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Working Papers

Research Department

WORKING PAPER NO. 97-26

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October 1997

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October 1997

[1]Previous versions of this paper were presented at the Midwest Macroeconomics Conference, the SED Conference, the RSAI Conference, FRB System Regional Meeting, FRB Minneapolis and Philadelphia, Clemson University and the University of Pennsylvania. We thank many seminar and conference participants for their useful comments. We also thank Victoria Coupland and Michael McLaughlin for excellent research assistance, and Sally Burke for helpful editorial comments. The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System.

Abstract

In this paper, we document a pronounced trend toward deconcentration of metropolitan employment during the postwar period in the United States. The employment share of initially more dense metro areas declined and those of initially less dense metro areas rose. Motivated by this finding, we develop a system-of-cities model in which increase in aggregate metropolitan employment causes employment to shift in favor of less dense metro areas because congestion costs increase more rapidly for the initially more dense metro areas. A calibrated version of the model shows that the more than twofold increase in employment experienced by MSAs during the postwar period was indeed a powerful force favoring deconcentration.

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Introduction

Economists have long recognized that, by locating in cities, firms can lower their costs of production by taking advantage of agglomeration economies. In some cases, firms benefit from being near similar firms, because it allows them dip into the city's pool of specialized workers or specialized products (localization economies). In other cases, firms benefit from the great variety of workers and services a big city offers (urbanization economies). In the not so distant past, many urban economists thought the benefits of city location was so strong that they predicted the continued concentration of economic activity in comparatively few metropolitan places (see Gottmann (1961)). Indeed, the ultimate vision was the development of "megalopolises," more or less continuous stretches of urban and suburban areas, encompassing a number of metropolitan places ("divergent growth").

Even while the predictions of megalopolises were being advanced, other forces were at work leading to a deconcentration rather than an increased concentration of economic activity. During the postwar period, employment in big metro areas grew less rapidly than employment in smaller metro areas. A flurry of papers in the early to mid-1980s attempted to account for this deconcentration of people and jobs as being related to greater preference for less urbanized living (Beale (1977,1982)); to reduction in urban agglomeration economies due to technological change (Garnick and Renshaw (1980), and Carlino (1985)); and to government policies (Coleman (1978) and Leven (1978)).

In this paper, we evaluate an alternative explanation that focuses on congestion costs as a factor underlying the spatial deconcentration of economic activity. The starting point for this explanation is the fact that in the United States aggregate metropolitan employment has more than doubled since 1950. If greater employment causes congestion costs to increase proportionately more for more dense urban areas, then even with unchanged preferences, technology, and government policies, increase in aggregate metropolitan employment could imply a more dispersed spatial distribution of employment.

To evaluate this explanation, we first document the extent of employment deconcentration in the United States during the postwar period. We do this by using county-level data to construct employment shares of the top 1, 5, 10, 20, 30, 50 and 70 percent most dense metropolitan statistical areas (MSAs) at roughly 10-year intervals beginning in 1951. These "Lorenz curves" for employment distribution clearly show that employment was highly concentrated in all years. They also show that the extent of concentration has declined over the postwar period: the "Lorenz curves" for later years lie completely inside those for earlier years. For instance, we estimate that in 1951, the top 1 percent most dense MSAs accounted for 14 percent of total employment and the top 50 percent most dense MSAs for 69 percent. By 1994, these employment shares had fallen to 6 percent and 64 percent, respectively.

Next, we develop a general equilibrium model of employment determination for an economy with a fixed number of locations and fixed land areas for each location. We show that an increase in total employment in this model may cause the equilibrium employment share of dense locations to decline and those of less dense locations to rise. Then, guided by micro studies in the urban and regional economics literature, we select numerical values for key model parameters and calibrate the model to reproduce the employment density of each MSA in 1951. We then use our calibrated model to predict the effects of an increase in aggregate metropolitan employment of the magnitude seen in the United States during the postwar period. We find that an increase in aggregate metropolitan employment of this magnitude is a strong force favoring deconcentration of employment. Indeed, our calibrated model generates *more* deconcentration of employment than is actually observed over this period. This finding is robust to variations in the values of the parameters governing the strength of agglomeration economies and congestion costs and to the details of the calibration step.

Our finding that aggregate metropolitan employment growth should have caused more deconcentration of employment than actually observed suggests that changes in technology or policies may have actually retarded the dispersal of employment. Thus, rather than thinking about changes in technology and policies as reasons for deconcentration they might be considered as possible candidates to explain the better-than-predicted growth of the relatively dense metro areas.

Our study complements recent studies on urban growth. Glaeser, Schienkman and Shleifer (1995) examine population growth in a cross-section of US cities between 1960 and 1990. For the most part, Glaeser et al focus on cities rather than MSAs, but their result for MSAs indicate that MSAs in the Northeast grew more slowly than MSAs elsewhere. Glaeser et. al's finding is broadly consistent with ours since the Northeast contains a relatively large share of the nation's dense metropolitan areas.

More recently, Eaton and Eckstein (1997) examined growth in urban areas in France and Japan and found that all cities grow at the same rate regardless of population size. They refer to this as evidence of parallel growth among the urban areas they study. In a related paper, Black and Henderson (1997) examined the evolution of urban population and the number of metropolitan areas in the United States between 1900 and 1950. They also characterize their finding as one of parallel growth of metropolitan areas of different sizes and types maintaining (with the entry of new metro areas) a stable relative size distribution of metropolitan areas over time. Our finding of employment deconcentration for the postwar United States seems different from the findings of these two studies. However, it is difficult to compare these studies with ours because we relate urban employment growth to urban employment *density* rather than just urban population size. As we discuss later, Black and Henderson's findings may be consistent with our view of employment deconcentration.

2 Facts

Data

We use *County Business Patterns* (CBP) data for the years 1951, 1959, 1969, 1979, 1989 and 1994 to document the evolution of the spatial distribution of aggregate metropolitan employment. The data consist of full- and part-time employees covered by the Federal Insurance Contributions Act (FICA).[1] Generally, employees of establishments totally exempt from FICA, such as most government employees, self-employed persons, and railroad employees, are excluded from *County Business Patterns*. Our data set consists of 2,618 of the 3,137 counties and county equivalents (boroughs, independent

[1] *County Business Patterns* data reflect employees on the payrolls of covered firms during the first quarter of the year. With the exception of 1951 and 1994, the first quarter for all other years in our sample occurred about one year before business-cycle peaks. The first quarter of 1951 occurred two years before a business-cycle peak. At this writing, a peak in the expansionary phase of the business cycle that began in the second quarter of 1991 has not been reached. Nonetheless, five of the six time periods between 1951 and 1994 occurred at about roughly the same phase of business-cycle expansions, and all six periods occurred during an expansionary phase of the cycle.

cities, parishes, etc.) comprising the United States.[2] Data on variables other than employment (population and land area of counties) were taken from *The City and County Data Book.*

Although counties represent a finer level of geographical detail, we chose *metropolitan statistical areas* (MSAs) as the geographical unit for our analysis. An MSA typically consists of a central city of at least 50,000 people, as well as any contiguous counties that are metropolitan in character, as determined by the percentage of its nonagricultural labor force and by the amount of commuting between the counties and the central city.

For each of the six years, we constructed a common set of MSAs by combining counties according to the 1983 classification of metropolitan areas. This procedure gave us a sample consisting of employment and other data for 297 MSAs. Although some of these MSAs would not qualify as MSAs in earlier years (according to the MSA definition implicit in the 1983 classification of metro areas), for ease of exposition we continue to refer to these "urban locations"of earlier years as MSAs.

