WORKING PAPERS
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WORKING PAPER NO. 12-19
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August 2012

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA
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The author thanks Cyril Monnet, Mitchell Berlin, and Costas Azariadis for excellent discussions.

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Abstract

We show the existence of an inherent instability associated with a purely private monetary system due to the role of endogenous debt limits in the creation of private money. Because the bankers' ability to issue liabilities that circulate as a medium of exchange depends on beliefs about future credit conditions, there can be multiple equilibria. Some of these equilibria have undesirable properties: Self-fulfilling collapses of the banking system and persistent fluctuations in the aggregate supply of bank liabilities are possible. In response to this inherent instability of private money, we formulate a government intervention that guarantees that the economy remains arbitrarily close to the constrained efficient allocation. In particular, we define an operational procedure for a central bank capable of ensuring the stability of the monetary system.

Keywords: Private money; endogenous debt limits; central banking

JEL classification: E42, E51, E58

1. INTRODUCTION

A fundamental question in monetary economics is the following: Can we rely on private agents to provide an efficient and stable monetary framework? Some economists have argued that the government monopoly in the creation of money is itself a source of instability and that private markets are capable of providing a sound monetary framework. Others have argued that the government control over the monetary system is necessary for achieving stability. These concerns go back at least to Friedman (1960), Klein (1974), Hayek (1976), and Friedman and Schwartz (1986). In particular, there has been much emphasis on two polar views. Friedman (1960) has argued that the government should be the sole issuer of money because private creation of government money substitutes will necessarily lead to excessive volatility and, consequently, an unstable monetary system. In the other extreme, we have the argument made in Hayek (1976) that private agents through private markets can effectively achieve desirable outcomes, even in the field of money and banking. According to this view, the government monopoly in the creation of money is itself a source of instability, so it should be abolished.

Recent developments in monetary theory have allowed economists to provide a rigorous theoretical framework to discipline the debate. Some prominent papers analyzing the issue include Champ, Smith, and Williamson (1996), Cavalcanti, Erosa, and Temzelides (1999), Cavalcanti and Wallace (1999a, 1999b), Williamson (1999), Azariadis, Bullard, and Smith (2001), Antinolfi, Huybens, and Keister (2001), Li (2001), Martin and Schreft (2006), Li (2006), Mills (2007), and He, Huang, and Wright (2008), among many others. Our paper contributes to this literature in two ways. First, we demonstrate the existence of an inherent instability associated with a monetary system in which privately issued notes are the only available medium of exchange. As will be clear, the role of endogenous debt limits in the creation of private money will be crucial for understanding our results. Second, as a response to this intrinsic instability, we describe the role that publicly supplied money will play in stabilizing the economy. Thus, our results suggest that the argument in favor of a purely private monetary system can be misleading and that a specific form of monetary

intervention is socially desirable.

In our theoretical framework, each individual banker's ability to issue liabilities that can be used as a medium of exchange will depend on future credit conditions. The banker's willingness to redeem any previously issued note today will depend on the value attached to his business, given that the punishment for reneging on his liabilities is the loss of his note-issuing privileges. This value is determined by future credit conditions such as future credit limits and future prices for his bank notes. Thus, more favorable credit conditions in future periods (higher future credit limits and higher future prices for his bank notes) imply that he will be less inclined to renege on his promises. Because the value of his business today depends on beliefs about future credit conditions, there will be multiple equilibria. In particular, we show that some of these equilibria have undesirable properties: Some of them are characterized by a self-fulfilling collapse of the banking system in which each banker's balance sheet will persistently shrink over time, while others display large fluctuations in the amount of bank notes in circulation.

Specifically, we show that any equilibrium characterized by large fluctuations in the creation and circulation of bank liabilities is necessarily inefficient, giving us a rationale for considering some form of government intervention. We formulate an operational procedure for a monetary authority or central bank that guarantees the local determinacy of equilibrium, in which case the economy remains arbitrarily close to the constrained efficient allocation. Then, we argue that the local determinacy of equilibrium is sufficient to guarantee the stability of the monetary system.

The model developed here builds on the ideas in Kehoe and Levine (1993). In particular, we have used the formulation for economies with sequential trade described in Alvarez and Jermann (2000). These models are characterized by the absence of commitment, and the threat of exclusion from future credit markets is sufficient to induce cooperation in the credit system. The value attached to the ability to trade in credit markets in future periods is an important element to determine the set of feasible trades in the present. In our analysis, we build on this idea to formulate the decision problem of a banker who is able to issue liabilities that may circulate in the economy.

The physical structure of our model and the pattern of trade it implies build on Lagos and Wright (2005). In particular, buyers and sellers trade sequentially in centralized and decentralized markets. They are anonymous and lack any commitment, which implies that any form of trade credit in decentralized exchange is infeasible. Thus, they need a medium of exchange to conduct their transactions. Following the analysis of a private monetary system in Cavalcanti, Erosa, and Temzelides (1999) and Cavalcanti and Wallace (1999a, 1999b), we assume that a subset of private agents is able to issue notes that circulate in the economy, and we refer to these agents as the bankers. The bankers are able to create notes that can be used as a medium of exchange because they have the ability to make their actions publicly observable (they are endowed with a record-keeping technology that allows them to publicly report their trades). This possibility, combined with the existence of a mechanism to punish any banker who defaults on his liabilities, results in a monetary arrangement in which the bankers' IOUs circulate as a medium of exchange.

Cavalcanti, Erosa, and Temzelides (1999) have shown that a private monetary system can be stable in the context of a standard random-matching model. They introduce a mechanism for note exchange that makes the creation of private money possible. Such a mechanism imposes that any banker who fails to redeem his notes will lose his note-issuing privileges. As a result, it is possible to obtain cooperation due to the threat of exclusion from the business of issuing notes, disciplining the amount of notes issued by any individual banker. Thus, their analysis also builds on the idea that, in the absence of commitment, some form of punishment for misbehavior is sufficient to guarantee cooperative behavior. In particular, they show that a version of the law of reflux holds, guaranteeing that bankers do not overissue notes. Because of the difficulty of solving for an equilibrium in such an environment, the authors restrict their analysis to stationary equilibria. Our goal is precisely to emphasize non-stationary allocations, paying particular attention to the dynamics. And we show that the analysis of non-stationary equilibria matters for the conclusions regarding the stability and efficiency of a private monetary system.

Azariadis, Bullard, and Smith (2001) have characterized the dynamic properties of a private monetary system and a hybrid system in which privately and publicly issued notes

coexist. The authors construct an overlapping generations model in which trade is imperfectly coordinated due to spatial separation. As a result, private debt can circulate as a medium of exchange. In contrast to their analysis, our framework emphasizes the role of endogenous debt limits. This emphasis results in very different conclusions. In particular, we show that a purely private monetary system can result in very large fluctuations that, in some cases, drive the economy to the autarkic allocation, as a result of a self-fulfilling collapse of the banking system. Moreover, we show the existence of a central bank intervention that is capable of keeping the economy arbitrarily close to the constrained efficient allocation. Thus, we find that the local determinacy of equilibrium is sufficient to guarantee a stable monetary framework.

