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**RISING INDEBTEDNESS AND HYPERBOLIC DISCOUNTING: A WELFARE ANALYSIS**

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# Rising Indebtedness and Hyperbolic Discounting: A Welfare Analysis*

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**Abstract**

Is the observed rapid increase in consumer debt over the last three decades good news for consumers? This paper quantitatively studies macroeconomic and welfare implications of relaxing borrowing constraints when consumers exhibit a hyperbolic discounting preference. In particular, I construct a calibrated general equilibrium life-cycle model with uninsured idiosyncratic earnings shocks and a quasi-hyperbolic discounting preference and examine the effect of relaxation of the borrowing constraint which generates increased indebtedness. The model can capture the two contrasting views associated with increased indebtedness: the positive view, which links increased indebtedness to financial sector development and better insurance, and the negative view, which associates increased indebtedness with consumers' *over-borrowing*. I find that while there is a welfare gain as large as 0.4% of flow consumption from a relaxed borrowing constraint, which is consistent with the observed increase in aggregate debt between 1980 and 2000 in the model with standard exponential discounting consumers, there is a welfare *loss* of 0.2% in the model with hyperbolic discounting consumers. This result holds in spite of the observational similarity of the two models; the macroeconomic implications of a relaxed borrowing constraint are similar between the two models. Cross-sectionally, although consumers of high and low productivity gain and medium productivity consumers suffer due to a relaxed borrowing constraint in both models, the welfare gain of low-productivity consumers is substantially reduced (and becomes negative in the case of strong hyperbolic discounting) in the hyperbolic discounting model due to the welfare loss from over-borrowing. Finally, I find that the optimal (social welfare maximizing) borrowing limit is 15% of average income, which is substantially lower than both the optimal level implied by the exponential discounting model (37%) and the level of the U.S. economy in 2000 implied by the model (29%).

JEL Classification: D91, E21, E44, G18
Keywords: Hyperbolic discounting, Temptation, Self-Control Heterogeneous agents, Incomplete markets, General equilibrium

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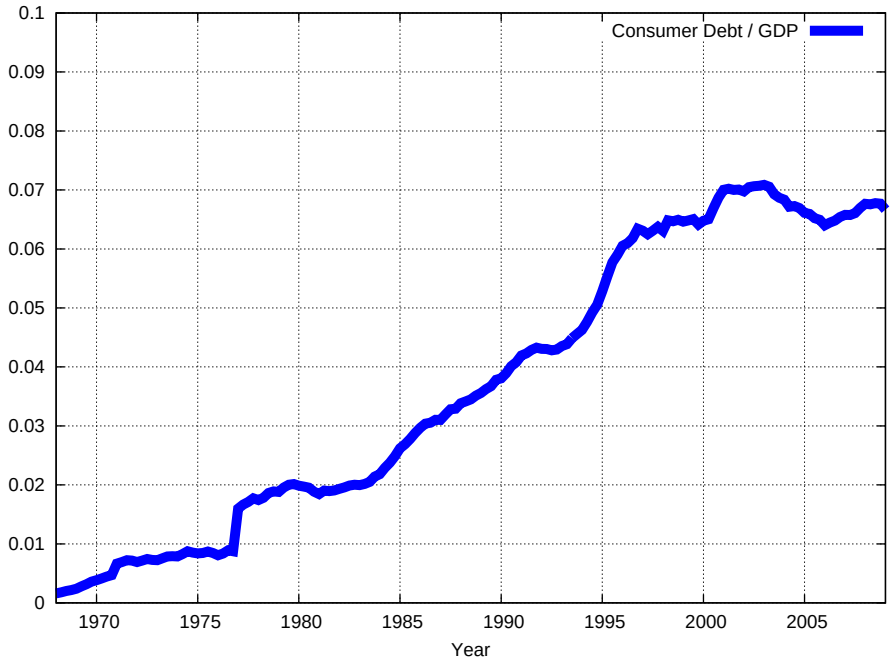



**Figure 1: Total unsecured consumer debt over GDP, Source: Federal Reserve Board, G.19**

# 1 Introduction

Since the early 1980s, there has been a substantial increase in the indebtedness of U.S. consumers, although there might be a reversal of the trend with the ongoing deep recession. Total household debt in the U.S. increased from 43% of GDP in 1982 to 62% in 2000.[1] Both unsecured and secured debt increased. The aggregate balance of unsecured debt increased from 5% of total income in the early 1980s to 9%.[2] Figure 1 shows the trend of unsecured consumer debt relative to GDP.[3] It was close to zero before 1970, rose to 2% by 1980, and has stabilized around 7% since 2000. While an increase in indebtedness is often seen as a result of an innovation in the financial sector and thus is linked to a positive welfare effect, there are two concerns. First, increased indebtedness is closely related to under-saving, which slows down capital accumulation. Second, there is a popular perception that consumers might be *over-borrowing* and *over-consuming*. The second concern has not been studied, since it cannot be systematically captured by models with a time-consistent preference. This is the main contribution of the current paper.

[1] Smith (2009).

[2] Livshits et al. (2007a).

[3] Unsecured consumer debt is measured as the revolving consumer credit in the G.19 series of the Federal Reserve Board (FRB). In the FRB data, total consumer credit consists of non-revolving and revolving credit. Revolving credit mainly consists of loans for automobiles, mobile homes, and boats but also includes some unsecured credit. Livshits et al. (2007a) constructed an unsecured consumer credit data series that includes not only revolving credit but also part of non-revolving credit. However, the difference between the revolving credit and the unsecured consumer credit they constructed is small (less than one percentage point as percentage of disposable income) for the period where more reliable data are available (after 1989).

In this paper I examine the macroeconomic and welfare implications of a relaxed borrowing constraint in the general equilibrium life-cycle model with hyperbolic discounting consumers. In particular, I assume that the increased indebtedness is due to a relaxed borrowing constraint that consumers face, calibrate the borrowing constraints so that the induced indebtedness matches observed aggregate debt level, and study the macroeconomic and welfare implications associated with the relaxed borrowing limit. The hyperbolic discounting model has become popular for analyzing consumers' behavior, especially behavior associated with borrowing and defaulting, since many researchers argue that the hyperbolic discounting model not only is consistent with experimental evidence but also is able to replicate some dimensions of consumers' behavior that the standard exponential discounting model cannot. By introducing a time-inconsistent preference into a standard macroeconomic model, and calibrating the model, I can systematically and quantitatively investigate the welfare implications associated with an increased indebtedness in the hyperbolic discounting model.

The model developed here is built on the incomplete market general equilibrium models initially developed by Huggett (1996) and Aiyagari (1994). Since this class of models allows researchers to explicitly study heterogeneity either due to ex-ante heterogeneity or due to market incompleteness within a standard general equilibrium macroeconomic framework, the models have been extended in a variety of ways and widely used to analyze a variety of issues, including business cycles, wealth inequality, optimal taxation, Social Security reform, and asset pricing. The model developed in the current paper introduces a hyperbolic discounting preference into the standard incomplete market general equilibrium model. The model is closest to the one by İmrohoroğlu et al. (2003), but they do not focus on the heterogeneity, while the heterogeneous welfare effect on different types of consumers is a key aspect of the analysis in the current paper.

The model developed in the current paper is also built on the recent developments in the quantitative model with hyperbolic discounting consumers. The pioneer work is Laibson (1997). I will review the literature on hyperbolic discounting in Section 2.1. Most existing works with hyperbolic discounting consumers are built on the partial equilibrium consumption-savings model of Deaton (1991) and Carroll (1997), while the general equilibrium plays an important role in the main results of the paper here.[4] In addition, the current paper is one of the few that focus not only on the implications of the allocation of hyperbolic discounting, but also on the welfare implications.[5]

There are five main findings. First, when calibrated using the same strategy, models with exponential discounting and hyperbolic discounting are observationally similar in terms of average life-cycle profiles of consumption, savings, and borrowing. The finding echoes what Angeletos et al. (2001) find and is closely related to the observationally equivalence result of Barro (1999). Even though a hyperbolic discounting preference induces consumers to over-consume through a low short-term discount factor compared with the exponential discounting model, the long-term discount factor must be calibrated to be higher in the hyperbolic discounting model to match the aggregate capital stock, which basically nullifies the effect of the low short-term discount factor. Second, although average life-cycle profiles are observationally similar, the two models have some

[4] Notable exceptions are İmrohoroğlu et al. (2003) and Krusell et al. (2009).
[5] Notable exceptions are Krusell et al. (2009) and Petersen (2004).

different cross-sectional implications; the model with hyperbolic discounting consumers generates *(i)* more borrowing-constrained consumers, *(ii)* higher dispersion of wealth, and *(iii)* higher consumption volatility. Third, not only are the models observationally similar, in particular the average life-cycle profiles, in the stationary equilibrium, the aggregate response to a relaxed borrowing constraint is both qualitatively and quantitatively similar between the two models. Because of the observational similarity, it is hard to distinguish the two models by the response in terms of macroeconomic aggregates. Fourth, even though macroeconomic implications are similar, the two models have sizably different welfare implications. More specifically, while a relaxed borrowing constraint has a positive effect to social welfare in the model with exponential discounting consumers, hyperbolic discounting consumers on average suffer from a relaxed borrowing constraint mainly because of *over-consumption*. The problem is serious from a policy perspective because the two models are hard to distinguish from the allocation but have contrasting welfare implications. Finally, the *optimal* level of borrowing limit is substantially lower at about 15% of the average total income in the model with hyperbolic discounting compared with the standard exponential discounting model, whose optimal borrowing limit is about 37%. This is a direct implication of the *over-borrowing* feature of the model with hyperbolic discounting. When consumers exhibit a hyperbolic discounting preference, there is a positive welfare effect of restricting borrowing by consumers.

I review the related literature, particularly on hyperbolic discounting models and on the increased indebtedness of the U.S. consumers since the early 1980s, in Section 2. Section 3 develops the model. Section 4 describes how the model is calibrated for quantitative exercises. Since the model is solved numerically, Section 5 gives an overview of the computational algorithm. More details can be found in Appendix A.2. Section 6 presents the main results of the paper. Section 7 concludes.

# 2 Related Literature

## 2.1 Hyperbolic Discounting

Strotz (1956) first argue that people are more impatient with respect to short-run trade-offs than to long-run trade-offs, and formalize the dynamic inconsistency problem using a game played against future selves.[6] Phelps and Pollak (1968) use the quasi-hyperbolic discounting function in the context of intergenerational time preferences; the consumer in the current generation discounts the utility of the next generation with a higher discount rate than the rate applied to the subsequent generations. The quasi-hyperbolic discounting used in the current paper was first used by Laibson (1997) in the context of a life-cycle model with a time-inconsistent preference.[7]

[6] Angeletos et al. (2001) provide a good summary of the literature.

[7] The actual discount factor function used in the "hyperbolic discounting" models are strictly not hyperbolic, but it is called by that name because originally a hyperbolic function was used. Krusell and Smith (2000) call the preference *quasi-geometric* discounting, which offers a more precise description of the actual discount factor function typically used in the literature. The preference is also called *present bias*.

