PERCEPTIONS AND MISPERCEPTIONS OF FISCAL INFLATION

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Abstract. The Great Recession and worldwide financial crisis have exploded fiscal imbalances and brought fiscal policy and inflation to the forefront of policy concerns. Those concerns will only grow as aging populations increase demands on government expenditures in coming decades. It is widely perceived that fiscal policy is inflationary if and only if it leads the central bank to print new currency to monetize deficits. Monetization can be inflationary. But it is a *mis*perception that this is the only channel for fiscal inflations. Nominal bonds, the predominant form of government debt in advanced economies, derive their value from expected future nominal primary surpluses and money creation; changes in the price level can align the market value of debt to its expected real backing. This introduces a fresh channel, not requiring monetization, through which fiscal deficits directly affect inflation. The paper begins by pointing out similarities and differences between the Weimar Republic after World War I and the United States today. It describes various ways in which fiscal policy can directly affect inflation and explains why these fiscal effects are difficult to detect in time series data.

Keywords: monetary-fiscal interactions; fiscal theory; monetization
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1. Introduction

Not so long ago, macroeconomists interested in understanding inflation and its determinants were comfortable sweeping fiscal policy under the carpet, implicitly assuming that the fiscal adjustments required to allow monetary policy to control inflation would always be forthcoming. This sanguine view is reflected in recent graduate textbooks, which make scant mention of fiscal policy, and in the economic models at central banks, which all but ignore fiscal phenomena. It is also reflected in the widespread adoption of inflation targeting by central banks, but the nearly complete absence of the adoption of compatible fiscal frameworks.

The Great Recession and accompanying worldwide financial crisis have brought an abrupt halt to the benign neglect of fiscal policy. Figure 1 underlies the sudden shift in attitude among economists and policy makers alike. Fiscal deficits worldwide, but particularly in advanced economies, shot up and public debt as a share of GDP ballooned to nearly 100 percent in advance economies. As central banks lowered nominal interest rates toward their zero bound, they moved to quantitative actions that dramatically expanded the size and riskiness of their balance sheets. With both fiscal *and monetary* authorities taking fiscal actions, professional

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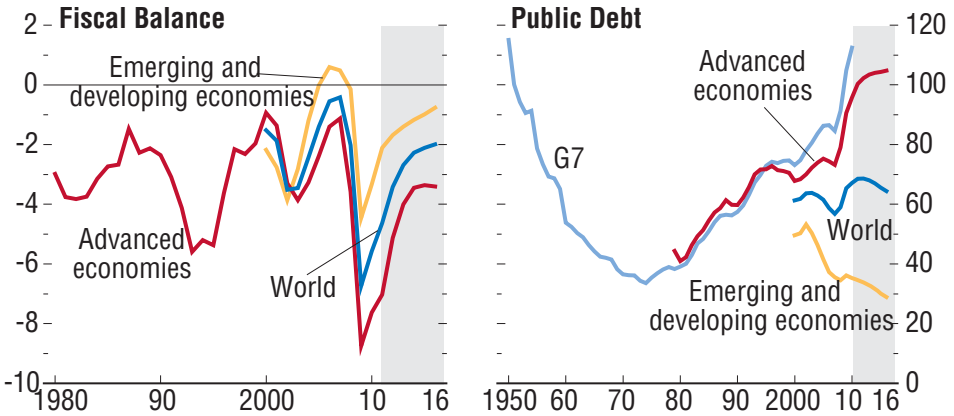

FIGURE 1: In percent of GDP. Source: International Monetary Fund (2011)

and policy focuses have now shifted to fiscal matters and the interactions of monetary and fiscal policies.

With the shift in focus has come enhanced interest in the potential channels through which fiscal policy can affect inflation. And, in light of the facts in figure 1, a pressing question is, "Do profligate fiscal policies threaten the progress many countries have made toward achieving low and stable inflation?" In the conventional monetary paradigm that underlies central bank models and, we conjecture, the thinking of central bankers, the answer is, "No, so long as the central bank steadfastly refuses to print new currency to finance deficits."

This paradigm maintains that there is no mechanism by which fiscal policy can be inflationary that is independent of monetary policy and money creation. Sargent and Wallace (1981) model this conventional view and dub it "unpleasant monetarist arithmetic." In their setup, fiscal policy runs a chronic primary deficit—spending exclusive of debt service less tax revenues—that is independent of inflation and government debt and a simple quantity theory demand for money holds, so the price level adjusts to establish money market equilibrium. The economy faces a fiscal limit because the private sector's demand for bonds imposes an upper bound on the debt-GDP ratio. Sargent and Wallace's government bonds are real: claims to payoffs denominated in units of goods.

If primary deficits are exogenous—one notion of "profligate" fiscal policy—and the exogeneity is immutable, then monetary policy loses its ability to control inflation. Conventional reasoning drives the result. If monetary policy initially aims to control inflation by setting money growth independently of fiscal policy, then eventually the exogenous deficit will drive debt to the fiscal limit. At the limit, if government is to remain solvent, monetary policy has no alternative but to print money to generate the seigniorage revenues needed to meet interest payments in the debt.[1] Eventually, money growth must rise and, by the quantity theory, so must inflation. Long-run monetary policy is driven by the need to stabilize debt

[1]We are assuming that in the long run the economy's growth rate is below the real interest rate on debt.

and the inflation rate is determined by the size of the total fiscal deficit, including interest payments.

This conventional paradigm reflects common perceptions of fiscal inflations. But it is a *mis*perception to believe that fiscal policy can affect inflation only if monetary policy monetizes deficits.

The tight connection between seigniorage financing and inflation in Sargent and Wallace's model stems from the assumption that bonds are real, or perfectly indexed to the price level. Higher real debt requires the government to raise more real resources—like seigniorage—to fully back the debt. But in practice only a small fraction of government debt issued by advanced economies is indexed. Even in the United Kingdom, which has the thickest market for indexed government bonds, about 80 percent of outstanding debt is nominal. Ninety percent of U.S. treasuries are nominal and fractions are still higher elsewhere.

Recognizing that bonds are denominated in nominal terms introduces a direct channel from fiscal policy to inflation. Called the *fiscal theory of the price level*, this channel does not rely on "monetizing deficits" or on insufficient inflation-fighting resolve by the central bank.[2] Instead, it springs from the fact that a nominal bond is a claim to a *nominal* payoff—dollars, euros, or shekels—and that the *real* value of the payoff depends on the price level.

Higher nominal debt may be fully backed by real resources—real primary surpluses and seigniorage—or it may be backed only by *nominal* cash flows. When real resources fully back the debt, the conventional paradigm prevails and fiscal policy is inflationary only if the central bank monetizes deficits. But when the government cannot or will not raise the necessary real backing, the fiscal theory creates a direct link between current and expected deficits and inflation.[3]

Even though the data in figure 1 have sent some policy makers into apoplexy, they are but the tip of the fiscal stress iceberg. Table 1 describes the real problem. Aging populations and promised government old-age benefits that far outstrip revenues provisions imply massive "unfunded liabilities." Plans to bring current deficits under control do little to address the coming fiscal stress. We have no special insights into the political solutions to this unprecedented fiscal problem, but we can shed light on the economic consequences—particularly for inflation—of alternative private-sector beliefs about how the fiscal stress will be resolved.

We work from the premise that central bankers have learned the unpleasant monetarist arithmetic lesson, so explicit monetization of deficits is off the table in advanced economies. For the most part, we also exclude outright default on the government liabilities of those countries. On-going developments in Europe vividly illustrate the lengths to which policy makers will go to avoid default, and policy makers in the United Kingdom, the United States, and elsewhere hold similar views.

[2]Leeper (1991), Sims (1994), Woodford (1995), and Cochrane (1999) describe the fiscal theory and its implications.

[3]The terms "fiscal theory" and "quantity theory" are unfortunate because they suggest that these are distinct models of price-level determination. As we show, the price level and inflation *always* depend on both monetary and fiscal policy behavior. The fiscal and quantity "theories" emerge under alternative monetary-fiscal regimes, as Gordon and Leeper (2006) show.

Country	Aging-Related Spending
Australia	482
Canada	726
France	276
Germany	280
Italy	169
Japan	158
Korea	683
Spain	652
United Kingdom	335
United States	495
Advanced G-20 Countries	409

TABLE 1: Net present value of impact on fiscal deficit of aging-related spending, in percent of GDP. Source: International Monetary Fund (2009).

There remain two possible resolutions to fiscal stress. First, government could successfully persuade the public that future revenue and spending adjustments will occur. With fiscal policy taking care of itself, we return to the sanguine world in which central banks retain control of inflation. Numbers in table 1 underscore how large those adjustments must be. Economic theory tells us that those policies must also be credible to firmly anchor expectations on the necessary fiscal adjustments.