A recent paper by Gu and Wright (2011) also emphasizes the role of endogenous debt limits in determining the dynamics of pure credit economies. These authors describe equilibria of an economy characterized by an intertemporal absence of double coincidence and lack of commitment, similar to ours. Their model is one of bilateral credit in which endogenous credit limits arise because of the agents' inability to commit to their promises, in which case credit limits today will depend on future conditions. In particular, they show that the set of equilibrium allocations can be very large, with some of them displaying interesting dynamics: Both deterministic and stochastic cycles as well as chaos are possible outcomes. In our analysis of a private monetary system, we find that some of the properties they find in their model are also observed in ours, despite the fact that private IOUs circulate in our model but not in theirs. We take the analysis one step further and ask the question: Is there an intervention that allows us to rule out such undesirable equilibria? Thus, our contribution is to formulate a government intervention designed to eliminate the possibility of having non-stationary equilibria with undesirable properties.

The paper is structured as follows. In Section 2, we present the basic framework, and we discuss its main ingredients in Section 3. In Section 4, we formulate and solve the planner's problem. In Section 5, we characterize equilibrium allocations under a purely private monetary system. In Section 6, we discuss the role of a monetary authority or central bank in stabilizing the economy. Section 7 concludes.

2. MODEL

Time $t = 0, 1, 2, ...$ is discrete, and the horizon is infinite. Each period is divided into two subperiods: day and night. There are two physical commodities: a daytime good and a nighttime good. There are three types of agents: buyers, sellers, and bankers. There is a $[0, 1]$ continuum of each type. Buyers, sellers, and bankers are infinitely lived.

The bankers want to consume the daytime good but cannot produce such a good. They have access to a storage technology that allows them to store the daytime good. This means that the daytime good can be either immediately consumed or stored as an inventory to be consumed in the following day subperiod. The storage technology returns $\beta^{-1} > 1$ units of the daytime good at date $t+1$ for each unit of the daytime good invested at date t. Finally, the nighttime good cannot be stored and must be immediately consumed.

Buyers and sellers want to consume and are able to produce the daytime good. Specifically, they produce the daytime good using a divisible technology that gives one unit of the good for each unit of effort they exert. Only buyers want to consume the nighttime good, and only sellers are able to produce the nighttime good. In particular, sellers are endowed with a divisible technology that requires one unit of effort to produce one unit of the nighttime good.

We now explicitly describe preferences. Let $x_t^b \in \mathbb{R}$ denote a buyer's daytime net consumption, and let $q_t^b \in \mathbb{R}_+$ denote his nighttime consumption. His preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^b + u\left(q_t^b\right) \right],$$

where $\beta \in (0, 1)$. The function $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing, and strictly concave, with $u'(0) = \infty$. Let $x_t^s \in \mathbb{R}$ denote a seller's daytime net consumption, and let $n_t^s \in \mathbb{R}_+$ denote his nighttime effort level. His preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c\left(n_t^s\right) \right],$$

where $c : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, increasing, and convex. Let $x_t^n \in \mathbb{R}_+$

denote a banker's daytime consumption. Each banker has preferences given by

$$\sum_{t=0}^{\infty} \beta^t x_t^n.$$

Buyers and sellers are anonymous: There is no technology to verify their identities, and their trading histories are privately observable. Unless otherwise stated, all types of agents lack any commitment.

In the day subperiod, there is a perfectly competitive (Walrasian) market in which agents trade the daytime good. In the night subperiod, only buyers and sellers trade. Following the literature, we refer to this night market as the decentralized market. For simplicity, we will use competitive pricing to determine the terms of trade in this market, even though this assumption makes the decentralized market less decentralized. Despite this, a medium of exchange remains essential as long as we maintain the intertemporal double coincidence problem and anonymity; see Rocheteau and Wright (2005) for a discussion.

Bankers are endowed with a technology that allows them to make their actions publicly observable at no cost. We assume the existence of a mechanism that guarantees that the bankers who renege on their promises be punished. Precisely, we assume that a banker who defaults on his liabilities can no longer have his actions publicly observable. Moreover, any assets he holds when he defaults will be immediately seized. This means that a defaulter will lose his note-issuing privileges.

3. DISCUSSION OF THE MODEL

Our framework builds on Lagos and Wright (2005).[1] In the decentralized night market, the absence of recordkeeping, together with people's inability to commit to their promises, implies that a buyer and a seller can trade only if a medium of exchange is made available. Because the bankers can make their actions publicly observable, they are able to issue notes that can be used as a medium of exchange provided that people believe that they will be

[1] Precisely, our model builds on Monnet and Sanches (2012), who also build on Lagos and Wright (2005). An alternative tractable framework that also creates an essential role for a medium of exchange is the large household model in Shi (1997).

willing to redeem them at par at a future date. In this case, the sellers are willing to accept these notes as a means of payment, so the buyers are willing to purchase these notes to use them as a means of payment.

In this respect, the availability of public knowledge of the banker's actions is crucial for allowing people to identify the states of the world in which the banker will be willing to repay his creditors. In the decentralized night market, a seller does not trust a buyer's IOU because he knows that the latter cannot be punished in case of default. But the same seller may accept a banker's IOU because the banker can be punished if he fails to redeem his IOUs. Thus, there will be some states of the world in which the banker will be willing to redeem his notes at par, and everybody knows in which states this will happen. Figure 1 shows how a banker's note will circulate in the economy.

The banker's willingness to repay his creditors today depends on future credit conditions. If future credit conditions are more favorable for him (because of higher future credit limits and higher future prices for his bank notes), then the banker will be less inclined to renege on his promises, given that the failure to redeem his previously issued notes will result in the loss of his note-issuing privileges. As a result, his ability to borrow funds today through the sale of notes increases because his creditors know that he will have more to lose in case of default. This means that the creation of bank notes at any given date will crucially depend on beliefs about future credit conditions. And this is the key to understanding our results.

The pattern of trade in the model implies that the bankers' IOUs will be periodically redeemed at the beginning of each period (in the day subperiod). Such a predictable pattern of note redemptions will allow us to fully characterize non-stationary allocations, but it will obviously ignore interesting distributional effects. Our goal here is to emphasize the role of endogenous debt limits in determining the dynamic behavior of economies in which a medium of exchange is required to support trade, and we think that leaving out distributional effects is excusable, even though we think such distributional effects may be interesting.

4. CONSTRAINED EFFICIENT ALLOCATIONS

In this section, we characterize constrained efficient allocations. To account for the fact that only the bankers have access to the record-keeping technology, we impose the restrictions that, in the day subperiod, only the buyer can produce for the banker and only the banker can transfer resources to the seller. The planner must take this pattern of transfers as given when choosing an allocation. This restriction on the feasible transfers takes into account the role of bankers as providers of a record-keeping device to support trade in the night subperiod.

To facilitate the description of the planner's problem, let $y_t^b \in \mathbb{R}_+$ denote the buyer's production in the day subperiod, given that we expect the buyer to be a net producer of the daytime good. We also expect the seller to be a net consumer of the daytime good, so we can restrict attention to the case in which $x_t^s \in \mathbb{R}_+$.

