Monetary Policy Expectations at the Zero Lower Bound

Michael Bauer   Glenn Rudebusch

Federal Reserve Bank of San Francisco

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Inference about monetary policy at the ZLB

Motivating questions

- How can we estimate expected future short rates when the current short rate is at the ZLB?
- What are the added benefits, if any, of macroeconomic variables in the information set?
- How tight is the ZLB constraint at any given time?
- How should we estimate the time until the future liftoff of the policy rate from zero?

This paper

Contribution
Address the above questions using dynamic term structure models (DTSMs) that respect the ZLB constraint

- Use shadow-rate DTSMs (Black, 1995)
- Demonstrate advantages over affine Gaussian DTSMs
    - better cross-sectional fit, more accurate short-rate forecasts, avoid violations of ZLB
- Discuss the role of shadow short rates and shadow yields
- Measure the tightness of the ZLB constraint
- Provide accurate estimates of duration until liftoff
- Document benefits of including macro variables

Dynamic term structure models

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- Affine Gaussian DTSM
  - Short rate: $r_t = \delta_0 + \delta_1' X_t$
  - VAR for X_t under risk-neutral ($\mathbb{Q}$) and real-world ($\mathbb{P}$) probability measures
  - Risk adjustment links cross section to time series

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- Bond prices and yields
  - $y_t^m = m^{-1} \sum_{i=0}^{m-1} E_t^{\mathbb{P}} r_{t+i} + YTP_t^m$
  - Affine model: yields and term premia are linear functions of X_t
  - Shadow-rate model: no analytical solution, approximation needed

Data

- Monthly sample from January 1985 to December 2012
- T-bill rates (H.15) and smoothed zero-coupon yields (GSW, 2007)
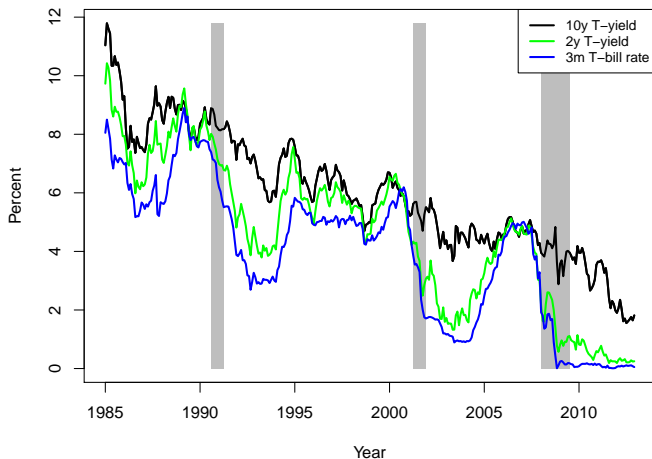
- Unemployment gap (BLS/CBO) and core CPI inflation (BLS)

Estimation

- State-space system
    - Measurement equation: $Y_t = g(X_t) + e_t \quad e_t \stackrel{iid}{\sim} N(0, \sigma_e^2 I_J)$
    - Transition equation: $X_t = \mu + \phi X_{t-1} + \Sigma \varepsilon_t$

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- ZLB affects factor-yield mapping
    - Affine model: $g(X_t) = A + BX_t$
        - Kalman filter
    - Shadow-rate model: $g(\cdot)$ nonlinear and unknown
        - Extended Kalman filter
        - Monte Carlo simulation for g and its Jacobian — slow but reasonably accurate.
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- Maximum likelihood estimation
    - For shadow-rate model, evaluation of likelihood fn. costly
    - Shortcut: use affine-model estimates from pre-ZLB subsample
        - 1985–2007 models have identical implications
        - Adding recent years leaves parameter estimates unchanged

Risk factors and normalizations

Risk factors and normalizations

- Yields-only models
  - Risk factors are PCs of model-implied (shadow-) yields
    $\Rightarrow X_t$ is latent
  - Canonical form of Joslin, Singleton, Zhu (2011)
  - Affine models: YA(2), YA(3)
  - Shadow-rate models: YZ(2), YZ(3)

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- Macro-finance models
  - Add macro factors: measures of economic activity and inflation
  - Spanning of macro risks by (shadow-) yields
  - Canonical form of Joslin, Le, Singleton (forthcoming JFE)
  - Affine models: MA(1), MA(2)
  - Shadow-rate models: MZ(1), MZ(2)

