In Search of a Nominal Anchor: What Drives Inflation Expectations?*

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Abstract

According to both central bankers and economic theory, anchored inflation expectations are key to successful monetary policymaking. Yet, we know very little about the determinants of those expectations. While policymakers may take some comfort in the stability of long-run inflation expectations, the latter is not an inherent feature of the economy. What does it take for expectations to become unanchored? We explore a theory of expectations formation that can produce episodes of unanchoring. Its key feature is state-dependency in the sensitivity of long-run inflation expectations to short-run inflation surprises. Price-setting agents act as econometricians trying to learn about average long-run inflation. They set prices according to their views about future inflation, which hence feed back into actual inflation. When expectations are anchored, agents believe there is a constant long-run inflation rate, which they try to learn about. Hence, their estimates of long-run inflation move slowly, as they keep adding observations to the sample they consider. However, in the spirit of Marcet and Nicolini (2003), a long enough sequence of inflation suprises leads agents to doubt the constancy of long-run inflation, and switch to putting more weight on recent developments. As a result, long-run inflation expectations become unanchored, and start to react more strongly to short-run inflation surprises. Shifts in agents' views about long-run inflation feed into their price-setting decisions, imparting a drift to actual inflation. Hence, actual inflation can show persistent swings away from its long-run mean. We estimate the model

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York, or the Federal Reserve System.

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using actual inflation data, and only short-run inflation forecasts from surveys. The estimated model produces long-run forecasts that track survey measures extremely well. The estimated model has several uses: 1) It can tell a story of how inflation expectations got unhinged in the 1970s; it can also be used to construct a counterfactual history of inflation under anchored long-run expectations. 2) At any given point in time, it can be used to compute the probability of inflation or deflation scares. 3) If embedded into an environment with explicit monetary policy, it can also be used to study the role of policy in shaping the expectations formation mechanism.

"Public confidence that inflation will remain low in the long run has numerous benefits. Notably, if people feel sure that inflation will remain well controlled, they will be more restrained in their wage-setting and pricing behavior, which (in something of a virtuous circle) makes it easier for the Federal Reserve to confirm their expectations by keeping inflation low. At the same time, by reducing the risk that inflation will come loose from its moorings, well-anchored inflation expectations may afford the central bank more short-term flexibility to respond to economic disturbances that affect output and employment."

— Ben Bernanke, former Governor of the Federal Reserve, October 7, 2004.[1]

1 Introduction

According to both central bankers and economic theory, anchored inflation expectations are key to successful monetary policy. As long as long-run expectations are anchored, monetary policy has more leverage to respond to short-run disturbances to the economy. Yet, we know very little about the determinants of those expectations. While policymakers may take some comfort in the stability of long-run inflation expectations, it should not be taken for granted — it is not an inherent feature of the economy.

What does it take for expectations to become unanchored? In this paper we explore a theory of expectations formation based on learning, which embeds a well-defined notion of anchored and unanchored expectations. Its key feature is state-dependency in the sensitivity of long-run inflation expectations to short-run inflation surprises. When estimated on actual inflation and **short-run** inflation forecasts from surveys, the model captures long-term inflation expectations extremely well. Hence, despite its simplicity, the model provides a useful framework to think about the actual expectations formation process.

In the model, price-setting agents act as econometricians trying to learn about average long-run inflation. In response to short-run inflation surprises, they update their views about future inflation, and set prices accordingly. Expectations about the entire future path of inflation feed back into actual inflation. The key dinstinguishing property of the model is that the sensitivity of long-run inflation expectations to short-run inflation surprises is state-dependent. This allows for an explicit criterion to determine whether expectations are anchored.

[1] Speech available here: http://www.federalreserve.gov/Boarddocs/speeches/2004/200410072/default.htm.

When expectations are anchored, agents believe there is a constant long-run inflation rate, which they try to learn about. Their estimates of long-run inflation move relatively more slowly, as they keep expanding the sample they use in estimation. For the reader familiar with the jargon from the learning literature, this amounts to a decreasing-gain algorithm. As a result, the sensitivity of long-run expectations to short-run inflation surprises decreases over time, along with the learning gain.

In the spirit of Marcet and Nicolini (2003), however, a sizeable enough sequence of inflation surprises leads agents to doubt the constancy of long-run inflation and start putting more weight on recent inflation developments — they switch to a constant-gain learning algorithm. As a result, long-run inflation expectations start to react more strongly to short-run inflation surprises, and they become unanchored. Shifts in agents' views about long-run inflation feed into their price-setting decisions, imparting a drift to actual inflation. Hence, actual inflation can show persistent swings away from its long-run mean.

The reduced-form representation of the model features a time-varying drift in inflation. For that reason, our framework makes contact with the literature on inflation dynamics that assumes an exogenous, time-varying inflation drift (e.g., Cogley and Sbordone 2006, Cogley, Primiceri, and Sargent 2007). However, in our model inflation drift is determined endogenously, and its persistence and volatility depend on the state of the economy. Specifically, innovations to the drift are related to innovations to actual inflation, and so is the time-variation in inflation persistence.

The restrictions alluded to in the previous paragraph constrain our model significantly, relative to similar models with an exogenous inflation drift. More fundamentally, we discipline our model through our estimation exercise. Specifically, we estimate the model using actual inflation data, and **only short-run** inflation forecasts from surveys. We do so for various countries for which we have inflation forecasts from survey data. Hence, the dynamics of anchoring and unanchoring, which depend on the sequence of inflation surprises, are pinned down by observations of short-term forecast errors from surveys. These restrictions could be expected to handicap the model in terms of fitting data on inflation and inflation expectations, relative to models with an exogenous inflation drift. Perhaps surprisingly, our estimated model fits the data essentially as well as specifications with an exogenous drift.

Importantly, our model fits long-run inflation forecasts from survey data – which we do not

use in the estimation – extremely well. In that sense, it provides a framework to think about the actual expectations formation process. The estimated models provide stories for the joint evolution of inflation and the term structure of inflation expectations for different countries, which can enhance our understanding of the role of long-run expectations in determining actual inflation. This is so because of the feedback from agents' expectations into actual inflation, through their price-setting decisions. So, for example, the model suggests that the unanchoring of inflation expectations was key to the high inflation of the 1970s. It also provides an estimate of when inflation expectations became anchored again, which accords with common wisdom. Despite the reduced-form nature of the model that we take to the data, we also find surprising that the few parameters that call for a "structural" interpretation are somewhat comparable across countries.

While in this paper we focus on "inflation surprises" as the fundamental innovations to the expectations formation process, this model of expectations formation can be embedded into an environment with structural shocks and explicit monetary policy. This would allow us to evaluate the role of policy in shaping the response of long-run expectations to different shocks.

1.1 Literature review

[To be added]

2 A Simple Model of Inflation Determination

To develop some ideas fundamental to the paper, we first present a simple reduced-form model of expectations formation and inflation determination. Because of nominal price rigidities, pricing decisions depend upon expectations of future inflation. This creates a link from the inflation expectations to actual inflation. This link is derived explicitly in a model with Calvo (1983) pricing in section 2.3. That model illustrates how our assumptions regarding firm beliefs about inflation are quite close to rational expectations — departing from them only because of the need to estimate the long-run average rate of inflation.

2.1 A Reduced-form Model

When setting prices agents perceive the law of motion of inflation to be

$$\text{PLM:} \qquad \pi_t = \bar{\pi}_t + \varphi_t, \tag{1}$$

where π_t is inflation; φ_t is a zero-mean stationary short-run component of inflation dynamics; and $\bar{\pi}_t$ is the average level of inflation expected to prevail in the long run. Agents estimate this long-run mean and, possibly, the law of motion for φ_t.

For simplicity adopt the following anticipated-utility assumption: although agents realize they will be update their estimate of $\bar{\pi}_t$ as new data become available, when making decisions in any period t, they expect $\bar{\pi}_t$ to remain constant — see Kreps (1998) and Sargent (1999). Expectations of future inflation are computed as

$$\hat{E}_t \left[\pi_{t+T}\right] = \bar{\pi}_t + \hat{E}_t \left[\varphi_{t+T}\right]$$

where the operator $\hat{E}_t$ denotes subjective expectations.

Because of nominal rigidities, newly set prices remain in place for at least a few periods. And because of strategic complementarity, when choosing a price in period t firms are concerned about future expected inflation. This renders actual inflation dependent upon expectations of future inflation. Suppose this dependence arises only through the perceived long-run inflation average. The actual law of motion of inflation is then

$$\text{ALM:} \qquad \pi_t = T\bar{\pi}_t + \varphi_t, \tag{2}$$

where $T < 1$ is a parameter which controls the feed back from inflation expectations to actual inflation. In general this coefficient will depend on the details of the firm's optimal price-setting problem. The perceived law of motion and the actual law of motion for inflation differ only in that the time-varying intercept in the actual law of motion for inflation is $T\bar{\pi}_t$ rather than $\bar{\pi}_t$. If T is close to unity this difference will be small.

It remains to specify the process by which agents update their estimates of average long-run inflation. Following Marcet and Nicolini (2003) beliefs are revised according to the learning algorithm

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_{t-1}^{-1} \times f_{t-1}, \tag{3}$$

where

$$f_t = \pi_t - \hat{E}_{t-1}\pi_t.$$

A complicated simultaneity is resolved by assuming that the $\bar{\pi}_t$ estimate (which affects current decisions) depends on the previous-period's forecast error. This is a standard assumption in the learning literature. The gain coefficient $k_t > 1$ is determined by

$$k_t = \begin{cases} \bar{g}^{-1}, \text{ if } \left| \hat{E}\pi_t - \mathbb{E}_{t-1}\pi_t \right| > v\sqrt{\mathbb{MSE}} \\ \\ k_{t-1} + 1, \text{ otherwise} \end{cases}, \tag{4}$$

where $\bar{g}, \nu > 0$ are parameters; $\hat{E}_{t-1}\pi_t$ is the agent's one-period-ahead forecast; $\mathbb{E}_{t-1}\pi_t$ is the model-consistent one-period-ahead expectation; and

$$\mathbb{MSE} = \mathbb{E}\left[\pi_t - \mathbb{E}_{t-1}\pi_t\right]^2$$

is the mean-squared error associated with one-period-ahead model-consistent expectations.

The state-dependent gain k_t captures agents' attempts to protect against structural change. A constant gain $\bar{g}$ produces better forecasts when the economic environment changes, but it does not converge in a stationary environment. In contrast, a decreasing gain estimator, such as ordinary-least squares, converges in stationary environments. The proposed learning algorithm uses ordinary least squares in periods of relative stability and switches to constant-gain when instability is detected — that is when

$$\left| \hat{E}_{t-1}\pi_t - \mathbb{E}_{t-1}\pi_t \right| > v\sqrt{\mathbb{MSE}}.$$

The parameter ν determines how alert agents are to model misspecification. A stable environment here is an economy where inflation does not vary too much. In this case the distance between an agent's forecast and the model-consistent forecast tends to be small. The updating rule (3) can also interpreted in terms of the Kalman filter. The model of the inflation drift is

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \eta_t \tag{5}$$

where the variance of η_t can take two values corresponding to different regimes: $\sigma_\eta^2 = \{\bar{\sigma}_\eta^2, 0\}$. In the first regime $\bar{\pi}_t$ drifts according to a random walk. The constant-gain algorithm can be interpreted as the Kalman updating of this model. The second regime corresponds to a constant mean for inflation, giving a decreasing gain — see Bullard (1992) for a discussion. The updating rule (3) is, however, not optimal because it does not fully internalize the regime switching in the variance of η_t.

Three assumptions are maintained throughout the paper.

Assumption 1. The process φ_t is exogenous and determined by

$$\begin{aligned} \varphi_t &= s_t + \mu_t \\ s_t &= \rho_s s_{t-1} + \epsilon_t \end{aligned}$$

where μ_t and ϵ_t are i.i.d. disturbances, normally distributed with mean zero and variances σ_μ^2 and σ_ϵ^2.

Assumption 2. Agents' conditional expectations for φ_t coincide with the model-consistent expectation, that is

$$\hat{E}_t\left[\varphi_{t+T}\right] = \mathbb{E}_t\left[\varphi_{t+T}\right] \text{ for } T \geq t.$$

To obtain more intuition on the properties of this learning algorithm, use assumptions (1) and (2) to write

$$\begin{aligned} \left|\hat{E}_{t-1}\pi_t - \mathbb{E}_{t-1}\pi_t\right| &= \left|(1-T)\,\bar{\pi}_t\right| \\ &= \left|(1-T)\left(\bar{\pi}_{t-1} + k_{t-1}^{-1} \times f_{t-1}\right)\right| \\ &= \left|(1-T)\left[\bar{\pi}_0 + \sum_{\tau=0}^{t} k_\tau^{-1} f_\tau\right]\right| \end{aligned}$$

given $\bar{\pi}_0$, f_0, k_0. The distance $\left|\hat{E}_{t-1}\pi_t - \mathbb{E}_{t-1}\pi_t\right|$ tends to be large when forecast errors happen to be of the same sign for several periods. This pushes $\bar{\pi}_t$ away from its long-run mean, and drives a wedge between the perceived and true model of the economy.

