

U.S. Investment 1901-2005: Incumbents, Entrants, and Q^*

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Abstract

We show that investment by new firms responds to Tobin's Q much more elastically than does investment by incumbent firms. To explain this fact we build a model in which the investment-supply curve of incumbent firms is highly elastic and positively related to Q . However, when variation in Q is caused by shifts in this supply curve, the equilibrium relation between Q and investment that it traces out is negative. That alone causes a negative equilibrium relation between Q and investment. At high levels of Q , however, the investment of incumbents is further reduced, or crowded out, by the positive response of the investment by entering firms to the rise in Q . We fit a composite-capital version of the model to data, and we then repeat the exercise for a physical-capital version.

1 Introduction

Investment in new firms appears to be significantly more elastic with respect to Tobin's Q than investment of established firms. We document this fact with aggregate measures. We show, however, that aggregate investment is negatively related to Q when variation in the latter is caused by shifts in the supply curve for new capital. A rise in Q has the side effect of drawing in more new firms, and this crowds out incumbent investment.

Most economic models imply that investment should respond positively to movements in Tobin's Q . Yet, measured investment of firms shows little response to movements in measured Tobin's Q . So little, in fact, that one needs puzzlingly high capital-adjustment costs to explain the pattern. The puzzle is there in aggregate data and at the firm level.

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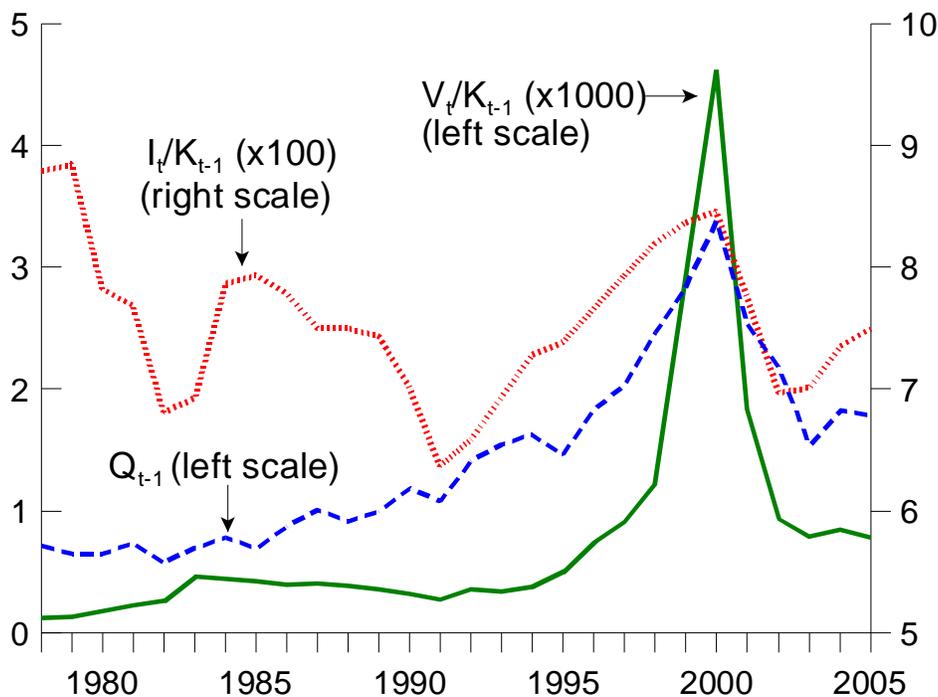


Figure 1: VENTURE INVESTMENT, AGGREGATE INVESTMENT, AND Q 1978-2005

No such puzzle exists for investment in new firms, however. Venture capitalists invest almost exclusively in young start-up firms. Venture investment as we know it today did not really get off the ground until the 1980s. Figure 1 shows that such investment responds elastically to Q , quite in contrast to aggregate investment which bears little relation to Q . The correlations with Q are 0.844 and 0.186, respectively.¹

The same is true for IPOs; while IPOs are an imperfect and delayed measure of investment in new firms, they are better than any other century-long time series that is available.² They are shown along with Q and aggregate investment in Figure 2.³

¹In Figure 1, data on venture capital investment are from the “Venture Xpert Database” of Thompson Venture Economics, Inc., and represent flows over each calendar year from 1978 through 2005. K_t is measured as the year-end stock of private fixed assets from the detailed fixed assets tables of the Bureau of Economic Analysis (2006, Table 6.1, line 1). I_t is gross private fixed investment from the National Income and Product Accounts. For Tobin’s Q , we use fourth quarter observations underlying Hall (2001) for 1978-1999, and ratio splice estimates prepared by Andrew Abel to Hall’s series for 1999 to 2005.

²The available data on incorporations and establishments are dominated by businesses such as cleaners, corner stores, etc., and therefore do not have much to do with our model.

³In Figure 2, K_t is the end-of-year stock of private fixed assets from the BEA (2006, Table 6.1, line 1) for 1925 through 2005. For 1900-1924, we begin with annual estimates from Goldsmith (1955, Vol. 3, Table W-1, col. 2, pp. 14-15) that include reproducible, tangible assets (i.e., structures, equipment, and inventories), and then subtract government structures (col. 3), public inventories

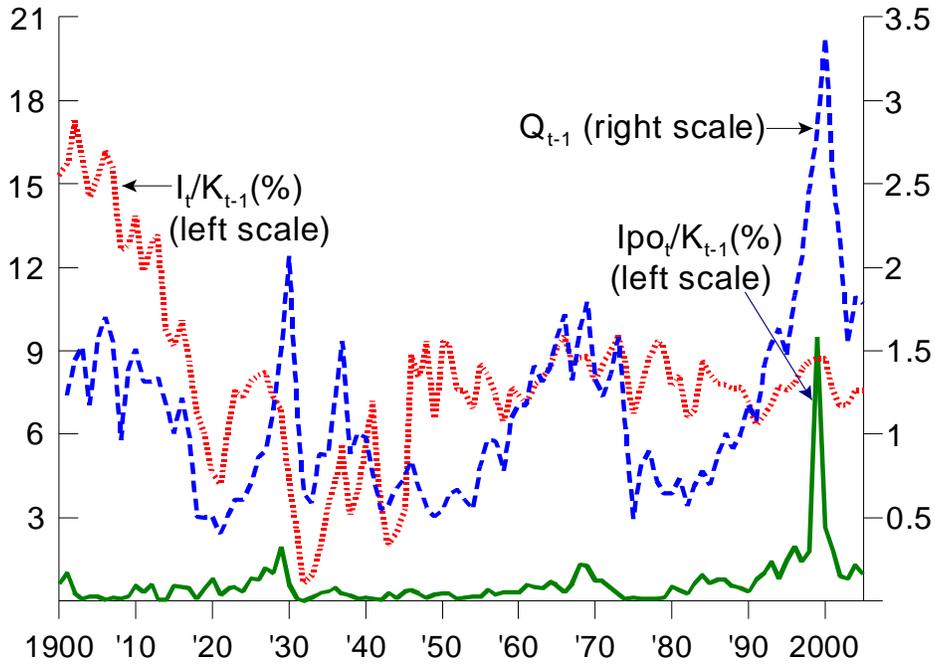


Figure 2: IPOs, AGGREGATE INVESTMENT, AND Q : 1900-2005

Over the past 115 years, and respective correlations are 0.574 and 0.305, while for 1954-2005 the correlations are 0.635 and 0.246.

We solve the puzzle by treating the investment- Q relation as the result of shocks to the supply of new capital to incumbents. An unfavorable shock to the supply of incumbents' capital raises the equilibrium Q and reduces the investment of in-

(col. 17), and monetary gold and silver (col. 18). We ratio-splice the result to the BEA series. I_t for 1929-2005 is gross private domestic investment from the BEA (2006, Table 5.2.5, line 4), to which we join estimates from Kendrick (1961, Table A-IIb, column 5, pp. 296-7) for 1900-1928. IPOs are measured as the year-end market value of the common stock of all firms that enter the University of Chicago's Center for Research in Securities Prices (CRSP) database in each year from 1925 through 2005, excluding American Depositary Receipts. The CRSP files include only listings from the New York Stock Exchange (NYSE) from 1925 until 1961, with American Stock Exchange and NASDAQ firms joining in 1962 and 1972 respectively. This generates large entry rates in 1962 and 1972 that for the most part do not reflect initial public offerings. Because of this, we compute the average of the entry rates in 1961 and 1963 and in 1971 and 1973, and assign these averages to the years 1962 and 1972 respectively. For 1900-1924 we obtain market values of firms that list for the first time on the NYSE using the pre-CRSP database of stock prices, par values, and book capitalizations developed by Jovanovic and Rousseau (2001, see footnote 1, p. 1). For Q , we continue to use Hall's (2001) fourth quarter data back to 1950, and then ratio splice the "equity Q " measure underlying Wright (2004) to take the series back to 1900. Note that Hall's measure of Q exceeds Wright's by factor of more than 1.5 in 1950, when the splice occurs, producing Q s before 1950 that are considerably higher than Wright's original estimates.

cumbents. Entrants, however, are not subject to this shock, and their investment rises. We shall follow the vintage-capital tradition and stress heterogeneous investment technologies. The heterogeneity will be over different vintages of firms. Instead of a fully-specified vintage-capital model, however, we model an economy in which capital is homogeneous, but in which entrants and incumbent firms have different investment technologies. Entrants will face convex capital-adjustment costs in the tradition of Lucas (1967) and Lucas and Prescott (1971), whereas incumbents will have constant but random costs of investment. The treatment will be a stochastic version of Prescott and Boyd (1987).

As in the vintage capital model, the value of old capital is in our model determined by the state of the investment technology – the technology of the latest vintage determines the market value of capital of earlier vintages. We assume, additionally, that shocks to the investment technology of incumbent firms determine their own Tobin’s Q *and* the equilibrium value of creating new firms. This small change in the vintage-capital model is important for explaining the pattern shown by Figures 1 and 2. Nevertheless, as in the vintage-capital model, high stock prices signal an unfavorable shock to the incumbent firms’ technology for creating new capital,⁴ and as a result, aggregate investment is decreasing in Q .

Two papers that stress investment of new productive units in a business-cycle context are Campbell (1998) and Bilbiie, Ghironi and Melitz (2006). These papers treat the capital of incumbents as fixed at its entering value, however, and so neither decomposes aggregate investment as we shall do here.

2 Model

Aggregate output is zk . There is one capital good, k , but two ways to augment it: via investment of incumbents, X , and via investment of entrants, Y , so that capital evolves as

$$k' = (1 - \delta)k + X + Y. \tag{1}$$

The aggregate resource constraint expresses the consumption of the representative agent as

$$C = zk - qX - h\left(\frac{Y}{k}\right)k \tag{2}$$

The RHS of (2) is linear homogeneous in (k, X, Y) .

⁴This feature would be present in Greenwood, Hercowitz and Krusell’s (1997) model if final-goods producers had convex adjustment costs to investment in addition to having to buy their capital from the capital-goods sector. Jovanovic and Rousseau (2008) show that old firms and firms with old capital trade at a discount, especially in sectors where technological progress is rapid. Thus more efficient new capital devalues old capital. Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001) have used this logic to argue that technological progress caused Q to fall below 1 in the mid 70s and remain there for 10 years.

On the RHS of (2), the two forms of investment cost are subtracted from output. The first cost, qX , is interpreted as the investment costs borne by incumbent firms which face a constant cost, q , per unit of capital created. We assume that q is random. In our model, q reflects the efficiency of entrants relative to incumbents. In the spirit of vintage-capital models, when q is high, the cost of entering capital is low relative to the cost of expanding the capital of incumbents. The parameter q will be regarded as a shock that is not reflected in the BLS measures of capital-goods prices. Greenwood, Hercowitz, and Krussell (1997) argue in a two-sector model that $1/q$ is the productivity of the capital-goods sector and is directly related to the measured price of capital. In our model, however, q is a shock *relative* to the price of capital that the BLS measures and uses to construct its estimated stock of capital. The model normalizes the measured cost of capital to unity.

The other cost, $h\left(\frac{Y}{k}\right)k$, represents the costs of creating entering capital. We follow Prescott and Boyd (1987) and assume that every unit of entering capital is ‘spun out’ by incumbents.⁵ In return for providing the investment needed to create that spin-out, the incumbent pays commensurately lower earnings to the workers that will end up managing the spin-out. This arrangement can work because, as Becker (1993) explains, general training should be financed by the worker.⁶

The determination of investment.—We shall show that the ex-dividend price of capital will equal q . The investment rate of entering capital is fully determined by, and increasing in q , as shown in Figure 3. Incumbent investment will take up the slack between desired total investment and the investment of entrants. There will be a second shock, z , and the supply of savings and, hence, the residual incumbent investment will depend on both q and z . Figure 3 illustrates the effect of a rise in q when z is held constant. The ‘interiority’ requirement that $x > 0$ implies that we can determine y from the intersection of the entrants’ investment-demand curve $h'(y)$ with q . As q rises while z stays fixed, two things happen: First, savings declines and with it total investment i must fall from i_1 to i_2 . Second, the supply of entrants rises from y_1 to y_2 , thereby crowding out even more incumbent investment.

The economy has no external effects or monopoly power and equilibrium can be represented by a planner’s problem. This problem has a traditional one-sector representation with a single cost of adjusting the economy-wide capital stock. That adjustment cost function will be a reduced form, representing the outcome of a static allocation problem, namely one of minimizing the cost of providing a certain amount of new capital conditional on the realization of q alone.

⁵We follow the terminology of Franco and Filson (2006) who write “In the finance literature, ‘spin-offs’ are created when an existing firm creates new firms from one of its divisions. Since the new firms we study are independent start-ups and avoid confusion by using the term ‘spin-out’. In Klepper and Sleeper (2000) a ‘spin-off’ describes what we denote as a ‘spin-out’.”

