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FINANCIAL DEVELOPMENT AND ECONOMIC GROWTH

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Abstract

We develop a theory of financial development based on the costs associated with the provision of external finance. These costs are assumed to arise within an environment where informational asymmetries between borrowers and lenders are costly to resolve. When borrowing is limited, producers with access to financial intermediary loans obtain higher returns to investment than other producers. This creates incentives for others to undertake the technology adoption necessary to access investment loans. Over time, as increasing numbers of producers gain access to external finance, borrowers' net worth rises relative to debt. This reduces the costs of financial intermediation and raises the overall return on investment. The theory is consistent with recent evidence that financial development reduces the costs associated with the provision of external finance and increases the rate of economic growth. Furthermore, the theory predicts that financial development raises the return on loans and reduces the spread between borrowing and lending rates.

1 Introduction

Cross-country studies have uncovered a contemporaneous correlation between the level of financial development and economic growth. King and Levine (1993) show that this correlation exists across a variety of measures that capture both the efficiency and the extent of the financial system. Further, the initial level of financial development predicts subsequent growth, and this result is robust to the introduction of additional explanatory variables. Thus, finance may not be merely concurrent with development, as first shown by Goldsmith (1969); financial development may cause economic growth.

Most studies of financial development have placed primary emphasis on the provision of external finance. It is commonly suggested that improvements in the ability of firms to finance investment using debt or equity lead to significant increases in production or growth.¹ Recent efforts to address the issue of causality lend support to this view. For example, Rajan and Zingales (1998) present industry-level evidence that links financial development to growth through the supply of external finance.² They find that financial development raises growth disproportionately in industries with relatively high external dependence. Demirgüç-Kunt and Maksimovic (1998) use firm characteristics to compute benchmark growth rates under the assumption of limited access to external finance. They show that the fraction of firms whose actual rate of growth exceeds their benchmark is rising in the level of financial development. Related research, seeking to identify the independent component of financial develop-

ment, has led to additional evidence for external finance as a leading channel through which financial development promotes economic growth. La Porta, Lopez-de-Silanes, Shleifer and Vishny (1997) find that, across countries, legal origins help determine the extent of investor protection, the rights given to creditors and shareholders, and thus affect the supply of external finance. Using their measures of creditor protection to isolate the predetermined component of banking sector development, Levine (1998) finds that this channel of financial development explains economic growth.³

To address these findings, we construct a dynamic general equilibrium model of financial development and growth. We depart from previous theory by focusing on the costs of borrowing through financial intermediaries in an economy with limited firm access to external finance. Specifically, in an environment where producers operate risky production opportunities and, if they have access to loans, borrow to finance investment, we assume that it is costly for lenders to verify production. Financial intermediaries arise to operate debt contracts in which the costly state verification of production occurs only when a borrower reports that he cannot fully repay his loan. In this setting, higher levels of internal investment, financed by producers' wealth or net worth, improve the likelihood of full loan repayment by reducing indebtedness. Effectively, net worth serves as collateral against debt. In equilibrium, producers invest their entire net worth, which determines the efficiency of financial intermediation, and thus the returns to both borrowers and lenders.

In our model, producers operating firms with access to investment loans experi-

ence higher returns to production and thus faster growth. This induces those unable to borrow to undertake the expenditures necessary to engage in debt contracts. The extent of the financial system rises. Both the more rapid growth for producers with access to external finance and the increase in the number of such producers serve to increase borrowers' net worth relative to debt. The consequent decline in indebtedness improves the likelihood that firms will be able to repay their debt, thereby reducing the frequency of verification and the costs of lending. The mean return on investment improves. Thus, financial development is associated with increases in the extent of the financial system that raise the rate of growth by reducing the costs of external finance. Driving these improvements in the efficiency of financial intermediaries is a decline in the level of indebtedness associated with the typical investment loan. The model exhibits joint causality between economic growth and financial development. Economic growth promotes financial development by increasing borrowers' collateralizable net worth. Financial development in turn raises the return on investment and, therefore, the rate of growth.

We add to existing theory by developing a framework within which to examine the role of external finance in the finance-growth relationship. In our model, financial development occurs as improvements in borrowers' net worth reduce the costs of financial contracts. This mechanism has been studied in business cycle models that examine the role of the credit channel in the propagation of shocks.⁴ Extending its application to a model of financial development, we are able to derive a theoretical

foundation for the empirical findings discussed above. Furthermore, our study yields several predictions: the reductions in the cost of financial contracts that cause financial development also imply a rise in the return on debt, a decline in the spread between borrowing and lending rates, and a decline in the premium commanded by producers with access to investment loans. These results may help focus further empirical examination of the finance and growth linkage.

We share with existing theory, in particular the important contribution of Greenwood and Jovanovic (1990), a common emphasis that associates financial development with increases in the extent of the financial system. In both models, financial development implies increases in the fraction of agents that have access to intermediaries. In the Greenwood and Jovanovic model, intermediaries allocate savings more efficiently than households; in our model, intermediaries help fund investment in firms.⁵ To the best of our knowledge, ours is the first formal analytical framework to provide a nontrivial role for external finance, and thereby endogenize the returns to borrowing and lending, in a model where both economic growth and financial development are endogenous.

To implement our analysis, in section 2, we first assume universal firm access to investment loans and develop a benchmark economy characterized by costly financial intermediation and endogenous economic growth. This allows us to present the basic mechanism, isolate the effect of collateral on the costs associated with financial intermediation, and examine the interaction between intermediation and growth.

Next, we study the process of financial development. Section 3 characterizes an economy's transition path from an initial state, where not all firms have access to external finance, to the limiting case outlined in section 2. Section 4 concludes.

2 A basic model of financial intermediation and growth

We develop a benchmark model of costly financial intermediation in an economy where growth is endogenous and there is universal firm access to external finance. To separate individuals into borrowers and lenders, we assume that in every period, with probability $1 - \varphi$, $\varphi \in (0, 1)$, an agent is allocated a production opportunity. Each producer operates a linear technology that is subject to an idiosyncratic productivity shock; capital is the sole input in production. Investment is financed through two distinct sources. First, the producer can use his own wealth for investment. Second, this internally financed investment may be augmented by borrowing. Individuals who fail to obtain production opportunities appoint intermediaries to write debt contracts with producers. This allows those without investment projects to allocate their savings. Lenders are unable to freely observe producers' productivity shocks. Incentive compatibility then requires that debt contracts involve, with some positive probability, the costly state verification of the output of producers reporting sufficiently low levels of productivity. As all lenders may operate the technology that allows

verification, any lender can become an intermediary, and financial intermediation is competitive.