One may wonder how agents can switch to a constant-gain algorithm based on a criterion that involves model-consistent expectations — which they are assumed not to know. This assumption is made in the "as if" tradition in economics. Agents indeed do not know $\mathbb{E}_{t-1}\pi_t$, but they conclude that there must be something unusual happening when their forecasts "go astray" in the sense that $\left|\hat{E}_{t-1}\pi_t - \mathbb{E}_{t-1}\pi_t\right|$ becomes too large. This assumption captures agents' efforts to infer model instability by using statistical tools to detect time-variation in their model's intercept. They key here is that criterion to switch gains depends on past

forecast errors. In turn, the size of forecast errors depends on the gain being used by agents. This opens the door for multiple learning equilibria, a model feature discussed at length later in the paper.

We consider parameter restrictions for which learning converges to a constant inflation rate. The model therefore implies a unique steady state. Our third assumption is then that the economic environments we study have a time invariant long-run mean of inflation that is eventually learned by market participants.

Assumption 3. The data-generating process for inflation has a time-invariant mean $\pi = 0$.

2.2 Models with an Exogenous Inflation Drift

Our framework relates to a recently popular class of model which posits inflation is, in part, determined by an exogenous source of drift. These include reduced-form models as in Stock and Watson (1993, 2007), Cogley, Primiceri, and Sargent (2010), and Kozicki and Tinsley (2012), models of the Phillips curve such as Cogley and Sbordone (2008), and various DSGE models of the kind proposed by Smets and Wouters (2007), Cogley, Primiceri, and Sargent (2010) and Del Negro, Giannoni, and Schorfheide (2015). In these models $\bar{\pi}_t$ evolves exogenously according to

$$\bar{\pi}_t = \rho_{\bar{\pi}} \bar{\pi}_{t-1} + e_t \tag{6}$$

with autoregressive coefficient close to unity. In contrast, in the model studied here the inflation drift is determined *endogenously* and depends on agents' past inflation forecast errors. To see this, re-write the learning algorithm (3) as

$$\begin{aligned} \bar{\pi}_{t+1} &= \bar{\pi}_t + k_t^{-1} \left(\pi_t - \hat{E}_{t-1} \pi_t \right) \\ &= \rho_{\bar{\pi},t} \bar{\pi}_t + \tilde{e}_t \end{aligned} \tag{7}$$

where

$$\begin{aligned} \rho_{\bar{\pi},t} &= \left[1 + k_t^{-1} \left(T_\pi - 1 \right) \right] \\ \tilde{e}_t &= k_t^{-1} \left(\varphi_t - \mathbb{E}_{t-1} \varphi_t \right) = k_t^{-1} \left(\epsilon_t + \mu_t \right). \end{aligned}$$

The second equation is obtained by substituting (2) for inflation and by using assumptions 1 and 2. The evolution of $\bar{\pi}_t$ under (7) differs in two key respects from (6). First, the autocorrelation coefficient, $\rho_{\bar{\pi},t}$, and the innovation volatility, $\tilde{e}_t$, are time-varying and depend on the state of the economy. Second, the innovations to $\bar{\pi}_t$ depend on inflation forecast errors. In the general model developed in the sequel, both persistent shocks to the economy and policy mistakes create unexpected movements in inflation, which can trigger movement in the inflation drift.

2.3 A model with explicit price-setting decisions

To derive an explicit link from expectations to actual inflation consider the theory of price setting proposed by Calvo (1983), as implemented by Yun (1996), and extended to arbitrary subjective expectations by Preston (2005). Firm i maximizes the present discounted value of profits

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[Y_T(i) \left(\frac{P_t(i)}{P_T} - MC_T \right) \right]$$

where $Q_{t,T}$ is the discount factor, MC_t is the real marginal cost and

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta_{p,t}} Y_t$$

is the demand curve that each firm faces. The elasticity of demand across differentiated goods, $\theta_{p,t}$, is time-varying. Each period, with probability α, the firm's price is reset optimally. Alternatively, with probability $1-\alpha$, the existing price is mechanically indexed to a weighted average of past inflation and the perceived long-run inflation rate

$$\bar{\Pi}_t^p = \bar{\pi}_t^{1-\gamma_p} \pi_{t-1}^{\gamma_p}.$$

To keep things simple, assume a constant steady-state mean of zero inflation. In log-linear deviation from steady state, the first-order conditions of this problem deliver the optimal price-setting decision

$$\hat{p}_t^* = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[(1-\alpha\beta)(mc_T + u_T) + \alpha\beta \left(\pi_{T+1} - \gamma_p \pi_T - (1-\gamma_p)\bar{\pi}_t \right) \right] \tag{8}$$

where u_t denotes a cost push shock arising from time-varying elasticity of demand. With a slight abuse of notation all variables are now interpreted as log-deviations from steady-state,

with $\hat{p}_t^* = \ln\left(P_t\left(i\right)/P_t\right)$ for all i. The optimal price depends on the expected future sequence of marginal costs and also inflation, adjusted for the economic effects of indexation. The second term arises from strategic complementarity in price setting, which engenders a tight connection between expectations of future inflation and current price decisions. Aggregating across firms, and using the fact that the aggregate price index satisfies the log-linear approximation

$$\hat{p}_t^* = \frac{\alpha}{1-\alpha}\pi_t$$

delivers the following data-generating process for inflation

$$\pi_t = \gamma_p\pi_{t-1} + (1-\gamma_p)\bar{\pi}_t + \tag{9}$$

$$\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\left(mc_T + u_T\right) + (1-\alpha)\,\beta\left(\pi_{T+1} - \gamma_p\pi_T - (1-\gamma_p)\bar{\pi}_t\right)\right]$$

where $\kappa = (1-\alpha)(1-\alpha\beta)/\alpha$.

The model is closed with a theory marginal cost determination. To keep matters simple, assume marginal costs evolve according to

$$\lambda\pi_t + mc_t = \tau_t, \tag{10}$$

where $\lambda > 0$ is a parameter, and τ_t is exogenous and evolves according to

$$\tau_t = \rho_\tau\tau_t + \epsilon_t^\tau \tag{11}$$

where $0 < \rho_\tau < 1$. The assumption is interpreted as permitting some feed back from inflation to marginal costs, as a reduced-form way to model the effects of monetary policy. When $\lambda = 0$ marginal costs would evolve exogenously, independently of monetary policy. The specific form of the assumption also nests a popular policy specification in the monetary economics literature. In the canonical New Keynesian model, relation (10) can be interpreted as deviations from a targeting rule, derived under optimal discretion, with a constant target of zero inflation. The deviations are given by the exogenous process τ_t. The parameter λ is shown below to be an important determinant of the drift in inflation in the true data-generating process.

What do agents know? To keep the framework as simple as possible, and in line with the previous section, assume they know everything except for the long-run mean of inflation.

Agents estimate a long-run inflation mean $\bar{\pi}_t$ in each period using the filtering algorithm (3). They form expectations of marginal costs and inflation using the correct law of motion for the marginal cost

$$mc_t = \tau_t - \lambda \pi_t,$$

and the following equilibrium *perceived* law of motion (PLM) for inflation:

$$\pi_t = \left(1 - \gamma_p\right) \bar{\pi}_t + \gamma_p \pi_{t-1} + \omega \tau_t + e_t,$$

where $\omega = \tilde{\kappa}\left(1 - \tilde{\beta}\rho_\tau\right)^{-1}$ denotes the coefficient that obtains under rational expectations equilibrium with

$$\tilde{\kappa} = \frac{\kappa}{1 + \lambda\kappa} \; ; \quad \tilde{\beta} = \frac{\beta}{1 + \lambda\kappa}$$

and e_t is an *i.i.d.* innovation. The basic idea here is that firms understand the rational expectations dynamics up to the drift term. Uncertainty about long-run inflation does not affect the rational expectations coefficients on lagged inflation or the exogenous component of marginal costs. They are determined by a standard fixed-point problem.

Evaluating expectations in (9) delivers the *actual* law of motion (ALM)

$$\pi_t = \left(1 - \gamma_p\right) T \bar{\pi}_t + \gamma_p \pi_{t-1} + \omega \tau_t + \tilde{u}_t$$

where

$$\begin{aligned} T &= \left(\frac{1}{1 + \lambda\kappa} - \frac{\tilde{\beta}\left(1 - \alpha\right)}{1 - \alpha\beta}\right) + \\ &\left(\tilde{\beta}\left(1 - \alpha\right) - \tilde{\kappa}\alpha\beta\lambda\right) \frac{1 - \gamma_p\alpha\beta}{1 - \gamma_p} \left(\frac{1}{1 - \alpha\beta} - \frac{\gamma_p}{1 - \alpha\beta\gamma_p}\right) \end{aligned}$$

and $\tilde{u}_t = \frac{1}{1+\lambda\kappa} u_t$ is the cost push shock. The ALM displays a time-varying intercept that differs from the PLM's. The parameter T captures the degree to which beliefs about inflation are self-fulfilling. In the case that $T = 1$ there is a self-confirming equilibrium in the language of Sargent (1999) — beliefs about inflation are confirmed by observed data. More generally when $T < 1$, beliefs are only partially validated observed data. Stronger responses of marginal costs to inflation, reflected in a higher value of the parameter λ, imply less feed back from beliefs to inflation — see Ferrero (2007) and Orphanides and Williams (2005). Similarly, price setting that exhibits a lower degree of indexation to beliefs about long-run inflation, $\bar{\pi}_t$, reflected in a higher γ_p, imply smaller feedback effects from learning.

3 Data and Estimation

This section details the data and estimation strategy. The idea is to estimate the model using data on inflation and various measures of short-term inflation expectations. The empirical success of the model is evaluated on several dimensions. One important source of external model validation regards model predictions for long-term forecasts of inflation which are not used in estimation. A key result is the empirical model replicates well survey measures of long-term inflation expectations for the sample of countries considered. Because of survey data limitations for international samples, we first discuss the benchmark data used to estimate the US model. Subsequent discussion turns to the international samples and specific issues confronting statistical inference.

3.1 The Benchmark Model

For all countries we estimate the following model

$$\begin{aligned}
\pi_t &= \left(1-\gamma_p\right) T\bar{\pi}_t + \gamma_p \pi_{t-1} + s_t + \mu_t \\
\bar{\pi}_t &= \bar{\pi}_{t-1} + k_{t-1}^{-1} \times f_{t-1} \\
f_t &= \left(1-\gamma_p\right)(T-1)\,\bar{\pi}_t + \mu_t + \epsilon_t \\
s_t &= \rho_s s_{t-1} + \epsilon_t,
\end{aligned}$$

where ϵ_t and μ_t are normally distributed with zero mean and variances σ_s^2 and σ_μ^2, and the gain k_t determined by (4). Writing the system in state-space form provides

$$\boldsymbol{\xi}_t = F(\mathbf{k}_{t-1}^{-1})\boldsymbol{\xi}_{t-1} + S_C\boldsymbol{\epsilon}_t$$

where the time-varying gain $\mathbf{k}_t$ is pre-determined at time t and

$$\boldsymbol{\xi}_t = \begin{bmatrix} \pi_t \\ \bar{\pi}_t \\ f_t \\ s_t \end{bmatrix} \quad \text{and} \quad \boldsymbol{\epsilon}_t = \begin{bmatrix} \mu_t \\ \epsilon_t \end{bmatrix}$$

with the matrices $F\left(\mathbf{k}_{t-1}^{-1}\right)$ and S_C defined in the appendix.

The model is estimated using full-information Bayesian methods. We seek to estimate the following vector of structural parameters

$$\bar{\theta} = \left(\begin{array}{cccccccc} \pi^* & \nu & \bar{g} & \gamma_p & T & \rho_s & \sigma_s^2 & \sigma_\mu^2 \end{array} \right)'$$

together with the variance of the observation errors to be discussed. The US and international models are distinguished by both the adopted priors and also the specific set of observable data used in estimation.

3.2 US Data

The model is estimated on quarterly data. For all countries inflation is measured by the CPI. This choice is driven by the broad availability of survey-based forecasts for this price index. Short-term expectations are measured using mean consensus forecasts from different surveys of professional forecasters. For the US, estimation employs two sources of short-term CPI inflation forecasts. The Livingston survey provides the longest series for short-term forecasts. The data set contains forecasts for the price level. From those, two series measuring 6-month-ahead CPI inflation forecasts are constructed. The first series, available starting in 1955, computes the inflation forecasts from a base period, which is the last monthly price level known at the time the survey was fielded. As this might not capture the information set of the forecasters, we use an additional series for the 6-months-ahead forecast where the forecasted inflation rate is computed using the price level forecast for the month in which the survey was taken (which is only available starting in 1992). These forecasts are available twice a year, in the second and fourth quarter. We also use one- and two-quarters-ahead CPI inflation forecasts from the Survey of Professional Forecasters, which are available at a quarterly frequency since 1981Q3. **Figure 1** shows the available data used for the estimation.