Because the first resolution is well understood, the paper focuses on a variety of alternative policy scenarios in which aspects of the second resolution—price-level changes induced by the fiscal theory—come into play. We focus on the fiscal theory because it seems to be poorly understood and quickly discarded by central bankers. For example, in their discussion of the implications of fiscal stress for central banks, Cecchetti, Mohanty, and Zampolli (2010, footnote 23) acknowledge the fiscal theory, but immediately dismiss it as "untested and controversial." As we point out below, the fiscal theory is no more or less "testable" than the quantity theory or its recent offspring, the new Keynesian/Taylor rule model of inflation. And it is "controversial," we believe, because it is relatively new and its implications and economic mechanisms have not yet been fully absorbed by monetary economists and policy makers.

1.1. **What We Do.** The policy scenarios we consider arise from the possibility that an economy may hit its fiscal limit, the point at which taxes and spending can no longer adjust to stabilize debt. A well-known case of the fiscal limit is the Weimar Republic in the early 1920s. The ensuing hyperinflation is viewed as a classic example of unpleasant arithmetic [Sargent (1986)]. Section 2 briefly recounts the fiscal conditions in Germany after World War I, finding some parallels to the United States today. Of course, there are also important differences, so we are by no means arguing that hyperinflation and political turmoil lie in America's future. But several preconditions that made Weimar Germany ripe for inflation also seem to be satisfied in the United States.

Section 3 uses a simple model to illustrate how the price level is determined in the conventional paradigm and in the fiscal theory. The conventional policy mix—Regime M—has monetary policy target inflation and fiscal policy stabilize the value of debt. An alternative mix—Regime F—is available when governments issue nominal bonds. That mix assigns monetary policy to stabilize debt and fiscal policy to control the price level, giving rise to the fiscal theory equilibrium.

In Regime M, deficit-financed tax cuts or spending increases do not affect aggregate demand because the private sector expects the resulting increase in government debt to be exactly matched by future tax increases or spending reductions. Expansions in government debt do not raise wealth. This fiscal behavior relieves monetary policy of fiscal financing concerns, freeing the central bank to target inflation.

Regime F posits different policies that align closely to actual behavior in many countries recently. Suppose that higher deficits do not create higher expected surpluses and that central banks either peg short-term nominal interest rates or raise them only weakly with inflation. Because a tax cut today does not portend future tax hikes, individuals initially perceive the increase in nominal debt to be an increase in their real wealth. They try to convert higher wealth into consumption goods, raising aggregate demand. Rising demand brings with it rising prices, which continue to rise until real wealth falls back to its pre-tax-cut level and individuals are content with their original consumption plans. By preventing nominal interest rates from rising sharply with inflation, monetary policy prevents debt service from growing too rapidly, which stabilizes the value of government bonds. In this stylized version of the fiscal theory, monetary policy can anchor expected inflation on the inflation target, but fiscal policy determines actual inflation.

The section goes on to describe how the maturity structure of nominal government bonds can alter the time series properties of inflation and it lays out the precise role that monetary policy plays in a fiscal equilibrium. A fiscal theory equilibrium is consistent with a wide range of patterns of correlation in data, including a positive correlation between inflation and money growth, a negative correlation between inflation and the debt-GDP ratio, and any correlation between inflation and nominal debt growth and deficits.

Having established that under Regime F policies monetary policy does not control inflation, we turn to plausible scenarios in which the central bank does not control inflation even in Regime M. One example arises when the public believes the economy may hit its fiscal limit at some point in the future. Even if monetary policy aggressively targets inflation in the years before the limit, it cannot determine the inflation rate and it cannot even anchor expected inflation. A second type of fiscal limit stems from the risk of sovereign default. When the central bank sets the interest rate of short-term government bonds, a higher probability of default feeds directly into current inflation. Finally, in a monetary union, the member nation whose fiscal policies are profligate will determine the union-wide price level, even if other member countries run fiscal policies that consistently target real debt.

Finally, in section 5 the paper turns to consider the empirical implications of monetary-fiscal policy interactions. That section lays out some observational equivalence results that arise in models of section 3. Restrictions on policy behavior and/or exogenous driving processes are crucial in discerning whether observed time series on inflation, debt, and deficits are generated by a Regime M or a Regime F equilibrium.

Central bankers who aim to hit an inflation target, need to know whether the economy resides in Regime M or in Regime F. Observational equivalence informs us that existing research may not be able to address this fundamental issue without first confronting the observational equivalence problem. Until we tackle this formidable empirical challenge, we cannot use data to distinguish perceptions from misperceptions about fiscal inflation.

2. 1920s Weimar Republic and 2011 United States

Perhaps the best-known historical example of a fiscal inflation is the German Weimar Republic immediately after World War I. There are some similarities between the fiscal situations in Germany in the early 1920s and the United States today.[4] We do not wish to draw the parallels too strongly because, as we discuss, important differences also exist. But some preconditions that made Germany ripe for fiscally-induced inflation also prevail, in modified form, in post-recession, post-financial crisis America.[5]

This section succinctly reviews the similarities and discusses the differences between 1920s Germany and present-day United States to provide a context for the theoretical derivations that follow.

2.1. **Similarities.** We begin by highlighting three similarities between the two countries.

Exploding Government Spending Obligations.

German reparations payments from the Treaty of Versailles were denominated in "hard currency," gold or U.S. dollars. German tax receipts were in marks. As the mark depreciated from a peak of 2.56 U.S. cents per mark in June 1920 to 0.01 cent per mark in December 1922 [Sargent (1986, Table G.2)], in real terms reparations payments grew exponentially. Allied powers, particularly France, were uncompromising in their insistence on timely and complete payments, so from the perspective of German fiscal authorities, the payments were an immutable force beyond their influence.

Legislated transfer payments—Social Security, Medicare, and Medicaid in the United States—may, from a political viewpoint, be nearly as immutable as Germany's reparations. Certainly, they are projected to grow nearly as quickly: the Congressional Budget Office's long-term projections have these transfers growing from 10.5 percent of GDP in 2011 to about 25 percent in 70 years, with publicly held federal debt rising to 113 percent of GDP in one scenario and 947 percent in another scenario [Congressional Budget Office (2010)]. Much of the growth is driven by health care costs, whose inflation rate consistently exceeds overall inflation.

[4]Parallels also exist between the other hyperinflation countries in Europe—Austria, Hungary, and Poland—and other advanced economies today—for example, Japan and the United Kingdom.

[5]By pointing out these similarities, we are *not* suggesting that the economic calamity that Germany incurred is likely to be replayed in the United States and we most assuredly *do not* imagine that the political developments that followed the German hyperinflation will occur in the United States. Our parallels apply only to economic conditions.

Internet gold bugs are drawing analogous parallels, but with the inevitable—and self-serving—conclusion that hyperinflation looms ahead and investments in precious metals are a safe hedge against high inflation. We do not endorse those views.

How immutable these entitlements programs are is a political question. Entitlements have long been considered the "third rail" of American politics, but some political leaders are now making proposals to scale back entitlements growth [Ryan (2010), National Commission on Fiscal Responsibility and Reform (2010)]. After much ballyhoo, these proposals appear to be dead-on-arrival now that elected officials have gauged the full measure of the electorate's opposition to substantial entitlements reform. Congressman Paul Ryan's own party has not endorsed his "Roadmap for America" and the presidential National Commission's proposal failed to garner the votes needed to send the plan to Congress for legislative action.

The U.S. government's promised future entitlements expenditures, like Germany's reparation payments, continue to grow as a share of the economy.

Unlikely Events Become Likely.

Going into World War I, many Germans felt the war would be brief and victorious. They patriotically bought war bonds, feeling assured the bonds would pay off with interest after the war was won. Defeat was a remote possibility. Hyperinflation and its accompanying economic disruptions were not on anyone's mind.

Large segments of the German populace, particularly the influential and wealthy segments, laid the blame for Germany's post-war economic troubles on the Versailles treaty and on vindictive allied powers like France, which was known to have pushed for still tougher sanctions on the vanquished countries. Nurske (1946) maintains that those segments attributed inflation solely to reparations payments and may have wanted inflation to continue unchecked to demonstrate Germany's inability to meet the payments. Political opponents of the Weimar government may also have been happy with high and rising inflation and the distortions the inflation created. Hyperinflation had clear winners and losers and opponents likely welcomed the economic and social turmoil that hyperinflation engendered [Layton (2005)].

In the United States, proposals to resolve the long-term fiscal imbalance, represented by Ryan (2010) and National Commission on Fiscal Responsibility and Reform (2010), have been dramatic and widely criticized as somewhat unbalanced. By focusing on fundamental changes to entitlements, rather than wholesale rethinking of the U.S. monetary-fiscal framework, the proposals rewrite the "social contract" that many Americans believe they have with their government. Some vocal elements in the U.S. political realm insist that dismantling the social safety net and greatly limiting the role of government is the only way to achieve sustainable policies.