⁶Other models in this spirit are Chari and Hopenhayn (1991), Franco and Filson (forthcoming) and Chatterjee and Rossi-Hansberg (2007).

at a value of y too low to satisfy the demand for saving in any state s , which rules out $x = 0$.⁸ Combining the two FOCs leads to

$$h'(y) = q. \quad (5)$$

as illustrated in Figure 3. Therefore $y(q) \equiv (h')^{-1}(q)$ is increasing in q .

If y always satisfies (5), we can write (3) as⁹

$$V(s, k) = \max_{k' \geq (1-\delta)k} \left\{ U(zk - q(k' - (1-\delta)k) - y(q)k) - h(y(q)k) + \beta \int V(s', k') dF(s', s) \right\}$$

whence

$$\begin{aligned} V_k &= U'(C)(z + q(1 - \delta + y(q)) - h(y(q))) = U'(C_t)(z_t + q_t(1 - \delta + y_t) - h(y_t)) \\ &= U'(C_t) \left(\frac{D_t}{k_t} + q_t \left(\frac{k_{t+1}}{k_t} \right) \right) \end{aligned}$$

because $1 - \delta + y_t = \frac{k_{t+1}}{k_t} - x_t$. Therefore the price of a unit of capital satisfies¹⁰

$$q_t = \beta \int \frac{U'(C_{t+1})}{U'(C_t)} \left(\frac{D_{t+1} + q_{t+1}k_{t+2}}{k_{t+1}} \right) dF(s_{t+1}, s_t). \quad (6)$$

This is the discounted expected value of the firm's earnings and in the standard decentralization as done by Lucas (1978) would be the price of capital that shareholders pay. The replacement cost of capital is unity, and therefore q is also what is known as Tobin's Q . We should think of q_t as the price of capital at the *end* of the period, since a purchase at date t at that price does not entitle the holder to period- t dividends.

Let preferences be

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

⁸This requirement will be considered explicitly when we solve for the growth rate in the deterministic case in Section 2.2.

⁹Evidently, the model has a standard one-sector representation. Let $I = X + Y$ and let

$$f(i, q) \equiv \min_x \{qx + h(y)\} \quad \text{subject to } x + y = i.$$

Then the economy is equivalent to one in which, instead of (1) and (2), we have

$$C = zk - f\left(\frac{I}{k}, q\right)k \quad \text{and} \quad k' = (1 - \delta)k + I.$$

¹⁰If x_{t+1} and y_{t+1} both were to equal zero, then we would get the more intuitive expression

$$q_t = \beta \int \frac{U'(C_{t+1})}{U'(C_t)} (z_{t+1} + (1 - \delta)q_{t+1}).$$

in which case the value function must satisfy

$$V(s, k) = v(s) k^{1-\sigma}, \quad (7)$$

where

$$v(s) = \max_{x,y} \left\{ \frac{(z - qx - h[y])^{1-\sigma}}{1-\sigma} + (1 - \delta + x + y)^{1-\sigma} v^*(s) \right\} \quad (8)$$

and

$$v^*(s) = \beta \int v(s') dF(s', s).$$

If the processes z and q are positively persistent and if they are mutually independent, v and v^* are strictly increasing in z and strictly decreasing in q .

The investment policies.—By (5) we know that y depends on q alone and is increasing in q .¹¹ Regarding x , we have

Proposition 1 x is (i) strictly increasing in z , and (ii) strictly decreasing in q .

Proof. (i) The RHS of (9) is strictly increasing in z directly and increasing through v^* , and it is strictly decreasing in x . The LHS of (9) does not depend on z . Then the implicit function theorem yields the first claim. (ii) From (7),

$$V_k(s, k) = (1 - \sigma) v(s) k^{-\sigma},$$

and then (4) reads

$$\begin{aligned} q &= \frac{1-\sigma}{U'} ([1 - \delta + x + y] k)^{-\sigma} \beta \int v(s') dF(s', s) \\ &= (1 - \sigma) \left(\frac{z - qx - h(y)}{1 - \delta + x + y} \right)^\sigma v^*(s) \\ &= (1 - \sigma) \left(\frac{z - qx - h((h')^{-1}(q))}{1 - \delta + x + (h')^{-1}(q)} \right)^\sigma v^*(s), \end{aligned} \quad (9)$$

which allows us to solve for x as a function of $v^*(s)$. Since $(h')^{-1}(q)$ is increasing in q , the term $(\cdot)^\sigma$ is strictly decreasing in q , and so is v^* . Therefore the RHS of (9) is strictly decreasing in q . As we mentioned under (i), the RHS of (9) is strictly decreasing in x . Since the LHS of (9) is increasing in q , the implicit function theorem then delivers the second claim. ■

¹¹The simplest way to overturn this exclusion restriction and to get q to depend on z would be to assume that h depends on z .

2.2 Decentralization 1: A corporate venturing economy

We can think of $h\left(\frac{Y}{k}\right)k$ as the cost of discovering new investment opportunities adding up to Y units of tomorrow's capital. Experience helps in such discovery and therefore k lowers its cost. This fits the activity of venture investment by corporations. These corporations retain ownership of the dividends that these investments will provide in the future. See Gompers (2002) and Dushnitsky and Shapira (2006) for overviews of this type of investment. Thus corporate investment breaks down into the cost of reinvesting in routine activities is qX and the cost of establishing new ventures is $h\left(\frac{Y}{k}\right)k$.

We shall assume that the firm maximizes the value of its current shareholders, measured in units of current consumption. As Lucas and Prescott (1971) explain, the firm takes as given the next-period value of its capital, its ex-dividend value today being q per unit of capital created. The firm's maximization problem then is

$$w = \max_{x,y} \{z - qx - h(y) + (1 - \delta + x + y)q\}. \quad (10)$$

This means that the firm's y must solve (5), but x must be obtained from the household savings decision and the identity that savings = investment = $x + y$. The linear technology for creating k does not yield any rents because the unit price of capital adjusts to equal the average and marginal cost of creating it. Therefore (10) reduces to

$$w = z + q(1 - \delta) + \max_y \{qy - h(y)\}.$$

As in Lucas (1978), let ownership of firms be the only means of saving. The representative household maximizes discounted expected utility subject to the constraint

$$qn' + c = (z - x + q)n$$

where n is the number of units of capital of the representative firm that the household owns and where $c = C/k$. The household's decision problem is

$$v(n, s) = \max_{n'} \left\{ U(n(z - x + q) - qn') + \beta \int v(n', s') dF(s', s) \right\}.$$

Assuming differentiability of v , the FOC is

$$qU'(C) = \beta \int v_n dG = \beta \int (z' - x' + q')U'(C') dF$$

where we have applied the envelope theorem. Equilibrium requires that $n = k$, and $D = (z - x)k$. Substituting this information into the FOC leads to (6). Therefore equilibrium and the optimum coincide.

2.3 Decentralization 2: A spin-out economy

We can, instead, think of $h\left(\frac{Y}{k}\right)k$ as the cost (either direct or in the form of foregone output) of providing the firm's workers with the training needed to discover new investment opportunities, but where the implementation of these opportunities is done by the workers after they leave the firm. The parent corporation now does not own the dividends that these investments will provide. Rather, it will charge for the training that it provides by paying lower wages. This decentralization will be essentially that of Prescott and Boyd (1987), and in the spirit of the analyses of general human capital in Becker (1993). People live for two periods and have preferences $E\{U(c_Y) + \beta U(c_O)\}$ where c_Y is consumption when young and c_O consumption when old. The young inherit all the capital that the firm creates, but a certain fraction of them, Y_t/k_{t+1} , start new firms. The rest stay and continue to operate the existing entity.

2.3.1 The version with size of firm exogenous

Output can be produced only if an old worker (the 'manager') and a young 'worker' are present. Population is constant and each firm is composed of an equal number of old and young workers. Let k be the human capital of the managers. Let $k' = k + I$ be the total human capital given to each young worker. Net output per unit of k is $z - f(i, q)$ where, as discussed in footnote 10, the firm's investment-cost minimization problem is

$$f(i, q) \equiv \min_{x, y} \{qx + h(y)\} \text{ subject to } x + y = i. \quad (11)$$

That is, the firm provides the capital as cheaply as possible so that y still solves (5). We then say that a fraction

$$\frac{y(q)}{1 - \delta + i(s)}$$

of the workers start new companies, while the rest stay with the same company.

Let k or, rather, the firms be the only asset. A firm is owned by its manager(s). No other assets exist in this economy but because returns are constant there is no borrowing and lending among firms in the equilibrium that we shall describe.

Because the young care about lifetime expected utility, the manager will choose the wage-training package that provides his one worker the equilibrium utility as efficiently as possible. Suppose that the manager consumes $kp(s)$. We take the function $p(s)$ as given for now, and we assume it to be independent of k . The lifetime utility of the worker, ϕ , is

$$\phi(s, k, p(\cdot)) = \max_i \left\{ U(k(z - f(i, q) - p(s))) + \beta \int U(k(1 - \delta + i)p(s')) dF(s', s) \right\}.$$

Since $\frac{\partial f}{\partial i} = q$, this gives rise to the investment policy $i(s)$ satisfying

$$q = \frac{1}{U'(C_Y)} \beta \int p(s') U'(C_O) dF(s', s) \quad (12)$$

With $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$, (12) reads

$$q = \frac{(1 - \delta + i)^{-\sigma}}{(z - f(i, q) - p(s))^{-\sigma}} \beta \int [p(s')]^{1-\sigma} dF(s', s), \quad (13)$$

and it is therefore indeed feasible, as we assumed, that $p(s)$ and i should be independent of k . Eq. (13) Therefore it is as if the worker buys the firm from the manager at a fee $kp(s)$ and maximizes his utility with the expectation that he will receive $k'p(s')$ next period.

Proposition 2 *If*

$$p(s) = z - x(s) - h(y[s]) + q(1 - \delta + i(s)) \quad (14)$$

(6) and (12) are the same and the spin-out economy coincides with a planner's optimum.

Proof. Since $z - x(s) - h(y[s]) + q(1 - \delta + i(s)) = \frac{D+qk'}{k}$, updating both sides by a period and substituting in the RHS of (12) implies (6). ■

There appear to be many other equilibria in this OLG economy, each corresponding to a different weight of the young and old in the sharing of output. This is a point that Cozzi (1998) has already made in a similar setting. An equilibrium to this situation is the one that we already displayed, namely, one in which everyone consumes a constant fraction of output in each period. In that case, (12) and (4)

$$-qU' + \beta \int V_k dF = 0, \quad (15)$$

are the same, because when everyone consumes the same amount, $\int V_k dF = E_t U'(C_{t+1})$, and therefore $p_t = q_t$.

Example.—Let $\sigma = 1$. Then (13) reads $q = \beta \frac{z - f(i, q) - p(s)}{1 - \delta + i}$, whence we get

$$i + \frac{\beta}{q} f(i, q) = \frac{\beta}{q} (z - p(s)) - (1 - \delta). \quad (16)$$

Since $\frac{\partial f}{\partial i} = q$, the LHS has a derivative w.r.t. i equal to $1 + \beta$ as long as the solutions for x and y are interior. Therefore, $i = \mu - \frac{\beta}{q(1+\beta)} p(s)$, where $\mu > 0$ is a constant. Therefore, equilibria with a larger share of the old imply less investment and less growth. On reflection we should have expected this, because (i) If this were a savings problem, a change in the return on capital would have exactly offsetting income and substitution effects, and (ii) A larger p implies a subtraction from the output left over for the consumption and investment of the young. Therefore there is a second negative income effect that causes investment to fall because consumption is a normal good.

2.3.2 The version with size of firm endogenous

Endogenizing firm-size.—We follow Prescott and Boyd (1987) and reach a unique solution by introducing a firm-size margin. To narrow down the equilibrium, they append a decision about firm size as follows: Let n be the firm's employment and let the firm's output be

$$\text{output} = kQ(n)$$

where k is the quality of the manager, as in Lucas (1978a). Let k , X , and Y be capital and investments per worker. Let the firm's investment costs depend only on the totals accumulated¹², nX and nY , so that these costs are $qnX - h(n\frac{Y}{k})k$. The firm's revenue per unit of k then is

$$zQ(n) - qx - h(ny),$$

where Q is increasing and strictly concave. For $n \neq 1$, the firm's subproblem (11) becomes

$$f(i, q, n) \equiv \min_y \{qn(i - y) + h(ny)\} \quad (17)$$

and when evaluated at $n = 1$, the FOC is still (5).

Since the firm's output is shared among n workers, the worker-participation constraint now reads

$$\max_i \left\{ U \left(\frac{k}{n} (zQ(n) - f(i, q, n) - p(s)) \right) + \beta \int U(k(1 - \delta + i)p(s')) dF(s', s) \right\}.$$

Normalizing $Q(1) = 1$, when evaluated at $n = 1$, the FOC w.r.t. i is still (12). Finally, the FOC w.r.t. the new choice variable, n , is

$$-(z - f(i, q, 1) - p(s)) + zQ'(1) - q(x + y) = 0,$$

in light of (5). Then since $q(x + y) - f(i, q, 1) = qy - h(y)$,

$$p(s) = z(1 - Q'(1)) + qy - h(y) > 0. \quad (18)$$

The inequality follows because (i) $1 - Q'$ is the average minus the marginal product of a unit measure of workers and (ii) Since $h' = q$, $qy - h$ is the marginal minus average cost.

Example: Once again, let $\sigma = 1$, $Q(n) = n^\alpha$, and $h(y) = \gamma y^2/2$. Then

$$p(s) = (1 - \alpha)z + \frac{q^2}{2\gamma}$$

¹²This is different from eq. (1) of Prescott and Boyd.

$Q'(1) = \alpha$, and $h(y) = \gamma y^2/2$. In that case, since (16) is still valid, we substitute into it for $p(s)$ from (18) to get

$$i + \frac{\beta}{q} f(i, q) = \alpha \frac{\beta}{q} z - (1 - \delta) - \beta \frac{q}{2\gamma}$$

which has exactly one solution for i whenever the parameters are chosen so that RHS is always positive.

