The Dynamics of Sovereign Debt Crises and Bailouts

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VERY PRELMINARY
COMMENTS WELCOME

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Abstract

Inspired by the European debt crisis of 2010, this paper provides a theoretical framework to analyze the dynamics of the sovereign debt

*This is preliminary indeed. The results have not yet been checked with sufficient care, despite multiple presentations of earlier drafts. We are thankful to our discussants so far. We are particularly thankful for an insightful discussion by Fernando Broner at a CREI conference in December 2011, which led to some important changes regarding the bailout analysis in the paper. Address: Francisco Roch, Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, IL 60637, U.S.A, email: froch@uchicago.edu. Harald Uhlig, Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, IL 60637, U.S.A, email: huhlig@uchicago.edu. The research of Harald Uhlig has been supported by the NSF grant SES-0922550. Harald Uhlig has an ongoing consulting relationship with a Federal Reserve Bank, the Bundesbank and the ECB.

crisis of a member country in a monetary union and the role of various bailout mechanisms. To do so, we draw on three sets of literatures. First, Arellano (2008) has shown how "bad luck" can lead to a sovereign debt crisis. Second, Cole-Kehoe (1996,2000) have shown how multiplicity of equilibria, when debt enters a "crisis zone" can lead to dramatic changes in debt pricing. Finally, the impatience of policy makers as in Beetsma and Uhlig (1999) provides a reason why a country would be in a crisis zone in the first place. We analyze the unfolding and the debt dynamics, if debt pricing is left to markets alone. Next, we discuss the dynamics, if there is intervention by some bail-out mechanism. We characterize the minimal actuarily fair intervention that restores the "good" equilibrium of Cole-Kehoe, relying on the market to provide residual financing.

Keywords:

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1 Introduction

In 2010, doubts spread on financial markets that Greece will be able to repay its sovereign debt. The prospect of a Greek sovereign default within the Euro zone led to fears of a contagion to other Euro zone member countries, notably Ireland, Portugal and perhaps Spain. Furthermore, a sovereign default within the Euro zone was judged to possibly endanger the European monetary system, its common currency as well as disrupt payment systems within Europe. Due to these concerns, the finance ministers of Europe approved a rescue package for Greece and created the European Financial Stability Facility (EFSF) in May 2010, in order to prevent a default as well as to return yield spreads to pre-crisis levels. The spreads, however, have remained persistently high and markets appear to judge the prospect of a default and/or an exit from the Euro zone as probable: events may meanwhile have surpassed this description. A survey of the issues and empirics of the situation can be found in Lejour et al (2010) and Arghyrou and Kontonikas (2011).

This paper is motivated by these developments and seeks to understand the dynamics of sovereign debt crises in a union of countries. There are too many pieces here to combine in one single paper. Instead, we shall provide a rather narrow focus. We proceed by first shedding light on sovereign default crises, when combining three key elements of the existing literature. Next, we discuss the dynamics, if there is intervention by some bail-out mechanism. We characterize the minimal actuarily fair intervention that restores the "good" equilibrium of Cole-Kehoe, relying on the market to provide residual financing. "Fair value" here means that the resources provided by the bail-out fund earn the market return in expectation, and therefore does not require drawing on tax payer money on average. We believe this is an important benchmark from which to consider and study more deeply subsidized bailout mechanisms.

The analysis of the dynamics of a sovereign debt crisis builds on and moderately extends three branches of the literature in particular. First, Arellano (2008) has analyzed the dynamics of sovereign default under fluctuations in income, and shown that defaults are more likely when income is low[1]. Tirole (2002) has analyzed the need for liquidity provision in financial crises. Second, Cole and Kehoe (1996,2000) have pointed out that debt crises may be self-fulfilling: the fear of a future default may trigger a current rise in default premia on sovereign debt and thereby raise the probability of a default in the first place. Both theories imply, however, that countries would have a strong incentive to avoid default-triggering scenarios in the first place. For example, Greiner et al (2007) have calculated that current debt levels in EMU member countries are probably sustainable in principle. We therefore build on the political economy theories of the need for debt contraints in a monetary union of short-sighted fiscal policy makers as in to provide a rationale for a default-prone scenario.

Indeed, there certainly has not been a lack of analysis and warning of academic economists about the risk of a debt build-up and ensuing problems in a monetary union. Beetsma and Uhlig (1999) point out that "*it is hard to imagine the ECB standing by idly, while the debt pileup in a member country ... leands to debt downgrading or default*". Beetsma and Bovenberg (1999,2001) point out that "*monetary unification boosts the accumulation of*

[1]That may sound unsurprising, but is actually not trivial. Indeed the recursive contract literature typically implies incentive issues for contract continuation at high rather than low income states, see e.g. Ljungqvist-Sargent (2004).

public debt" and that "*international [fiscal] risk sharing may be undesirable because it weakens fiscal discipline*". Uhlig (2003) analyzes the scenario of a sovereign default in a monetary union and its repercussion for ECB policy. Cooper, Kempf and Peled (2010) warn that "*regional governments, anticipating central bank financing of their debt obligations, have an incentive to create excessively large deficits.*". This brief survey just scratches the surface: the existing literature is undoubtedly considerably larger.

The literature now moves beyond the stage of warning and instead embarks on sorting through the wreckage. For example, Fink and Scholl (2011) analyze conditionality and bailouts in a quantitative model of sovereign risk. Gennaioli, Martin and Rossi (2011) examine the implication for bank balance sheets. This paper seeks to make a contribution to this evolving literature.

2 A model of sovereign default dynamics

The model combines Arellano (2008) with Cole and Kehoe (2000), follows much of their specifications and studies the scope for minimalistic "fair value" bailouts. We assume that there is a single fiscal authority, which finances government consumption $c_t \geq 0$ with tax receipts $y_t \geq 0$ and assets $B_t \in \mathbf{R}$ (with positive values denoting debt, in reverse of the notation used in Arellano (2008)), in order to maximize its utility

$$U = \sum_{t=0}^{\infty} \beta^t (u(c_t) - \chi_t \delta_t) \tag{1}$$

where β is the discount factor of the policy maker, $u(\cdot)$ is a strictly increasing, strictly concave and twice differentiable felicity function, χ_t is an exogenous one-time utility cost of default and $\delta_t \in \{0, 1\}$ is the decision to default in period t. We shall assume that tax receipts y_t are exogenous[2], while

[2]It may be interesting to endogenize tax collection!

consumption, the level of debt and the default decisions are endogenous and chosen by the government.

In Arellano (2008) as well as Cole and Kehoe (2000), this is the utility of the representative household, y_t is total output and c_t is the consumption of the household, i.e. the fiscal authority is assumed to maximize welfare. The structure assumed here is mathematically the same, and consistent with that interpretation. It is also consistent with our preferred interpretation, where the utility function represents the preferences of the policy maker. For example, given the uncertainty of re-election, a policy maker may discount the future more steeply than would the private sector. Spending may be on groups that are particularly effective in lobbying the government. Finally, y_t should then be viewed as tax receipts, not national income.

A more subtle, though not essential difference is the cost of a default, modeled here as a one-time utility cost χ_t, while it is modelled as a fractional loss in output in Arellano (2008) with Cole and Kehoe (2000). Note, however, that $c_t = y_t$ in default, and that at least for log-preferences, $u(c_t) = \log(c_t)$, a proportional decline in consumption each period following the default can equivalently be written as a one-time loss in utility.

In each period, the government enters with some debt level B_t and the tax receipts y_t as well as some other random variables are realized. Traders on financial markets are assumed to be risk neutral and discount future repayments of debt at some return R, and price new debt B_{t+1} according to some market pricing schedule $q_{m,t}(B_{t+1})$. There may be international assistance ("bailout") in issuing new debt: we shall analyze this only from the receiving country at this point. Denote the assisted pricing schedule with $q_{a,t}(B_{t+1}) \geq 0$. Given the pricing schedule, the government then first makes a decision whether or not to default on its existing debt. If so, it will experience the one-time exogenously given default utility loss χ_t, be excluded from debt markets until re-entry, and simply consume its output, $c_t = y_t$ in this as well as all future periods, while excluded from debt markets. We assume that re-entry to the debt market happens with probability $0 \leq \alpha < 1$, drawn iid each period, and that re-entry starts with a debt level of zero. If the government does not default, it will choose consumption and the new debt level according to the budget constraint

$$c_t + (1-\theta)B_t = y_t + q_t(B_{t+1})(B_{t+1} - \theta B_t) \tag{2}$$

where[3]

$$q_t(B_{t+1}) = \max\{q_{m,t}(B_{t+1}), q_{a,t}(B_{t+1})\} \tag{3}$$

where $0 < \theta \leq 1$ is a parameter, denoting the fraction of debt that currently needs to be repaid. The parameter θ allows to study the effect of altering the maturity structure: the lower θ, the longer the maturity of government debt. The remainder of the debt θB_t will be carried forward, with the government issuing the new debt $B_{t+1} - \theta B_t$. In line with the policy of the European Financial Stability Facility, the assistance is given for the new debt only. There may be additional restrictions outside the formulation above: we shall return to their discussion in section 4.

