

Technical Appendix: PRISM-II Documentation

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1 Introduction

This document describes the second-generation DSGE model (PRISM-II) that is developed and maintained by the Real Time Data Research Center (RTDRC) and by the Research Department of the Federal Reserve Bank of Philadelphia. PRISM-II is a medium-scale DSGE model—inspired by Gertler et al. (2008)—that features the various nominal and real frictions that were present in the first-generation PRISM, but that in addition explicitly incorporates a role for unemployment arising from labor market search frictions. This document lays out the model and explains the estimation procedure.

2 Model

The economy consists of an intermediate goods sector, a representative household, a retail sector, and a government.

2.1 Intermediate Goods Sector

The production technology of each of the firms in the intermediate goods sector is assumed to take the Cobb-Douglas form:

$$Y_t = K_t^\alpha Z_t^{1-\alpha} (h_t n_t)^{1-\alpha}, \quad (1)$$

where Y_t is the intermediate good, K_t is the current-period effective units of physical capital, Z_t is total factor productivity (TFP), n_t is employment, and h_t represents hours of work per worker. The TFP series obeys:

$$\ln Z_t - \ln Z_{t-1} = (1 - \rho_z) \ln \gamma_z + \rho_z (\ln Z_{t-1} - \ln Z_{t-2}) + \varepsilon_{z,t},$$

where $\ln \gamma_z$ is the unconditional mean of the stochastic process $z_t = \ln Z_t - \ln Z_{t-1}$. The objective of each firm is to maximize the present discounted value of the stream of profits, $\Pi(\cdot)$, written as:

$$\Pi(n_{t-1}, W_t; Z_t) = \max_{n_t, h_t, v_t, K_t} p_t^w Y_t - W_t h_t n_t - \frac{c_t^v v_t^{1+\epsilon^v}}{1 + \epsilon^v} - r_t^k K_t + \mathbb{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Pi(n_t, W_{t+1}; Z_{t+1}),$$

where p_t^w is the price of the intermediate good, W_t is real wage per hour, $\frac{c_t^v v_t^{1+\epsilon^v}}{1 + \epsilon^v}$ represents hiring costs as a function of the number of job openings v_t ; c_t^v is a scale parameter of the hiring cost function and equals $c^v Z_t$, ϵ^v is its elasticity parameter, r_t^k is the rental rate of capital, β is the discount factor, and Λ_t is marginal utility of the representative household's consumption. The real wage W_t is a state variable due to the dependence

on its past, as discussed below. This optimization problem is subject to the following law of motion for employment:

$$n_t = n_{t-1} - sn_{t-1} + v_t q(\theta_t), \quad (2)$$

where s is a constant separation rate and $q(\theta_t)$ is the job filling rate. The first-order conditions (FOCs) to the problem are:

$$r_t^k = \alpha \frac{p_t^w Y_t}{K_t}, \quad (3)$$

$$\frac{c_t^v}{q_t} v_t^{\epsilon^v} = (1 - \alpha) \frac{p_t^w Y_t}{n_t} - W_t h_t + \mathbb{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} (1 - s) \frac{c_{t+1}^v}{q_{t+1}} v_{t+1}^{\epsilon^v}, \quad (4)$$

$$W_t = (1 - \alpha) \frac{p_t^w Y_t}{h_t n_t}. \quad (5)$$

Equation (3) is the FOC for the demand of capital, Equation (4) is the job creation (labor demand) condition, and Equation (5) characterizes the firm's demand for hours from each worker.

2.2 Labor Flows and Stocks

The search friction is represented by the following aggregate matching function:

$$m_t \tilde{u}_t^\phi v_t^{1-\phi},$$

where m_t denotes the time-varying matching efficiency and \tilde{u}_t is the number of job seekers in the current period, which is written as:

$$\tilde{u}_t = 1 - n_{t-1} + sn_{t-1}. \quad (6)$$

Equation (6) assumes that workers who lost their job at the beginning of period t enter the matching market in the same period. We separately define the unemployment rate u_t as:

$$u_t = 1 - n_t. \quad (7)$$

Given the above matching function, the job filling rate $q(\theta_t)$ is written as:

$$q(\theta_t) = \frac{m_t \tilde{u}_t^\phi v_t^{1-\phi}}{v_t} = m_t \left(\frac{v_t}{\tilde{u}_t} \right)^{-\phi} = m \theta_t^{-\phi}. \quad (8)$$