The observation equation for US data is then

$$Y_t^{US} = \left[\begin{array}{c} \pi_t \\ \mathbf{E}_t^{SPF} \pi_{t+1} \\ \mathbf{E}_t^{SPF} \pi_{t+2} \\ \mathbf{E}_t^{LIV_1} \left(\frac{1}{2} \sum_{i=1}^{2} \pi_{t+i} \right) \\ \mathbf{E}_t^{LIV_2} \left(\frac{1}{2} \sum_{i=1}^{2} \pi_{t+i} \right) \end{array} \right] = \pi^* + H_t' \xi_t + R_t o_t$$

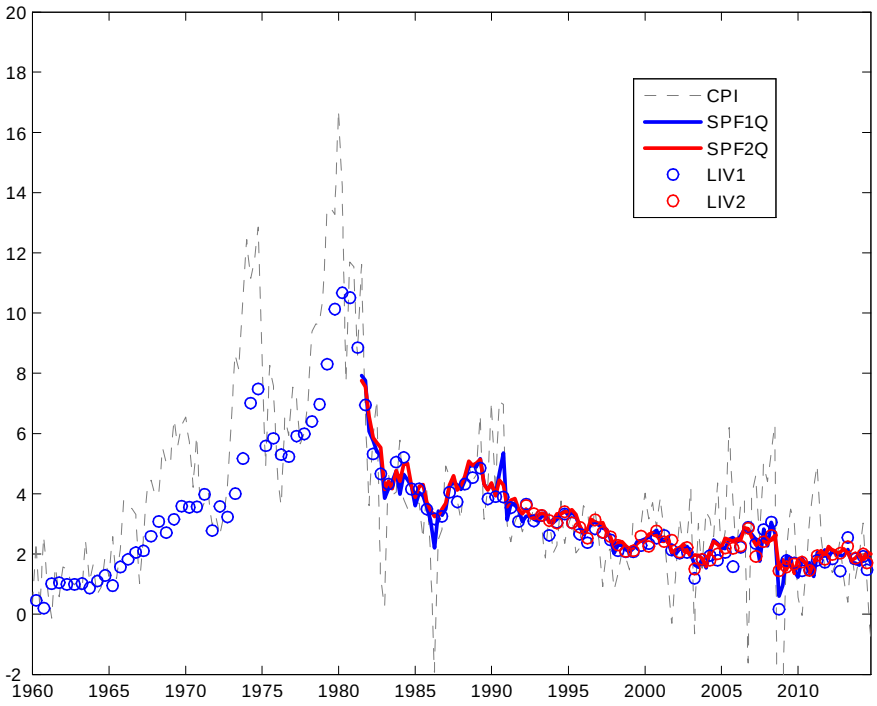

Figure 1: The figure shows the evolution of CPI inflation (dashed grey line) and consensus (mean) survey-forecasts for inflation. In detail, the one- and two-quarters ahead forecasts from SPF are shown by the solid blue and red lines, respectively. The blue and red circles show the six months-ahead forecasts from the Livingston survey.

where the survey forecasts labelled "LIV_1" uses the actual price level as a base, and "LIV_2" uses the forecasted price-level as a base. The vector o_t includes observation errors for all variables. We include an observation error on CPI inflation as well as on the survey-based forecasts as it allows filtering a measure of underlying inflation that drives short-term inflation expectations. The required forecasts are mapped into model forecasts in obvious fashion. For example

$$\mathbf{E}_t^{SPF}\pi_{t+1} = \hat{E}_t\pi_{t+1} = \left(1-\gamma_p\right)\bar{\pi}_t + \gamma_p\pi_t + \omega\rho_\tau\tau_t,$$

where, again, $\hat{E}_t$ denotes price-setters' forecasts in the model. The matrices H_t and R_t are time-varying because of missing observations.

The priors for the US are detailed in **table 1** below.[2] The priors on all parameters besides $\bar{g}$ and ν are fairly loose and reflect common choices in Bayesian estimation of DSGE models.

[2]The priors on observation errors for both US and foreign countries, omitted in the table, are all Inverse gamma with mean of 0.1 and unitary variance.

The priors on $\bar{g}$ and ν reflect some theoretical bounds which we deem as reasonable. For example, the parameter $\bar{g}$ is interpreted as giving weight $(1-\bar{g})^N$ to the N^{th} old observation in the sample. For example, $\bar{g} = 0.1$ (the mean of the distribution) gives a weight close to zero to observations older than ten years. Most of the probability mass is placed over the interval $\bar{g} \in [0.05; 0.3]$. Lower values of $\bar{g}$ would be too hard to distinguish from a decreasing gain in a sample as long as the one we have, while higher values imply forecasters discount heavily even most recent data (less than 2 years). The parameter ν measures when agents begin to doubt the constancy of the inflation rate. We assume agents have a good forecasting model and therefore assume they would react if their forecast depart by a small amount from the true model. As a result we impose a fairly tight prior on ν attributing most of the probability mass on values between $0.01 - 0.03$.

Given an estimate of the posterior mode, the Metropolis-Hasting algorithm is used to simulate the posterior distribution

$$P\left(\theta^{US}|Y_t^{US}\right) = L(Y_t^{US}|\theta^{US})P(\theta^{US}).$$

3.3 International Data

The sample of international data comprises the following countries: France, Germany, Japan, UK and Sweden. Inflation expectations are measured using data available from Consensus Economics. While the Consensus Economics data set includes short-term and long-term professional survey forecasts for a wide set of countries, it presents two challenges for estimation. First, Consensus Economics forecasts are made on a year-over-year basis. This formulation prevents a clean identification of the mechanism of the model, which links one-step-ahead forecast errors to the beliefs about long-term inflation, $\bar{\pi}_t$. Second, in contrast with the US forecast data, for most countries expectations data are only available from 1991, providing a limited time series for estimation. The following discussion details how each of these complications is handled.

Mapping data to model concepts. Estimation employs available Consensus Economics inflation forecasts for the one- and two-years-ahead horizons. Because of the year-over-year specification, these forecasts give most weight to quarterly forecasts up to four quarters ahead. They are reasonably classified as short-term forecasts. Subsequent discussion shows these forecasts differ significantly from the long-term forecasts we aim to explain.

To map the data concept into the model concept, note that year-over-year forecasts can be approximated as a weighted average of quarterly forecasts at different horizons, with "tent-shaped" weights. For the estimation, six sets of forecasts are used. The first two are forecasts for the current year, made in the first and second quarter of the year. These forecasts can be expressed as

$$\begin{aligned}
\mathbf{E}_t^{Cons}\pi_{year1,Q2} &\approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t-3+i} \\
\mathbf{E}_t^{Cons}\pi_{year1,Q1} &\approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t-4+i}
\end{aligned}$$

where $w(i)$ selects the i'th element of the vector $w = \left(\begin{array}{ccccccc} \frac{1}{16} & \frac{2}{16} & \frac{3}{16} & \frac{4}{16} & \frac{3}{16} & \frac{2}{16} & \frac{1}{16} \end{array} \right)$. The various expectations can be easily computed using knowledge of the structural model. The forecasts for the next calendar year are taken in each quarter of the current year. They can be expressed as

$$\begin{aligned}
\mathbf{E}_t^{Cons}\pi_{year2,Q4} &\approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t-2+i}; \quad \mathbf{E}_t^{Cons}\pi_{year2,Q3} \approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t-1+i} \\
\mathbf{E}_t^{Cons}\pi_{year2,Q2} &\approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t+i}; \quad \mathbf{E}_t^{Cons}\pi_{year2,Q1} \approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t+1+i},
\end{aligned}$$

where the weights remain the same. The observation equation for each foreign country is then

$$Y_t^F = \left[\begin{array}{c} \pi_t \\ \mathbf{E}_t^{Cons}\pi_{year1,Q2} \\ \mathbf{E}_t^{Cons}\pi_{year1,Q1} \\ \mathbf{E}_t^{Cons}\pi_{year2,Q4} \\ \mathbf{E}_t^{Cons}\pi_{year2,Q3} \\ \mathbf{E}_t^{Cons}\pi_{year2,Q2} \\ \mathbf{E}_t^{Cons}\pi_{year2,Q1} \end{array} \right] = \pi^F + H_t^{F\prime}\xi_t + R_t^F o_t$$

Because forecasts are different in each quarter, we have only one observation for each forecast per year. The time-varying matrices H_t^F and R_t^F again handle these missing observations. Looking at the weights, a key observation is that what is common to all these forecasts is that most of the weight (a fraction ranging from from 10/16 to 12/16) is given to quarterly forecasts ranging from one to four-quarters ahead.

Confronting a short sample. To handle the short available sample for the international data we employ posterior information from the US model to shape the priors adopted in estimation for these countries. In particular, the US posterior is used as a prior for all parameters except for the steady-state inflation rate and all observations errors. For these latter parameters, we use the same prior distributions specified for the US. One final assumption is made. When simulating the posterior distribution of foreign parameters, the foreign likelihood function is scaled by the parameter λ^F so that

$$P^F\left(\theta^F|Y_t^F, Y_t^{US}, \theta^{US}\right) = L(Y_t^*|\theta^{US}, \theta^F)^{\lambda^F} L(Y_t^{US}|\theta^{US}) p(\theta^{US}) p(\theta^F).$$

Smaller values of the parameter λ^F imply model predictions are more closely tied to the US posterior distribution. There are two reasons to proceed in this way. First, by choosing a low value of λ^F we can evaluate how the model estimated on US data performs in terms of capturing long-term forecasts in different countries. Second, we estimated the model on international data using CPI inflation starting in the mid-to-late 1950s (depending on the country), but survey data are available only starting in 1991. Because of this "unbalanced" sample, and the other data limitations discussed above, the data are not too informative about the mechanism of the model. In response to that we downweight the foreign Likelihood function. The presented results consider the cases of $\lambda^F = 0.2$ and $\lambda^F = 0.5$. The first case delivers a posterior distribution of the common parameters that is very close to the distribution for the US model. The second case gives substantial weight on country-specific data. The results are discussed in section 3.6.

3.4 US: The Benchmark Model

Table 1 reports the prior and posterior distribution of the parameters. For the most part the data are informative about model parameters, with the exception of ν, which has a fairly tight prior. The learning algorithm parameters ν and $\bar{g}$ have median estimates 0.02 and 0.12. This implies a switch from the decreasing-gain algorithm to the constant-gain algorithm occurs when the scaled subjective forecasts depart from the model-consistent forecasts by more than 2.2 per cent. The constant gain parameter implies that long-term inflation expectations are quite sensitive to short-term forecast errors. Indeed, a gain of this value implies a weight of approximately zero on ten-year-old data and a weight of about 0.1 on five-year-old data. The model implies a high degree of self-referentiality, with the parameter T taking a value of 0.91

— shifting beliefs about long-term inflation are in large part self-fulfilling. The remaining structural parameter γ_p is quite low, revealing subjective beliefs are more important than price indexation as a source of persistence.

Table 1. *Prior Distributions and Posterior Parameter Estimates for the US*

	Prior			**Posterior**				
	Dist.	Mean	SD	Mode	Mean	SD	5%	Med.	95%
$4\pi^*$	Normal	2.0	1.2	2.21	2.49	.29	1.96	2.50	2.90
ν	Gamma	.02	.006	.019	.022	.006	.013	.022	.033
g	Gamma	.10	.050	.124	.126	.028	.083	.124	.174
T	Beta	.7	.150	.952	.906	.041	.823	.914	.957
ρ_s	Beta	.7	.150	.887	.879	.028	.834	.880	.925
γ_p	Beta	.5	.260	.124	.140	.029	.095	.138	.191
σ_s	IGamma	.5	4	.087	.088	.009	.074	.088	.103
σ_μ	IGamma	.5	4	.377	.358	.036	.300	.357	.419

Figure 2 shows the one-quarter-ahead forecast errors predicted by the model as compared with the data. The model does a good job in capturing the evolution of forecast errors, which largely appear to be contained in the 95% bands.

Recall that we discipline the estimation of the model with *short-run inflation forecasts only*. Hence, a comparison of the long-run inflation forecasts produced by the model with long-run survey forecasts is a legitimate "test" of the model as a description of long-run inflation expectations. Results of the estimated model are presented in **Figure 3**, showing long-run inflation forecasts produced by the model, and various measures of long-run inflation expectations that were not used in the estimation.

The figure reveals that the model captures surprisingly well the dynamics of long-term inflation expectations as measured by survey data. The result is remarkable as the model has very few degrees of freedom since the forecasts errors, f_t, driving long-term forecasts are observed. In fact, observation errors on the survey forecasts are relatively small, with annualized standard deviations in the range of 1.18 for actual inflation and a range of $0.08-0.3$ for survey forecasts (modal estimates).