As noted previously, the employment coverage of CBP is not complete. To get some indication of the number of workers missing from our data but present in our MSAs we compared the total number of workers in our MSAs to total nonfarm payroll employment in each of the six years. By this measure, we appear to have anywhere between 63 to 68 percent of all workers in our MSAs. Since rural areas must have accounted for *some* nonfarm employment, this measure is in the nature of a lower bound.

The Distribution of Employment and the Pattern of Employment Deconcentration

We use our sample to document the degree of spatial inequality of aggregate U.S. employment and how that inequality has changed over time. We do this by ranking MSAs in each year by their *employment densities* in that year and then observing what share of total CBP employment is accounted

[2] We have less than complete coverage of counties for a variety of reasons. Some counties that were separately identified in later years in our sample were combined with other counties in 1951. Rather than exclude these counties from our data set, the counties that were combined in 1951 were combined for all periods in our data set. In addition, the independent cities in Virginia and the independent cities of St. Louis, MO, and Washington, DC,which are treated by the Census Bureau as separate counties, were dropped from our analysis.

for by the top 1 percent to 70 percent of most dense MSAs in each year. If the spatial distribution of workers *not* covered by CBP is similar to those of covered workers, these shares should be a reasonable estimate of the spatial distribution of *all* U.S. workers.

Table 1: Evolution of Employment Distribution Across MSAs,1951-94

Year	1%	5%	10%	20%	30%	50%	70%
1951	.1367	.2969	.4225	.5028	.5925	.6932	.7495
1959	.1265	.2958	.4109	.5016	.5991	.6923	.7561
1969	.1105	.2734	.3925	.4919	.5896	.6944	.7512
1979	.0824	.2353	.3301	.4877	.5736	.6671	.7366
1989	.0732	.2268	.3327	.5002	.5735	.6761	.7418
1994	.0554	.1966	.2859	.4572	.5358	.6417	.7097
%change 1951-94	-59.0	-33.8	-32.3	-9.1	-9.6	-7.4	-5.3

Table 1 shows that dense MSAs account for a disproportionately large share of aggregate metropolitan employment in each of our years. For instance, in 1951 the 1 percent, or three most dense MSAs, accounted for 14 percent of aggregate metropolitan employment; the 10 percent, or 30 most dense MSAs, for 42 percent; 20 percent, or 60 densest MSAs for one-half of aggregate metropolitan employment. Thus, the distribution of aggregate metropolitan employment is remarkably concentrated.

Table 1 also shows that the distribution of employment has changed over time. While the three most dense MSAs accounted for 14 percent of total national employment in 1951, the proportion fell to just 5.5 percent by 1994, a 59 percent decline. As the last row of Table 1 shows, the percentage of aggregate metropolitan employment accounted for by the 30 densest MSAs fell 32 percent between 1951 and 1994. The shares of the 20 percent to 70 percent most dense MSAs also fell between 1951 and 1994, but to a lesser extent than for the 10 percent most dense MSAs. In this sense, the postwar period in the United States has been characterized by *deconcentration of aggregate metropolitan employment.*

Figure 1, showing plots of the entire employment distribution curves for MSA employment for1951, 1959, 1969, 1979, 1989, and 1994 makes this deconcentration trend abundantly clear: observe that the distribution curves for successive "decades" lie inside each other.[3]

[3] In Figure1, employment in each percentile is divided by total MSA employment for that year instead of total CBP employment.

As noted in the introduction, our finding of employment deconcentration for the postwar U.S. seems different from Black and Henderson's finding of "parallel growth" for the 1900 to 1950 period in the U.S. While differences in time period and unit of observation (Black and Henderson examine population rather than employment) may play a role, it is possible that the differences may be more apparent than real. Black and Henderson's notion of "parallel growth" involves (in part) the entry of new metropolitan areas. Because we use the 1983 MSA classification of metro areas, some locations in our sample weren't MSAs in the earlier years. Thus, if classification of existing locations into MSAs is taken to mean "entry" of new MSAs, the number of metro areas grew in our sample as well. Thus, our data may be consistent with their notion of "parallel growth." Equivalently, Black and Henderson's findings may be consistent with our notion of employment deconcentration in that their "new" urban areas were also locations that were initially less dense.

The Issue

In this paper we wish to examine the extent to which the increase in aggregate metropolitan employment was responsible for the pattern of employment deconcentration documented above. As noted earlier, our hypothesis is that locations that were relatively dense in 1951 faced higher congestion costs at the margin than less dense locations. This suggests that over time, the employment share of locations that were dense in 1951 would fall and that of locations that were relatively less dense in 1951 would rise.

Figure 2 shows plots of the evolution of employment share between 1951 and 1994 for locations ranked by their 1951 employment densities. We band locations into 10 groups: the first nine groups have 30 MSAs each and the tenth has 27. As is evident, there is a clear tendency for the employment share of less dense locations to rise at the expense of more dense locations.[4] Thus, it seems at least reasonable to pursue the question posed in this paper.

To evaluate the role of an increase in aggregate metropolitan employment in employment deconcentration, we need a model. The model we use in this paper is inspired by the system-of-cities model described in Henderson (1987). The key conceptual difference between Henderson's system-of-cities

[4] A similar tendency is discernable in ungrouped data as well. This information is presented later in the paper.

model and ours is that we take the number of urban locations ("cities") in the economy as exogenously given and fixed in land area. We do this to conform to our data in which the number and areas of each location (MSA) is fixed. A second important difference is that we allow a location's employment density to play an important role in the model. In all other respects, our model is considerably simpler than Henderson's. The simplicity is forced on us because we have to use estimates of agglomeration economies and congestion costs available from micro studies to render our model numerical. We believe that these estimates, which are crude and imprecise, cannot be used to "calibrate" a system-of-cities model any more sophisticated than the one we present in the next section.

3 The Model

There are M locations indexed by $i = 1, 2, 3, ..., M$ and a large number of individuals who live and work in these locations. The technological opportunities available to these people, their preferences and endowments, and market equilibrium conditions are described below.

Technology

There is one internationally traded good and M local goods. The production function for producing the internationally traded good in location i is:

$$Y = \lambda \phi_i \beta(N_i) N_i^{\alpha} K_i^{1-\alpha} \ , \ 0 < \alpha < 1 \tag{1}$$

where N_i and K_i are the labor and capital used in location i, λ is an economy-wide technology index, and ϕ_i is an index that captures the impact of location on city i's production capabilities. For instance, the production advantages conferred by being a port would be captured by a high ϕ_i, as would proximity to output and input markets. Finally, $\beta(N_i)$ is a function that takes into account the production advantages of agglomeration:

$$\beta(N_i) = \max\{N_{\min}^{\nu}, N_i^{\nu}\}, \ N_{\min} \geq 0, \nu > 0 \tag{2}$$

This specification of the agglomeration function has two important features. First, it restricts agglomeration economies to be a constant below some "threshold" level of employment, $N_{\min}$. Second, it asserts that in the range where agglomeration benefits are increasing in local employment, the elasticity of agglomeration benefits with respect to change in employment is a constant.

Each location also produces a local good that can be consumed only by local residents. The local good in location i is produced using a technology that is linear in the traded good:

$$G_i = (\xi_i \Gamma_i)^{-1} Y \tag{3}$$

Here ξ_i is an index specific to location i and is meant to capture the effect of location on the production of the local good. For instance, housing being an important local good, ξ_i could reflect the costs of producing housing in location i. The other factor Γ_i is a function that models the costs imposed by employment density on the production of the local good:

$$\Gamma_i = e^{\gamma(N_i/A_i)}, \ \gamma > 0 \tag{4}$$

where A_i is land area of location i. Thus, according to (3) and (4), higher employment density makes the production of the local good less efficient. An important property of the Γ_i function is that its elasticity with respect to employment density is increasing in density:

$$\frac{d(\ln \Gamma_i)}{d \ln(N_i/A_i)} = \gamma(N_i/A_i) \tag{5}$$

Endowments and Preferences

There are two types of individuals, those who own capital and those who do not. Both types have one unit of labor that they inelastically supply to firms in one of the M locations. We assume (for tractability) that owners of capital are immobile and their location exogenously given. There is a measure $N_i^F \geq 0$ of owners in location i. The total measure of all individuals is given by N.