Note that θ_t is the ratio between the number of job openings and the number of job seekers, and hence it represents the labor market tightness. Similarly, the job finding rate is written as:

$$f(\theta_t) = \frac{m_t \tilde{u}_t^\phi v_t^{1-\phi}}{\tilde{u}_t} = m_t \theta_t^{1-\phi}. \quad (9)$$

From the household's point of view, the stock of employment evolves according to:

$$n_t = (1 - s)n_{t-1} + [1 - (1 - s)n_{t-1}]f(\theta_t). \quad (10)$$

The matching efficiency series obeys:

$$\ln m_t = (1 - \rho_m) \ln \bar{m} + \rho_m \ln m_{t-1} + \varepsilon_{t,m}. \quad (11)$$

Time-varying matching efficiency is useful to explicitly allow for unemployment fluctuations that cannot be accounted for by other shocks. Furlanetto and Goshenny (2016) also introduce the matching efficiency shock to the model similar to ours and argue that it plays an important role in explaining labor market fluctuations.

2.3 Household

It is assumed that members of the representative household pool their incomes from all sources, thus allowing each member to be insured against unemployment risk. The household value function is written as follows:

$$V(C_{t-1}, K_{t-1}^p, H_{t-1}, I_{t-1}, \chi_t, \zeta_t) = \max_{C_t, K_t^p, h_t, H_t, I_t, \nu_t} \chi_t \left[\ln(C_t - lC_{t-1}) - \bar{h} \frac{h_t^{1+\nu}}{1+\nu} n_t \right] + \beta \mathbb{E}_t V(C_t, K_t^p, H_t, I_t, \chi_{t+1}, \zeta_{t+1}). \quad (12)$$

This optimization problem is subject to the following constraints:

$$C_t + I_t + \frac{H_t}{r_t P_t} = W_t h_t n_t + (1 - n_t) B_t + r_t^k \nu_t K_{t-1}^p + D_t + T_t - \mathcal{A}(\nu_t) K_{t-1}^p + \frac{H_{t-1}}{P_t} \quad (13)$$

$$K_t^p = (1 - \delta) K_{t-1}^p + \zeta_t \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right) \right] I_t, \quad (14)$$

$$K_t = \nu_t K_{t-1}^p, \quad (15)$$

$$\ln \chi_t = \rho_\chi \ln \chi_{t-1} + \varepsilon_{t,\chi}, \quad (16)$$

$$\ln \zeta_t = \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{t,\zeta}, \quad (17)$$

where C_t is consumption, K_t^p is physical capital, H_t is nominal bond holdings, I_t is gross investment, ν_t is the utilization rate of the capital stock, χ_t is the intertemporal preference shock, ζ_t is the investment specific technology shock, l is a habit parameter, \bar{h} is a scale parameter for the disutility of hours worked, and $1/\nu$ is the Frisch (intensive-margin) elasticity of labor supply, B_t is a flow value of unemployment (UI benefits), r_t^k is the rental rate of capital, P_t is the price level of the final good, r_t is the gross nominal interest rate, D_t is dividends paid by the retail sector, T_t is the lump sum transfers from the government, $\mathcal{A}(\cdot)$ represents the cost of capital utilization, $\mathcal{S}(\cdot)$ is the adjustment cost function for investment. It is assumed that $B_t = bZ_t$. We choose \mathcal{A} such that the utilization rate ν_t is normalized to one along the balanced growth path and has no resource costs, i.e., we set $\mathcal{A}(1) = 0$, $\mathcal{A}'(1) = \bar{r}_k$. We denote the elasticity by $\xi_A \equiv \mathcal{A}'(1)/\mathcal{A}''(1)$. Note also that $\mathcal{S}(\gamma_z) = \mathcal{S}'(\gamma_z) = 0$ and that $\mathcal{S}''(\gamma_z) = \xi_S$.