We will characterize efficient allocations under the assumption that both the buyer and the seller can fully commit to the planner's allocation. Thus, the only incentive problem that the planner has to explicitly account for is the banker's desire to deviate from his proposed allocation. Let $\bar{U}^n \in \mathbb{R}$ denote the required utility level exogenously assigned to each banker, and let $\bar{U}^s \in \mathbb{R}$ denote the required utility level exogenously assigned to each seller. The planner's problem then consists of choosing an allocation

$$\left\{y_t^b, x_t^s, x_t^n, q_t^b, n_t^s\right\}_{t=0}^{\infty}$$

to maximize the lifetime utility of the buyer,

$$\sum_{t=0}^{\infty} \beta^t \left[-y_t^b + u\left(q_t^b\right)\right],$$

subject to the daytime resource constraints,

$$x_t^s + x_t^n = y_t^b, \text{ for each date } t \geq 0,$$

the nighttime resource constraints,

$$q_t^b = n_t^s, \text{ for each date } t \geq 0,$$

the constraint guaranteeing that the seller gets at least the initially promised utility level $\bar{U}^s \in \mathbb{R}$,

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c\left(n_t^s\right)\right] \geq \bar{U}^s,$$

the constraint guaranteeing that the banker gets at least the initially promised utility level $\bar{U}^n \in \mathbb{R}$,

$$\sum_{t=0}^{\infty} \beta^t x_t^n \geq \bar{U}^n,$$

and, finally, the banker's individual rationality constraints,

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(y_\tau^b - x_\tau^s\right) \geq y_t^b, \text{ for each date } t \geq 0.$$

Notice that, in the day subperiod, the banker is entitled to receive the transfer y_t^b from the buyer. Also, the planner instructs the banker to transfer the amount $x_t^s \leq y_t^b$ to the seller. The banker can refuse to make such a transfer to the seller and increase his daytime consumption by the amount x_t^s. The punishment for such a deviation will be exclusion from the transfer system designed by the social planner, resulting in the autarkic payoff. The banker's individual rationality constraint guarantees that he will prefer not to deviate from the planner's allocation.

We can rewrite the planner's problem as follows:

$$\max_{\left\{x_t^s, y_t^b, q_t^b\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[-y_t^b + u\left(q_t^b\right)\right], \tag{1}$$

subject to the following constraints:

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c\left(q_t^b\right)\right] \geq \bar{U}^s, \tag{2}$$

$$\sum_{t=0}^{\infty} \beta^t \left(y_t^b - x_t^s\right) \geq \bar{U}^n, \tag{3}$$

$$-x_t^s + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \left(y_\tau^b - x_\tau^s\right) \geq 0, \text{ for each date } t \geq 0. \tag{4}$$

Let $\lambda \in \mathbb{R}_+$ denote the Lagrange multiplier associated with (2), let $\gamma \in \mathbb{R}_+$ denote the Lagrange multiplier associated with (3), and let $\beta^t \mu_t \in \mathbb{R}_+$ denote the Lagrange multiplier

associated with (4) at date t. The first-order conditions for an interior solution are given by

$$-1+\gamma+\sum_{\tau=0}^{t-1}\mu_\tau=0, \tag{5}$$

$$\lambda-\gamma-\sum_{\tau=0}^{t}\mu_\tau=0, \tag{6}$$

$$u'\left(q_t^b\right)-\lambda c'\left(q_t^b\right)=0, \tag{7}$$

together with the complementary slackness conditions,

$$\lambda\left\{\sum_{t=0}^{\infty}\beta^t\left[x_t^s-c\left(q_t^b\right)\right]-\bar{U}^s\right\}=0,$$

$$\mu\left[\sum_{t=0}^{\infty}\beta^t\left(y_t^b-x_t^s\right)-\bar{U}^n\right]=0,$$

$$\mu_t\left[-x_t^s+\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}\left(y_\tau^b-x_\tau^s\right)\right]=0.$$

Combining (5) with (6), we find that

$$\lambda=1+\mu_t$$

for each date $t\geq 0$. This means that we must have $\mu_t=\bar{\mu}\geq 0$ for all $t\geq 0$, which implies

$$\frac{u'\left(q_t^b\right)}{c'\left(q_t^b\right)}=1+\bar{\mu},$$

for each date $t\geq 0$. Thus, it follows that $q_t^b=q^b$ for each date $t\geq 0$. Thus, a necessary condition for any constrained efficient allocation is to have a constant value for the production of the nighttime good.

If $\bar{\mu}=0$, then we have $q_t^b=q^*$ for all $t\geq 0$, with q^* satisfying

$$\frac{u'\left(q^*\right)}{c'\left(q^*\right)}=1,$$

in which case the banker's individual rationality constraint is not binding. Another possibility is to have $\bar{\mu}=\beta^{-1}-1$, in which case the allocation $q_t^b=\hat{q}<q^*$ for all $t\geq 0$, with $\hat{q}$ satisfying

$$\frac{u'\left(\hat{q}\right)}{c'\left(\hat{q}\right)}=\frac{1}{\beta},$$

together with $x_t^s = (1-\beta)^{-1}\bar{U}^s + c(\hat{q})$ and $y_t^b = (1-\beta)^{-1}\left(\bar{U}^s + \bar{U}^n\right) + c(\hat{q})$ for all $t \geq 0$, is a solution to the planner's problem provided that we choose

$$\bar{U}^n = \frac{1-\beta}{\beta}\left[\bar{U}^s + (1-\beta)\,c(\hat{q})\right].$$

In this case, the banker's individual rationality constraint binds at each date. We refer to this solution as a constrained efficient allocation.

5. PRIVATE MONETARY SYSTEM

In this section, we describe the equilibrium outcome of an economy without government intervention. In this economy, the money supply is completely endogenous: The bankers issue private debt that can be used as a medium of exchange so that the aggregate money supply depends entirely on the banking sector's willingness to expand its balance sheet.

To finance his investments at date t, the banker raises funds by selling notes to buyers. Then, he uses these funds to acquire property titles on the storage technology. At date $t+1$, he collects the proceeds from his investments and repays his creditors, consuming or reinvesting the remaining profits. Specifically, a note issued by a banker at date t gives him ϕ_t units of the daytime good and is a promise to repay one unit of the daytime good at date $t+1$ to the note holder. Each banker has a technology that allows him to create perfectly divisible notes at zero cost. Notes issued by one banker are perfectly distinguishable from those issued by any other banker so that counterfeiting is not a problem.

Throughout the paper, we restrict attention to symmetric equilibria in which all notes trade at the same price. This means that the notes issued by any pair of bankers will be perfect substitutes (as long as people believe both bankers will be willing to redeem them at par). Let ϕ_t denote the common price of a newly issued note in terms of the date-t daytime good so that $1/\phi_t$ gives the real rate of return for anyone who holds a note from date t to date $t+1$. Every agent in the economy will take the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ as given when making his individual decisions.

As we have seen, each banker's trading history will enter the public record of transactions if and only if he has not previously defaulted on his liabilities. This means that a banker

will be able to issue notes only if he has always redeemed his notes at par. If a banker deviates, then he will lose his note-issuing privileges and will have his assets seized.