2.1 State space representation

We shall restrict attention to the following state-space representations of the equilibrium. At the beginning of a period, the aggregate state

$$s = (B, d, z) \tag{4}$$

describes the endogenous level of debt B, the default status d and some exogenous variable $z \in Z$. We assume that z follows a Markov process and that all decisions can be described in terms of the state s. The probability

[3]The next constraint may need to depend on whether $B_{t+1} - \theta B_t$ is positive or negative

measure describing the transition for z to z' shall be denoted with $\mu(dz' \mid z)$. More specifically, we shall assume that z is given by

$$z = (y, \chi, \zeta, \psi) \tag{5}$$

We assume that $y \in [y_L, y_H]$ with $0 < y_L \leq y_H$ either has a strictly positive and continuous density $f(y \mid z_{\text{prev}})$, given the previous Markov state z_{prev} or is nonrandom. We assume likewise that $\chi \in [\chi_L, \chi_H]$ with $0 \leq \chi_L \leq \chi_H$ either has a strictly positive and continuous density $g(\chi \mid d, z_{\text{prev}})$, given the previous Markov state z_{prev} and the default state d or is nonrandom. We assume that $\zeta \in [0, 1]$ is uniformly distributed and denotes a "crisis" sunspot and where $\psi \in [0, 1]$ is uniformly distributed and denotes a "bailout" sunspot. We assume that the distributions of the four entries in z is independent of each other, given the previous state. For most parts, we shall assume that z is iid, and that therefore the distributions for y and χ also do not depend on z_{prev}. For notation, we shall use $y(s)$ to denote the entry y in the state s, etc..

If the government does not default ($\delta = 0$), the period-per-period budget constraint is

$$c + (1 - \theta)B(s) = y(s) + q(B'; s)(B' - \theta B(s)) \tag{6}$$

where B' is the new debt level chosen by the government and where $q(B'; s)$ is the pricing function for the new debt B'.

If the government defaults ($\delta = 1$), the budget constraint is

$$c = y(s) \tag{7}$$

We assume that the government will be excluded from debt markets until it is given the possibility for re-entry. We assume that re-entry to the debt market happens with probability $0 \leq \alpha < 1$, drawn iid each period, and that re-entry starts with a debt level of zero. Technically, this means that $d = 0$

in the state s will be turned to $d = 1$ in the state s' following a default, and that $d = 1$ is followed by $d = 1$ with probability $1 - \alpha$ and with $d = 0$ with probability α. There is no other role for d. The default decision of the government is endogenous and (assumed to be) a function of the state s, $\delta = \delta(s)$.

We can now provide a recursive formulation of the decision problem for the government. The value function in the default state and after the initial default utility loss is given by

$$v_D(z) = u(y(z)) + \beta(1 - \alpha)E\left[v_D(z') \mid z\right] + \alpha E\left[v_N D(s' = (0, 0, z')) \mid z\right] \quad (8)$$

Given the debt pricing schedule $q(B; s)$, the value from not defaulting is

$$\begin{aligned} v_{ND}(s) = \max_{c,B'} \{ & u(c) + \beta E\left[v(s') \mid z\right] \mid \\ & c + (1 - \theta)B(s) = y(s) + q(B'; s)(B' - \theta B(s)) \\ & s' = (B', d(s), z') \} \end{aligned}$$

The overall value function is given by

$$v(s) = \max_{\delta \in \{0,1\}} (1 - \delta)v_{ND}(s) + \delta(v_D(z(s)) - \chi(s)) \quad (9)$$

Given parameters, a law of motion for z as well as the assisted debt pricing function $q_a(B; s) \geq 0$, an **equilibrium** is defined as measurable mappings $q_m(B'; s), q(B'; s)$ in B' and s as well as $c(s), \delta(s)$ and $B'(s)$ in s, such that

1. Given the pricing function $q(B'; s)$, the government maximizes its utility with the choices $c(s), \delta(s)$ and $B'(s)$, subject to the budget constraint (6) and subject to the exclusion from financial markets for all periods, following a default.

2. The market pricing function $q_m(B'; s)$ is consistent with risk-neutral pricing of government debt.

3. The pricing function satisfies

$$q(B'; s) = \max\{q_m(B'; s), q_a(B'; s)\} \tag{10}$$

2.2 Debt pricing

This subsection of the analysis follows closely the analysis in Cole and Kehoe (2000) and Arellano (2008), adapted to the model at hand. Traders on financial markets are assumed to be risk neutral and to discount future debt repayments at some return R. This shall be generalized in section **??**. If there has been a default in the past (i.e. if $d = 1$), then traders assume that the country will also always default in the future[4]. The market price for debt following a default is therefore identical to zero,

$$q_m(B'; s) = 0, \text{ if } d(s) = 1 \tag{11}$$

Given a level of debt B and no past defaults, let

$$D(B) = \{z \mid \delta(s) = 1 \text{ for } s = (B, 0, z)\} \tag{12}$$

be the default set, and let

$$A(B) = \{z \mid \delta(s) = 0 \text{ for } s = (B, 0, z)\} \tag{13}$$

be the set of all z, such that the government will not default and instead, continue to honor its debt obligations: both are (restricted to be) a measurable set, according to our equilibrium definition. The disjoint union of $D(B)$ and $A(B)$ is the entire set Z. Define the market price for debt, in case of no current default, i.e.

$$\bar{q}_m(B'; s) = \frac{1}{R} \int_{z' \in A(B)} (1 - \theta + \theta q_m(B(s' = (B', 0, z')))) \, \mu(dz' \mid z) \tag{14}$$

[4]For that, we also need that $\chi = 0$ for all s with $d = 1$, as a slight and entirely technical modification to the iid assumption stated above

Here and below, we use the notation $B(s' = (B', 0, z'))$ to denote the new debt level $B(s')$, given the new state $s' = (B', 0, z')$. A shorter, more accurate, but perhaps more confusing notation would simply be $B((B', 0, z'))$. Due to risk neutral discounting, this is the market price of debt, if there is no default "today". Define the probability of a continuation **next period** per

$$P(B'; s) = \text{Prob}(z' \in A(B') \mid s) = E\left[1_{\delta(s')=0} \mid s\right] \tag{15}$$

If $\theta = 0$, i.e., if all debt has the maturity of one period only, then

$$\bar{q}_m(B'; s) = \frac{1}{R} P(B'; s) \tag{16}$$

As there may be international assistance in issuing new debt, define

$$\bar{q}(B'; s) = \max\{\bar{q}_m(B'; s), q_a(B'; s)\} \tag{17}$$

We need to check, whether there could be a default "today". We shall impose the following assumption.

Assumption A. 1 *Given a state s, either $q_m(B'; s) = \bar{q}_m(B'; s)$ for all B' or $q_m(B'; s) = 0$ for all B'.*

This assumption rules out equilibria, where, say, the market expects a current default, if the government tries to finance some future debt level B', but not for others[5]

We now turn to analyzing the possibility for a self-fulfilling expectation of a default. Define the value of not defaulting, if the market prices are consistent with current debt repayment,

$$\begin{aligned}\bar{v}_{ND}(s) = \max_{c,B'} \{ & u(c) + \beta E\left[v(s') \mid z\right] \mid \\ & c + (1-\theta)B(s) = y(s) + \bar{q}(B'; s)(B' - \theta B(s)) \\ & s' = (B', d(s), z')\}\end{aligned}$$

[5]Cole and Kehoe (2000) finesse this issue with more within-period detail, having the government first sell new debt at some pricing schedule, before taking the default decision.

where it should be noted that the continuation value function is as before, i.e. given by (9). Define the value of not defaulting, if the market prices are consistent with a current default, or, more generally, if the assisted market price exceeds the market price at all chosen debt levels :

$$\begin{aligned}\underline{v}_{ND}(s) = \max_{c,B'}\{&u(c) + \beta E\left[v(s') \mid z\right] \mid \\ &c + (1-\theta)B(s) = y(s) + q_a(B';s)(B' - \theta B(s)) \\ &s' = (B', d(s), z')\}\end{aligned}$$

With that, define two bounds for the current debt levels B, see also figure 2. Above the upper bound $B \geq \bar{B}(z)$, the government finds it optimal to default today, even if the market was willing to finance future debt in the absence of a default now, i.e. even if $q(B';s) = \bar{q}(B';s)$. Above the lower bound $B \geq \underline{B}(z)$, the government finds it optimal to default, if the market thinks it will do so and therefore is unwilling to finance further debt, $q_m(B';s) = 0$. I.e., let

$$\bar{B}(z) = \inf\{B \mid \bar{v}_{ND}(s = (B,1,z)) \leq v_D(z(s)) - \chi(s = (B,1,z))\} \tag{18}$$

as well as

$$\underline{B}(z) = \inf\{B \mid \underline{v}_{ND}(s = (B,0,z)) \leq v_D(z(s)) - \chi(s = (B,0,z))\} \tag{19}$$

Whether or not there will be a default at some debt level B between these bounds will be governed by the sunspot random variable ζ. As in Cole-Kehoe (2000), we shall assume that the probability of a default in this range is some exogenously given probability π.

Assumption A. 2 *For some parameter $\pi \in [0,1]$, and all s with $\underline{B}(z) \leq B(s) \leq \bar{B}(z)$, we have $q_m(B';s) = \bar{q}_m(B';s)$, if $\zeta(s) \geq \pi$ and $q_m(B';s) = 0$, if $\zeta < \pi$.*

Note that the assumption relates endogenous objects to each other.