Following Laibson (1997), there has been a surge in research on the hyperbolic discounting preference. Laibson (1997) studies the role of an illiquid asset like housing in providing an imperfect commitment device for time-inconsistent consumers. Laibson et al. (2003) uses a hyperbolic discounting model to explain why many people carry a balance on credit cards with a high interest rate even if they also carry a positive balance of a liquid asset. Angeletos et al. (2001) compare the implications of models with exponential and hyperbolic discounting consumers and argue that the hyperbolic discounting model replicates various dimensions of consumption and savings behavior better than the standard exponential discounting model. Laibson et al. (2007) use simulated method of moments to jointly estimate key parameters associated with the hyperbolic discounting model. Malin (2008) studies the role of a savings floor when consumers exhibit a hyperbolic discounting preference. İmrohoroğlu et al. (2003) study the macroeconomic and welfare effects of having an unfunded Social Security program in the model with a hyperbolic discounting consumers. Barro (1999) studies the neoclassical growth model with a hyperbolic discounting preference. Tobacman (2009) investigates the wealth distribution of such a model.

Although the hyperbolic discounting preference potentially has welfare implications very different from those in the standard exponential discounting preference, not many papers quantitatively study the welfare implications of the macroeconomic model with a hyperbolic discounting preference. Notable exceptions are İmrohoroğlu et al. (2003), Krusell et al. (2009) and Petersen (2004). İmrohoroğlu et al. (2003) study how the welfare implications of Social Security reform are different between the standard exponential discounting model and hyperbolic discounting models. Krusell et al. (2009) show that in the model with temptation and self-control, which can be considered as the generalization of the hyperbolic discounting preference studied in the current paper, a savings subsidy (or negative capital income tax) is optimal. This is because a savings subsidy prevents consumers from over-consuming and under-saving and thus helps consumers overcome the temptation for current consumption. Petersen (2004) studies the welfare implications of various tax policies in the life-cycle general equilibrium model.

## 2.2 Increasing Indebtedness in the U.S.

The two papers most closely related to the current paper are Campbell and Hercowitz (2009) and Obiols-Homs (2009). Both papers investigate the welfare consequences associated with the rising debt in the U.S., but both use the standard exponential discounting preference. Campbell and Hercowitz (2009) study the welfare implications of the observed increase in consumer debt, in particular, secured debt. They use the calibrated general equilibrium model of impatient borrowers (who have a perpetually lower discount factor) and patient savers (with a perpetually higher discount factor) and argue that the welfare effect is negative for borrowers and positive for savers. Even though borrowers enjoy a welfare gain from relaxation of the down-payment constraint, a negative effect from a higher equilibrium interest rate dominates the welfare gain from better consumption smoothing. Naturally, savers gain substantially from the higher interest rate. As discussed by Smith (2009), a problem of the paper is that the model is highly stylized, including the assumption of the different discount factor. It is not clear if the quantitative result of the paper is robust in a more general environment. Obiols-Homs (2009) studies the cross-section of

the welfare effect of a relaxed borrowing limit, as in the current paper, in the general equilibrium model with infinitely lived consumers. The author finds that the welfare effect is non-linear, in particular U-shaped, with respect to individual productivity. With the exponential discounting model, the same property is obtained in the current paper.

Livshits et al. (2007a) investigate jointly the reasons behind the increase in unsecured loans and in consumer bankruptcies. They find that combination of a decline in the transaction cost of lending and the cost of filing for bankruptcy replicates what has occurred since the early 1980s quite well. Benton et al. (2007) study over-borrowing from the point of view of behavioral economics using survey evidence.

# 3 Model

The model is based on the general equilibrium life-cycle model of Huggett (1996), with the quasi-hyperbolic discounting preference of Laibson (1996).

## 3.1 Demographics

Time is discrete. In each period, the economy is populated by $I$ overlapping generations of consumers. In time $t$, a measure $(1+\nu)^t$ of consumers are born. $\nu$ is the constant population growth rate. Each generation is populated by a mass of measure-zero consumers. Consumers are born at age 1 and could live up to age $I$. There is a probability of early death. Specifically, $s_i$ is the probability with which an age $i$ consumer survives to age $i+1$. With probability $(1-s_i)$, an age $i$ consumer does not survive to age $i+1$. $I$ is the maximum possible age, which implies $s_I = 0$.

Consumers retire at age $1 < I_R < I$. Consumers with age $i < I_R$ are called *workers*, and those with age $i \geq I_R$ are called *retirees*. $I_R$ is fixed; there is no retirement choice.

## 3.2 Preference

The preference of consumers is time separable and characterized by an instantaneous utility function and two discount factors. The instantaneous utility function $u(c)$ is standard; it is strictly increasing and strictly concave in $c$.

I use a quasi-hyperbolic discounting preference, which was first analyzed by Phelps and Pollak (1968) and used in a quantitative macroeconomic model in Laibson (1996) and Laibson et al. (2007). According to their set-up, in period $t$, instantaneous utility in period $t$, $t+1$, $t+2$, $t+3$, $t+4$,..., is discounted by 1, $\beta$, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$,... Since $\beta$ is used to discount utility from the current period and the next, while $\delta$ is used to discount future utility from the next period on, $\beta$ and $\delta$ are called short-term and long-term discount rates, respectively. Notice that the standard exponential

discounting is a special case with $\beta = 1$; in this case, future utility is discounted at the constant rate of $\delta$.

For an age $i$ consumer, the expected life-time utility $U_i$ can be defined as follows:

$$U_i = u(c_i) + \beta \, \mathbb{E} \sum_{j=i+1}^{I} \delta^{j-(i+1)} u(c_j) \tag{1}$$

The important feature of this class of preference is that the preference exhibits *time inconsistency*. For example, the discount factor applied between period $t+1$ and $t+2$ in period $t$ is $\delta$, while the discount factor between the same periods changes to $\beta$ in period $t+1$. When $\beta \in (0,1)$, the preference implies a *present bias*; if there is no constraint or commitment device, consumers over-consume and under-save or over-borrow in retrospect.

## 3.3 Technology

There is a representative firm that has access to the following constant scale production technology:

$$Y = ZF(K, L) \tag{2}$$

where $Y$ is output, $Z$ is the level of total factor productivity, $K$ is capital stock, and $L$ is labor supply. Capital depreciates at a constant rate $\kappa$ per period.

## 3.4 Endowment

Consumers are born with zero assets. Each consumer is endowed with one unit of time each period and inelastically supplies labor, since leisure is not valued. Labor productivity of a consumer is characterized by $e(i, p)$, where $i$ captures the life-cycle profile of labor productivity, and $p$ is the uninsured shock to labor productivity. $p$ is assumed to have finite support; $p \in \{p_1, p_2, ..., p_N\}$. Each newborn consumer draws its initial $p$ from an i.i.d. distribution where $\pi_p^0$ is the probability attached to $p$. After the initial $p$ is drawn, $p$ follows a first-order Markov process with $\pi_{p,p'}$ as the transition probability from $p$ to $p'$.

## 3.5 Market Arrangements

Capital and labor are traded competitively. Consumers are not allowed to trade state-contingent securities but can borrow or save using asset $a$, subject to a borrowing constraint $\underline{a}$.

## 3.6 Government

The government has three roles in the model: *(i)* running the Social Security program, *(ii)* collecting a proportional income tax, *(iii)* collecting accidental bequests using estate taxes and redistributing the proceeds with a lump-sum transfer.

The government runs a simple pay-as-you-go Social Security program. The government imposes a flat payroll tax with the tax rate of $\tau_S$ to all workers and uses the proceeds to finance Social Security benefits $b_i$ of current retirees. It is assumed that all retirees receive the same amount of benefits regardless of their contribution, and the government budget associated with the Social Security program balances each period. Formally, $b_i = 0$ for $i < I_R$ and $b_i = \overline{b}$ for $i \geq I_R$. $\overline{b}$ is the constant amount of Social Security benefit.

The government collects a proportional general income tax with tax rate $\tau_I$. Both capital and labor income are taxed at the same rate. The proceeds are not redistributed or valued by consumers.

Because of the stochastic death, there are accidental bequests in the model. I assume that the government collects all of the accidental bequests using 100% of the estate taxes and redistributes the proceeds equally to the surviving consumers every period. $tr$ denotes the lump-sum transfer under the program.

## 3.7 Consumer's Problem

I define the problem of a consumer recursively. As I will state formally in the next section, I focus on the stationary equilibrium. Therefore, we can drop the aggregate state variables from the individual consumer's problem. The problem of an age $i$ consumer with the current productivity shock $p$ and asset position $a$ can be characterized by the following Bellman equation:

$$\widetilde{V}(i,p,a) = \max_{c,a'} \left[ u(c) + \beta\ \delta\ s_i \sum_{p'} \pi_{p,p'} V(i+1,p',a') \right] \tag{3}$$

subject to

$$c + a' = (a + tr)(1 + r(1-\tau_I)) + e(i,p)(1-\tau_I-\tau_S)w + b_i \tag{4}$$

$$a' \geq \underline{a} \tag{5}$$

$a' = \widetilde{g}_a(i,p,a)$ is the optimal decision rule associated with the Bellman equation above. Notice that the value function on the left-hand side, $\widetilde{V}(i,p,a)$, is different from the one on the right-hand side, which is $V(i,p,a)$. This is due to the time-inconsistency problem associated with the quasi-hyperbolic discounting preference. The value function is updated with the following equation:

$$V(i,p,a) = \left[ u(c) + \delta\ s_i \sum_{p'} \pi_{p,p'} V(i+1,p',a') \right] \tag{6}$$

where

$$a' = \widetilde{g}_a(i,p,a) \tag{7}$$
$$c = (a+tr)(1+r(1-\tau_I)) + e(i,p)(1-\tau_I-\tau_S)w + b_i - \widetilde{g}_a(i,p,a) \tag{8}$$

Mechanically, the consumer chooses the optimal asset level $a'$ with the discounting factor $\beta\delta$ but the actual value function is evaluated with the discount factor $\delta$. İmrohoroğlu et al. (2003) distinguish the two cases in terms of what hyperbolic discounting consumers expect about their own future decisions. According to their classification, a *naive* consumer wrongly thinks that future selves make decisions in a time-consistent manner (using only the discount factor $\delta$). On the other hand, a *sophisticated* consumer thinks that future selves are time-inconsistent (using both $\beta$ and $\delta$). The way I formalize the consumer's problem, which is the same as in Laibson (1996) and Laibson et al. (2007), can be classified as *sophisticated* consumers according to the classification of İmrohoroğlu et al. (2003). Angeletos et al. (2001) find that *naive* and *sophisticated* hyperbolic discounting consumers behave similarly in a life-cycle model.

## 3.8 Equilibrium

I will define below the recursive competitive equilibrium where the demographic structure is stationary, even though the size of the population is growing at a constant rate $\nu$. In the equilibrium with a stationary demographic structure, prices $\{r,w\}$ are constant over time.

Let $\mathbf{M}$ be the space of individual state. $(i,p,a) \in \mathbf{M}$. Let $\mathscr{M}$ be the Borel $\sigma - algebra$ generated by $\mathbf{M}$, and $\mu$ the probability measure defined over $\mathscr{M}$. I will use a probability space $(\mathbf{M}, \mathscr{M}, \mu)$ to represent a type distribution of consumers.