Not long ago, Social Security and Medicare were sacrosanct. Budget discussions set them aside as inviolable. Now Americans put significant probability on future cuts in benefits. A survey that asked American workers how confident they are that entitlements programs will continue to provide benefits of at least or equal to the benefits received by current retirees found that fully 70

percent were either "not at all" or "not too" confident.[6] Policy changes once thought impossible now are viewed as probable.

Policy Uncertainty.

The Treaty of Versailles left unspecified the magnitude and timing of reparation payments. Even when the allied powers finally set the figure at 132 billion gold marks (£6.6 billion) at a rate of 2.5 billion marks per year, actual payments proceeded in fits and starts until the Dawes Plan in the summer of 1924. That plan extended loans to Germany and phased in reparation payments, which didn't reach their full amounts until 1928-1929. Sargent (1986) argues that uncertainty about the reparations the German government owed left expectations of fiscal policy unanchored and impaired efforts to stabilize the mark.

Uncertainty is the hallmark of monetary and fiscal policies in the United States and, arguably, in most advanced economies. Sargent (2006) depicts policy uncertainty by replacing the usual probability triple, $(\Omega, \mathcal{F}, \mathcal{P})$, with $(?, ?, ?)$. He argues that the "prevailing notions of equilibrium" all assume agents have a complete description of the underlying uncertainty, while Knightian uncertainty or ambiguity about future policy might be a better description of reality. Even among countries that have explicit inflation targets, few countries have adopted the fiscal arrangements or rules that are consistent with assigning central banks the task of controlling inflation. If tensions arise between monetary and fiscal goals, existing policy frameworks are silent on how the tensions will be resolved.

This is the situation in "normal" times. Most advanced economies are leaving normal times behind as they head into a prolonged era of fiscal stress. Aging populations and promised old-age benefits, with no plans in place to finance the benefits, have created massive "unfunded liabilities" [table 1]. Will they be funded? If so, how and when? Are they actually liabilities or will benefits be reduced? If so, how and when? The sheer magnitude of the "unfunded liabilities" make simple extrapolations of past policy behavior untenable. The coming fiscal stress and attendant uncertainties have released U.S. fiscal expectations from their moorings.

Economic troubles for the Weimar Republic stemmed from the country hitting its "fiscal limit." Government expenditures had been cut dramatically, but revenues simply did not keep pace with reparations payments. After the Versailles treaty, Germany was regarded as a pariah state, unable to borrow from abroad. This is an extreme form of a fiscal limit that left the government no alternative but to print money to finance expenditures. At the fiscal limit, German monetary policy was completely subjugated to fiscal needs. The Weimar's monetary-fiscal story maps well into Sargent and Wallace's (1981) unpleasant monetarist arithmetic view of fiscal inflation in which deficits are systematically monetized, at least in the early years of the Republic's existence.

[6]The 70 percent figure applies to both Social Security and Medicare when polled separately [Helman, Copeland, and VanDerhei (2011, figures 44 and 46)].

2.2. **Differences.** In considering the similarities, it is important to recognize that critical differences also exist between the Weimar Republic and the United States in 2011.

Economic tensions grew rapidly in Germany, culminating in a massive hyperinflation that ended abruptly [Sargent (1986)]. For a number of reasons, tensions can simmer for quite a while in the United States and are likely to show up in economic time series only very gradually.[7] First, U.S. government spending commitments extend over several decades and do not begin to rise rapidly for some years. There is no reason to expect the United States to hit its fiscal limit, wherever it may be, as quickly as the Weimar government hit its limit.

Second, the United States continues to serve the enviable role as a safe haven for investments. The dollar is still the world's reserve currency, with no immediate threats on the horizon. U.S. treasuries are perceived as possibly the safest asset in the world. As long as the U.S. government can place its debt without incurring risk premia, it can avoid the fiscal limit that the Weimar Republic was up against from its inception.

Third, the United States is politically stable, notwithstanding its current penchant for political gridlock. In contrast, the Weimar government was under constant—often violent—attacks from extreme elements on both the right and the left, so it was impossible to achieve a stable political environment. Although U.S. politics have become increasingly polarized in the past 20 years, there is no reason to expect that chaos and class warfare that marred the Weimar. In recent years, the polarization in American politics has produced gridlock; when fiscal compromises cannot be reached, fiscal problems remain unresolved. But this is *déjà vu* all over again, reminiscent of the 1990s, which eventually produced tax and spending reforms that generated substantial fiscal surpluses.

Fourth, unlike Germany's reparations payments, which were owed to foreign governments, U.S. entitlements obligations are internal and they *can* be reduced. Early signs that substantial entitlements reform will occur have not been especially promising. But the national debate is young and the need for reform does not yet seem politically or economically urgent.

Fifth, unlike Weimar Germany after the Versailles treaty, the United States has a healthy tax base and large taxing capacity. Because the Weimar Republic was the first effort to create a unified central government in Germany, when the government came into existence there was essentially no centralized tax system [Graham (1930)]. Much of the government's early legislative effort went into establishing a reliable source of revenues. With effectively little tax base, the government had no option but to turn to money creation to finance domestic spending. For the United States, as with entitlements cuts, the government *can* exploit its taxing capacity; whether it will is a political matter.

Finally, the United States, like other advanced economies, has established an operationally independent central bank, which is designed in large part to insulate monetary policy from fiscal financing needs and other political pressures. Chinks have recently appeared in the Federal Reserve's armor, particularly in the wake of the financial crisis, with some elected officials even calling for the Fed's abolition and a return to the gold standard. But those calls are coming from the fringes, and the Fed's operational independence does not seem to be in serious jeopardy.

[7]Formal models that illustrate the gradualness appear in the simulations in Davig, Leeper, and Walker (2010, 2011).

2.3. **Inflation Threats in the United States?** An argument that holds substantial currency among economists and policy makers is that central banks learned the lessons of the Weimar and many other high inflations that, for example, Fischer, Sahay, and Végh (2002) document. First, too-rapid money growth generates inflation. Second, operationally separating the central bank from the fiscal authority ensures that the finance ministry cannot require the central bank to provide any specific cash flows or seigniorage revenues. The understanding of the connection between money growth and inflation, coupled with the operational independence of the central bank, the argument goes, permits the monetary authorities today to achieve their policy objectives.

This argument builds on Friedman's (1970) aphorism that "inflation is always and everywhere a monetary phenomenon" and it makes an implicit and essential assumption: fiscal policy will always behave in the "appropriate" manner. Sims (1999, p. 424) defines "appropriate" fiscal behavior in his description of central bank independence: "A truly independent central bank is one that can act, even under inflationary or deflationary stress, without any worry about whether the necessary fiscal backing for its actions will be forthcoming." That is, if in pursuit of its objectives a central bank were to encounter balance sheet difficulties, an independent bank would be automatically recapitalized by the fiscal authority.

Sims's point also relates to Wallace's (1981) Modigliani-Miller theorem for open market operations: the impacts of central bank asset swaps depend on fiscal policy behavior. In Wallace's paper, open-market sales of bonds have no effects on equilibrium allocations and prices. Under alternative assumptions on fiscal behavior, such monetary contractions may reduce inflation and under Sargent and Wallace's (1981) assumptions, the contractions raise inflation.

The theory presented below introduces an additional dimension to the monetary-fiscal interactions that Wallace considers: the channel for price-level determination that operates through nominally denominated outstanding government debt and expected future primary fiscal surpluses. Because this channel is more subtle than Sargent and Wallace's monetization mechanism, fiscal policy can affect inflation even if an operationally independent central bank dutifully avoids buying government bonds with newly printed fiat money.

3. Simple Model of Monetary-Fiscal Interactions

We present a simple analytical model of price-level and inflation determination that is designed to illustrate the role that the interactions between monetary and fiscal policies play in the inflation process. Throughout the analysis we restrict attention to rational expectations equilibria, so the results can be readily contrasted to prevailing views, which also are based on rational expectations.

The model draws from Leeper (1991), Sims (1994), and Woodford (2001) to lay the groundwork for how monetary and fiscal policies *jointly* determine equilibrium. These results are well known, but the broader implications of thinking about macro policies jointly are not fully appreciated.

An infinitely lived representative household is endowed each period with a constant quantity of non-storable goods, y. To keep the focus away from seigniorage considerations, we initially

examine a cashless economy, which can be obtained by making the role of fiat currency infinitesimally small. The next section brings money back into the picture. Government issues nominal one-period bonds, allowing us to define the price level, P, as the rate at which bonds exchange for goods.

The household chooses sequences of consumption and bonds, $\{c_t, B_t\}$, to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1 \tag{1}$$

subject to the budget constraint

$$c_t + \frac{B_t}{P_t} + \tau_t = y + z_t + \frac{R_{t-1}B_{t-1}}{P_t} \tag{2}$$

taking prices and $R_{-1}B_{-1} > 0$ as given. The household pays taxes, τ_t, and receives transfers, z_t, each period, both of which are lump sum.