Empirical performance: affine vs. shadow-rate models

Cross-sectional fit

- Shadow-rate models fit the yield curve substantially better over the recent ZLB period

Violations of ZLB

- Affine models violate ZLB during 2009–2012
  - Forward curves and short-rate expectations dip below zero
  - Probability of negative future rates while mean positive

Forecast accuracy

- Shadow-rate models better OOS than affine model
- Macroeconomic information improves performance

Shadow short rate

Common interpretation:

the short rate we would observe without the ZLB constraint

- Krippner (2012)
  - Calibrates restrictive two-factor yields-only model
  - Estimates very negative shadow rate, around -8%
  - Argues that it reflects the stance of monetary policy

- Bullard (2012)
  - Accepts Krippner's estimates and argument
  - Compares shadow rate to prescription of Taylor rule
  - Concludes monetary policy is too easy

- Wu and Xia (2013)
  - During ZLB period replace FFR by shadow-rate in FAVAR
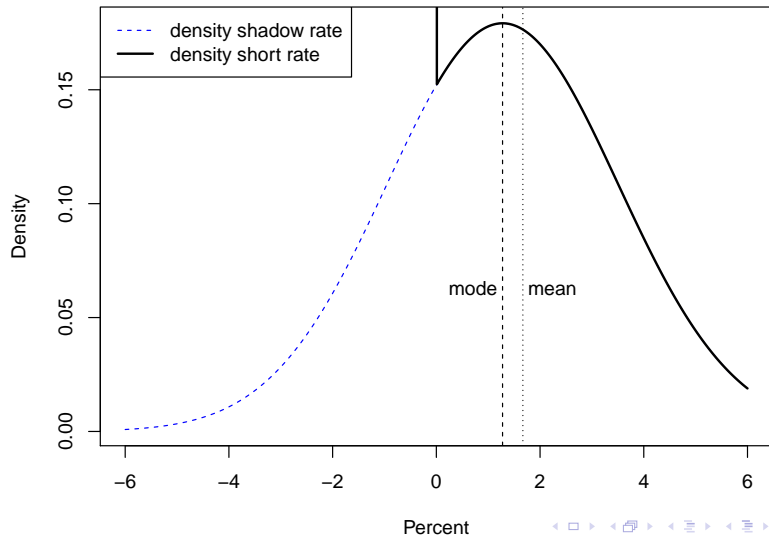
Shadow short-rate estimates
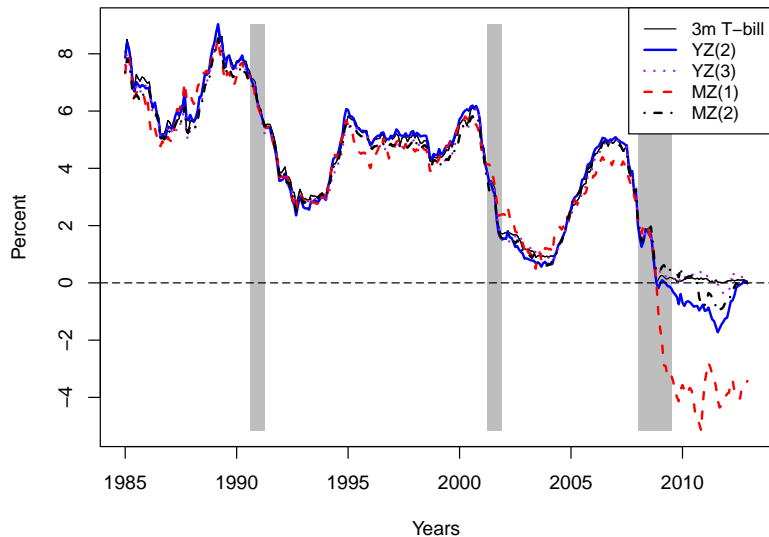
Shadow short rates

- Highly model-dependent
  - Number of risk factors
    (Kim and Singleton, 2012; Christensen and Rudebusch, 2013)
  - Numerical value of lower bound
- Is used to fit yields at the short end
  - Mainly reflects information in shortest maturities
- Interpretation as "policy stance" difficult
  - Monetary policy affects entire term structure

Key insight

Don't look at *current* shadow short rates, but instead consider distribution of *future* shadow short rates.