The first-order conditions of this problem are:

$$\Lambda_t = \chi_t \frac{1}{C_t - lC_{t-1}} - \beta \mathbb{E}_t \chi_{t+1} \frac{l}{C_{t+1} - lC_t}, \quad (18)$$

$$\Lambda_t = r_t \beta \mathbb{E}_t \left(\frac{\Lambda_{t+1} P_t}{P_{t+1}} \right), \quad (19)$$

$$\bar{h} \chi_t h_t^\nu = \Lambda_t W_t \quad (20)$$

$$\mathcal{A}'(\nu_t) = r_t^k, \quad (21)$$

$$\omega_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[(1 - \delta) \omega_{t+1} + r_{t+1}^k \nu_{t+1} - \mathcal{A}(\nu_{t+1}) \right], \quad (22)$$

$$\omega_t \zeta_t \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right) \right] = \omega_t \zeta_t \frac{I_t}{I_{t-1}} \mathcal{S}'\left(\frac{I_t}{I_{t-1}}\right) + 1 - \beta \mathbb{E}_t \omega_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \zeta_{t+1} \mathcal{S}'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2, \quad (23)$$

where ω_t represents Tobin's Q.

2.4 Wages

To determine the wage, first define earnings \bar{W}_t as:

$$\bar{W}_t = h_t W_t. \quad (24)$$

We assume that the worker and the firm bargain over \bar{W}_t . To derive the expression for \bar{W}_t , we write the values of employment (N_t), unemployment (U_t), and a filled job (J_t) as follows:

$$\begin{aligned} N_t &= \bar{W}_t - \frac{\bar{h}\chi_t h_t^{1+v}}{\Lambda_t} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[(1-s + sf(\theta_{t+1}))N_{t+1} + s(1-f(\theta_{t+1}))U_{t+1} \right], \\ U_t &= B_t + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[f(\theta_{t+1})N_{t+1} + (1-f(\theta_{t+1}))U_{t+1} \right], \\ J_t &= (1-\alpha) \frac{p_t^w Y_t}{n_t} - \bar{W}_t + (1-s) \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} J_{t+1}. \end{aligned}$$

The interpretation is straightforward. If employed this period, the worker receives \bar{W}_t , and in the following period, she obtains the value N_{t+1} if either she did not lose the job with probability $1-s$, or finds a job within the same period after separation, which occurs with probability $sf(\theta_{t+1})$. In the third equation, the first two terms correspond to the firm's flow profits and the next term captures the future value after imposing the free entry condition.

Following Hall (2005), we allow for equilibrium wage (earnings) rigidity of the following form:

$$\bar{W}_t = \rho^w z_t \bar{W}_{t-1} + (1-\rho^w) \bar{W}_t^f, \quad (25)$$

where \bar{W}_t^f is (hypothetical) period-by-period flexible Nash bargained wage (i.e., "reference" wage); ρ^w measures the degree of its rigidity. We can obtain the flexible Nash bargained wage payment \bar{W}_t^f by using the surplus sharing rule:

$$\eta J_t = (1-\eta)(N_t - U_t),$$

where η is the bargaining power of the worker. Using the three value functions above in this equation, one can get:

$$\bar{W}_t^f = \eta(1-\alpha) \frac{p_t^w Y_t}{n_t} + (1-\eta) \left[\frac{\bar{h}\chi_t h_t^{1+v}}{\Lambda_t} + B_t \right] + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[\eta(1-s)c_t^v \theta_{t+1} v_{t+1}^{\epsilon_v} \right]. \quad (26)$$

Note that Equation (25) implies the following indexation of nominal earnings:

$$P_t \bar{W}_t = \rho^w \pi_t z_t P_{t-1} \bar{W}_{t-1} + (1-\rho^w) P_t \bar{W}_t^f. \quad (27)$$

2.5 Hours Per Worker

From Equations (5) and (20), we have the following equilibrium condition for hours per worker.

$$(1-\alpha) \frac{p_t^w Y_t}{n_t} = \frac{\bar{h}\chi_t h_t^{1+v}}{\Lambda_t}. \quad (28)$$

As described in the previous section, earnings \bar{W}_t are determined through bargaining, while Equation (28) determines hours per worker. The implied hourly wage rate is then determined by Equation (24).