Government spending is zero each period, so the government chooses sequences of taxes, transfers, and debt to satisfy its flow constraint

$$\frac{B_t}{P_t} + \tau_t = z_t + \frac{R_{t-1}B_{t-1}}{P_t} \tag{3}$$

given $R_{-1}B_{-1} > 0$, while the monetary authority chooses a sequence for the nominal interest rate.

After imposing goods market clearing, $c_t = y$ for $t \geq 0$, the household's consumption Euler equation reduces to the simple Fisher relation

$$\frac{1}{R_t} = \beta E_t \left(\frac{P_t}{P_{t+1}} \right) \tag{4}$$

The exogenous (fixed) gross real interest rate, $1/\beta$, makes the analysis easier but is not without some lose of generality, as Davig, Leeper, and Walker (2010) show in the context of fiscal financing in a model with nominal rigidities. This is less the case in a small open economy, so one interpretation of this model is that it is a small open economy in which government debt is denominated in terms of the home nominal bonds ("currency") and all debt is held by domestic agents.

The focus on price-level determination is entirely for analytical convenience; it is not a statement that inflation is the only thing that macro policy authorities do or should care about. Because price-level determination is the first step toward understanding how macro policies affect the aggregate economies, the key insights derived from this model extend to more complex environments.

How the price level gets determined depends on monetary-fiscal policy behavior. At a general level, macroeconomic policies have two tasks to perform: control inflation and stabilize government debt. Monetary and fiscal policy are perfectly symmetric with regard to the two tasks and two different policy mixes can accomplish the tasks. The conventional assignment of tasks—Regime M—instructs monetary policy to target inflation and fiscal policy to target real debt (or the debt-GDP ratio). But an alternative assignment—Regime F—also works: monetary policy is tasked with maintaining the value of debt and fiscal policy is assigned to control inflation. We now describe these two regimes in detail.

3.1. **Regime M: *Active Monetary/Passive Tax Policy.*** This policy regime reproduces well-known results about how inflation is determined in the canonical model of monetary policy, as presented in textbooks by Galí (2008) and Woodford (2003), for example. This regime—denoted active monetary and passive fiscal policy—combines an interest rate rule in which the central bank aggressively adjusts the nominal rate in response to current inflation with a tax rule in which the tax authority adjusts taxes in response to government debt sufficiently to stabilize debt.[8] In this textbook, best-of-all-possible worlds, monetary policy can consistently hit its inflation target and fiscal policy can achieve its target for the real value of debt.

To derive the equilibrium price level for the model laid out above, we need to specify rules for monetary, tax, and transfers policies. Monetary policy follows a conventional interest rate rule, which for analytical convenience, is written somewhat unconventionally in terms of the inverse of the nominal interest and inflation rates

$$R_t^{-1} = R^{*-1} + \alpha \left(\frac{P_{t-1}}{P_t} - \frac{1}{\pi^*} \right), \qquad \alpha > 1/\beta \tag{5}$$

where π^* is the inflation target and $R^* = \pi^*/\beta$ is the steady state nominal interest rate. The condition on the policy parameter α ensures that monetary policy is sufficiently hawkish in response to fluctuations in inflation that it can stabilize inflation around π^*.

Fiscal policy adjusts taxes in response to the state of government debt

$$\tau_t = \tau^* + \gamma \left(\frac{B_{t-1}}{P_{t-1}} - b^* \right), \qquad \gamma > r = 1/\beta - 1 \tag{6}$$

where b^* is the real debt (or debt-GDP) target, τ^* is the steady state level of taxes, and $r = 1/\beta - 1$ is the net real interest rate. Imposing that γ exceeds the net real interest rate guarantees that any increase in government debt creates an expectation that future taxes will rise by enough to both service the higher debt and retire it back to b^*.

For now we shall assume that government transfers evolve exogenously according to the stochastic process

$$z_t = (1-\rho)z^* + \rho z_{t-1} + \varepsilon_t, \qquad 0 < \rho < 1 \tag{7}$$

where z^* is steady-state transfers and ε_t is a serially uncorrelated shock with $E_t \varepsilon_{t+1} = 0$.

Equilibrium inflation is obtained by combining (4) and (5) to yield the difference equation

$$\frac{\beta}{\alpha} E_t \left(\frac{P_t}{P_{t+1}} - \frac{1}{\pi^*} \right) = \frac{P_{t-1}}{P_t} - \frac{1}{\pi^*} \tag{8}$$

Aggressive reactions of monetary policy to inflation imply that $\beta/\alpha < 1$ and the unique bounded solution for inflation is

$$\pi_t = \pi^* \tag{9}$$

[8] Applying Leeper's (1991) definitions, "active" monetary policy targets inflation, while "passive" monetary policy weakly adjusts the nominal interest rate in response to inflation; "active" tax policy sets the tax rate independently of government debt and "passive" tax policy changes rates strongly enough when debt rises to stabilize the debt-GDP ratio. Of course, fiscal policy could, instead, be associated with setting transfers instead of taxes.

so equilibrium inflation is always on target, as is expected inflation.[9]

If monetary policy determines inflation—and the path of the price level, $\{P_t\}$—how must fiscal policy respond to disturbances in transfers to ensure that policy is sustainable? This is where passive tax adjustments step in. Substituting the tax rule, (6), into the government's budget constraint, (3), taking expectations conditional on information at $t-1$, and employing the Fisher relation, (4), yields the expected evolution of real debt

$$E_{t-1}\left(\frac{B_t}{P_t} - b^*\right) = E_{t-1}(z_t - z^*) + (\beta^{-1} - \gamma)\left(\frac{B_{t-1}}{P_{t-1}} - b^*\right) \tag{10}$$

Because $\beta^{-1} - \gamma < 1$, debt that is above target brings forth the expectation of higher taxes, so (10) describes how debt is expected to return to steady state following a shock to z_t. In a steady state in which $\varepsilon_t \equiv 0$, debt is $b^* = (\tau^* - z^*)/(\beta^{-1} - 1)$, equal to the present value of primary surpluses.

Another perspective on the fiscal financing requirements when monetary policy is targeting inflation emerges from a ubiquitous equilibrium condition. In any dynamic model with rational agents, government debt derives its value from its anticipated backing. In this model, that anticipated backing comes from tax revenues net of transfer payments, $\tau_t - z_t$. The value of government debt can be obtained by imposing equilibrium on the government's flow constraint, taking conditional expectations, and "solving forward" to arrive at

$$\frac{B_t}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j (\tau_{t+j} - z_{t+j}) \tag{11}$$

This intertemporal equilibrium condition, (11), provides a new perspective on the crux of passive tax policy. Because P_t is nailed down by monetary policy and $\{z_{t+j}\}_{j=1}^{\infty}$ is being set independently of both monetary and tax policies, any increase in transfers at t, which is financed by new sales of nominal B_t, *must* generate an expectation that taxes will rise in the future by exactly enough to support the higher value of real B_t/P_t.

In this model, the only potential source of an expansion in debt is disturbances to transfers. But passive tax policy implies that this pattern of fiscal adjustment must occur regardless of the reason that B_t increases: economic downturns that automatically reduce taxes and raise transfers, changes in household portfolio behavior, changes in government spending, or central bank open-market operations. To expand on the last example, we could modify this model to include money to allow us to imagine that the central bank decides to tighten monetary policy exogenously at t by conducting an open-market sale of bonds. If monetary policy is active, then the monetary contraction both raises B_t—bonds held by households—and it lowers P_t; real debt rises from both effects. This can be an equilibrium *only if* fiscal

[9]As Sims (1999) and Cochrane (2011a) emphasize, echoing Obstfeld and Rogoff (1983), there are actually a continuum of *explosive* solutions to (8), each one associated with the central bank threatening to drive inflation to positive or negative infinity if the private sector's expectations are not anchored on π^*. Cochrane uses this logic to argue that fundamentally only fiscal policy can uniquely determine inflation. Sims argues, in a monetary model that supports a barter equilibrium, that only a fiscal commitment to a floor value of real money balances can deliver a unique equilibrium. Determinacy comes from the fiscal authority committing to switch from a passive stance if the price level gets too high to adopt a policy that redeems government liabilities at a fixed floor real value. If the fiscal commitment is believed, in equilibrium, this fiscal "backstop" will never need to be used and only stable price-level paths will be realized. Both Cochrane and Sims argue that there is nothing monetary policy alone can do to eliminate the explosive price-level paths.

policy is expected to support it by passively raising future real tax revenues. That is, given active monetary policy, (11) imposes restrictions on the class of tax policies that is consistent with equilibrium; those policies are labeled "passive" because the tax authority has limited discretion in choosing policy. Refusal by tax policy to adjust appropriately undermines the ability of open-market operations to affect inflation in the conventional manner, as in Wallace (1981).