Probability density of future short rate

Distribution under $\mathbb{Q}$-measure, on December 31, 2012, four-year horizon, model MZ(2)

Shadow yields vs. fitted yields

- Fitted yields $\approx m^{-1} \sum_{i=0}^{m-1} E_t^{\mathbb{Q}} r_{t+i}$
  - Reflect expectations of future short rates

- Shadow yields $\approx m^{-1} \sum_{i=0}^{m-1} E_t^{\mathbb{Q}} s_{t+i}$
  - Reflect expectations of future shadow short rates

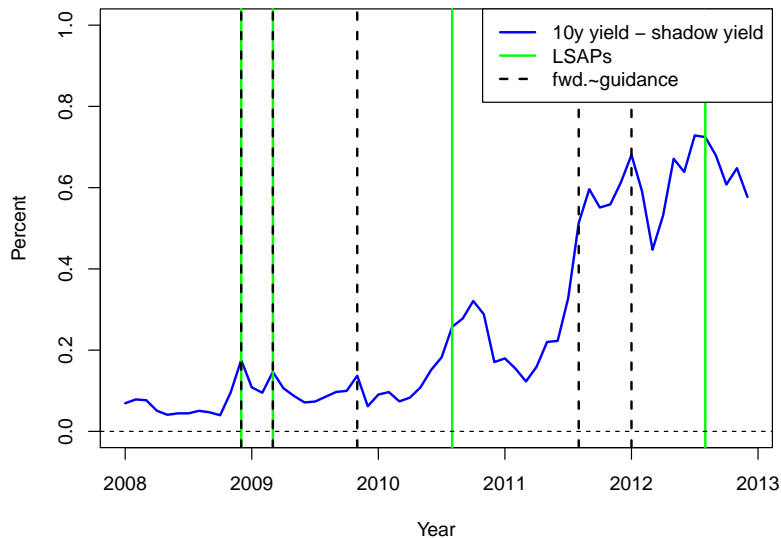- Wedge due to ZLB constraint on r_t
  - Option value of physical currency

Shadow yield curves

Actual, fitted and shadow yields for model MZ(2)

June 30, 2011

December 31, 2012

Percent

Maturity

× actual yields
fitted yield curve
shadow yield curve
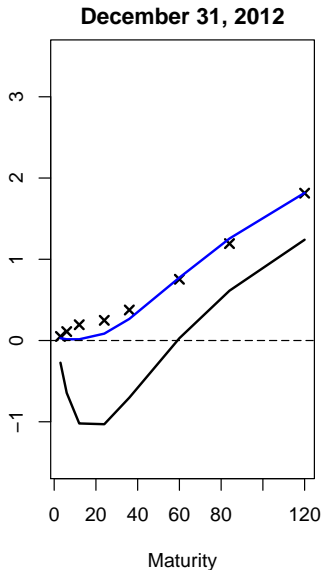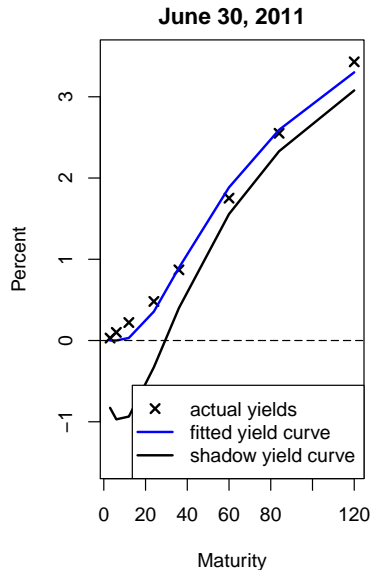
ZLB wedge

Ten-year fitted yield minus shadow yield, model MZ(2)

How tight is the ZLB constraint?

- ZLB wedge
  - Difference between yields and shadow yields
  - Reflects probability mass at zero
  - Corresponds to option value of physical currency

- Empirical observations
  - ZLB constraint has become tighter between June 2011 and December 2012
    - Finding consistent with Swanson and Williams (2012)
  - Tightness typically increased around monetary policy actions
  - Substantial, continuous increase in tightness from 2008 to 2012

- Alternative, more intuitive measure:
  - Time until policy liftoff

Liftoff from the ZLB

Key question

What is our best guess for the duration until liftoff?