We assume that the costs of verification are proportional to the quantity of output produced. Implicit in this assumption is the belief that larger investments are associated with more complex production processes and generate higher verification costs. This proportional cost structure prevents the economy from resolving the informational asymmetries that drive financial intermediation, by simply accumulating wealth, as would occur in the more common case of fixed costs.⁶ Furthermore, this assumption allows the benchmark economy to exhibit balanced growth.

We assume, for simplicity, that all individuals have ex-ante identical preferences with logarithmic period utility and a constant subjective discount factor, $\beta \in (0, 1)$. To diversify the idiosyncratic risk associated with risky investment projects, producers participate in mutual insurance arrangements.⁷ Furthermore, given a large number of debt contracts, intermediaries can perfectly diversify risk for lenders and thus are not subject to monitoring themselves. Taken together, our assumptions on risk-sharing allow us to abstract from any potential effect of the inequality of wealth across producers and to focus instead on the effect of the difference in average wealth between producers and lenders upon the costs of financial contracting. Moreover, as seen in the next section, the insurance assumption allows us to capture the level of financial development through a single state variable describing the fraction of wealth held by lenders.

The sequence of events within each period is as follows. At the start of the period, the production lottery occurs. Lenders appoint financial intermediaries to manage their wealth. Producers enter into debt contracts with intermediaries and arrange group insurance. Next, each producer privately observes and reports his own productivity. At this time, debt contracts are honored and costly state verification may occur. Afterward, lenders receive the return on their deposits, while producers observe each others' output and honor their insurance agreements. Finally, given their wealth, agents determine the level of their current consumption. Note that, while no producer may usefully report another's output to any nonproducer, each is nonetheless able to observe others' productivity; thus producers can arrange for contingent trades among themselves. We assume that no agent can be identified by a lender in subsequent periods; this rules out multi-period lending arrangements. Finally, we assume the complete enforceability of all contractual arrangements.

Let H index the set of all agents, and define H_P (H_L) as the current set of producers (lenders). For any producer $i \in H_P$, output is given by the stochastic production technology $z(i)I(i)$, where $I(i)$ is total current capital invested in i 's production technology, and $z(i)$ is a shock that is independently and identically distributed across i , with support $[0, \alpha]$, $0 < \alpha < \infty$, and distribution function $F(z)$. $F(z)$ is \mathcal{C}^2 and $F'(z) > 0, \forall z \in (0, \alpha)$. We assume full depreciation of investment capital and define the unconditional expectation of productivity as $\zeta \equiv \int z dF(z)$. With $z(i)I(i)$ as output, auditing by any individual $j \in H_L$ will incur a cost of $\lambda z(i)I(i)$, where $\lambda \in (0, 1)$.

Thus, in the event that output is verified, net output is $(1 - \lambda) z(i) I(i)$.⁸ We informally refer to the producer's level of productivity, $z(i)$, as the state in some of the following discussion.

For any producer, let k represent the level of internal resources invested in production, and let b define the quantity of debt borrowed through a financial intermediary. Total investment in the producer's current project is $I = k + b$. The contract will allocate debt to maximize the expected profit of the producer, given his ability to diversify risk, subject to the intermediary's cost of lending. It will specify that the borrower must announce his state after production, and it will determine which states are to be verified and what the repayment for each is to be. We impose incentive-compatibility; the contract must ensure truthful reporting of the state by the borrower. Given our assumptions, the debt contract will be a standard debt contract.⁹

Feasibility constrains the repayment of the borrower to be no greater than output in any state. Consequently, asymmetry of information between borrower and lender provides the former with an incentive to misrepresent actual productivity in an effort to reduce his payment. This private information problem is overcome by the verification of all reported states involving sufficiently low (given the cost of lending and the amount borrowed) values of z . Within the remaining set of states, repayment must be state-invariant to be consistent with truthful reporting of productivity; define γ so that this cost is $\gamma(k + b)$. In any verified state, the intermediary receives all output net of verification costs, $(1 - \lambda) z(k + b)$. Note that k is collateralized net worth, as

its value, alongside the value of debt, is confiscated whenever the borrower is unable to repay the loan. To induce the borrower to truthfully report low productivity and request verification, the fixed repayment that prevails in the absence of verification must be infeasible for the borrower when actual productivity warrants verification. This implies that the repayment per unit investment in the absence of verification, γ , is also the highest verified level of productivity.

In summary, the competitive financial intermediary will solve the problem described by equations (1) and (2), given R and k . The objective function in (1) represents the borrower's expected return, while (2) ensures that the intermediary's expected return allows him to repay lenders the competitive return, R , on debt. For now, we suppress the borrower's individual rationality constraint; this is examined in lemma 2 below.

$$\max_{\gamma, b} \int_{\gamma}^{\alpha} (z - \gamma)(k + b)dF(z) \quad (1)$$

$$\text{subject to } \left[\int_0^{\gamma} (1 - \lambda)z dF(z) + \gamma(1 - F(\gamma)) \right] (k + b) \geq Rb \quad (2)$$

Define $J(\gamma)$ to be the intermediary's expected revenue per unit invested

$$J(\gamma) \equiv \int_0^{\gamma} (1 - \lambda)z dF(z) + \gamma[1 - F(\gamma)] \quad (3)$$

and note that $J'(\gamma) = 1 - F(\gamma) - \lambda\gamma F'(\gamma)$. We restrict the set of admissible distribution functions $F(z)$ by the following assumption.

Assumption A

$$F'(z) + zF''(z) \geq 0, \forall z \in (0, \alpha)$$

Our assumption implies that the expected cost of verification is not only increasing but also weakly convex in the range of states subject to auditing. This ensures a unique interior solution to the debt contract. It follows from this assumption that $J(\gamma)$ is strictly increasing on $(0, \bar{\gamma})$, where $\bar{\gamma} \in (0, \alpha)$ is the unique value such that $J'(\bar{\gamma}) = 0$, and is strictly concave. Thus the contract will never involve a level of repayment per unit invested, γ , above $\bar{\gamma}$. There is always a lower value of γ that would achieve the same revenue and be preferred by the borrower, as it implies less verification and consequently higher expected profit.