Individuals who do not own capital locate to maximize utility. The utility that a mobile individual gets from living in location i is given by:

$$U_i = \pi(N_i) g_i^\theta c^{1-\theta}, \ 0 < \theta < 1, \tag{6}$$

where g_i and c are the individual's consumption of the local and traded good, respectively. The function $\pi(N_i)$ is an amenity index that takes account of agglomeration benefits for consumers:

$$\pi(N_i) = \max\{N_{\min}\eta, N_i\eta\}, \ N_{\min} > 0, \eta > 0 \tag{7}$$

This specification parallels the specification of agglomeration benefits in the production of the traded good and has similar properties. For simplicity, we assume that the employment level beyond which agglomeration benefits for consumers begin to increase with local employment is the same level as for producers.

Equilibrium Conditions

Let the traded good be the *numeraire*. Let the price of the local good in location i be p_i, the wage rate w_i, and let the world rental rate on capital be r. We will focus on equilibria in which there is a positive measure of mobile individuals residing in each location. Formally, we require:

$$N_i > N_i^F \tag{8}$$

Utility maximization implies that a mobile individual in location i chooses $g_i = \theta(w_i/p_i)$ and $c = (1-\theta)w_i$. Thus, the indirect utility of a mobile individual residing in location i is:

$$V_i = \pi(N_i)\theta^\theta(1-\theta)^{(1-\theta)}p_i^{-\theta}w_i$$

Given (8), in equilibrium workers migrate until utility across locations is equalized. This implies:

$$\pi(N_i)\theta^\theta(1-\theta)^{(1-\theta)}p_i^{-\theta}w_i = \overline{V} \tag{9}$$

We assume that all local goods are supplied competitively. The producers of these goods take the employment density in each location as given. Therefore, the price of the local good in location i will equal its marginal cost:

$$p_i = \xi_i e^{\gamma(N_i/A_i)} \tag{10}$$

Turning to the traded good, a firm that locates in location i takes the level of local employment and the product wage in that location as given. It also takes r as given. In equilibrium, the product wage in each location must be such that the profit from producing the traded good is zero in all locations. These zero-profit conditions are:

$$w_i = [\alpha\alpha(1-\alpha)^{(1-\alpha)}\lambda\phi_i\beta(N_i)]^{\frac{1}{\alpha}}r^{\frac{-(1-\alpha)}{\alpha}} \tag{11}$$

Finally, the sum of labor supply across all locations must equal the exogenously given total supply of labor in the economy:

$$\sum_{i=1}^{M} N_i = N \tag{12}$$

Denote $\theta^{\theta}(1-\theta)^{(1-\theta)}\ \alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\lambda^{1/\alpha}$ by $H(\alpha,\theta,\lambda)$ and the product of location-specific factors, $\xi_i^{-\theta}\cdot\phi_i^{1/\alpha}$, by S_i. Then, substituting equations (10) and (11) into equation (9) yields:

$$H(\alpha,\theta,\lambda)\cdot S_i\cdot\beta(N_i)^{\frac{1}{\alpha}}\cdot\pi(N_i)\cdot e^{-\gamma\theta(\frac{N_i}{A_i})}\cdot r^{\frac{-(1-\alpha)}{\alpha}}=\overline{V} \tag{13}$$

In what follows, the M equations in (13) and equation (12) are used to solve for the $M+1$ unknowns, N_i, $i=1,2,3,...,M$, and $\overline{V}$. This procedure assumes that the unobserved distribution of immobile individuals can always be specified to satisfy the inequalities in equation (8) for the levels of N_i calculated using the M equations in (13) and equation (12). It is possible to proceed this way because of the assumption that these cities can import or export the traded good and the capital stock from each other or the rest of the world. If the open-economy assumption is dropped, it would be necessary to impose economywide resource balance conditions for the traded good and the capital stock. The distribution of immobile workers will then matter for the determination of equilibrium.[5]

4 Properties of the Model

The first objective is to explain how the utility of a mobile worker in a given location is affected by the employment density in that location and how the equilibrium employment density of a location is determined given the utility level that mobile workers can get elsewhere. The second objective is to examine how changes in economywide variables, such as λ, r, and N, affect the equilibrium employment density in each location. The material

[5]If immobile workers in city i own k_i^F units of capital per person and their utility function has the same form as that of mobile individuals, the total demand for the traded good in city i is $Y_i^D=[(1-\theta)+\theta\xi_i e^{\gamma(N_i/A_i)}][w_iN_i+r(N_i-N_i^F)k_i^F]$. The supply of the traded good in city i is $Y_i=[(1-\alpha)/r]^{(1-\alpha)/\alpha}[\lambda\phi_i\beta(N_i)]^{1/\alpha}N_i$. For a closed economy, $\Sigma_iY_i^D$ must equal Σ_iY_i, which means that knowledge of N_i^F and k_i^F is needed to determine the equilibrium employment levels.

in this section provides the background to understand the results of the computational experiments reported in the next section.

Equilibrium Employment Density for a Single Location

Let ν/α be denoted by μ, $\theta\gamma$ by δ, and $(N_{\min}/A_i)$ by $\overline{D}_i$. Then, using (2) and (7), the l.h.s. of equation (13) may be written as a function of density D:

$$V_i(D) = H(\alpha, \theta, \lambda) \cdot S_i \cdot A_i^{\mu+\eta} \cdot \max\{D^{\mu+\eta}, \overline{D}_i^{\mu+\eta}\} \cdot e^{-\delta\, D} \cdot r^{\frac{-(1-\alpha)}{\alpha}}$$

It is convenient to work with the logarithmic transform of $V_i(D)$. Let $\ln(D)$ be denoted by d, $\ln(\overline{D}_i)$ by $\bar{d}_i$,and $\ln(H(\alpha, \theta, \lambda) \cdot S_i \cdot A_i^{\mu+\eta} \cdot r^{\frac{-(1-\alpha)}{\alpha}})$ by s_i. Then:

$$\ln(V_i(e^d)) \equiv v_i(d) = s_i + (\mu + \eta) \cdot \max\{d, \bar{d}_i\} - \delta \cdot e^d \tag{14}$$

The function $v_i(d)$ is continuous over the entire range $(-\infty, +\infty)$ and differentiable everywhere except $\bar{d}_i$. In the $(-\infty, \bar{d}_i)$ range, where agglomeration economies are insensitive to changes in local employment, the function is strictly decreasing and strictly concave:

$$\partial v_i/\partial d = \partial^2 v_i/\partial d^2 = -\delta \cdot e^d < 0 \tag{15}$$

In the $(\bar{d}_i, +\infty)$ range, where agglomeration economies increase with local employment, the behavior of v_i reflects the interaction of congestion costs and agglomeration economies. The first and second derivatives with respect to d over this range are:

$$\partial v_i/\partial d = (\mu + \eta) - \delta \cdot e^d \tag{16}$$

$$\partial^2 v_i/\partial d^2 = -\delta \cdot e^d < 0 \tag{17}$$

Thus, $v_i(d)$ continues to be strictly concave but with regard to the sign of the first derivative two possibilities exist: (i) $\overline{D}_i \geq (\mu+\eta)/\delta$ or, equivalently, $\bar{d}_i \geq \ln[(\mu+\eta)/\delta]$. In this case (16) implies that the $v_i(d)$ function is strictly decreasing over the range $(\bar{d}_i, +\infty)$. (ii) $\bar{d}_i < \ln[(\mu+\eta)/\delta]$. In this case (16) implies that $v_i(d)$ initially increases, reaching a local maximum at $d = \ln((\mu+\eta)/\delta)$, and then declines.

