begin{bmatrix} \varepsilon_{D,t} \\ \varepsilon_{F,t} \end{bmatrix}.$$

Innovations $\varepsilon_{D,t}$ and $\varepsilon_{D,t}$ are serially independent, with covariance matrix

$$V_{\varepsilon} = \begin{bmatrix} V_{\varepsilon^D \varepsilon^D} & V_{\varepsilon^D \varepsilon^F} \\ V_{\varepsilon^F \varepsilon^D} & V_{\varepsilon^F \varepsilon^F} \end{bmatrix}.$$

Monetary injections in the model are $X_t^D = M_{s,t+1}^D - M_{s,t}^D$ and $X_t^F = M_{s,t+1}^F - M_{s,t}^F$, where $M_{s,t}^D$ and $M_{s,t}^F$ are per own-country-household stocks of domestic and foreign currencies. The exogenous money growth rates $\chi_t^D = \frac{X_t^D}{M_{s,t}^D}$ and $\chi_t^F = \frac{X_t^F}{M_{s,t}^F}$ depend on the monetary regimes generating monetary policy. We now turn to the nature of monetary policy.

3.3 Monetary Policy and Beliefs

The autoregressive process governing domestic and foreign money growth, following that used in the estimation procedure above, is assumed to be:

$$\left(\begin{pmatrix} X_t^D \\ X_t^F \end{pmatrix} - \begin{pmatrix} \bar{X}_t^D \\ \bar{X}_t^F \end{pmatrix}\right) = \Psi \cdot \left(\begin{pmatrix} X_{t-1}^D \\ X_{t-1}^F \end{pmatrix} - \begin{pmatrix} \bar{X}_{t-1}^D \\ \bar{X}_{t-1}^F \end{pmatrix}\right) + \begin{pmatrix} e_t^D \\ e_t^F \end{pmatrix} \tag{13}$$

with

$$\begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix} \in \left\{ \begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix}_L, \begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix}_H \right\}$$

and where Ψ is an autoregression matrix, and V is a covariance matrix. $\begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix}_L$ and $\begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix}_H$ represent the long-term rate of money expansion in the two countries in each possible regime, which we denote by subscript L for low growth in the two-country system or H for high growth. The long-term money expansion rate depends on which monetary policy regime is in place. Monetary policy regimes switch over time according to the transition law:

$$p_{ij} = Pr\left[\begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix}_t = \begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix}_j \mid \begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix}_{t-1} = \begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix}_i\right]$$
$$i, j \in \{L, H\}.$$

Agents in the economy know the parameters of the transition laws. The vector of serially-independent control errors is drawn from regime-dependent normal distribution functions $N(0, \sigma_a^2)$ for regimes $a = L, H$. Innovations e_t^D and e_t^F are independent of labor productivity innovations.

We assume that agents cannot directly observe the current and past monetary policy regimes. Instead, they form beliefs based on the known parameters of the process governing monetary growth rates and on current and past observed actual money growth rates. Let b_t denote the belief that the current regime is characterized by low money growth. Thus, $b_t = Prob\left\{ \begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix}_t = \begin{pmatrix} \bar{X}^D \\ \bar{X}^F \end{pmatrix}_L \right\}$. Beliefs are updated rationally using the Bayesian recursion:

$$b_t = \frac{g_l(b_{t-1}, X_t)}{g_l(\cdot) + g_h(\cdot)}$$

with

$$g_l = b_{t-1} * p_{ll} * f_{ll}(\epsilon_t^{ll}) + (1 - b_{t-1}) * p_{hl} * f_{hl}(\epsilon_t^{hl})$$

$$g_h = b_{t-1} * p_{lh} * f_{lh}(\epsilon_t^{lh}) + (1 - b_{t-1}) * p_{hh} * f_{hh}(\epsilon_t^{hh})$$

where ϵ_t^{ij} is the innovation vector implied by the money growth process under the assumption that the regime was i last period and is j this period and $f_{ij}(\cdot)$ is the normal pdf for ϵ_t^{ij}.

Given this belief structure, money growth expectations and inflation expectations could adjust sluggishly, depending on parameter values. For example, if agents have been operating in a high money growth regime and there is suddenly a switch to the low growth regime, it may take a long string of relatively low money growth observations to appreciably alter the probability agents assign to actually being in the low growth regime. Such sluggishness in expectations in the face of changes in policy regimes will influence the economic outcomes of policies such as a disinflation planned by a less-than-fully-credible monetary authority. With sluggish expectations, it may take a prolonged period for variables to fully adjust to a planned disinflation. The extent to which beliefs influence the economic outcomes of changes in money growth rates is considered in our quantitative evaluation of the model.

3.4 The Economy's State and Equilibrium

The state of the world economy in period t is characterized by values for $M_{s,t}^D, M_{s,t}^F, \kappa_t^D, \kappa_t^F, A_t^D, A_t^F, K_t^D, K_t^F, b_t$, and, S_t. $M_{s,t}^D(M_{s,t}^F)$ and $\kappa_t^D(\kappa_t^F)$ are per domestic (foreign) household money and capital stocks. $A_t^D(A_t^F)$ and $K_t^D(K_t^F)$ are the domestic (foreign) representative household's beginning currency and capital stocks. S_t denotes the vector of innovations to money growth and labor productivities in the home and foreign country while b_t denotes a vector of belief probabilities over the two states of global monetary policy.

An equilibrium involves state-contingent prices, wages, interest and exchange rates, and optimal household decision rules satisfying market clearing and aggregate consistency conditions. Market clearing conditions are: $\tilde{H}_t^D = H_t^D$, $\tilde{H}_t^F = H_t^F$ for labor; $Y_t^D = C_{D,t}^D + C_{D,t}^F + I_t^D$, $Y_t^F = C_{F,t}^F + C_{F,t}^D + I_t^F$ for goods; $\tilde{L}_t^D = L_t^D$, $\tilde{L}_t^F = L_t^F$ for loans; and $A_t^D + X_t^D = M_{D,t}^D + M_{D,t}^F$, $A_t^F + X_t^F = M_{F,t}^F + M_{F,t}^D$ for foreign exchange. Aggregate consistency requires that $A_t^D = M_{S,t}^D$, $A_t^F = M_{S,t}^F$ for money stocks, and $K_t^D = \kappa_t^D$, $K_t^F = \kappa_t^F$ for capital stocks.

3.5 Household Decisions and Qualitative Results

Since the choice problem facing domestic and foreign households is of the same form, we focus on the domestic household's problem. The household maximizes utility measure (9) subject to trading opportunities and constraints in (1)-(8), technology (12) and the technology shock process, and money shock process (13).

Consider a case of full information, in which households and firms have full knowledge of all current-period shocks prior to making consumption and investment decisions. Let $V^D(A_t^D, K_t^D, S_t)$ be the value function corresponding to the domestic household's problem. $V(\cdot)$ satisfies the functional equation:

$$
\begin{aligned}
V^D(A_t^D, K_t^D, S_t) \quad &= \quad max_{(N_t^D, K_{t+1}^D, M_{F,t}^D, \tilde{H}_t^D, L_t^D)}\{\mu(C_t^D, 1 - \tilde{H}_t^D) + \\
&\qquad \beta_D \int V^D(A_{t+1}^D, K_{t+1}^D, S_{t+1})\Phi(S_{t+1} \mid S_t)\}
\end{aligned}
$$