Lemma 1 establishes existence and uniqueness of the solution to the debt contract. The zero expected profit condition for the intermediary requires that (2) bind. Furthermore, given the proportionality of total expected verification costs to the level of output, the optimal value of γ is independent of k , the investment directly financed from the producer's net worth. However, k , which serves as collateral within the contract, determines loan size. For any given quantity of debt, b , and given a level of productivity, z , producers with greater levels of k will have higher output. This raises the intermediary's expected profit, given γ . Thus, given that intermediation is competitive, the loan size rises with k . The solution to the debt contract implies that γ satisfies $R = T(\gamma)$ where

$$T(\gamma) \equiv J(\gamma) + \frac{J'(\gamma)}{1 - F(\gamma)} \int_{\gamma}^{\alpha} (z - \gamma) dF(z). \quad (4)$$

It is straightforward to show that $T'(\gamma) < 0$.

Lemma 1 *Given assumption **A**, R and k , (a) there exists a unique $\gamma \in (0, \bar{\gamma})$ that solves every debt contract, where γ is the solution to $R = T(\gamma)$ and (b) $b = \frac{J(\gamma)}{R - J(\gamma)}k$.*

Proof. See appendix.

An important implication of lemma 1 is that all producers face the same distribution of returns on their share of investment, k . To see this, note that a producer's profit, given z , is $\max(0, (z - \gamma)(k + b))$. The expected return to producers, per unit of self-financed investment,

$$Q = \int_0^{\alpha} \max(0, (z - \gamma)) \frac{(k + b)}{k} dF(z), \quad (5)$$

may be written independently of k as

$$Q = R \frac{1 - F(\gamma)}{J'(\gamma)}. \quad (6)$$

We now prove that producers' individual rationality constraints are satisfied. Each individual with a production opportunity will choose to participate in a debt contract rather than producing autarchically ($Q > \zeta$) or freely disposing of his project and becoming a saver ($Q > R$).

Lemma 2 *The contract implies that $Q > \zeta > R$.*

Proof. See appendix.

From lemma 2 we know that in any period, $t = 0, 1, \dots$, an individual with a production opportunity will supply his entire savings, S_t , as internal finance into investment, $k = S_t$, and, given insurance, will earn ex post the expected return Q . Those without such investment projects will earn R as lenders. Let L_t be an indicator function that equals 1 if an agent obtains a production opportunity and 0 otherwise. After production, an agent allocates his wealth, $(QL_t + (1 - L_t)R)S_t$, between consumption, C_t , and savings, S_{t+1} . This allocation solves the following lifetime expected utility maximization problem, where uncertainty originates through the randomness of production opportunities and affects the return to savings.

$$\max_{\{C_t, S_{t+1}\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \log C_t \quad (7)$$

subject to

$$C_t + S_{t+1} \leq (QL_t + (1 - L_t)R)S_t, t = 0, 1, \dots, \quad (8)$$

$$S_0 \text{ given.} \quad (9)$$

Having stated the individual's problem, we use a recursive approach hereafter in characterizing behavior. Suppressing time subscripts, let $V(S)$ represent the expected lifetime utility of currently having a production opportunity and $W(S)$ be the expected lifetime utility of a current lender. With probability $1 - \varphi$, the agent will

remain a producer in the next period; otherwise he will become a lender. V solves the Bellman equation in (10) (primes indicate the value of a variable in the next period).

$$V(S) = \max_{S'} \left(\log(QS - S') + \beta \left[\varphi W(S') + (1 - \varphi)V(S') \right] \right) \quad (10)$$

Lenders' value functions, W , are defined by a similar functional equation:

$$W(S) = \max_{S'} \left(\log(RS - S') + \beta \left[\varphi W(S') + (1 - \varphi)V(S') \right] \right). \quad (11)$$

Standard methods (see Khan (1999)) may be used to prove that both lenders and producers save at the rate β ; the unit coefficient of relative risk aversion associated with logarithmic preferences implies that savings is unresponsive to the return risk generated by occupational uncertainty. Thus $S' = \beta QS$ for (10), and $S' = \beta RS$ in (11).

We now study equilibrium. Define the aggregate demand for debt as $B = \sum_{i \in H_P} b_i$, $K = \sum_{i \in H_P} k_i$ as the economywide level of internally financed investment and A_L as the aggregate supply of loans. The stationarity of the environment permits us to describe equilibrium as determined by a value of R such that: (a) $\gamma = T^{-1}(R)$ solves each contract (1) - (2) given k_i , $i \in H_P$; (b) Q satisfies (6); (c) taking R and Q as given, producers and lenders solve (10) and (11) respectively and (d) markets clear in every period, the aggregate demand for debt equals the total supply of loans by lenders,

$$B = \sum_{i \in H_L} S_i. \quad (12)$$

Let A represent aggregate savings and recall that $k_i = S_i$, for each $i \in H_P$, so

$$A = \sum_{i \in H_P} k_i + \sum_{i \in H_L} S_i. \quad (13)$$

Assuming a large number of agents, the fraction of the economy that does not obtain a production opportunity is constant and equal to φ in every period. This also represents the fraction of total savings held by lenders, $\sum_{i \in H_L} S_i = \varphi A$. Using this condition and (13) in (12), we derive the aggregate equilibrium debt to investment ratio $\frac{B}{B+K} = \varphi$. This implies that the equilibrium R is that which determines the value of γ such that the debt to investment ratio common across contracts, $\frac{b}{k+b}$, equals the fraction of savings held by lenders. Lemma 3 shows how this equilibrium condition may be used to directly determine γ .

Lemma 3 *In equilibrium γ solves $J(\gamma)/\varphi = T(\gamma)$ where $D_\varphi \gamma > 0$.*

Proof. See appendix.

Using lemmas 1 and 3 and equation (3), we rewrite (5) as

$$Q = \frac{1}{1-\varphi} \left(\zeta - \lambda \int_0^\gamma z dF(z) - R\varphi \right).$$

Since any agent receives Q with frequency $(1-\varphi)$ and R otherwise, the economy-wide average return on savings will be ρ as shown in (14).

$$\rho = \zeta - \lambda \int_0^\gamma z dF(z) \quad (14)$$

Thus the average return in the economy falls below the marginal product of investment by the amount of output lost due to verification. The rate of growth of the economy, given that individuals uniformly save at the rate β , is $\beta\rho - 1$.