The equilibrium will therefore look as follows (up to breaking indifference at the boundary points):

1. If $B > \bar{B}(z)$, the government will default now and not be able to sell any debt. The market price for new debt will be zero.
2. If $\underline{B}(z) \leq B \leq \bar{B}(z)$, the government will
   (a) default with probability π (more precisely, for $\zeta(z) < \pi$), and the market price for new debt will be zero,
   (b) continue with probability $1-\pi$ (more precisely, for $\zeta(z) \geq \pi$), and the market price for new debt will be $\bar{q}_m(B'; s)$.
3. If $B < \underline{B}(z)$, the government will not default, and the market price for debt will be given by $\bar{q}_m(B'; s)$.

Following Cole and Kehoe (2000), we shall use the term "crisis zone" for the maximal range for new debt, for which there might be a "sunspot" default next period, i.e. for

$$B' \in \mathcal{B} = \left[\min \underline{B}(z), \max \bar{B}(z)\right]$$

Note that safe debt will be priced at q^* satisfying

$$q^* = \frac{1}{R}(1 - \theta + \theta q^*)$$

and is therefore given by

$$q^* = \frac{1-\theta}{R-\theta} \tag{20}$$

Conversely, given some price q, one can infer the implicit equivalent safe rate

$$R(q) = \theta + \frac{1-\theta}{q} \tag{21}$$

3 No bailouts

In this section, we exclude assisted debt issuance, i.e. we assume that $q_a(B'; s) \equiv 0$. We therefore furthermore assume, that the bailout sunspot $\psi(s)$ is "irrelevant", i.e. all functions are independent of ψ: it may not be necessary to assume so, but it seems unnecessary to consider it. We finally shall assume that z is iid.

The following results are essentially in Arellano (2008) and states that default incentives increase with higher debt.

Proposition 1 *Suppose z is iid and that all functions are independent of ψ. If default is optimal for $s^{(1)} = (B^{(1)}, 0, z)$, then default is optimal for $s^{(2)} = (B^{(2)}, 0, z)$, whenever $B^{(2)} > B^{(1)}$.*

This is proposition 1 in Arellano (2008).

The next proposition states that lower tax receipts y increases default incentives.

Proposition 2 *Suppose z is iid and that all functions are independent of ψ. Default incentives are stronger, the lower are tax receipts. I.e., for all $y^{(1)} \leq y^{(2)}$, if $z^{(2)} = (y^{(2)}, \chi, \zeta, \psi) \in D(B)$, then so is $z^{(1)} = (y^{(1)}, \chi, \zeta, \psi) \in D(B)$.*

This is the non-trivial insight and proposition 3 in Arellano (2008) and follows similarly from the concavity of $u(\cdot)$. A graphical representation is in figure 1. In that figure, a pricing function $q(B'; s)$ is taken as given. We are typicallyk considering two pricing functions in particular. Due to the possibility of a sunspot, the pricing function may be $q = \bar{q}_m(B'; s)$ or $q \equiv 0$. The latter results in a larger default set in the latter case. A graphical representation is in figure 2.

By comparison to proposition 2, the next proposition is certainly more trivial and obvious, and states that less "shame" χ of defaulting results in higher incentives to default.

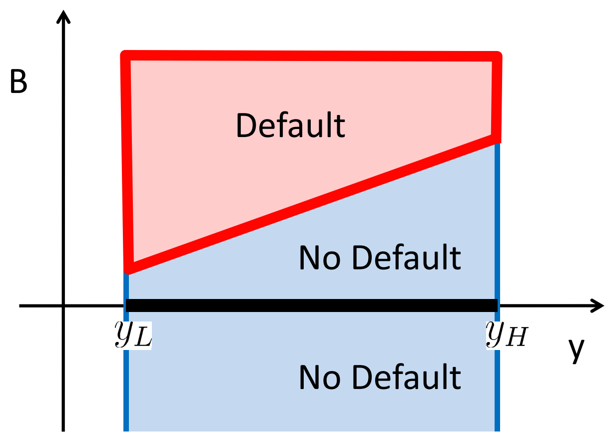


Figure 1: *Relationship between debt, income and the default decision, at a given pricing function* $q(B'; s)$

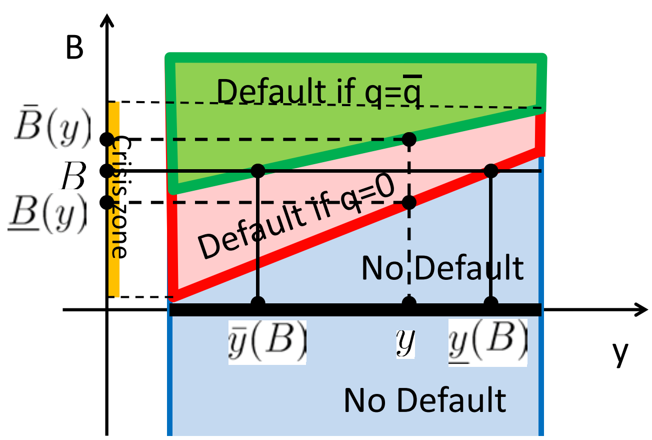


Figure 2: *Relationship between debt, income and the default decision, for the two pricing functions* $q = \bar{q}_m(B'; s)$ *and* $q \equiv 0$

Proposition 3 *Suppose z is iid and that all functions are independent of ψ. Default incentives are stronger, the lower is the utility penalty from defaulting. I.e., for all $\chi^{(1)} \leq \chi^{(2)}$, if $z^{(2)} = (y, \chi^{(2)}, \zeta, \psi) \in D(B)$, then so is $z^{(1)} = (y, \chi^{(1)}, \zeta, \psi) \in D(B)$.*

With these results, we can derive the dependence of the pricing function on the debt level.

Proposition 4 *Suppose that $q_a(B'; s) \equiv 0$, i.e. no bailouts. Then $q(B'; s)$ is decreasing in the debt level B'. If y and/or χ is random with a strictly positive and continuous density, then $q(B'; s)$ is continuous in B' with a nonpositive derivative in B', except for finitely many points.*

Proof: *To be completed. Note, that changes in B' "smoothly" move into the default areas, when y and/or χ is random with a strictly positive and continuous density.* •

A graphical representation of the pricing function $q = \bar{q}_m(B'; s)$ is in figure 3 for the case of $\theta = 0$, i.e. one-period bonds. If the next period debt level is below the lowest level, at which a default could possibly be expected, $B' \leq \min \underline{B}(z)$, then the debt is safe and will be discounted at R. As B' increases beyond this level, there will be some states of nature in the future, for which a default may occur: these defaults become gradually more likely with increases in B', as one can infer from figure 2. Once the debt level is so high, that a default must surely occur tomorrow, then the current price level must be zero as well. The pricing function depends on the sunspot default probability tomorrow in a subtle way, as figure 4 shows. With a zero probability of a "sunspot" default, the debt B' needs to exceed $\min \bar{B}(z)$ in order for the price $\bar{q}_m(B'; s)$ to decline. Indeed, $\bar{B}(z)$ itself depends on π and should intuitively rise, as π falls (since q is shifting upwards): this is indicated by the shift also of $\max \bar{B}(z)$ in that figure.

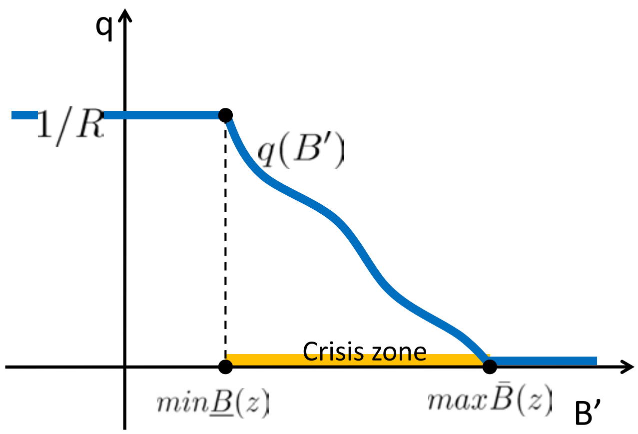


Figure 3: *The market price* $q(B') = \bar{q}_m(B'; s)$ *as a function of future debt* B'.

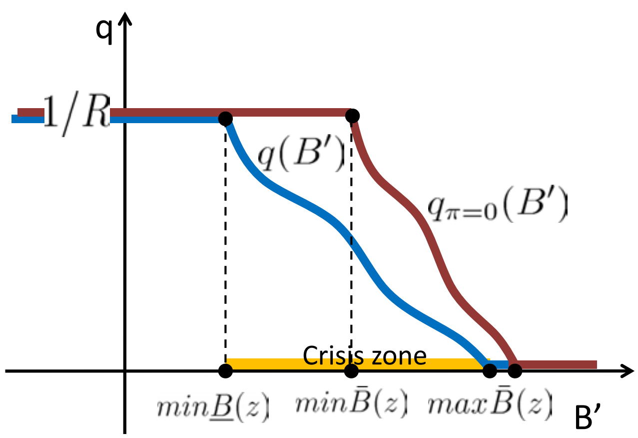


Figure 4: *The market price* $q(B') = \bar{q}_m(B'; s)$ *for nonzero "sunspot" default probability* π *as well as for* $\pi = 0$.