A^D_{t+1} is given by wealth evolution (8). Binding cash constraints in (2)-(3), and (5) are used to eliminate $C^D_{D,t}$, $C^D_{F,t}$, and H^D_t as separate decisions. Also, from the foreign exchange market allocation (1) we have $A^D_t - N^D_t + \alpha_1 W^D_t \tilde{H}^D_t = M^D_{D,t} + e_t M^D_{F,t}$. Consequently, choice of $M^D_{D,t}$ is implied by choices of N^D_t, $\tilde{H}^D_t$, and $M^D_{F,t}$, since A^D_t is predetermined and e_t and W^D_t are taken by the household. Optimality conditions for N^D_t, K^D_{t+1}, $M^D_{F,t}$, $\tilde{H}^D_t$, L^D_t are:

$$-\mu_{C^D_{D,t}} \frac{1}{P^D_t} + \beta_D \int \mu_{C^D_{D,t+1}} \frac{R^D_{L,t}}{P^D_{t+1}} \Phi(S_{t+1} \mid S_t) = 0 \tag{14}$$

$$-\int [\mu_{C^D_{D,t+1}} \frac{P^D_t}{P^D_{t+1}} + \beta_D \mu_{C^D_{D,t+2}} \frac{P^D_{t+1}}{P^D_{t+2}} \left\{ f^D_{K^D_{t+1}} + 1 - \delta_D \right\}] \Phi(S_{t+1} \mid S_t) = 0 \tag{15}$$

$$-\mu_{C^D_{D,t}} \frac{1}{P^D_t} + \mu_{C^D_{F,t}} \frac{1}{e_t P^F_t} = 0 \tag{16}$$

$$-\mu_{l^D_t} + \mu_{C^D_{D,t}} \alpha_2 \frac{W^D_t}{P^D_t} - (1-\alpha_1)\beta_D \int \frac{\mu_{C^D_{D,t+1}}}{P^D_{t+1}} \Phi(S_{t+1} \mid S_t) = 0 \tag{17}$$

$$f^D_{H^D_t} - \frac{W^D_t}{P^D_t} R^D_{L,t} = 0 \tag{18}$$

where $\mu_{l^D_t}$ is the period t marginal utility of leisure, and the period t marginal products of domestic labor and capital are denoted respectively by $f^D_{H^D_t}$ and $f^D_{K^D_t}$. Condition (14), derived from the deposit choice, relates the nominal interest rate, anticipated inflation, and the household's intertemporal marginal rate of substitution. Equation (15) governs the capital investment decision. Equation (16) is derived from decisions about consumption and the domestic and foreign currency balances to use in acquiring consumption goods. Equation (17), derived from the work effort supply choice, equates the real wage and intratemporal marginal rate of substitution between a consumption quantity and leisure. Equation (18), from the firm's loan demand decision, equates labor's marginal product and the real cost of an additional labor unit (real wage and interest cost of borrowing currency to hire labor). Beliefs enter into the Euler equations by way of the integration over the transition function $\Phi(S_{t+1} \mid S_t)$, i.e., by way of the expectations.

If agents make their cash allocation decisions after observing money shocks, nominal interest rates depend only on Fisherian fundamentals–the real rate and expected inflation. However, a positive shock to money growth in a country will increase expected inflation, and with a relatively small effect on the real rate, the nominal interest rate will rise. If the nominal interest rate rises, borrowing costs of firms will rise, leading to reduced employment and output. These responses of interest rates and output to a positive money shock run counter to conventional wisdom and to evidence from VAR impulse response functions. Consequently, we consider an environment in which agents choose their allocation of currency between shopping balances and balances used in the foreign exchange market prior to observing contemporaneous money shocks.

When cash allocation decisions are made before observing money shocks, a liquidity effect, along with Fisherian fundamentals, helps determine nominal interest rates. Recall that financial intermediaries are the recipients of monetary injections in each country. Since households cannot adjust their portfolios after a money shock hits, intermediaries, flush with cash, will induce firms to borrow and disproportionately absorb any money injection through lower nominal interest rates. If nominal rates end up lower in equilibrium, employment will increase since the cost of borrowing to hire labor has fallen. The equilibrium outcome for interest rates and real activities depends on the relative strengths of the anticipated inflation effect and liquidity effect. As Christiano (1990, 1991) shows in a closed economy, and Shlagenhauf and Wrase (1995a, 1995b) show in an open economy, the liquidity effect can dominate in a version of the model with empirically plausible parameter values. However, the liquidity effect lacks persistence. The cash allocation rigidity that gives rise to the liquidity effect vanishes in the period following a money shock and consequently, so, too, do most of the effects of the shock. Our interests are in examining how a driving process with regime switches in money growth, along with sluggishness of expectations that arise from regime switching and Bayesian updating of beliefs about monetary regimes, affects the dynamics of the liquidity effect and the dynamics of nominal and real exchange rate responses to monetary innovations.

4 Quantitative Results

The model is solved, parameterized, and simulated to evaluate its quantitative implications. Solving the model involves combining domestic and foreign Euler equations with equilibrium and aggregate-consistency conditions. Since closed-form solutions cannot be obtained, given the nonlinear nature of the model, we solve the model using the method of undetermined coefficients in Christiano (1990). The procedure involves: (i) transforming variables to induce stationarity; (ii) linearizing optimality conditions by taking a first-order Taylor approximation about the nonstochastic steady state and imposing equilibrium and aggregate consistency conditions; (iii) conjecturing recursive laws of motion for choice variables that are linear in the state variables; and (iv) determining coefficient values for the linear decision rules using the method of undetermined coefficients. Details of the solution procedure, including how we track actual money growth and beliefs that agents form about monetary regimes through time, are included in a separate technical appendix (Sill and Wrase, 1999).

Values of parameters ρ^j, v^j, β^j, γ^j, α^j, δ^j, θ^j, μ^j , for $j = D, F$, and parameters of the shock processes that we use in simulations are summarized in

Table 5.

Table 5			
	Parameter	*Domestic Country*	*Foreign Country*
Preferences			
Subjective discount factor	β	0.990	0.990
Utility curvature	ρ	-1.000	-1.000
Home-good consumption share	υ	0.500	0.500
Leisure share	γ	0.760	0.760
Technology			
Capital share	α	0.350	0.350
Depreciation	δ	0.025	0.025
Scale factor	θ	1.000	1.000
Trend technology growth	μ	0.004	0.004

ρ^D and ρ^F, which determine curvatures of period utility functions, are each set to -1. The share υ^D (υ^F) of domestic (foreign) goods in the domestic (foreign) household's Cobb-Douglas consumption composite is set to 0.5, a value used in a number of recent studies, including Stockman and Tesar (1990) and Schlagenhauf and Wrase (1995a, 1995b). Leisure shares in momentary utility for both countries are set to $\gamma^D = \gamma^F = 0.76$, which, together with the model's other parameter values, implies a steady-state allocation in each country of roughly 26 percent of nonsleep time to market activity. Discount rates β^D and β^F are set to 0.99, which implies a nonstochastic steady-state real interest rate of 1 percent per quarter in each country, close to the average return on capital over the last century in the U.S.

The production technologies we use have Cobb-Douglas capital-labor substitution. Labor's share for both countries is set to $1 - \alpha^D = (1 - \alpha^F) = 0.65$, standard values in closed-economy real-business-cycle models, and values consistent with postwar U.S. data. Capital depreciation rates δ^D and $\delta^F$are each set to 0.025, implying annual depreciation of 10 percent, close to the average depreciation rate for the U.S. over the postwar period. Technology variables θ^D and θ^F are simply scale variables that we set to unity. Average per capita real GDP growth is set at 0.0041, roughly equal to growth rates found in actual data in the countries we are considering.

The stochastic process governing money growth shocks in the two countries is in (13). Values for the parameters in the money growth process are the maximum likelihood estimates in Table 1, obtained using Hamilton's regime-switching model and data on growth rates of the per capita monetary base for Canada and the U.S. While most of our analysis does not focus on effects of technology shocks, we include them to allow considerations of the model's business cycle statistics relative to the actual data counterparts. Based on estimates in Schlagenhauf and Wrase (1995a, 1995b), the stochastic process governing technology shocks is parameterized using values $T_{DD} = T_{FF} = 0.9870$, and $T_{DF} = T_{FD} = 0.000$. Standard deviations of domestic and foreign productivity innovations are 0.0136 and 0.0126, respectively, with a correlation of 0.2630 between innovations.

4.1 Transitory Money Shocks

The first quantitative exercise performed is an analysis of the responses of model variables to a temporary (one period) one-standard-deviation increase in money growth. The two-country money growth system is assumed to have been in a low money growth regime for a long time (long enough so that beliefs have settled down) when the shock occurs. We begin by considering a transitory easing of U.S. money growth. Figure 4 shows the evolution of beliefs about remaining in a low growth regime. The shock occurs in period 3 in the figure. While in the period of the shock there is a significant assignment of probability to the monetary system's having shifted to high growth (showing up as a downward movement in the probability that agents assign to being in the low growth regime, probability b_t), the belief quickly reverts as subsequent money growth realizations are consistent with a continuation of the low growth environment. The rapid recovery of beliefs is consistent both with the infrequent shifts in regimes in the money driving process and with the transitory nature of the shock.

Figure 5 plots the responses of variables to a transitory expansionary shock to U.S. money growth, in the top set of figures, or Canadian money growth, in the bottom set of figures. Note that exchange rates in the top set of figures is quoted in terms of the U.S. vis-á-vis Canada, while the reverse is true in the bottom set. To focus on the role played by monetary regime uncertainty and beliefs in the model, we used a particular parameterization of the flow of funds in the model in generating the impulse responses. That parameterization is a case in which workers' wage receipts are allowed to flow contemporaneously to the foreign exchange market ($\alpha_1 = 1$) and in which firms do not fund investment by borrowing ($\alpha_2 = 0$). Impulse responses are generated under two different information assumptions. The plot labeled "full info" is for the case where monetary policy is completely credible and agents know about the true state of the monetary regime. The plot labeled "uncertainty" is for the case where agents are uncertain about the true monetary regime and, therefore, must form beliefs. For both cases it is assumed that household portfolio decisions are made prior to observing the period's money shock.