**Definition 1** (Stationary recursive competitive equilibrium)**.** *A stationary recursive competitive equilibrium is a set of prices $\{r,w\}$, government policy variables $\{b_i, tr\}$, aggregate capital stock $K$, aggregate labor supply $L$, value functions $V(i,p,a)$ and $\widetilde{V}(i,p,a)$, optimal decision rule $\widetilde{g}_a(i,p,a)$, and the stationary measure after normalization with respect to population growth, $\mu$, such that:*

1. *Given the prices and policy variables, $V(i,p,a)$ and $\widetilde{V}(i,p,a)$ are a solution to the consumer's optimization problem defined in Section 3.7, and $\widetilde{g}_a(i,p,a)$ is the associated optimal decision rules.*

2. *The prices $r$ and $w$ are determined competitively, i.e.,*

$$r = ZF_K(K,L) - \kappa \tag{9}$$
$$w = ZF_L(K,L) \tag{10}$$

3. *Measure of consumers $\mu$ is consistent with the demographic transition, the stochastic process of shocks, and the optimal decision rules, after normalization with respect to population growth.*

4. *Aggregate capital and labor are consistent with the individual optimal decisions, i.e.:*

$$K = \frac{1}{1+\nu} \int_{\mathscr{M}} \widetilde{g}_a(i,p,a)\, d\mu \tag{11}$$

$$L = \int_{\mathscr{M}} e(i,p)\, d\mu \tag{12}$$

5. *Government satisfies period-by-period budget balance with respect to the social security program, i.e.,*

$$\int_{\mathscr{M}} b_i\, d\mu = \int_{\mathscr{M}} e(i,p)\, w\, \tau_S\, d\mu \tag{13}$$

6. *Government satisfies period-by-period budget constraint with respect to the estate tax and lump-sum transfer, i.e.,*

$$\int_{\mathscr{M}} tr\, d\mu = \frac{1}{1+\nu} \int_{\mathscr{M}} (1-s_i)\, \widetilde{g}_a(i,p,a)\, d\mu \tag{14}$$

# 4 Calibration

## 4.1 Demographics

One period is set as one year in the model. Age 1 in the model corresponds to the actual age of 20. $I$ is set at 81, meaning that the maximum actual age is 100. $I_R$ is set at 45, implying that the agents become retired at the actual age of 65. The population growth rate, $\nu$, is set at 1.2% annually. This growth rate corresponds to the average annual population growth rate of the U.S. over the last 50 years. The survival probabilities $\{s_i\}_{i=1}^{I}$ are taken from the life table in Social Security Administration (2007).[8] In order to guarantee the maximum age of $I = 81$, $s_{81} = 0$ is imposed. Figure 9 in Appendix A.1 shows the conditional survival probabilities used.

## 4.2 Preference

For the period utility function, the following constant relative risk aversion (CRRA) functional form is used:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{15}$$

[8] Table 4.C6 of Social Security Administration (2007). An average of the survival probabilities of males and females is used.

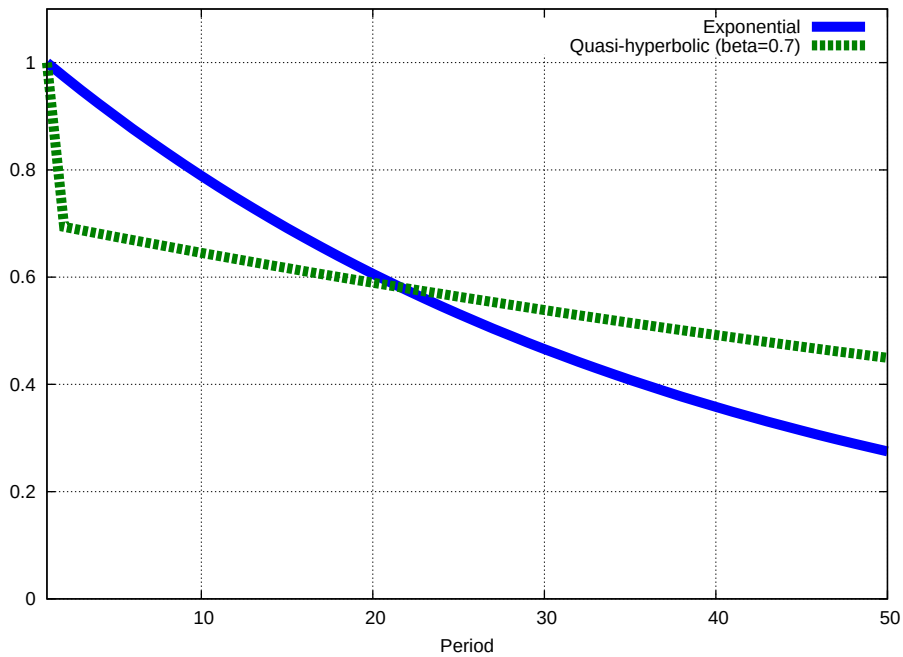



**Figure 2: Comparison of discount factors**

$\sigma$ is set at 1.5, which is the commonly used value in the literature. It is also the point estimate of Laibson et al. (2007).

Discount factors $\beta$ and $\delta$ are calibrated to be different for different model economies, but the calibration strategy is the same. For all cases, I set the short-term discount factor $\beta$ at first and calibrate the long-term discount factor $\delta$ so that the capital-output ratio of the economy in the baseline case is 3.0, which is the historical average value of the U.S. economy. In other words, different model economies have different short-term discount factors $\beta$, but they have the same aggregate capital stock in equilibrium.[9]

In the model with the standard exponential discounting consumers, $\beta = 1$ by assumption. I found that with $\delta = 0.9740$ the stationary equilibrium of the model generates a capital-output ratio of 3.0. For the model with hyperbolic discounting consumers, I use $\beta = 0.70$ as the baseline value of the short-term discount factor and calibrate $\delta$ such that the model achieves the same capital-output ratio of 3.0. The short-term discount factor of 0.70 is the one-year discount factor typically obtained from laboratory experiments. Moreover, the benchmark point estimate of Laibson et al. (2007) is $\beta = 0.703$, or the annual short-term discount rate of about 40%. This procedure generates $\delta = 0.9910$. The calibrated value of $\delta$ is higher than 0.958, which is the value that Laibson et al. (2007) estimate jointly with $\beta$. A large part of the difference is due to the existence of the mortality shock in the current model, which Laibson et al. (2007) do not have. If I adjust $\delta$

[9] Both Angeletos et al. (2001) and Tobacman (2009) calibrate the long-term discount factor $\delta$ for the model with exponential discounting consumers such that the average wealth holding at age 63 (the age just before retirement) is the same as in the model with hyperbolic discounting consumers where $\beta$ and $\delta$ are jointly estimated.

that is obtained here by being multiplied by the average survival probability (0.9828), the *effective* long-term discount factor becomes 0.974. Figure 2 compares the discount factors of the standard exponential discounting and hyperbolic discount for periods 1 to 50. The calibrated $\beta$ and $\delta$ are used. Notice that the discount factor function drops substantially more from period 1 to 2 in the case of the hyperbolic discounting preference. On the other hand, the discount factor applied to utility in the distant future is higher for the hyperbolic discounting model. Laibson (1997) argue that housing, from which inhabitants can enjoy utility as long as they own it and live in it, has an extra value for hyperbolic discounting consumers, since the dividends can be enjoyed for a long period of time.

I also investigate the case when the discount rate is 80% annually, which is twice as high as in the baseline hyperbolic discounting case. An 80% annual discount rate implies the short-term discount factor $\delta$ of 0.56. Using the same calibration strategy, the economy with a low short-term discount factor yields $\delta = 1.0005$. Even though the long-term discount factor is above unity, the effective discount factor becomes less than one if the survival probability is taken into account.

## 4.3 Technology

The following standard Cobb-Douglas production function is assumed:

$$Y = ZF(K, L) = ZK^{\theta}L^{1-\theta} \tag{16}$$

$Z$ is pinned down such that, in the baseline steady state, the equilibrium wage is normalized to one. The procedure implies $Z = 0.896$. $\theta$ is set at 0.36. This value is consistent with the average capital share of income of the U.S. economy. Capital is assumed to depreciate at the constant rate of $\kappa = 0.06$ per year. Huggett (1996) calibrates $\kappa = 0.06$ by matching the depreciation-output ratio of the model economy to its U.S. counterpart.

## 4.4 Endowment

I assume the following standard multiplicative form of individual productivity.

$$e(i, p) = e_i \; p \tag{17}$$

$e_i$ represents the age-earnings profile and $p$ is the individual productivity shock. Since retirement age is fixed at $I_R$, $e_i = 0$ for $i \geq I_R$. To calibrate $\{e_i\}_{i=1}^{I_R-1}$, I follow Huggett (1996) and use the data on the median earnings of male workers of different age groups from Social Security Administration (2007).[10] The median earnings data are multiplied by the employment to population ratio of males in each age group. The employment to population ratio for each age group is obtained from McGrattan and Rogerson (2004).[11] Finally, the resulting age-productivity profile is smoothed out

[10] The earnings data are taken from Table 4.B6 of Social Security Administration (2007).
[11] Table 3, 4, and 5 of McGrattan and Rogerson (2004).

by fitting the age profile of the product of median earnings and the employment to population ratio to a quadratic function of age. The resulting earnings profile is shown in Figure 10 in Appendix A.1.

In order to calibrate the stochastic process for $p$, first, following Huggett (1996), I assume that the logarithm of $p$ is drawn from a normal distribution $N(0, \sigma_0^2)$ and follows an AR(1) process with the persistence parameter $\rho_p$ and the standard deviation of the innovation term $\sigma_\epsilon$. These assumptions imply that the earnings for each age group is log-normally distributed, which captures the empirical distribution well. In sum, the stochastic process for $p$ is characterized by a triplet $(\rho_p, \sigma_0^2, \sigma_\epsilon^2)$. Following Huggett (1996), I set $(\rho_p, \sigma_0^2, \sigma_\epsilon^2) = (0.96, 0.38, 0.045)$. These parameter values are consistent with existing estimates of the stochastic process of individual earnings shocks and jointly replicate the empirical earnings Gini of 0.42.

The AR(1) process obtained above is approximated using the algorithm of Tauchen (1986) with 18 abscissas. Among the 18 possible realizations of $p$, 17 are equally-spaced between $-4\sigma_p$ and $4\sigma_p$ where $\sigma_p$ is the standard deviation of the unconditional distribution of $p$. In order to capture to a certain extent the observed extreme concentration of earnings, the last abscissa is set at $6\sigma_p$.

### 4.5 Market Arrangements

In the baseline case, the borrowing limit $\underline{a}$ is set at zero; i.e., no borrowing is allowed. In experiments, I relax the borrowing limit. I will relax the borrowing limit to the extent where the resulting aggregate debt matches some target so that the aggregate amount of debt is the same between the model and the corresponding U.S. economy.

### 4.6 Government

The payroll tax rate for the Social Security contribution $\tau_S$ is set at 0.10, which is the average contribution to the Social Security program as a fraction of labor income in the U.S. The proportional income tax rate of $\tau_I = 0.2378$ is set to match the ratio of total (federal, state, and local) government consumption over total income. The historical average of the ratio for the U.S. is 0.195.

## 5 Computation

Since there is no analytical solution to the model, the model is solved numerically. The solution algorithm is standard. Both the value functions and the optimal decision rule are approximated using piecewise linear functions. The optimal decision with respect to saving is solved using a golden section search. The equilibrium prices (wage and interest rate) and the transfer are found using iteration. Details about the numerical procedure are found in Appendix A.2.