It is useful to analyze the first-order condition of the government, when considering its choice for the future debt level B', assuming that the debt pricing rule is "sufficiently nice". Define the level of consumption, resulting from a particular debt choice B',

$$c(B';s) = y(s) + q(B';s)(B' - \theta B(s)) - (1-\theta)B(s) \tag{22}$$

At the optimal choice, $B' = B'(s)$ and $c(B';s) = c(s)$. From there, consider marginally increasing the amount of debt B' . This yields a current utility gain

$$\left(\frac{\partial U}{\partial B'}\right)_{(I)} = u'(c(s))\left(q(B';s) + q_1(B';s)B'\right) \tag{23}$$

Per the envelope theorem for v_{ND}, i.e. conditional on a state s' of no default, the utility loss tomorrow is given by

$$\frac{\partial v_{ND}(s')}{\partial B'} = \beta u'(c(s'))(\theta - 1 - \theta q(B''(s');s')) \tag{24}$$

where we have used the hopefully intuitive notation $B''(s')$ to denote the debt choice next period, given next periods state s', instead the of the formally correct but possibly confusing notation $B'(s')$. Integrating the losses given by (24) yields

$$\begin{aligned}\left(\frac{\partial U}{\partial B'}\right)_{(II)} &= \beta\pi \int_{\{z|B'\leq \underline{B}(z)\}} u'(c(s' = (B',0,y,\chi,0,0)))(1-\theta+\theta q(B''(s');s'))\mu(dz) \\ &\quad +\beta(1-\pi)\int_{\{z|B'\leq \bar{B}(z)\}} u'(c(s' = (B',0,y,\chi,1,0)))(1-\theta+\theta q(B''(s');s'))\mu(dz) \\ &= \beta E\left[u'(c(s'))(1-\theta+\theta q(B''(s');s'))1_{\delta(s')=0}\right] \end{aligned}\tag{25}$$

where we have set $\zeta = 0$ and $\zeta = 1$ for the two crisis sunspot situations, and arbitrarily fixed $\psi = 0$.

However, the set of default states changes. To keep the analysis tractable, suppose that χ is not random but constant, while the distribution for y has a nontrivial, strictly positive and bounded density $f(y) = F'(y)$ on $[y_L, y_H]$.

With the help of proposition 2, the condition $B \leq \underline{B}(z)$ can equivalently written as $y \geq \underline{y}(B)$, while the condition $B \leq \bar{B}(z)$ can equivalently written as $y \geq \bar{y}(B)$ for some bounds $\bar{y}(B) \leq \underline{y}(B)$. Additionally, there is then the net loss in utility due to increasing the risk of default (or, technically, the differentiation with respect to the boundary of the integral),

$$\left(\frac{\partial U}{\partial B'}\right)_{(III)} + \beta(1-\pi)\left(v_{ND}(B', \bar{y}(B'), 1, 0) - v_D(\bar{y}(B'), 0, 0)\right) f(\bar{y}(B')) \frac{d\bar{y}(B')}{dB'} + \beta\pi\left(v_{ND}(B', \underline{y}(B'), \chi, 0)\chi + \chi v_D(\underline{y}(B'), \chi, 0)\right) f(\underline{y}(B')) \frac{d\underline{y}(B')}{dB'}$$

Note now, though, that the boundaries are defined by the condition that the expression in brackets equals zero, unless we are at the boundary of the interval $[y_L, y_H]$ and therefore the derivative of $\underline{y}(B')$ or of $\bar{y}(B')$ with respect to B' is zero.

The argument regarding this third part generalizes, in case χ is random too. we note this result as follows.

Proposition 5 *If the condition for optimality can be written as a first-order condition, it is*

$$\left(\frac{\partial U}{\partial B'}\right)_{(I)} = \left(\frac{\partial U}{\partial B'}\right)_{(II)} \tag{26}$$

where the two pieces are given by (23) and (25). Put differently,

$$q(B'; s) + q_1(B'; s)B' = \beta E\left[\frac{u'(c(s'))}{u'(c(s))}(1 - \theta + \theta q(B''(s'); s'))1_{\delta(s')=0}\right] \tag{27}$$

If $\theta = 0$ *(only short-term debt), then*

$$q(B'; s) + q_1(B'; s)B' = \beta E\left[\frac{u'(c(s'))}{u'(c(s))}1_{\delta(s')=0}\right] \tag{28}$$

or

$$1 - h(B'; s)B' = \beta R E\left[\frac{u'(c(s'))}{u'(c(s)} \mid \delta(s') = 0\right] \tag{29}$$

where the hazard rate $h(B'; s)$ *is given by*

$$h(B'; s) = -\frac{\partial E\left[\delta(s') = 0\right]/\partial B'}{E\left[\delta(s') = 0\right]} \tag{30}$$

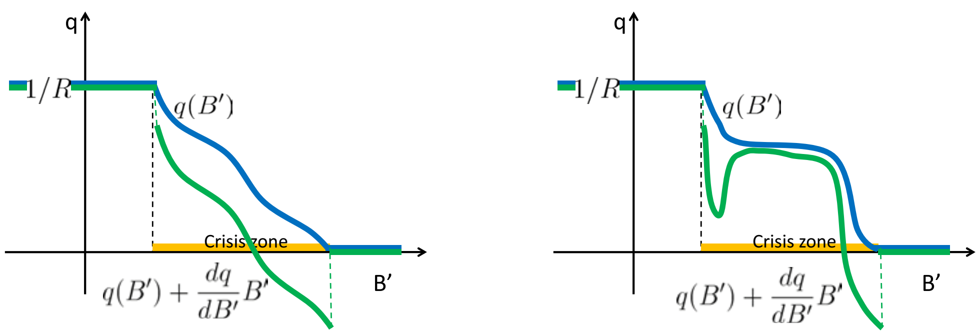
Proof: *For equation (28), note that* $q(B'; s) = E\left[\delta(s') = 0\right]/R$. •

There is an important tension here. Consider $\theta = 0$ and the first order condition (28). When increasing the debt level, the usual "consumption-versus-savings" first-order effect ought to be an increase in current consumption and a decrease in future consumption, leading to an decrease in current marginal utility and an increase in future marginal utility, resulting in some optimal level. This is offset by the decrease in resources gained per additional unit of debt on the left-hand side, due to the decrease in q_1 and the decrease in the no-default region on the right-hand side. It is not a priori clear, that there is a unique solution. Put differently, it is not a priori clear and perhaps even unlikely, that the budget set (6) is convex in the choices (c, B').

This issue and the first-order condition (28) are examined in figure 5: we shall focus entirely on the case $\theta = 0$, though this discussion can probably be generalized. The left column shows the "benign" case. In the upper left panel, the market price for new debt $q(B')$ declines at a reasonably even pace, so that the left hand side in equation (28) is monotonously decreasing, and even becomes negative, until debt reaches $\max_z \bar{B}(z)$. That left hand side is then compared to the rhs of (28) in the lower left panel. For the figure, it has been assumed that the rhs is rising in B': as discussed, even that may not be the case. The two curves intersect at a unique point. The right column shows one possible scenario, where multiple solutions to the first-order condition may emerge. Start from the upper right panel: there, $q(B')$ becomes rather flat for a portion of the new debt levels, implying a jump upwards in the left-hand side of (28). As a result, the right-hand side of equation (28) may now intersect the left-hand side of (28) multiple times, as shown in the bottom right panel.

Nonetheless, for the purpose of some discussions, it may be illuminating to proceed with examining the first-order condition, and assuming that it

The market price $q(B')$ vs the lhs of (28):



The two sides of (28):

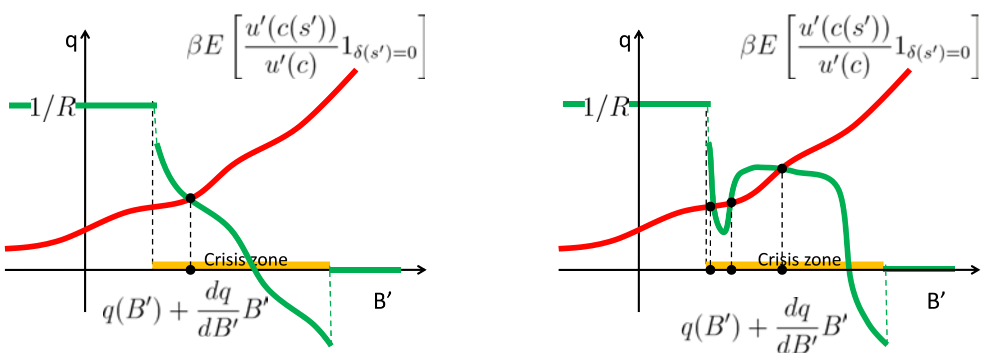


Figure 5: *Examining the first order condition (28)*

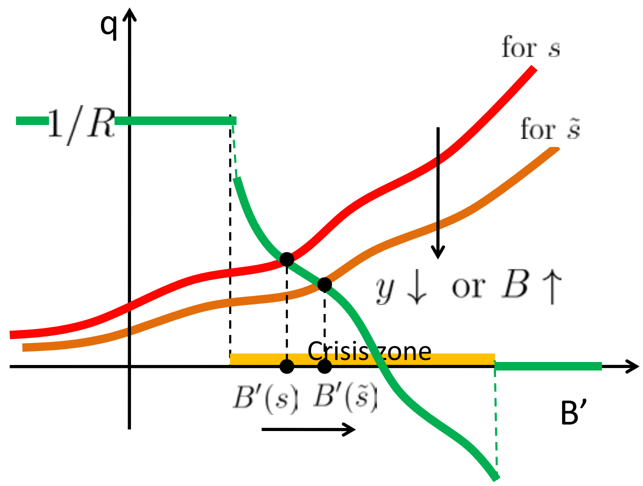


Figure 6: *The first-order condition (28) versus variations in the state s: implications for the new debt level B'.*

provides the unique solution, while keeping the caveat in mind, that this may not be right. We shall state this as an explicit assumption, in case it is necessary to make an explicit reference to it.