5.1. Buyer's Decision Problem

Let $w_t^b(a)$ denote the value function for a buyer with a portfolio of a notes at the beginning of the day market, and let $v_t^b(a)$ denote the value function for a buyer with a portfolio of a notes at the beginning of the night market. The Bellman equation for a buyer in the day subperiod is given by

$$w_t^b(a) = \max_{(x,a')\in\mathbb{R}\times\mathbb{R}_+} \left[x + v_t^b\left(a'\right)\right],$$

subject to the daytime budget constraint

$$x + \phi_t a' = a.$$

Here a' denotes his choice of note holdings at the end of the day market. Because of quasi-linear preferences, the value $w_t^b(a)$ is an affine function, $w_t^b(a) = a + w_t^b(0)$, with the intercept $w_t^b(0)$ given by

$$w_t^b(0) = \max_{a'\in\mathbb{R}_+} \left[-\phi_t a' + v_t^b\left(a'\right)\right]. \tag{8}$$

Let p_{t+1} denote the price of one unit of the date-t nighttime good in terms of the date-$(t+1)$ daytime good. The Bellman equation for a buyer with a portfolio of a' notes at the beginning of the night market is given by

$$v_t^b\left(a'\right) = \max_{q\in\mathbb{R}_+} \left[u(q) + \beta w_{t+1}^b\left(a' - p_{t+1}q\right)\right], \tag{9}$$

subject to the liquidity constraint

$$p_{t+1}q \leq a'. \tag{10}$$

Note that $p_{t+1}q$ gives the number of notes that the buyer needs to give up (his nominal expenditure) in order to purchase q units of the nighttime good. Using the fact that $w_t^b(a)$ is an affine function, we can rewrite the Bellman equation (9) as follows:

$$v_t^b\left(a'\right) = \max_{q\in\mathbb{R}_+} \left[u(q) - \beta p_{t+1}q\right] + \beta a' + \beta w_{t+1}^b(0).$$

The liquidity constraint (10) may either bind or not, depending on the buyer's note holdings. In particular, we have

$$\frac{dv_t^b}{da}\left(a'\right) = \begin{cases} \frac{1}{p_{t+1}} u'\left(\frac{a'}{p_{t+1}}\right) \text{ if } a' < p_{t+1}\hat{q}\left(p_{t+1}\right); \\ \\ \beta \text{ if } a' > p_{t+1}\hat{q}\left(p_{t+1}\right); \end{cases}$$

where $\hat{q}\left(p_{t+1}\right) = \left(u'\right)^{-1}\left(\beta p_{t+1}\right)$. The first-order condition for the optimal choice of note holdings is then given by

$$-\phi_t + \frac{dv_t^b}{da}\left(a'\right) \leq 0,$$

with equality if $a' > 0$. If $\phi_t > \beta$, then the optimal choice of note holdings will be given by

$$u'\left(\frac{a'}{p_{t+1}}\right) = \phi_t p_{t+1}. \tag{11}$$

Because of quasi-linear preferences, all buyers choose to hold the same quantity of notes at the end of the day market and, consequently, purchase the same amount of the nighttime good. Thus, condition (11) gives the aggregate demand for notes as a function of the relative price of the nighttime good p_{t+1} and the price of notes ϕ_t. A higher price for the bankers' notes reduces the amount of notes demanded. The effect of the relative price p_{t+1} on the demand for notes depends on the curvature of the utility function $u\left(q\right)$. If $-\left[u''\left(q\right)q\right]/u'\left(q\right) < 1$, then an increase in p_{t+1} reduces the demand for notes, holding ϕ_t constant. In this case, the substitution effect dominates. If $-\left[u''\left(q\right)q\right]/u'\left(q\right) > 1$, then an increase in p_{t+1} results in a higher demand for notes. In this case, the substitution effect is dominated.

5.2. Seller's Decision Problem

Let $w_t^s\left(a\right)$ denote the value function for a seller with a portfolio of a notes at the beginning of the day market, and let $v_t^s\left(a\right)$ denote the value function for a seller with a portfolio of a notes at the beginning of the night market. The Bellman equation for a seller in the day market is given by

$$w_t^s\left(a\right) = \max_{\left(x,a'\right)\in\mathbb{R}\times\mathbb{R}_+} \left[x + v_t^s\left(a'\right)\right],$$

subject to the daytime budget constraint

$$x + \phi_t a' = a.$$

Here a' denotes his choice of note holdings at the end of the day market. Similarly, the value $w_t^s(a)$ is an affine function, $w_t^s(a) = a + w_t^s(0)$, with the intercept $w_t^s(0)$ given by

$$w_t^s(0) = \max_{a' \in \mathbb{R}_+} \left[-\phi_t a' + v_t^s\left(a'\right) \right]. \tag{12}$$

The Bellman equation for a seller with a portfolio of a' notes at the beginning of the night market is given by

$$v_t^s\left(a'\right) = \max_{n \in \mathbb{R}_+} \left[-c(n) + \beta w_{t+1}^s\left(p_{t+1} n + a'\right) \right]. \tag{13}$$

Because the technology to produce the nighttime good is linear, n units of effort will yield n units of the good. This means that the seller will receive $p_{t+1} n$ notes in exchange for his supply of n units of the nighttime good. Using the fact that $w_t^s(a)$ is an affine function, we can rewrite the right-hand side of (13) as follows:

$$\max_{n \in \mathbb{R}_+} \left[-c(n) + \beta p_{t+1} n \right] + \beta a' + \beta w_{t+1}^s(0).$$

The first-order condition for the optimal choice of nighttime effort is then given by

$$c'(n) = \beta p_{t+1}. \tag{14}$$

Thus, condition (14) determines the nighttime effort decision as a function of the relative price of the nighttime good p_{t+1}. Note that a higher relative price induces the seller to produce more of the nighttime good. Finally, the first-order condition for the optimal choice of note holdings is given by

$$-\phi_t + \beta \leq 0,$$

with equality if $a' > 0$. This means that the seller will not hold notes at the end of the day market if $\phi_t > \beta$.

5.3. Banker's Decision Problem

Now we describe the decision problem of a banker. Let $w_t^n(b_{t-1}, i_{t-1})$ denote the value function for a banker with debt b_{t-1} and assets i_{t-1} at the beginning of date t. The banker's assets at the beginning of date t consist of titles on the storage technology acquired at date $t-1$, whereas the banker's debt refers to the amount of notes issued at date $t-1$. Thus, the banker's decision problem can be formulated as follows:

$$w_t^n(b_{t-1}, i_{t-1}) = \max_{(x_t, i_t, b_t) \in \mathbb{R}_+^3} \left[x_t + \beta w_{t+1}^n(b_t, i_t) \right], \tag{15}$$

subject to the daytime budget constraint

$$i_t + x_t + b_{t-1} = \beta^{-1} i_{t-1} + \phi_t b_t$$

and the debt limit

$$b_t \leq \bar{B}_t.$$

Here i_t denotes the amount of resources (units of the daytime good) that the banker decides to invest at date t. In other words, i_t gives the banker's assets at the beginning of date $t+1$. When making his investment decisions at each date, the banker takes as given the sequence of debt limits $\{\bar{B}_t\}_{t=0}^{\infty}$, the marginal return on his assets β^{-1}, and the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$.