A policy regime in which monetary policy is active and tax policy is passive produces the conventional outcome that inflation is always and everywhere a monetary phenomenon and a hawkish central bank can successfully anchor actual and expected inflation at the inflation target. Tax policy must support the active monetary behavior by passively adjusting taxes to finance disturbances to government debt—from whatever source, including monetary policy—and ensure policy is sustainable.

Although conventional, this regime is not the only mechanism by which monetary and fiscal policy can jointly deliver a unique bounded equilibrium. We turn now to the other polar case.

3.2. **Regime F: *Passive Monetary/Active Tax Policy.*** Passive tax behavior is a stringent requirement: the tax authority must be willing and able to raise taxes in the face of rising government debt. For a variety of reasons, this does not always happen, and it certainly does not happen in the automated way prescribed by the tax rule in (6). Sometimes political factors—such as the desire to seek reelection—prevent taxes from rising as needed to stabilize debt.[10] Some countries simply do not have the fiscal infrastructure in place to generate the necessary tax revenues. Others might be at or near the peak of their Laffer curves, suggesting they are close to the fiscal limit.[11] In this case, tax policy is active and $0 \leq \gamma < 1/\beta - 1$.

Analogously, there are also periods when the concerns of monetary policy move away from inflation stabilization and toward other matters, such as output stabilization or financial crises. These are periods in which monetary policy is no longer active, instead adjusting the nominal interest rate only weakly in response to inflation. Woodford (2001) cites the Federal Reserve's bond-price pegging policy during and immediately after World War II as an example of passive monetary policy. The recent global recession and financial crisis is a striking case when central banks' concerns shifted away from inflation. In some countries the policy rate was reduced to its zero lower bound. Then monetary policy is passive and, in terms of policy rule (5), $0 \leq \alpha < 1/\beta$.

We focus on a particular policy mix that yields clean economic interpretations: the nominal interest rate is set independently of inflation, $\alpha = 0$ and $R_t^{-1} = R^{*-1} \geq 1$, and taxes are set independently of debt, $\gamma = 0$ and $\tau_t = \tau^* > 0$. These policy specifications might seem extreme and special, but the qualitative points that emerge generalize to other specifications of passive monetary/active tax policies.

[10] Davig and Leeper (2006, 2011) generalize (6) to estimate Markov switching rules for the United States and find that tax policy has switched between periods when taxes rise with debt and periods when they do not.

[11] Trabandt and Uhlig (2006) characterize Laffer curves for capital and labor taxes in 14 EU countries and the United States to find that some countries—Denmark and Sweden—are on the wrong side of the curve, suggesting that those countries must lower tax rates to raise revenues.

One result pops out immediately. Applying the pegged nominal interest rate policy to the Fisher relation, (4) yields

$$E_t\left(\frac{P_t}{P_{t+1}}\right) = \frac{1}{\beta R^*} = \frac{1}{\pi^*} \tag{12}$$

so expected inflation is anchored on the inflation target, an outcome that is perfectly consistent with one aim of inflation-targeting central banks. It turns out, however, that another aim of inflation targeters—stabilization of actual inflation—which can be achieved by active monetary/passive fiscal policy, is no longer attainable.

Impose the active tax rule on the intertemporal equilibrium condition, (11),

$$\frac{B_t}{P_t} = \left(\frac{\beta}{1-\beta}\right)\tau^* - E_t\sum_{j=1}^{\infty}\beta^j z_{t+j} \tag{13}$$

and use the government's flow constraint, (3), to solve for the price level

$$P_t = \frac{R^* B_{t-1}}{\left(\frac{1}{1-\beta}\right)\tau^* - E_t\sum_{j=0}^{\infty}\beta^j z_{t+j}} \tag{14}$$

At time t, the numerator of this expression is predetermined, representing the nominal value of household wealth carried into period t. The denominator is the expected present value of primary fiscal surpluses from date t on, which is exogenous. So long as $R^* B_{t-1} > 0$ and the present value of revenues exceeds the present value of transfers, a condition that must hold if government debt has positive value, expression (14) delivers a unique $P_t > 0$.

We have done nothing mystical here, despite what some critics claim [for example, Buiter (2002) or McCallum (2001)]. In particular, the government is not assumed to behave in a manner that violates its budget constraint. Unlike competitive households, the government is not required to choose sequences of control variables that are consist with its budget constraint for all possible price sequences. Indeed, for a central bank to target inflation, it *cannot* be choosing its policy instrument to be consistent with any sequence of the price level; doing so would produce an indeterminate equilibrium. Identical reasoning applies to the fiscal authority: the value of a dollar of debt—$1/P_t$—depends on expectations about fiscal decisions in the future; expectations, in turn, are determined by the tax rule the fiscal authority announces. The fiscal authority credibly commits to its tax rule and, given the process for transfers, this determines the backing of government debt and, therefore, its market value.

Using the solution for the price level in (14) to compute expected inflation, it is straightforward to show that $\beta E_t(P_t/P_{t+1}) = 1/R^*$, as required by the Fisher relation and monetary policy behavior.[12] This observation leads to a sharp dichotomy between the roles of monetary and fiscal policy in price-level determination: monetary policy alone appears to determine *expected* inflation by choosing the level at which to peg the nominal interest rate, R^*, while conditional on that choice, fiscal variables appear to determine *realized* inflation. Monetary

[12]To see this, compute

$$E_{t-1}\frac{1}{P_t} = \frac{\left(\frac{1}{1-\beta}\right)\tau^* - E_{t-1}\sum_{j=0}^{\infty}\beta^j z_{t+j}}{R^* B_{t-1}}$$

To find expected inflation, simply use the date $t-1$ version of (14) for P_{t-1} and simplify to obtain $\beta E_t(P_t/P_{t+1}) = 1/R_{t-1} = 1/R^*$.

policy's ability to target expected inflation holds in this simple model with a fixed policy regime; as we show in section 4, when regime change is possible, monetary policy may not even be able to control expected inflation.

To understand the nature of this equilibrium, we need to delve into the underlying economic behavior. This is an environment in which changes in debt do *not* elicit any changes in expected taxes, unlike in section 3.1. First consider a one-off increase in current transfer payments, z_t, financed by new debt issuance, B_t. With no offsetting increase in current or expected tax obligations, at initial prices households feel wealthier and they try to shift up their consumption paths. Higher demand for goods drives up the price level, and continues to do so until the wealth effect dissipates and households are content with their initial consumption plan. This is why in expression (13) the value of debt at t changes with expected, but not current, transfers. Now imagine that at time t households receive news of higher transfers in the future. In the first instance, there is no change in nominal debt at t, but there is still an increase in household wealth. Through the same mechanism, P_t must rise to revalue current debt to be consistent with the new expected path of transfers: the value of debt falls in line with the lower expected present value of surpluses.

Cochrane (2009, p. 5) offers another interpretation of the equilibrium in which "'aggregate demand' is really just the mirror image of demand for government debt." An expectation that transfers will rise in the future reduces the household's assessment of the value of government debt. Households can shed debt only by converting it into demand for consumption goods, hence the increase in aggregate demand that translates into a higher price level.

Expression (14) highlights that in this policy regime the impacts of monetary policy change dramatically. When the central bank chooses a higher rate at which to peg the nominal interest rate, the effect is to *raise* the price level next period. This echoes Sargent and Wallace (1981), but the economic mechanism is different. In the current policy mix, a higher nominal interest rate raises the interest payments the household receives on the government bonds it holds. Higher $R^* B_{t-1}$, with no higher anticipated taxes raises household nominal wealth, triggering the same adjustments as above. In this sense, as in Sargent and Wallace, monetary policy has lost control of inflation.

This section has reviewed existing results on price-level determination under alternative monetary-fiscal policy regimes. In each regime a bounded inflation rate is uniquely determined, but the impacts of changes in policy differ markedly across the two regimes. We now turn to elaborate on a key difference between the fiscal theory and unpleasant arithmetic.

3.3. **Why the Fiscal Theory is Not Unpleasant Arithmetic.** It is not uncommon for policy makers to equate fiscal inflations to the mechanism that Sargent and Wallace (1981) highlighted and then to dismiss its relevance. As King (1995, p. 171–172) wrote about unpleasant arithmetic:

> "I have never found this proposition very convincing.... [A]s an empirical matter, the proposition is of little current relevance to the major industrial countries. This is for two reasons. First, seigniorage—financing the deficit by issuing currency rather than bonds—is very small relative to other sources of

> revenues. Second, over the past decade or so, governments have become increasingly committed to price stability.... This sea change in the conventional wisdom about price stability leaves no room for inflation to bail out fiscal policy."

Later in the same commentary, King [p. 173] acknowledges that "...periodic episodes of *unexpected* inflation...have reduced debt-to-GDP ratios." This observation is consistent with the fiscal theory, though King does not attribute the inflation to fiscal news.