- Example: December 2012
- FOMC
    - no liftoff until UR $< 6.5\%$
    - 13/19 participants predict liftoff in 2015
    - central tendency UR forecast for 2015 is 6.0–6.6%
- Surveys in January 2013
    - BCFF: 80% expect UR to decrease below 6.5% in 2015 or later
    - Primary Dealers: 84% expect liftoff to occur in 2015 or later
- Reasonable liftoff estimate: 30 months (June 2015) or more

Common practice: use forward curve

- Forward rates or (Eurodollar/fed funds) futures rates
- These reflect $E_t^{\mathbb{Q}} r_{t+h}$
- Liftoff: horizon where forward curve crosses 25 basis points
- Examples
  - Wall Street commentary
  - Dealer research notes
  - Ueno, Baba, Sakurai (2006) – Euroyen futures

Liftoff estimate based on forward rates

Forward rates $(E_t^{\mathbb{Q}} r_{t+h})$ on December 31, 2012, for model MZ(2)
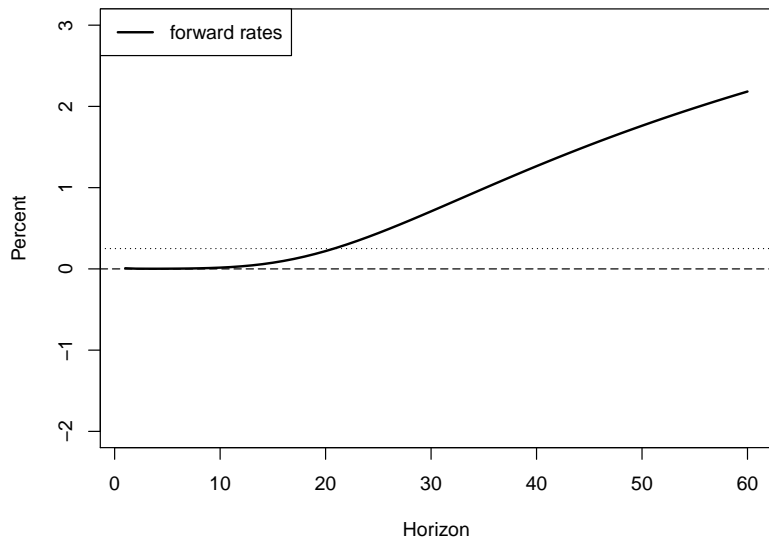
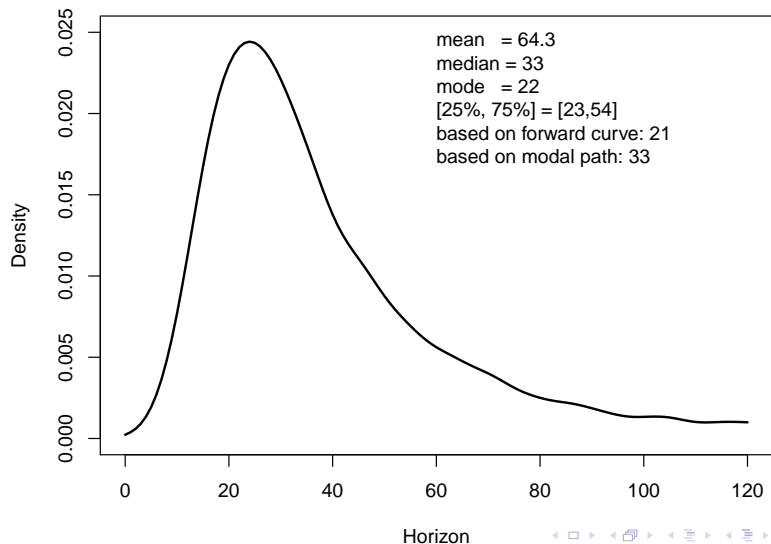# Distribution of liftoff horizon

- Duration/liftoff horizon = first hitting time (stochastic)
- Distribution of future shadow rates implies distribution of ZLB duration

- How to obtain the distribution?
  - Vasicek model: analytical (Linetsky, 2004; Ueno, Baba, Sakurai, 2006; Ichiue and Ueno, 2012)
  - Multifactor models: simulations

- Which central tendency to consider?
  - Mode (Linetsky, 2004; Ueno, Baba, Sakurai, 2006; Ichiue and Ueno, 2012)
  - Mean (Ichiue and Ueno, 2007)
  - Optimal forecast? Loss function?