Figures 3(a) and 3(b) illustrate these two cases. These two cases arise because agglomeration economies are related to employment while congestion costs are related to employment density. Thus, it is possible for a compact location to become quite dense before it attains the employment level beyond which agglomeration economies increase with size. In this case, which is the first case noted above, agglomeration economies can slow down the decline in utility that occurs with increasing density but they cannot reverse it (Figure 3(a)). In contrast, a location that is large in land area might attain $N_{\min}$ before it gets too dense. In this case, which is the second case noted above, increasing agglomeration economies will overcome the utility-depressing effect of increasing density for some range of employment density (Figure 3(b)).

The shape of the $v_i(d)$ function bears directly on the possibility of multiple equilibrium employment densities for a single location. The densities corresponding to the points where the $v_i(d)$ function intersects the horizontal "outside-opportunities" line are all equilibrium density levels. In 3(a), where the $v_i(d)$ function is monotonically declining, there is only one equilibrium density level. In contrast, there are *three* equilibrium density levels in 3(b). As usual, the middle equilibrium, d_i^M, is unstable: a small increase or decrease in density, by raising or lowering the utility level above or below what mobile workers can get in other locations, will induce further increases or decreases in density. The other two equilibria, d_i^L and d_i^H, are stable.

Equilibrium Employment Densities for the System-of-Cities

The purpose of this section is to explain how equilibrium employment densities respond to changes in economywide variables in the model. One complicating factor is the possibility of multiple equilibria. To do comparative statics, we must specify a procedure for selecting one equilibrium when more than one exists.

The procedure we employ involves two restrictions. First, we restrict attention to stable equilibria only. This is justifiable on the ground that unstable equilibria (i.e., equilibria in which some location is in an unstable equilibrium) are "razor's edge" cases. Second, we assume that in response to a change in a parameter, the economy moves to the new stable equilibrium that is closest to the initial equilibrium. The justification for this restriction is less clear, but it seems to be a reasonable procedure for studying the effects of environmental changes, such as technological change and increase in aggregate metropolitan employment, that occur in small increments over time.[6]

With these remarks in mind, we first make precise the conditions that a system-of-cities equilibrium must satisfy, including the stability restriction.

Definition 1 *The collection* $\{d_i^*, v^*, s_i, \bar{d}_i, N\}$ *is a system-of-cities equilibrium if it satisfies the following conditions:*

$$v^* = s_i + (\mu + \eta) \cdot \max\{d_i^*, \bar{d}_i\} - \delta \cdot e^{d_i^*} \;\forall\; i = 1, 2, \ldots, M \tag{18}$$

$$\sum_{i=1}^{M} A_i \cdot e^{d_i^*} = N \tag{19}$$

$$d_i^* \in (-\infty, \bar{d}_i) \cup (\ln[(\mu + \eta)/\delta], +\infty) \;\forall\; i = 1, 2, \ldots, M \tag{20}$$

Conditions (18) and (19) are the equal utility and aggregate labor resource balance conditions, respectively. These correspond to equations (13) and (12) of the previous section. The stability requirement is incorporated in condition (20). For the case where $\bar{d}_i < \ln[(\mu + \eta)/\delta]$, this condition prohibits d_i^* to lie in the closed interval $[\bar{d}_i, \ln[(\mu + \eta)/\delta]]$; as shown in Figure 3(b), this interval corresponds to the domain of d for which the $v_i(d)$ function is upward sloping. For the case where $\bar{d}_i \geq \ln[(\mu + \eta)/\delta]$, (20) does not impose any restriction at all, since d_i^* can then lie anywhere on the real line. No restriction is needed because in this case the $v_i(d)$ function is decreasing in d over the entire real line.[7]

The first proposition characterizes the nature of a system-of-cities equilibrium. It gives sufficient conditions under which one location will be more dense than another. The three propositions that follow it give the main comparative statics results (all proofs are collected in the appendix).

Proposition 1 *Consider two locations* i *and* j *for which* $S_i > S_j$. *If* $A_i \geq A_j$ *and* $\bar{d}_i$ *and* $\bar{d}_j$ *are both greater than* $\ln[(\mu + \eta)/\delta]$, *in any system-of-cities equilibrium* $d_i^* > d_j^*$.

[6] There remains the issue of selecting the initial equilibrium density for a city in the comparative statics exercise. As explained in section 5, in the comparative statics exercise we perform, this issue is resolved by selecting equilibrium densities that match employment densities for each city for 1951. In other words, we use history to select the initial equilibrium.

[7] The presence of the stability restriction implies that employment density of a location may change discontinuously in response to a change in an environmental factor such as aggregate employment. Consequently, such a system-of-cities equilibrium may fail to exist for *some* parameter values. However, this difficulty is not encountered in any of the computational experiments conducted in section 6.

The conditions under which one location will be more dense than another are quite intuitive. All else remaining the same, a location that is more efficient than another in producing one or both goods (i.e., has a higher ϕ and/or lower ξ) will attract more workers. If such a location is also larger in land area, it can support a bigger workforce and enjoy at least as much, if not more, agglomeration economies for any given level of congestion. This may further increase in its density.

Proposition 2 *The equilibrium employment density in each location is independent of the rental rate r and of the economywide total factor productivity index λ.*

Changes in λ and r shift every $v_i(d)$ function up or down by the same amount for all locations. Not surprisingly, the only effect of such a change is to alter the utility level that mobile individuals receive in each location by some identical amount. Thus, there is no reason for the mobile workers to attempt to relocate and no reason for employment densities to change.

The next proposition focuses on the consequence of an increase in the aggregate number of workers. We provide conditions under which there is an inverse relationship between the initial employment density of a location and the growth rate of employment in that location.

Proposition 3 *Let $\{d_i^1, v^1, s_i, \bar{d}_i, N^1\}$ be the initial system-of-cities equilibrium and suppose that $\{d_i^2, v^2, s_i, \bar{d}_i, N^2\}$ is the new system-of-cities equilibrium for $N^2 > N^1$. Let $\rho_i = (N_i^2/N_i^1)$ denote the (gross) growth rate of employment in location i. If $d_i^1 > \bar{d}_i > \ln[(\mu+\eta)/\delta]$ for all i, then $\rho_i > 1$ for all i and $d_i^1 > d_j^1$ implies $\rho_i < \rho_j$.*

This inverse relationship between density and employment growth can be intuitively explained as follows. If in each location agglomeration benefits are sensitive to local employment (i.e., $d_i^1 > \bar{d}_i$ for all i), then a 1 percent increase in employment in each location would raise agglomeration benefits by $(\mu+\eta)$ percent in each location. On the other hand, since a 1 percent increase in a location's employment implies a 1 percent increase in its density (because the area of each location is being held fixed), the cost of living would rise by (approximately) $\delta \cdot D_i^1$ in each location. Thus, the cost of living will rise relatively more in dense areas. If utility of a mobile worker is a decreasing function of density in each location (i.e., $\bar{d}_i > \ln[(\mu+\eta)/\delta]$ for all i), then a one percent increase in total employment must, in equilibrium, result in a less than 1 percent increase in employment in dense locations and a more than 1 percent increase in employment in less dense ones.

The next proposition establishes the link between an inverse relationship between initial density and local employment growth and deconcentration of employment. First, we define "deconcentration of employment."