2.1 Summary of results for the basic model

This model of financial contracts and growth provides a simple framework wherein the rate of growth is affected by the costs of external finance. Each period's production lottery leaves a fraction of individuals without direct access to a production opportunity. These lenders offer investment loans to producers through financial intermediaries. Given any required return on such loans, R , debt contracts determine the unit value of investment opportunities, Q , for producers. In equilibrium, the wealth of lenders relative to producers determines R to clear the market for debt.

The rationing of production opportunities implies a premium for producers relative to lenders, $Q > R$. An implication of the higher return to producers is that they completely invest their wealth into the risky production opportunity. Consequently, a weighted average of Q and R determines the mean return on investment, ρ , with the weights corresponding to the fraction of agents in the economy engaged in production and lending. As seen in (14), ρ is falling in the cost of verification, λ , per unit output. More importantly, it is declining in the fraction of individuals who are

lenders, φ . In particular, from (14) we have $D_\gamma \rho = -\lambda \gamma F'(\gamma) < 0$; therefore, using lemma 3, $D_\varphi \rho = (D_\gamma \rho) D_\varphi \gamma < 0$.

A higher value of φ implies a relative scarcity of investment opportunities and thus relatively high levels of lending associated with each available producer. In effect, a higher value for this parameter places a larger fraction of economywide resources in the hands of lenders. Fewer producers and more lenders lead to a higher equilibrium level of indebtedness associated with the generic investment loan. The rise in debt relative to total investment raises the likelihood that any particular firm will be unable to repay its loan and will be subject to the costly auditing of output. This increases γ , the range of productivity levels subject to verification, and raises the expected costs of verification for financial intermediaries. The return to lending falls, as does the average return on wealth in the economy. Hence φ serves as a measure of indebtedness across production units. Higher levels of φ imply reduced net worth, which serves as collateral, per unit debt. The reduction in collateral relative to debt raises the costs of lending and reduces the return on investment and the aggregate rate of economic growth. Thus, the principal result of the benchmark model is that the level of indebtedness characterizing financial contracts affects the overall mean return on investment and thus the rate of economic growth.

3 Financial development

We examine financial development by generalizing the basic model to allow for endogenous changes in the extent of access to financial intermediaries and external finance. This focus on the extent of the financial system is motivated by the common emphasis on the provision of external finance discussed above. Hence, we associate incomplete financial development with nonuniversal firm access to external finance. This limit on the number of producers participating in loan arrangements with financial intermediaries will raise the level of indebtedness and result in higher costs of borrowing and lending, relative to the previous model. However, higher profits for producers able to externally finance investment will induce others to undertake costly expenditures that, if successful, will allow them the use of investment loans. Over time, both the fraction of producers with access to external finance and the proportion of total wealth held by such individuals will rise. This will increase the level of collateralized net worth relative to debt within the typical investment loan. The associated fall in indebtedness will reduce the costs of financial intermediation for the reasons discussed at the end of the previous section, and in the limit the basic model developed there will describe this economy.

To limit the number of firms able to borrow, we assume that loan contracts require a borrower to possess a commitment technology. Ownership of this technology, which is specific to each agent and nontransferable, allows the holder to credibly commit to a loan contract and, in particular, to allow verification by lenders when necessary.

Producers operating without this commitment and verification technology are unable to borrow; the infeasibility of multi-period contractual arrangements implies that they would not repay loans, so they must self-finance all investment. The set of all individuals, borrowers and lenders, with access to the commitment technology is hereafter labeled the intermediated group, since producers in this set may borrow from financial intermediaries. The remainder of the economy are the unintermediated group. As before, producers engage in risk-sharing, though now within each distinct group. Note that the possession of a commitment device is relevant for an agent only when he wishes to borrow. Individuals are unaffected in periods when they are operating as lenders. Lenders in both the intermediated and unintermediated group deposit their wealth with financial intermediaries. However, these resources are lent only to firms in the intermediated group.

The extent of financial intermediation increases endogenously over time through technology adoption by individuals. Specifically, all agents currently without access to investment loans may invest resources to adopt a commitment technology. The investments made toward technology adoption occur at the end of a period, after production, and the payoff is uncertain. The probability of successfully acquiring the commitment technology, $\theta\delta$, where $\theta \in (0, 1)$, is increasing in the fraction of wealth invested, δ . This assumption captures the essential elements needed for a study of the interaction between financial and economic development: obtaining access to external finance is costly, with the probability of an individual's successfully gaining access

rising in the level of resources invested. Further, the assumption of constant cost per unit wealth will highlight the role of collateralized net worth in the process of financial development. Alternative cost structures with decreasing unit costs would yield predictable effects of economic development upon financial development.¹⁰

Let K represent the total savings or collateralized net worth of producers in the intermediated group. Define the ratio of the total savings of lenders, A_L , to the sum of their savings and the savings of these producers as $\chi = \frac{A_L}{K+A_L}$. Assuming that wealth held by producers in the unintermediated group is a nontrivial fraction of total producer net worth, χ exceeds φ with $\varphi \leq \chi < 1$. For producers with access to external finance, we will show that $1 - \chi$ is the equilibrium fraction of investment financed using own resources or collateralized net worth. As a result, χ is an aggregate state variable that determines the efficiency of financial intermediation, and thus the returns to producers and lenders, at any point in time. Define $R(\chi)$ to be the return to lenders and $Q(\chi)$ to be the return for producers within the intermediated group, per unit net worth. As we shall prove below, the aggregate state evolves over time according to a continuous process $\chi' = X(\chi)$ where $X : [\varphi, 1) \rightarrow [\varphi, 1)$.

In this economy with endogenous financial development, financial intermediation and production are similar to that described in section 2. When they have access to a production opportunity, agents become producers, investing their own resources in their production opportunity and augmenting their internal investment with debt financing if they belong to the intermediated group. Financial contracts between

producers and intermediaries continue to be described by (1) and (2), although, as already mentioned, R is now a function of χ . Thus the characterization of debt contracts provided by lemma 1 continues to apply, and all contracts involve a common value of γ that is independent of k . However, this value, which solves $T(\gamma) = R(\chi)$, now depends on χ ; $\gamma = \Gamma(\chi)$. Next, given $\Gamma(\chi)$, the return to producers in the intermediated group, $Q(\chi)$, is given by (6). Producers without access to loans earn a return of ζ on their investment. Lemma 2 continues to apply, and $Q(\chi) > \zeta > R(\chi)$. Therefore, all agents who obtain production opportunities, in either sector, invest their entire savings into their projects.