As Figure 5 shows, the effects on all variables, especially Canadian variables, are minor. In the U.S., the one-period-lived portfolio rigidity leads to dominance of liquidity over anticipated inflation effects on nominal interest rates. Given the rigidity, financial intermediaries in the U.S. become flush with currency when the expansionary U.S. money shock occurs. To entice firms to disproportionately absorb the extra liquidity, downward pressure on interest rates arises. And since this downward liquidity effect pressure outweighs Fisherian anticipated inflation effects in the period of the shock, the U.S. nominal interest rate falls. Because the decline in the U.S. nominal interest rate is larger than the very minor decline for Canada, the U.S.-Canada interest differential decreases on impact of the shock.

The decline in U.S. firms' borrowing costs serves to stimulate labor input acquisition and, hence, output. In addition, the expansionary U.S. money shock feeds extra liquidity into the foreign exchange market because of an increase in U.S. workers' wage receipts, which are channeled to that market. This leads to small impact nominal and real depreciations of the U.S. dollar.

Following the period of the shock, agents reshuffle their portfolios and anticipated inflation effects dominate, leading to an increase in the U.S. interest rate followed by a return to the initial level as the inflationary effects of the shock die out. Coincident with the increase in the U.S. interest rate, firms' borrowing costs are bid up. This leads to a reduction in labor input acquisition and output. And because U.S. household workers' wage receipts decline, there is a decline in U.S. dollars channeled to the foreign exchange market. This leads to the nominal and real appreciation of the U.S. dollar that follows the depreciation of the period of the shock.

The case in which there is a transitory easing of Canadian monetary policy, depicted in the bottom set of pictures in the figure, shows a small decline in the Canadian nominal interest rate and small increase in Canadian output as the liquidity effect dominates the anticipated inflation effect on interest rates. In the U.S., there is a small decline in the nominal interest rate that is not perceptible in the figure and a small positive output effect. Because the Canadian nominal interest rate falls by more than the U.S. nominal rate, the nominal interest differential shown at the bottom of Figure 5 widens. The effects on U.S. nominal interest rates and output are very small. Larger impact effects of the expansionary Canadian money shock are found in the nominal Canadian dollar depreciation and the real depreciation. Following the period of the shock, the Canadian dollar experiences nominal and real appreciations.

Note, in Figure 5, that the difference between the responses when agents face regime uncertainty and form beliefs and the responses when agents are fully informed about regimes is small. For most variables, there is virtually no difference in responses across information assumptions. The biggest differences, in the case of the Canadian money shock, show up in some additional persistence in the effects on nominal interest rates, output levels, and the real exchange rate of the transitory shock when agents are uncertain about regimes. This reflects the sluggishness in adjustment of inflation expectations, as it takes time for agents to revise their beliefs that a regime switch to high money growth is, in fact, not occurring. The general lack of significant differences in responses across information variants is not surprising, however. As with the belief response shown in Figure 4, the lack of long-lived, significant effects of uncertainty and beliefs on the impulse responses follows from the infrequent nature of regime switching in the money driving process as well as the transitory nature of the impulses. We will see, though, that regime uncertainty and beliefs are more important when the money shocks are of a more permanent nature.

4.2 Changes in Regime

To analyze long-lived changes in money growth rates, we perform the following experiment. The economy is assumed to begin in a global high money growth regime. We then impose a transition to a low money growth regime and analyze the effects on model variables. Notice, given our estimates of the money driving process in Table 2, that the regime shift consists of a more substantial reduction in Canadian money growth (from 2.6279, on average, to 0.7918 per quarter) than in U.S. money growth (from 1.7656 to 1.1365). The same parameterization is used as in the transitory shock experiments ($\alpha_1 = 1$ and $\alpha_2 = 0$).

Figure 6 shows the effect on beliefs of the permanent disinflation. It takes approximately six quarters for beliefs to fully adjust to the new monetary regime. Figure 7 plots the effects of the disinflation policy on interest rates, output levels, and nominal and real exchange rates. The nominal exchange rate in the figure is expressed in U.S. dollars per Canadian dollar. Liquidity effects on interest rates, output, and exchange rates in the period of the regime switch are easily outweighed by anticipated inflation effects. Subsequent effects stem primarily from reductions in anticipated Canadian and U.S. inflation and, in the case of uncertainty, beliefs. The nominal appreciation of the Canadian dollar reflects the more substantial deceleration of Canadian money growth relative to the United States. Canadian appreciation shows up initially also in the real exchange rate The subsequent real depreciation shown in the figure is merely an artifact of unbalanced nominal growth across the two countries in the sample.

Reductions in anticipated inflation serve to reduce nominal interest rates, in Fisherian fashion, in each country, which, in turn, stimulates output. Given the relatively larger deceleration of money growth in Canada than in the U.S., nominal interest rates decline more in Canada, and the U.S.-Canada nominal interest rate differential therefore rises, as shown at the bottom of the figure.

Note that, in contrast to the transitory shock experiments, there now is increased persistence in interest rate and output adjustments in the uncertainty case relative to the full information case. It takes about five quarters or more for interest rates and output levels to settle to their new long-run state when agents are uncertain about the monetary regime. This compares to about two quarters for the full information case. However, the absolute difference between the two paths is rather small. And beliefs matter even less for nominal exchange rates.

Why does uncertainty about the monetary regime generate additional persistence in interest rate adjustment in the case of a regime switch? Recall that the system had been in a high growth regime for some time. As Euler equation (14) indicates, households in that regime based their deposit and consumption decisions partly on expectations of relatively high inflation rates. When the regime switches to low growth, uncertain agents only gradually ratchet their inflation expectations down and, so, make adjustments in their deposit and consumption plans that are not as large as the adjustments by agents who are fully informed about regimes. Given the switch to the low money growth regime, intermediaries in the short term are infused with deposits from uncertain agents that are larger than those received from fully informed agents. As a result, nominal interest rates are lower in the adjustment of the economy with agents who are uncertain about regimes and are learning that a regime switch occurred relative to the economy with fully informed agents. Given the relatively lower path for nominal interest rates, the path of output adjustment has higher output levels for several quarters as agents who are uncertain about regimes gradually update their beliefs.

Because beliefs play a role in adding to persistence in effects of long-term changes in money growth rates in the model, it is instructive to consider some sensitivity analysis. In particular, consider effects on beliefs of different degrees of uncertainty about monetary regimes. Intuitively, following a shock, one would expect beliefs about being in a particular regime to adjust to the actual regime in place faster, the more pronounced the difference in regimes and the lower the amount of noise in the observed process. As Moran (1997) shows formally in the context of a model similar to ours, this intuition shows up in the following properties of our belief structure: the larger the difference in average money expansion across regimes, the faster beliefs converge to the actual regime in place following a shock. And the higher the variability of monetary control errors within a regime, the slower is belief convergence. These properties can be seen by examining beliefs formed in our model in three cases, summarized in Figure 8. In each case we assume that the economy is initially in a high growth monetary regime and then makes a transition to a low growth regime. The transition occurs in quarter 10 in the figure.

Case 1 shows belief b_t , the probability assigned by an agent in the model to the current regime being characterized by low money growth, for the baseline parameterization of the model. Case 2 shows belief b_t when the difference in average money growth rates across regimes is widened (by one order of magnitude). Comparing the two cases reveals that widening the average money growth spread across regimes makes regimes more detectable to agents, allowing beliefs to adjust more rapidly to the regime switch. It takes only one quarter in case 2 for agents to identify that a regime switch from high to low average money growth has occurred.

Case 3 shows belief b_t when the difference in average money growth rates across regimes is the same as case 1, but the standard deviation of monetary control errors in each regime is increased (by one order of magnitude for each country) relative to case 1. Comparing the two cases reveals that increasing noise within each regime makes it more difficult for agents to detect regimes, preventing beliefs from rapidly adjusting to the regime switch. It takes longer in case 3 than case 1 for agents to identify the regime switch that occurred.

Examination of the three cases in Figure 8 shows that, depending on the nature of the different regimes, beliefs can exhibit behavior ranging from instantaneous adjustment to sluggish adjustment lasting over 10 quarters. Given our parameterization of the money growth process using our estimates from actual data, we find, however, that uncertainty about regimes and belief formation in the model does very little to help account for observed persistent effects of monetary innovations on key economic variables.