If $\phi_t > \beta$, then the banker finds it optimal to borrow up to his debt limit, i.e., he will choose $b_t = \bar{B}_t$. Because the return paid on his notes (his cost of funds) is lower than the return on his assets, he makes a positive profit by borrowing and investing the proceeds in the storage technology. Note also that, because the return on his assets equals his rate of time preference, he is indifferent between immediately consuming and reinvesting the proceeds from his previously accumulated profits (his retained earnings). Therefore, a solution to the banker's optimization problem is $i_t = \phi_t \bar{B}_t$, which means that the banker invests all funds he has borrowed at date t but does not use his own funds. Thus, the balance sheet of a typical banker will have no equity, only debt. In this case, the banker's

consumption at date t is simply given by

$$x_t = \bar{B}_{t-1}\left(\beta^{-1}\phi_{t-1} - 1\right).$$

Thus, at each date t, the banker's discounted lifetime utility is given by

$$w_t^n\left(\bar{B}_{t-1}, \phi_{t-1}\bar{B}_{t-1}\right) = \sum_{\tau=t}^{\infty} \beta^{\tau-t}\bar{B}_{\tau-1}\left(\beta^{-1}\phi_{\tau-1} - 1\right).$$

This means that his lifetime utility at any point in time depends on future debt limits as well as future prices for his notes. Specifically, higher debt limits and higher prices for his notes will increase his lifetime utility, by increasing his future revenues and profit margins.

5.4. Aggregate Note Holdings

Let a_t denote the date-t aggregate note holdings. For any price $\phi_t > \beta$, the liquidity constraint (10) will bind, in which case the value of the notes in circulation must equal the value of the aggregate production in the night market:

$$a_t = p_{t+1}q_t. \tag{16}$$

Using (14) to substitute for p_{t+1}, we obtain the following equilibrium condition:

$$\frac{u'(q_t)}{c'(q_t)} = \frac{\phi_t}{\beta}. \tag{17}$$

This condition determines the production of the nighttime good as a function of the price of the bankers' notes. We can use (17) to implicitly define $q_t = q(\phi_t)$, in which case $q'(\phi_t) < 0$ for any ϕ_t. Thus, a higher price for the bankers' notes results in a lower amount produced and traded in the night market. The aggregate note holdings as a function of the price ϕ_t are given by

$$a(\phi_t) = \frac{c'[q(\phi_t)]\, q(\phi_t)}{\beta}. \tag{18}$$

Notice that $a'(\phi_t) < 0$ for any ϕ_t. This means that a higher price for the bankers' notes results in a lower demand for these notes.

5.5. Equilibrium

To define a symmetric equilibrium, we need to specify the sequence of debt limits $\left\{\bar{B}_t\right\}_{t=0}^{\infty}$ in such a way that the bankers are willing to supply the amount of notes other agents demand and are willing to fully repay their note holders. We take two steps to define a sequence of debt limits satisfying these two conditions. First, for any given sequence of prices $\{\phi_t\}_{t=0}^{\infty}$, we set

$$\bar{B}_t = a\left(\phi_t\right) \tag{19}$$

at each date t. This condition guarantees that each banker is willing to supply the amount of notes in (18) at the price ϕ_t so that the market for the bankers' notes will clear at each date. Recall that each banker finds it optimal to borrow up to his credit limit. Then, given this choice for the individual debt limits, we need to verify whether a particular choice for the price sequence $\{\phi_t\}_{t=0}^{\infty}$ implies that each banker does not want to renege on his liabilities at any date. As we have seen, a banker who reneges on his liabilities will lose his note-issuing privileges. Moreover, he will have his assets seized upon default. Thus, a particular price sequence $\{\phi_t\}_{t=0}^{\infty}$ is consistent with the solvency of each banker if and only if

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} a\left(\phi_{\tau-1}\right)\left(\beta^{-1}\phi_{\tau-1}-1\right) \geq a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1}-1\right)+\phi_t a\left(\phi_t\right)$$

holds at each date t. As in Alvarez and Jermann (2000), these solvency constraints allow the banker to borrow as much as possible without inducing him to default on his liabilities. The left-hand side gives the banker's beginning-of-period lifetime utility. The right-hand side gives the short-term payoff the banker gets if he decides *not* to invest the resources he has borrowed at date t. In this case, he can increase his current consumption by the amount $a\left(\phi_t\right)\phi_t$, but he will permanently lose his note-issuing privileges at date $t+1$, resulting in the autarkic payoff from date $t+1$ onward.

As in Alvarez and Jermann, we want to allow the bankers to borrow as much as they can and at the same time make sure that they do not want to default. If we define debt limits

that are not too tight, then we can rewrite the solvency constraints as follows:

$$-\phi_t a(\phi_t) + \beta w_{t+1}^n = 0, \tag{20}$$

where w_t^n denotes the banker's lifetime utility at the beginning of date t,

$$w_t^n = \sum_{\tau=t}^{\infty} \beta^{\tau-t} a\left(\phi_{\tau-1}\right)\left(\beta^{-1}\phi_{\tau-1} - 1\right).$$

We can also rewrite (15) as follows:

$$w_t^n = a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1} - 1\right) + \beta w_{t+1}^n. \tag{21}$$

Note that the term $a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1} - 1\right)$ gives the banker's current profit. It depends on the amount of notes issued at the previous date as well as the price at which the banker sold his notes at that time. Specifically, at date $t-1$, the banker received the amount $a\left(\phi_{t-1}\right)\phi_{t-1}$ in exchange for his notes. He invested this amount in the storage technology, obtaining the revenue $\beta^{-1}a\left(\phi_{t-1}\right)\phi_{t-1}$ at date t. Because each note is a promise to pay one unit of the daytime good, his current liabilities are $a\left(\phi_{t-1}\right)$. Thus, his profit will be given by the difference between the revenue $\beta^{-1}a\left(\phi_{t-1}\right)\phi_{t-1}$ and the repayment $a\left(\phi_{t-1}\right)$. As we have seen, he will immediately consume any profit he makes.

Combining (20) with (21), we obtain the following equilibrium law of motion for the price of notes:

$$\phi_t a(\phi_t) = a\left(\phi_{t-1}\right). \tag{22}$$

The formal definition of a perfect-foresight competitive equilibrium is now straightforward.

Definition 1 *A symmetric competitive equilibrium is a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ satisfying $\phi_t \geq \beta$ and (22) at each date t.*

Note that (20) indicates that the current amount of resources devoted to each banker (the supply of credit to the banking system) depends on his future credit limits and future prices of bank notes. Future credit conditions will be more favorable for the banker if future credit limits and future prices of bank notes are higher. In this case, the value attached to

his note-issuing privileges is higher, making it more costly for him to renege on his promises. If future credit conditions are less favorable for the banker (because of lower future credit limits and lower future prices of bank notes), then he will be more inclined to renege on his promises. This means that the supply of credit to the banking system today depends entirely on future credit conditions.