3 Micro issues

This section discusses micro issues.

3.1 Micro evidence on q

The model argues that high values of Tobin's Q are caused entirely by spontaneous rises in the cost of capital formation, q , as shown in (2). Thus the critical shock here is q . This is the true cost of capital relative to its measured cost. The measured cost is the price of capital relative to consumption which we have normalized at unity.

A major adjustment cost of capital is the cost of getting to use it optimally. These costs are likely to involve the use and creation of human capital. Adler and Clark (1991) study the causes of the productivity growth that follows after new capital is installed. Foregone-output costs are eliminated as (a) the new capital is de-bugged,¹³ and (b) people are trained to use it.¹⁴

If it is mainly skilled labor that participates in easing the firm's adjustment to its new k , it seems that the q should be related to the cost of skilled labor, transformed so that it is stationary. The most famous transformation of the cost of skilled labor is the skill premium which one arrives at by dividing that cost by the cost of unskilled labor. Figure 4 shows the relation between Q and the skill premium for the post-WW2 period, while Figure 5 shows it for the entire century.¹⁵

¹³"...approximately one quarter of the engineering changes for the product under analysis ...were motivated by design errors. Some of these errors would be uncovered in production through testing, while other design changes might reasonably be expected to be made under the impetus of market experience. The bulk of the ECs, however, were not motivated by design errors, but instead by ease-of-manufacture and cost-reduction concerns. These changes are primarily prompted by production experience. So we might expect part of the experience effect captured in the learning curve to reflect deliberate engineering changes." (p. 271)

¹⁴Training of 'direct' personnel (nonsupervisory, shop-floor personnel) should be a second conduit for learning. The investment of training time should lead to some improvement in worker performance, and thus, productivity. So part of the experience effect should be due to training.

¹⁵We construct the post-WWII skill premium in Figure 4 as the "returns to college" for all men from Goldin and Katz (1999, Figure 1, p. 32, and Table 7, p. 45), which are available on a decadal basis from 1939 to 1989 with a final observation in 1995. These returns are constructed as differences in mean log wages adjusted for the age and experience composition of the workforce. To build the

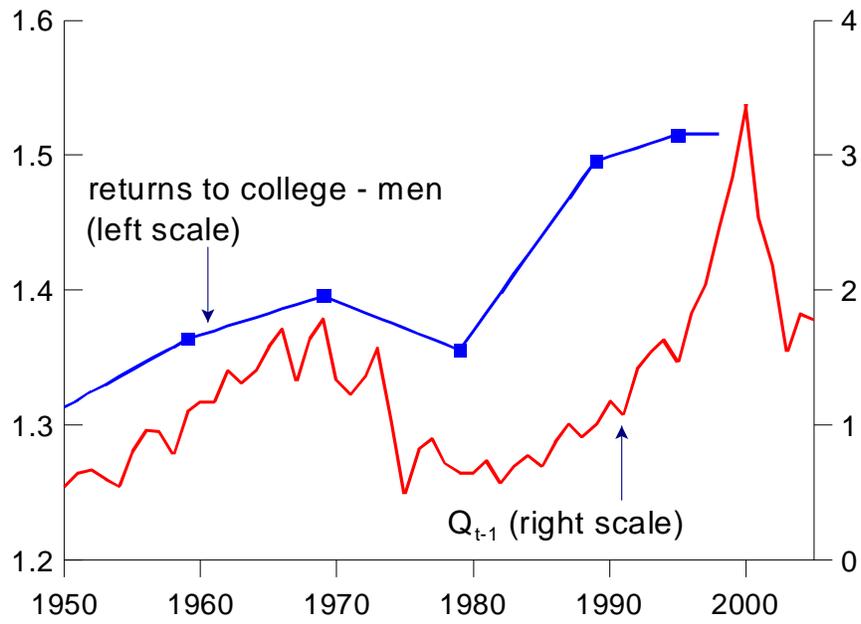


Figure 4: Q AND THE SKILL PREMIUM SINCE 1950

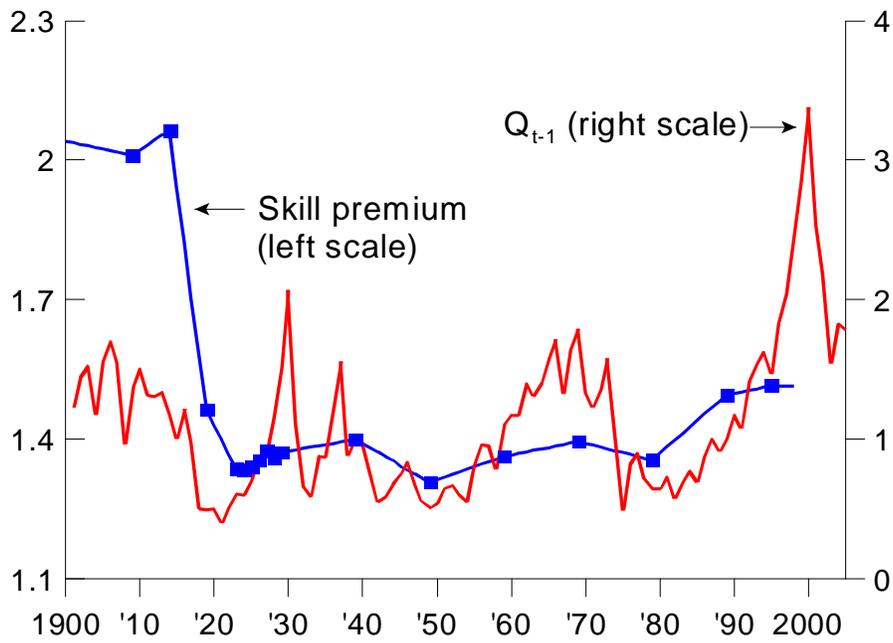


Figure 5: Q AND THE SKILL PREMIUM SINCE 1900

If a higher skill premium raises the marginal cost of capital for incumbents, should it not also do that for entrants? Indeed, should it not affect the marginal cost of capital for entrants even *more* than incumbents since, after all, it is the entrants that bring in the new technology? One answer is that, even though $h(y)$ does not depend on q directly, in equilibrium we have $h'(y) = q$ so that in equilibrium the marginal costs of entrants and incumbents are always the same. But a rise in q raises the entrants' average cost of capital more than proportionally. At least this is true in the quadratic specification that we shall use, namely when $h(y) = \frac{\gamma}{2}y^2$. The FOC $q = h'(y)$ then yields $y = q/\gamma$, in equilibrium

$$h(y[q]) = \frac{\gamma}{2} \left(\frac{q}{\gamma} \right)^2 = \frac{q^2}{2\gamma}.$$

By contrast, the average cost of capital for incumbents is q .¹⁶

3.2 The homogeneity of k

The model assumes that capital is homogeneous; the two ways of creating it deliver the same thing, as stated by (1). After entry, the capital of the latest vintage does not change relative to that of earlier vintages. The loss of market share to entrants is therefore shared by incumbents of all vintages equally.

Figure 6 shows how well over time the IPO-ing firms of each decade performed relative to firms that had IPO'd before them. There is some downward trend in these series because the stock market has grown over time. Thus, for example, the IPOs of the 1890s which included AT&T and General Electric, were large relative to the value of the stock market in 1890 partly because fewer firms were listed in 1890. The take-away fact is that the lines are mostly flat, meaning that vintages mostly hold their own relative to earlier vintages, thereby supporting the assumption that k is homogeneous.

These measures look at surviving value, and not returns. If dividend policies differed by cohort so that if, for example, the 1890's cohort tended to pay less dividends

century-long series shown in Figure 5, we join this series with the ratio of the earnings of all clerical workers, excluding supervisors, to the earnings of production workers in manufacturing, which are available in 1895, 1909, 1914, 1919, 1923-29, and 1939 from Goldin and Katz (1999, Table 2, p. 38). When we make the ratio splice in 1939 after linearly interpolating between the available observations to form a continuous series, the return to college is 40 percent, while the clerical to manufacturing wage premium is 15 percent. This means that our synthetic "skill premium" in Figure 5 is about 22 percent higher for the 1900-1939 period than the actual ratio of clerical to manufacturing wages.

¹⁶Jerzmanowski and Nabar (2007) argue that financial deregulation freed up financing, including venture capital, which raised the demand for skilled people to develop new firms, thereby raising the skill premium. In this way they link venture activity and the skill premium. In contrast, we do not explain the premium, but simply think of it as one measure of the cost of creating new productive capacity of all sorts. The rise in this cost rises Q and draws entrants in, as well as VC activity.

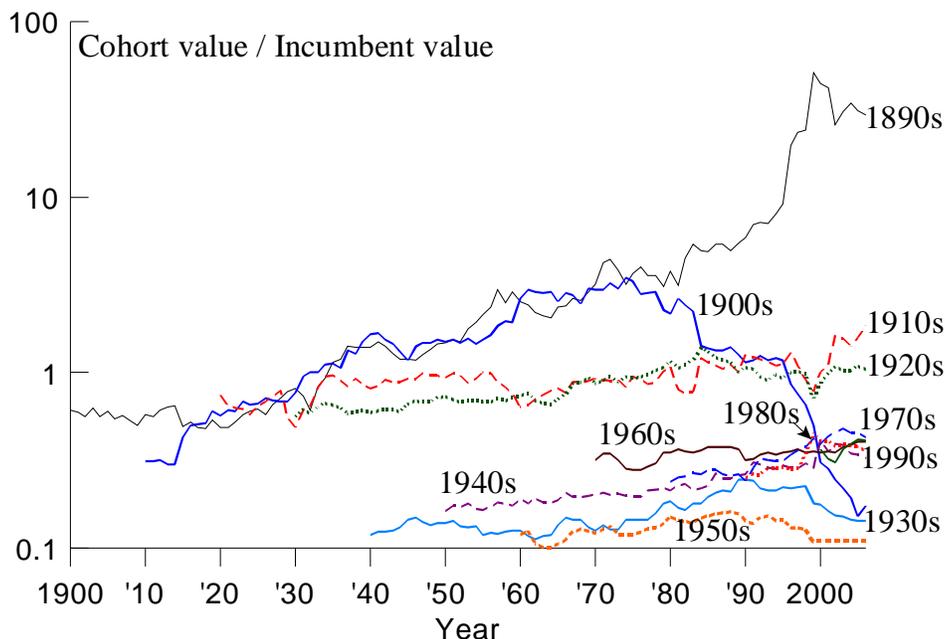


Figure 6: SHARES OF DECADAL VINTAGES RELATIVE TO EARLIER VINTAGES

and reinvest more of its earnings, changes in value would reflect partly such a difference in reinvestment policies. An alternative measure is cohort returns. Gerdes (1999) has studied the returns patterns by stock-market cohort and found that older (but not very old) vintages have a higher return than the representative rebalanced index. Our vintage value plot in Figure 6 shows no tendency for the value of any particular decade to appreciate more than the others, with the exception of the 1890s cohort: The 1890s cohort includes GE and AT&T and has, since the 1970s, done much better than the other cohorts, especially the 1900-1910 cohort.

The stability of the values of the different vintages is not inconsistent with two anomalies in the Finance literature. Fama and French (2000) say there is substantial mean reversion in earnings at the firm level. This could be true and yet earnings could be stable at the level of the cohort as our model implies. For instance, z could have a mean-reverting but firm-specific component, but the law of large numbers would remove the influence of this component on the relative valuation of cohorts. When divided by k_t , aggregate earnings, zk_t , are also mean reverting in the model because z is stationary. The second is the systematic IPO underpricing which implies that new firms are initially undervalued when they IPO, a phenomenon that inflates the stock-market returns of young capital relative to old capital.

The model assumes that firms are homogeneous. If firms had different (q, z) realizations and if no other change was made to the model, it would imply much more turbulence among firms than we observe: All X would be undertaken exclusively by

those firms that had the highest- z and lowest- q combination. To prevent this from happening we would need to add adjustment costs to x . This would introduce a positive slope to the supply curves of x in Figure 3 which would now be firm specific. But the model's aggregate implications would remain much the same.

3.3 Entering vs. incumbent investment

Who creates 'entering' capital and who owns it? To determine the market arrangements that lead such capital to come into being, we need to check the evidence on this point and how others have modeled it. The literature on entry (Carroll and Hannan 2000) has distinguished three types of firm entry:

- De Novo entry = Entry by a new firm
- Spin-off entry = Technically, entry by a new firm
- De Alio or 'lateral' entry = Entry by an existing firm.

Entry, however, can be into a new industry or into an already established or 'old' industry. Each type of entry carries its own investment. Therefore 'entry investment' is of three types. All investment that is not 'entry investment' will be termed 'incumbent investment.' In other words, incumbent investment is the investment of an existing firm in an industry where it has operated in the past.¹⁷ To clarify these distinctions, consider the following table:

		Origin of investment	
		New Firm	Old Firm
Destination of investment	New Industry	a	b
	Old industry	c	d

In the table, the symbols a , b , c , and d denote total amounts of investment of the various kinds. Specifically, then, we may associate these investment types with the types of firms that carry out that investment as follows:

- $a + c =$ De Novo + Spin-off investment;
- $b =$ lateral investment;
- $d =$ Incumbent investment+ lateral investment.

In terms of possible meanings of the symbols defined in our model, we have:

(i) $X = d$, and $Y = a + b + c$;

¹⁷In fact, some incumbent investment may be hard to classify because of the multiproduct nature of production in most plants. A firm may build a plant that produces some of an 'old' product, and some of a new product, in which case it is ambiguous whether that investment represents item b or item d .

(ii) $X = b + d$, and $Y = a + c$.