For transitory shocks, beliefs add almost nothing to the impulse responses implied by the model, as is to be expected. Yet even for actual regime switches, the quantitative effects on responses of variables to money shocks are not altered substantially by adding regime uncertainty and belief formation to the model. This leads us to believe that there is little quantitative significance to adding beliefs to the study of monetary policy changes as in the analyses of Moran (1997, 1998) or, for the case of domestic effects of Canadian monetary policy regime switches, the analysis of Andolfatto and Gomme (1997). However, given our interest in exchange rate effects of regime uncertainty and beliefs, it remains of interest to consider effects in our model of regime switching and beliefs on the nature of the process governing exchange rates. In particular, it is of interest to determine whether the nature of the money growth driving process matters for exchange rate behavior in the model and whether beliefs also matter.

4.3 Long Swings: Regime Switches in Exchange Rates

Given the evidence provided earlier of regime switching behavior in the nominal and real exchange rates between Canada and the U.S., we now consider the ability of the model to generate exchange rate series displaying regime switches, which we will refer to as long swings. We ask two questions. First, absent a driving process for money growth with regime switching, are there long swings in nominal and real exchange rate series generated by the model? Second, with the regime switching money growth process that we estimated from Canadian and U.S. data, does the ability of the model to generate long swings depend on whether agents face regime uncertainty and form beliefs? To answer these questions, we perform experiments in which the model is simulated under alternative driving processes and belief structures, and the resulting time series of nominal and real exchange rates are tested for long swings. The same cash-flow parameterization is used as in the money shock experiments ($\alpha_1 = 0$ and $\alpha_2 = 1$). The test is whether there is evidence of nonlinearity in the first difference of the log of the nominal exchange rate, or real exchange rate, using the procedures that were used to characterize the actual data series earlier.

We simulated each model variant 100 times and tested each simulated time series of the nominal and the real exchange rate for nonlinearity. The results in Table 6 show the fraction of tests returning a result that the relevant series is better modeled with a Markov-switching process than a simple linear process.

Table 6

Money Growth Process	e_t *Beliefs*	e_t *No Beliefs*	$e_t * \frac{P_t^F}{P_T^D}$ *Beliefs*	$e_t * \frac{P_t^F}{P_T^D}$ *No Beliefs*
Linear	n.a.	0.22	n.a.	0.00
Switching	0.67	0.62	0.01	0.00

The higher the fraction, the more confidence we can place on the existence of long swings being present in the model's exchange rate dynamics. Columns in the table with *Beliefs* in the heading identify cases in which we simulate the model assuming that, when relevant, agents face regime uncertainty and form beliefs according to the Bayesian learning mechanism outlined earlier. Columns with headings marked *No Beliefs* identify cases in which agents in the model know with certainty the monetary regime in place in each period.

Entries marked n.a. mean not applicable, because beliefs are irrelevant when a linear process is used, since there are no regime switches or regimes over which to form beliefs.

There are three noteworthy features of the results. First, consistent with actual data on the nominal exchange rate between the Canadian and U.S. dollar, there is evidence of long swings in the model's nominal exchange rate provided that a money growth process with regime switching is used. Consequently, the nature of the monetary driving process is important for the model to account for long swings in nominal exchange rates. When the model is subjected to a driving process consistent with our evidence of regime switching in money growth rates in Canada and the U.S., long swings in nominal exchange rates are detected. Absent regime switching in the money growth process, the model does not show evidence of long swings.

The second noteworthy feature of the results is that independent of the presence or absence of regime switching in the money growth process and of regime uncertainty, there is no evidence of long swings in real exchange rates in the model. The model is consequently unable to account for the long swings that we identified in the observed real exchange rate between Canada and the U.S. Evidence of long swings in the nominal, but not real, exchange rate in the model suggests that a departure from the assumption of perfectly flexible prices could help explain real exchange rates.

The third notable feature of the long swings tests is that adding beliefs adds to the model's ability to account for long swings in the nominal exchange rate between Canada and the U.S., but only slightly. Nonlinearities in the process driving money growth in the model are the proximate cause of long swings in the model, and the dynamics are not altered substantially whether or not beliefs are present. This, perhaps, should not be surprising in light of the infrequent shifts in regimes evident in both the money growth rates and exchange rates for Canada and the U.S.

In summary, the long swing test results suggest that adding a driving process for money growth with regime switches helps the model produce long swings in nominal exchange rates, but not real exchange rates, and adding beliefs provides only a small amount of additional assistance.

4.4 Exchange Rate Variability and Business Cycle Moments

We now consider the ability of the model, with and without belief formation, to account for observed business cycle fluctuations, measured by second moments of key variables. The actual and model-generated data moments are displayed in Table 7. We parameterize the money growth driving process in the model using our estimates from Canadian and U.S. monetary base data, identifying the U.S. as the model's domestic country, country D, and Canada as the model's foreign country, country F. For comparability, moments are calculated for actual and simulated time series that have been Hodrick-Prescott filtered. For each variable listed in the table, the initial two columns of numbers are actual data moments for the sample period 1960:2-1996:2 for Canada and the U.S. For model-generated data, with and without beliefs, indicated by the respective headings $Beliefs$ and No, results are provided for alternative parameter combinations governing flows of cash among different agent types. Recall that when α_1 is set to 1 (0), all (none) of the workers' nominal wage receipts are allowed to flow to the foreign exchange market in a period. When α_2 is set to 1 (0), firms finance physical investment using funds obtained from intermediaries (from goods market receipts).

Table 7						
	Canada	U.S.	$\alpha_1 = 1$ $\alpha_2 = 0$		$\alpha_1 = 1$ $\alpha_2 = 1$	
			$Beliefs$	No	$Beliefs$	No
St. Dev.						
Y_t^D		0.015	0.014	0.014	0.014	0.014
Y_t^F	0.017		0.013	0.013	0.013	0.013
$\frac{I_t^D}{Y_t^D}$		3.212	2.218	2.718	2.687	2.692
$\frac{I_t^F}{Y_t^F}$	5.382		2.855	2.860	2.829	2.779
$\frac{C_t^D}{Y_t^D}$		0.731	0.532	0.539	0.545	0.549
$\frac{C_t^F}{Y_t^F}$	0.751		0.553	0.565	0.568	0.578
R_t^D		0.003	0.005	0.005	0.005	0.005
R_t^F	0.004		0.005	0.004	0.006	0.005
P_t^D		.016	0.021	0.021	0.023	0.022
P_t^F	0.014		0.024	0.025	0.025	0.026
e_t	0.025	0.025	0.028	0.028	0.031	0.031
$e_t \frac{P_t^F}{P_t^D}$	0.027	0.027	0.015	0.015	0.015	0.015
AR(1)						
R_t^D		0.739	0.161	0.162	0.081	0.055
R_t^F	0.751		0.249	-0.095	0.268	-0.059
e_t	0.866	0.866	0.645	0.646	0.604	0.611
$e_t \frac{P_t^F}{P_t^D}$	0.868	0.868	0.234	0.205	0.204	0.190

	$\alpha_1 = 0$ $\alpha_2 = 1$		$\alpha_1 = 0$ $\alpha_2 = 0$		No $Money$ $Shocks$ $\alpha_1 = 1$ $\alpha_2 = 0$	
	$Beliefs$	No	$Beliefs$	No	$Beliefs$	No
St. Dev.						
Y_t^D	0.017	0.017	0.014	0.014	0.014	0.014
Y_t^F	0.019	0.018	0.015	0.014	0.014	0.013
$\frac{I_t^D}{Y_t^D}$	2.192	2.22	2.590	2.555	2.671	2.687
$\frac{I_t^F}{Y_t^F}$	1.888	1.967	2.505	2.669	2.883	2.892
$\frac{C_t^D}{Y_t^D}$	0.941	0.880	0.695	0.674	0.529	0.532
$\frac{C_t^F}{Y_t^F}$	0.819	0.849	0.671	0.693	0.551	0.547
R_t^D	0.019	0.019	0.120	0.012	0.004	0.004
R_t^F	0.037	0.035	0.025	0.023	0.005	0.003
P_t^D	0.028	0.029	0.024	0.024	0.012	0.011
P_t^F	0.027	0.028	0.022	0.023	0.014	0.015
e_t	0.027	0.028	0.026	0.025	0.007	0.007
$e_t \frac{P_t^F}{P_t^D}$	0.027	0.026	0.019	0.018	0.015	0.015
AR(1)						
R_t^D	-0.092	-0.099	-0.129	-0.130	-0.015	-0.006
R_t^F	0.079	-0.040	0.136	-0.058	0.324	-0.004
e_t	0.795	0.815	0.833	0.831	0.900	0.882
$e_t \frac{P_t^F}{P_t^D}$	0.018	-0.002	0.146	0.086	0.224	0.201

Consider, first, nominal and real exchange rates. Nominal and real exchange rates between Canada and the U.S. have standard deviations above standard deviations of output levels and are strongly autocorrelated. Standard deviations of nominal and real exchange rates are roughly 1.5 times the standard deviations of Canadian and U.S. GDP. As Table 7 reveals, the same is true in certain parameterizations of the model. Nominal exchange rate variabilities implied by all of the parameterizations are roughly in line with the variability found in actual data. Real exchange rates are generally less volatile in the model than in the actual data. In all parameterizations, price levels implied by the model are more volatile than their actual data counterparts. Evidently, price flexibility in the model allows prices to absorb too much of the effects of money and technology shocks.