2.6 Retail Sector

There is a continuum of monopolistically competitive retailers indexed by j on the unit interval. Retailers buy the intermediate goods at price p_t^w , differentiate them with a technology that transforms them into consumption goods, and then sell them to the household. Each retailer faces the following demand function:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\epsilon_t} Y_t, \quad (29)$$

where ϵ_t is the elasticity of substitution, which is related to the markup μ_t as follows:

$$\epsilon_t = \frac{1 + \mu_t}{\mu_t}. \quad (30)$$

The variable μ_t evolves according to:

$$\ln \mu_t = (1 - \rho_\mu) \ln \bar{\mu} + \rho_\mu \ln \mu_{t-1} + \varepsilon_{t,\mu}. \quad (31)$$

The firm sets its price subject to a quadratic price adjustment cost, maximizing the following expression:

$$\Pi_{jt}^R(P_{jt-1}) = \max_{P_{jt}} \frac{P_{jt}}{P_t} Y_{jt} - p_t^w Y_{jt} - \frac{\tau}{2} \left(\frac{P_{jt}}{\pi_{t-1}^\psi (\pi^*)^{1-\psi} P_{jt-1}} - 1 \right)^2 Y_t + \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{jt}^R(P_{jt}),$$

where $\pi_t = \frac{P_t}{P_{t-1}}$; π^* is central bank's target inflation rate; ψ is a degree of backwardness. The first-order condition under the symmetric equilibrium is:

$$1 - \epsilon_t - \tau \pi_t \left(\frac{\pi_t}{\pi_{t-1}^\psi (\pi^*)^{1-\psi}} - 1 \right) \frac{1}{\pi_{t-1}^\psi (\pi^*)^{1-\psi}} + p_t^w \epsilon_t + \mathbb{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \tau \left(\frac{\pi_{t+1}}{\pi_t^\psi (\pi^*)^{1-\psi}} - 1 \right) \frac{Y_{t+1}}{Y_t} \frac{\pi_{t+1}}{\pi_t^\psi (\pi^*)^{1-\psi}} = 0. \quad (32)$$

2.7 Government

The central bank sets the nominal interest rate as follows:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r} \right)^{\rho_r} \left[\left(\prod_{j=0}^3 \frac{\pi_{t-j}}{\pi^*} \right)^{r_\pi} \left(\frac{Y_t}{Y_{t-4}} \frac{1}{\gamma_z^4} \right)^{r_{gy}} \right]^{1-\rho_r} \kappa_t, \quad (33)$$

where r is the steady-state nominal interest rate, ρ_r is the degree of monetary policy inertia embedded in the monetary policy equation, r_π is the response of the nominal interest rate to deviations of inflation from the inflation target (π^*), r_{gy} is the response of the nominal interest rate to deviations of output growth from the growth rate of the economy at the steady-state (γ_z), and κ_t is an exogenous monetary policy shock. The monetary policy shock is assumed to follow:

$$\ln \kappa_t = \rho_\kappa \ln \kappa_{t-1} + \varepsilon_{t,\kappa}. \quad (34)$$

The government expenditures G_t obeys:

$$G_t = \left(1 - \frac{1}{x_t} \right) Y_t, \quad (35)$$

where x_t varies according to:

$$\ln x_t = (1 - \rho_x) \ln \bar{x} + \rho_x \ln x_{t-1} + \varepsilon_{t,x}. \quad (36)$$

2.8 Resource Constraint

The following resource constraint closes the model.

$$Y_t = C_t + I_t + \frac{c_t^v v_t^{1+\epsilon^v}}{1+\epsilon^v} + A(\nu_t) K_{t-1}^p + \frac{\tau}{2} \left(\frac{\pi_t}{\pi_{t-1}^\psi (\pi^*)^{1-\psi}} - 1 \right)^2 Y_t. \quad (37)$$

3 Detrended Model

The model is rendered stationary by detrending the level equations above by TFP, Z_t . The lower case letters represent stationary variables.

3.1 Intermediate Goods Sector

- Production function:

$$y_t = k_t^\alpha (h_t n_t)^{1-\alpha}. \quad (38)$$

- TFP:

$$\ln z_t = (1 - \rho_z) \ln \gamma_z + \rho_z \ln z_{t-1} + \varepsilon_{z,t}, \quad (39)$$

where

$$z_t = \frac{Z_t}{Z_{t-1}}.$$

- Demand for capital:

$$r_t^k = \alpha p_t^w \frac{y_t}{k_t}. \quad (40)$$

- Job creation condition:

$$\frac{c^v}{q_t} v_t^{\epsilon^v} = (1 - \alpha) \frac{p_t^w y_t}{n_t} - \bar{w}_t + (1 - s) \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{c^v}{q_{t+1}} v_{t+1}^{\epsilon^v}, \quad (41)$$

where $\lambda_t = \Lambda_t Z_t$.