We are not sure about whether b should be part of X or part of Y . Thus we are flexible on how to interpret the model's symbols empirically. Some comments now on how we shall match the data to the concepts of cost in (2):¹⁸

1. *Who creates entering capital?*—The cost of an incumbent firm's capital is qX , and that of entering firms is $h \left(\frac{Y}{k}\right) k$. We think of q as the unit cost of routine investment, and of h as the cost of doing new things. New ideas vary in quality and the good ones yield a higher return. The best ideas are exploited first, hence h' is upward sloping. The microfoundations of spin-outs involve asymmetric information. Klepper and Thompson (2006) argue that spin-outs occur because people disagree over management decisions – people leave firms to develop their ideas on their own because coworkers would otherwise implement the idea suboptimally. Chatterjee and Rossi-Hansberg (2007) argue that the best ideas leave the firm.¹⁹ Finally, Franco and Filson (2006) argue that a firm's span-of-control limit pushes some of its employees out to start their own firms using the training they received on the job. The models differ, but they all treat entry as essentially a withdrawal of capital from existing firms, and that is also how we shall treat it.
2. *Externalities in investment.*—Both decentralizations assume a representative firm that treats the cost $h \left(\frac{Y}{k}\right) k$ as internal. If Y stands for all new capital in new firms, this means that new firms can be formed only by paying the cost $h \left(\frac{Y}{k}\right) k$. Each new firm, therefore, is a product of activity done in incumbent firms.²⁰ Only the first two categories are covered by the RHS of (2), but perhaps the third does not matter quantitatively. This is analytically extremely convenient, for it implies that equilibrium can be calculated via the planner's optimum.²¹
3. *Who owns entering capital?*—In the first decentralization the claims to the profits of the new firms will be held by incumbent firms, resembling corporate

¹⁸Alongside the sort of macro model we have built here, models of the various entry margins are developed and tested – see Wang (2007) for a systematic step in this direction.

¹⁹Some evidence on this is in Prusa and Schmitz (1994).

²⁰This assumption may not hold in fact; some ventures are started by people with no relevant labor-market experience.

²¹One could, instead, treat a part of the costs as external, such as

$$kh \left(\frac{Y}{k}\right) G \left(\frac{\mathbf{Y}}{\mathbf{k}}\right),$$

with k the component internal to the firm, and (\mathbf{Y}, \mathbf{k}) as the external part. The firm's problem would remain linearly homogeneous, and equilibrium would remain qualitatively the same but, in general, would no longer be efficient.

venturing activity that most large firms undertake.²² In the second decentralization the claims to the earnings will be owned by ex-employees of incumbent firms. The second decentralization is of greater empirical significance because only ten percent of venture investments are made by corporations – Gompers (2004, Chart 4). Fitting the model to composite capital

4 Fitting aggregate data

To fit the model’s implications we shall need to measure q , z , X , and Y , and we shall need to construct k . Because our measures of entering investment are for spending on a broad range of VC assets or for the valuation of IPOs, we shall need to construct a composite-capital series for k .

Calculating composite k .—We do this by combining a series for physical capital with a series for human capital. Consider two types of investment: In physical capital, $I_{K,t}$, and in human capital, $I_{H,t}$. Composite k evolves as

$$k_{t+1} = (1 - \delta)k_t + I_{H,t} + I_{K,t}. \quad (19)$$

We use the standard definition of physical investment as the measure for $I_{K,t}$ and discuss the measurement of $I_{H,t}$ below. Physical capital is defined as the year-end stock of private fixed assets (see footnotes 1 and 3 for data sources). The raw data on human capital are from Murphy and Tamura. They provide the number of low-skill and high-skill agents from 1901 to 2000, which we define as $N_{U,t}$ and $N_{S,t}$. Our skill premium series (see footnote 14) is given as $\lambda_t = \frac{w_{S,t}}{w_{U,t}}$, where $w_{U,t}$ and $w_{S,t}$ are wages per person at date t .²³ We proceed as follows:

1. Calculate the real stock of human capital, H_t^R , as $H_t^R \equiv N_{U,t} + \lambda_t N_{S,t}$.
2. Convert H_t^R into nominal units by multiplying it by the CPI, P_t , to get $\hat{H}_t \equiv P_t H_t^R$.
3. So calculated, \hat{H} grows more slowly than K (the stock of physical capital) over the century. The nominal indexes in Figure 7 show this for both quantities. We therefore adjust the average growth of \hat{H} by the exact amount needed to ensure

²²Corporate venture investment is remarkably similar to that of independent VCs. The main difference that is relevant here is that corporations have a slight tendency to avoid seed rounds and early rounds (Dushnitsky and Shapira 2007, Table 2). In 1983 Corporations made only 5 percent of all VC investments, but by 1994 this was up to 12 percent (Gompers 2002, Table 3).

²³Since the numbers of high- and low-skilled individuals ends in 2000 and their wage ratios end in 1998, we use the average annual growth rates of each over the proceeding five years to bring these series forward to 2005.

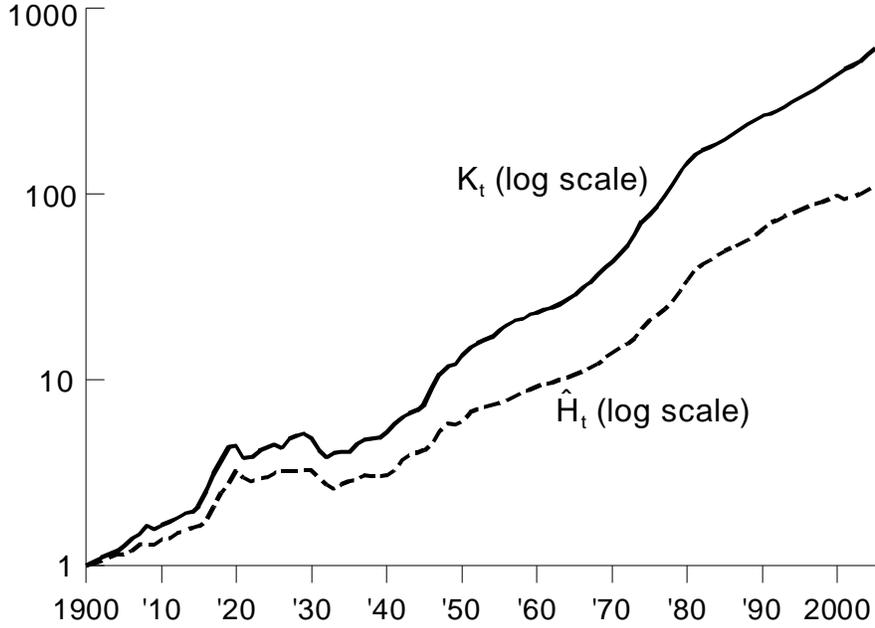


Figure 7: INDEXES OF NOMINAL K AND \hat{H} , 1900-2005

that it grows as fast as the K series over the century. This is done by solving for g_H in the equation

$$\frac{\hat{H}_{2005}}{\hat{H}_{1900}} e^{105g_H} = \frac{K_{2005}}{K_{1900}},$$

and then defining

$$\tilde{H}_t = \hat{H}_t e^{(t-1900)g_H}.$$

4. We calculate $\tilde{I}_{H,t}$ from the \tilde{H} series as

$$\tilde{I}_{H,t} = \tilde{H}_{t+1} - (1 - \delta)\tilde{H}_t. \quad (20)$$

5. We treat the average level of H as an unobservable. This introduces a free scaling parameter H_0 whereby we let $I_{H,t} = H_0 \tilde{I}_{H,t}$, and $H_{1901} = H_0 \tilde{H}_{1901}$. The resulting series for H_t calculated via $H_{t+1} = (1 - \delta)H_t + I_t$ is proportional to H_0 .
6. To start the iteration using equation 19, we set $k_{1901} = K_{1901}^\theta H_{1901}^{1-\theta}$, where $\theta = 0.25$. We then iterate on equation 19 to get a series for the composite capital measure, k .

Calculating z .—Since output is zk , we measure z by the ratio of private output over the course of a given year to private capital at the start of that year.²⁴ Having obtained composite k , we then set $z_t = GDP_t/k_t$ for all t . This measure is not accurate for the period from the start of U.S. involvement in World War II until several years after the war because, as Gordon (1969) explains, capital used by private firms was sometimes classified as government capital, and this would cause \hat{z} to be biased upwards. For this reason we exclude the 1941-1953 period from our regression analysis below.

Calculating q .—In light of (6), we measure q by Tobin’s Q.

Calculating Y .—We cannot measure Y_H and Y_K separately, only their sum Y . We entertain two measures for Y_t . The first is total investment of venture capital funds. The second is the volume of IPOs. We now discuss each in turn.

Y as VC investment.—Venture funds concentrate on high-tech startup investment, and very little else. Occasionally venture capital (VC) funds are involved in taking mature companies public, but this is rare and the investment involved is small. Thus, while our series for VC-investment excludes new firms that are not backed by venture capital, at least it excludes anything that one might call incumbent investment. On the other hand, not all the investments that VCs make are on plant and equipment – some of them are used to pay rent on office space, and on salaries, raw materials, etc. The correlation between this measure of investment and the investment by the portfolio companies themselves on physical capital can be understood in terms of the following breakdown: (i) How fast does a portfolio company spend its VC funding – sometimes called the “burn rate,” and (ii) What fraction of that spending gets used on physical capital. Regarding (i), since VC-funding rounds are rarely farther apart than a year, and since the frequency of our data is in any event annual, the amount of time-averaging implied by this is minimal as long as companies spend most of the money from one round by the time that the next round begins. Regarding (ii), this is likely to depend very much on the sector. In biotechnology, for example, the high cash burn rate is typically due to R&D. In internet-related businesses, it often goes on advertising. In either case it is not physical capital that is created but rather something intangible that we hope to capture with our measure of human capital.²⁵

Figure 8 shows a strong and positive univariate relation between the log of V_t/k_{t-1} defined as the ratio of total VC investment per \$1000 of beginning-of-year *composite*

²⁴Private output, defined as gross domestic product less government expenditures on consumption and investment, are from the Bureau of Economic Analysis (2006) for 1929-2005, to which we ratio splice Kendrick’s (1961, Table A-IIb, pp. 296-7, col. 11) estimates of gross national product less government.

²⁵Clayton (2002) lists the following 6 broad types of intangibles: (i) Teams of employees, (ii) Intellectual property, including patents, copyrights, trademarks, licenses, and domain names, (iii) Contracts, agreements, licenses, (iv) Customer lists (v) Data, trade secrets, (vi) Work in process. VC investment creates all these forms of capital. IPO valuations of the company will also capture this capital. Thus both measures overstate the creation of physical capital.

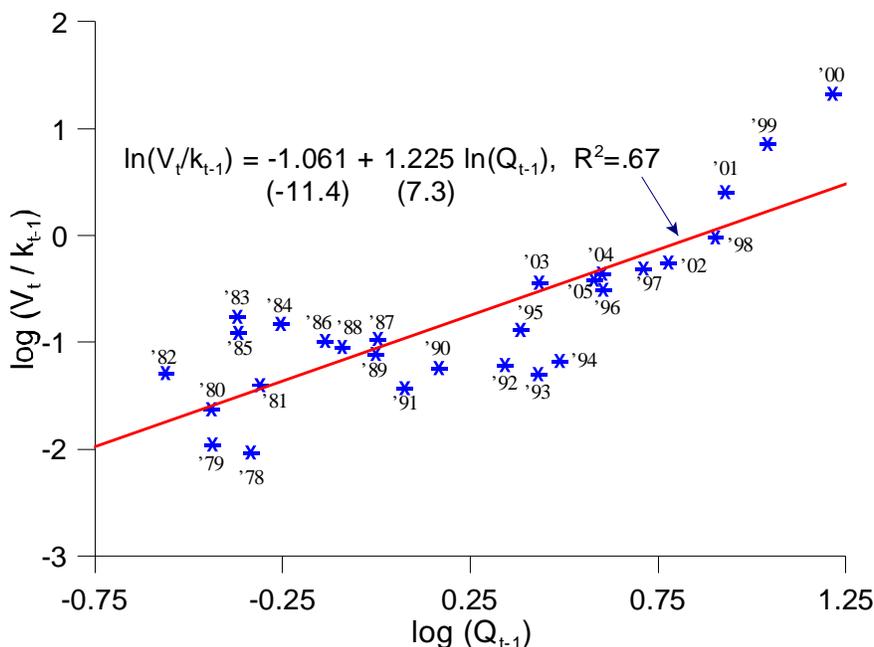


Figure 8: THE RELATION BETWEEN VENTURE INVESTMENT (WITH COMPOSITE CAPITAL) AND Q , 1978-2005

capital, and the log of Q at the start of each year from 1978 to 2005.

In Figure 8 our measures of “entering investment” do not seem to show the same pattern as the “Class-1” (i.e., low payout) sample of Fazzari, Hubbard and Petersen (1988) or the “immature firms” sample of Chirinko and Schaller (1995). These studies work entirely with publicly-owned firms, and the authors chose their sub-samples in order to identify firms that were likely to be liquidity constrained. These authors found a smaller Q -elasticity of investment among the low-payout and immature firms than among high-payout and mature firms. It is clear that our data on entering investment cover firms that are not so liquidity constrained. That is certainly so for the VC-backed investment data. A VC-backed firm may feel that it is liquidity constrained, but the VC funds backing it usually are not, because the investment “overhang” (i.e., the amount by which monies committed to the VC fund exceed those actually used to fund their investments) typically exceeds an entire year’s worth of investment (Thompson Venture Economics). Roughly speaking, then, VC-backed firms have indirect and immediate access to about one-year’s worth of investment. Indirectly then, one may be tempted to infer liquidity constraints, but this would be unwarranted. VC-backed firms are usually much more focused on state-of-the-art technologies than other small firms, and much more likely to eventually go public and for that reason if for no other, much more likely to be sensitive to variation in Tobin’s Q .