The size of the share of investment spending financed by borrowing, governed by the setting for α_2, has only minor effects on the business cycle dynamics of the model. Anticipated inflation has a negative effect on investment expenditures subject to cash-in-advance constraints, as discussed in Stockman (1981) and Christiano (1991). This effect is most pronounced in the model when α_2 is set to unity. However, holding fixed the share of wage receipts flowing to the foreign exchange market, governed by α_1, results of the model with $\alpha_2 = 1$ are not significantly different from the opposite extreme of setting $\alpha_2 = 0$. Anticipated inflation effects on investment in the model are evidently not very strong.

Table 7 also reveals that the size of the share of workers' wage receipts flowing to the foreign exchange market has some significant effects on the model's cyclical dynamics. Holding fixed α_2, increasing the share of wages flowing to the foreign exchange market leads to smoother consumption and nominal interest rates, as well as a smoother real exchange rate series. When $\alpha_1 = 0$, consumption volatility relative to output volatility is slightly above what we observe in the Canadian and U.S. data. Increasing α_1 from zero to unity brings consumption volatility slightly below the actual data counterparts. The consumption smoothing effect of increasing α_1 stems from an increased ability of wage receipts in a period to flow to the foreign exchange market, where agents set up cash balances to use for contemporaneous consumption purchases. In general equilibrium, allowing agents to use wage receipts for contemporaneous consumption expenditures contributes to their ability to smooth consumption.

While increasing α_1 to unity brings the model's implied consumption variability slightly below that found in actual data, it also provides a tremendous improvement in the model's implication for nominal interest rate variability. When wage receipts are not allowed to channel into foreign exchange trading, the variabilities of nominal interest rates in the model are counterfactually high. As equation (18), governing a typical worker's labor supply decision, indicates, the effects of anticipated inflation on labor supply depends crucially on the degree to which wage receipts can be used contemporaneously for consumption. Further, equation (19), governing labor demand of a firm, shows that the nominal interest rate in a country reflects conditions in the labor market, including labor supply and the real wage. In equilibrium, when flows of wage receipts that ultimately lead to contemporaneous consumption expenditures are shut down, influences of shocks in the model on conditions in the labor market lead to counterfactual implications for nominal interest rate dynamics. However, when these flows are allowed by increasing the value of α_1, the variability of the nominal interest rate in the model moves toward variabilities we observe in the Canadian and U.S. data.

Table 7 also shows that implications of the model with regime uncertainty and beliefs for business cycle statistics are virtually indistinguishable from those of the model with agents who are fully informed about regimes. This follows from two features of the model. First, technology shocks dominate the dynamics of the model economy. This can be seen by comparing moments of model-generated data when α_1 is set to 1 and α_2 to 0 in the first two columns of moments of model-generated data with the final two columns. The first two columns are from the model with both technology and money shocks. The final two columns are from the model with only technology shocks. As in many recent monetary business cycle models with flexible prices, business cycle moments implied by the model are not significantly different if money shocks are removed from the system altogether. While this may point to more serious attention to sticky price models in accounting for business cycle dynamics, there has been very little quantitative work to date showing that such models better account for actual data properties when both money and technology shocks are included.

Another possibility is to consider monetary effects in a model like the one above in which agents trade currencies to facilitate trades in assets as well as goods. Agents in our model trade currencies solely to facilitate transactions in the goods market. Consequently, there is a strong tie between agents' intratemporal marginal rates of substitution between domestic- and foreign-produced goods and exchange rates. Since this substitution margin involves international relative quantities of goods within a period, there is no strong link in the model between exchange rates and expectations of future values of nominal variables. Linking currency trades and, hence, exchange rates to asset trades could potentially make beliefs matter more in the model, as expectations about future monetary developments play a more important role in exchange rate determination. Solving and simulating the dynamics of such a model is, however, challenging since it is necessary to account for relative wealth dynamics in a more complicated fashion than in the model used here.

The second feature of the model leading to the similarity of results with or without beliefs is that, to the extent that money shocks play a role, agents' beliefs about monetary regimes do not have large enough effects in the model on nominal or real variables to influence the business cycle statistics we address. While beliefs may provide some assistance in explaining persistence in effects of long-lived money shocks on variables, accounting for beliefs, in the form of learning about regimes, seems less important in accounting for business cycle fluctuations in nominal and real variables. Such a conclusion is consistent with findings of other research on empirical implications of regime switching processes and learning. Evans and Lewis (1995) and Kaminsky (1993), among others, arrive at similar conclusions in their analyses of regime shifts, labeled "peso problems," and learning. Their findings are that while regime switching in driving processes seems to matter empirically along certain dimensions, learning contributes little to small sample effects of potential regime switches. Our findings are consistent with this literature: switching in the process characterizing Canadian and U.S. money growth seems to matter in actual data and in our model, but the addition of learning to the model contributes only slightly in helping account for some of the persistence in effects of long-lived money shocks on key economic variables.

5 Conclusion

This paper has presented and discussed evidence on effects of monetary innovations on real and nominal variables in Canada and the U.S., properties of nominal and real exchange rates between the two countries, and business cycle properties of the two economies. Given evidence of regime switching behavior both in the process governing money growth rates in the two countries and in the processes governing exchange rates, we constructed and analyzed an open economy monetary model in which agents face monetary regime switches. In addition, agents in the model do not know with certainty the true state of monetary policy. Rather, they form beliefs based on observed money growth rates, which encompass monetary regime switches and random monetary control errors.

Quantitative implications of parameterized versions of the model are evaluated relative to properties of data drawn from the actual Canadian and U.S. economies. In particular, we consider the ability of the model, with and without regime uncertainty and belief formation, to account for effects of monetary innovations on key economic variables, including interest rates, exchange rates, and real activity. The results indicate that, in order to account for nonlinearities in exchange rates, in the form of long swings, it helps to have a process driving money growth that itself contains nonlinearities in the form of monetary regime switches. In accounting for observed persistence in effects of money shocks on key macroeconomic variables, marginal help is provided by including a Bayesian process by which agents form beliefs about monetary regimes. Beliefs do not play a significant role in accounting for business cycle properties of nominal and real variables.

ENDNOTES

1. In the models there is no distinction between the terms of trade and the real exchange rate. Moments for the two international relative price measures are provided in Table 1 to illustrate that, in general, using alternative measures, international relative prices possess high volatilities.

2. The notational conventions are: A subscript denotes the country of origin of a good or money balance. A superscript denotes the residence of the household choosing the variable. A tilde "$\tilde{\ }$" denotes a quantity supplied; household choice variables without tildes are quantities demanded.

3. In simulations, parameter values are used for which agents drive cash constraints to bind as strict equalities. That is, the gross nominal interest rates exceed unity.