# 6 Main Results

In Section 6.1, I compare the life-cycle profiles of economies with standard exponential discounting and hyperbolic discounting. The purpose is to investigate the (non-)difference in the life-cycle profiles generated by the time-inconsistent preference. In Section 6.2, I investigate the cross-sectional distribution of wealth in both economies. In Sections 6.3 and 6.4, I investigate the effect of the changes in the borrowing limit in the model with exponential discounting and with hyperbolic discounting. For the hyperbolic discounting model, two economies with different short-term discount factors (0.70 and 0.56) are investigated. I will analyze the changes in the macroeconomic aggregates and distribution in Section 6.3 and study the welfare implications of changes in the budget constraint in Section 6.4.

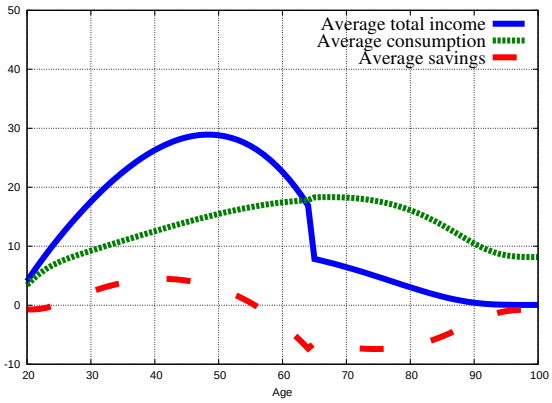
## 6.1 Life-Cycle Profiles

Figure 3 compares the life-cycle profiles of the model economies with the standard exponential discounting on the left and with the hyperbolic discounting (with the short-term discount factor of $\beta = 0.70$) on the right. The top two panels compare the average profiles of total income, consumption, and savings over the life-cycle. The middle two panels compare the average profile of asset holdings in two model economies. What is most striking is that there is little difference between the two model economies in terms of the average life-cycle profile. In both economies, the average consumption profile is flatter than the income profile. Consumers save during the working period and dissave during the retirement period.

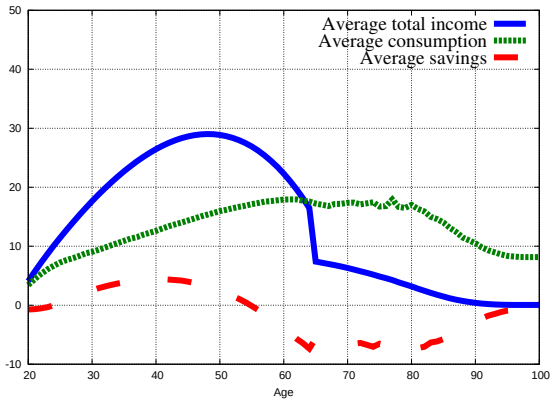
The bottom two panels of Figure 3 compare the cross-sectional variance of earnings and consumption over the life-cycle in two model economies. Since there is no labor-leisure decision, the profile for earnings is common between the two figures. The variance of consumption increases with age in both model economies, which is a feature of the life-cycle model with a highly persistent earnings shock. The only noticeable difference between the two figures is that the consumption variance increases faster in the model with hyperbolic discounting consumers especially toward the end of the working period. The feature is due to the extreme assumption of the constant Social Security benefit. If the amount of benefit is positively but imperfectly correlated with the contribution, which is the case in the U.S. economy, the spike of the consumption variance at the time of retirement substantially weakens.

## 6.2 Wealth Inequality

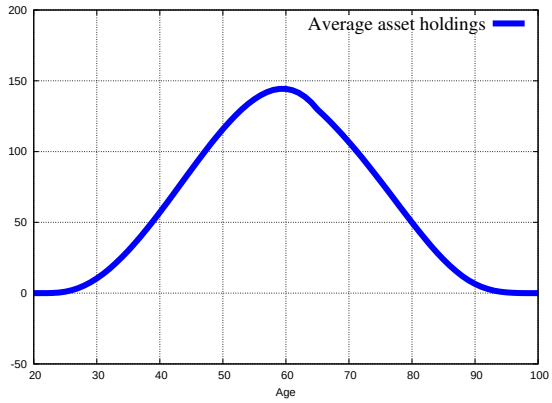
Table 1 compares the statistics related to the wealth inequality of the models and the data. Figure 4 compares the Lorenz curves for the U.S. economy and model economies with exponential and hyperbolic discounting. The statistics are based on the Survey of Consumer Finances (SCF) 1998 wave, computed by Budría et al. (2002). When the parameters are calibrated such that the macroeconomic aggregates are similar across different models, the hyperbolic discounting prefer-



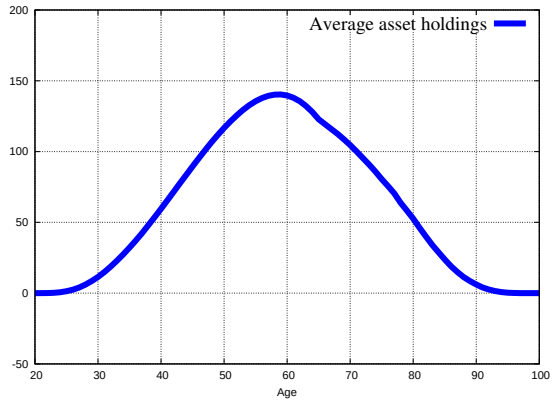

(a) Exponential: income consumption, and savings



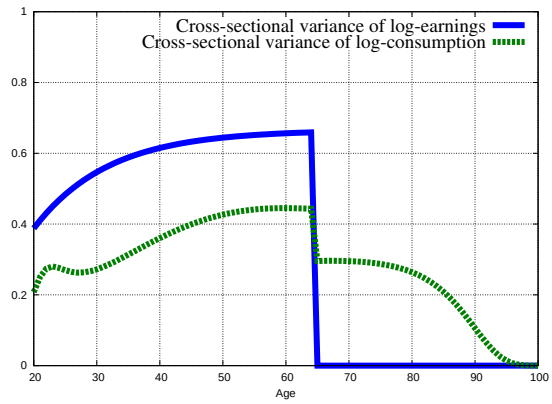

(b) Hyperbolic: income consumption, and savings



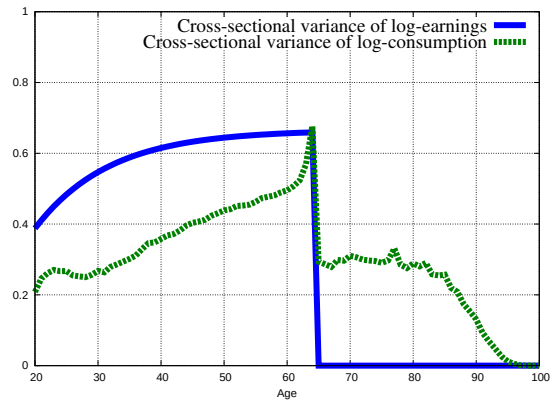

(c) Exponential: asset holdings




(d) Hyperbolic: asset holdings




(e) Exponential: cross-sectional variance of earnings and consumption




(f) Hyperbolic: cross-sectional variance of earnings and consumption

**Figure 3: Comparison between exponential discounting and hyperbolic discounting models**

**Table 1: Wealth distribution**

| Economy | Gini | Mean/ Median | Prop $\leq 0$ | Proportion of wealth held by 1st | 2nd | 3rd | 4th | 5th | top 1% |
|---|---|---|---|---|---|---|---|---|---|
| U.S.[1] | 0.803 | 4.03 | 0.099 | -0.3 | 1.3 | 5.0 | 12.2 | 81.7 | 34.7 |
| Exponential | 0.726 | 3.61 | 0.234 | 0.0 | 0.7 | 5.8 | 19.1 | 74.4 | 11.9 |
| Hyperbolic ($\beta = 0.70$) | 0.733 | 4.02 | 0.284 | 0.0 | 0.4 | 5.4 | 19.1 | 75.1 | 11.9 |
| Hyperbolic ($\beta = 0.56$) | 0.734 | 4.24 | 0.308 | 0.0 | 0.3 | 5.1 | 19.5 | 75.0 | 11.9 |

[1] Source: Survey of Consumer Finances 1998, statistics computed by Budría et al. (2002).

ence generates a slightly higher wealth inequality. The Gini index for total wealth in the baseline hyperbolic discounting model is 0.733, which is slightly higher than 0.726, the Gini index for the exponential discounting model. In terms of the skewness of the wealth distribution, the hyperbolic discounting model generates a substantially higher skewness; the mean/median ratio of wealth is 3.61 for the exponential discounting model, while it is 4.02 in the hyperbolic discounting model. The value for the hyperbolic discounting model is close to the empirical value of 4.03.

The reason behind the higher wealth inequality is that more consumers are consuming all of their income and saving nothing in the hyperbolic discounting model. The proportion of consumers with non-positive assets (which means zero, because borrowing is prohibited for now) is 0.284 in the hyperbolic discounting model, compared with 0.234 for the exponential discounting model. In terms of the top end of the wealth distribution, both the exponential discounting model and the hyperbolic discounting model fail to replicate the extreme wealth concentration at the top of the distribution; the proportion of wealth held by the top 1% wealthiest is the same across the models at 11.9%, and it is far below the empirical value of 34.7%.[12] The features of the hyperbolic discounting model become stronger in the model with a very low short-term discount factor ($\beta = 0.56$), but the additional change in the Gini index is small (from 0.733 to 0.734).

Tobacman (2009) also compares the wealth inequality implied by models with exponential and hyperbolic discounting. In the baseline case with both liquid and illiquid assets, the model with hyperbolic discounting exhibits a Gini index of 0.508, which is slightly higher than the value for the exponential discounting model (0.488). The magnitude of the difference is comparable to what is obtained here. In the models with only liquid assets like the current model, however, he finds a sizable difference in terms of wealth inequality between exponential and hyperbolic discounting models; the wealth Gini index with $\underline{a} = 0$ is 0.720 for the hyperbolic discounting model, while it is 0.630 for the exponential discounting model.

[12] The failure to replicate the extreme concentration of wealth is partly because the individual productivity process is calibrated based on the Panel Study of Income Dynamics (PSID), where very rich households are substantially under-represented. For more on models of wealth distribution, see Quadrini and Ríos-Rull (1997).