We now describe the savings problem facing producers and lenders. Let $V(S, \chi; 0)$ represent the value function of producers, and $W(S, \chi; 0)$ the value function for lenders, in the unintermediated group. Define $V(S, \chi; 1)$ and $W(S, \chi; 1)$ as the value functions for producers and lenders in the intermediated group, respectively. Given the known law of motion for χ , expected lifetime welfare for producers in the unintermediated group satisfies the following functional equation.

$$\begin{aligned}
V(S, \chi; 0) = \max_{S', \delta} & \left(\log(\zeta S(1 - \delta) - S') + \beta\theta\delta \left(\varphi W(S', \chi'; 1) + (1 - \varphi)V(S', \chi'; 1) \right) \right. \\
& \left. + \beta(1 - \theta\delta) \left(\varphi W(S', \chi'; 0) + (1 - \varphi)V(S', \chi'; 0) \right) \right)
\end{aligned}
\tag{15}$$

As before, S' is savings, while ζS is current wealth given ζ , the return on investment for producers without recourse to external finance. In devoting δ fraction of his wealth

to obtaining access to loans, the producer has $\theta\delta$ probability of permanently joining the intermediated group. Regardless of his group, in the next period he will be a lender with probability φ , and otherwise will remain a producer. The lifetime utility maximization problem for lenders is similar, with RS denoting current wealth:

$$W(S, \chi; 0) = \max_{S', \delta} \left(\log(R(\chi)S(1 - \delta) - S') + \beta\theta\delta \left(\varphi W(S', \chi'; 1) + (1 - \varphi)V(S', \chi'; 1) \right) + \beta(1 - \theta\delta) \left(\varphi W(S', \chi'; 0) + (1 - \varphi)V(S', \chi'; 0) \right) \right). \quad (16)$$

Access to external finance, once achieved, is permanent, and there is no need to invest additional resources towards retaining the use of the commitment technology. Thus $\delta = 0$ for agents in the intermediated group. Since the producers in this set are able to supplement their own savings when investing, they earn the return $Q(\chi)$ on savings. Given current wealth QS , their value function satisfies the following equation.

$$V(S, \chi; 1) = \max_{S'} \left(\log(Q(\chi)S - S') + \beta \left(\varphi W(S', \chi'; 1) + (1 - \varphi)V(S', \chi'; 1) \right) \right) \quad (17)$$

Lenders in this group have current wealth RS , but otherwise face a welfare maximization problem identical to that of producers in the intermediated group.

$$W(S, \chi; 1) = \max_{S'} \left(\log(R(\chi)S - S') + \beta \left(\varphi W(S', \chi'; 1) + (1 - \varphi)V(S', \chi'; 1) \right) \right) \quad (18)$$

Due to the constant unit elasticity of intertemporal substitution characteristic of logarithmic preferences, all agents save β fraction of their wealth, net of any investments made by those attempting to join the intermediated group.¹¹ Furthermore, this unit elasticity and the common future probability of obtaining investment projects imply that all individuals without the commitment technology allocate the same fraction of their wealth toward technology adoption. As a result, the optimal value of δ may be expressed independently of the level of savings, as indicated by (19), which applies to all individuals in the unintermediated group. Thus, the savings rate for any individual in the intermediated group is β , as before, while for those in the unintermediated group it is $\beta(1 - \delta)$.

$$\frac{1}{(1-\beta)(1-\delta)} = \beta\theta \left(\left[\varphi W(1, \chi'; 1) + (1 - \varphi)V(1, \chi'; 1) \right] - \left[\varphi W(1, \chi'; 0) + (1 - \varphi)V(1, \chi'; 0) \right] \right) \quad (19)$$

The left-hand side of (19) represents the loss in lifetime welfare from the resources devoted to technology adoption, while the right-hand side represents the discounted expected gain, per unit wealth. The larger this expected gain, the higher will be the fraction of resources devoted to joining the intermediated group. δ is a function of χ' , since this determines future returns for producers with access to investment loans,

and thus the gain from being able to use these loans. Since $\chi' = X(\chi)$, we write $\delta = \Delta(\chi)$.

We now study equilibrium using the approach of section 2. Define A_1 (A_0) as total savings within the intermediated (unintermediated) group, and $A = A_1 + A_0$ as aggregate savings. The sum of investment across firms with access to external finance is $K + B$. As before, in equilibrium, the aggregate quantity of debt $B = A_L$, implying that the debt to investment ratio $\frac{B}{K+B} = \chi$. Summing across groups, lenders' total savings is $A_L = \varphi A$, again as before. However, now, recalling that production opportunities are independently and identically distributed across agents, the net worth of producers in the intermediated group is $K = (1 - \varphi) A_1$. This gives

$$\chi = \frac{\varphi A}{(1 - \varphi) A_1 + \varphi A}. \quad (20)$$

Since $0 < A_1 \leq A$, it follows that $\varphi \leq \chi < 1$. Generalizing lemma 3 by replacing φ with χ , the relevant debt to investment ratio, we find that $\Gamma(\chi)$ will satisfy (21) in equilibrium.

$$\frac{J(\Gamma(\chi))}{\chi} = T(\Gamma(\chi)) \quad (21)$$

Lemma 4 *In equilibrium, $\Gamma(\chi)$ solves (21) and $\Gamma'(\chi) > 0$, $Q'(\chi) > 0$ and $R'(\chi) < 0$.*

Proof. See appendix.

When χ exceeds φ , the relatively low levels of collateralized net worth across borrowers raises indebtedness within the typical loan contract, relative to the model of section 2. This increases the possibility of insolvency and increases γ , raising the likelihood of costly verification, and thus the costs of financial contracts.

We have seen that financial equilibrium is entirely a function of the debt to investment ratio, χ , which is a measure of the indebtedness of the typical borrower-producer relative to total resources in production. We complete our study by examining the evolution of χ over time. Solving the equilibrium condition (20) for A_1/A allows us to write the proportion of wealth in the intermediated group as a function of χ :

$$a(\chi) \equiv \frac{A_1}{A} = \frac{(1-\chi)\varphi}{(1-\varphi)\chi} \quad (22)$$

where $a(1) = 0$, $a(\varphi) = 1$ and $a'(\chi) = -\frac{\varphi}{(1-\varphi)\chi^2} < 0$.