Distribution of liftoff horizon

Distribution of liftoff horizon, based on Monte Carlo simulation, on December 31, 2012, for model MZ(2)

mean   = 64.3
median = 33
mode   = 22
[25%, 75%] = [23,54]
based on forward curve: 21
based on modal path: 33

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  - Closely corresponds to median of liftoff distribution
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  - ▶ We focus on $\mathbb{Q}$-measure dynamics
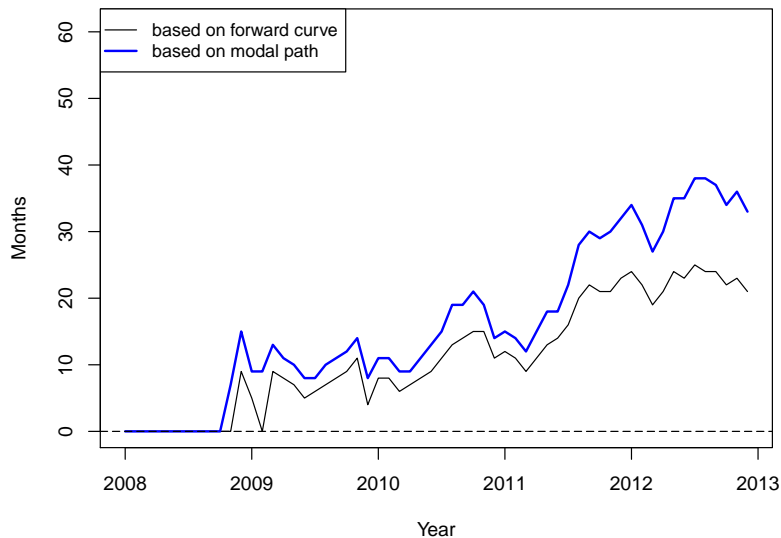
Duration estimate based on modal path