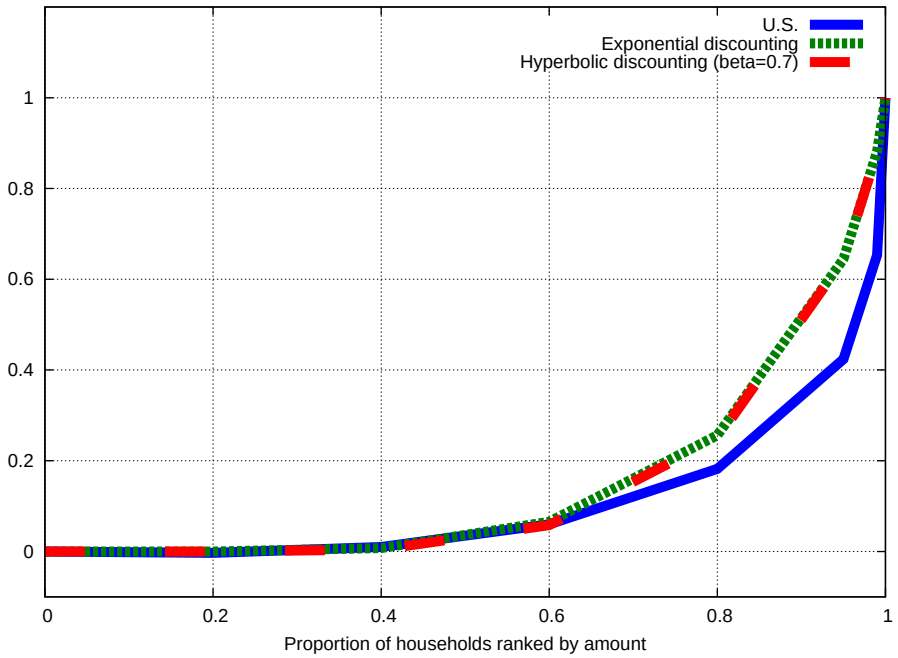



**Figure 4: Lorenz curve for the wealth distribution**

## 6.3 Rising Indebtedness: Macroeconomic Implications

I will investigate the macroeconomic and welfare implications of the increased indebtedness in the models with exponential and hyperbolic discounting. In particular, I assume that the increased indebtedness is due to a relaxed borrowing constraint that consumers face, calibrate the borrowing constraints so that the induced indebtedness matches observed aggregate debt level, and inquire the macroeconomic and welfare implications associated with the relaxed borrowing limit. This section studies the macroeconomic implications and Section 6.4 analyzes the welfare implications. The focus is on the difference between the implications of the standard exponential discounting model and those of the hyperbolic discounting models. I show in the previous section that, when calibrated to the same set of targets, in particular aggregate wealth, the steady state implications are similar between the models with different preference specifications. If, in addition, the macroeconomic and welfare implications of increased indebtedness are also similar between the exponential and hyperbolic discounting models, there is no need to use the non-standard hyperbolic discounting preference for an analysis of increased indebtedness. What I will show is that this is not the case; in particular, the welfare implications are very different between the models with different preference specifications.

Table 2 summarizes the macroeconomic implications of rising aggregate debt from 1970 to 1980 and 2000. The first panel summarizes the results of the exponential discounting model. The three rows correspond to the economies in the three time periods: The first one corresponds to the *1970 economy*, where borrowing does not exist, i.e., $\underline{a} = 0$. This is the economy calibrated in Section 4. The second economy is called the *1980 economy*, where the borrowing limit is relaxed such that

**Table 2: Macroeconomic implications of rising debt**

| Economy[1] | $\underline{a}$[2] | D/Y | K[3] | Y[3] | r% | wage | Var(c)[4] |
|---|---|---|---|---|---|---|---|
| **Exponential discounting model ($\beta = 1.00$)** | | | | | | | |
| 1970 | 0.000 | 0.000 | 1.000 | 1.000 | 6.01 | 1.000 | 0.481 |
| 1980 | −0.103 | −0.020 | 0.986 | 0.995 | 6.11 | 0.995 | 0.465 |
| 2000 | −0.337 | −0.070 | 0.959 | 0.985 | 6.33 | 0.985 | 0.455 |
| **Hyperbolic discounting model ($\beta = 0.70$) with $\underline{a}$ of exponential discounting model** | | | | | | | |
| 1970 | 0.000 | 0.000 | 1.000 | 1.000 | 5.99 | 1.000 | 0.489 |
| 1980 | −0.103 | −0.023 | 0.989 | 0.996 | 6.09 | 0.996 | 0.474 |
| 2000 | −0.337 | −0.082 | 0.960 | 0.985 | 6.32 | 0.985 | 0.468 |
| **Hyperbolic discounting model ($\beta = 0.70$)** | | | | | | | |
| 1970 | 0.000 | 0.000 | 1.000 | 1.000 | 5.99 | 1.000 | 0.489 |
| 1980 | −0.090 | −0.020 | 0.988 | 0.996 | 6.09 | 0.996 | 0.475 |
| 2000 | −0.290 | −0.070 | 0.964 | 0.987 | 6.29 | 0.987 | 0.467 |
| **Hyperbolic discounting model ($\beta = 0.56$)** | | | | | | | |
| 1970 | 0.000 | 0.000 | 1.000 | 1.000 | 5.95 | 1.002 | 0.504 |
| 1980 | −0.080 | −0.020 | 0.988 | 0.996 | 6.04 | 0.998 | 0.492 |
| 2000 | −0.271 | −0.070 | 0.965 | 0.987 | 6.22 | 0.990 | 0.484 |

[1] 1970: Economy with no borrowing. 1980: economy calibrated to $D/Y = 2\%$. 2000: Economy calibrated to $D/Y = 7\%$. $70-80$: Difference between 1970 and 1980 economies. For the second panel, borrowing constraints that are obtained in the exponential discounting model are used in the hyperbolic discounting model with $\beta = 0.70$.

[2] Borrowing limit relative to total income.

[3] Level in 1970 economy normalized to one.

[4] Average of cross-sectional variances of consumption for all age groups.

the aggregate amount of debt in the new stationary equilibrium is 2% of output. This target corresponds to the aggregate amount of unsecured debt in the U.S. economy in the early 1980s. The third economy is the *2000 economy*, where the borrowing limit is further relaxed such that the economy exhibits an aggregate amount of debt as large as 7% of output. You can see from the second column in the first panel that, for the exponential discounting model, a borrowing limit of the size of 10.3% and 33.7% of average income is needed to generate aggregate debt of 2% and 7% of GDP, respectively. Capital stock declines as the borrowing limit is relaxed. In the 2000 economy, equilibrium capital stock is 4% lower than in the 1970 economy without borrowing. Since the labor is inelastically supplied, the decline in capital stock generates a decline in output. Output in the 2000 economy is about 1.5% lower than in the 1970 economy. The equilibrium interest rate goes up from 6% in 1970 to 6.3% in 2000 as capital becomes more scarce, and wage declines as capital stock declines. Since a relaxed borrowing constraint implies better consumption smoothing,

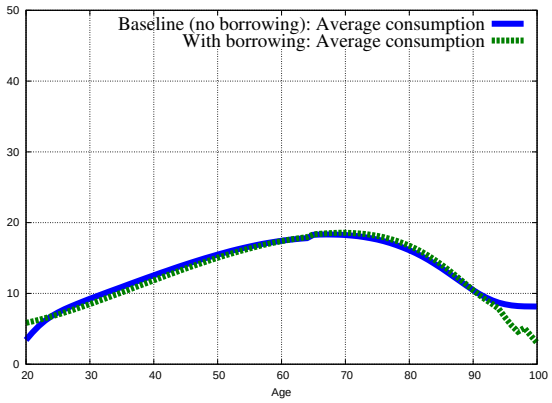
consumption variance declines as the borrowing constraint is relaxed; consumption variance drops from 0.48 in the 1970 economy to 4.55 in the 2000 economy.

Figure 5 compares the 1970 (no borrowing) and 2000 (7% debt to output ratio) economies with both exponential and hyperbolic discounting consumers. Panels on the left side correspond to the exponential discounting models, and panels on the right side are associated with hyperbolic discounting models. Among the panels on the left side, the most notable change in the figures is the change in the cross-sectional variance of log-consumption. In panel (e), it is easy to see that the variance declines substantially for young consumers, at the expense of an increased variance for later stages of life. Since some young consumers insure themselves better in the economy with borrowing, the aggregate debt for the young on average increases (panel (c)). The average consumption profile becomes flatter with borrowing (panel (a)).

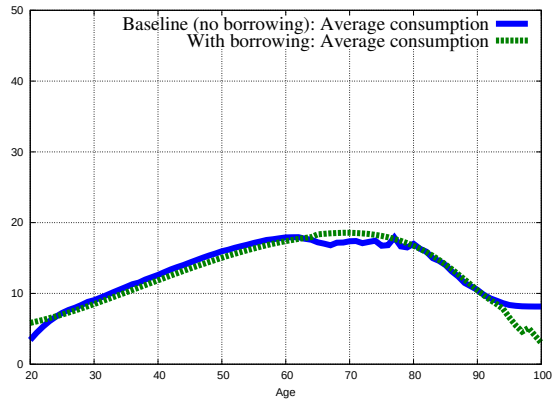
The second panel in Table 2 summarizes the results for the baseline hyperbolic discounting model ($\beta = 0.70$), but with the borrowing constraints obtained from the exponential discounting model. You can see in the column $\underline{a}$ that the borrowing constraints used in the second panel are the same as those used in the first panel. Most changes are quite similar between the first and the second panel. But there is one important difference: the response of the aggregate debt is substantially stronger in the hyperbolic discounting model. When the borrowing constraint is relaxed such that the debt increased by 2% of GDP in the exponential discounting model, the aggregate debt over GDP increased by 2.3% in the hyperbolic discounting model. When the debt over GDP ratio increased to 7% in the exponential discounting model, the same borrowing constraint induces an 8.2% debt over GDP ratio in the hyperbolic discounting model.

In the third panel, I implement the same procedure as in the first panel for the baseline hyperbolic discounting model ($\beta = 0.70$). Since the response of aggregate debt to a relaxed borrowing constraint is stronger in the hyperbolic discounting model, the borrowing constraints that induce a 2% and 7% debt to GDP ratio are expected to be tighter than in the exponential discounting model. By applying the same procedure to the hyperbolic discounting model, I am investigating the difference in the macroeconomic implications of an increased indebtedness depending on the model used for the analysis. As expected, the borrowing constraints for the 1980 and 2000 economies are tighter than for the exponential discounting model, 9% and 29% of average income, respectively. Naturally, the responses of all the macroeconomic aggregates other than the debt to GDP ratio (which is controlled) are weaker in the third panel. The right panels in Figure 5 exhibit the life-cycle profile of hyperbolic discounting models in 1970 (no debt) and 2000 (7% debt over GDP). The changes by allowing debt are similar to those in the left panels, which are associated with exponential discounting models.