3.2 Labor Market Flows

The equations here are mostly the same as in the previous section, but are listed below for completeness.

- Job filling rate:

$$q_t = m_t \theta_t^{-\phi}. \quad (42)$$

- Job finding rate:

$$f_t = m_t \theta_t^{1-\phi}. \quad (43)$$

- Employment evolution:

$$n_t = (1 - s)n_{t-1} + [1 - (1 - s)n_{t-1}]f(\theta_t). \quad (44)$$

- The number of job seekers:

$$\tilde{u}_t = 1 - n_{t-1} + s n_{t-1}. \quad (45)$$

- The unemployment rate:

$$u_t = 1 - n_t. \quad (46)$$

- Matching efficiency:

$$\ln m_t = (1 - \rho_m) \ln \bar{m} + \rho_m \ln m_{t-1} + \varepsilon_{t,m}. \quad (47)$$

3.3 Household

- Effective capital services:

$$k_t = \frac{\nu_t}{z_t} k_{t-1}^p. \quad (48)$$

- Evolution of physical capital:

$$k_t^p = (1 - \delta) \frac{1}{z_t} k_{t-1}^p + \zeta_t \left[1 - S\left(z_t \frac{i_t}{i_{t-1}}\right) \right] i_t. \quad (49)$$

- Capital utilization:

$$\mathcal{A}'(\nu_t) = r_t^k. \quad (50)$$

- Tobin's Q:

$$\omega_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t z_{t+1}} \left[(1 - \delta) \omega_{t+1} + r_{t+1}^k \nu_{t+1} - \mathcal{A}(\nu_{t+1}) \right]. \quad (51)$$

- Investment:

$$\omega_t \zeta_t \left[1 - S\left(z_t \frac{i_t}{i_{t-1}}\right) \right] = \omega_t \zeta_t z_t \frac{i_t}{i_{t-1}} \mathcal{S}'\left(z_t \frac{i_t}{i_{t-1}}\right) + 1 - \beta \mathbb{E}_t \omega_{t+1} \frac{\lambda_{t+1}}{\lambda_t z_{t+1}} \zeta_{t+1} \mathcal{S}'\left(z_{t+1} \frac{i_{t+1}}{i_t}\right) \left(z_{t+1} \frac{i_{t+1}}{i_t}\right)^2. \quad (52)$$

- Consumption:

$$\lambda_t = \frac{\chi_t z_t}{c_t - l c_{t-1}} - \beta h \mathbb{E}_t \frac{\chi_{t+1}}{c_{t+1} z_{t+1} - l c_t}. \quad (53)$$

- Euler equation:

$$1 = r_t \beta \mathbb{E}_t \left(\frac{1}{z_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \right). \quad (54)$$

- Preference shock:

$$\ln \chi_t = \rho_\chi \ln \chi_{t-1} + \varepsilon_{t,\chi}. \quad (55)$$

- Investment specific technology shock:

$$\ln \zeta_t = \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{t,\zeta}. \quad (56)$$

3.4 Wages

- Earnings

$$\bar{w}_t = h_t w_t. \quad (57)$$

- Nash bargained earnings:

$$\bar{w}_t^f = \eta(1 - \alpha)p_t^w \frac{y_t}{n_t} + (1 - \eta) \left[b + \frac{\bar{h}\chi_t h_t^{1+v}}{\lambda_t} \right] + \beta(1 - s)\eta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} c^v \theta_{t+1} v_{t+1}^{\epsilon_v}. \quad (58)$$

- Actual earnings:

$$\bar{w}_t = \rho^w \bar{w}_{t-1} + (1 - \rho^w) \bar{w}_t^f. \quad (59)$$

3.5 Hours Per Worker

- Hours per worker

$$(1 - \alpha) \frac{p_t^w y_t}{n_t} = \frac{\bar{h}\chi_t h_t^{1+v}}{\lambda_t}. \quad (60)$$