A fiscal theory equilibrium can be consistent with *any* average rate of inflation and money creation. This point emerges clearly in Leeper's (1991) local analysis around a given deterministic steady state: on average inflation could be zero, yet monetary and fiscal shocks generate all the results shown in section 3.2. In the model above, the unconditional mean of inflation is π^*, the inflation target, and in a monetary version of the model, π^* determines average seigniorage revenues.

A key difference between the fiscal theory and unpleasant arithmetic is that the former operates only in an economy with nominal government debt, whereas the latter is typically discussed under the assumption of real debt. Without a fully fleshed-out model, the distinction between nominal and real debt can be understood by examining the corresponding intertemporal equilibrium conditions—the analogs to (13). We add fiat currency to make a point about the role of seigniorage revenues. For nominal debt the equilibrium condition is

$$B_{t-1} = P_t \sum_{j=0}^{\infty} \beta^j E_t \left[\tau_{t+j} - z_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right] \tag{15}$$

while for real debt, v_t, it is

$$v_{t-1} = \sum_{j=0}^{\infty} \beta^j E_t \left[\tau_{t+j} - z_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right] \tag{16}$$

Both conditions involve the expected present value of primary surpluses plus seigniorage. The fiscal theory is about how changes in this expected present value lead to changes in P_t. Unpleasant arithmetic is about how increases in v_{t-1} induce increases in expected future seigniorage, $(M_{t+j} - M_{t+j-1})/P_t$.

To understand the differences, consider a hypothetical increase in P_t, holding all else fixed. In (15), higher P_t raises the nominal backing to debt, so it implies higher cash flows in the form of nominal primary surpluses: more nominal debt can be supported with no change in real surpluses or seigniorage. In (16), higher P_t lowers the real backing to debt because it reduces seigniorage revenues and *real* cash flows.

This makes clear why the fiscal theory is *not* about seigniorage: even if real balances are arbitrarily small or the economy is on the wrong side of the seigniorage Laffer curve, under the fiscal theory, higher P_t increases the backing of debt by raising the nominal cash flows associated with primary surpluses. In this case, as (16) shows, higher P_t does nothing to affect the backing of real debt.

3.4. **Regime F: Two-Period Government Debt.** To get a richer sense of inflation dynamics in the passive monetary/active fiscal regime, suppose that the government issues

nominal bonds with a maximum maturity of two periods. Let $B_t(j)$ denote the face value of zero-coupon nominal bonds outstanding at the end of period t, which mature in period j and let $Q_t(j)$ be the corresponding nominal price for those bonds. At the beginning of period t, the returns, $R_t(t+1)$ and $R_t(t+2)$, are known with certainty and are risk free. Clearly, $R_t(t+1)^{-1} = Q_t(t+1)$, $R_t(t+2)^{-1} = Q_t(t+2)$ $Q_t(t) = 1$ and $B_t(j) = 0$ for $j \leq t$.

The government's flow budget constraint is

$$\frac{Q_t(t+1)B_t(t+1)}{P_t} + \frac{Q_t(t+2)B_t(t+2)}{P_t} + x_t = \frac{B_{t-1}(t)}{P_t} + \frac{Q_t(t+1)B_{t-1}(t+1)}{P_t} \tag{17}$$

where x_t is the primary surplus inclusive of seigniorage revenues, defined as

$$x_t \equiv \tau_t - z_t + \frac{M_t - M_{t-1}}{P_t} \tag{18}$$

where M_t is the nominal quantity of fiat money outstanding.

We bring money in by positing a simple, interest inelastic, demand for money[13]

$$\frac{M_t}{P_t} = f(c_t) \tag{19}$$

that, in equilibrium, implies that real money balances are constant

$$\frac{M_t}{P_t} = k \tag{20}$$

In a frictionless economy with a constant real interest rate, the household's Euler equation deliver the one- and two-period nominal bond prices

$$Q_t(t+1) = \beta E_t \left(\frac{P_t}{P_{t+1}} \right) \tag{21}$$

$$Q_t(t+2) = \beta E_t Q_{t+1}(t+2) \left(\frac{P_t}{P_{t+1}} \right) \tag{22}$$

Using (21) in (22) yields

$$Q_t(t+2) = \beta^2 E_t \left(\frac{P_t}{P_{t+2}} \right) \tag{23}$$

Take expectations of the government budget constraint, impose the asset-pricing relations and the transversality condition, which requires the expected present value of the market value of debt to be zero, to obtain the intertemporal equilibrium condition

$$\frac{Q_t(t+1)B_t(t+1) + Q_t(t+2)B_t(t+2)}{P_t} = \sum_{i=1}^{\infty} \beta^i E_t x_{t+i} \tag{24}$$

Combining (24) with (17) yields

$$\frac{B_{t-1}(t) + Q_t(t+1)B_{t-1}(t+1)}{P_t} = \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} \tag{25}$$

[13]This specification may be obtained from a cash-in-advance model or from money-in-utility/transactions-cost models in which the interest elasticity is driven to the zero limit. Because money is essential, the model cannot generate equilibria with explosive inflation, as in Sims (1994) and Cochrane (2011a).

The left side of (25) is the market value of debt outstanding at the beginning of period t. Two terms in this value—the face value of outstanding nominal bonds, $B_{t-1}(t)$ and $B_{t-1}(t+1)$—are carried into period t from period $t-1$, so they are predetermined at t. But two other terms—the price of two-period bonds issued at $t-1$ and sold at t, $Q_t(t+1)$, and the price level, P_t—are determined at period t and respond to shocks and news that arrive at t.

Using the equilibrium relationship in (21) in (25) makes clear the tradeoffs that monetary policy faces when primary surpluses are fixed

$$\frac{B_{t-1}(t)}{P_t} + \beta B_{t-1}(t+1) E_t \frac{1}{P_{t+1}} = \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} \tag{26}$$

Monetary policy faces two limiting cases. It can lean strongly against current inflation to fix P_t, but then it must permit future inflation, $E_t(1/P_{t+1})$, to adjust. Alternatively, it can stabilize expected inflation at $t+1$, but then it must allow P_t to adjust. The tradeoff between current and future inflation depends on the ratio $B_{t-1}(t+1)/B_{t-1}(t)$, the ratio between the outstanding quantities of two-period to one-period bonds, a role for the maturity structure of government debt that Cochrane (2001) emphasizes. As debt becomes of increasingly short maturity, this ratio falls and a larger change in expected inflation is required to compensate for a given change in current inflation.

3.4.1. *Fiscal Expansions and Inflation.* We employ the two equilibrium conditions, (20) and (26), to derive the implications for inflation of alternative policy environments. We pose monetary policy as controlling the one-period nominal bond price, $Q_t(t+1)$, which is equivalent to controlling the short-term nominal interest rate, $R_t = 1/Q_t(t+1)$.

For this exposition, we make the simplifying assumption that the primary surplus, $\{\tau_t - z_t\}$ is exogenous or at least independent of the price level and the value of outstanding government debt. This may seem like an extreme and implausible assumption in light of Hall and Sargent's (2011) accounting that since World War II adjustments in primary surpluses have been an important determinant of U.S. debt-GDP dynamics. Of course, Hall and Sargent's is an accounting exercise that does not aim to establish that fluctuations in government debt *caused* subsequent surplus adjustments that were designed to stabilize debt.[14] But even if we make the bold assumption of causality, Hall and Sargent do not find that surpluses *always* adjust to rationalize the value of debt. Other evidence, whose causal interpretation is also in question, suggests that U.S. fiscal policy has fluctuated between regimes in which policies systematically raise future surpluses in response to high debt and regimes in which surpluses evolve largely independently of debt [Davig and Leeper (2006)].

As we argued above, the fiscal stress advanced economies face smacks more of the European situation in the 1920s than the experiences of advanced economies' since World War II. Given the political economy forces at play, simple extrapolations of policy behavior over the past several decades into coming decades are tenuous at best. Assuming that fiscal policy will go through periods in which surpluses are set independently of debt or that private decision makers *believe* such periods are possible—even likely—is a reasonable working assumption.

[14] Bohn (1998) is often cited as evidence that establishes this causality, but his methods cannot distinguish between estimates of a behavioral relation for fiscal policy and an equilibrium relation between surpluses and debt [Li (2008)].

Exogenous surpluses are a tractable way to examine the qualitative nature of equilibria in which debt is not systematically stabilized by primary surpluses.

We take the primary fiscal surplus sequence, $\{\tau_t - z_t\}$, as exogenous and imagine that information arrives at t that causes agents to revise downward their views about current or expected surpluses.

The first term on the right side of (26) may be written as $x_t = \tau_t + s_t - z_t$. In equilibrium—imposing equilibrium condition (20)—seigniorage is

$$s_t = \frac{M_t - M_{t-1}}{P_t} = k - \frac{M_{t-1}}{P_t} \tag{27}$$

Then the second equilibrium condition, (26), becomes

$$\frac{B_{t-1}(t) + M_{t-1}}{P_t} + \beta B_{t-1}(t+1) E_t \frac{1}{P_{t+1}} = k + \tau_t - z_t + \sum_{i=1}^{\infty} \beta^i E_t x_{t+i} \tag{28}$$

For a given debt maturity structure, summarized by the ratio $B_t(t+2)/B_t(t+1)$, monetary policy behavior determines the mix of current and expected inflation that arises from lower current or anticipated surpluses.