The volume of IPOs.—An IPO represents a transition rate from one financing status to another. It arguably measures the rate at which firms acquire easier financing, since one motive for IPOs, it is said, is to gain access to liquidity. It is apparent that there is a “timing” aspect to the IPO decision; The owners of the firm will want to exercise the option to have an IPO when market values are high and the owners can get the most for their shares. Rather than a measure of real investment, one can argue that what we measure is simply the exercise of a financial option. On the other hand, evidence also shows (Chemmanur, He and Nanda 2005, Figure 3) that a firm’s investment rises by the non-trivial factor of 1.4 around the time of IPO (± 2 years), which means that IPOs also measure a rise in investment. But such investment is less ‘entry’ investment than VC-backed investment is because the firms are often a lot older – see Figure 1 of Jovanovic and Rousseau (2001b). Our second measure for y_t is the total year-end market value of firms that had an IPO during year t , which we denote by *IPO capitalization*. Though with a lag, this measure will include not only VC investments but also corporate venturing investments (Gompers 2002).²⁶ It is remarkable how similar the corporate-venturing portfolio firms are to the independent VC-backed portfolio companies. They are concentrated only slightly in investment rounds that come later than the typical rounds of independent-VC-backed investments, but are otherwise quite similar in terms of the size or the investments and industries covered. Corporate-backed ventures are more likely to reach IPO (Gompers 2002, Table 6), which runs counter to popular perception that corporations invest in young startups in order to acquire them once they reach a viable stage. Recalling that q_t is the end-of period price of capital, the date- t value of entering-firms’ capital relative to the value of *beginning*-of-period capital is

$$\varepsilon_t^* = \frac{q_t y_t k_t}{q_{t-1} k_t} = \frac{q_t}{q_{t-1}} h'^{-1}(q_t) \quad (21)$$

(from (5)). Then the model says that z should not enter this equation once q_t and q_{t+1} are held fixed.

Functional form.—The functional form for adjustment costs will be assumed to be quadratic: $h(y) = \frac{\gamma}{2} y^2$, in which case $y = \frac{q}{\gamma}$. Although we do not have x in closed form as a function of q , we shall assume that it, too, is adequately represented by a linear function of q . Since $y_t = q_t/\gamma$, and so (21) reads

$$\varepsilon_t^* = \frac{q_t^2}{\gamma q_{t-1}} \quad (22)$$

or, taking logs, $\ln \varepsilon_t^* = -\ln \gamma + 2 \ln q_t - q_{t-1}$. We also note that z_t does not enter this regression. We measure $q_{t+1} y_t k_t$ as the end-of- t value of all the firms that listed during year t , and $q_t k_t$ is the value of firms listed at the beginning of year t . Figure 9 once

²⁶During the late 1960s and early 1970s, more than 25 percent of the Fortune 500 firms set up divisions that emulated venture capitalists.

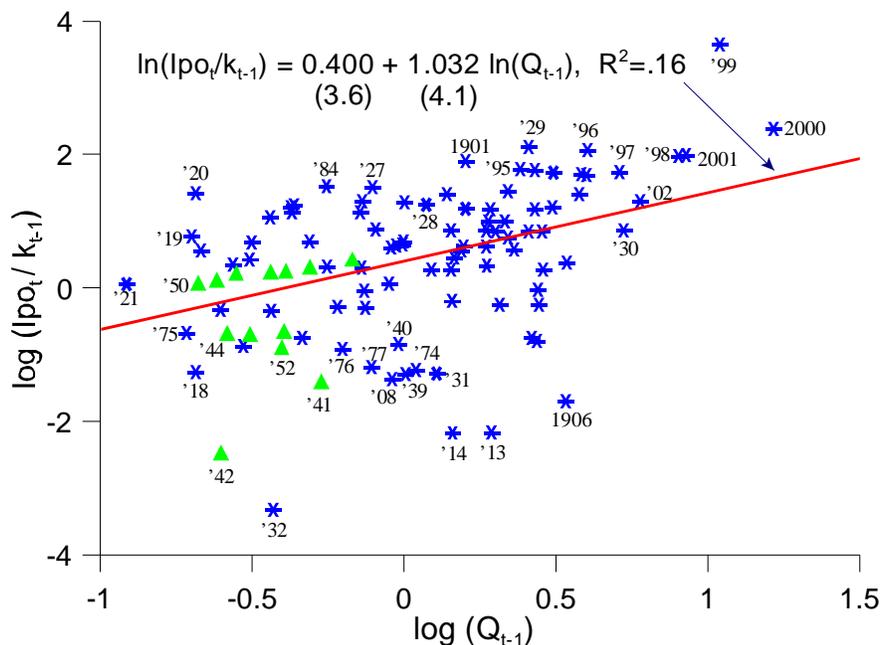


Figure 9: THE RELATION BETWEEN IPOs VALUES (WITH COMPOSITE CAPITAL) AND Q , 1901-2005

again shows a strong and positive univariate relation between this second measure of y and Q .²⁷

Calculating composite x and y .—We suppose that the ratio X/Y is the same for physical and for human capital. That is,

$$\frac{X_K}{Y_K} = \frac{X_H}{Y_H}.$$

Then we are able to calculate

$$x = \frac{X_K + X_H}{k} \quad \text{and} \quad y = \frac{Y_K + Y_H}{k}.$$

Using our estimate of composite incumbent investment, x , we proceed to examine the cross-section relationship with aggregate composite investment, $i = x + y$, and Q and compare the result with the Q -sensitivity of entry investment alone. This would be the right way to measure i if all entering investment was in the form of foregone output, but venture investment, for one, is measured entry investment. To account for this, we instead form i by defining $I_t = X_t + 2Y_t$ and then deflating by the composite capital stock at the start of the year (i.e., k_{t-1}). We do this because roughly half of the firms that have IPOs in the United States are VC backed. Indeed, given

²⁷The years from 1941 to 1953 appear as light triangles in Figure 10 and Figure 11 (below), and are not included in fitting the regression relationships shown.

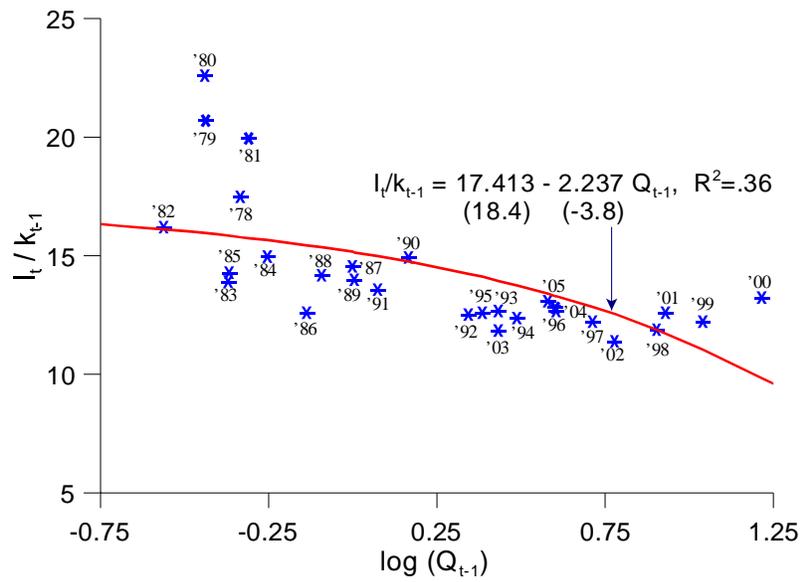


Figure 10: THE RELATION BETWEEN AGGREGATE COMPOSITE INVESTMENT AND Q , 1978-2005

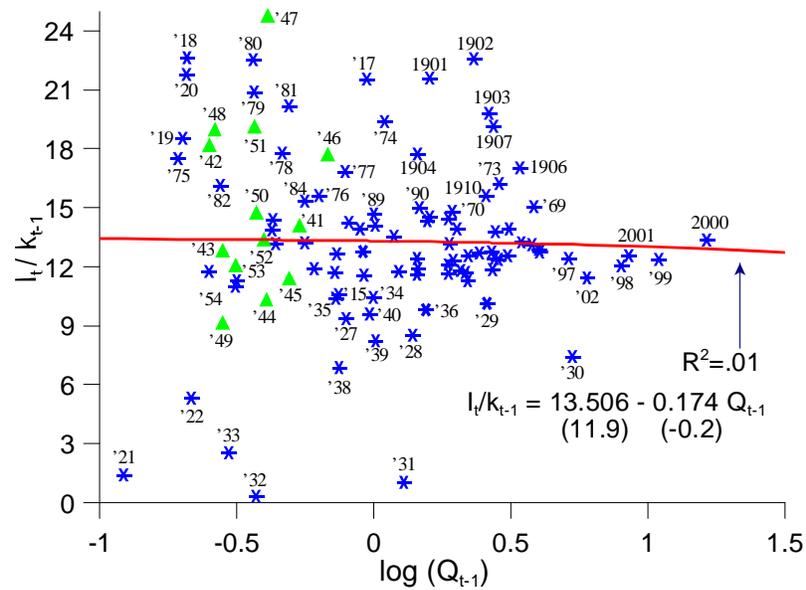


Figure 11: THE RELATION BETWEEN AGGREGATE COMPOSITE INVESTMENT AND Q , 1901-2005

that far fewer non-VC-backed firms ever go public, investment in these companies is probably higher than that in VC-backed firms, meaning that our I_t is probably still an underestimate of total composite investment. Figure 10 shows the negative and statistically significant cross-section relation that the model implies between I_t/k_{t-1} and the log of Q_{t-1} for the 1978-2005 period (i.e., the period covered by Figure 8). We form a second measure of composite I_t as the simple sum of incumbent investment and IPOs. Figure 11 shows that, using our second measure of investment, the relation is weaker for the century as a whole.

Regression results for the data with composite capital.—Proposition 1 tells us that the two investment policies, x and y , differ qualitatively in their dependence on q and z . The investment of entrants, y , should depend positively on q , and not at all on z . Investment of incumbents, x , should be decreasing in q and increasing in z . Table 1 reports the regressions that test these implications. In both the 1978-2005 and 1901-2005 periods, the coefficients on Q_{t-1} are positive and statistically significant for entrant investment, though z_t is also significant over the whole century. At the same time, the coefficients on Q_{t-1} are always negative in the incumbent and aggregate investment regressions, and z_t is always positive and statistically significant, just as the model would predict.

Table 1
Entering Capital and Aggregate Investment Regressions

	1978-2005				1901-2005			
	$\ln V_t/K_{t-1}$	X_t/K_{t-1}	I_t/K_{t-1}		$\ln Ipo_t/K_{t-1}$	X_t/K_{t-1}	I_t/K_{t-1}	
const.	-1.061 (-11.38)	-1.692 (-0.60)	-13.023 (-2.96)	-13.391 (-3.04)	0.400 (3.56)	-3.780 (-2.07)	-10.406 (-6.08)	-10.743 (-6.39)
Q_{t-1}	1.225 (7.28)	1.185 (4.76)	-0.442 (-1.07)	-0.338 (-0.77)	1.032 (4.07)	0.997 (4.01)	-0.687 (-1.50)	-0.247 (-0.55)
z_t		-0.363 (-0.22)	1.617 (7.00)	1.634 (7.06)		1.484 (2.29)	1.427 (15.11)	1.432 (15.43)
R^2	.67	.67	.79	.79	.16	.20	.72	.73
N	28	28	28	28	92	92	92	92

Note: T-statistics in parentheses. All variables are scaled as in Figures 1 and 2 and enter as log levels in the entry regressions and as simple levels in the X_t/K_{t-1} and I_t/K_{t-1} regressions. The regressions for 1901-2005 exclude the years 1941-53.

Simulations: criterion when choosing parameters.—the parameters are chosen so as to solve the following problem:

$$\max_{\{\beta, \delta, \sigma, \gamma, H_0\}} \left\{ \begin{array}{l} \lambda_\beta(\beta - 0.95)^2 + \lambda_\delta(\delta - 0.085)^2 + \lambda_\sigma(\sigma - 3)^2 + \lambda_x(\bar{x}_{\text{Fitted}} - \bar{x}_{\text{Data}})^2 \\ + \lambda_y(\bar{y}_{\text{Fitted}} - \bar{y}_{\text{Data}})^2 + \lambda_g \left(\sum_t [x_{\text{Fitted},t} + y_{\text{Fitted},t} - \delta - 0.019]^2 \right) \end{array} \right\} \quad (23)$$

Clearly, the choice of the weights λ_i affect the optimally chosen parameters. We chose the weights in such a way that the first three terms are of roughly the same order, the last three terms are of roughly the same order, but so that the last three terms are also roughly an order of magnitude more important than the first three. We apply this criterion to the long series (1901 – 2005) to solve for the parameters. The plots of the short series are done with the same set of parameters with the exception of γ .²⁸ The estimated parameters were

	1901-2005	1978-2005	λ -weights
β	0.952	0.952	1000
δ	0.067	0.673	1000
σ	4.10	4.10	0.1
γ	470	1865	
H_0	83	83	
θ	0.25	0.25	
mean(x)			1000
mean(y)			100,000
growth			10

Figure 12 plots the series for z_t that results with the above parameter settings as well as an approximation to this series using Tauchen’s procedure. Figure 13 plots the model prediction and the data for 1901-2005.

The short series.—For the years 1978-2005, we extract z from the series calculated above and then use Tauchen’s procedure to get a finer approximation to this part of the z series. The approximation is not as good due to the large drop in z at the start. We leave the parameter set unchanged, however, so that we are simply focusing on the model’s predictions for the shorter series, and comparing it to the VC series instead of the IPO series. Figure 14 plots the series for z_t that results with the above parameter settings as well as an approximation to this series using Tauchen’s procedure. Figure 15 plots the model prediction and the data for 1978-2005.