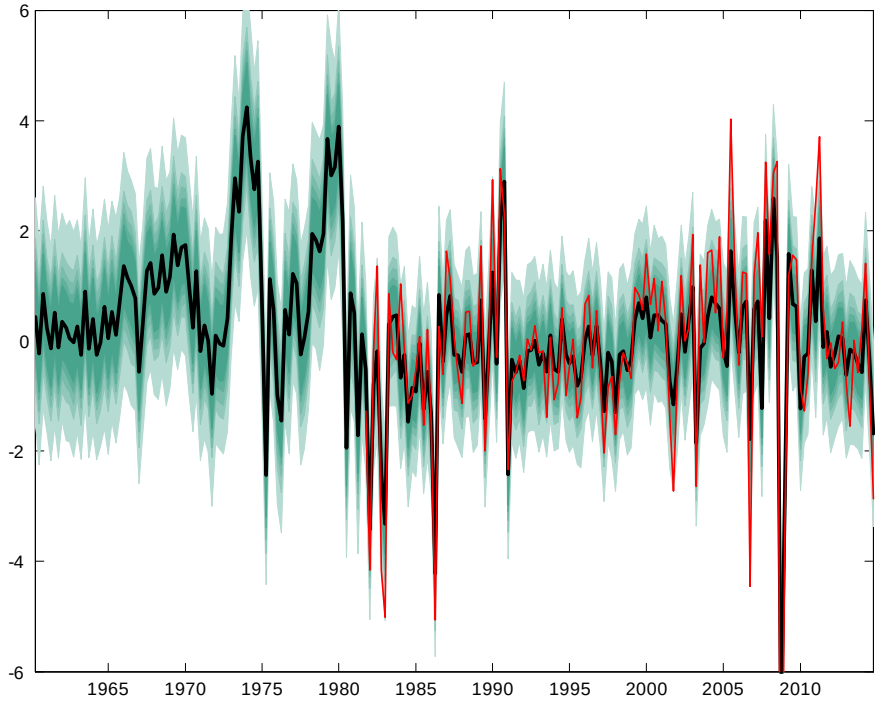

Figure 2: The figure compares the model's predicted one-quarter-ahead forecast errors with the data. In detail, the black solid line is the median (point-wise) prediction from the model. The solid line is the one-quarter-ahead forecast from SPF. The green shaded area denotes the 95% bands.

Figure 4 shows both the path of long-term forecasts together with the path of the learning gain implied by the estimated model at the parameters' posterior mode. The estimated model tells a story in which inflation expectations got unanchored in the 1970s, after a series of short-run inflation surprises. Agents kept using a constant-gain learning algorithm for quite a while – until the late 1990s. At that point, they reverted back to a decreasing gain procedure, and as a result the sensitivity of long-run inflation expectations decreased. This explains why long-run inflation expectations have been remarkably stable in the face of relatively volatile inflation in the last decade.

3.5 Alternative Models of Inflation Drift

Next, we compare the model predictions to two alternative empirical models estimated using the same data. The first model is the exogenous drift model discussed in section 2.2. Rather than assume beliefs about long-term inflation are generated by the learning algorithm (3) and

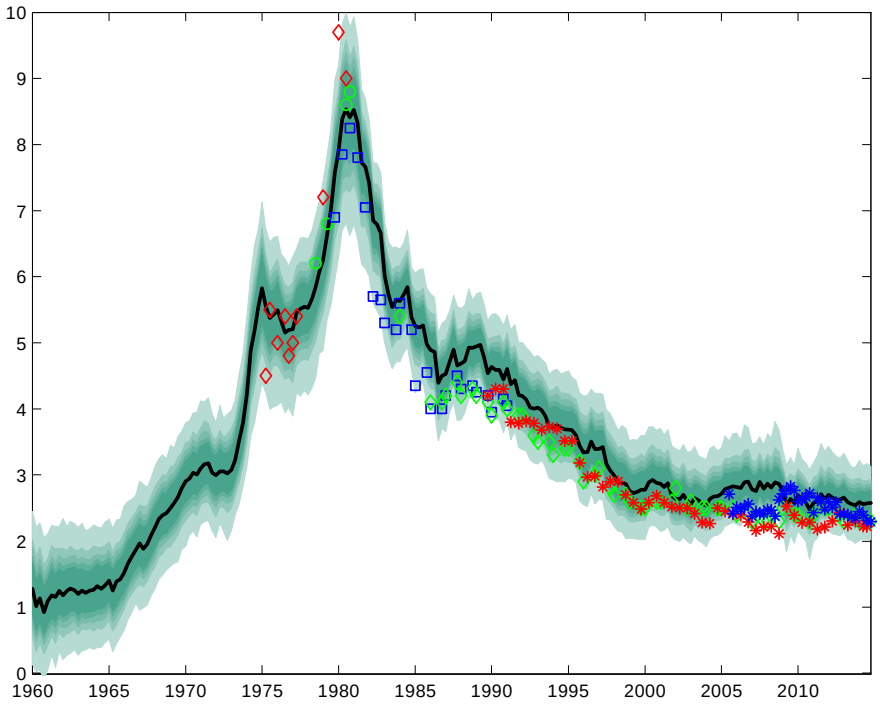

Figure 3: The figure compares the long-term forecast predicted by the model to different measure of survey-based long-term forecasts. In detail, the black line is the (median) predicted five-to-ten years forecasts from the model's PLM, with the green shaded area showing the 95% bands. Turning to the survey data. The red diamonds show 5-10 years CPI forecasts from the Michgan households survey, the green circles show the 1-10years CPI inflation forecasts from the Decision-Makers Poll of Portfolio Managers, the blue squares show the 1-10years CPI from Blue Chip Economic Indicators. The green diamonds show the 5-10years forecasts from Blue Chip Economic Indicators. The red stars show the 5-10years forecasts from Consensus Economics. Finally, and the blue stars are 5-10years implied forecasts from SPF.

(4), assume the inflation drift is specified by the exogenous process

$$\bar{\pi}_t = \rho_\pi \bar{\pi}_{t-1} + \varepsilon_t.$$

To guarantee identification, fix the persistence of $\bar{\pi}_t$ to $\rho_\pi = 0.995$. Remaining model features are unchanged, with actual inflation determined by

$$\pi_t = \left(1 - \gamma_p\right) \bar{\pi}_t + \gamma_p \pi_{t-1} + s_t + \mu_t.$$

Table 2 shows prior and posterior estimates for this model. The parameters in common with the benchmark model take fairly similar values. However, the steady-state annual inflation rate is estimated to be somewhat lower (2.13 compared to 2.5 percent). The standard deviation on the exogenous inflation drift is reasonably small at 0.049.

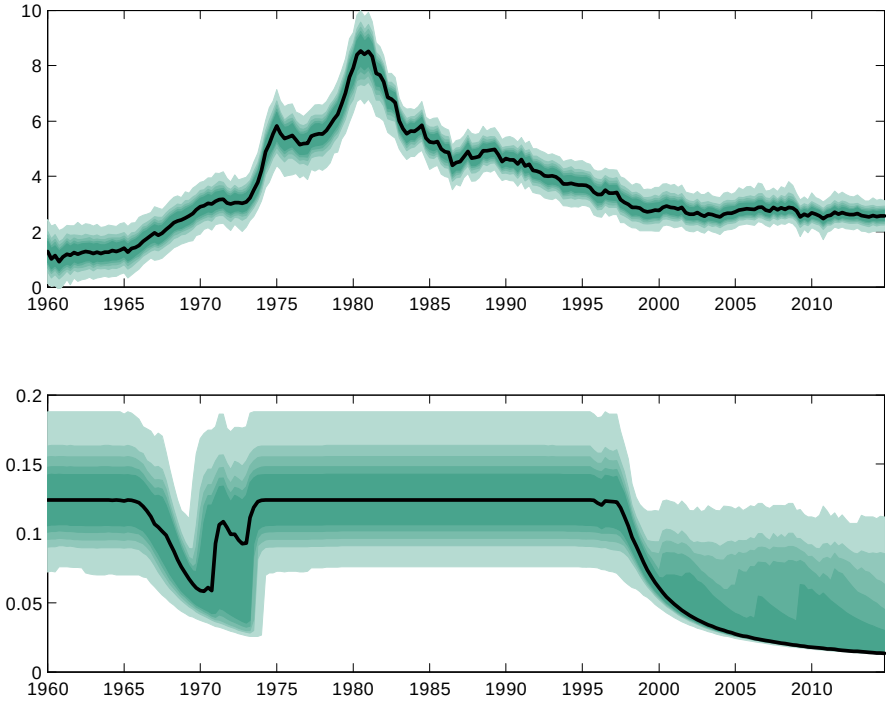

Figure 4: The figure shows the model predictions for long-term forecasts and the learning gain k_t^{-1}. The top panel shows the median prediction for the 5-10years forecast (black line) , with the 95% bands in green. The bottom panel shows the median prediction for the learning gain k_t^{-1}, again with the 95% bands in green.

Table 2. *Prior and Posterior Parameter Estimates Exogenous Drift Model*

	Prior			**Posterior**				
	Dist.	Mean	SD	Mode	Mean	SD	5%	Med.	95%
$4\pi^*$	Normal	2.0	1.2	2.13	2.13	.360	1.54	2.13	2.72
σ_{π^*}	IGamma	.1	1	.047	.049	.009	.035	.049	.064
ρ_s	Beta	.7	.150	.900	.891	.037	.827	.894	.948
γ_p	Beta	.5	.260	.082	.089	.017	.065	.087	.117
σ_s	IGamma	.5	4	.088	.088	.010	.072	.088	.105
σ_μ	IGamma	.5	4	.487	.480	.032	.426	.481	.528

Figure 5 shows model predictions for the long-term forecasts under an exogenous drift. While the broad contours of long-run inflation expectations are similar in each model, the

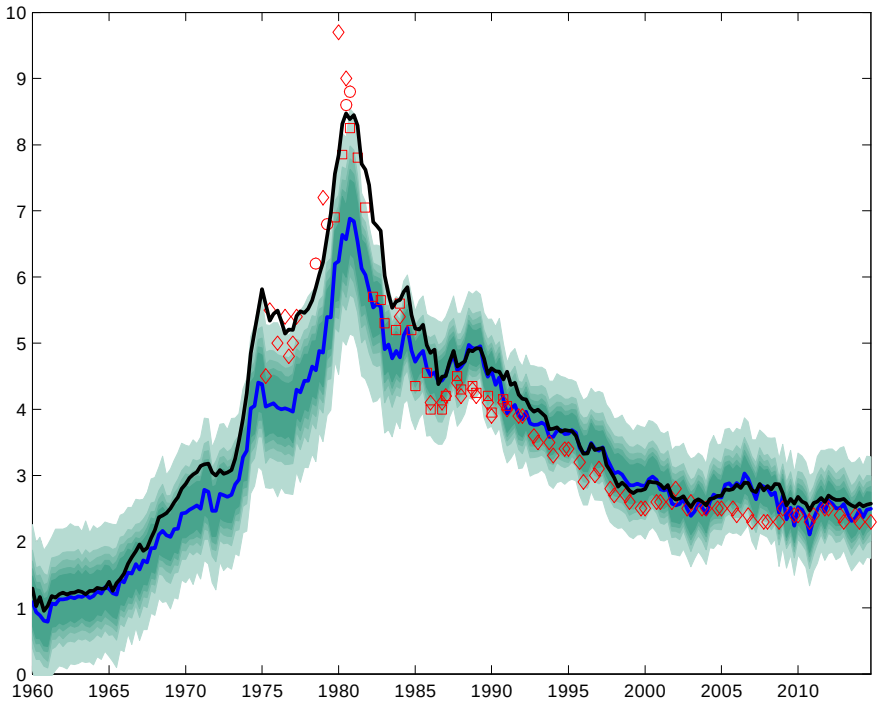

Figure 5: The figure show the predicted evolution of long-term forecasts from the model with exogenous drift. In detail, the blue line is the median prediction from the model with exogenous drift, while the black line is the median prediction from the baseline model. The shaded green area denoted the 95% bands from the model with exogenous drift.

benchmark model clearly exhibits a superior ability to match sharp movements in the survey data. Most notable being the rise in long-term inflation expectations in the mid to late 1970s, and the subsequent decline during the 1980s as a result of the program of disinflation under Chairman Volcker. In periods of relative stability in prices, such as the 1990s and 2000s, the exogenous and endogenous drift models have remarkably similar predictions for long-term inflation outcomes.

The second alternative model assumes firms continue to face a filtering problem to infer long-run inflation, but do so using only a constant gain. There is no switching to decreasing gain. **Table 4** shows prior and posterior estimates for this model. Again, the parameters that are common across this and the benchmark model are quite similar. The most significant difference concerns the estimated constant-gain coefficient, which takes the value 0.095. The 95 percentile of the posterior distribution only just incorporates the constant-gain estimate from the benchmark model.