Let the average rate of return for individuals in the unintermediated group be R_0 and that for those in the intermediated group be R_1 . In the former set, φ fraction of the population are lenders and earn $R(\chi)$, while the remainder, producers, earn ζ .

$$R_0(\chi) = \varphi R(\chi) + (1-\varphi)\zeta. \quad (23)$$

In contrast, producers in the intermediated group earn $Q(\chi)$.

$$R_1(\chi) = \varphi R(\chi) + (1-\varphi)Q(\chi) \quad (24)$$

Since producers with access to investment loans earn higher returns on their production opportunities, $Q(\chi) > \zeta$, the mean return on savings is higher for those in the intermediated group, $R_1(\chi) > R_0(\chi)$. As shown below, this alone is sufficient to raise the net worth of borrowers relative to lenders. Moreover, the inflow of wealth associated with successful technology adoption serves to reinforce this growth in the fraction of wealth held by producers using investment loans.

Suppressing dependence on χ , the law of motion for wealth held by the set of agents in the unintermediated group is $A'_0 = \beta(1 - \delta)R_0(1 - \theta\delta)A_0$, where $\delta = \Delta(\chi)$ and $\beta(1 - \delta)$ is the net savings rate. Fraction $\theta\delta$ of this set transfers to the intermediated group; the strong law of large numbers implies that the same fraction of wealth is transferred. Recall that the savings rate across the rest of the population, those belonging to the intermediated group, is β . This implies that the law of motion for total wealth held in this set is $A'_1 = \beta R_1 A_1 + \beta\theta\delta(1 - \delta)R_0 A_0$. We may write

$$a(\chi') = \frac{A'_1}{A'_1 + A'_0} = \frac{R_1 a + \theta\delta(1 - \delta)R_0(1 - a)}{R_1 a + (1 - \delta)R_0(1 - a)}. \quad (25)$$

This allows us to infer the law of motion for the aggregate state variable, $\chi' = X(\chi)$. Assume $\chi = \varphi$; then $a(\chi) = a(\chi') = 1$ and $\chi' = \chi$. Thus $\chi = \varphi$ is a steady state. The steady state economy is described by the model of section 2. However, when $\chi > \varphi$, $a(\chi) < 1$, and the proportion of wealth held by agents in the intermediated group grows, since $R_1 > R_0 > 0$. Thus $a(\chi') > a(\chi)$ or $\chi' < \chi$. We have proven the following proposition.

Proposition 1 χ evolves over time according to $\chi' = X(\chi)$, such that, given any initial value $\chi_0 \in (\varphi, 1)$, χ converges monotonically to φ .

The period by period reductions in χ imply equivalent decreases in borrowing producers' indebtedness. Lemma 4 indicates that the equilibrium value of γ falls. We define this process, given its implications for the efficiency of intermediation, as financial development. Financial development raises the return on debt. However, increases in its relative supply reduce the premium on collateralized net worth, and Q declines.

We are now in a position to study the impact of financial development on growth. The average return on wealth in the economy is $\rho(\chi) \equiv [1 - a(\chi)]R_0(\chi) + a(\chi)R_1(\chi)$. Our existing results allow us to obtain an analogue of (14):

$$\rho(\chi) = \zeta - \frac{\lambda\varphi}{\chi} \int_0^{\Gamma(\chi)} z dF(z). \quad (26)$$

Growth in the economy, $g(\chi) \equiv A'/A$, may be calculated using the fact that $A' = A'_1 + A'_0$, along with the laws of motion for A_1 and A_0 .

$$g(\chi) = \beta \left(a(\chi)R_1(\chi) + (1 - \Delta(\chi))(1 - a(\chi))R_0(\chi) \right) \quad (27)$$

Examining (26), we see that the average return on wealth is the expected return from the production technology net of verification costs, adjusted for the fraction of production subject to intermediation. Over the course of financial development, χ

falls to its steady state value of φ , so the rate of return given by (26) will eventually equal that in (14). While χ exceeds φ , access to loans is limited, and there is less costly intermediation per unit investment. However, this is associated with relatively low levels of internal resources held by borrowers. Scarcity of collateralized net worth implies higher costs of financial intermediation: $\Gamma(\chi) > \Gamma(\varphi)$. The limited access to intermediation reduces verification costs, while the increased indebtedness of those who are able to borrow raises costs. At any given level of χ , the effect of a small reduction on the overall rate of return and on growth is ambiguous without additional restrictions upon the distribution of productivity, $F(z)$. Rather than exploring sufficient conditions for monotonicity of $\rho(\chi)$ and $g(\chi)$, we prove that the process of financial development, represented by a reduction in χ , increases both on average.¹²

Proposition 2 *Financial development increases both the mean return on savings and the rate of economic growth on average.*

Proof. See appendix.

The savings rate, which is independent of wealth or occupation in either group, is at least as high in the intermediated group as in the unintermediated group; $\beta \geq \beta [1 - \Delta(\chi)]$. Additionally, the return to production, per unit net worth, is also larger for this first set; $Q(\chi) > \zeta$. As a result, the shift in production from the unintermediated to the intermediated group raises the mean economywide return to producers, as well as the overall return to investment. Moreover, on average, the rate of growth of producer wealth rises.

We conclude this section by examining the effect of financial development on the spread between borrowing and lending rates. The return on debt is the opportunity cost of internal funds for a producer, while the borrowing rate measures his cost of externally financing investment. The interest rate spread provides a measure of the external finance premium, the additional cost of external funds. To calculate the expected real cost of borrowing, we must take into account the probability of verification and the resultant confiscation of output, as well as the nonverified repayment of γ . We call C the real expected loan rate, which is expected output net of expected profit of the producer per unit debt. $Cb = \int_0^\alpha z(k+b) dF(z) - \int_0^\alpha (z-\gamma)(k+b) dF(z)$. Using (3) and (21) we have:

$$C(\chi) = \frac{J(\Gamma(\chi)) + \lambda \int_0^{\Gamma(\chi)} z dF(z)}{J(\Gamma(\chi))} T(\Gamma(\chi)). \quad (28)$$

Define $M(\chi)$ to be the markup between the borrowing rate, $C(\chi)$, and the lending rate, $R(\chi)$; $M(\chi) = C(\chi)/R(\chi)$. The following result implies that this interest rate spread is increasing in χ .