3.6 Retail Sector

- Inflation:

$$1 - \epsilon_t - \tau \pi_t \left(\frac{\pi_t}{\pi_{t-1}^\psi (\pi^*)^{1-\psi}} - 1 \right) \frac{1}{\pi_{t-1}^\psi (\pi^*)^{1-\psi}} + p_t^w \epsilon_t + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \tau \left(\frac{\pi_{t+1}}{\pi_{t-1}^\psi (\pi^*)^{1-\psi}} - 1 \right) \frac{y_{t+1}}{y_t} \frac{\pi_{t+1}}{\pi_{t-1}^\psi (\pi^*)^{1-\psi}} = 0. \quad (61)$$

- Elasticity of substitution:

$$\epsilon_t = \frac{1 + \mu_t}{\mu_t}. \quad (62)$$

- Markup:

$$\ln \mu_t = (1 - \rho_\mu) \ln \bar{\mu} + \rho_\mu \ln \mu_{t-1} + \varepsilon_{t,\mu}. \quad (63)$$

3.7 Government and Resource Constraint

- Monetary policy:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r} \right)^{\rho_r} \left[\left(\prod_{j=0}^3 \frac{\pi_{t-j}}{\pi^*} \right)^{r_\pi} \left(\frac{y_t}{y_{t-4}} \prod_{j=0}^3 \frac{z_{t-j}}{\gamma_z} \right)^{r_{gy}} \right]^{1-\rho_r} \kappa_t. \quad (64)$$

- Monetary policy shock:

$$\ln \kappa_t = \rho_\kappa \ln \kappa_{t-1} + \varepsilon_{t,\kappa}. \quad (65)$$

- Government expenditures:

$$g_t = \left(1 - \frac{1}{x_t} \right) y_t. \quad (66)$$

- The government expenditure shock:

$$\ln x_t = (1 - \rho_x) \ln x + \rho_x \ln x_{t-1} + \varepsilon_{t,x}. \quad (67)$$

- The resource constraint:

$$y_t = c_t + i_t + \frac{c^v v_t^{1+\epsilon^v}}{1 + \epsilon^v} + \mathcal{A}(\nu_t) \frac{k_{t-1}^p}{z_t} + \frac{\tau}{2} \left(\frac{\pi_t}{\pi_{t-1}^\psi (\pi^*)^{1-\psi}} - 1 \right)^2 y_t. \quad (68)$$

4 Empirical Application

We estimate the log-linearized version of the model described using standard Bayesian method implemented in Adjemian et al. (2011). For the current model, the sample period starts in 1971Q3. We re-estimate the model every quarter as we receive more data. The sample period for the results below ends at the third quarter of 2018.

4.1 Calibrated Parameters

Some parameters are calibrated prior to the estimation either directly or through steady-state restrictions. Table 1 summarizes these parameters. The capital share parameter α and the depreciation rate of the physical capital δ are set to 0.33 and 0.025, respectively, both of which are standard in macro. The value of the discount factor β is selected to be 0.9996. This pins down the nominal interest rate, given inflation expectations and growth along the balanced growth path. The economy is assumed to grow 0.4 percent per quarter along the balanced growth path, and thus $\gamma_z = 1.004$.

The quarterly employment separation probability s is set to 0.195. In the model, those workers that separate at the beginning of the period may find a job within the same period, which occurs with probability f_t . The steady-state value of f_t is targeted to 0.75 and thus the probability that an employed worker at the beginning of the period ends up in the unemployment pool at the end of the period is 0.0488. Note also that $s = 0.195$ and $f = 0.75$ imply the unemployment rate equals 6.1 percent at the steady state. The scale parameter of the matching function is set to 0.75 because the steady-state value of labor market tightness θ is normalized to 1, which implies $\bar{m} = f$. The elasticity of the matching function with respect to \tilde{u}_t is set to 0.5. The hiring cost function is assumed to be quadratic (thus $\epsilon^v = 1$) as in Gertler et al. (2008). The level of unemployment benefits b is set to 0.2145. This value is computed by imposing the restriction that the worker's flow outside value including the value of not-working, measured in terms of the consumption good amounts to 71 percent of the steady-state earnings level (see the expression in the square bracket in (58)). This value has often been used in the literature (e.g., Hall and Milgrom (2008)). The inverse of the elasticity of intensive-margin labor supply is fixed at 2. The labor-supply elasticity of 0.5 is in line with the evidence in micro-econometric studies.