Table 3. *Prior and Posterior Parameter Estimates for the US (Model with constant gain)*

	Prior				Posterior				
	Dist.	Mean	SD	Mode	Mean	SD	5%	Med.	95%
$4\pi^*$	Normal	2.0	1.2	2.15	2.21	.38	1.59	2.21	2.84
g	Gamma	.10	.050	.094	.095	.018	.066	.095	.127
T	Beta	.7	.150	.903	.906	.057	.777	.889	.961
ρ_s	Beta	.7	.150	.900	.899	.029	.850	.900	.946
γ_p	Beta	.5	.260	.151	.159	.031	.112	.157	.213
σ_s	IGamma	.5	4	.085	.085	.009	.072	.085	.101
σ_μ	IGamma	.5	4	.360	.359	.037	.299	.358	.422

Figure 6 shows this model's prediction for the long-term forecasts. The constant gain model performs better than the exogenous drift model, though still struggles to match the peak movements in inflation expectations in the late 1970s. Model predicted long-term inflation expectations also appear to be overly sensitive to short-run inflation forecasts in the mid 2000s, reflecting the constant-gain algorithm places relatively little weight on older data, even in periods of relative stability.

3.5.1 Are households forecasts consistent with the model?

[To be added]

3.6 International evidence

Because of the limited survey data available on foreign countries, the foreign posterior distribution for all parameters, except average inflation and the observation errors, is obtained using the US posterior distribution as a prior. Moreover, the foreign likelihood receives a weight λ^F less than 1. **Table 2** show posterior estimates for the different countries for $\lambda^F = 0.5$. The parameter ν is omitted because, as in the case of the US its posterior distribution does not deviate from the prior. The key observation is that, with the exception of the constant gain estimate for France, the parameters' estimates are remarkably close to those of the US.

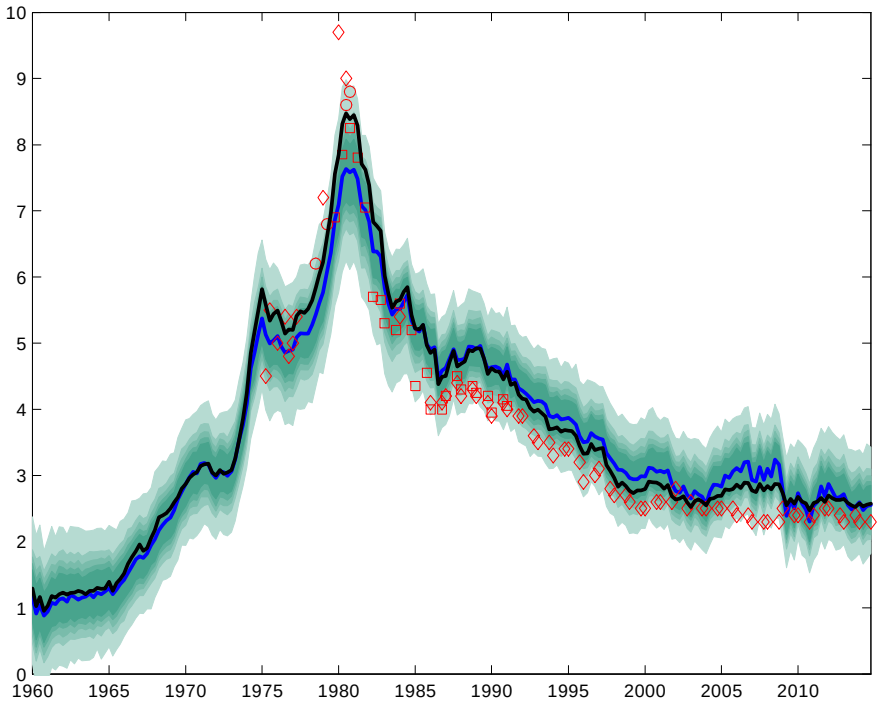

Figure 6: The figure show the predicted evolution of long-term forecasts from the model with constant gain. In detail, the blue line is the median prediction from the model with constant gain, while the black line is the median prediction from the baseline model for the 5-10years forecast. The shaded green area denoted the 95% bands from the model with constant gain. Finally the red lines denote the long-term forecasts from the surveys.

Table 5. *Posterior Parameter Estimates for selected countries*

	US			**Japan**			**France**		
	5%	Med.	95%	5%	Med.	95%	5%	Med.	95%
$4\pi^*$	1.96	2.50	2.90	1.12	1.81	2.55	1.35	1.81	2.24
g	.083	.124	.174	.081	.139	0.193	.188	.269	.360
T	.823	.914	.957	.847	.924	.963	.923	.953	.973
ρ_s	.834	.880	.925	.867	.909	.956	.705	.799	.879
γ_p	.095	.138	.191	.093	.140	.203	.060	.106	.156

	Germany			**Sweden**			**UK**		
$4\pi^*$	1.32	1.91	2.62	1.50	2.07	2.63	1.48	2.08	2.66
g	.112	.155	.216	.039	.072	.150	.028	.052	.079
T	.847	.915	.956	.730	.892	.953	.560	.830	.937
ρ_s	.830	.872	.908	.891	.956	.977	.963	.976	.991
γ_p	.099	.150	.215	.084	.119	.177	.082	.103	.131

We now turn to describing the model's ability to predict the evolution of long-term forecast in the set of countries that we consider. For each country we present three sets of figures. The first figure illustrates the evolution of short-term forecasts used in the estimation. The second figure shows the 5-10 year forecasts predicted by the model estimated with both $\lambda^F = 0.5$ and $\lambda^F = 0.2$, and contrasts these findings with the 5-10 year forecast from Consensus Economics. The third figure illustrates the evolution of the learning gain. As mentioned above, all models are estimated using actual data from the late 1950s. However the figures illustrate model predictions starting in 1980, since no survey forecast in our sample is available before 1991.

Rather than discussing the results country-by-country, our results are summarized by the following three general remarks. First, the model characterizes surprisingly well the evolution of long-term forecasts in the countries we consider: for most of the sample the survey-based forecasts are inside the 95% bands. Even more surprisingly, the model's predictions using $\lambda^F = 0.2$, which delivers a posterior distribution similar to the US, does as well, with the exception of France. For this country the posterior estimates of the gain $\bar{g}$ are higher, which helps predict the evolution of long-term forecasts in the early 1990s.

Second, most countries have experienced some form of anchoring of expectations during the 1990s, but some countries like Japan and Germany have experienced episodes of unanchoring in the past 15 years, while other countries like France, Sweden and the UK have displayed more stable expectations.

Third, the comparison between the model predicted paths and survey-based forecasts suggests that in some countries short-term forecast errors might not have been the only determinant of long-term expectations. For example Japan in the period 2000-2012 has shown higher long-term inflation expectations than predicted by the model, possibly reflecting the central bank of Japan unconventional policies implemented to fight deflation, or concerns about long-term fiscal sustainability. Sweden also displays a faster decline in survey-based long-term forecasts compared to model predictions in the early 1990s. This coincides with the announcement and adoption of an inflation targeting regime in 1992-1995. These episodes suggest that the model might be used to filter the evolution of long-term expectations in absence of announcement effects or other policy-related factors that might shift expectations beyond short-term inflation forecast errors. However, the results, for both the US and the other countries, show that measures of short-term inflation expectations go a long-way in

explaining the dynamics of long-term inflation expectations. [more to be added.]

4 Counterfactuals

[To be added]

5 Discussion

5.1 Role of policy

[To be added]

5.2 Individual rationality

A desirable feature of the learning algorithm describe above is that it should asymptotically converge to the truth $\bar{\pi}_t = 0$, given that no actual regime switch occurs in the model. This is guaranteed provided the switching parameter ν is not too 'small'. From, the convergence properties of the model described above, for t sufficiently large it is possible to find a ν such that the condition $\left(1 - \gamma_p\right)(1 - T)\,\bar{\pi}_t > v$ is satisfied almost surely. As ν is estimated, we would need to verify that the parameter in fact satisfies this restriction. This corresponds to the *Asymptotic Rationality* requirement in Marcet and Nicolini (2003). What happens if ν is too large? In that case never switches to constant gain, preventing what we call learning equilibria.. However, during the transition, agents make large and persistent forecast errors. In the paper we estimate ν and, predictably enough, it will be consistent with switches to the constant gain algorithm. Another property of the learning algorithm that we imposed is what Marcet and Nicolini (2003) refer to as *Internal Consistency*. This refers to the fact that agents have to be satisfied with the parameters of the learning algorithm (in particular the focus in of the gain $\bar{g}$). It is going to be formalized in later version of the notes but the idea is that $\bar{g}$ is a best response for each agent in the model. Any agent facing the actual law of motion for inflation and choosing an alternative gain should not be able improve her forecasting performance meaningfully. Hence, she would like to stick with the equilibrium gain. If such a gain exist (and we shall verify that the estimated gain satisfies this property), then as the algorithm switches to constant gain, the economy is in a learning equilibrium where inflation expectations drift and agents have no incentive to deviate from the learning

rule they use. Intuitively, the closer T is to one, the more likely the economy has a learning equilibrium other than $\bar{g}^{-1} \to 0$ (which corresponds to the rational expectations equilibrium).

Summing up, agents' estimate eventually converges to the correct inflation mean. However, during the transition the economy can switch to a period of volatile long-term inflation expectations as agents' algorithm switches to a constant gain In detail, a sufficiently large shock to inflation (or a sequence) is likely to induce a switch to constant gain. This in turn generates strong feedbacks from inflation expectations to inflation, increasing volatility and reinforcing the choice of a constant gain algorithm (learning equilibrium). Eventually, a sequence of shocks leads the long-term estimate of inflation close to the truth. The algorithm switches to OLS, inducing stability into the inflation process. As the gain becomes sufficiently low (and $\bar{\pi}_t$ sufficiently close to its true value), the condition $\left(1-\gamma_p\right)(1-T)\,\bar{\pi}_t$ is satisfied at all times and the learning process converges. However, during the transition process, agents can switch few times between OLS and tracking, as a large shock can hit inflation for still relatively high values of the decreasing gain, prompting a switch back to the constant gain.

Learning equilibria. In this section we evaluate whether the estimated gain is consistent with a specific equilibrium concept, which we define as a learning equilibrium (this is the Internal consistency requirement in Marcet and Nicolini, see above). We evaluate this through simulation.

To make the whole business simple, let us focus on the model where there is not switching. In principle we should evaluate the learning equilibrium for both ν and $\bar{g}$, but for simplicity we focus on the constant gain. Then what matters is how good the predictor is once you switch to the constant gain. (more on this.)

One can then compute or simulate and compare the mean square errors associated to different predictors. As in Marcet and Nicolini, the learning algorithm is Internally Consistent if

$$\mathbf{E}\left[f_t\left(\bar{g}\right)\right]^2 \leq \min_{\bar{g}'} \mathbf{E}\left[f_t'\left(\bar{g},\bar{g}'\right)\right]^2 + \epsilon$$

for ϵ small. It is easy enough to show that it holds in our paper, as shown in the figure below (we can also show through simulation of the nonlinear algorithm but the result would be fairly similar). So learning equilibria exists for these parameter values. These equilibria depend on the strength of feedback between expectations and actual inflation, which in this

case is captured by the evolution of $\bar{\pi}_t$

$$\bar{\pi}_t = \left[\bar{g}^{-1}\left(1-\gamma_p\right)(T-1)+1\right]\bar{\pi}_{t-1} + \bar{g}^{-1}\cdot \text{Shocks}.$$

with $\left[\bar{g}^{-1}\left(1-\gamma_p\right)(T-1)+1\right]$ close to unity the *actual drift* of inflation is very close to a random-walk, the agents' *perceived drift*. As mention before, the drift depends on of strongly the marginal cost responds to inflation. [Results to be added]

6 Conclusion

[TO BE ADDED]

7 Appendix

7.1 State-Space model and Likelihood

The model is described by the following equations (described in the main text)

$$\begin{aligned}
\pi_t &= \left(1-\gamma_p\right) T\bar{\pi}_t + \gamma_p \pi_{t-1} + s_t + \mu_t \\
\bar{\pi}_t &= \bar{\pi}_{t-1} + k_{t-1}^{-1} \times f_{t-1} \\
f_t &= \left(1-\gamma_p\right)(T-1)\,\bar{\pi}_t + \mu_t + \epsilon_t \\
s_t &= \rho_s s_{t-1} + \epsilon_t.
\end{aligned}$$