Current Inflation

Suppose initially that the central bank pegs the short-bond price at $Q_t(t+1) = Q^*$ for all t, effectively pegging expected inflation through the Euler equation, (21). Then (28) becomes

$$\frac{W_{t-1}}{P_t} = \widehat{EPV}_t(x) \tag{29}$$

where $W_{t-1} \equiv B_{t-1}(t) + M_{t-1} + Q^* B_{t-1}(t+1)$ and $\widehat{EPV}_t(x) \equiv k + \tau_t - z_t + \sum_{i=1}^{\infty} \beta^i E_t x_{t+i}$. By pegging the bond price, the central bank forces the full adjustment to news about lower surpluses to occur through increases in the current price level that revalue the outstanding nominal government liabilities. For an incremental change in surpluses, $d\widehat{EPV}_t(x)$, the change in the price level is

$$dP_t = -\frac{W_{t-1}}{[\widehat{EPV}_t(x)]^2} d\widehat{EPV}_t(x) \tag{30}$$

so the rise in the price level is increasing in total nominal government liabilities outstanding and decreasing in the initial market value of those liabilities.

A higher price level raises nominal money demand. To maintain the pegged bond price at Q^*, the central bank must expand the nominal money stock by $dM_t = k dP_t$, which ensures that the money market clears at t. It does this by buying outstanding bonds with newly issued M_t. With Q^* pegged, this open-market purchase can occur in either one- or two-period bonds, to the same effect. As ever, characterizing monetary policy as controlling the nominal interest rate entails a supporting open-market policy.

Expressed in proportional changes, the equilibrium is

$$\frac{dP_t}{P_t} = \frac{dM_t}{M_t} = -\frac{d\widehat{EPV}_t(x)}{\widehat{EPV}_t(x)} \tag{31}$$

The supporting open-market policy is not the textbook case of $\Delta M_t = -\Delta B_t$, in which new money is swapped for bonds, dollar-for-dollar. Instead, given the new equilibrium price level from (30) and the associated new equilibrium level of money balances, $dM_t = kdP_t$, the new level of nominal bonds outstanding must be consistent with the government's flow budget constraint. Denote the face value of government bonds outstanding at t by $\mathcal{B}_t \equiv B_t(t+1) + Q^* B_t(t+2)$. In equilibrium, the change in $\mathcal{B}_t$ consistent with the government's budget constraint and the equilibrium in (31) may be expressed as

$$\frac{d\mathcal{B}_t}{\mathcal{B}_t} = \left(\frac{k + \tau_t - z_t}{Q^* \mathcal{B}_t / P_t}\right) \frac{d\widehat{EPV}_t(x)}{\widehat{EPV}_t(x)} \tag{32}$$

News at t that primary surpluses will be lower in the future raises P_t. To maintain equilibrium in the money market and allow the short-term bond price to be pegged at Q^*, the central bank passively expands M_t in proportion to the rise in prices. In general, this is not the end of the policy adjustments because the higher price level that arises from news about *future* surpluses leaves the government's budget out of balance by revaluing outstanding debt obligations. As (32) makes clear, in equilibrium the face value of government bonds may rise or fall—more or fewer bonds will be in the hands of the public in period t—as a consequence of the news of lower future surpluses. If the current (modified) primary surplus—$k+\tau_t-z_t$—is positive, the face value of bonds declines; if it's negative, the face value rises.

The empirical implications of this equilibrium underscore the difficulties associated with casual views about the patterns of correlation that a fiscal inflation produces. To summarize, news of lower future surpluses creates the following pattern of correlations:

- negative correlation between inflation and market value of initial government liabilities, W_{t-1}/P_t;
- positive correlation between inflation and money growth;
- *any* correlation between nominal debt growth and inflation (or money growth);
- higher inflation and money growth predicts *future* fiscal deficits, contradicting the Granger-causality results of King and Plosser (1985).

Evidently, monetary policy behavior—the pegging of short bond prices—plays a central role in this equilibrium. But that role is not the traditional one of monetizing debt and there will be no evidence in time series data that inflation is being produced by high current budget deficits or open-market purchases of government bonds, although there will be strong evidence that inflation is proportional to money growth.

Future Inflation

By pegging the short-term nominal rate in *every* period, the central bank also pegs the long-term (two-period) interest rate. This forces all adjustments to fiscal news into the current price level and leaves expected price levels unchanged. A different monetary policy can force all adjustments into future prices, leaving the current price level unchanged.

Rewrite equilibrium condition (28) as

$$\frac{B_{t-1}(t) + M_{t-1}}{P_t} + \beta[B_{t-1}(t+1) + M_t] E_t \frac{1}{P_{t+1}} = \widetilde{EPV}_t(x) \tag{33}$$

where $\widetilde{EPV}_t(x) \equiv (1+\beta)k + \tau_t - z_t + \tau_{t+1} - z_{t+1} + \sum_{i=2}^{\infty} \beta^i E_t x_{t+i}$.[15]

We seek an equilibrium in which $dP_t = 0$, implying that $dM_t = 0$ also. In such an equilibrium, news that revised down the expected present value, $\widetilde{EPV}_t(x)$, affects expected inflation according to

$$d\left(E_t \frac{P_t}{P_{t+1}}\right) = \frac{1}{\beta[(B_{t-1}(t+1) + M_t)/P_t]} d\widetilde{EPV}_t(x) \tag{34}$$

Lower expected primary surpluses produce higher expected inflation.

The central bank implements the equilibrium in which lower expected surpluses raise future, but not current, prices by adjusting the one-period nominal interest rate appropriately. First write the equilibrium change in expected prices in (34) in terms of $E_t(P_t/P_{t+1})$ and note that the Euler equation implies that $Q_t(t+1) = \beta E_t(P_t/P_{t+1})$. Monetary policy pushes into the future the inflationary consequences of anticipated fiscal expansions by setting policy as

$$dQ_t(t+1) = \frac{1}{\left(\frac{B_{t-1}(t+1)+M_t}{P_t}\right)} d\widetilde{EPV}_t(x) \tag{35}$$

If the expected present value of surpluses falls, the central bank reduces the price of one-period bonds, raising the one-period nominal interest rate. That is, monetary policy leans against expected fiscal expansion.

At $t+1$, when the higher price level is realized, M_{t+1} must rise proportionately. The equilibrium displays patterns of correlation analogous to those above and conventional empirical approaches to fiscal policy and inflation will have a difficult time finding evidence that fiscal expansions are inflationary. Data will contain overwhelming support, however, for positive money growth/inflation correlation.

3.5. **Regime F: Long-Term Government Debt.** Inflation dynamics become still more interesting when we posit that the government issues only consols, a perpetuity that never matures.[16] The government's flow budget constraint is

$$\frac{Q_t B_t}{P_t} + x_t = \frac{(1+Q_t)B_{t-1}}{P_t} \tag{36}$$

We also have the Euler equation for consols

$$Q_t = \beta E_t \frac{P_t}{P_{t+1}}(1+Q_{t+1}) \tag{37}$$

We are assuming an endowment economy in which the endowment is constant each period.

Iterate on the flow constraint, (36), impose (37) and the transversality condition, and combine the result with the flow budget constraint to yield the intertemporal equilibrium condition

$$\frac{(1+Q_t)B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t x_{t+j} = EPV_t(x) \tag{38}$$

[15]To obtain (33) we used $\beta E_t s_{t+1} = E_t[(M_{t+1} - M_t)/P_{t+1}] = \beta[k - M_t E_t(1/P_{t+1})]$.

[16]This exposition draws on Cochrane (2001, 2011b).

The intertemporal equilibrium condition implies a convenient expression linking, *in equilibrium*, the bond price, the current price level, and the expected present value of surpluses

$$\frac{d(1+Q_t)}{1+Q_t} - \frac{dP_t}{P_t} = \frac{dEPV_t(x)}{EPV_t(x)} \tag{39}$$

From (37), the price of the consol can be expressed in terms of the entire expected future path of inflation rates

$$Q_t = \sum_{j=1}^{\infty} \beta^j E_t \frac{P_t}{P_{t+j}} \tag{40}$$

$$= \sum_{j=0}^{\infty} E_t \left(\prod_{i=0}^{j} \frac{1}{R_{t+i}} \right) \tag{41}$$

where R_t is the one-period nominal interest rate controlled by the central bank. The associated short-term nominal bond is priced as $1/R_t = \beta E_t(P_t/P_{t+1})$.