Lemma 5 $M(\chi)$ is increasing in χ .

Proof. See appendix.

This establishes that the external finance premium falls as the economy financially develops.

3.1 Summary of results for financial development model

We have examined financial development in an economy where the extent of financial intermediation, initially limited, endogenously increases over time. The lack of universal firm access to investment loans implies that the debt to investment ratio across firms in the intermediated group, χ , exceeds its long-run value of φ . This yields a relatively high level of indebtedness, which in turn drives up the costs associated with the provision of external finance. However, the gain, per unit producer net worth, of having access to external finance, $Q(\chi) - \zeta$, provides an incentive for agents in the unintermediated group to attempt technology adoption. Over time, more producers are able to borrow, and the fraction of wealth held by the intermediated group rises. As the wealth of borrowers relative to lenders rises, collateralizable net worth, relative to debt, increases within the typical borrowing arrangement, and χ decreases. Consequently, the likelihood that a producer will be unable to repay his investment loan, and will induce costly verification, is reduced; $\Gamma(\chi)$ falls. As a result, the costs of providing external finance fall and intermediation becomes more efficient. Thus, as the provision of external finance broadens, financial development occurs. On average, this raises the return on investment and the economywide rate of growth. The increases in the availability of investment loans are also associated with a rise in the rate of growth of firms. Note the interaction between finance and growth. Growth causes financial development, in part by raising the wealth of existing borrowers and in part by providing incentives for others to acquire the ability to borrow. Finan-

cial development, in turn, raises the return to investment and thereby drives higher growth.

Our focus above has been on the costs of external finance or, equivalently, the efficiency of financial contracting. Much of the empirical evidence estimates increases in the level of financial development using measures of the extent of the financial system. Financial development is commonly associated with a rise in the level of external finance, relative to GDP. The process of financial development described here implies such a long-run rise in the provision of external finance. In particular, the long-term increase in the rate of economic growth implies that borrowing, which represents external finance in this model, will rise relative to output. To see this, note that when total predetermined savings or investment is A , gross domestic product is ζA . The fraction of this output that will comprise external finance, given a debt to investment ratio of χ , is equal to the current savings of next period's lenders, $\varphi g(\chi) A$. The ratio of external finance relative to GDP is then $\frac{g(\chi)\varphi}{\zeta}$. Proposition 2 indicates that this ratio rises over time. The rise in the rate of economic growth associated with reductions in indebtedness, χ , implies increases in the provision of external finance. Thus, we see that increases in the efficiency of financial intermediation raise the rate of economic growth and, in turn, these improvements in growth cause further increases in the extent of the financial system.

4 Concluding remarks

There is by now a large quantity of empirical evidence that financial development is correlated with and perhaps causes economic growth. Attempts to evaluate the importance of the financial system for growth have emphasized the external finance channel. Increases in the supply, or reductions in the costs, of external finance appear to stimulate economic growth by increasing productivity across firms. We have developed a dynamic general equilibrium model to address these empirical findings. In contrast to previous theoretical work, our model economy includes a nontrivial role for external finance in the financial development process. Our theory is based on the costs of external finance that arise when there are informational asymmetries between borrowers and lenders that require costly state verification of the returns to investment. Financial development is associated with reductions in these costs as the resources of borrowers increase. The implications of changes in net worth for the costs of investment finance have also been emphasized by studies of the role of the credit channel in the propagation of business cycles. In particular, the financial accelerator model examines how procyclical changes in the collateralized net worth of borrowers amplify business cycle fluctuations.¹³ By comparison, in the financial development model studied here, both economic growth and increases in the provision of external finance steadily raise borrowers' net worth relative to their debt. This raises the efficiency of financial contracting, which in turn increases the mean return on investment and, thus, the rate of economic growth. Our explicit focus on

the role of external finance in the relationship between financial development and economic growth has led to potentially useful implications for empirical work. Financial development generated through the external finance channel raises the return on loans and reduces the spread between the borrowing rate and the return on loans. Such results may help direct future empirical analysis seeking to distinguish between theories of financial development or to establish the quantitative importance of the external finance channel in a general theory of financial development.

Notes

1. Examples include Bagehot's writings (1873, pages 1 - 20) on 19th century English financial markets, Cameron's (1967, chapter III, page 75) work comparing the Scottish and English banking systems, Hicks' (1969, page 141 - 148) study of the Industrial Revolution, and the discussion in McKinnon (1973, pages 10 - 14).
2. Many common measures of financial development are based on the provision of external finance. For example, King and Levine (1993) use domestic credit relative to GDP as a measure of the extent of the financial system, while Rajan and Zingales (1998) add stock market capitalization. Examining these sources of external finance, Levine and Zervos (1998) find that both banking sector development and active, large stock markets are significant predictors of economic growth. See Levine (1997) for a recent survey of this and related evidence.
3. This work is further extended by Levine, Loayza and Beck (1998) and Beck, Levine and Loayza (1999).
4. An early example of a formal model is Bernanke and Gertler (1989).
5. See also Bencivenga and Smith (1991) and Greenwood and Smith (1997).
6. Examples of costly state verification with fixed costs include Townsend (1979), Gale and Hellwig (1985), Williamson (1986), Boyd and Smith (1994) and Gertler

and Gilchrist (1994). An assumption of fixed verification costs in our model would imply higher returns to wealthier producers. This would complicate our analysis by destroying the equilibrium approach we follow below and would introduce the entire distribution of wealth into the aggregate state vector.

7. There is considerable evidence of effective risk-sharing across individuals, even in environments that are relatively undeveloped. Examining risk-sharing within Indian villages, Townsend (1994) concludes that full insurance is “a surprisingly good benchmark” (p. 539). Similarly, Udry (1993), in a study of risk-pooling in rural Northern Nigeria, finds no evidence of informational asymmetries between households engaging in mutual insurance arrangements.
8. We assume that the auditing technology and legal framework prevent stochastic auditing. Boyd and Smith (1994) show that the welfare gains from stochastic auditing are likely to be small.
9. A formal derivation of the debt contract is provided in the Technical Appendix to Khan (1999), available from the author upon request. Townsend (1979) and Gale and Hellwig (1985) provide analyses of insurance and debt contracts under the assumption of costly state verification. Williamson (1986) examines equilibrium financial intermediation using a costly state verification paradigm.
10. Abstracting from borrowing and lending, Greenwood and Jovanovic (1990) provide a detailed analysis of the effects of fixed costs in a related environment.