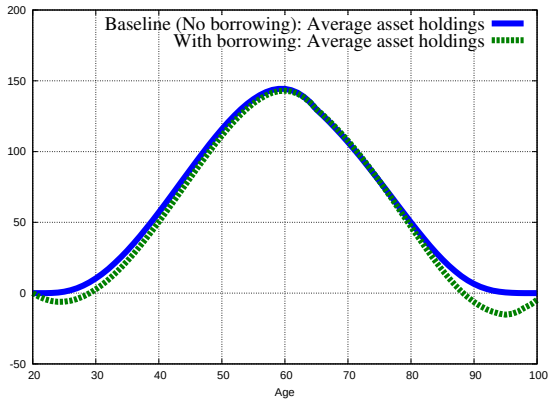
The last panel in Table 2 summarizes the macroeconomic implications of an increased indebtedness for the economy with strong hyperbolic discounting ($\beta = 0.56$). As in the case for the baseline hyperbolic discounting model ($\beta = 0.70$), the borrowing constraints are calibrated so that the economy generates a 2% and 7% aggregate debt to GDP ratio in 1980 and 2000 economies, respectively. As the second column shows, the borrowing constraints have to be even tighter than in the baseline hyperbolic discounting model because of the stronger response of debt to a relaxation of the borrowing constraint. As we saw in the baseline hyperbolic discounting model,



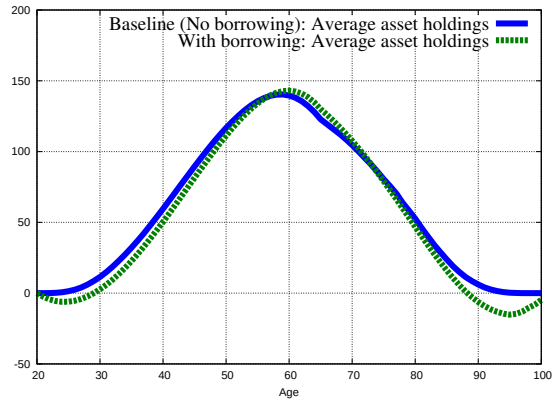

(a) Exponential: consumption



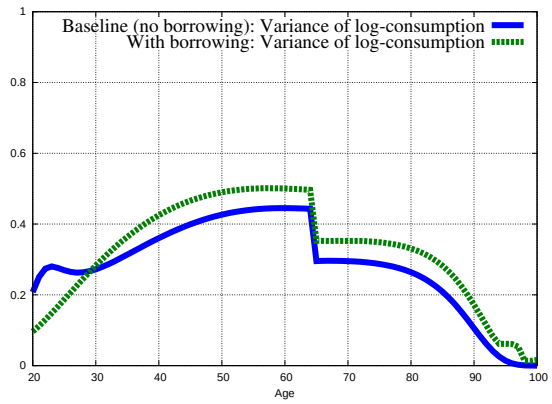

(b) Hyperbolic: consumption



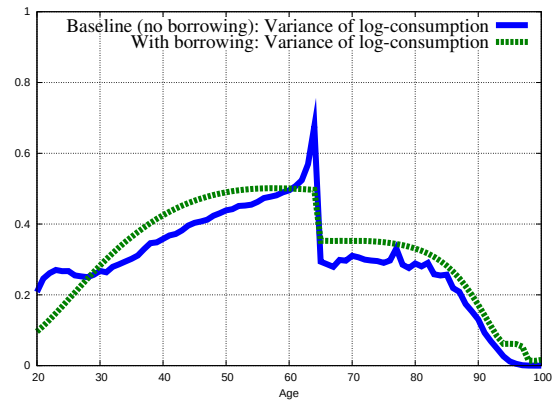

(c) Exponential: asset holdings




(d) Hyperbolic: asset holdings




(e) Exponential: cross-sectional variance of log-consumption




(f) Hyperbolic: cross-sectional variance of log-consumption

**Figure 5: Comparison of the models with and without borrowing**

**Table 3: Macroeconomic effect of rising debt: partial and general equilibrium effects**

| Economy[1] | GE[2] | $\underline{a}$[3] | D/Y | K[4] | Y[4] | r% | wage | Var(c)[5] |
|---|---|---|---|---|---|---|---|---|
| **Exponential discounting model ($\beta = 1.00$)** | | | | | | | | |
| 1970 | – | 0.000 | 0.000 | 1.000 | 1.000 | 6.01 | 1.000 | 0.481 |
| 1980 | PE | −0.125 | −0.020 | 0.973 | 0.990 | 6.01 | 1.000 | 0.461 |
| 1980 | GE | −0.125 | −0.020 | 0.986 | 0.995 | 6.11 | 0.995 | 0.465 |
| 2000 | PE | −0.337 | −0.070 | 0.931 | 0.975 | 6.11 | 0.995 | 0.446 |
| 2000 | GE | −0.337 | −0.070 | 0.959 | 0.985 | 6.33 | 0.985 | 0.455 |
| **Hyperbolic discounting model ($\beta = 0.70$)** | | | | | | | | |
| 1970 | – | 0.000 | 0.000 | 1.000 | 1.000 | 5.99 | 1.000 | 0.489 |
| 1980 | PE | −0.090 | −0.020 | 0.980 | 0.993 | 5.99 | 1.000 | 0.472 |
| 1980 | GE | −0.090 | −0.020 | 0.988 | 0.996 | 6.09 | 0.996 | 0.475 |
| 2000 | PE | −0.290 | −0.070 | 0.938 | 0.977 | 6.09 | 0.996 | 0.460 |
| 2000 | GE | −0.290 | −0.070 | 0.964 | 0.987 | 6.29 | 0.987 | 0.467 |
| **Hyperbolic discounting model ($\beta = 0.56$)** | | | | | | | | |
| 1970 | – | 0.000 | 0.000 | 1.000 | 1.000 | 5.95 | 1.002 | 0.504 |
| 1980 | PE | −0.080 | −0.020 | 0.972 | 0.990 | 5.95 | 1.002 | 0.488 |
| 1980 | GE | −0.080 | −0.020 | 0.988 | 0.996 | 6.04 | 0.998 | 0.492 |
| 2000 | PE | −0.271 | −0.070 | 0.946 | 0.980 | 6.04 | 0.998 | 0.479 |
| 2000 | GE | −0.271 | −0.070 | 0.965 | 0.987 | 6.22 | 0.990 | 0.484 |

[1] 1970: Economy with no borrowing. 1980: economy calibrated to $D/Y = 2\%$. 2000: Economy calibrated to $D/Y = 7\%$.

[2] GE: general equilibrium. PE: partial equilibrium. For the 1980 economy, prices are fixed at the 1970 level. For the 2000 economy, prices are fixed at the 1980 level.

[3] Borrowing limit relative to total income.

[4] Level in the 1970 economy normalized to one.

[5] Average of cross-sectional variances of consumption for all age groups.

the size of the response of macroeconomic aggregates is even weaker than in the hyperbolic discounting model with a higher (lower) discount factor (rate). For example, the cross-sectional log-consumption variance declines by only 2 percentage points, while the consumption variance drops by 2.6 percentage points and 2.2 percentage points in the economies with exponential discounting and baseline hyperbolic discounting, respectively.

What is the role of general equilibrium in shaping the macroeconomic implications discussed above? Table 3 shows the decomposition between the partial and the general equilibrium effect for both the exponential and the hyperbolic discounting models. In Table 3, rows associated with *GE* (General Equilibrium) are the same as in Table 2. Rows associated with *PE* (Partial Equilibrium) show the macroeconomic effects *without* the general equilibrium effect. For the 1980

economy, prices are fixed at the level of the 1970 economy, and the borrowing limit is relaxed to the 1980 level. For the 2000 economy, prices are fixed at the 1980 level, and the borrowing limit is relaxed to the 2000 level. For all economies, the general equilibrium effect is clear: without the general equilibrium effect, macroeconomic responses are quantitatively stronger. In other words, the general equilibrium effect partly offsets the responses. Without the general equilibrium effect, both capital stock and output decrease even more, and the log-consumption inequality declines to a larger extent, too. There is no difference between the exponential and hyperbolic discounting models in terms of the role of the general equilibrium effect.

## 6.4 Rising Indebtedness: Welfare Implications

In this section, I will investigate the welfare implications of increased indebtedness. Before starting the analysis, two issues related to the welfare analysis in the current environment need to be addressed. First, since the model used here features a heterogeneous agent model with life-cycle and uninsured idiosyncratic shocks, there is no obvious way to define *social welfare*. I investigate social welfare in two ways. First, I use the ex-ante expected life-time utility in the stationary equilibrium as social welfare. The virtue of this welfare criterion is that this naturally takes into account both the welfare gain or loss from changes in aggregate consumption (efficiency effect) and the welfare gain or loss due to changes in the degree of insurance (insurance effect). For this reason, the social welfare function is used to investigate the optimal capital income taxation by Conesa et al. (2009). Because of the heterogeneity, it is also important to look at the heterogeneity of the welfare effect for different types of consumers. To that end, I also investigate the expected life-time utility in the stationary equilibrium for consumers with different initial productivity $p$. Since the productivity shock is highly persistent, looking at the welfare implications for consumers with different initial $p$ roughly corresponds to studying the heterogeneous effects on consumers with different productivity potentials.

Second, when the hyperbolic discounting model is interpreted as the dynamic game between the current and future selves, welfare of which self should be used? In particular, when the ex-ante expected welfare is evaluated before a consumer is born, does the consumer discount utility in period 2 by $\beta\delta$, as the consumer does in period 1, or just $\delta$, as the consumer is considered to do *before* period 1? The two options for the social welfare function can be defined as follows:

$$\mathbb{E}V = \sum_p d_p^0 \, V(1,p,0) \tag{18}$$

$$\mathbb{E}\widetilde{V} = \sum_p d_p^0 \, \widetilde{V}(1,p,0) \tag{19}$$

$\mathbb{E}V$ is defined in the updating equation (6) and corresponds to the case where the second period utility is discounted only by $\delta$, while $\mathbb{E}\widetilde{V}$ is defined in the Bellman equation (3) and corresponds to the case where utility in period 2 is discounted by $\beta\delta$. I use $\mathbb{E}V$ as the social welfare function for the following reasons. First, Krusell et al. (2009) show that the hyperbolic discounting model can be interpreted as the extreme case of the model with preference which exhibits temptation

**Table 4: Welfare implications of rising debt**

| | | Welfare gain[3] | | | |
|---|---|---|---|---|---|
| | | | Initial productivity | | |
| Economy[1] | GE[2] | $\mathbb{E}V$ | Low | Mid | High |
| **Exponential discounting model ($\beta = 1.00$)** | | | | | |
| 1970-1980 | PE | +1.60 | +4.53 | +1.40 | +0.02 |
| 1970-1980 | GE | +1.22 | +3.96 | +1.03 | +0.08 |
| 1980-2000 | PE | +1.46 | +4.71 | +1.26 | +0.04 |
| 1980-2000 | GE | +0.60 | +3.34 | +0.41 | +0.16 |
| **Hyperbolic discounting model ($\beta = 0.70$)** | | | | | |
| 1970-1980 | PE | +0.72 | +2.18 | +0.60 | −0.61 |
| 1970-1980 | GE | +0.46 | +1.69 | +0.35 | +0.12 |
| 1980-2000 | PE | +0.25 | +2.06 | +0.17 | −0.32 |
| 1980-2000 | GE | −0.20 | +1.12 | −0.28 | +0.60 |
| **Hyperbolic discounting model ($\beta = 0.56$)** | | | | | |
| 1970-1980 | PE | +0.24 | +1.09 | +0.11 | +1.76 |
| 1970-1980 | GE | +0.10 | +0.58 | +0.02 | +4.86 |
| 1980-2000 | PE | −0.17 | +0.82 | −0.22 | −3.34 |
| 1980-2000 | GE | −0.36 | −0.05 | −0.41 | +0.20 |

[1] 1970: Economy with no borrowing. 1980: economy calibrated to $D/Y = 2\%$. 2000: Economy calibrated to $D/Y = 7\%$.

[2] GE: general equilibrium. PE: partial equilibrium. For the 1980 economy, prices are fixed at the 1970 level. For the 2000 economy, prices are fixed at the 1980 level.

[3] Welfare gain measured by the percentage increase in flow consumption associated with relaxing the borrowing limit.

and self-control. When this interpretation is employed, the utility of an individual is naturally defined as $\mathbb{E}V$.[13] Second, the important consideration in the current paper is the welfare loss due to the relaxed borrowing constraint and over-consumption. The notion of welfare loss from excess consumption naturally supports the use of the *ex-post* utility function. Malin (2008) uses the same welfare criteria in a three period model to study the optimality of savings floors when preference exhibits a present-bias.