Definition 2 *Let $z \in \{1, 2\}$ index two different distributions of employment across the M locations. Let $L^z(m)$ be the fraction of total employment accounted for by the m most dense locations in the distribution z. Then employment is more deconcentrated (or less concentrated) in the second distribution relative to the first if $L^2(m) \leq L^1(m)$ for all m and $L^2(m) < L^1(m)$ for some m.*

Proposition 4 *Let $\{d_i^1, v^1, s_i, \bar{d}_i, N^1\}$ be the initial system-of-cities equilibrium and suppose that $\{d_i^2, v^2, s_i, \bar{d}_i, N^2\}$ is the new system-of-cities equilibrium for $N^2 > N^1$. If $\bar{d}_i > \ln[(\mu + \eta)/\delta]$ for all i and $d_i^1 > d_j^1$ implies $\rho_i < \rho_j$, then the new distribution of employment will be less concentrated than the initial distribution of employment.*

Taken together, these two propositions provide sufficient conditions for an increase in total employment to cause deconcentration of employment. These conditions are quite restrictive, so it is important to emphasize that they are sufficient conditions only. In effect, these conditions restrict agglomeration economies to play only a limited role in the determination of employment across locations. However, even if these conditions do not hold for all locations, deconcentration will occur as long as locations for which these conditions do hold account for a significant fraction of total employment. Indeed, in the numerical model examined in section 6, we encounter several violations of this inverse relationship (usually because the $\bar{d}_i < \ln[(\mu + \eta)/\delta]$ condition is violated), but the deconcentration property is still preserved.

5 Parameter Selection and Calibration

The numerical specification of the model described by (18) - (20) involves choosing values for three groups of parameters. These are (i) the agglomeration-related parameters, μ, η, and $\bar{d}_i$, (ii) the congestion-related parameter δ, and (iii) the location-specific factors s_i.[8]

We use existing micro studies to guide our selection of the agglomeration- and congestion-related parameters. The location-specific factors s_i are determined by calibrating the model to reproduce the actual 1951 employment densities for all 297 locations in our data set. As will become clear below, the ability of our model to match employment densities in 1951 depends importantly on the values of the agglomeration and congestion parameters. Thus, the two steps are not independent of each other. In view of this, we use available micro studies to put bounds on the agglomeration and congestion parameters and then select from within these bounds to carry out the calibration step.

Relationship Between Employment and Population Across MSAs

Before we can proceed with parameter selection and calibration, there is an important preliminary step. The available estimates of agglomeration and congestion parameters give the strength of these effects in relation to changes in local *population* rather than local employment. Therefore, to use these estimates we need to know how employment and population are related in our locations.[9]

We use the employment data for 1979 and the population data for 1980 to gauge the relationship between employment and population for our 297 MSAs. Figure 4 plots the logarithm of location employment in 1979 against the logarithm of location population in 1980. The solid line is one that best describes this relationship in a least-squares sense. It has an intercept of -2.2 and a slope of 1.0865. Thus, the employment to population ratio shows a slight tendency to rise with population. Figure 5 plots the employment to

[8] Each s_i is a sum of both location-specific as well as economywide variables. However, since the s_i's differ across locations only because of differences in the location-specific variables, we refer to the s_i's as location-specific factors.

[9] Note that it is not advisable to use (the easily available) *aggregate* employment to population ratio for this purpose. For one thing, the demographics of large and small MSAs tend to be systematically different, so there are systematic differences in the employment to population ratio across MSAs of different sizes. Furthermore, as noted earlier, the County Business Pattern data do not cover all workers, so that the employment to population ratio for any location in our data set is most likely lower than the employment to population ratios reported for aggregate data.

population ratios against the logarithm of location employment. The median employment to population ratio, indicated by the solid line, is 0.32. We use these facts below.

Estimates of Agglomeration-Related Parameters

Recall that $\mu = \nu/\alpha$ and $\bar{d}_i = \ln(N_{\min}/A_i)$. Since we have observations on the land area of each MSA (A_i),we need estimates of α, ν, $N_{\min}$, and η only.

Let's begin with α, the exponent to labor input in the production function for the traded good. Under perfect competition, the equality of wages and marginal product of labor implies that the share of value-added absorbed by compensation to workers is α. Furthermore, since workers are employed only in the production of the traded good, the share of compensation in value-added is α for every enterprise in our model. Average α, as measured by labor's share in U.S. GDP, has been about 2/3 for the post-WWII period. Since this estimate is relatively precise, we set $\alpha = 0.67$.

In order to bound ν and $N_{\min}$, we turn to studies that attempt to estimate the degree of agglomeration economies for U.S. cities. As discussed in Moomaw (1981), there are essentially two ways of obtaining such an estimate. In the first method, the zero-profit condition for firms is used to deliver a relationship between a location's nominal wage and such characteristics as its population size, industry mix, etc. In this approach, an estimate of the coefficient on population size is an estimate of the strength of agglomeration economies. In the second method, the production function is estimated directly using data on value-added, employment, capital stock, population size, industry mix, etc. Again, the coefficient on population size provides an estimate of the strength of agglomeration economies.

Turning first to estimates obtained using the zero-profit condition, note that the zero-profit condition (11) in conjunction with agglomeration function (2) implies:

$$\ln w_i = \text{constant } + \alpha^{-1} \ln \phi_i + \nu \cdot \alpha^{-1} n_i \cdot \chi_i + \nu \cdot \alpha^{-1} n_{\min} \cdot (1 - \chi_i)$$

where n_i is the log of employment, $n_{\min}$ is log of $N_{\min,}$and χ_i is an indicator function that takes on the value 1 when n_i exceeds $n_{\min}$ and 0 otherwise. Sveikauskas (1975) estimated a relationship of this form for each of 14 two-digit manufacturing industries. He used SMSA population rather than employment as a regressor and ignored the possibility of thresholds (i.e., assumed that χ is 1 for every observation). He obtained estimates of $\nu \cdot \alpha^{-1}$ that range from 0.0116 to 0.0855 with a median value of around 0.048 (Table IV, p. 404).[10] Using the estimated relationship between log employment and log population, and a labor share of 2/3, Sveikauskas' estimates imply a median estimate of ν of about 0.03.[11]

However, Sveikauskas' estimates of $\nu \cdot \alpha^{-1}$ suffer from (at least) two opposite biases. First, he used only a limited number of variables to control for location-specific factors ϕ_i. Recall that Proposition 1 suggests that there is likely to be a positive dependence between ϕ_i and n_i in equilibrium so that omission of relevant location-specific factors will bias the estimates of $\nu \cdot \alpha^{-1}$ upward. On the other hand, Sveikauskas did not consider the possibility that agglomeration economies may be insensitive to changes in population up to a certain level (i.e., the possibility of a threshold like $N_{\min}$), which may have biased his estimate of $\nu \cdot \alpha^{-1}$ downward.[12] Still, the extent of the downward bias is much less certain than the upward bias that almost certainly exists and is likely to be quite significant. Therefore, it is reasonable to conclude that 0.03 is an upper bound estimate of ν. Thus, we proceed on the assumption that Sveikauskas' estimates suggest a value of ν between 0.03 and, say, 0.015. This implies a value of μ somewhere in the 0.0225 to 0.045 range.

Turning to production function estimates, observe that the location-specific production function (1) in conjunction with the agglomeration function (2) implies the following relationship:

$$y_i = \text{constant} + \ln \phi_i + \nu n \cdot \chi_i + \nu n_{\min} \cdot (1 - \chi_i) + (1 - \alpha) k_i \qquad (21)$$

where y_i is log of location i's average labor productivity and k_i is the log of its capital per worker ratio. As before, χ is an indicator variable that takes on the value 1 if n_i exceeds $n_{\min}$ and zero otherwise. Segal (1976)

[10] The data pertain to all SMSAs in 1967.

[11] Moomaw (1981) adjusted Sveikauskas' estimates of $\nu \cdot \alpha^{-1}$ for the *observed* labor share in each industry and reported estimates of ν that range from 0.006 to 0.0485 with a median value of 0.0266. However, as noted by Ciconne and Hall (1996), the Census of Manufactures data overstate value-added per worker in larger cities and hence underestimate the worker's share of value-added for those cities.