Proposition 2 *$\phi_t = 1$ for all $t \geq 0$ is the unique non-autarkic stationary equilibrium.*

Proof. It is straightforward to verify that the constant sequence $\phi_t = 1$ for all $t \geq 0$ satisfies (22). The uniqueness of this *interior* solution follows immediately from the fact that $a'(\phi) < 0$ for any ϕ. **Q.E.D.** ■

In this equilibrium, the credit limits and prices of notes will be constant over time. People do not expect future credit conditions to change over time and, as a result, the amount of notes issued at each date as well as their price will remain constant over time. In particular, people expect that future credit limits will always be given by $a(1)$ and that the price of notes will always be equal to one. People know that, as long as the amount raised from the sale of notes equals $a(1)$ for each banker at each date, he or she will find it optimal to make good on his or her promises. As a result, no banker will ever default along the equilibrium path.

From our characterization of constrained efficient allocations, we can easily conclude that the stationary equilibrium we have just described is constrained efficient. In other words, it is a solution to the planner's problem for an appropriate choice of the initially required utility levels $\bar{U}^s$ and $\bar{U}^n$.

Proposition 3 *The non-autarkic stationary equilibrium $\phi_t = 1$ for all $t \geq 0$ is constrained efficient.*

This means that a purely private monetary system is capable of implementing a constrained efficient allocation. Thus, in principle, we could conclude that there is no externality necessarily associated with the creation and circulation of private money. However,

as we will see, beliefs about future credit conditions can adversely influence the prices of bank notes in future periods, resulting in a situation in which the amount of bank notes in circulation will persistently decline over time. As a result, the amount of goods produced and traded in the decentralized night market will shrink over time, which is necessarily inefficient.

Specifically, depending on the initial condition ϕ_0, we can construct other equilibria in which credit limits and prices fluctuate over time. In these equilibria, the amount of resources devoted to the bankers at the current date will continue to depend on future credit limits and future prices of notes. The only difference is that future credit limits may be higher or lower than the current one and future prices maybe higher or lower than the current one. The dynamics will be completely driven by expectations about future credit conditions.

5.6. Self-fulfilling Collapses

In this subsection, we characterize equilibria for which credit limits monotonically decrease over time, which means that credit conditions constantly deteriorate over time. We interpret this kind of equilibrium as a self-fulfilling collapse of the banking system characterized by a persistent decline in the demand for bank notes driven by expectations that future credit conditions will persistently deteriorate. As we will show, this kind of equilibrium will have an adverse impact on the real economy. In particular, the quantities produced and traded in the decentralized night market will persistently decline over time, which is necessarily inefficient.

To facilitate our exposition, suppose that $u(q) = (1-\sigma)^{-1}\left(q^{1-\sigma} - 1\right)$. Assume further that $0 < \sigma < 1$. In this case, the revenue of each banker, given by $\beta^{-1}\phi a(\phi)$, is a decreasing function of the price ϕ because

$$\phi a'(\phi) + a(\phi) < 0$$

for any ϕ. As a result, we have

$$\frac{d\phi_t}{d\phi_{t-1}} = \frac{a'\left(\phi_{t-1}\right)}{\phi_t a'(\phi_t) + a(\phi_t)} > 0,$$

which means that (22) defines an implicit mapping $\phi_t = f\left(\phi_{t-1}\right)$ that is strictly increasing. In particular, we have

$$\left.\frac{d\phi_t}{d\phi_{t-1}}\right|_{\phi_{t-1}=\phi_t=1} = \frac{a'(1)}{a'(1)+a(1)} > 1,$$

which means that the mapping $\phi_t = f\left(\phi_{t-1}\right)$ crosses the 45-degree line from below at $\left(\phi_{t-1}, \phi_t\right) = (1,1)$. Thus, for any initial condition $\phi_0 > 1$, the equilibrium price trajectory is strictly increasing and unbounded. Along this equilibrium path, the individual debt limits, given by $\bar{B}_t = a(\phi_t)$, decrease monotonically over time and converge to zero. This means that the equilibrium allocation approaches the autarkic allocation as $t \to \infty$. As a result, liquidity becomes scarcer and more expensive over time, and buyers and sellers will be able to trade smaller amounts of goods in the decentralized night market.

We can interpret this kind of equilibrium as a self-fulfilling collapse of the banking system. As we have seen, the determination of equilibrium quantities and prices totally depends on the beliefs of people regarding future credit conditions. Because people believe that future credit limits and future prices will persistently decrease over time, the amount of funds devoted to each banker will be lower today. This also means that the number of bank notes in circulation today will be lower. In fact, the number of notes in circulation will monotonically decrease over time, resulting in a decreasing amount of goods traded in the night market. From a buyer's standpoint, his demand for notes will decrease over time because these notes will become more expensive in future periods. Because liquidity in the economy is becoming more expensive over time, he will continuously reduce his note holdings and will be able to purchase ever smaller amounts of goods from sellers.

As we have seen, any constrained efficient allocation implies a constant quantity of goods produced and traded in the decentralized night market, which requires a stable supply of bank notes over time. As a result, the kind of non-stationary equilibrium we have characterized in this subsection is necessarily inefficient.

5.7. Endogenous Cycles

In this subsection, we show that it is possible to have equilibria in which credit conditions and real quantities fluctuate over time around the constrained efficient stationary equilibrium. In particular, we can have two types of cycles: damped cycles that asymptotically converge to the stationary equilibrium or explosive cycles that may never converge.

Suppose now that $u(q) = (1-\sigma)^{-1}\left(q^{1-\sigma} - 1\right)$ with $\sigma > 1$. In this case, the revenue of each banker is an increasing function of the price ϕ because

$$\phi a'(\phi) + a(\phi) > 0$$

for any ϕ. As a result, we have

$$\frac{d\phi_t}{d\phi_{t-1}} = \frac{a'\left(\phi_{t-1}\right)}{\phi_t a'(\phi_t) + a(\phi_t)} < 0,$$

which means that (22) defines an implicit mapping $\phi_t = f\left(\phi_{t-1}\right)$ that is strictly decreasing. In this case, for any initial condition $\phi_0 \neq 1$, we will have endogenous cycles. The price of notes will be below one in one period and then above one in the next.

If we restrict attention to the local dynamics, there will be two possibilities: damped oscillations or explosive oscillations. If we have

$$\left.\frac{d\phi_t}{d\phi_{t-1}}\right|_{\phi_{t-1}=\phi_t=1} = \frac{a'(1)}{a'(1) + a(1)} \in (-1, 0),$$

then, for any initial condition $\phi_0 \neq 1$ within a small neighborhood of one, we will observe damped oscillations. The price of notes will oscillate over time but will asymptotically converge to one. Along this equilibrium path, the individual credit limits fluctuate over time within a fixed bounded interval, having a slightly higher value than $a(1)$ in one period and then a slightly lower value than $a(1)$ in the next, asymptotically converging to $a(1)$. The same thing happens to the amount of notes in circulation and the quantity of goods that buyers trade with sellers in the night market.

So far, we have assumed a specific functional form for the utility function to facilitate our exposition. It turns out that, for this specific utility function, the case in which

$$\left.\frac{d\phi_t}{d\phi_{t-1}}\right|_{\phi_{t-1}=\phi_t=1} = \frac{a'(1)}{a'(1) + a(1)} < -1, \tag{23}$$

necessarily implies that an equilibrium does not exist because the price of bank notes must, at any time, be greater than or equal to the discount factor β. However, for more general preferences, it is possible to have equilibria in which condition (23) holds and the price of notes remains always above the discount factor, in which case we will observe explosive oscillations along the equilibrium path. More precisely, for any initial condition $\phi_0 \neq 1$ within a small neighborhood of one, the equilibrium price of notes will eventually exit such a neighborhood.