Forward rates ($E_t^{\mathbb{Q}} r_{t+h}$), shadow forward rates ($E_t^{\mathbb{Q}} s_{t+h}$), and modal path, in December 31, 2012, for model MZ(2)
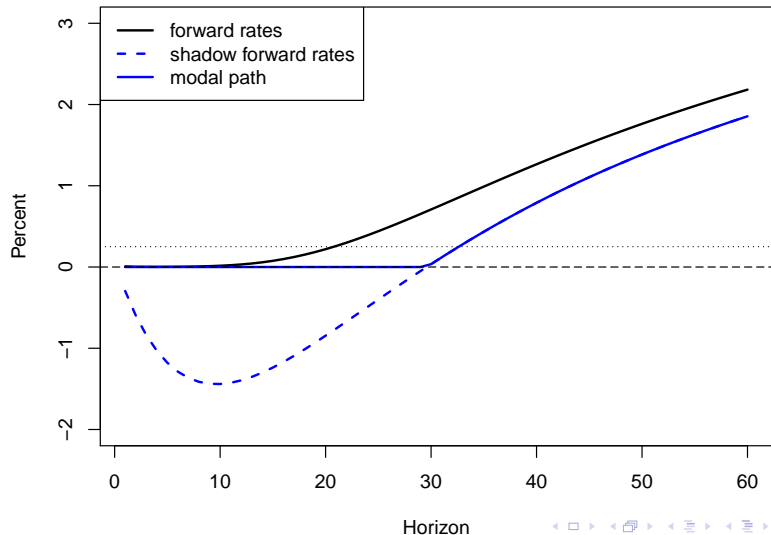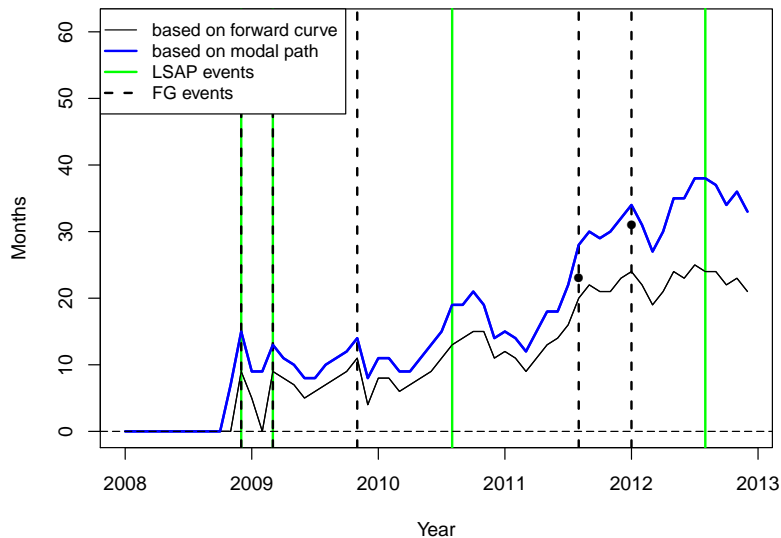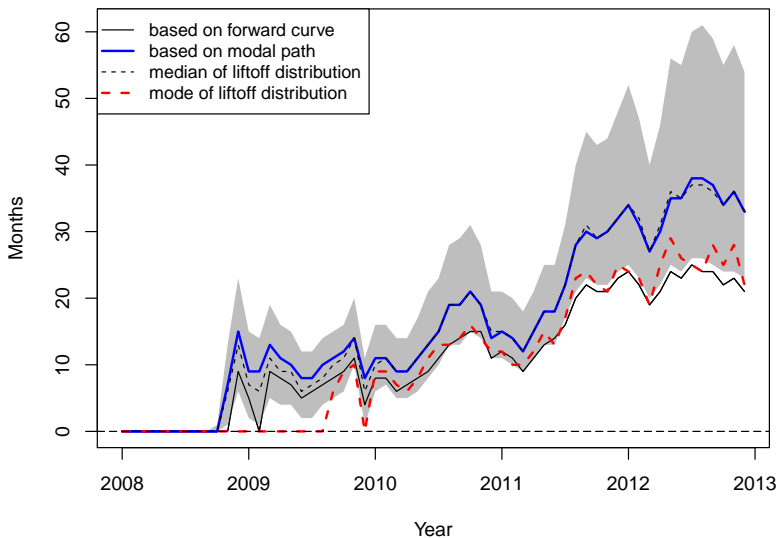
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Estimated horizon (in months) until policy liftoff, model MZ(2)

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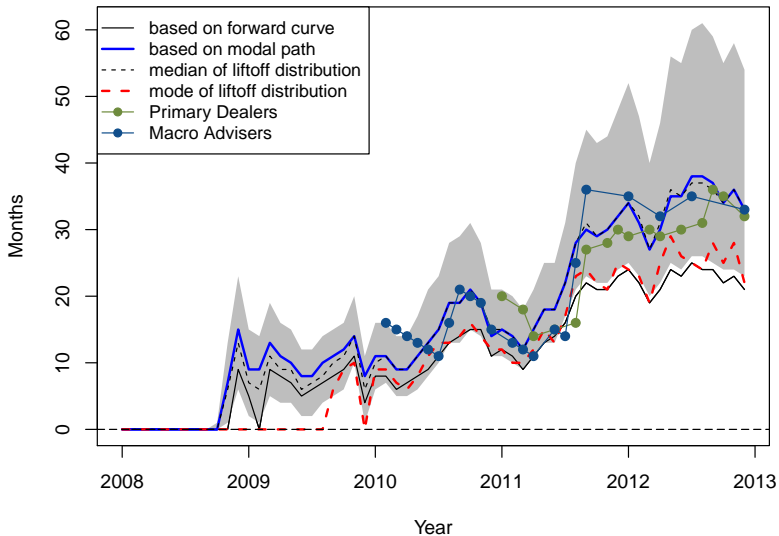
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Summary

- Analyzing interest rates at the ZLB
  - Shadow-rate DTSMs offer substantial advantages over Gaussian affine DTSMs
  - Added benefit of macroeconomic information

- How tight is the ZLB constraint?
  - Wedge between the yields and shadow yields measures option value of ZLB
  - Tightness has increased substantially 2008-2012

- Estimating policy liftoff
  - Bias when using short-rate expectations/forward curve
  - Instead consider shadow-rate expectations/modal path

- Measuring the stance of monetary policy at the ZLB?
  - Shadow-rate model provides univariate measures of policy stance