Table 4 summarizes the welfare implications of increased indebtedness in both the exponential and the hyperbolic discounting models. Figure 6 and Figure 7 show the welfare gain of moving from the 1980 economy to the 2000 economy for consumers with different initial productivity $p$. Figure 6 compares the heterogeneous welfare effect with and without a general equilibrium effect in the model with exponential discounting consumers. Figure 7 compares the heterogeneous welfare

[13] See Krusell et al. (2009) and Nakajima (2009) for more detailed discussions.

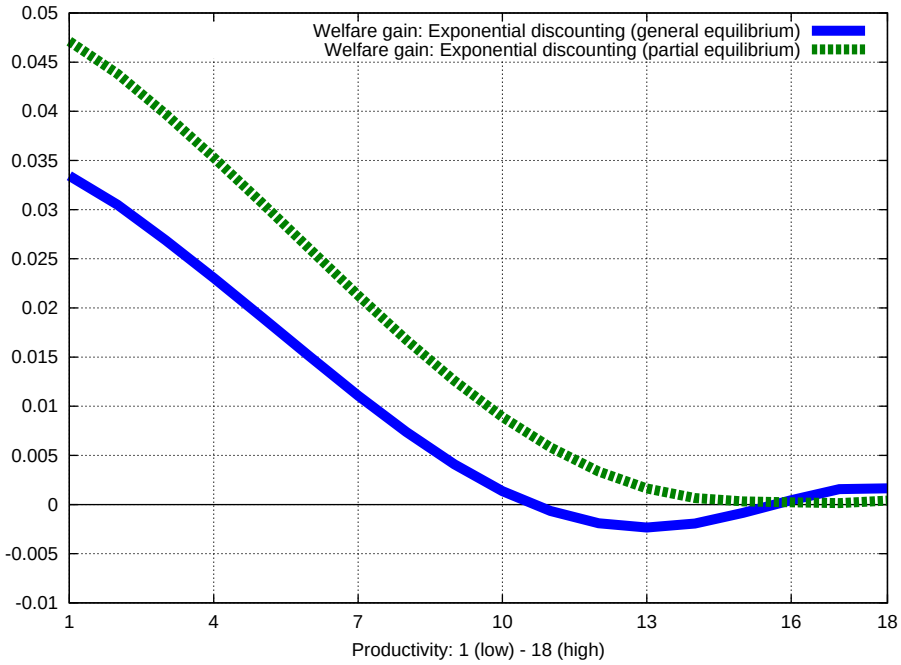



**Figure 6: Heterogeneity of welfare gain: Partial and general equilibrium**

effect in models with exponential and hyperbolic discounting consumers, taking into account the general equilibrium effect.

First, let us focus on the model with exponential discounting consumers (the top panel of Table 4 and Figure 6). In the exponential discounting model, a relaxed borrowing constraint yields a welfare gain in terms of ex-ante expected life-time utility of a consumer. By moving from the 1970 economy (no borrowing) to the 1980 economy ($D/Y = 2\%$), there is a gain in social welfare equivalent to as large as 1.2% of flow consumption. Moving from the 1980 economy to the 2000 economy ($D/Y = 7\%$) is associated with a social welfare gain of 0.6%. In both transitions, the general equilibrium effect offsets some of the gain, through lower output associated with lower capital stock. With respect to the welfare effect on consumers with different productivity potentials, three groups with different initial productivity are affected very differently. First, those with initial low productivity benefit most from the relaxed borrowing constraint. This is because the likelihood that they are constrained by the borrowing limit is highest for this group of consumers. However, they experience a welfare loss from the general equilibrium effect. In Figure 6, the line representing the welfare effect, which takes the general equilibrium effect into account, is located below the line representing the welfare effect without the general equilibrium effect, for consumers with low initial productivity. The reason is a lower equilibrium wage and a higher equilibrium interest rate, caused by a lower capital stock. Since the consumers in the group tend to borrow more often, and the main source of their income is labor income, both price effects hit consumers negatively. Second, the group with high initial productivity does not gain much from the partial equilibrium effect. For those with the highest initial productivity, the welfare gain without the general equilibrium effect is a mere 0.04% of flow consumption. Since it is not likely that they are constrained by

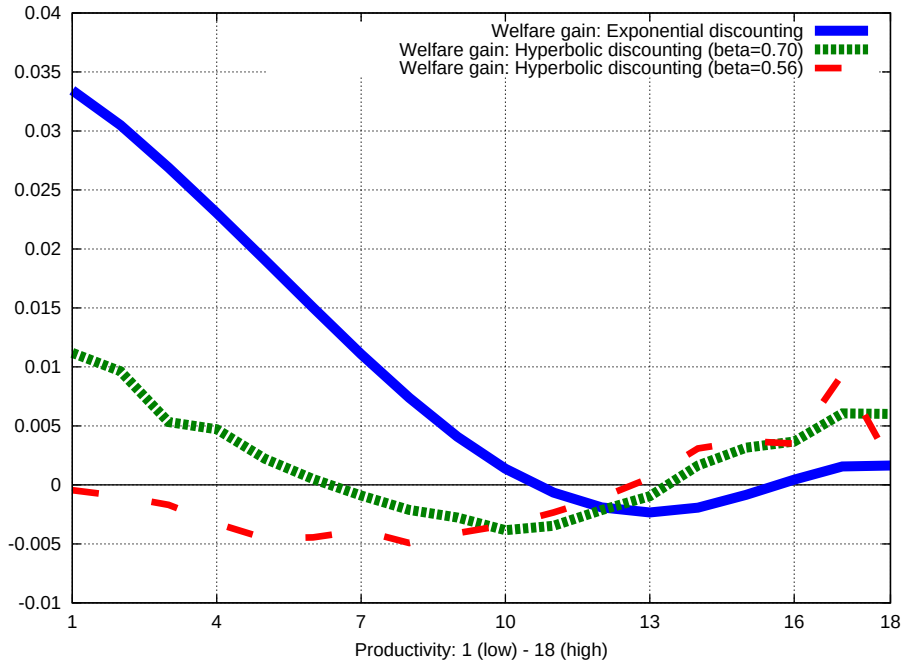



**Figure 7: Heterogeneity of welfare gain: Exponential and hyperbolic discounting**

the borrowing limit, they do not gain much from a relaxed borrowing constraint. However, they gain from the general equilibrium effect, albeit to a small extent. This is because they most likely remain savers throughout their lives, and they benefit from a high interest rate, although part of the gain is offset by a lower wage. This contrasting general equilibrium effect for high and low productivity consumers is exactly what Campbell and Hercowitz (2009) emphasize in a different but closely related environment. In the model by Campbell and Hercowitz (2009) with secured credit, high discount rate consumers who remain borrowers lose from the general equilibrium effect, and low discount rate consumers gain from the general equilibrium effect. In their setup, the general equilibrium effect is strong enough to incur welfare loss for consumers with low initial productivity (in their case, high discount rate consumers), but the effect is not strong enough to overturn the welfare gain from the partial equilibrium effect here. Finally, interestingly, consumers with moderate initial productivity suffer from the relaxed borrowing constraint. This is due to the combination of the weak welfare gain from the relaxed borrowing limit and the stronger negative welfare loss from a lower wage and a higher interest rate. As a result, the solid line in Figure 6, which represents the welfare effect for heterogeneous consumers, exhibits a U-shape. This is the same property that Obiols-Homs (2009) found in a slightly different environment.

How about the welfare implications of an increased indebtedness for the economy with hyperbolic discounting consumers? The middle and bottom panels of Table 4 summarize the welfare effects in hyperbolic discounting models with varying degrees of the strength of hyperbolic discounting. Figure 7 compares the heterogeneity of welfare effects across consumers with different initial productivity in the models with exponential and hyperbolic discounting. Regarding social welfare defined as the ex-ante expected life-time utility, although the welfare effect is small but

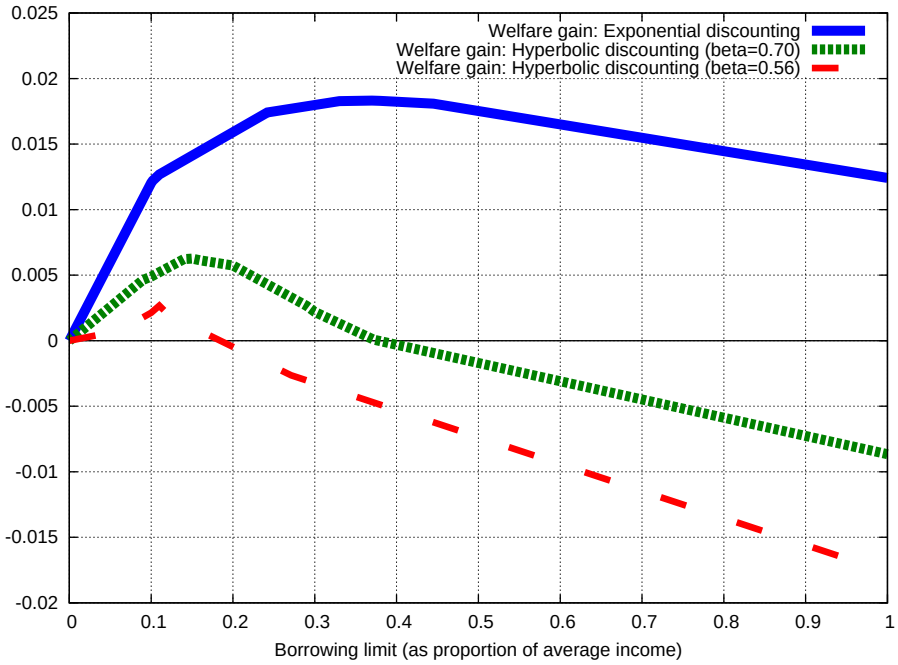



**Figure 8: Level of borrowing limit and social welfare**

positive (0.5% of flow consumption), if the 1970 economy and the 1980 economy with the baseline ($\beta = 0.70$) hyperbolic discounting are compared, there is a social welfare loss of 0.2% of flow consumption between the 1980 and 2000 economies. In the case of stronger ($\beta = 0.56$) hyperbolic discounting, the welfare loss between the 1980 and 2000 economies is as large as 0.4% of flow consumption. Cross-sectionally, although the U-shape is still observed in Figure 7, there are significant differences. The difference is especially striking for consumers with low initial productivity. Their gain from having a relaxed borrowing limit is significantly smaller in the case of the baseline hyperbolic discounting and negative in the case of strong hyperbolic discounting. The key reason is the negative welfare effect of over-borrowing. Those who are close to the borrowing constraint benefit from having a less strict borrowing constraint, which facilitates consumption smoothing across time and states, but suffer from borrowing more than is optimal in retrospect.