It can be cast in the following state-space form

$$\boldsymbol{\xi}_t = F(\mathbf{k}_{t-1}^{-1})\boldsymbol{\xi}_{t-1} + S_C\boldsymbol{\epsilon}_t,$$

where

$$\boldsymbol{\xi}_t = \begin{bmatrix} \pi_t \\ \bar{\pi}_t \\ f_t \\ s_t \end{bmatrix}; \quad \boldsymbol{\epsilon}_t = \begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix},$$

$$F(\mathbf{k}_{t-1}^{-1}) = \begin{bmatrix} \gamma_p & \left(1-\gamma_p\right)T & \mathbf{k}_{t-1}^{-1}\left(1-\gamma_p\right)T & \rho_s \\ 0 & 1 & \mathbf{k}_t^{-1} & 0 \\ 0 & \left(1-\gamma_p\right)(T-1) & \mathbf{k}_{t-1}^{-1}\left(1-\gamma_p\right)(T-1) & 0 \\ 0 & 0 & 0 & \rho_s \end{bmatrix}$$

and

$$S_C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

In compact terms

$$\boldsymbol{\xi}_t = F(\mathbf{k}_{t-1}^{-1})\boldsymbol{\xi}_{t-1} + S_C\boldsymbol{\epsilon}_t.$$

The model can be then estimated using the standard Kalman filter recursions

$$\begin{aligned}
\xi_{t|t} &= \xi_{t|t-1} + P_{t|t-1}H_t \left(H_t' P_{t|t-1} H_t + R_t\right)^{-1} \left(Y_t - \pi^* - H_t' \xi_{t|t-1}\right) \\
P_{t|t} &= P_{t|t-1} - P_{t|t-1}H_t \left(H_t' P_{t|t-1} H_t + R_t\right)^{-1} H_t' P_{t|t-1} \\
\mathbf{k}_t &= \begin{cases} \bar{g}^{-1}, \text{ if } \left|(T-1)\left(1-\gamma_p\right) e_2 \xi_{t|t}\right| > v\sqrt{\mathbb{MSE}} \\ \\ \mathbf{k}_{t-1} + 1, \text{ otherwise} \end{cases} \\
\xi_{t+1|t} &= F(\mathbf{k}_t^{-1}) \xi_{t|t} \\
P_{t+1|t} &= F(\mathbf{k}_t^{-1}) P_{t|t} F(\mathbf{k}_t^{-1})' + S_C \Sigma_\epsilon S_C'
\end{aligned}$$

where $e_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$ and Σ_ϵ denotes the variance-covariance matrix of $\boldsymbol{\epsilon}_t$.

Finally, the likelihood is computed as

$$\begin{aligned}
&2\pi^{-n/2} \left|H_t' P_{t|t-1} H_t + R_t\right|^{-1/2} \times \\
&\exp\left\{-\frac{1}{2}\left(Y_t - \pi^* - H_t' \xi_{t|t-1}\right)' \left(H_t' P_{t|t-1} H_t + R_t\right)^{-1} \left(Y_t - \pi^* - H_t' \xi_{t|t-1}\right)\right\},
\end{aligned}$$

where Y_t denotes the vector of observables.

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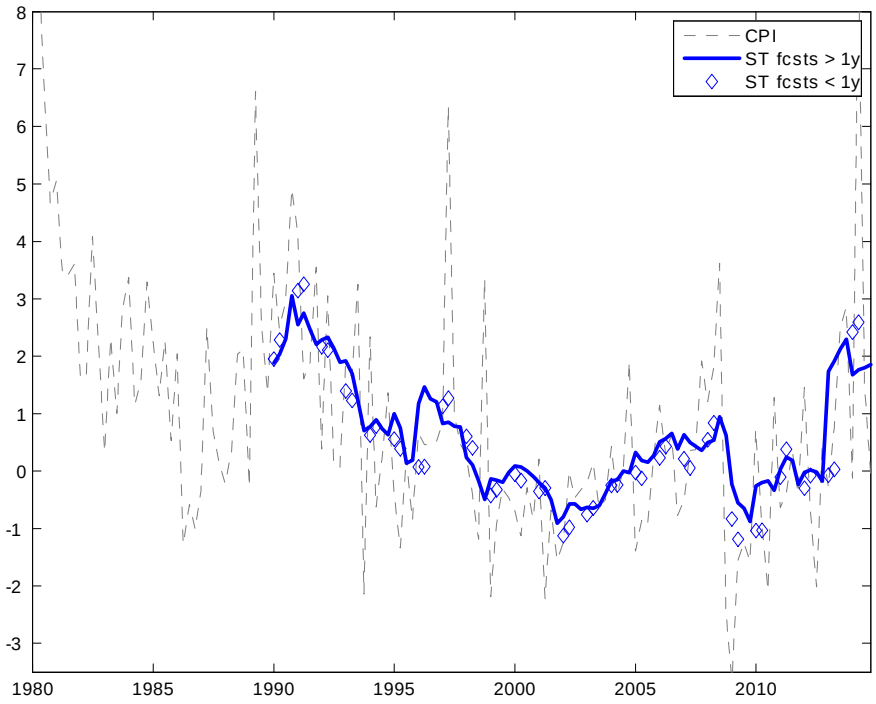

Figure 7: The figure shows the short-term forecast used in the estimation for Japan. The grey dashed line denotes CPI inflation, the blue diamonds show inflation forecasts with a forecst horizon of less than a year. The blue solid line denotes yearly forecasts that give nost weight to quarterly forecasts ranging from one to four quarters ahead.

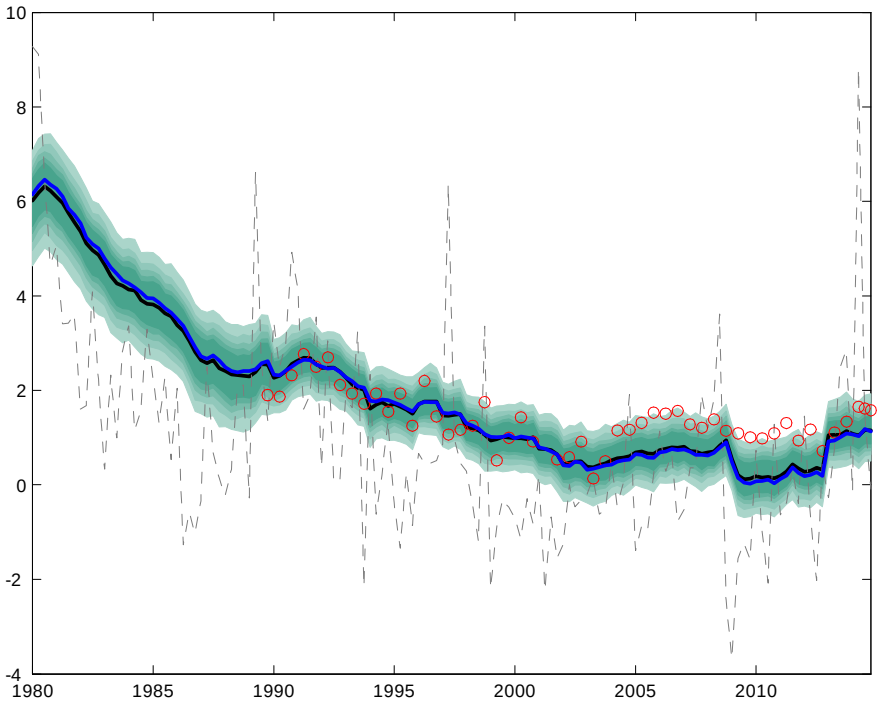

Figure 8: This figure shows the predicted long-term forecast for Japan. In detail, the grey dashed line denotes CPI inflation, the black solid line denoted the median model predicted path for the 5-10years forecast using estimates with $\lambda^F = 0.5$, and blue solid line shows the predicted forecast using estimates with $\lambda^F = 0.2$. The green area denotes 95% bands corresponding to the estimates with $\lambda^F = 0.5$. Finally, the red circles show the 5-10years forecasts from Consensus Economics.

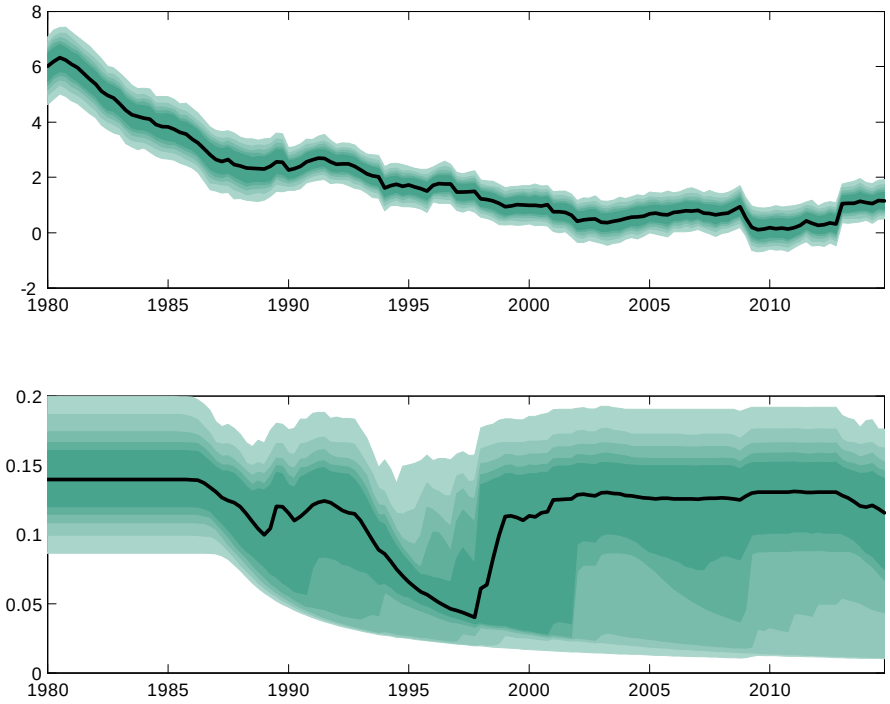

Figure 9: The figure shows the model predictions for long-term forecasts and the learning gain k_t^{-1} for Japan ($\lambda^F = 0.5$). The top panel shows the median prediction for the 5-10years forecast (black line) , with the 95% bands in green. The bottom panel shows the median prediction for the learning gain k_t^{-1}, again with the 95% bands in green.

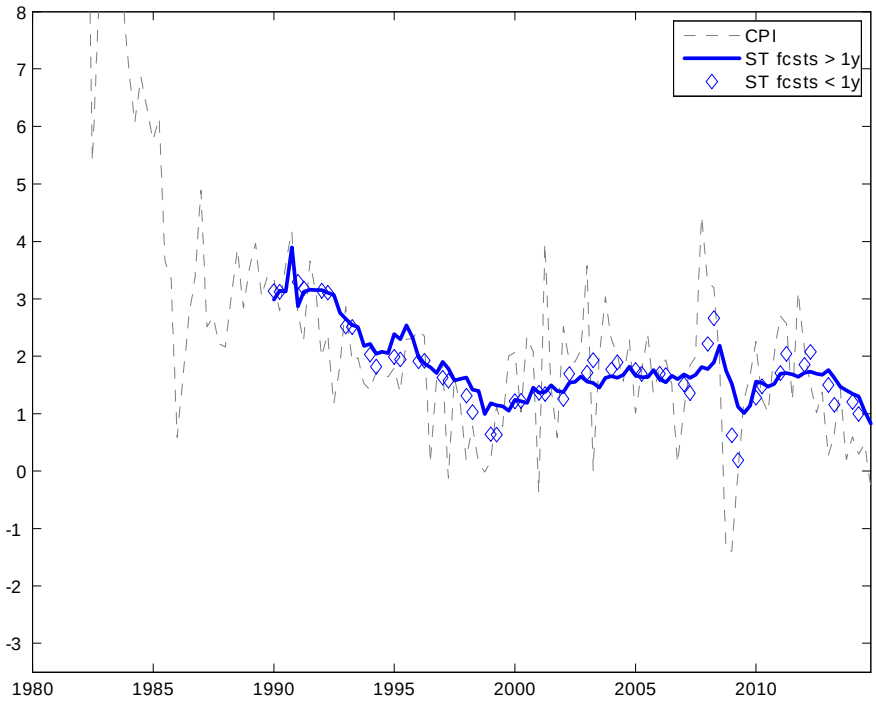

Figure 10: The figure shows the short-term forecast used in the estimation for France. The grey dashed line denotes CPI inflation, the blue diamonds show inflation forecasts with a forecst horizon of less than a year. The blue solid line denotes yearly forecasts that give nost weight to quarterly forecasts ranging from one to four quarters ahead.