11. A formal characterization of the optimal policy for (15) - (18) is straightforward but lengthy and included in Khan (1999).
12. Sufficient conditions under which $\rho(\chi)$ and $g(\chi)$ both increase monotonically as χ falls are derived in Khan (1999). One example that satisfies these conditions is the case of uniformly distributed productivity shocks, $F(z) = \frac{z}{\alpha}$, with $\varphi \geq 1/2$. The distribution assumption is sufficient for $\rho'(\chi) < 0$, and the restriction on φ then ensures that $g'(\chi) < 0$.
13. See, for example, Bernanke and Gertler (1989, 1990) or Gertler and Gilchrist (1994). Additional references are contained in the recent work of Bernanke, Gertler and Gilchrist (1998), which also provides a more detailed discussion.

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Appendix

Proof of lemma 1:

Part (a): Define $P(\gamma) \equiv \int_{\gamma}^{\alpha} (z - \gamma) \frac{R}{R - J(\gamma)} dF(z)$. The debt contract is equivalent to $\max_{\gamma \in [0, \alpha]} [P(\gamma)k]$. Note that the optimal value of γ is independent of k and $P'(\gamma) = \frac{R}{R - J(\gamma)} \left(\frac{J'(\gamma)}{R - J(\gamma)} \int_{\gamma}^{\alpha} (z - \gamma) dF(z) - [1 - F(\gamma)] \right)$. As $\zeta > R$ (see lemma 2), $P'(0) = \frac{\zeta}{R} - 1 > 0$ while $P'(\bar{\gamma}) = -\frac{R[1 - F(\bar{\gamma})]}{R - J(\bar{\gamma})} < 0$ and there exists $\gamma \in (0, \bar{\gamma})$ with $P'(\gamma) = 0$. For any such γ , assumption **A** guarantees $P''(\gamma) |_{P'(\gamma)=0} < 0$ hence it is unique. Finally, The first-order condition, $P'(\gamma) = 0$, may be rewritten as $R = T(\gamma)$ where $T(\gamma)$ is given by (4).

Part (b): Equation (2) may be rewritten to yield the optimal level of debt, $b = \frac{J(\gamma)}{R - J(\gamma)} k$. ■

Proof of lemma 2: Note that when $\gamma = 0$, $Q = T(0) = \zeta$. Given assumption **A**,

$$\frac{dQ}{d\gamma} = \frac{J(\gamma)}{[J'(\gamma)]^2} (\lambda(F'(\gamma) + \gamma F''(\gamma)) + F(\gamma)F'(\gamma) + \lambda\gamma(F'(\gamma))^2) > 0,$$

while $R = T(\gamma)$ with $T'(\gamma) < 0$. As lemma 1 proves $\gamma > 0$, we have $Q > \zeta > R$. ■

Proof of lemma 3: In equilibrium, $\frac{B}{B+K} = \varphi$. Summing b_i over H_P and substituting $R = T(\gamma)$, we have $B = \frac{J(\gamma)}{T(\gamma) - J(\gamma)} K$ or $J(\gamma)/\varphi = T(\gamma)$. Both $J(\gamma)$ and $T(\gamma)$ are continuous; $J'(\gamma) > 0$, $T'(\gamma) < 0$; $J(0)/\varphi = 0 < \zeta = T(0)$ and $J(\bar{\gamma})/\varphi = T(\bar{\gamma})/\varphi > T(\bar{\gamma})$. This proves existence of a solution, $J(\gamma)/\varphi = T(\gamma)$ with $\gamma \in (0, \bar{\gamma})$. Uniqueness follows from $\frac{d\gamma}{d\varphi} = \frac{J(\gamma)}{\varphi[J'(\gamma) - \varphi T'(\gamma)]} > 0$. ■

Proof of lemma 4: Substituting χ for φ in the proof of 3 yields $J(\gamma)/\chi = T(\gamma)$, which implies $\gamma = \Gamma(\chi)$ with $\Gamma'(\chi) = \frac{J(\Gamma(\chi))}{\chi[J'(\Gamma(\chi)) - \chi T'(\Gamma(\chi))]} > 0$. Next the proof of 2 establishes $Q'(\chi) = \frac{dQ}{d\gamma}\Gamma'(\chi) > 0$ and $R'(\chi) = T'(\Gamma(\chi))\Gamma'(\chi) < 0$. ■

Proof of proposition 2: We will prove that (1) $\lim_{\chi \rightarrow 1} \rho(\chi) < \rho(\varphi)$ and (2) $\lim_{\chi \rightarrow 1} g(\chi) < g(\varphi)$. To prove (1), note that $\lim_{\chi \rightarrow 1} \rho(\chi) = \varphi R(1) + (1 - \varphi)\zeta$, while $\rho(\varphi) = \varphi R(\varphi) + (1 - \varphi)Q(\varphi)$. Using lemma 4, we know that $R(1) < R(\varphi)$ while $Q(\varphi) > \zeta$. To prove (2), note that we have just established that $R_U(1) < R_I(\varphi)$. Since $0 \leq [1 - \Delta(\chi)] \leq 1$, the result follows from $\lim_{\chi \rightarrow 1} g(\chi) = \beta[1 - \Delta(X(1))]R_U(1) < \beta R_I(\varphi) = g(\varphi)$. ■

Proof of lemma 5: Using (22), and suppressing the dependence of γ upon χ , we have $I(\chi) = \frac{J(\gamma) + \lambda \int_0^\gamma z dF(z)}{J(\gamma)}$ and $I'(\chi) = \frac{J(\gamma)\gamma F'(\gamma) - J'(\gamma) \int_0^\gamma z dF(z)}{[J(\gamma)]^2} \Gamma'(\chi)$. Given lemma 4, the sign of $I'(\chi)$ is the same as the sign of the numerator. Taking the limit, $\lim_{\gamma \rightarrow 0} (J(\gamma)\gamma F'(\gamma) - J'(\gamma) \int_0^\gamma z dF(z)) = 0$. Differentiation of this term yields $J(\gamma)(F'(\gamma) + \gamma F''(\gamma)) - J''(\gamma) \int_0^\gamma z dF(z) \geq 0$ (> 0 if $\gamma > 0$). We conclude that $I'(\chi) > 0$. ■