[12] The reason for the downward bias is as follows. If a population "threshold" exists, y_i will be less sensitive to differences in population size. An estimation strategy that ignored "thresholds" (and used the full variation in population size) would assign a smaller coefficient on population to "fit" the relative insensitivity of y_i to population size.

estimated a relationship of this form on data from 58 SMSAs for 1967. He used SMSA population rather than employment as the size variable and used a term like $\nu \cdot \chi_i$, instead of $\nu n_i \cdot \chi_i + \nu n_{\min} \cdot (1 - \chi_i)$, to take into account differences in productivity stemming from differences in population size. He found measurable productivity differences for SMSAs with population above and below 2 million and estimated that difference to be about 8 percent in favor of SMSAs with a population over 2 million.

Because Segal used broad population categories to measure SMSA size, his estimate of an 8 percent productivity differential in favor of large SMSAs cannot be used to determine ν. On the other hand, his finding that productivity differences are discernible between the group of SMSAs with population greater than 2 million and the group of remaining SMSAs may be interpreted as evidence that $N_{\min}$ cannot be any greater than $750,000$ workers.[13] We proceed on the assumption that a plausible value of $N_{\min}$ lies somewhere in the $100,000$ to$750,000$ range.

We did not find any usable estimates of η. For want of something better, we set its value equal to 0.01.

Estimates of Congestion-Related Parameters

Recall that $\delta = \theta \cdot \gamma$, where θ is the share of local goods in the household budget and γ is the percentage change in the price of the local good due to a unit change in employment density.

In our model, the relationship between the price of the local good and employment density is given by equation (10). This equation implies the following relationship:

$$\ln p_i = \ln \xi_i + \gamma \cdot d_i$$

Roback (1982) estimated a relationship of this form using data from 73 SMSAs for 1973. She used the logarithm of the average residential site price as the dependent variable and various SMSA-specific factors and SMSA population density as regressors. The coefficient on the density variable in her regression is 2.0×10^{-4} (Table 3, p. 1272). Since the median MSA employment to population ratio for our locations is 0.32, Roback's estimate of the density coefficient implies a γ value of (approximately) 6.0×10^{-4}.

[13] We used the estimated relationship between log employment and log population in Figure 4 to determine the employment level corresponding to a population of 2 million.

To get a perspective on this estimate, consider an MSA with employment density of 4900 workers per square mile. This figure is about the upper limit of the employment densities observed in our data set. For such an MSA, a 1 percent increase in employment density would mean an additional 49 workers per square mile and an increase in the site price of residential land of approximately 2.94 ($= 49 \times 6.0 \times 10^{-4} \times 10^2$) percent.

Of course, Roback's density coefficient measures the proportionate increase in residential site price associated with an increase in population density alone (holding other factors, such as crime and pollution, constant). In reality, an increase in MSA population density is likely to be accompanied by increases in crime, pollution, and other congestion-related costs. Thus, it is reasonable to view her estimate as providing a lower bound on γ.

To estimate θ, we used the 1972-73 consumption expenditure shares of urban wage earners and clerical workers reported in Jacobs and Shipp (1990).[14] We summed the expenditure shares on food, shelter, utilities (including fuels and public services), public transportation, entertainment, and sundries. These categories amounted to 56.8 percent of total household expenditures (Table 2, p. 22). Since some of these components are not entirely local, we estimate θ to be 0.50. Since we believe this estimate to be relatively precise, we set θ to 0.50.

Taken together, these estimates imply a value of δ no smaller than 3.0×10^{-4}. We proceed on the assumption that a plausible value of δ is somewhere in the 3.0×10^{-4} to 6.0×10^{-4} range.

Calibration of MSA-Specific Factors

The final step is to determine the MSA-specific factors. This is done by choosing agglomeration and congestion parameters within the plausible ranges noted above and calibrating the model to match employment densities observed in 1951 for each of our 297 locations.

This step would be straightforward but for the fact that the observed employment density for each location must satisfy the stability restriction noted in the definition of system-of-cities equilibrium (equation (20)). For each location, this restriction depends on its land area, A_i, and on the values of the agglomeration- and congestion-related parameters. In particular, it does *not*

[14] Expenditure shares tend to change over time. Since our estimate of γ is derived from a study based on 1973 data, we used the expenditure shares for the closest available year.

depend on the location-specific factor. Therefore, once the agglomeration- and congestion-related parameters are chosen, these choices (along with A_i) determine the density zone that conflicts with stability. The difficulty is that there is no assurance that actual employment densities in 1951 will lie outside this "forbidden zone."

Figure 6 illustrates the problem for a value of $N_{\min} = 100,000$, $\mu = 0.045$ and $\delta = 3.0 \times 10^{-4}$. It gives a scatter plot of all 297 $(\bar{d}_i, d_i)$ pairs for 1951. The vertical line is erected at $\ln[(\mu + \eta)/\delta]$ and the diagonal line is the 45° line. For any scatter point lying below the 45°-line and to the left of the vertical line, $\bar{d}_i < d_i < \ln[(\mu + \nu)/\delta]$. Thus, any point in that triangular area violates the stability restriction. For these parameter choices, there are, evidently, quite a few locations whose employment densities in 1951 cannot be explained by our model.

However, these parameter choices imply the "worst-case-scenario" in terms of numbers of locations that violate the stability restriction. Relative to the plausible ranges for δ, μ, and $N_{\min}$, the selected values put the vertical line as far to the right as possible and each scatter-point as close to the horizontal axis as possible. For lower values for μ and/or higher values for δ, the vertical line would move to the left, which would reduce the number of observations falling in the "forbidden zone"; for higher values of $N_{\min}$ (and, therefore, higher values of $\bar{d}_i$ for each location), the scatter-plot would migrate upward, which would reduce the number of observations in the "forbidden zone" as well.

Since we wanted to retain the ability to vary μ and δ within their permissible range, we met the stability requirement by setting $N_{\min}$ high enough so that even with μ at its top value and δ at its bottom value there are no observations in the "forbidden zone." We found that an $N_{\min}$ value of 550,000 was sufficient to accomplish this.

For our baseline model, we set $N_{\min} = 550,000$ workers. We set $\mu = 0.034$ and $\delta = 4.5 \times 10^{-4}$, which are the midpoints of the respective ranges of μ and δ.

The calibration step was then performed as follows. We first normalized the location-specific factor for the densest location with more than $N_{\min}$ workers in 1951 to 1. Then, using this location's area and its actual employment density in 1951 (and the selected values for $N_{\min}, \mu, \eta$, and δ), we determined from equation (18) the equilibrium value of v^* for this location in 1951. For the remaining locations, we used their areas, their actual employment densities in 1951, and the computed value of v^* to solve for the unique location-specific factors from corresponding equations in (18).

6 Findings

Implications of Employment Growth in the Baseline Model

Figure 7(a) plots the actual employment densities for our locations in 1951 and the predicted employment densities for 1994.[15] The locations are ordered by actual employment density in 1951. Every location is predicted to have higher employment density in 1994 than in 1951. There is also a tendency for density to rise proportionately more for less dense areas. Note that, by construction, our model exactly reproduces the employment density for 1951.

Figure 7(b) plots the actual employment densities for 1994 along with those predicted by our model. Two features of this plot stand out. First, the predicted density plot is not as jagged as the actual one. Second, the predicted employment density is lower than actual employment density for locations that were dense in 1951 and is higher than actual employment density for locations that were relatively less dense in 1951.