Again, the kind of non-stationary equilibrium described in this subsection (endogenous cycles) is inefficient because it necessarily results in fluctuations in the quantity of goods produced and traded in the decentralized night market. In the case of damped cycles, we can at least guarantee that the economy actually converges to the constrained efficient allocation.

5.8. Discussion

As we have shown, a purely private monetary system is capable of implementing a constrained efficient allocation. However, the existence of other equilibria with undesirable properties for initial conditions arbitrarily close to $\phi_0 = 1$ implies that such a system is necessarily unstable. These equilibria arise because some beliefs about future credit conditions can adversely affect current and future prices of bank notes. Specifically, some of these equilibria will display large fluctuations in the supply of bank notes. Our welfare analysis has shown that any constrained efficient allocation necessarily implies a constant amount of goods produced and traded in the decentralized night market. Because equilibria displaying self-fulfilling collapses and explosive cycles involve large fluctuations in the quantity of private money created by the banking sector, they are socially undesirable so that there is a potential role for government intervention. In particular, an effective intervention will allow us to rule out the possibility of having non-stationary equilibria with undesirable properties.

6. GOVERNMENT INTERVENTION

Suppose that the government decides to issue the same kind of bank notes as those issued by individual bankers at each date. Assume further that the government has access to the technology to store the daytime good from one period to the next. Finally, assume that the government can fully commit to its future promises. These assumptions imply that the government can act as a banker, issuing notes and investing the proceeds from the sale of notes in the storage technology. The main differences from a private banker are that the government can fully commit to its promises and does not necessarily seek to maximize the discounted sum of its profits. Throughout the paper, we will assume that the government does not have the power to levy lump-sum taxes (nor any other form of tax). This means that we can really think of the government intervention as one in which it runs a central bank. In this section, we will characterize an operational procedure for such a bank that is designed to stabilize credit conditions around the constrained efficient stationary equilibrium.

Let $\bar{D}_t$ denote the amount of government notes that are issued at date t and that mature at date $t+1$. The government's budget constraint is given by

$$\beta^{-1} i_{t-1}^g + \phi_t \bar{D}_t = \tau_t + i_t^g + \bar{D}_{t-1},$$

where τ_t denotes the real value (in terms of the daytime good) of a lump-sum transfer to households (buyers and sellers) in the day subperiod at date t and i_t^g denotes the date-t amount of resources invested in the storage technology to meet repayments at future dates.

At each date, the government invests the proceeds from the sale of notes in the storage technology to meet the promised repayment in the following date, which implies $i_t^g = \phi_t \bar{D}_t$. Any profit from the sale of notes is transferred to private agents in the form of lump-sum transfers, which implies $\tau_t = \left(\beta^{-1}\phi_{t-1} - 1\right) \bar{D}_{t-1}$. Thus, a feasible monetary regime is given by any sequence $\left\{\bar{D}_t, \tau_t, i_t^g\right\}_{t=0}^{\infty}$ satisfying the government's budget constraint, together with the restrictions we have just mentioned.

6.1. Equilibrium

We restrict attention to equilibria in which households treat the notes issued by the government and those issued by private bankers as perfect substitutes. In this case, the function $a(\phi_t)$ defined in (18) continues to represent the aggregate demand for notes by the private sector. Then, the amount of resources devoted to the private banking system at date t is given by

$$\phi_t a(\phi_t) - \phi_t \bar{D}_t.$$

For simplicity, we can define a monetary regime only in terms of the sequence of publicly issued notes $\left\{\bar{D}_t\right\}_{t=0}^{\infty}$. Then, we can use the government's budget constraint to construct the lump-sum transfers and investment policies needed to implement such a particular sequence. Now we can define an equilibrium in the same way as before.

Definition 4 *Given the specification of a monetary regime $\left\{\bar{D}_t\right\}_{t=0}^{\infty}$, a symmetric competitive equilibrium is a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ satisfying $\phi_t \geq \beta$,*

$$a\left(\phi_{t-1}\right) - \bar{D}_{t-1} = \phi_t \left[a(\phi_t) - \bar{D}_t\right], \tag{24}$$

and

$$a(\phi_t) \geq \bar{D}_t \tag{25}$$

for each date t.

Now we want to study the existence of equilibrium in the presence of publicly issued notes. Consider initially a passive policy in which the government does not issue notes: $\bar{D}_t = 0$ for all $t \geq 0$. In this case, $\phi_t = 1$ for all $t \geq 0$ is a non-autarkic stationary equilibrium, and the properties of such an equilibrium are the same as those presented in the previous section.

Now consider regimes $\left\{\bar{D}_t\right\}_{t=0}^{\infty}$ in which the amount of publicly issued notes is not necessarily constant over time but remains forever within a small neighborhood of zero. Because the central bank chooses the quantity of its own notes that it wants to issue at each date, it is able to restrict the amount of its own notes in circulation to any bounded interval. In this case, we can study the local determinacy of equilibrium using standard textbook methods;

see, for instance, Azariadis (1993) and Woodford (2003). Again, to facilitate our exposition, suppose that $u(q) = (1-\sigma)^{-1}\left(q^{1-\sigma} - 1\right)$, with $\sigma \neq 1$. Define $\hat{\phi}_t \equiv \phi_t - 1$. Then, a linear approximation to (24) is given by

$$\hat{\phi}_t = b\hat{\phi}_{t-1} + \Delta_t,$$

where

$$b \equiv \frac{a'(1)}{a'(1) + a(1)},$$
$$\Delta_t \equiv \frac{\bar{D}_t - \bar{D}_{t-1}}{a'(1) + a(1)}.$$

If $|b| > 1$, then this equation can be solved forward to obtain a *unique* bounded solution

$$\hat{\phi}_t = -\frac{1}{b}\sum_{j=0}^{\infty}\left(\frac{1}{b}\right)^j \Delta_{t+1+j}. \tag{26}$$

In other words, there exists a sufficiently small neighborhood around $\phi = 1$ such that the unique equilibrium can be approximated by (26). This means that the equilibrium price of notes today depends on the future path of government policies with respect to the amount of publicly issued notes.

Because the equilibrium price of notes depends on the future path of government policies with respect to the amount of publicly issued notes, future credit conditions will remain relatively stable in the case in which the government keeps the amount of publicly issued notes within a sufficiently small neighborhood of zero. This means that future credit limits will remain within a small neighborhood of $a(1)$ and that future prices will remain within a small neighborhood of one, implying arbitrarily small fluctuations in the current supply of credit to the banking system and, consequently, the aggregate amount of notes issued. As a result, the quantities of goods produced and traded in the decentralized night market will remain within an arbitrarily small neighborhood of $\hat{q}$.

6.2. Discussion

Provided that the government keeps the amount of publicly issued notes within a sufficiently small neighborhood of zero, the unique equilibrium can be approximated by (26), in

which case the price of notes will remain within a small neighborhood of one. This means that the introduction of publicly issued notes eliminates the possibility of having a self-fulfilling collapse of the banking system, guaranteeing the stability of the monetary system. (Note that we have $b > 1$ for the case in which we obtain self-fulfilling collapses.)