Regression results with the model-generated data.—As a check on the model with composite capital, we repeat the regressions in Table 1 using the simulated data

²⁸Since we use an alternate data series for y (i.e., VC investment) and are working with the expression $y = q/\gamma$, we allow γ to change in the simulation for 1978-2005. We calculate the new γ from a simple regression of the y data on the fitted q values.

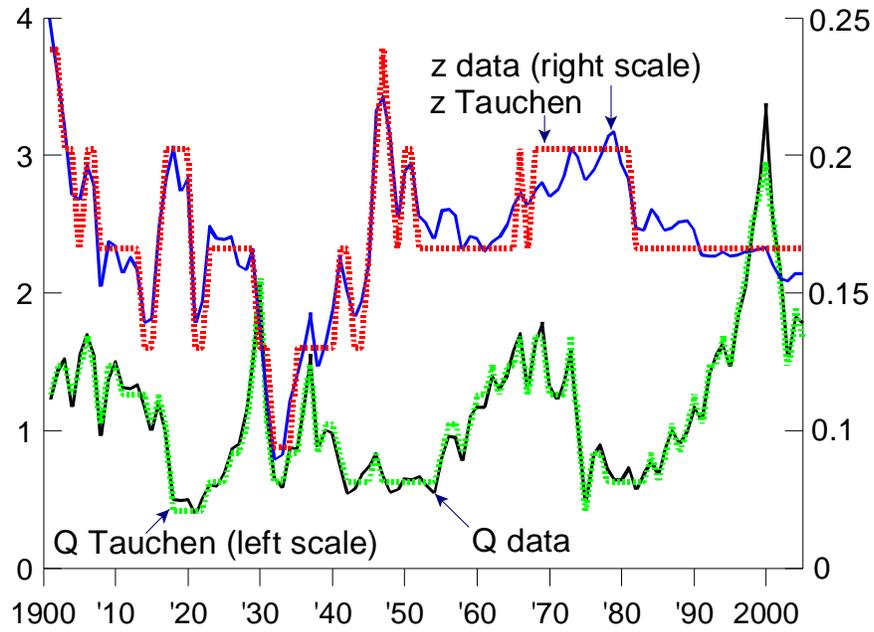


Figure 12: THE Q AND z SERIES AND THEIR DISCRETE APPROXIMATIONS, 1901-2005

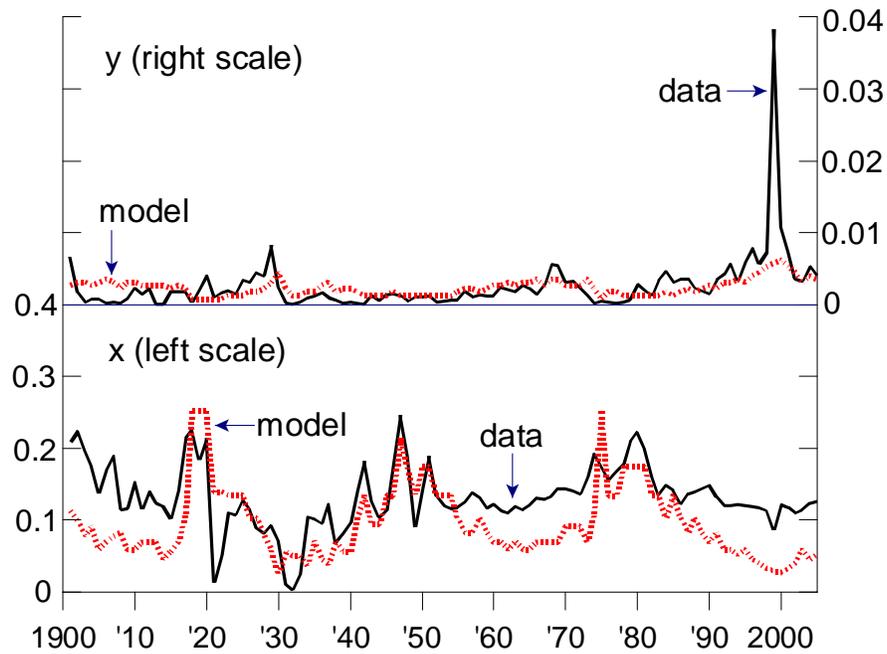


Figure 13: THE Y AND X SERIES AND THEIR SIMULATED VALUES (WITH COMPOSITE CAPITAL), 1901-2005

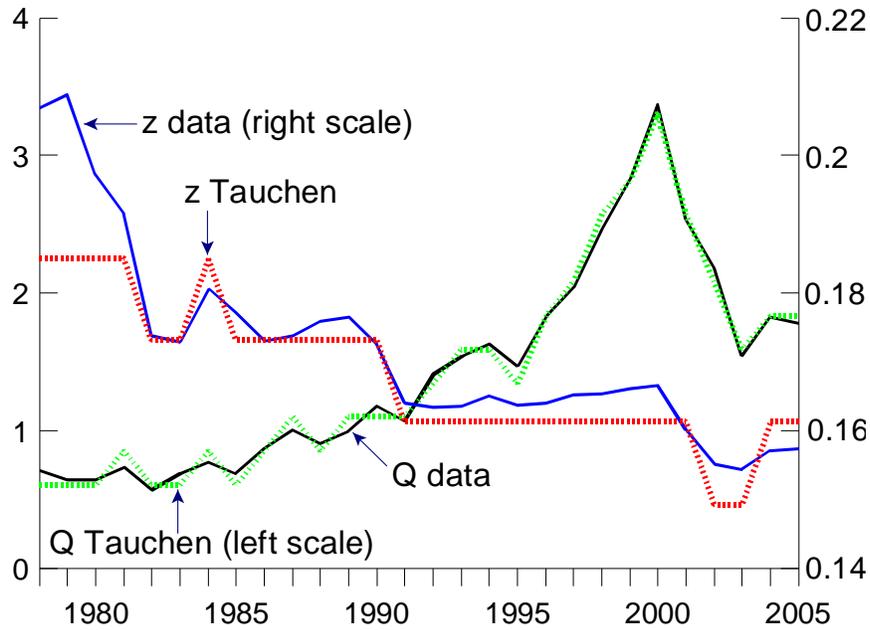


Figure 14: THE Q AND z SERIES AND THEIR DISCRETE APPROXIMATIONS, 1978-2005

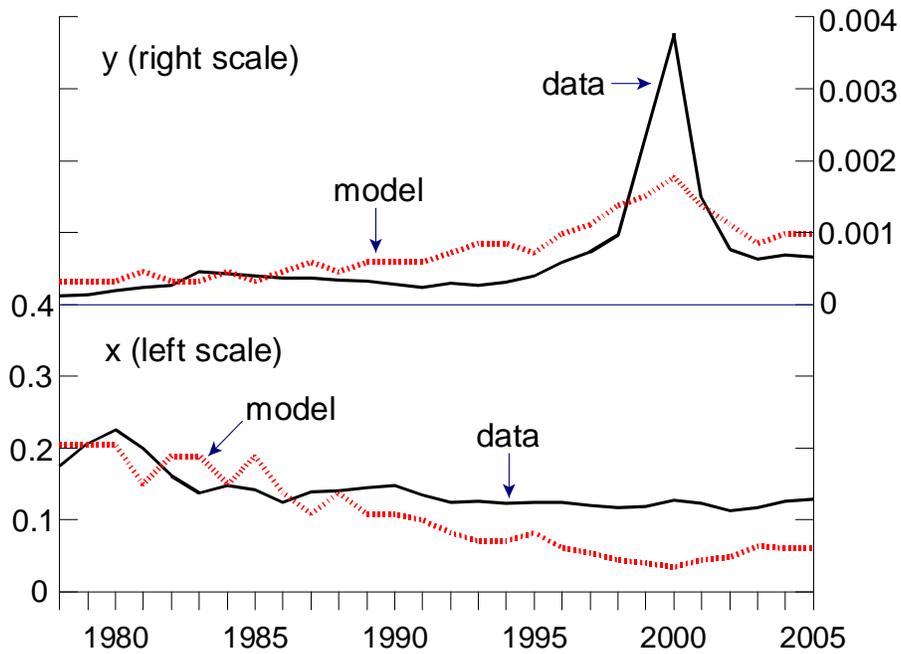


Figure 15: THE Y AND X SERIES AND THEIR SIMULATED VALUES (WITH COMPOSITE CAPITAL), 1978-2005

(i.e., the dashed lines in Figures 12, 13, 14, and 15). Since y must satisfy equation (5), if $h'(y)$ is quadratic, the simulation by definition generates a perfect fit between new investment (V_t/K_{t-1} and Ipo_t/K_{t-1}) and q , while there is no effect of z . We thus report only the investment (i.e., the XX_t/K_{t-1} and I_t/K_{t-1}) regressions in Table 2. In both time periods Q_{t-1} is strongly and negatively related to X_t/K_{t-1} and I_t/K_{t-1} as the model suggests. Indeed, the coefficients on Q_t are more strongly negative for X_t/K_{t-1} , which does not contain new investment, than they are for the broader I_t/K_{t-1} measure. At the same time, the predicted positive relations for z_t in \widehat{X}_t/K_{t-1} and I_t/K_{t-1} are statistically significant throughout.

Table 2
Entering Capital and Aggregate Investment Regressions,
Model-Generated Data with Composite Capital

	1978-2005		1901-2005	
	\widehat{X}_t/K_{t-1}	I_t/K_{t-1}	\widehat{X}_t/K_{t-1}	I_t/K_{t-1}
constant	-24.771 (-2.62)	-24.771 (-2.62)	2.269 (1.37)	2.269 (1.37)
Q_{t-1}	-4.398 (-6.11)	-4.345 (-6.04)	-7.052 (-14.46)	-6.840 (-14.04)
z_t	2.489 (4.82)	2.489 (4.82)	0.894 (9.75)	0.894 (9.75)
R^2	.89	.89	.76	.76
N	28	28	92	92

Note: T-statistics in parentheses. All variables are scaled as in Figures 1 and 2, and enter as simple levels. The regressions for 1901-2005 exclude the years 1941-53.

5 Fitting data with k as physical capital

In this section we fit the same model to the same data on output and Q , but now with k measured using data on physical capital alone. It follows that the implied z generated with this procedure will also be different. For the VC data in Figure 1, if the fraction spent on physical capital were fixed (and independent of Q), this series would accurately describe capital investment up to a factor of proportionality. For the IPO data, however, things are not as simple because IPO values are aggregates of past investments.

The physical-capital content of IPO values.—The value of an IPO-ing firm represents the market value of all of the firm’s assets, not just its physical capital. Further, the physical capital content of IPOs (and VC investment) is probably trending down over the century. Thus we will need to infer the fraction of the value attributable to physical capital. We proceed in three steps.

1. For the years for which Compustat has good coverage of exchange-listed firms (i.e., post 1977) we extract the share of capital in expenditures by sector. Pooling over years we then calculate for each sector the share

$$s_i \equiv \frac{E_K}{E_K + E_L}$$

where E_K is spending on K and E_L is the wage bill. In other words, we ignore intermediate goods and materials spending.

2. We have sectoral information for most IPOs for each year from 1901 to 2005. Let A_t be the set of IPOs for which such information is available in year t . For the years 1901-2005 we calculate the date- t share of physical capital in IPOs as

$$S_t \equiv \frac{1}{\sum_{i \in A_t} IPO_{i,t}} \sum_{i \in A_t} s_i IPO_{i,t}.$$

This assumes that the share of K in IPO value is the same as the share of K in costs.

3. Finally we assume that S_t was the same among the IPOs for which we have no sectoral information as it was for the ones that we do, and calculate the series

$$IPO_t \equiv S_t \sum_{i \in \{A_t \cup \sim A_t\}} IPO_{i,t}.$$

This is the calculated capital content of all IPOs at date t .

As in our analysis of investment in composite capital (i.e., physical and human) above, we multiply the VC series by a factor of two because roughly half of the firms that have IPOs in the United States are VC backed. We also must recognize, however, that only a fraction of what is called “entry investment” is spending on physical capital, while the rest is spent on labor and materials. On the other hand, a lot of the compensation in startups is in the form of deferred compensation, i.e., stock options. Therefore a fraction of a firm’s worth at private acquisition goes to purchase past-dated labor. To adjust for this, we next divide the VC series by a factor of four to capture physical capital’s share in output before proceeding with our regression and simulation analyses below.

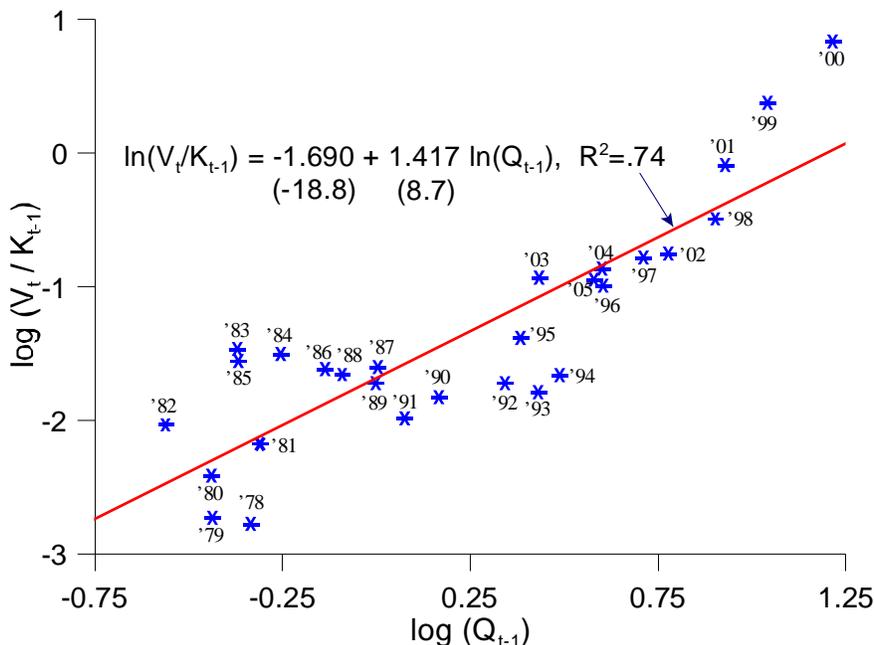


Figure 16: THE RELATION BETWEEN VENTURE INVESTMENT (WITH PHYSICAL CAPITAL) AND Q , 1978-2005

Figure 16 shows the strong and positive univariate relation between the log of V_t/K_{t-1} , defined as the ratio of total VC investment per \$1000 of beginning-of-year *physical* capital, and the log of Q at the start of each year from 1978 to 2005. Figure 17 shows that such a relation also holds when we measure V_t with IPO values over the entire century.