The first feature indicates that many more locations changed their rankings (relative to their rankings in 1951) in the data than in the model. In terms of our model, in which location-specific factors are calibrated to match 1951 employment densities, this feature underscores the importance of *changes* in location-specific factors in accounting for actual employment densities in 1994. Nevertheless, the fact that changes in rankings occur in the model at all (as evidenced by the spikes in predicted employment densities) is noteworthy. These spikes occur because of the feedback effects of agglomeration economies when employment in a location increases beyond $N_{\min}$ *and* that location's land area is large enough for $\bar{d}$ to be less than $\ln[(\mu + \eta)/\delta]$.

Figure 8 illustrates this for the Riverside-San Bernardino metropolitan area, one of the locations that experienced an employment spike in our model. The top horizontal line is the computed v^* in 1951 with d^{51}_{R-SB} as Riverside-San Bernardino's observed employment density in that year. The bottom

[15] In this and all subsequent experiments in this section, the computed equilibrium employment is within ±1 percent of total 1994 employment in our MSAs.

horizontal line is the predicted v^* in 1994 and d_{R-SB}^{94} is the predicted employment density for Riverside-San Bernardino. Observe that because the $v(d)$ function has an increasing segment, the equilibrium density in 1994 is considerably higher than it would be if that segment were absent.

The second feature indicates that there is more deconcentration of employment in the model than in the data. This can be seen most clearly by grouping locations and examining the employment shares of groups. As in Figure 2, we ranked locations by employment density in 1951 and then banded them into 10 groups: the first nine groups have 30 locations each and the final group has 27. Figure 9 plots the actual cumulative employment shares in 1951 and 1994 and the predicted cumulative employment shares in 1994 for each of these 10 groups.[16] It is quite evident that the model predicts more deconcentration than that which actually occurred.

We can get further insight into predictions of the model if we examine employment shares for each of these groups. Figure 10 is similar to Figure 9 but plots employment shares of each group instead of cumulative employment shares. Note that the predicted employment shares are less than actual employment shares for the first eight groups while it is the other way around for the last two groups.

Also, at least for this way of grouping the data, the model successfully captures the *qualitative* features of the actual pattern of deconcentration. The groups for which actual employment shares fell between 1951 and 1994 (groups 1 and 2) were also the ones for which predicted employment shares fell. And, with the exception of group three, groups for which actual employment shares rose between 1951 and 1994 were also the groups for which predicted employment shares rose.

The Effect of Variations in the Strength of Agglomeration Economies and Congestion Costs on Employment Deconcentration

In this section, we examine the sensitivity of the predictions of the baseline model to changes in δ, μ and $N_{\min}$.

[16] Recall that the shares are with respect to total employment as reported in the County Business Patterns. Since this includes employment in non-metro counties, cumulative employment share of the tenth group is less than 100 percent.

Sensitivity to Changes in δ (Congestion-related Parameter)

Figure 11 plots the predictions of the model when all agglomeration parameters are kept at their baseline values but the value of δ is varied. In this plot, the middle bar for each group is the baseline prediction while the first bar is the model's prediction when $\delta = 3.0 \times 10^{-4}$ and the third bar its prediction when $\delta = 6.0 \times 10^{-4}$.

To interpret this plot it is important to recognize that δ is not the *only* parameter that changes across the three simulations. Since δ is used in the calibration step, a change in δ also changes the settings of the location-specific factors. Thus, the bars for each group reflect differences in both δ *and* induced differences in location-specific factors.

To sort out the effects of these simultaneous changes, it is useful to consider the expression for the first-round (or impact) effect on the utility levels of mobile workers in 1994 of a change in δ. Using equation (18), this change can be written as:

$$v_i^{'}(1994) - v^*(1994) = (s_i^{'} - s_i) - (\delta' - \delta) \cdot e^{d_i^*(1994)}$$

where $v_i^{'}(1994)$ is the utility level that would prevail in location i if the employment density of location i is held fixed at the value predicted for it in 1994 by the baseline model (we denote this predicted equilibrium density by $d_i^*(1994)$) and $s_i^{'}$ is the ith location-specific factor when the congestion parameter is set to δ'.

This expression has two parts: the first part is the effect of the induced change in the location-specific factor and the second part is the direct effect of a change in δ. Figure 12 plots the direct impact effect of a decrease in δ from the baseline value of 4.5×10^{-4} to 3.0×10^{-4}. As we would expect, there is a substantial increase in the utility level of mobile workers in the very dense locations relative to other, less dense ones. If this were the only effect in operation, our model would generate an increase in the employment share of the most dense location along with (quite possibly substantial) decreases in employment shares of less dense locations.

Turning to the induced change in location-specific factors, our calibration step implies:

$$s_i^{'} - s_i = -(\delta' - \delta) \cdot (e^{d_1(1951)} - e^{d_i(1951)})$$

Figure 13 plots the induced changes in location-specific factors. Notice that the effect of induced changes in location-specific factors is roughly *opposite* to the direct effects. This is intuitive. A decrease in δ lowers congestion costs and dense locations gain more from that reduction than less dense cities. By itself, this would imply a greater concentration of workers in more dense locations. However, since the calibration step forces the model to reproduce the (unchanged) employment density for 1951, this increased attractive force of lower congestion cost must be countered by making dense locations less attractive relative to less dense ones. Hence, the calibration step increases the location-specific factors of less dense locations relative to more dense ones.

Figure 14 shows the overall impact effect of a reduction in δ. Notice that there is now much less effect on the utility level of mobile workers in the very dense locations. Indeed, the overall effect is to make several less dense locations very attractive (as indicated by the spikes in the chart). As result, the employment share of the least dense group rises at the expense of other groups.[17]

An important implication of the above analysis is that the findings of the baseline model are robust to changes in the congestion cost parameter. A 100 percent variation in δ hardly affects the predicted employment shares of the first eight groups of locations. The significant changes are confined to the two least dense groups. This comes about because the model is required to match observed employment densities in 1951 for *all* parameter selections. This requirement imposes severe constraints on how much the predicted employment shares can vary with changes in δ.

Sensitivity to Changes in μ and $N_{\min}$ *(Agglomeration-Related Parameters)*

Figures 15 plots the predictions of the baseline model when the value of μ is varied. As before, the middle bar in each group in each figure is the prediction of the baseline model. The first bar in each group is the model's prediction when $\mu = 0.023$ and the third bar the prediction when $\mu = 0.045$. Evidently variations in μ do not affect the employment share of the first nine

[17] It should be clear that the effects of an increase in δ will be a mirror opposite of these effects. The overall effect will be a decline in the utility of several less dense locations and, therefore, a fall in the employment share of the least dense group.

groups very much, but the employment share of the tenth group rises with increases in μ. Once again, the relative insensitivity of model prediction to changes in μ occurs because of the offsetting effects of induced changes in location-specific factors and the direct effect of changes in μ. In Figures 16 and 17 , we plot the direct effect and the induced changes in s_i of a reduction in μ from 0.034 to 0.023. The direct effect of a reduction in μ is to reduce agglomeration benefits of large MSAs which tends to reduce the utility level of more dense locations relative to less dense ones. However, the induced effect goes the other way. The overall impact effect of a reduction in μ, shown in Figure 18, is to leave the utility unchanged for most locations but reduce it more for less dense locations than for dense locations. Hence, the employment share of the least dense location falls relative to other locations.

Figure 19 plots the predictions of the model when $N_{\min}$ is varied. As before, the first bar in each group is the prediction of the baseline model. The first bar in each group is the model's prediction when $N_{\min}$ is lowered to 400,000 and the third bar its predictions when it's raised to 750,000. Once again, the effect on predicted employment shares is not very large for the first nine groups. The share of the least dense group decreases with increases in $N_{\min}$. Figures 20 and 21 show the direct impact effect and the induced changes in location-specific factors of a reduction in $N_{\min}$ to 400,000. As is evident, the direct and induced effects on the most dense locations are in opposite directions. The over all impact effect, shown in Figure 22, is largest for locations in the third, fourth, ninth, and tenth groups. Correspondingly, these groups gain employment share at the expense of other groups.