The steady-state price markup ($\bar{\mu}$) is set to 0.2. The steady-state level of the exogenous government expenditure process \bar{x} is set to 1.25, which implies the share of government expenditures in output being 19.3 percent. We fix the target inflation rate at 2 percent so that $\pi^* = 1.02^{\frac{1}{4}}$.

There are two parameters c^v and \bar{h} that are endogenously determined after the estimation is completed; we discuss these parameters here because they are not directly estimated. The scale parameter of the hiring cost function c^v is selected so that the job creation condition holds, given all the parameters and the targeted steady-state job filling rate at 0.75. Similarly, the scale parameter of the labor supply function is chosen such that hours of work equal 1/3 at the steady state.

Parameter	Description	Value
α	Capital share	0.33
β	Discount factor	0.9996
δ	Depreciation rate	0.025
\bar{m}	Scale parameter of matching function	0.75
ϕ	Elasticity of matching function	0.5
s	Separation probability	0.195
b	UI benefits	0.2145
ϵ^v	Curvature of hiring cost	1
ν	Inverse of elasticity of labor supply	2
γ_z	Steady state TFP growth	1.004
\bar{x}	Steady-state level of government expenditures	1.25
$\bar{\mu}$	Steady-state level of markup	0.2
π^*	Target inflation rate	$1.02^{\frac{1}{4}}$

Table 1: Calibrated Parameters

4.2 Data

We use the following macroeconomic series to estimate the remaining parameters. Real output in the model corresponds to NIPA real GDP. We compute real GDP by dividing the nominal GDP series by the chained-price GDP deflator. It is converted into per capita real GDP by dividing it by population 16 years or older. Consumption in the model corresponds to total personal consumption expenditures less durable-goods consumption in the data. Investment is defined as gross private domestic investment plus durable-goods consumption. We take nominal consumption and investment series and divide both series by the chained-price GDP deflator and 16+ population to obtain real per-capita series. We use a geometrically smoothed version of the population series to deal with small discontinuities. We compute quarter-to-quarter growth rates as log difference of real per capita variables and multiply the growth rates by 100 to convert them into percentages.

For labor market variables, we use the unemployment rate, the vacancy rate, and real earnings per worker in the estimation. Specifically, the logged quarterly series of the unemployment rate, taken from the Current Population Survey, is with the CBO estimate of the natural rate of unemployment. This series is linked with log-deviations of u_t from its steady-state level. We detrend the unemployment rate because it exhibits low frequency movements due to factors, such as demographic changes, that our model does not explicitly model. For the vacancy rate, we use the total number of job openings from the JOLTS (Job Opening and Labor Turnover Survey). Since this series is available only from December 2000 onwards, we splice it with the Conference Board’s help-wanted index series and extend the vacancy series backwards. We multiply the level of the latter series by a constant factor. The multiplicative factor is computed such that the average levels of the two series match up over the overlapping sample period (between December 2000 and December 2014). The total number of job openings is normalized by the labor force. Its quarterly average series is logged and HP filtered with the smoothing parameter set at 10^5 . Similar to the unemployment rate, the vacancy rate series exhibits a low frequency trend that our model is not designed to capture. We remove this slow moving trend via the HP filter. The detrended series is equated with log deviations of v_t from its steady-state level. We compute quarter-over-quarter growth rates of real earnings per worker, using the data available through the Productivity and Cost Program of the BLS. We first obtain the real hourly earnings index, the aggregate hours index, and the aggregate employment index. Quarter-over-quarter log differences

in these three indexes allow us to compute quarter-over-quarter log differences in real earnings per worker. We assume that this series is measured with some i.i.d. error and estimate the standard deviation (σ_{mew}) of the measurement error.

The effective Fed funds rate is used as the measure of the monetary policy rate. In quarters when the funds rate was constrained by the effective lower bound (ELB), we treat the funds rate as missing. Further, assuming that the expectation hypothesis of the term structure holds, we include the two-year treasury rate as a noisy measure of the expected funds rate over the next two years. We calibrate the noise to lie within a few basis points of the value implied by the expectation hypothesis, after taking out the average term premium. This measure of expected interest rates over the next two years ensures that the estimation is informed by variations in monetary policy expectations over the next two years even during the ELB period when the observations for the funds rate are missing.