It also eliminates the possibility of explosive oscillations. As we have seen, the case in which we can have explosive oscillations in the absence of intervention implies $b < -1$, which also guarantees the existence of a unique forward-looking solution given by (26). This means that our operational procedure for the central bank has the property of ruling out the possibility of having equilibria with undesirable characteristics.

Before we conclude, two points are worth mentioning. First, we do not need the central bank to have any form of privilege. Our proposed intervention simply requires the central bank to compete with private bankers in the business of note issuance in a specific way. Thus, the government monopoly over note issuance is not a fundamental characteristic of an efficient and stable monetary system.

Finally, we can see from (26) that government mismanagement of the currency can result in large aggregate fluctuations, consistent with the historical evidence presented in Friedman and Schwartz (1963). This highlights the importance of endowing the central bank with a specific procedure or rule for the conduct of monetary policy.

7. CONCLUSION

In this paper, we have analyzed the efficiency and stability of a private monetary system. In particular, we have studied the creation of private money within the Lagos-Wright framework. The key frictions in the environment are people's inability to commit to their future promises and the lack of a record-keeping technology for most of the traders in the economy. Those who have access to a record-keeping technology are able to issue liabilities that circulate as a medium of exchange and enjoy a higher lifetime utility than that associated with autarky precisely because of their note-issuing privileges. We have referred to these agents as the bankers.

In our analysis, debt limits are endogenously determined, so the current credit limit of any banker depends on beliefs about future credit conditions. Thus, his ability to issue bank notes is constrained by beliefs about future credit limits and future prices for his bank notes. As a result, there can be multiple equilibria under a purely private monetary system. Moreover, some of these equilibria have some undesirable properties. In some cases, we can observe a self-fulfilling collapse of the banking system in which the balance sheet of each banker persistently shrinks along the equilibrium path. In these equilibria, the amount of bank notes in circulation persistently declines over time as note holders permanently reduce their demand for these notes, adversely affecting real quantities. In particular, people will be able to trade ever smaller quantities of goods because of a persistent rationing of bank notes (a currency famine). It is also possible to have equilibria in which the aggregate amount of bank notes in circulation excessively fluctuates over time, resulting in cycles that sometimes can self-perpetuate. We have shown that all of these equilibria are necessarily inefficient, which naturally gives rise to the formulation of welfare-improving government policies.

To formulate a government intervention in our framework, we have deliberately eliminated any fiscal power the government can potentially have, so we can refer to such an intervention as the creation of a purely monetary authority or central bank. The government intervention we have described above has the role of stabilizing the economy in that it rules out the possibility of self-fulfilling collapses and explosive cycles. Moreover, such an intervention guarantees that the economy remains arbitrarily close to the constrained efficient allocation. Thus, it naturally provides an operational procedure for guiding the decisions of a monetary authority or central bank.

Our analysis has confirmed the conjecture that a purely private monetary system can be unstable. Even though the conjecture that competition among private agents for the creation of government currency substitutes is capable of implementing a constrained efficient allocation seems to be correct, according to our analysis, the claim that such a system can also be stable is certainly not true. Thus, the view that free banking can create a sound monetary framework ignores the role that endogenous debt limits play in the creation of

private money. Our analysis has confirmed the conjecture that the creation of a monetary authority or central bank, equipped with an operational procedure of the kind described above, is sufficient to ensure monetary stability. Finally, we have shown that it is not necessary to grant any form of market power to the central bank in order to achieve efficiency and stability. Thus, a monopoly over note issuance is not a fundamental characteristic of an efficient and stable monetary system.

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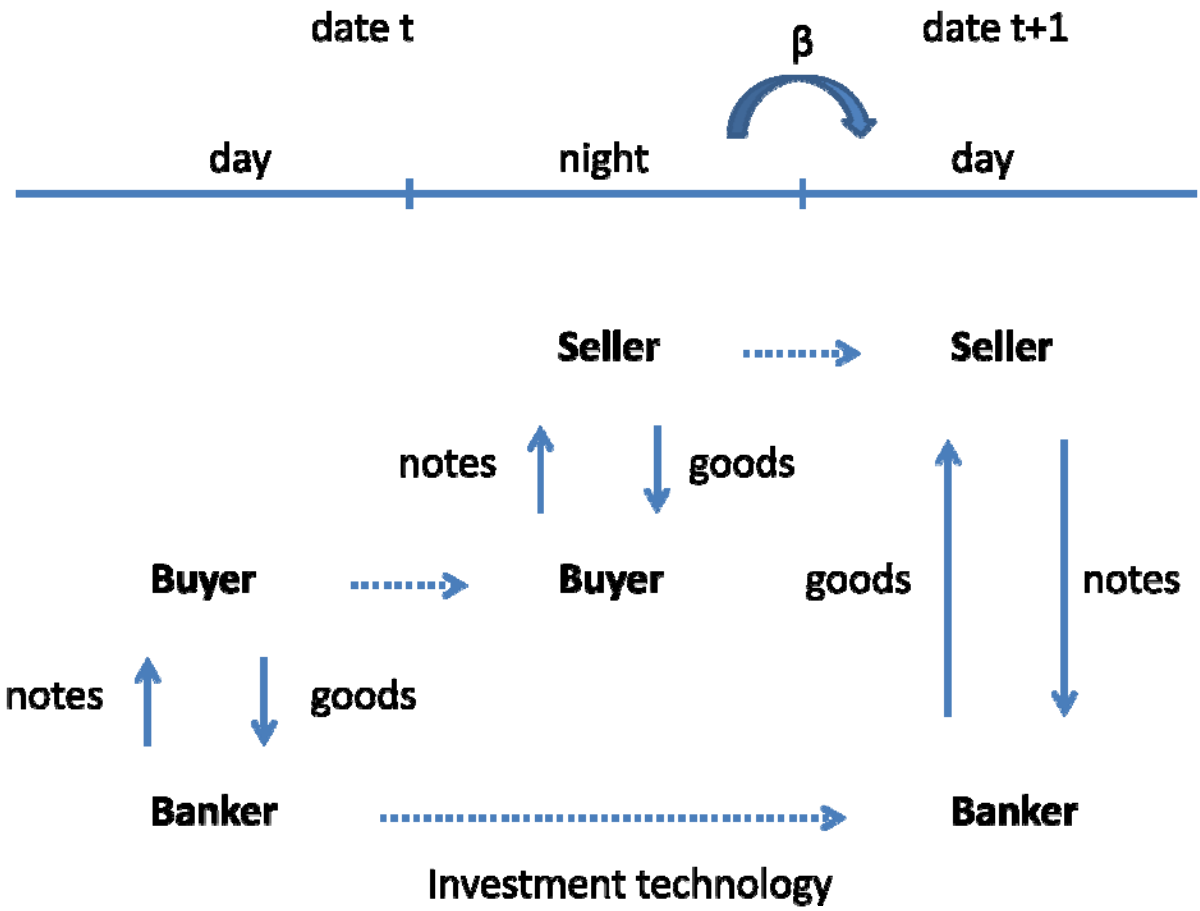
Figure 1 – Circulation of Bank Notes