Measures of x .—We also entertain two measures for x based on the aggregate data for investment in physical capital. The first is private investment, I_t , deflated by the private capital stock at the start of the year, K_{t-1} . The second measure subtracts Y from I to arrive at an estimate of X that we denote by \hat{X}_t .

Figure 18 shows a weak and statistically insignificant cross-section relation between I_t/K_{t-1} and the log of Q_{t-1} for the 1978-2005 period (i.e., the period covered by Figure 8). Using IPO values to form Y_t , Figure 19 shows that the relation is a bit stronger for the century as a whole.

Regression results for the data in Figures 1 and 2.—Proposition 1 tells us that the two investment policies, x and y , differ qualitatively in their dependence on q and z . The investment of entrants, y , should depend positively on q , and not at all on z . Investment of incumbents, x , should be decreasing in q and increasing in z . Table 3 reports the regressions that test these implications. For both time periods, the

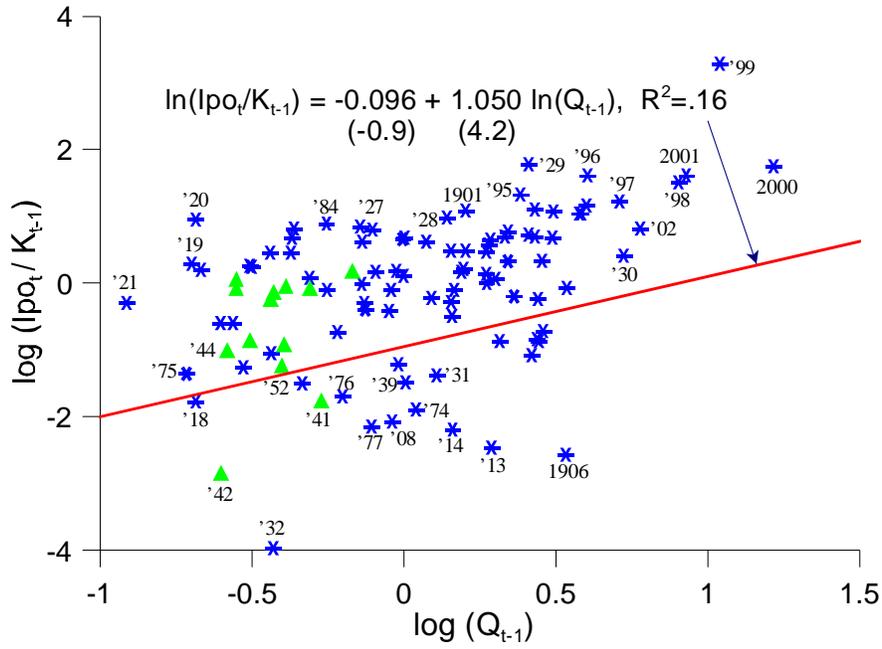


Figure 17: THE RELATION BETWEEN IPOs VALUES (WITH PHYSICAL CAPITAL) AND Q , 1901-2005

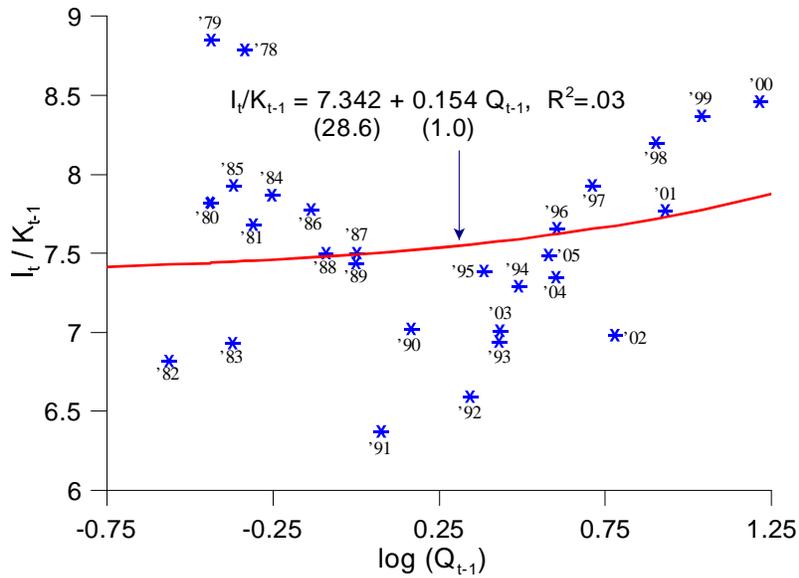


Figure 18: THE RELATION BETWEEN AGREGATE PHYSICAL INVESTMENT AND Q , 1978-2005

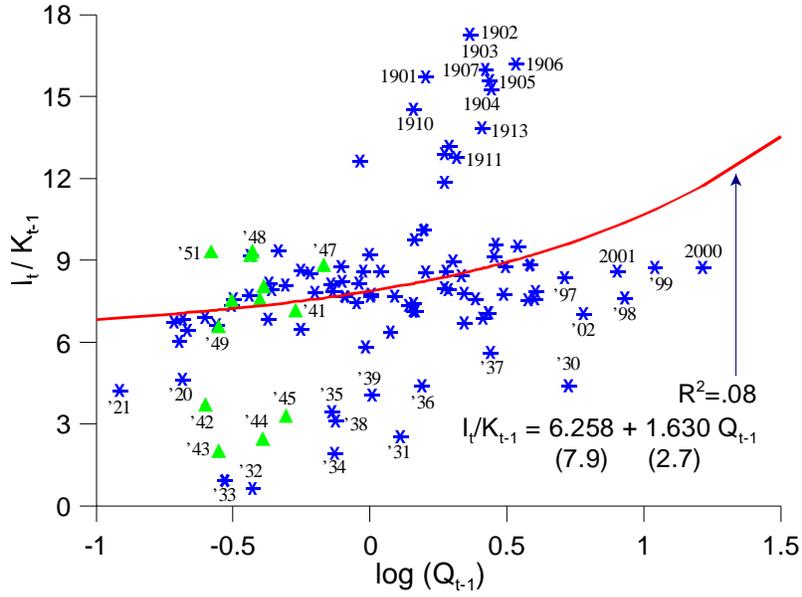


Figure 19: THE RELATION BETWEEN AGGREGATE PHYSICAL AND Q , 1901-2005

Table 3

Entering Capital and Aggregate Investment Regressions, Actual Data

	1978-2005				1901-2005			
	$\ln V_t/K_{t-1}$	\widehat{X}_t/K_{t-1}	I_t/K_{t-1}		$\ln Ipot/K_{t-1}$	\widehat{X}_t/K_{t-1}	I_t/K_{t-1}	
const.	-1.690 (-18.76)	19.256 (2.77)	2.864 (1.07)	3.040 (1.11)	-0.096 (-0.85)	-8.268 (-2.08)	-10.538 (-3.72)	-10.659 (-3.85)
Q_{t-1}	1.417 (8.72)	1.879 (8.98)	-0.147 (-0.70)	-0.082 (-0.38)	1.050 (4.15)	0.837 (3.10)	0.160 (0.29)	0.449 (0.84)
z_t		-5.780 (-3.01)	0.127 (1.70)	0.121 (1.58)		2.246 (2.05)	0.476 (6.16)	0.475 (6.29)
R^2	.74	.81	.12	.12	.16	.20	.33	.36
N	28	28	28	28	92	92	92	92

Note: T-statistics in parentheses. All variables are scaled as in Figures 1 and 2, and enter as log levels in the entry regressions and as simple levels in the \widehat{X}_t/K_{t-1} and I_t/K_{t-1} regressions. The regressions for 1901-2005 exclude the years 1941-53.

coefficients on q are positive and statistically significant in the y equations, whether we include z or not, but z turns out to be statistically significant for y when included in the regressions. In the x equations, the coefficients on Q_{t-1} are never statistically significant for \widehat{X}_t/K_{t-1} or I_t/K_{t-1} , but are negative as expected for the 1978-2005 period, while the coefficients on z_t are positive as expected in both time periods, and significantly so for 1901-2005. It is also interesting that the positive and significant coefficient on Q_{t-1} shown for the I_t/K_{t-1} equation in Figure 19 above does not survive when z_t is included in the regression. This is likely due to amelioration of an omitted variable bias, much like the explanatory power of Tobin's Q is reduced when cash flows are included in the investment regressions of Fazzari, Hubbard, and Petersen (1988). Though not as strong as our findings with the composite capital stock, we consider these results to be a qualified success for the model.

Simulations: criterion when choosing parameters.—We fit our two sets of data using physical capital only. Simulation 1 uses IPOs as the measure of new investment for the entire 20th century. Simulation 2 uses VC investment as the measure of investment by new firms and covers the post-1978 period.

Estimation routine.—The simulated policies are optimal for a discretized version of the (q, z) process. The parameters of the (q, z) processes were chosen to be AR(1) and fitted via the Tauchen-Hussey procedure. We let z take on 5 values and q take on 15 values. The statistical properties of the discretized processes were then presented to the Planner who chose the optimal policies. In each simulation we chose the parameters $\{\beta, \delta, \sigma, \gamma\}$ to solve the same problem as in (23), except that the parameter H_0 is absent. The estimated parameters were

	1901-2005	1978-2005	λ -weights
β	0.936	0.936	1000
δ	0.050	0.050	1000
σ	6.350	6.350	0.1
γ	439	3124	
mean(x)			1000
mean(y)			100,000
growth			10

Figure 12 plots the series for z_t that results with the above parameter settings as well as an approximation to this series using Tauchen's procedure. Figure 13 plots the model prediction and the data for 1901-2005.

The short series.—For the years 1978-2005, we extract z from the series calculated above and then use Tauchen's procedure to get a finer approximation to this part of the z series. We leave the parameter set basically unchanged, however, other than recomputing γ , so that we are simply focusing on the model's predictions for the shorter series, and comparing it to the VC series instead of the IPO series. Figure 22