4. As long as the gross loan rate exceeds unity, intermediaries lend all available cash to firms.

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Figure 1

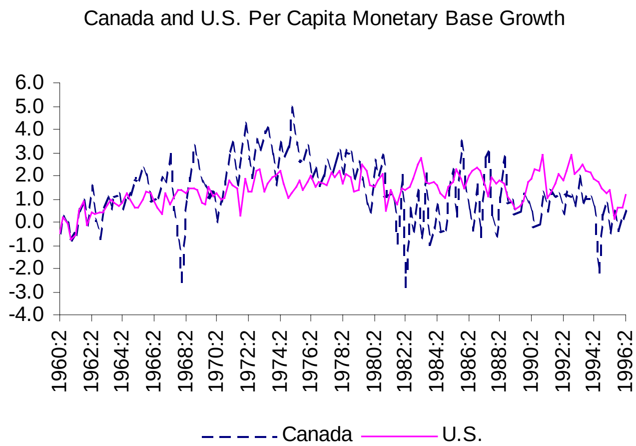
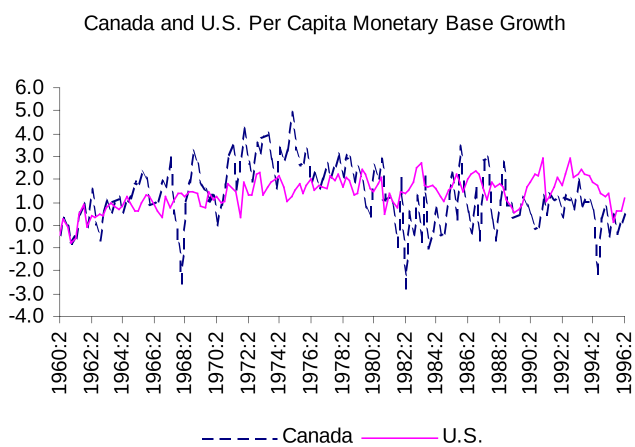


Figure 2



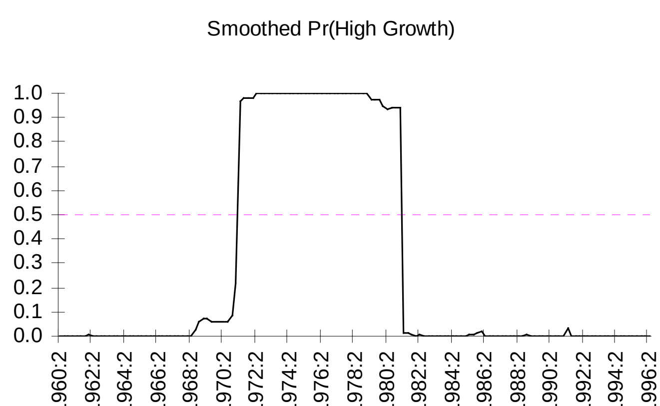
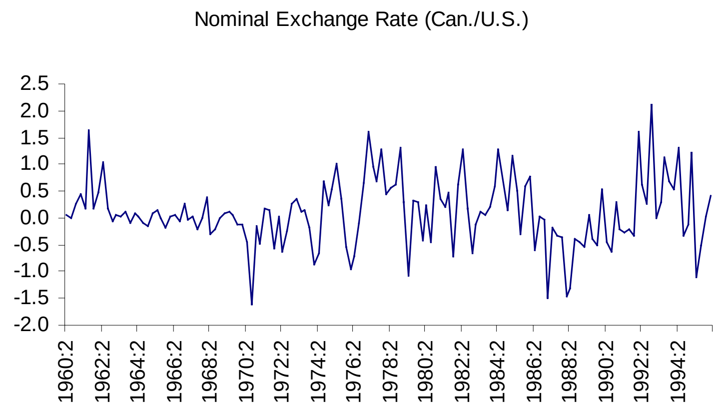


Figure 3


Nominal Exchange Rate (Can./U.S.)
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
1960:2
1962:2
1964:2
1966:2
1968:2
1970:2
1972:2
1974:2
1976:2
1978:2
1980:2
1982:2
1984:2
1986:2
1988:2
1990:2
1992:2
1994:2

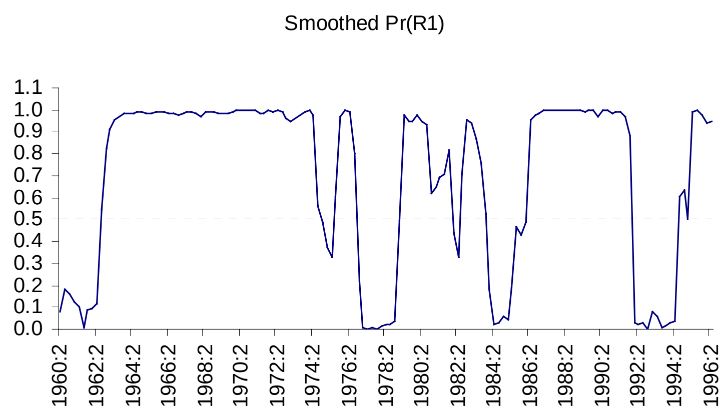
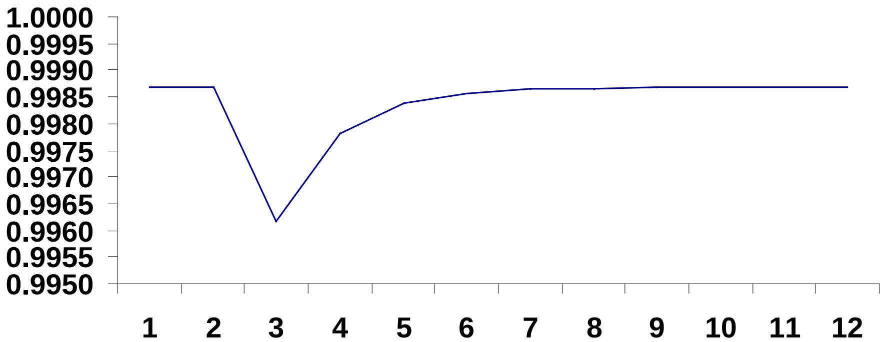

Smoothed Pr(R1)
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1960:2
1962:2
1964:2
1966:2
1968:2
1970:2
1972:2
1974:2
1976:2
1978:2
1980:2
1982:2
1984:2
1986:2
1988:2
1990:2
1992:2
1994:2
1996:2