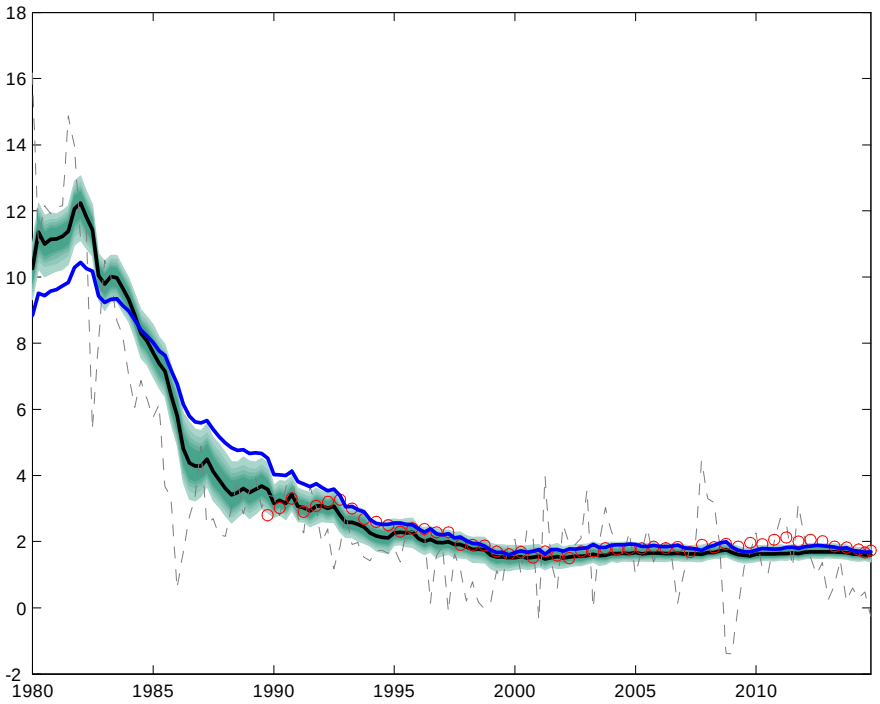

Figure 11: This figure shows the predicted long-term forecast for France. In detail, the grey dashed line denotes CPI inflation, the black solid line denoted the median model predicted path for the 5-10years forecast using estimates with $\lambda^F = 0.5$, and blue solid line shows the predicted forecast using estimates with $\lambda^F = 0.2$. The green area denotes 95% bands corresponding to the estimates with $\lambda^F = 0.5$. Finally, the red circles show the 5-10years forecasts from Consensus Economics.

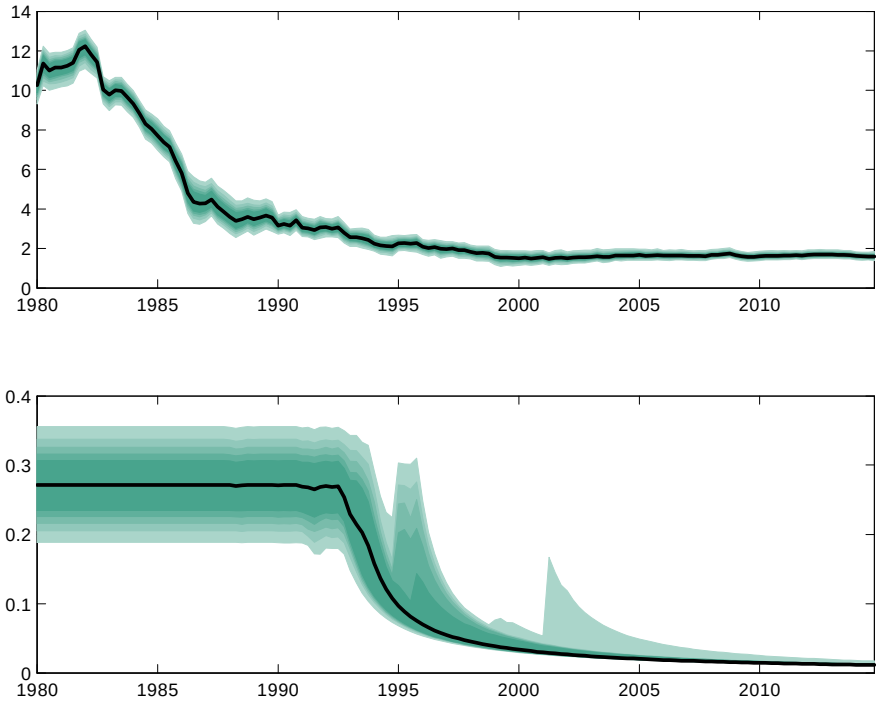

Figure 12: The figure shows the model predictions for long-term forecasts and the learning gain k_t^{-1} for France ($\lambda^F = 0.5$). The top panel shows the median prediction for the 5-10years forecast (black line) , with the 95% bands in green. The bottom panel shows the median prediction for the learning gain k_t^{-1}, again with the 95% bands in green.

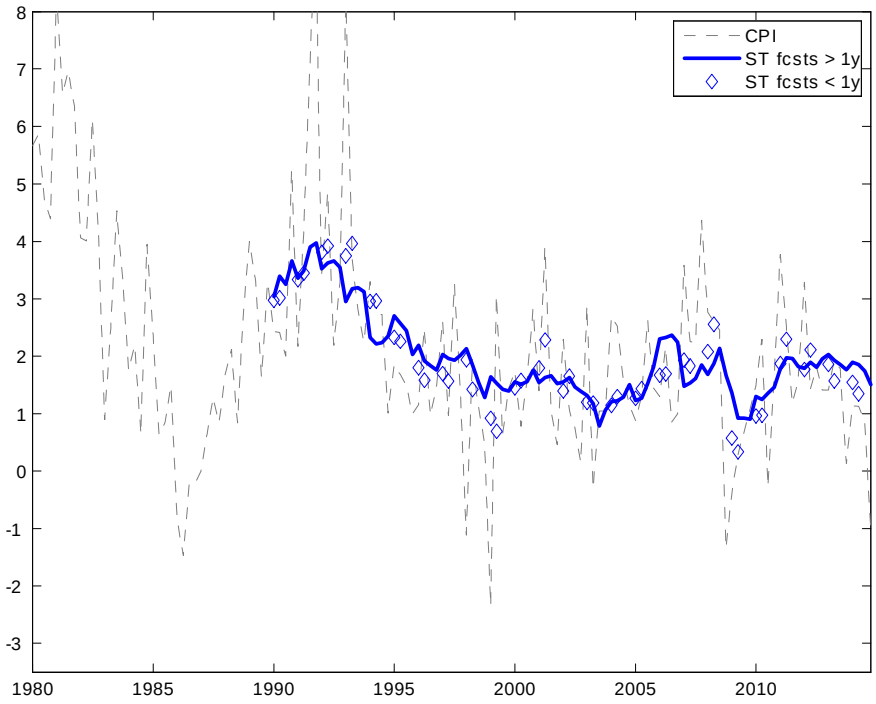

Figure 13: The figure shows the short-term forecast used in the estimation for Germany. The grey dashed line denotes CPI inflation, the blue diamonds show inflation forecasts with a forecst horizon of less than a year. The blue solid line denotes yearly forecasts that give nost weight to quarterly forecasts ranging from one to four quarters ahead.

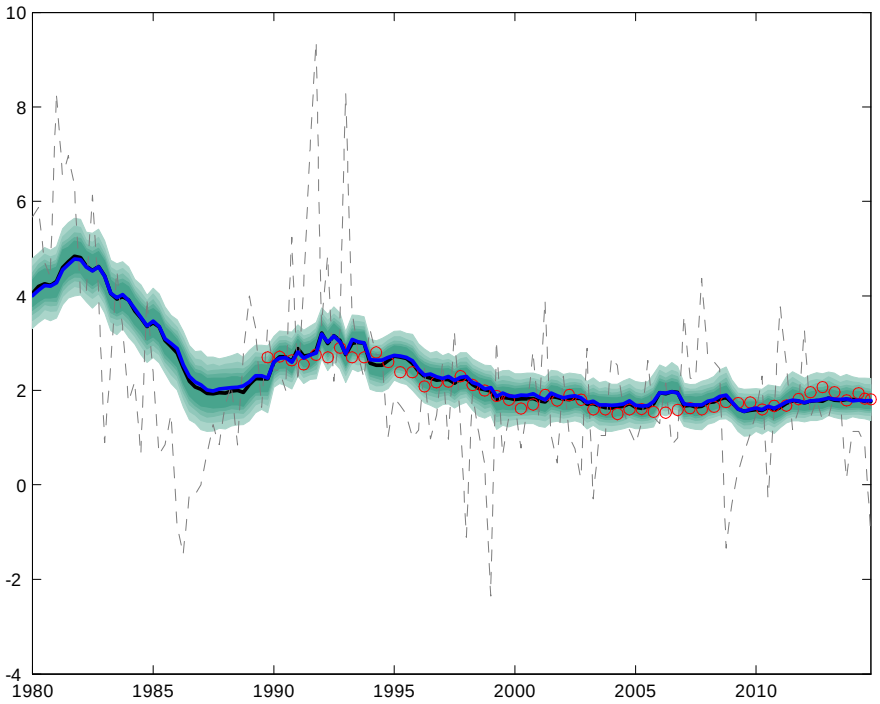

Figure 14: This figure shows the predicted long-term forecast for Germany. In detail, the grey dashed line denotes CPI inflation, the black solid line denoted the median model predicted path for the 5-10years forecast using estimates with $\lambda^F = 0.5$, and blue solid line shows the predicted forecast using estimates with $\lambda^F = 0.2$. The green area denotes 95% bands corresponding to the estimates with $\lambda^F = 0.5$. Finally, the red circles show the 5-10years forecasts from Consensus Economics.

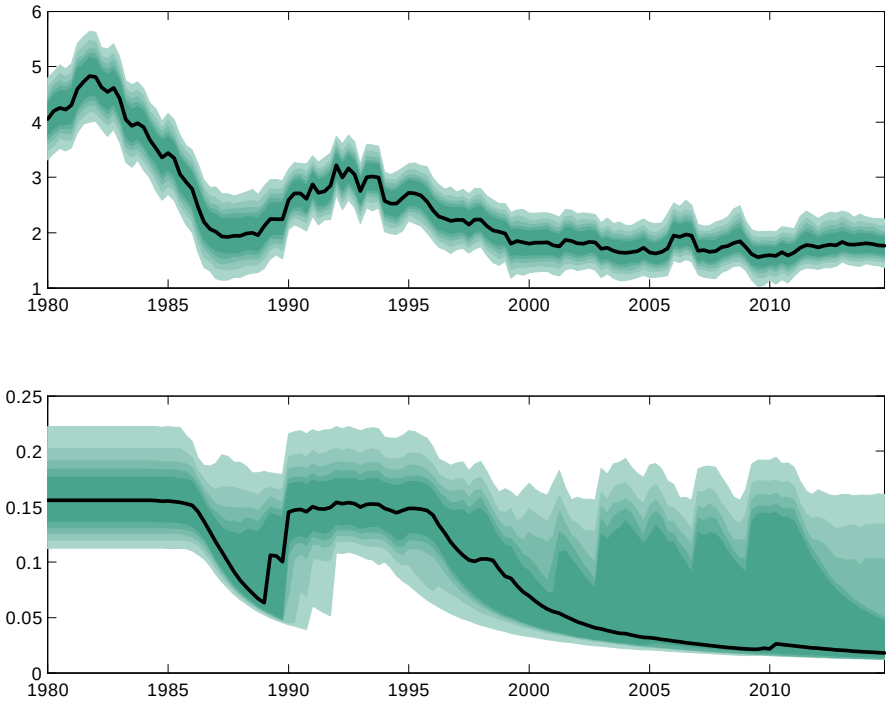

Figure 15: The figure shows the model predictions for long-term forecasts and the learning gain k_t^{-1} for Germany ($\lambda^F = 0.5$). The top panel shows the median prediction for the 5-10years forecast (black line) , with the 95% bands in green. The bottom panel shows the median prediction for the learning gain k_t^{-1}, again with the 95% bands in green.

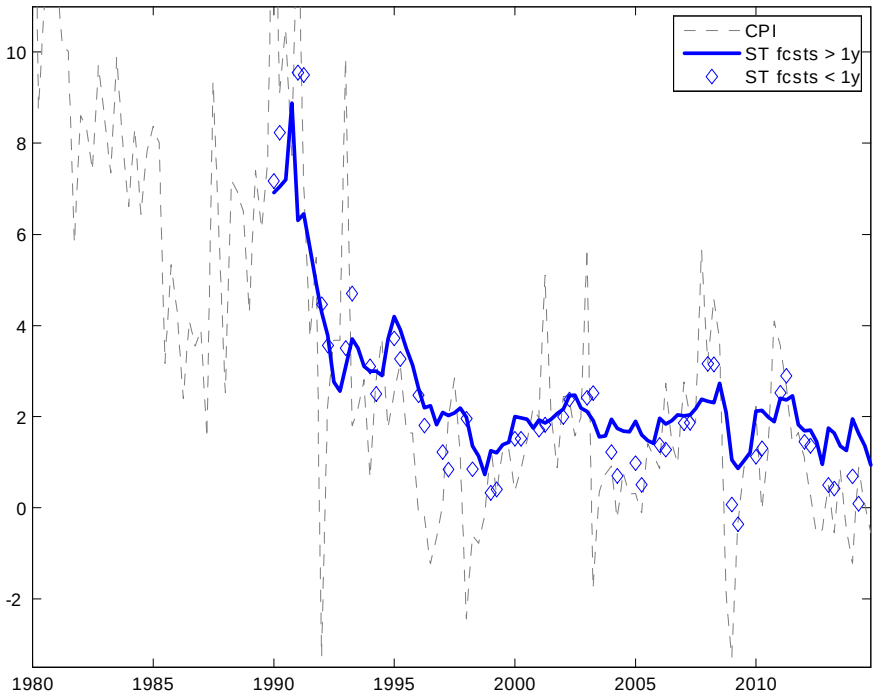

Figure 16: The figure shows the short-term forecast used in the estimation for Sweden. The grey dashed line denotes CPI inflation, the blue diamonds show inflation forecasts with a forecst horizon of less than a year. The blue solid line denotes yearly forecasts that give nost weight to quarterly forecasts ranging from one to four quarters ahead.