Assumption A. 3 *The first-order conditions given in proposition 5 characterize the solution, and the solution is unique.*

With that assumption, some further comparative statics is possible, as shown in figure 6. For lower y or for higher B, one obtains a lower level of current consumption, keeping future debt B' the same. This results in higher marginal utility $u'(c)$ or a lower rhs of the first-order condition (28).

Consider now the case, where χ is constant and where the fluctuations in income are very small[6]. In that case, the price is nearly flat at $q = (1-\pi)/R$

[6]This analysis is preliminary and rather speculative. Hopefully, we will succeed with a clean-up in a future version of this paper

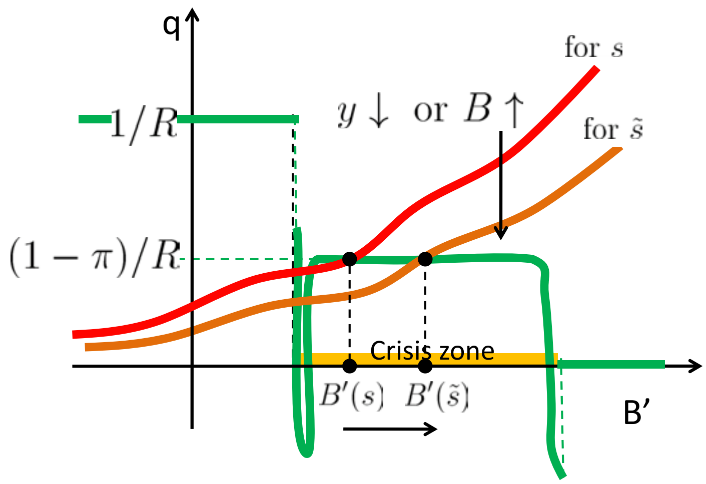


Figure 7: *The first-order condition (28) when income fluctuations are small.*

in the crisis zone, $\min \underline{B}(z) \leq B' \max \bar{B}(z)$. Figure 7 shows the resulting version of (28), corresponding essentially to the situation described in Cole and Kehoe (2000). The question is now, how large B' is, compared to the debt level B leading into this scenario. Consider the case where $\beta R = 1$. If income is literally constant, then consumption should be constant and the debt level should likewise remain constant, **except** that the country can also avoid the cost of default altogether[7] by "saving itself" out of the crisis zone, as shown in Cole and Kehoe (2000). The version of (28) for an initial debt level $B = 0$ is shown in figure 8: at constant income and $\beta R = 1$, the country will simply maintain that debt level rather than increase it.

Indeed, with a modest degree of income variation and for $\beta R = 1$, the country will choose to distance itself over time from the default zone as far as possible, saving for precautionary motives. The ensuing dynamics is shown

[7]This appears to clash with the first-order condition derived above. The issue will be cleared up in a future version of this paper.

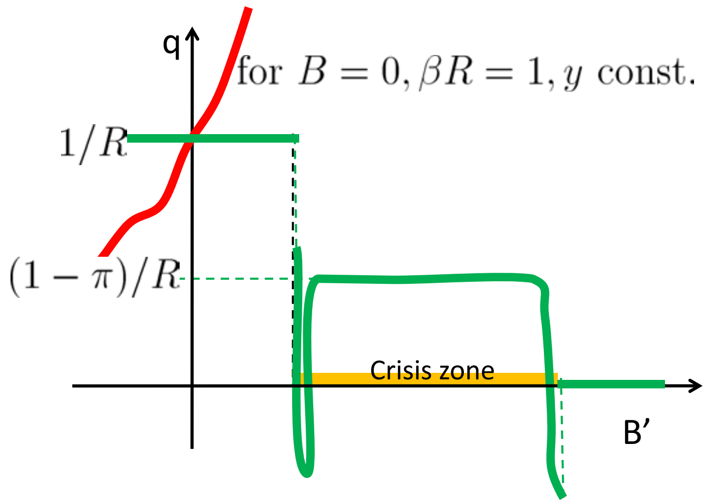


Figure 8: *The first-order condition (28) when income fluctuations are negligible and initial debt is zero.*

in figure 9. If $\beta R < 1$, but close to 1, then the asset accumulation will not “run away”, but still, the country will choose to accumulate large amounts of assets, as shown in figure 10. As a result, a sovereign debt crisis is highly unlikely. Here, it is therefore important to appeal to the political economy literature on sovereign debt accumulation, as in the literature cited in the introduction. If the government discounts the future sufficiently highly, i.e. if βR is considerably smaller than unity, then the country will possibly perch itself at a precarious point with an amount of debt in the crisis zone, as shown in figure 11. Indeed, reintroducing the income fluctuations in this picture results in a stationary distribution for the debt level, under suitable assumptions, as shown in figure 12.

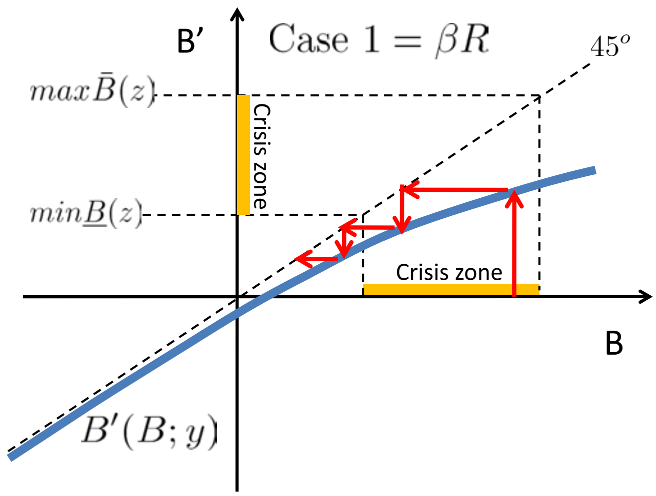


Figure 9: *The debt dynamics for small income fluctuations and $\beta R = 1$.*

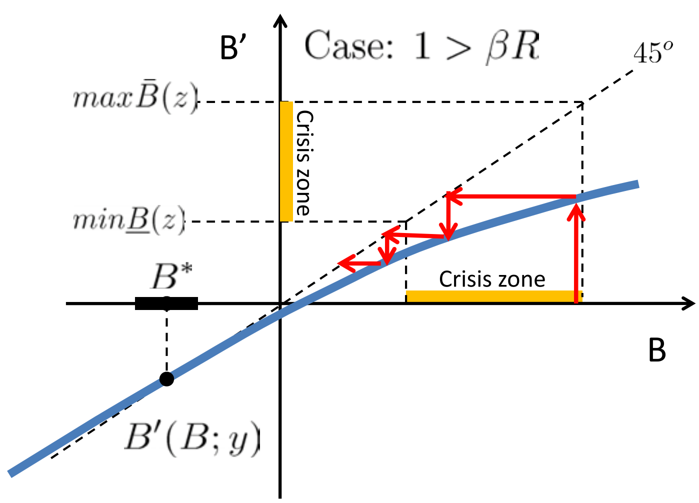


Figure 10: *The debt dynamics for small income fluctuations and βR below, but near 1.*

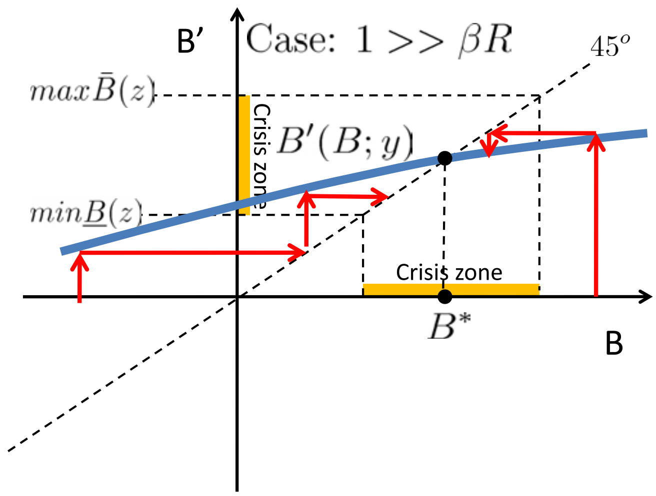


Figure 11: *The debt dynamics for small income fluctuations and βR far below 1.*

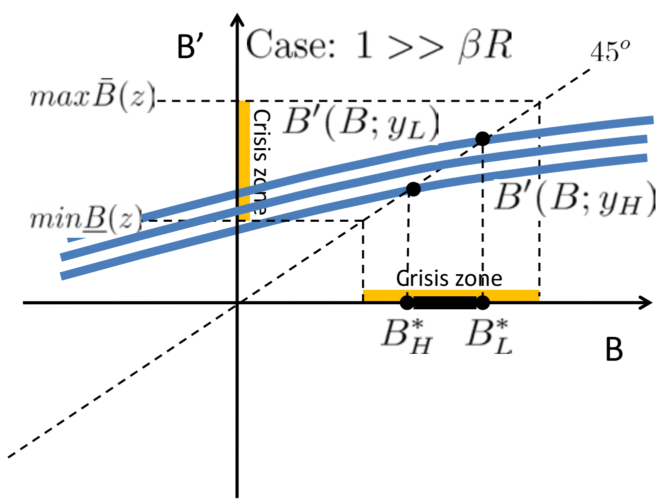


Figure 12: *The stationary debt dynamics for small income fluctuations and βR far below 1.*

4 Bailouts

Let us now consider the possibility for a bailout. While there are many possibilities to consider, we shall focus on the benchmark case of an actuarily fair bailout. More precisely, we envision a facility with sufficiently deep pockets (backed by, say, governments other than the one under consideration here), which aims at ensuring the selection of the "good" equilibrium, while earning the market rate of return in expectation on its bond holdings. I.e., we imagine that this bailout facility insists on actuarily fair pricing. It may well be that actual policy interventions amount to a subsidy or perhaps even a penalty. We view the actuarily fair "restoration-of-the-good-equilibrium" as an important benchmark. It might be interesting to consider other mechanisms, which are not actuarily fair, as well, and we do so in the appendix, see section A. An alternative is to examine the conditionality of such bailouts, combining help with insistence on fiscal discipline, see Fink-Scholl (2011).