Figure 4

Beliefs-Response to Transitory U.S. Money Growth Shock



Beliefs-Response to Transitory Canadian Money Growth Shock

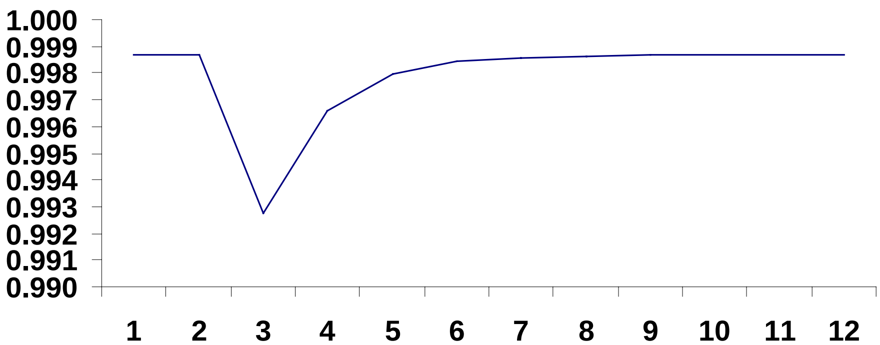


Figure 5

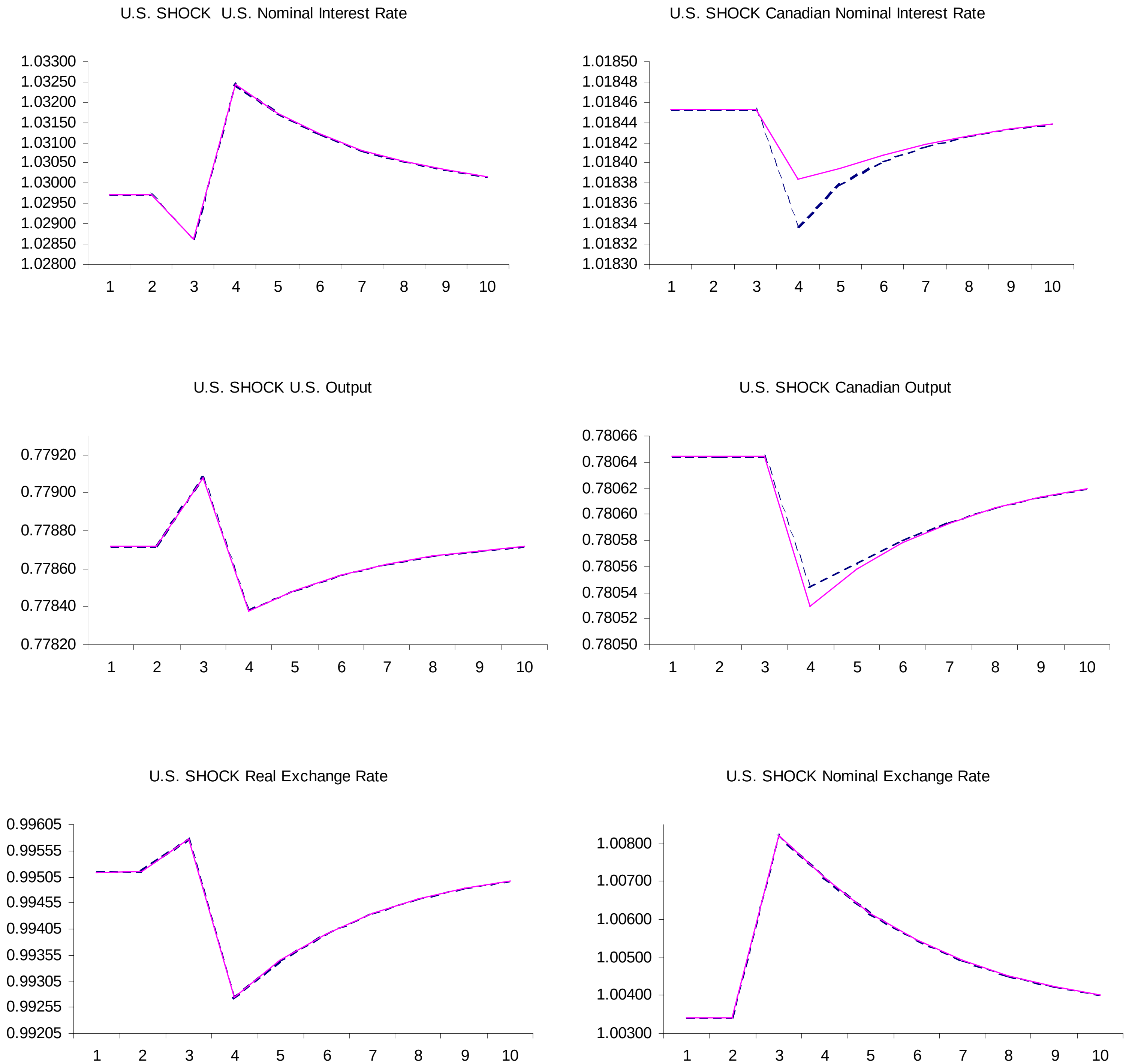
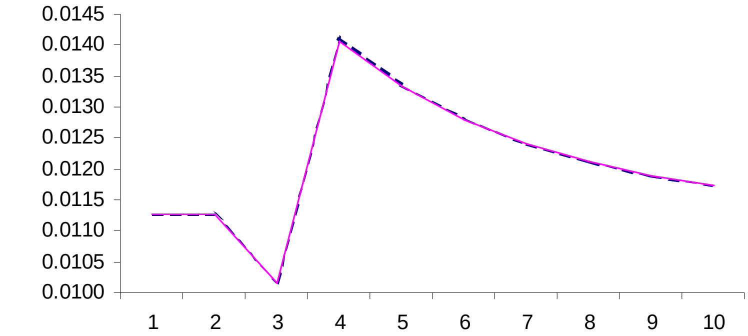
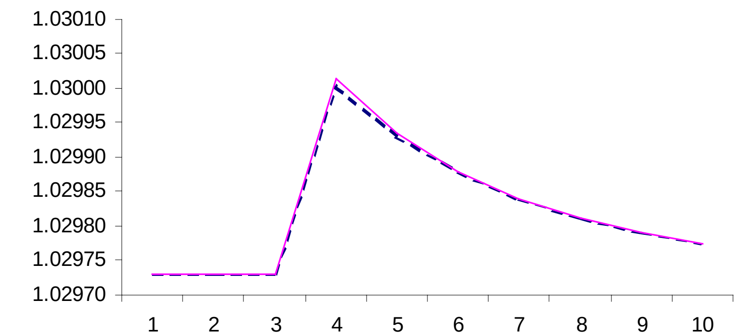

U.S. SHOCK U.S. Nominal Interest Rate
1.03300
1.03250
1.03200
1.03150
1.03100
1.03050
1.03000
1.02950
1.02900
1.02850
1.02800
1 2 3 4 5 6 7 8 9 10
U.S. SHOCK Canadian Nominal Interest Rate
1.01850
1.01848
1.01846
1.01844
1.01842
1.01840
1.01838
1.01836
1.01834
1.01832
1.01830
1 2 3 4 5 6 7 8 9 10
U.S. SHOCK U.S. Output
0.77920
0.77900
0.77880
0.77860
0.77840
0.77820
1 2 3 4 5 6 7 8 9 10
U.S. SHOCK Canadian Output
0.78066
0.78064
0.78062
0.78060
0.78058
0.78056
0.78054
0.78052
0.78050
1 2 3 4 5 6 7 8 9 10
U.S. SHOCK Real Exchange Rate
0.99605
0.99555
0.99505
0.99455
0.99405
0.99355
0.99305
0.99255
0.99205
1 2 3 4 5 6 7 8 9 10
U.S. SHOCK Nominal Exchange Rate
1.00800
1.00700
1.00600
1.00500
1.00400
1.00300
1 2 3 4 5 6 7 8 9 10

Figure 5 (continued)

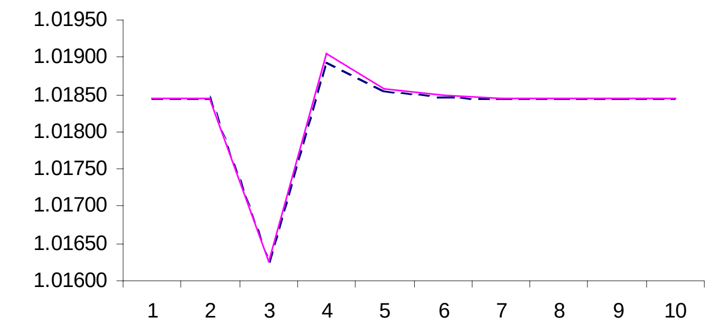
U.S. SHOCK Interest Differential (U.S.-Canada)



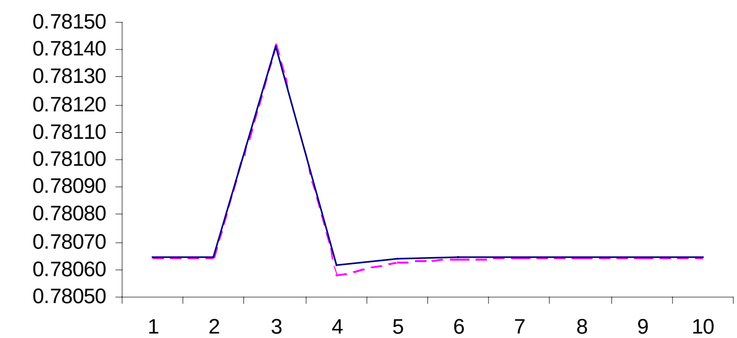
CANADA SHOCK U.S. Nominal Interest Rate



U.S. SHOCK Canadian Nominal Interest Rate



CANADA SHOCK U.S. Output



CANADA SHOCK Canadian Output

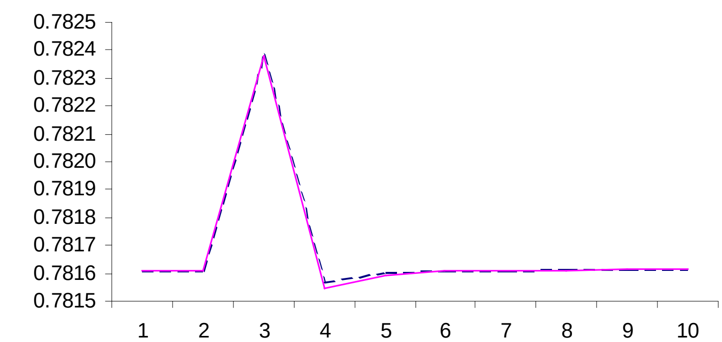
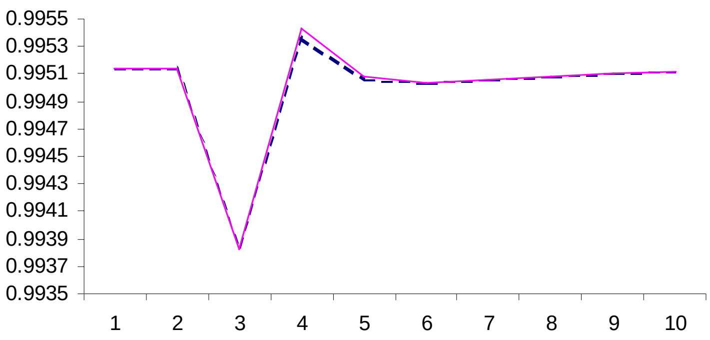
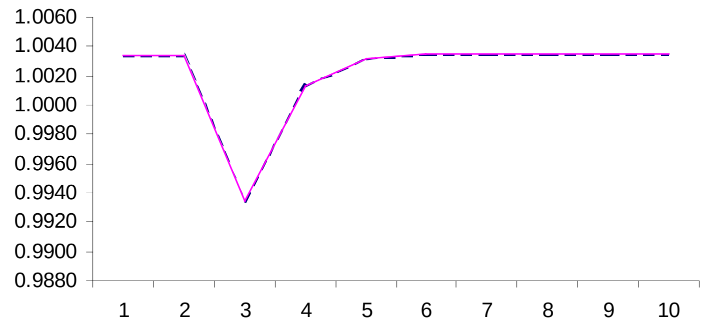