The Effect of Variations in Location-Specific Factors on Employment Deconcentration

We also investigated whether the predictions for 1994 are sensitive to changes in the way the location-specific factors are calibrated. Figure 23 compares the predictions for 1994 employment shares of the baseline model to those of a model in which the location-specific factors are calibrated to match observed employment densities in 1959. All other parameters are held at their baseline values. As is evident, this change has very little effect on the predictions for 1994.

7 Conclusion

This article looks at how U.S. metropolitan areas of different densities absorbed the increase in aggregate metropolitan employment over the postwar period. An examination of almost 300 metro areas shows that the initially more dense metro areas lost employment share to the initially less dense metro areas, a trend we label as deconcentration of employment.

Using a calibrated version of the system-of-cities model, we show that the more than twofold increase in aggregate metropolitan employment experienced by our MSAs during the postwar period was a powerful force favoring deconcentration. Increase in aggregate metropolitan employment leads to deconcentration because congestion costs rise faster for initially dense metro areas than for the initially less dense metro areas.

Many economists have speculated that deconcentration of employment represents nothing more than a continuation of the forces that led to suburbanization of people and jobs (Leven (1978), among others). While changes in preferences or government policies and a decline in urbanization economies have been shown to be important for suburbanization, there is little evidence that these forces are responsible for deconcentration. Our findings suggest that the forces underlying suburbanization are not needed to explain deconcentration.

If our finding that greater aggregate metropolitan employment generated *more* spatial deconcentration of employment than is actually observed in the postwar period is accepted, the next step will be to explain why this is so. In this regard, we believe that technological change may have actually favored employment growth in the more dense metropolitan areas as suggested by the "new" growth theory (Lucas (1988) and the studies by Eaton and Eckstein and Black and Henderson mentioned earlier). This literature has emphasized the role of cities in the process of inventions and innovations and it is possible that this ongoing process of technological change put denser MSAs technologically ahead of the less dense MSAs.

APPENDIX

Proof of Proposition 1

Since $\bar{d}_i$ and $\bar{d}_j$ are both greater than $\ln[(\mu+\eta)/\delta]$,it follows from equations (15) and (16) that $v_i(d)$ and $v_j(d)$ are both strictly decreasing functions. Next, observe that for any location k:

$$v_k(d) = s_k - (\mu + \eta) \cdot a_k + (\mu + \eta) \max\{d + a_k, n_{\min}\} - \delta \cdot e^d$$

where $n_{\min} = \ln(N_{\min})$ and $a_k = \ln(A_k)$. By assumption, $s_i - (\mu + \eta) \cdot a_i > s_j - (\mu + \eta) \cdot a_j$ (recall the definition of s_i). Also, by assumption $\max\{d + a_i, n_{\min}\} \geq \max\{d + a_j, n_{\min}\}$ for any d. Hence, $v_i(d) > v_j(d)$ for all d. Since both functions are decreasing, the function $v_i(d)$ must lie to the right of $v_j(d)$. Therefore, in any equilibrium, d_i^* must exceed d_j^*. ■

Proof of Proposition 2

Let $\{d_i^*, v^*, s_i, \bar{d}_i, N\}$ be a system-of-cities equilibrium for some r and λ. Suppose that λ and r change to λ' and r', respectively. This implies that s_i change to s_i'. But it follows from the definition of s_i that $s_i' = s_i + \Delta$ for some $\Delta \in R$. Now consider the collection $\{d_i^*, v^* + \Delta, s_i', \bar{d}_i, N\}$. It is clear that (i) this collection satisfies all equilibrium conditions in (1) and is therefore a system-of-cities equilibrium, and (ii) the equilibrium density for each location in this new collection is (trivially) the one that is closest to the initial equilibrium for each location. The result follows. ■

Proof of Proposition 3

To begin with, note that $\bar{d}_i > \ln[(\mu+\eta)/\delta]$ implies that the $v_i(d)$ function is strictly decreasing in d for all i so that there is a unique equilibrium density for each location in both the initial and the new equilibrium.

(i) Now, observe that, with no change in any A_i, an increase in total employment implies that equilibrium employment density must increase in at least one location. For specificity, suppose it rises for location i'. It follows that $v_{i'}^2 < v_{i'}^1$. Then, Definition 18 implies that $v_i^2 < v_i^1$ for all i and hence $d_i^2 > d_i^1$ for all i as well. Since $d_i^2 = \ln \rho_i + d_i^1$, $\rho_i > 1$ for all i.

(ii) Next, observe that for any pair of locations i and j, (18) implies:

$$v_i^2 - v_i^1 = v_j^2 - v_j^1 \tag{22}$$

Since $d_i > \bar{d}_i$ for all i, substituting (18) into (22) yields:

$$-\delta(e^{d_i^2} - e^{d_i^1}) + (\mu + \eta)(d_i^2 - d_i^1) = -\delta(e^{d_j^2} - e^{d_j^1}) + (\mu + \eta)(d_j^2 - d_j^1)$$

Using the fact that $d_i^2 = \ln \rho_i + d_i^1$ then gives:

$$-\delta \cdot e^{d_i^1}(\rho_i - 1) + (\mu + \eta) \ln \rho_i = -\delta \cdot e^{d_j^1}(\rho_j - 1) + (\mu + \eta) \ln \rho_j \qquad (23)$$

Now suppose that $d_i^1 > d_j^1$. Then, for any common value of ρ the l.h.s. of (23) is less than its r.h.s. Furthermore, since $d_i^1 > \ln[(\mu + \eta)/\delta]$ for all i, the l.h.s. and the r.h.s. of (23) are strictly decreasing for $\rho_i \geq 1$ and $\rho_j \geq 1$, respectively. It follows then that (23) implies $\rho_i < \rho_j$. ■

Proof of Proposition 4

To begin with, note that $\bar{d}_i > \ln[(\mu+\eta)/\delta]$ implies that the $v_i(d)$ function is strictly decreasing in d for all i so that there is a unique equilibrium density for each location in both the initial and the new equilibrium.

Since $\bar{d}_i > \ln[(\mu + \eta)/\delta]$ for all i implies that $v_i(d)$ function is strictly decreasing for all i, it follows from (18) that for any i and j, $d_i^1 > d_j^1$ implies that $s_i > s_j$, which, in turn, implies that $d_i^2 > d_j^2$. Therefore, the arrangement of locations that orders them in terms of decreasing density in the initial equilibrium also orders them in terms of decreasing density in the new equilibrium. Furthermore, since the inverse relationship between initial density and employment growth is assumed to hold, this arrangement also orders location in terms of increasing ρ_i. For the rest of this proof, assume that locations are ordered in this way.

Define ρ to be N^2/N^1 and let $1 \leq K < M$ be such that $\rho_i \leq \rho$ for all $i \leq K$ and $\rho_i > \rho$ for all $i > K$. Define N_i^s/N^s as ω^s, $s = 1, 2$. Since $\omega_i^2 < \omega_i^1$ if and only if $\rho_i < \rho$, and $L^s(m)$ is, by definition, $\sum_{i=1}^m \omega_i^s$, it follows that $L^2(m) < L^1(m)\ \forall\ m \leq K$ and $\sum_{i=m}^M \omega_i^2 > \sum_{i=m}^M \omega_i^1\ \forall\ m > K$. Now observe that the second inequality is equivalent to $1 - \sum_{i=m}^M \omega_i^2 < 1 - \sum_{i=m}^M \omega_i^1\ \forall\ m > K$. Since $L^s(m-1) + \sum_{i=m}^M \omega_i^s = 1$, it is also equivalent to $L^2(m-1) < L^1(m-1)\ \forall m > K$. ■

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