Even with our narrow focus, there are a variety of choices. The bailout facility may be there "in eternity" or just for several or even one period. We shall concentrate on the case, where the bailout is there "forever". The bailout facility may buying the entire debt issued by the country, or some smaller amount.

If the bailout facility buys the entire debt, then the solution is easy in principle. It should calculate the $\pi = 0$-equilibrium described above, price debt accordingly, $q_a = \bar{q}_{m,\pi=0}$, and let the country choose the debt level it wants, given this pricing schedule. Since the bailout facility is always there, also in the future, to guarantee the "good" equilibrium, the pricing is actuarily fair. While potentially attractive, one may argue, that this solution has two drawbacks. First, the facility might need to be very large. Second, it would need to calculate the pricing function $\bar{q}_{m,\pi=0}$, something that competitive markets may do better than governments.

We therefore consider the alternative of a minimal bailout facility. I.e., we characterize the minimal level of debt such a facility needs to guarantee buying at the "good equilibrium" price, so that markets must coordinate on the good $\pi = 0$ equilibrium. We will have the "good-equilibrium" pricing function $\bar{q}_{m,\pi=0}$ for the total debt, except that only a certain fraction $B'_a(s)$ of the total new debt $B'(s)$ needs to be purchasable from the facility, while the remainder $B'(s) - B'_a(s)$ must be purchased on the market. The facility can then simply price its own lending at market prices, which is a simple task in principle. What it needs to do, though, is to calculate the appropriate minimal guarantee level $B'_a(s)$ of lending. It is rather trivial, that any guarantee larger than $B'_a(s)$, but below the total level $B'(s)$ will do the trick just as well. We assume that the facility sets prices to the "good equilibrium" market prices, even if the market does not buy at all: this is only relevant "off-equilibrium". It is important in this construction, that the debt held by the facility is treated the same as the debt held by market participants. It would be interesting to consider extensions, where the debt held by the facility is junior or senior to market-held debt. Note that only a guarantee $B'_a(s)$ is needed. The country is indifferent between purchasing this debt from the facility or from the market, and so is the market. The guarantee just needs to be there, in the (now hypothetical) case that the market coordinates on the default outcome.

To characterize the guarantee, we need to re-examine the default decision. The bailout facility sets the guarantee $B'_a(s)$. Define

$$q_a(B'; s) = \begin{cases} \bar{q}_{m,\pi=0} & \text{if } B' \leq B'_a(s) \\ 0 & \text{if } B' > B'_a(s) \end{cases} \tag{31}$$

If $B'_a(s)$ is sufficiently high, we assume that the facility uses this pricing function as the relevant pricing function for the "off-equilibrium" situation, that the market does not provide additional lending. It might be interesting to consider other pricing functions for that scenario: they probably do influence the calculations of $B'_a(s)$.

Choose some small $\epsilon > 0$, to break indifference. Find $B'_a(s) \geq 0$ and the associated pricing function $q_a(B'; s)$ of (31), so that

$$\underline{v}_{ND}(s = (B, 0, z)) = v_D(z(s)) - \chi(s = (B, 0, z)) + \epsilon \text{ for all } 0 \leq B \leq \bar{B}(z) \tag{32}$$

where $\bar{B}(z)$ is the maximum level of current debt consistent with no default in the good $\pi = 0$ equilibrium. For $B > \bar{B}(z)$, define $B'_a(s) = 0$, but do note, that $q_a(B'; s) = 0$ for any $B' > 0$ per definition of $\bar{B}(z)$. In other words, the facility could also provide the (meaningless) guarantee of willing to buy any positive level of debt $B'_a(s)$ at a zero price, with our assumption (31).

Proposition 6 *Suppose $B'_a(s)$ and $q_a(B'; s)$ satisfy (31) and (32), for some $\epsilon > 0$. Then, $\underline{B}(z) = \bar{B}(z)$, i.e, there will not be a default, unless debt exceeds $\bar{B}(z)$.*

Proof: *to be completed.* •

In the iid case and with a constant embarrassment utility costs $\chi > 0$ of defaulting, a bit more can be said. In that case, some constant value $\beta \tilde{v}_D$

$$\beta E[v_D(z')] \equiv \beta \tilde{v}_D$$

is the continuation value from defaulting. Likewise, when receiving the full guarantee $B'_a(s)$, the continuation value of not defaulting is $\beta \tilde{v}_{ND}(B'_a(s))$, given by

$$\beta E[v(B'_a(s), 0, z')] = \beta \tilde{v}_{ND}(B'_a(s))$$

Criterion (32) becomes

$$\begin{aligned} & u(y(s)) - u\left(y(s) + q_a(B'_a(s); s)\left(B'_a(s) - \theta B(s)\right) - (1-\theta)B(s)\right) \\ = \ & \beta \tilde{v}_{ND}(B'_a(s)) - \beta \tilde{v}_D - \chi - \epsilon \end{aligned} \tag{33}$$

comparing the current utility gain from defaulting to the utility continuation loss from defaulting, including the embarrassment cost χ.

Proposition 7 *In the iid and constant-χ case, we have*

1. *For two states s_1, s_2, if $B(s_1) > B(s_2)$, then $B'_a(s_1) \geq B'_a(s_2)$.*

2. *If $B(s) > 0$, then*

$$q_a(B'_a(s); s)\left(B'_a(s) - \theta B(s)\right) < (1-\theta)B(s)$$

3. *For two states s_1, s_2, if $y(s_1) > y(s_2)$, then $B'_a(s_1) \leq B'_a(s_2)$.*

4. *For two states s_1, s_2, if $\chi(s_1) > \chi(s_2)$, then $B'_a(s_1) \leq B'_a(s_2)$.*

Proof: *To be completed (and perhaps modified). The first part appears to be obvious. The second part is a version of proposition 2 in Arellano (2008), but may need some additional assumptions. The third follows from the second.* •

5 A numerical example

This section presents the results of a numerical exercise, where the model is solved using value function iteration (see A for more details). First we discuss the functional forms and parametrization, and then we give the results.

The government's within period utility function has the CRRA form

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

We assume that the income process is a log-normal autoregressive process with unconditional mean μ

$$\log(y_{t+1}) = (1-\rho)\,\mu + \rho \log(y_t) + \varepsilon_{t+1}$$

tax rev.: 15 points, mimic AR(1)	y_t	0.9,...,1.1
AR(1)	ρ	0 (iid) or 0.9
std. dev.	σ_ϵ	3.4%
Interest rate	r	1%
Maturity structure	θ	0
Government's U: CRRA with risk av. =	σ	2
Exclusion	α	0.282
Discount factor	β	0.8
Cost	χ_1	0.15
Cost	χ_2	0.175
Cost	χ_3	0.2
SFC sunspot probability	π	0.005

Table 1: Parameter values.

with $E(\varepsilon) = 0, E(\varepsilon^2) = \sigma_\varepsilon^2$.

Table 1 summarizes the parameters used in this exercise. We set the risk aversion parameter, σ, equal to 2, which is within the range of accepted values in studies of business cycles. A period in the model refers to a quarter. The risk-free interest rate is set equal to 1 percent. As in Arellano (2008), the probability of regaining access to capital markets (ψ) is 0.282. The discount factor β is set to 0.8. We assume that the embarrassment cost, χ, can take three values with equal probability. For the benchmark analysis we set θ equal to 0 (one period bonds) and π equal to 0.005. We analyze both the case when income is iid ($\rho = 0$) and when it is very correlated ($\rho = 0.9$).