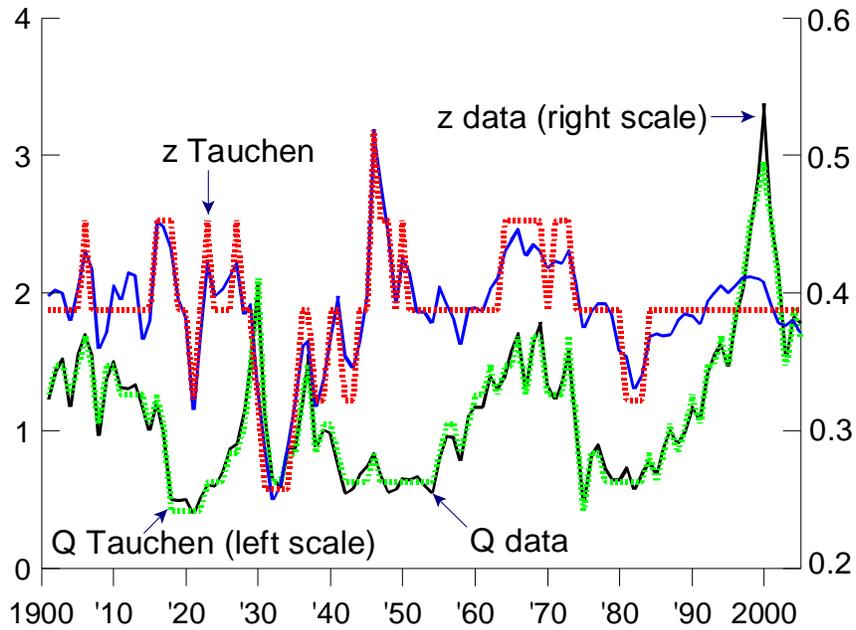


Figure 20: THE Q AND z SERIES AND THEIR DISCRETE APPROXIMATIONS, 1901-2005

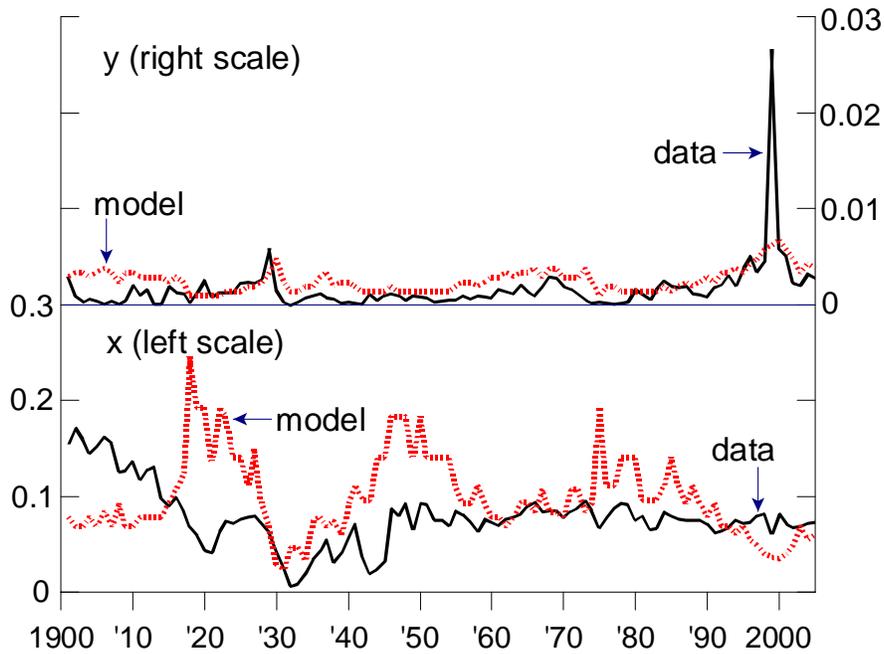


Figure 21: THE Y AND X SERIES AND THEIR SIMULATED VALUES (WITH PHYSICAL CAPITAL), 1901-2005

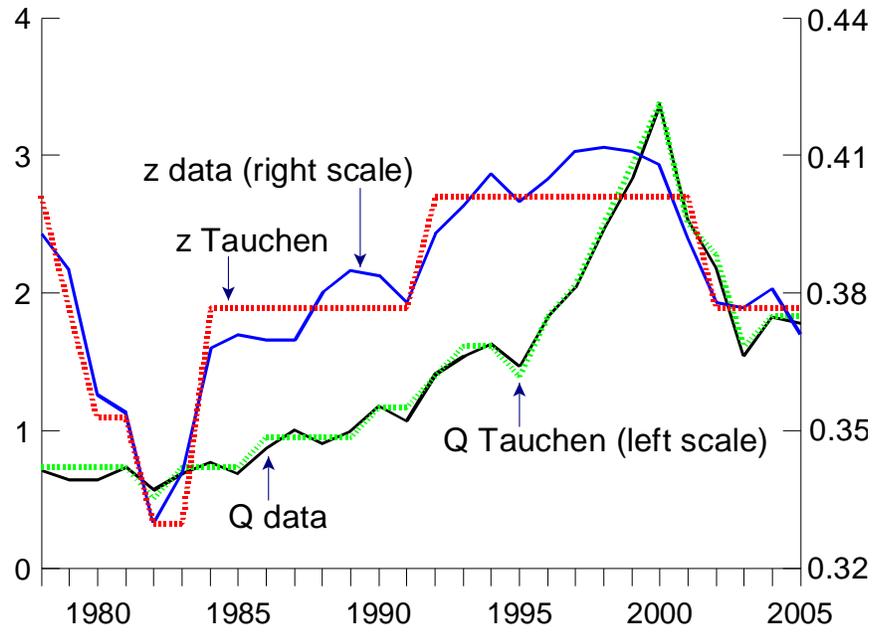


Figure 22: THE Q AND z SERIES AND THEIR DISCRETE APPROXIMATIONS, 1978-2005

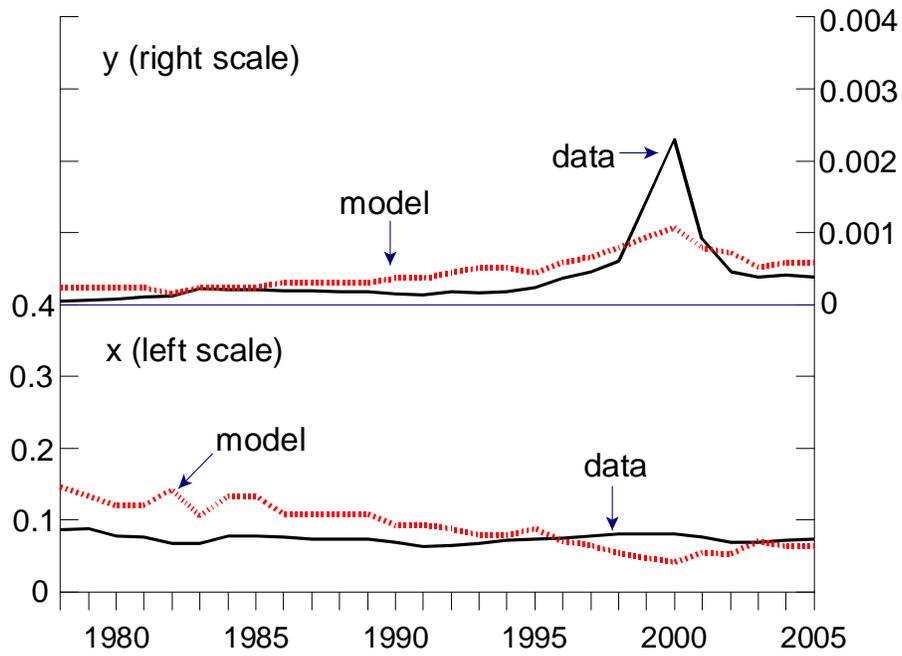


Figure 23: THE Y AND X SERIES AND THEIR SIMULATED VALUES (WITH PHYSICAL CAPITAL), 1978-2005

plots the series for z_t that results with the above parameter settings as well as an approximation to this series using Tauchen's procedure. Figure 23 plots the model prediction and the data for 1978-2005.

The discretized processes track the actual values extremely well.²⁹ In both simulations the model fits y pretty well, but it exaggerates the negative relation between Q ; the data seem to show an incumbent investment that is flat.

Regression results with the model-generated data.—As a final check on the model, we repeat the regressions in Table 3 using the simulated data (i.e., the dashed lines in Figures 20, 21, 22, and 23). Once again, equation (5) and a quadratic form for $h'(y)$ implies a perfect fit between new investment (V_t/K_{t-1} and Ipo_t/K_{t-1}) and q , so we report only the investment (i.e., the \widehat{X}_t/K_{t-1} and I_t/K_{t-1}) regressions in Table 4. In both time periods Q_{t-1} is strongly and negatively related to \widehat{X}_t/K_{t-1} and I_t/K_{t-1} as the model suggests. Indeed, the coefficients on Q_t are more strongly negative for \widehat{X}_t/K_{t-1} , which does not contain new investment, than they are for the broader I_t/K_{t-1} measure. At the same time, the predicted positive relations for z_t in \widehat{X}_t/K_{t-1} and I_t/K_{t-1} are statistically significant only for the 1901-2005 period.

Table 4
Entering Capital and Aggregate Investment Regressions,
Model-Generated Data with Physical Capital Only

	1978-2005		1901-2005	
	\widehat{X}_t/K_{t-1}	I_t/K_{t-1}	\widehat{X}_t/K_{t-1}	I_t/K_{t-1}
constant	11.001 (2.17)	11.001 (2.17)	7.019 (4.18)	7.019 (4.18)
Q_{t-1}	-3.966 (-10.43)	-3.934 (-10.34)	-5.567 (-11.25)	-5.339 (-10.79)
z_t	0.106 (0.75)	0.106 (0.75)	0.529 (5.69)	0.529 (5.69)
R^2	.86	.86	.63	.62
N	28	28	92	92

Note: T-statistics in parentheses. All variables are scaled as in Figures 1 and 2, and enter as simple levels. The regressions for 1901-2005 exclude the years 1941-53.

²⁹The simulations use the data from Figures 1 and 2 directly. We proxy z_t with private GDP as a percent of K_t , with the latter constructed as described in footnote 2.

6 Conclusion

We analyzed various measures of investment by new firms and we found that such investment responds to Tobin's Q much more elastically than does investment by incumbent firms, which responds hardly at all. We argue that this is because investment of new firms crowds out investment by incumbents more when Q is high than when Q is low. Paradoxically, the investment of incumbents is highly elastic and, for that very reason, easy to crowd out with little effect on stock prices.

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7 Appendix: Growth in the deterministic case

Lucas (1988) solved explicitly for the optimal and the equilibrium rate of growth. We can do that too, but only for some parameter values. When q and z are constant, the rate of growth of capital and output is $g \equiv x + y - \delta$. We can solve for g with the help of the following result:

Proposition 3 *When (q, z) are constant, y still solves (5) and x satisfies the implicit function*

$$q = \beta (1 - \delta + x + y)^{-\sigma} (z - qx - h(y) + q[1 - \delta + x + y]). \quad (24)$$

Proof. When q and z are constant, (6) becomes

$$q = \beta \left(\frac{C'}{C} \right)^{-\sigma} (z - qx - h(y) + q(1 - \delta + x + y)).$$

But C must grow at the same rate as k , namely $x + y - \delta$, and this leads to (24). ■

Solving for g in a special case.—For the case in which $\sigma = 1$ and $h(y) = \frac{2}{2}y^2$, we have $y = \frac{1}{\gamma}q$ and $h[y(q)] = \frac{1}{2\gamma}q^2$, and later on this Appendix shows that

$$x = \beta \frac{z}{q} - \frac{1}{2\gamma} (2 - \beta) q - (1 - \beta)(1 - \delta) \quad (25)$$

which is declining in q and increasing in z , β , and δ , and that

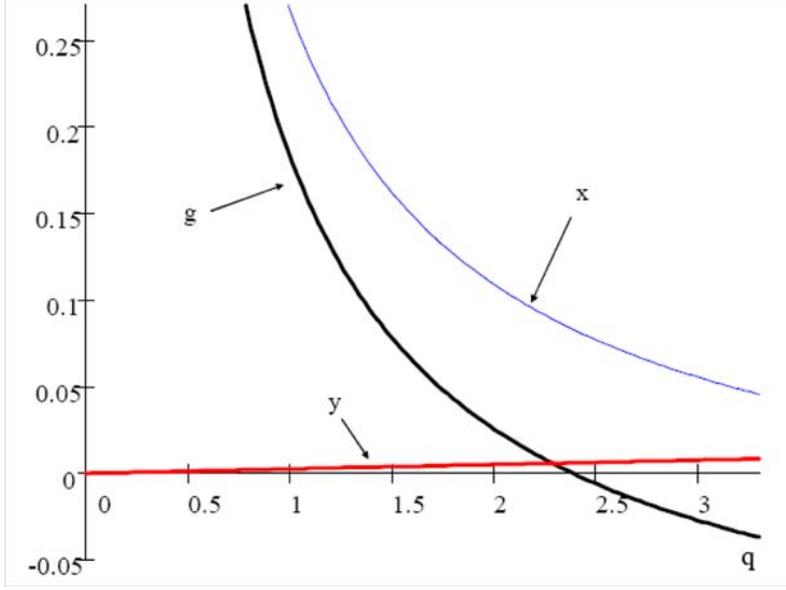
$$g = \beta \left(\frac{z}{q} + \frac{q}{2\gamma} \right) - (1 - \beta)(1 - \delta) \quad (26)$$

which is increasing in z and β , and decreasing in δ and in q , the latter being confined to the ‘admissible’ range in which $x > 0$.

Calibrated case.—If entrants investment is roughly one percent of GDP, then

$$\frac{Y}{zk} = \frac{y}{z} = \frac{q}{\gamma z} = 0.01. \quad (27)$$

Second, let the capital-output ratio be 3 so that $k/zk = 10$, implying that $z = 0.33$. The average value of q is 1.3. Then (27) implies that $\gamma = \frac{1.3}{(0.33)(0.01)} = 394$. We set $\beta = 0.95$, and $\delta = 0.10$ and obtain the following plot:



LONG-RUN GROWTH AND Q , $\gamma=394$, $\beta=0.95$, AND $\delta=0.10$.

The growth rate is the rate is the black line, and we also plot the investment of entrants, q/γ as the red line and of incumbents $x = i - y = \delta + g - y$ as the blue line and plot the result in the Figure.

Derivation of (25).—When $\sigma = 1$ and $h(y) = \frac{\gamma}{2}y^2$, (24) reads

$$q = \beta \left(1 - \delta + x + \frac{1}{\gamma}q\right)^{-\sigma} \left(z - qx - \frac{1}{2\gamma}q^2 + q \left[1 - \delta + x + \frac{1}{\gamma}q\right]\right)$$

so that

$$1 - \delta + x + \frac{1}{\gamma}q = \frac{\beta}{q(1 - \beta)} \left(z - qx - \frac{1}{2\gamma}q^2\right),$$

which, since $1 + \frac{\beta}{1 - \beta} = \frac{1}{1 - \beta}$, implies that

$$\begin{aligned} x &= \beta \left(\frac{z}{q} - \frac{1}{2\gamma}q\right) - (1 - \beta) \left(1 - \delta + \frac{1}{\gamma}q\right) \\ &= \beta \frac{z}{q} + \beta \frac{1}{2\gamma}q - (1 - \beta)(1 - \delta) - \frac{1}{\gamma}q \end{aligned}$$

i.e., (25).

Derivation of (26).—Since $g = x + \frac{q}{\gamma} - \delta$,

$$\begin{aligned} g &= \frac{2q}{2\gamma} + \beta \frac{z}{q} - \frac{1}{2\gamma}(2 - \beta)q - (1 - \beta)(1 - \delta) - \delta \\ &= \beta \frac{z}{q} + \frac{\beta}{2\gamma}q - (1 - \beta) + \delta(1 - \beta) - \delta, \end{aligned}$$

i.e., (26).

The ‘admissible’ range of qs for which $x > 0$.—Since x is decreasing in q , (25) states that the upper bound on q , call it q^u , solves the equation $x = 0$, i.e.,

$$\frac{1}{2\gamma} (2 - \beta) q^2 + (1 - \beta) (1 - \delta) q - \beta z = 0,$$

which means that (letting $b = (1 - \beta) (1 - \delta)$),

$$q^u = \frac{\gamma}{2 - \beta} \left(-b + \sqrt{b^2 + \frac{2(2 - \beta)\beta z}{\gamma}} \right) < \frac{\gamma}{2 - \beta} \sqrt{\frac{2(2 - \beta)\beta z}{\gamma}} < \gamma \sqrt{\frac{2\beta z}{\gamma}} < \sqrt{2\gamma z},$$

where the first inequality follows because for any $n > 0$, $\sqrt{b^2 + n} < b + \sqrt{n}$. Now differentiating in (26), this implies that

$$\frac{\partial g}{\partial q} = -\frac{z}{q^2} + \frac{1}{2\gamma} < -\frac{z}{2\gamma z} + \frac{1}{2\gamma} = 0.$$