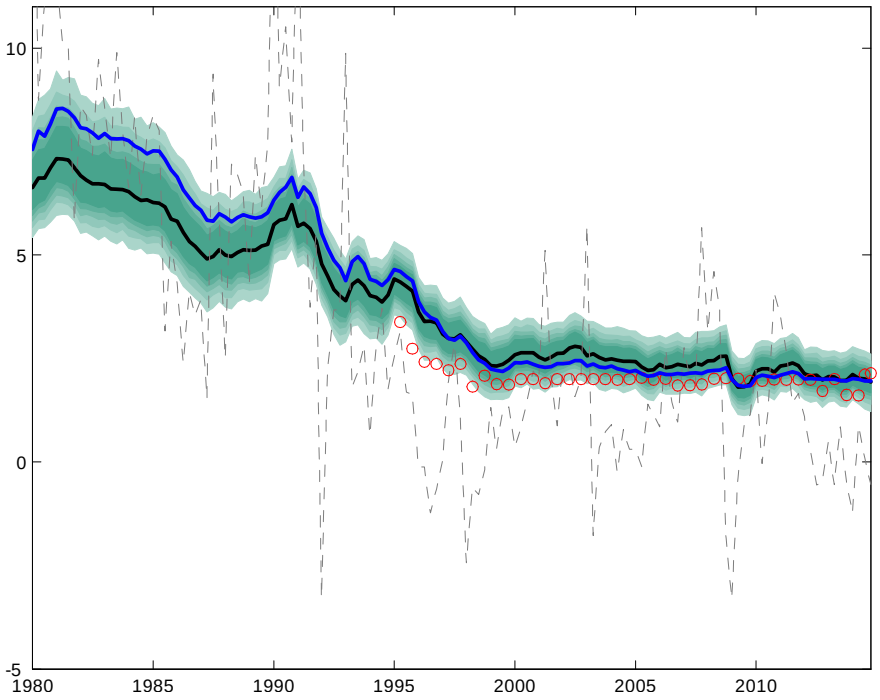

Figure 17: This figure shows the predicted long-term forecast for Sweden. In detail, the grey dashed line denotes CPI inflation, the black solid line denoted the median model predicted path for the 5-10years forecast using estimates with $\lambda^F = 0.5$, and blue solid line shows the predicted forecast using estimates with $\lambda^F = 0.2$. The green area denotes 95% bands corresponding to the estimates with $\lambda^F = 0.5$. Finally, the red circles show the 5-10years forecasts from Consensus Economics.

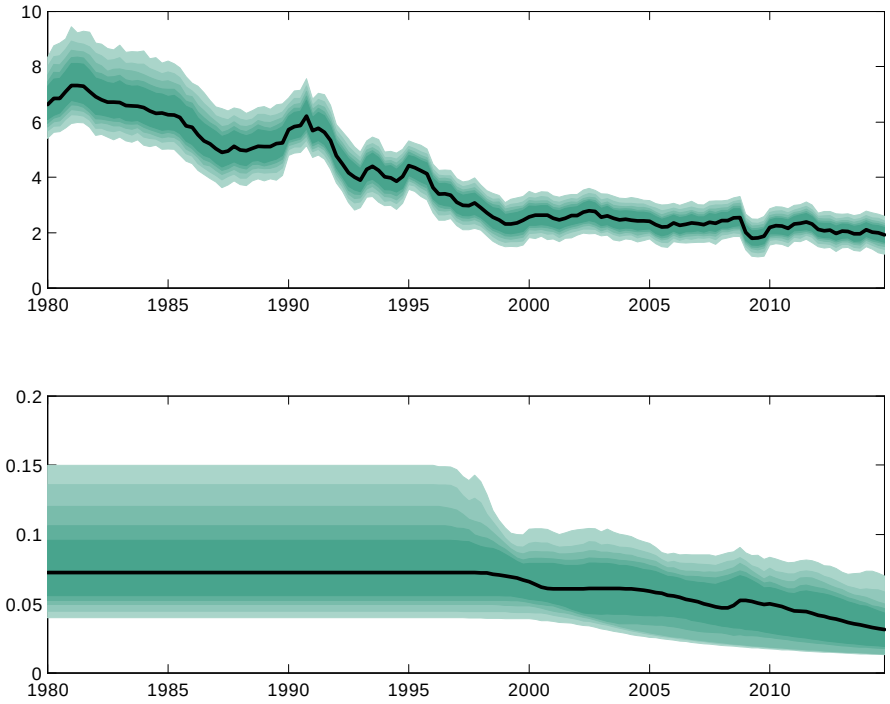

Figure 18: The figure shows the model predictions for long-term forecasts and the learning gain k_t^{-1} for Sweden ($\lambda^F = 0.5$). The top panel shows the median prediction for the 5-10years forecast (black line) , with the 95% bands in green. The bottom panel shows the median prediction for the learning gain k_t^{-1}, again with the 95% bands in green.

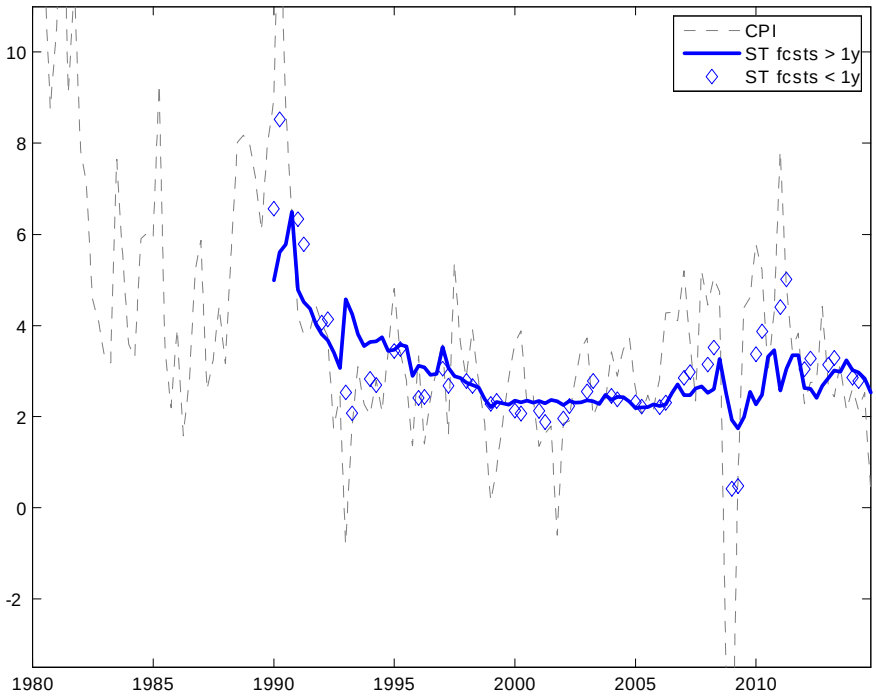

Figure 19: The figure shows the short-term forecast used in the estimation for UK. The grey dashed line denotes CPI inflation, the blue diamonds show inflation forecasts with a forecst horizon of less than a year. The blue solid line denotes yearly forecasts that give nost weight to quarterly forecasts ranging from one to four quarters ahead.

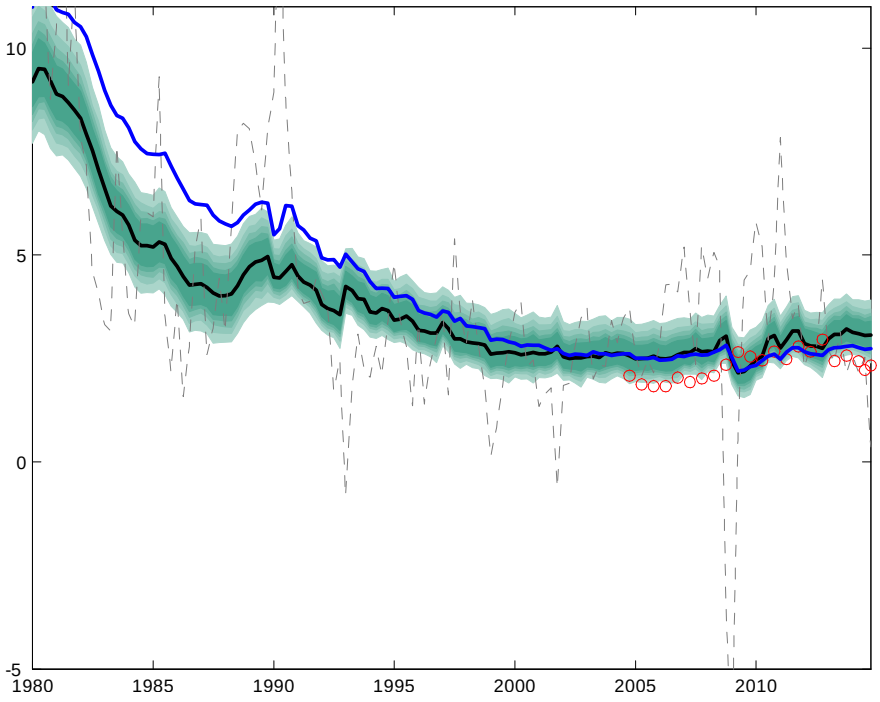

Figure 20: This figure shows the predicted long-term forecast for UK. In detail, the grey dashed line denotes CPI inflation, the black solid line denoted the median model predicted path for the 5-10years forecast using estimates with $\lambda^F = 0.5$, and blue solid line shows the predicted forecast using estimates with $\lambda^F = 0.2$. The green area denotes 95% bands corresponding to the estimates with $\lambda^F = 0.5$. Finally, the red circles show the 5-10years forecasts from Consensus Economics.

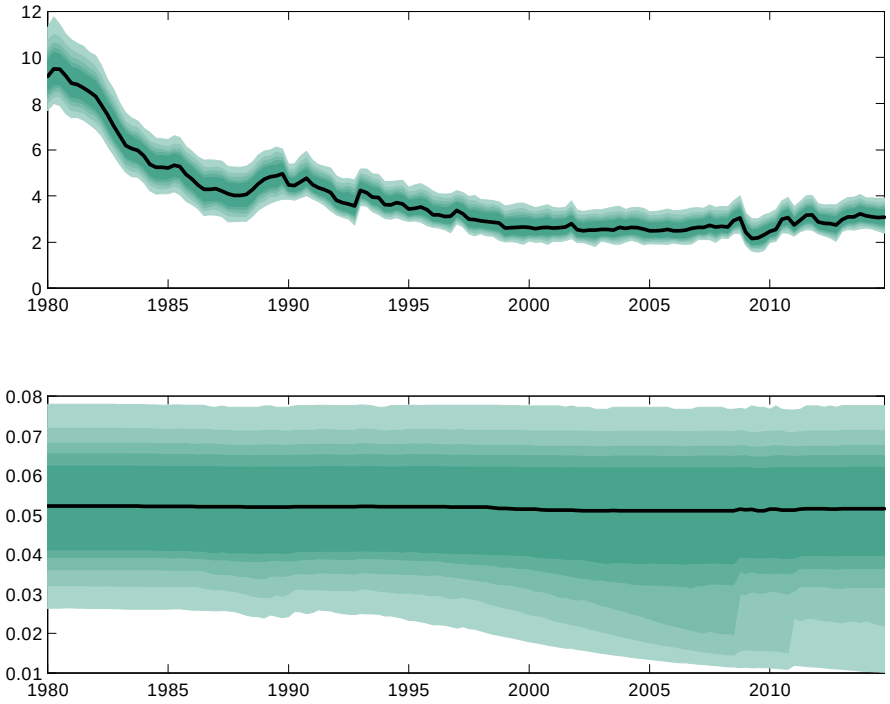

Figure 21: The figure shows the model predictions for long-term forecasts and the learning gain k_t^{-1} for UK ($\lambda^F = 0.5$). The top panel shows the median prediction for the 5-10years forecast (black line) , with the 95% bands in green. The bottom panel shows the median prediction for the learning gain k_t^{-1}, again with the 95% bands in green.