Figure 13 shows the minimal guarantee level $B'_a(s)$ of lending (as a fraction of the total new debt $B'(s)$) as a function of the initial debt $B(s)$ for the three different values that the embarrassment cost can take. First, the

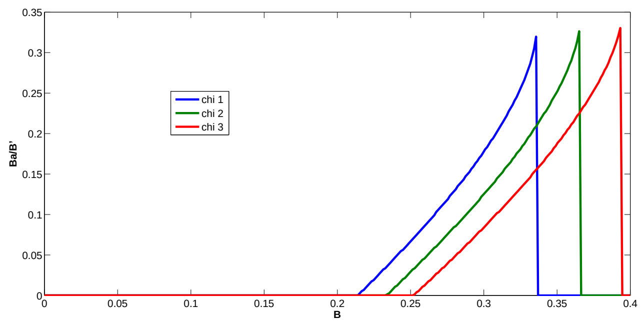


Figure 13: *iid case.* B_a/B' *as function of* B*, different* χ

figure shows that the facility only provides assistance $(B_a'(s) > 0)$ only for those $B(s)$ corresponding to the crisis zone: for levels of $B(s)$ outside the crisis zone, the government will either default or repay independently of the sunspot shock and thus there is no need for the facility. Second, notice that for the three cases the fraction $B_a'(s)/B'(s)$ is increasing in $B(s)$. This can be interpreted as the graphical representation of the first part in Proposition 7 and the intuition is straightforward: a higher $B(s)$ implies that the government needs to rollover a larger amount of debt, which generates stronger incentives to default. Thus, in order to avoid a default, the minimal guarantee needs to be larger. Third, notice that for a given level of $B(s)$ the fraction $B_a'(s)/B'(s)$ is decreasing in χ. This can be thought of the graphical representation of the fourth part in Proposition 7: when the embarrassment cost of defaulting is low the government has higher incentives to default, and thus the facility needs to provide a larger level of assistance.

A graphical representation of the third part in Proposition 7 is in Figure 14: for a given level of $B(s)$, the minimal guarantee level decreases as the level of tax receipts increases. For high levels of income, the government has low default incentives and thus the facility needs to provide a low guarantee. Another way of representing this result is showin in Figure 16. Figure 15

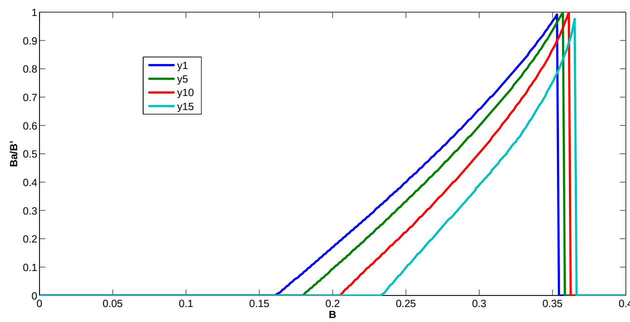


Figure 14: *iid case,* χ_2. B_a/B' *as function of* B, *different* Y

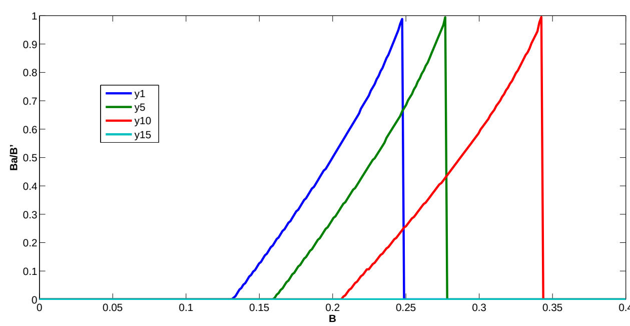


Figure 15: $\rho = 0.9$, χ_2. B_a/B' *as function of* B, *different* Y

shows the counterpart of Figure 14 but for the case that income is highly correlated $(\rho = 0.9)$. Figure 16 shows how the guarantee varies for different levels of the correlation parameter, ρ.

6 Conclusions

We have analyzed the dynamics of sovereign debt defaults, drawing on insights from three literatures, particularly Arellano (2008), Cole-Kehoe (2000) and Beetsma-Uhlig (1999). More precisely, we have analyzed the dynamics of sovereign debt, when politicians discount the future considerably more than

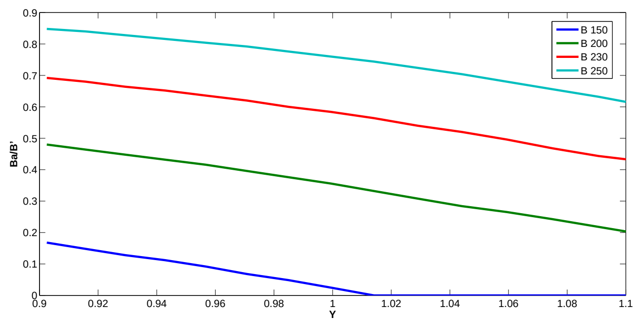


Figure 16: *iid case* (χ_2). B_a/B' *as function of* Y

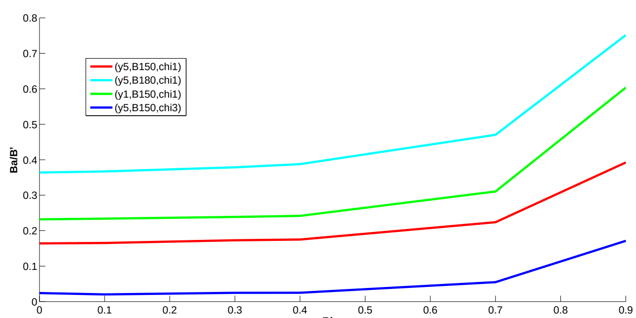


Figure 17: *Varying* ρ. B_a/B' *as function of* ρ

private markets and when there are possibilities for both a “sunspot-”driven default as well as a default driven by worsening of economic conditions or weakening of the resolve to continue with repaying the country debt.

We have shown how this can lead to a scenario, where the country perches itself in a precarious position, with the possibility of defaults imminent. We characterized the minimal actuarily fair intervention that restores the “good” equilibrium of Cole-Kehoe, relying on the market to provide residual financing.

References

[1] Allen, Franklin and Douglas Gale (2007), *Understanding Financial Crisis*, Clarendon Lectures in Finance, Oxford University Press, Oxford.

[2] Arghyrou, Michael G. and Alexandros Kontonikas (2011), “The EMU sovereign-debt crisis: fundamentals, expectations and contagion”, European Commision, Economic and Financial Affairs, Economic Papers 436.

[3] Arellano, Cristina (2008), “Default Risk and Income Fluctuations in Emerging Economies,” American Economic Review, vol. 98 (3), pp. 690-712.

[4] Beetsma, Roel and Harald Uhlig (1999), “An Analysis of the Stability and Growth Pact,” The Economic Journal, vol 109, pp. 546-571.

[5] Cole, Harold L. and Timothy J. Kehoe (1996), “A self-fulfilling model of Mexico’s 1994-1995 debt crisis,” Journal of International Economics, vol. 41, pp. 309-330.

[6] Cole, Harold L. and Timothy J. Kehoe (2000), “Self-Fulfilling Debt Crises,” Review of Economic Studies, vol. 67, no. 1, pp. 91-116.

[7] Cooper, Russell, Hubert Kempf and Dan Peled (2010), “Regional debt in monetary unions: is it inflationary?”, European Economic Review, vol. 54 (3), 345-358.

[8] Fink, Fabian and Almuth Scholl (2011), “A quantitative model of sovereign debt, bailouts and conditionality”, University of Konstanz, draft.

[9] Gennaioli, Nicola, Alberto Martin and Stefano Rossi (2011), “Sovereign Default, Domestic Banks and Financial Institutions”, CREI, University of Pompeu Fabra, draft.

[10] Greiner, Alfred, Uwe Köller and Willi Semmler (2007), “Debt Sustainability in the European Monetary Union: Theory and empirical evidence for selected countries,” Oxford Economic Papers, vol. 59(2), pp. 194-218.

[11] Lejour, Arjan, Jasper Lukkezen and Paul Veenendaal (2010), “Sustainability of government debt in the EU”, draft, CPB Netherlands Bureau for Economic Policy Analysis.

[12] Ljungqvist, Lars and Thomas J. Sargent (2004), *Recursive Macroeconomic Theory*, 2nd edition, MIT Press, Cambridge, MA.

[13] Tirole, Jean (2002), *Financial Crises, Liquidity and the Monetary System*, Princeton University Press, Princeton, NJ.

[14] Uhlig, Harald (2003), “One money, but many fiscal policies in Europe: what are the consequences?”, in M. Buti, Monetary and Fiscal Policies in EMU, Cambridge University Press, 2003, pp. 29-56.

[15] Uhlig, Harald (2010), “A model of a systemic bank run,” Journal of Monetary Economics, vol 57(1), 78-96.

A Other bailout mechanisms

Let us now consider the possibility for a bailouts, which may not necessarily be actuarily fair, as an extension of the discussion in the main body of the paper, and as these may be important for certain policy discussions. We shall focus on a few benchmark cases and explore their implications. First, suppose that, for a single period, debt can be sold at some fixed "assisted" price $0 < q_a < 1/R$ to some outside facility, provided the total amount B' of debt does not exceed some upper limit $\bar{B}_a$. This is a bailout and a stylized version of the one-time rescue for Greece or a one-time intervention by the European Financial Stability Facility. The resulting situation is shown in figure 18. The green line denotes the market price for existing debt sold to private lenders, while the blue line denotes the line, at which debt can be sold to the outside facility. The new debt level $B'_a(s)$ now exceeds the old debt level. Essentially, given the bailout, there is no longer quite the same pressure for the government of the country to cut back on government spending, due to the impending financial crisis. Indeed, we have seen how the attempts of government cut backs in Greece and Portugal have run into fierce local resistance: a luxury, that certainly would not have been there, if these countries needed to keep borrowing on private markets only and wished to avoid a default. As this is a one-time bailout, the resulting debt dynamics is given by figure 11, starting towards the right end, and indicated with the red arrow there (indeed, that arrow only applies in this situation: without the bailout, there would have been an assured default at that debt level outside the crisis zone).