Figure 5 (continued)

CANADA SHOCK Real Exchange Rate



CANADA SHOCK Nominal Exchange Rate



CANADA SHOCK Interest Differential (U.S.-Canada)

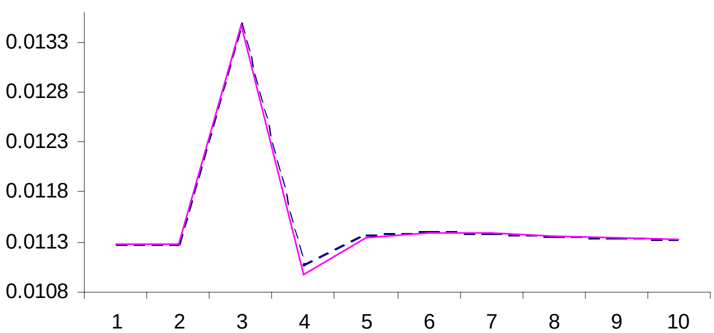


Figure 6

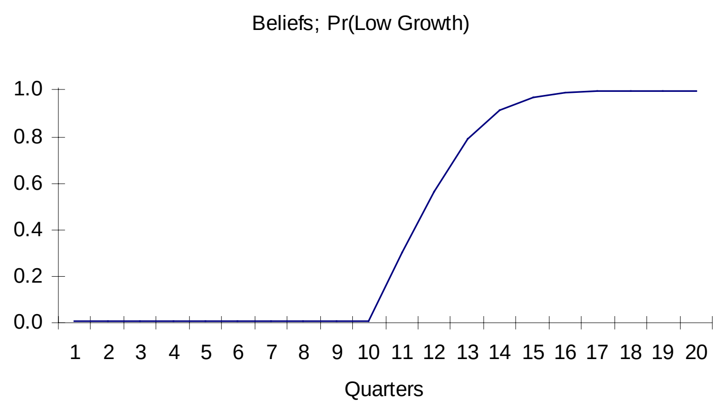
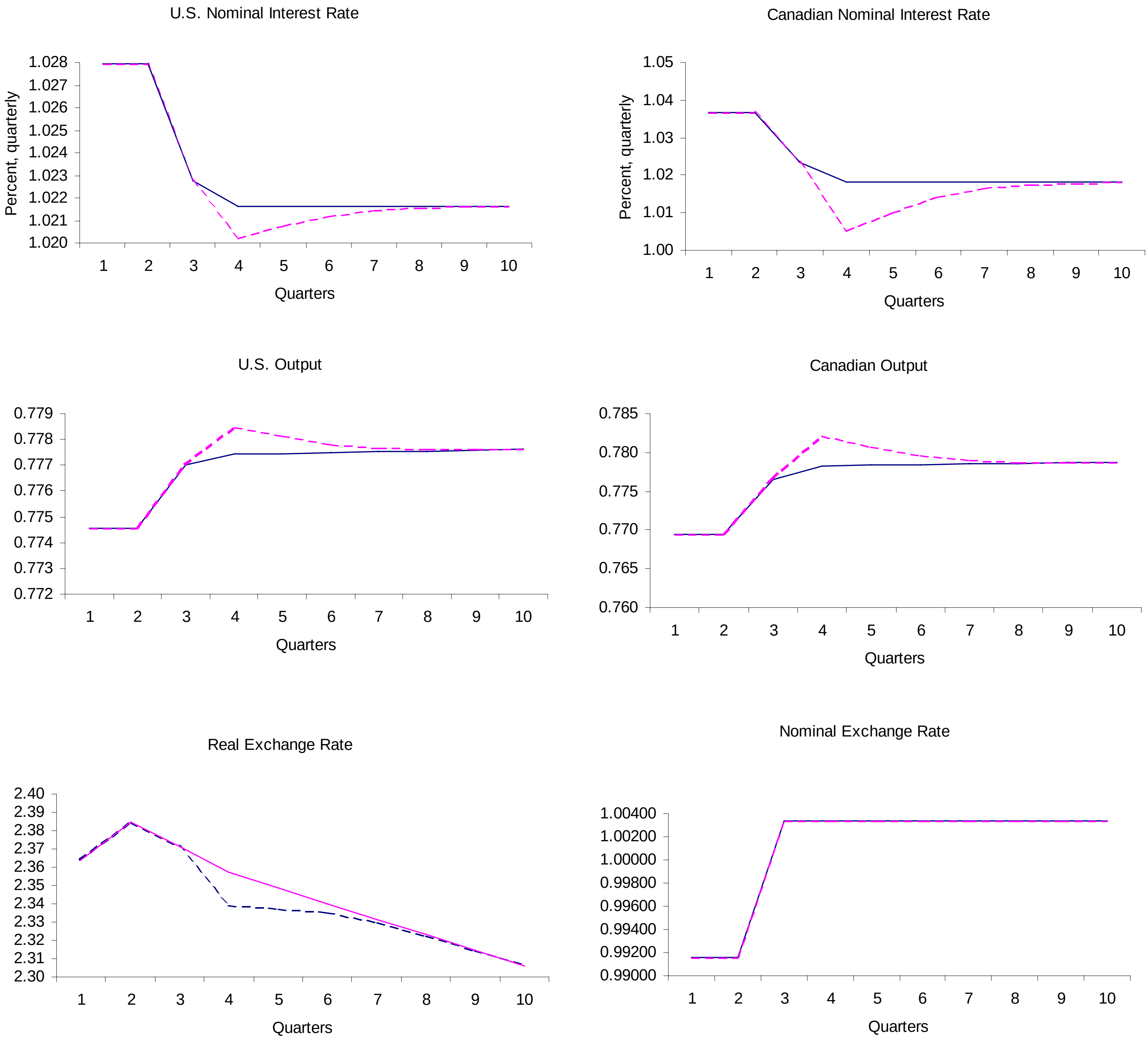

Beliefs; Pr(Low Growth)
1.0
0.8
0.6
0.4
0.2
0.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Quarters

Figure 7



U.S. Nominal Interest Rate
Percent, quarterly
1.028
1.027
1.026
1.025
1.024
1.023
1.022
1.021
1.020
1 2 3 4 5 6 7 8 9 10
Quarters
Canadian Nominal Interest Rate
Percent, quarterly
1.05
1.04
1.03
1.02
1.01
1.00
1 2 3 4 5 6 7 8 9 10
Quarters
U.S. Output
0.779
0.778
0.777
0.776
0.775
0.774
0.773
0.772
1 2 3 4 5 6 7 8 9 10
Quarters
Canadian Output
0.785
0.780
0.775
0.770
0.765
0.760
1 2 3 4 5 6 7 8 9 10
Quarters
Real Exchange Rate
2.40
2.39
2.38
2.37
2.36
2.35
2.34
2.33
2.32
2.31
2.30
1 2 3 4 5 6 7 8 9 10
Quarters
Nominal Exchange Rate
1.00400
1.00200
1.00000
0.99800
0.99600
0.99400
0.99200
0.99000
1 2 3 4 5 6 7 8 9 10
Quarters

Figure 7 (continued)

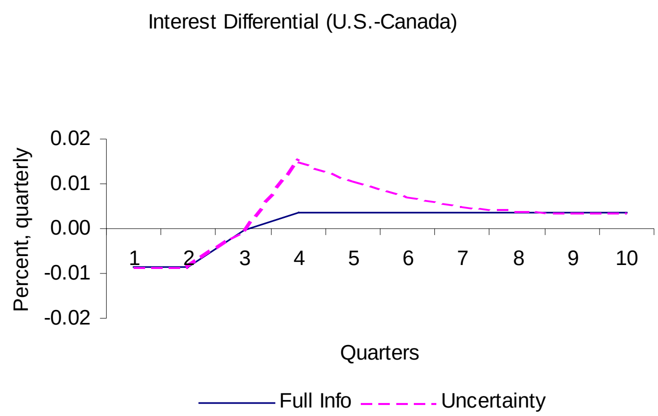


Figure 8