Lastly, we use core-PCE inflation as the observable measure of inflation. We detrend the inflation rate by a measure of long-term PCE inflation expectations. Although trend inflation is constant at 2 percent in the model, trend inflation is likely to be time varying over longer sample periods and we capture this trend via long-term PCE expectations. For the period after 2007Q1, we use long-term PCE inflation expectations available through the SPF (Survey of Professional Forecasters). For the period between 1991Q4 and 2006Q4, we use CPI inflation expectations available also in the SPF. For the overlapping sample period, CPI inflation expectations are 20 basis points higher than PCE inflation expectations. We splice the two series after subtracting 20 basis points from the CPI inflation expectations for 1991Q4 to 2006Q4. Prior to 1994Q4, we use other sources to compute the long-term CPI inflation expectations. From 1979Q4 to 1991Q3, we use inflation expectations available from the Livingston and Blue Chip surveys (all available from the Federal Reserve Bank of Philadelphia). Whenever available, we use the Livingston survey and otherwise use the Blue Chip survey. If neither is available, we linearly interpolate between the combined surveys. Before 1979Q3, we use the historical break-even rates for inflation expectations computed by the Federal Reserve Bank of New York. In our estimation, the detrended core-PCE inflation rate is linked to the deviation of the inflation rate from its steady-state value (2 percent) in the model.

5 Estimated Parameters

The estimation results are presented in Tables 2 and 3. Our choice of prior distributions is standard. Posterior means are also roughly in line with the existing literature. The model introduces real wage rigidity, and the parameter ρ^w is indeed estimated to be fairly high at 0.88. The estimation results for exogenous processes are also roughly in line with the existing literature. The estimated parameter values for the matching efficiency process are similar to those estimated by Furlanetto and Goshenny (2016), although their model is different from ours and they use different observables to estimate the shock process. Another notable result is that in our estimation, the markup shock is estimated to be highly persistent and quite volatile. We find that this shock contributes significantly to overall variations of the model.

Parameter	Density	Prior		Posterior	
		Mean	Std	Mean	90% Intv.
τ	Gamma	50.00	10.00	80.07	[65.18 , 97.59]
ψ	Beta	0.50	0.20	0.06	[0.01 , 0.12]
ℓ	Beta	0.50	0.20	0.95	[0.94 , 0.97]
ρ_w	Beta	0.50	0.10	0.88	[0.83 , 0.94]
κ	Gamma	2.00	2.00	12.41	[6.53 , 19.29]
η	Beta	0.50	0.20	0.73	[0.61 , 0.85]
r_π	Normal	1.50	0.25	2.62	[2.35 , 2.88]
r_{gy}	Normal	0.40	0.30	0.53	[0.44 , 0.62]
r_ρ	Beta	0.50	0.20	0.85	[0.83 , 0.87]

Table 2: Estimated Structural Parameters

Parameter	Distribution	Prior		Posterior	
		Mean	Std	Mean	90% Intv.
ρ_m	Beta	0.50	0.20	0.93	[0.89 , 0.97]
ρ_χ	Beta	0.50	0.20	0.38	[0.28 , 0.48]
ρ_ζ	Beta	0.50	0.20	0.81	[0.77 , 0.85]
ρ_μ	Beta	0.50	0.20	0.98	[0.96 , 1.00]
ρ_x	Beta	0.50	0.20	0.99	[0.99 , 1.00]
ρ_z	Beta	0.50	0.20	0.44	[0.34 , 0.54]
σ_z	Inverse Gamma	0.01	2.00	0.0054	[0.0047 , 0.0061]
σ_m	Inverse Gamma	0.01	2.00	0.0220	[0.0202 , 0.0239]
σ_χ	Inverse Gamma	0.01	2.00	0.0880	[0.0607 , 0.1148]
σ_ζ	Inverse Gamma	0.01	2.00	0.1112	[0.0684 , 0.1535]
σ_κ	Inverse Gamma	0.01	2.00	0.0027	[0.0025 , 0.0030]
σ_x	Inverse Gamma	0.01	2.00	0.0060	[0.0055 , 0.0065]
σ_μ	Inverse Gamma	0.01	2.00	0.0615	[0.0512 , 0.0716]
σ_{mew}	Inverse Gamma	0.01	2.00	0.0092	[0.0084 , 0.0100]

Table 3: Estimated Exogenous Parameters

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