It may be more interesting to consider a permanent version of this facility: all future borrowing by the country at hand can be done at some fixed price $0 < q_a < 1/R$, provided the total amount B' of debt does not exceed some upper limit $\bar{B}_a$. In that case, the pricing is given by figure 19.

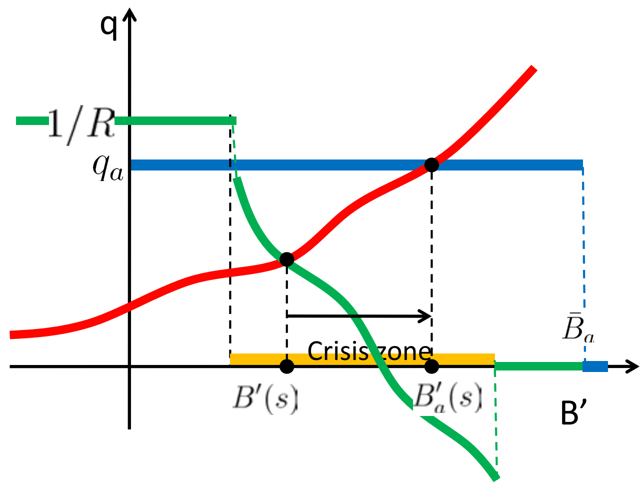


Figure 18: *The choice of the debt level in case of a one-time assistance or bailout.*

The existence of the borrowing guarantee now removes the doubt of private lenders that the country will be able to borrow tomorrow. As a result, the country debt becomes safe and will be discounted at the usual safe rate R. The mere promise of the permanent facility results in a markedly reduced market interest on the country debt, provided the promised facility is fully credible.

This may appear to be a wonderful solution. This is so only at first blush, however. Note that the borrowing increases from $B'(s)$ to $B'_a(s)$. Indeed, the country will once again find its perch in the crisis zone of probabilistic default: this time, however, triggered by the debt limit imposed by the facility[8]. The country will borrow privately at the safe return R, until it gets near the imposed debt limit. At that point, credibility on private credit markets

[8]Without a debt limit, the country will choose to run a Ponzi scheme, borrowing forever more without ever repaying.

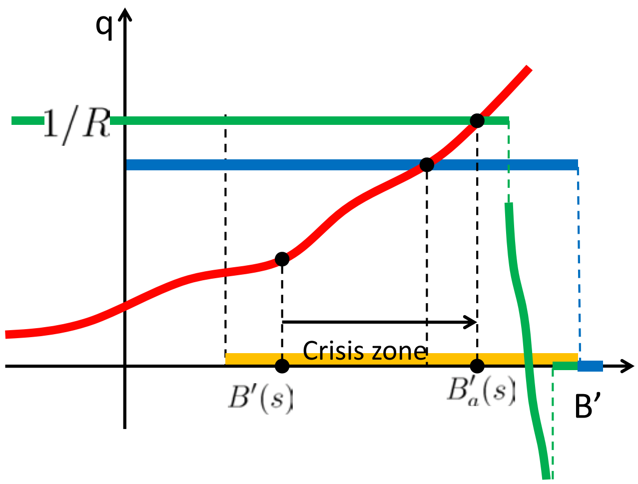


Figure 19: *The choice of the debt level in case of a permanent assistance or bailout.*

collapses as a default is now viewed as likely, the country will borrow one last time, but this time from the facility at the reduced price, and will default in the next period. The proof is by contradiction: if it would not default in the next period (or if such a default would be very unlikely), then it would borrow privately, rather than at the "penalty rate" from the facility. The ensuing debt dynamics is shown in figure 20.

Both scenarios are in conflict with the observation, however, that yields on, say, Greece, Portugese and Irish debt are high and continue to be high, i.e. that there continue to be default fears by private markets. While it is conceivable, that we are simply in that "terminal" period described in the previous scenario, an alternative view here is that the bailout is probabilistic. This can be modelled in analogy to the default sunspot above. I.e., assume some bailout probability $0 < \omega < 1$. If the "bailout sunspot" ψ is below ω, $\psi < \omega$, then the country can borrow at the price $0 < q_a < 1/R$ from

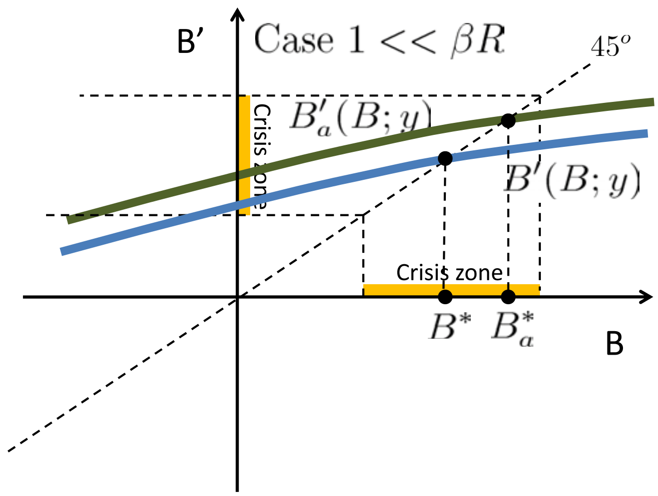


Figure 20: *The stationary debt dynamics for small income fluctuations and a permanent bailout facility.*

the outside facility, provided the total amount B' of debt does not exceed some upper limit $\bar{B}_a$. If the "bailout sunspot" ψ exceeds ω, $\psi \geq \omega$, then the country must rely on private markets alone.

This will have the effect shown in figure 21. The level of debt at which a country will now prefer a default in those periods when no borrowing from the facility is possible, has increased compared to the "no bailout ever" scenario, as the country can hope for the option of borrowing from that facility in the future. Therefore, the crisis zone shifts to the right. The debt dynamics is shown in figure 22. Essentially, this is now a shifted version of the debt dynamics without that facility: rather than repaying the debt, the country shifts to higher debt levels, and the probability of a default is essentially the same as it was before. This takes a bit of time, of course. The facility therefore provides a temporary, but not a permanent resolution of the fiscal crisis. The debt is once again traded at a premium, as before, except that

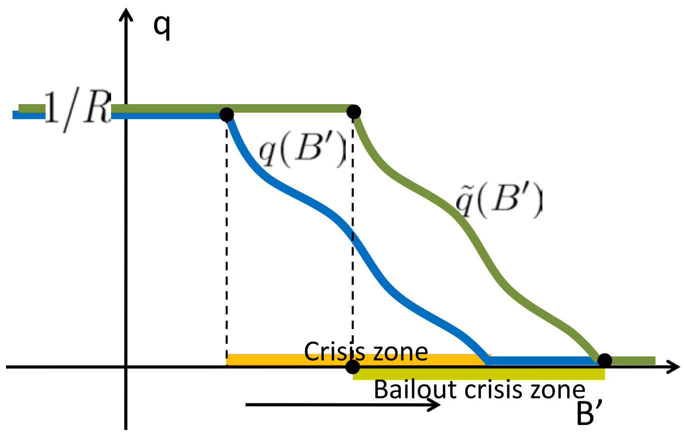


Figure 21: *Comparing the no-bailout private market pricing function* $q(B')$ *with the pricing function* $\tilde{q}(B')$ *in case of probabilistic bailouts.*

the probabilistic bailout means that these higher premium will be afforded at a higher debt level, than without that facility, while avoiding the default.

In essence, these scenarios show that the bailout facility only postpones the day of reckoning. It provides temporary relieve to the country in its desire to maintain a high level of government consumption, but leaves the default situation in a very similar and precarious situation as before, once the initial relief is “used up”.

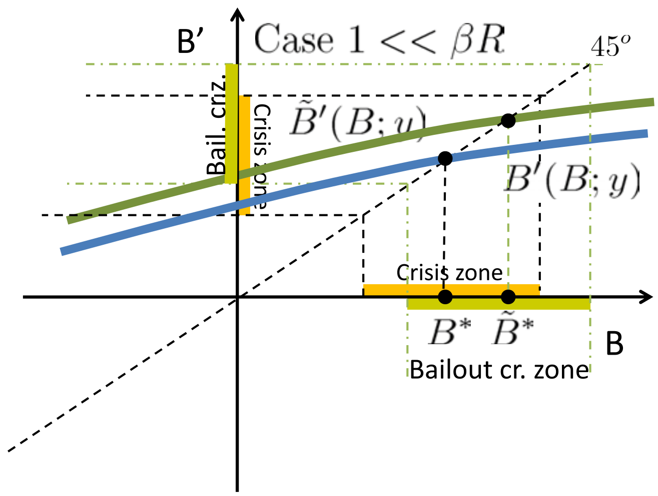


Figure 22: *The stationary debt dynamics for small income fluctuations and probabilistic bailout facility.*

A Computational algorithm

We solve the model numerically using a discrete state space method similar to Aguiar and Gopinath (2006). I discretize the endowment space into 15 equally spaced grids, and the asset space into 300 grids. I assume that the χ cost can take 3 values with equal probability: $0 < \chi_1 <.\chi_2 < \chi_3$.

The computational algorithm consists of the following value function iteration:

1. Assume an initial bond price schedule $q^0 = \frac{1}{1+r}$.
2. Use this price function and initial guess for the value functions to solve for the optimal value functions and policy functions.
3. Update the bond price schedule and repeat the previous steps until the price functions converge.