The discussion in this section implies that the *optimal* level of the borrowing constraint differs, potentially substantially, across models with different preference assumptions. Here I define *optimal* as the level of the uniform borrowing limit that is associated with the highest social welfare defined as the ex-ante expected life-time utility. The way optimality is defined here is closely related to the way the optimal capital income tax rate is defined in Conesa et al. (2009). Figure 8 exhibits social welfare under different levels of the borrowing constraint in the models with exponential and hyperbolic discounting. Three things are worth pointing out. First, the solid line, which is for the model with exponential discounting consumers, is located above the other lines, which are associated with the hyperbolic discounting models, implies that the welfare gain is always higher in the exponential discounting model, conditional on the same level of borrowing constraint. Second, all lines are hump shaped, because the equilibrium effect from a lower capital

stock dominates at some point for all economies. Third, the optimal level of the borrowing limit is decreasing in the strength of the hyperbolic discounting. This is because hyperbolic discounting preference implies smaller (or negative) welfare gain from relaxed borrowing constraint for low productivity consumers.

For the exponential discounting model, the level of the uniform borrowing limit that maximizes social welfare is 37% of average income. Interestingly, this level turns out to be very close to the level in the 2000 economy (34%), which generates 7% debt-to-output ratio in the steady state. In other words, the model with exponential discounting consumers implies that the 2000 economy is close to the optimal level in terms of the strength of the borrowing constraint. On the other hand, in the baseline ($\beta = 0.70$) hyperbolic discounting model, the optimal level is 15% of average income. This optimal level is substantially lower than in the hyperbolic discounting model, because of the welfare loss from over-borrowing, and the stronger general equilibrium effect. Not only that, the optimal level is substantially lower than 29%, which is the borrowing limit of the 2000 economy. In other words, according to the baseline hyperbolic discounting model, the 2000 U.S. economy features an excessively relaxed borrowing constraint. In the case of stronger hyperbolic discounting ($\beta = 0.56$), the optimal borrowing limit declines further to 11% of the average income.

Just as commitment by using an illiquid asset is valued in Laibson (1997) and forced saving might be welfare-improving in Malin (2008), the existence of a tight borrowing constraint prevents consumers from over-consuming and thus potentially has a welfare-improving role. If the borrowing constraint is tightened to the optimal level of 15% in the baseline hyperbolic discounting model, there is a sizable welfare gain (about 0.4% of flow consumption).

# 7 Conclusion

In this paper, I investigate the macroeconomic and welfare implications of rising indebtedness in the U.S. using the model with hyperbolic discounting consumers. There are five main findings. First, when calibrated to the same set of targets, models with exponential and hyperbolic discounting exhibit similar life-cycle profiles. Even if the short-term discount factor of the hyperbolic discounting models induce consumers to borrow or dissave more and consume more in the current period than in the exponential discounting model, the long-term discount rate must be calibrated much higher than the exponential discounting model. Second, there are some differences cross-sectionally; basically, hyperbolic discounting models expand the wealth distribution and push more consumers to borrowing or zero saving. Third, not only are the two models observationally similar in terms of aggregates, the two models have similar predictions in terms of macroeconomic changes in response to a relaxed borrowing constraint. Fourth, although the macroeconomic implications of the relaxed borrowing limit are similar between the two models, the welfare implications are very different; Hyperbolic discounting models imply significantly lower or even negative welfare effects associated with rising indebtedness. In particular, I find that when debt increases to the same extent as in the period between 1980 and 2000, there is a negative effect on social welfare as large as 0.2% of flow consumption in the economies populated with hyperbolic discounting

consumers. The difference in the welfare implications is very striking between the hyperbolic discounting model and the standard exponential discounting model, since the standard model with exponential discounting consumers implies a welfare gain (0.4% of flow consumption). Finally, the optimal borrowing limit is substantially lower in the hyperbolic discounting model. One could interpret that the restriction on borrowing could be welfare-improving according to the hyperbolic discounting model.

There are two ways to interpret the set of findings in the current paper. First, even though the models with exponential and hyperbolic discounting are observationally similar in many dimensions, they have very different welfare implications. Therefore, from the welfare perspective, even though there is little need to use the non-standard hyperbolic preference to study macroeconomic implications, it is important to find other and better ways to distinguish between the two models. Second, it might be possible to use the survey evidence to distinguish between the two models. If we know how much consumers suffer from over-borrowing and over-consuming, we could map the results into the strength of hyperbolic discounting, in particular, the level of the short-term discount factor.

One interesting and important extension from the current paper is associated with consumer bankruptcy. The increase in consumer debt has been accompanied by a substantial increase in consumer bankruptcy. Recently, the consumer bankruptcy law was modified to make bankruptcy more costly and not available to some consumers in order to discourage abuse of the law.[14] The standard equilibrium models of consumer bankruptcy[15] imply that a tougher bankruptcy law might benefit consumers by allowing stronger commitment to repay. But it is not clear if the intuition holds when consumers could suffer from *over-consuming*, as some argue. Nakajima (2009) investigates this issue.

[14] See White (2007) for details.
[15] See Livshits et al. (2007b) and Chatterjee et al. (2007).

# Appendix A

## A.1 Calibration Appendix

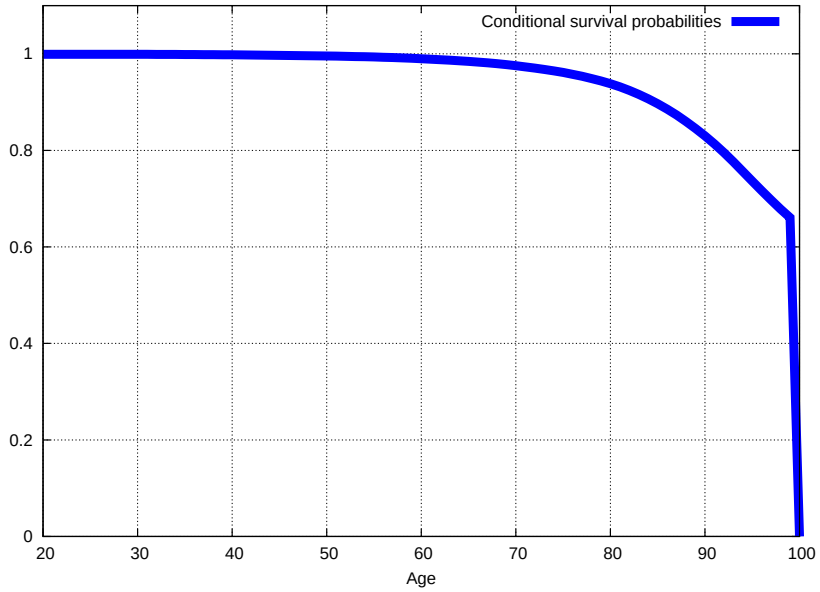



**Figure 9: Conditional survival probabilities**

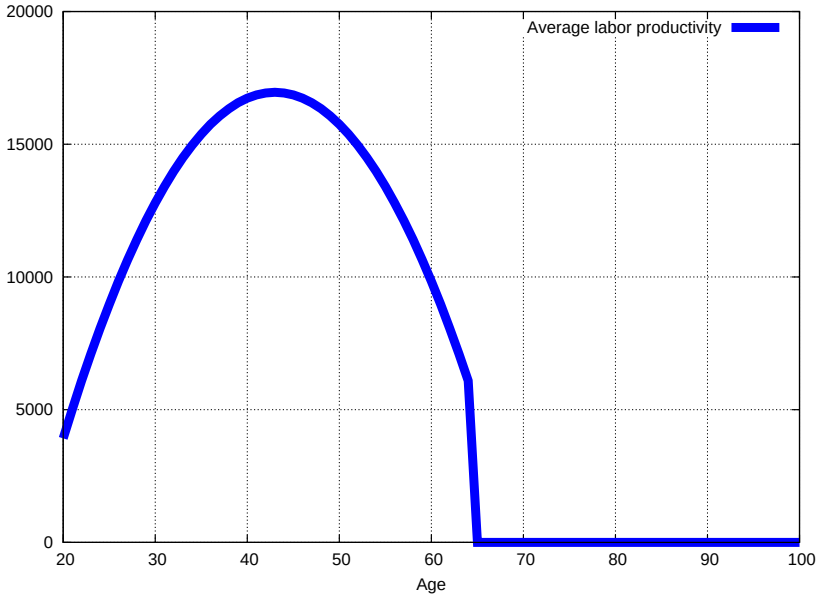



**Figure 10: Average life-cycle profile of labor productivity**

## A.2 Computational Appendix

I will describe below the computational algorithm to solve the stationary equilibrium of the model. I will focus on the model with hyperbolic discounting consumers. The solution method for the exponential discounting model is straightforward.

**Algorithm 1** (Computation algorithm for solving stationary equilibrium)**.**

1. *Set the initial guess of the aggregate capital $K^0$ and per-consumer transfer $tr^0$. Notice that since there is no labor-leisure decision, aggregate labor supply $L^*$ can be computed independently. Similarly, the Social Security benefit $\overline{b}$ can be computed independently from the solution algorithm.*

2. *Given $K^0$ and $L^*$, compute the interest rate $r$ and the wage $w$. The transfer used in the iteration is equal to the guess, i.e., $tr = tr^0$.*

3. *Given $\{tr, r, w\}$, solve the consumer's optimization problem using backward induction.*

   (a) *Set $V(I+1, p, a) = 0$ for all $p$ and $a$.*

   (b) *Solve the problem of the consumer of age $I$, using the Bellman equation (3) for all $p$ and $a$. Future value function is interpolated using piece-wise linear functions. Optimal savings level is found using golden section search. Notice that the Bellman equation is NOT used for updating the value function. This step yields the optimal decision rule $\widetilde{g}_a(I, p, a)$.*

   (c) *Use the optimal decision rule $\widetilde{g}_a(I, p, a)$ just obtained and (6) to update the value and obtain $V(I-1, p, a)$ $\forall p, a$.*

   (d) *With $V(I-1, p, a)$ at hand, we can solve the problem of age $I-1$ consumers. Keep going back in the same way until the value function and the optimal decision rule for age 1 (initial age) consumers are obtained.*

4. *Using the obtained optimal decision rule $\widetilde{g}_a(i, p, a)$, simulate the model.*

   (a) *Set the type distribution for the newborns. In particular, all newborns have $i = 1$ and $a = 0$. The distribution of $p$ is subject to $\{\pi_p^0\}$.*

   (b) *Update the type distribution using the stochastic process for $p$ and the optimal decision rule $\widetilde{g}_a(I, p, a)$. The optimal decision rule is interpolated using piece-wise linear approximation.*

   (c) *Keep updating until age $I$ (last age).*

5. *Compute the aggregate capital stock $K^1$ and total amount of accidental bequests implied by the simulated distribution. Notice that agents survive according to the survival probability, and there is population growth, which makes size of the younger population larger. Make these adjustments when computing the aggregate capital stock and accidental bequests. Specifically,*

*when the measure of age* 1 *consumers is normalized to one, measures of age* $i$ *consumers,* $\widetilde{\mu}_i$ *can be represented as follows:*

$$\widetilde{\mu}_i = \frac{1}{(1+\pi)^{i-1}} \prod_{j=0}^{i-1} s_j \tag{20}$$

*where* $s_0 = 1$. *Once the aggregate amount of accidental bequests is computed, we can compute the per-consumer lump-sum transfer* $tr^1$.

6. *Compare* $\{K^0, tr^0\}$ *and* $\{K^1, tr^1\}$. *If they are sufficiently close, we can assume that* $\{K^0, tr^0\}$ *was the consistent guess and stop. Otherwise, update* $\{K^0, tr^0\}$ *and go back to step 2.*

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