Using (39), (40) and (41), a given percentage decrease in the expected present value of surpluses can be apportioned into any mix of current and expected inflation rates consistent with (38) and (40). Monetary policy behavior determines the precise pattern of expected inflation rates through its setting of current and expected short-term nominal interest rates.[17]

Consols, though not a realistic maturity structure for government bonds, help to make clear the range of possible inflation processes that a fiscal theory equilibrium can produce. First, inflation effects are larger when they are concentrated in only a few periods and smaller when they are spread over many periods. Second, because only the present value of inflation is pinned down by (38) and (40), news of lower future surpluses can generate *any* path of expected inflation: it can rise or fall in various periods, so long as the present value of expected inflation adjusts to satisfy (38) and (40). Third, because many paths of the surplus are consistent with a given expected present value, the expected surplus can also rise or fall in various periods, as long as the deliver the expected present value.

4. How Fiscal Policy Can Undermine Monetary Control of Inflation

This section examines situations in which fiscal policy can undermine monetary control of inflation. We provide three scenarios in which Regime M fails to target inflation. These scenarios are by no means exhaustive, but simply illustrate the extent to which monetary and fiscal policy must coordinate in order to effectively control the price level. One example draws on Davig, Leeper, and Walker (2010), Leeper (2011), and Leeper and Walker (2011) and assumes Regime M is operative until a fiscal limit is hit at date T. A fiscal limit is the point at which tax rates, either through political or economic constraints, can no longer adjust to passively raise future tax revenues. A second example introduces risky sovereign debt to show that a higher probability of default feeds directly into higher current inflation. The third scenario is a two-country monetary union in which one country follows Regime F with

[17]Because in this policy regime the equilibrium price level is uniquely determined by (38), together with equilibrium $\{Q_t\}$, monetary policy may be treated as setting the sequence of short rates, $\{R_t\}$, exogenously in any pattern desired, without fear of generating indeterminacy of equilibrium.

	Regime 1 $t = 0, 1, \ldots, T-1$	Regime 2 $t = T, T+1, \ldots$
Monetary Policy	$R_t^{-1} = R^{*-1} + \alpha\left(\frac{P_{t-1}}{P_t} - \frac{1}{\pi^*}\right)$	$R_t^{-1} = R^{*-1}$
Tax Policy	$\tau_t = \tau^* + \gamma\left(\frac{B_{t-1}}{P_{t-1}} - b^*\right)$	$\tau_t = \tau^{\max}$

TABLE 2: Monetary-Fiscal Policy Regimes Before and After the Fiscal Limit at Date T

the central bank pegging the nominal interest rate. We demonstrate in this case that even if the other country implements Regime M, then inflation in the monetary union is determined by the Regime F country, *regardless of the country's size*. This analysis draws on work by Sims (1997), Bergin (2000), and Daniel and Shiamptanis (2011)].

4.1. **Fiscal Limit.** This section modifies the simple model in section 3 by assuming the economy at some known future date T reaches a fiscal limit. Section 3.2 emphasizes the reluctance of increasing taxes to stabilize debt in the face of growing transfer payments. We model this by assuming that at date T, taxes reach their maximum, $\tau^{\max}$.[18]

Leading up to T, policy is in the active monetary/passive fiscal regime described above, but from date T on, tax policy has no option but to become active, with $\tau_t = \tau^{max}$ for $t \geq T$. If monetary policy remained active, neither authority would stabilize debt and debt would explode. Existence of a bounded equilibrium requires that monetary policy switch to being passive, which stabilizes debt. Table 2 summarizes the assumptions about policy behavior.

We assume that government transfers evolve exogenously according to the stochastic process

$$z_t = (1-\rho)z^* + \rho z_{t-1} + \varepsilon_t, \qquad 0 < \rho < 1 \tag{42}$$

where z^* is steady-state transfers and ε_t is a serially uncorrelated shock with $E_t\varepsilon_{t+1} = 0$.

The intertemporal equilibrium condition now is the sum of two distinct parts

$$\frac{B_0}{P_0} = E_0 \sum_{j=1}^{T-1} \beta^j s_j + E_0 \sum_{j=T}^{\infty} \beta^j s_j \tag{43}$$

where the function for the primary surplus, s_t, changes at the fiscal limit according to

$$s_t = \begin{cases} \tau^* - \gamma(B_{t-1}/P_{t-1} - b^*) - z_t, & t = 0, 1, ..., T-1 \\ \tau^{\max} - z_t, & t = T, ..., \infty \end{cases} \tag{44}$$

[18]In this model with lump-sum taxes there is no upper bound for taxes or debt, so long as debt does not grow faster than the real interest rate. But in a more plausible production economy, in which taxes distort behavior, there would be a natural fiscal limit—the peak of the Laffer curve. See Davig, Leeper, and Walker (2010) for further discussion and Bi (2011) for an application of an endogenous fiscal limit to the issue of sovereign debt default.

Expression (43) decomposes the value of government debt at the initial date into the expected present value of surpluses leading up to the fiscal limit and the expected present value of surpluses after the limit has been hit. Date T is assumed to be known.[19]

Evaluating the second part of (43) and letting $\tau^{\max} = \tau^*$, after the limit is hit at T

$$\begin{aligned} E_0 \sum_{j=T}^{\infty} \beta^j s_j &= E_0 \left(\frac{B_{T-1}}{P_{T-1}} \right) \\ &= \frac{\beta^T}{1-\beta}(\tau^* - z^*) - \frac{(\beta\rho)^T}{1-\beta\rho}(z_0 - z^*) \end{aligned} \tag{45}$$

The first part of (43) is given by

$$\begin{aligned} E_0 \sum_{j=1}^{T-1} \beta^j s_j &= \sum_{j=1}^{T-1} \left(\frac{\beta}{1-\gamma\beta} \right)^j [(\tau^* - \gamma b^*) - E_0 z_j] \\ &= (\tau^* - \gamma b^* - z^*) \sum_{j=1}^{T-1} \left(\frac{\beta}{1-\gamma\beta} \right)^j - (z_0 - z^*) \sum_{j=1}^{T-1} \left(\frac{\beta\rho}{1-\gamma\beta} \right)^j \end{aligned} \tag{46}$$

Pulling together (45) and (46) yields equilibrium real debt at date $t = 0$ as a function of fiscal parameters and the date 0 realization of transfers

$$\begin{aligned} \frac{B_0}{P_0} &= (\tau^* - \gamma b^* - z^*) \sum_{i=1}^{T-1} \left(\frac{\beta}{1-\gamma\beta} \right)^i - (z_0 - z^*) \sum_{i=1}^{T-1} \left(\frac{\beta\rho}{1-\gamma\beta} \right)^i \\ &+ \left(\frac{\beta}{1-\gamma\beta} \right)^{T-1} \left[\frac{\beta^T}{1-\beta}(\tau^{\max} - z^*) - \frac{(\beta\rho)^T}{1-\beta\rho}(z_0 - z^*) \right] \end{aligned} \tag{47}$$

This expression determines the equilibrium value of debt at $t = 0$ and, by extension, at each date in the future. We make three observations. First, this economy will *not* exhibit Ricardian equivalence for τ^{max} sufficiently small and sufficiently large increases in transfers. In the derivations above, we set $\tau^{max} = \tau^*$, but a sufficient condition for our results to go through is given by $\tau^{max} < \tau^* + \gamma(B_{t-1}/P_{t-1} - b^*)$ for all realizations of z_t. The fiscal rule after T implies that positive innovations to transfers will not be entirely offset by future changes in tax rates. Only in the absence of the fiscal limit or if τ^{max} is sufficiently large will Ricardian equivalence hold. This occurs despite the fact that in the absence of a fiscal limit such a tax rule delivers Ricardian equivalence, as it did in section 3.1. Second, higher transfers at time 0, z_0, which portend a higher future path of transfers because of their positive serial correlation, reduce the value of debt. This occurs for the reasons that section 3.2 lays out: higher expected government expenditures reduce the backing and, therefore, the value of government liabilities. Finally, how aggressively tax policy responds to debt before hitting the fiscal limit, γ, matters for the value of debt. The Ricardian equivalence that exists in the permanent active monetary/passive tax regime implies that the *timing* of taxation is irrelevant: how rapidly taxes stabilize debt has no bearing on the value of debt so long as debt is sustainable.

[19] Davig, Leeper, and Walker (2010) and Leeper and Walker (2011) relax this assumption by modeling T is a random variable. In this case, there are expectational spillover effects which further strengthen the arguments made in this section.

To calculate the price level at $t = 0$, use the government's flow budget constraint and the fact that $s_0 = \tau_0 - z_0$, with taxes following the rule shown in table 2 to solve for P_0:

$$P_0 = \frac{R_{-1}B_{-1}}{b_0 + \tau_0 - z_0} \tag{48}$$

Given $R_{-1}B_{-1} > 0$, (48) yields a unique $P_0 > 0$. Entire sequences of equilibrium $\{P_t, R_t^{-1}\}_{t=0}^{\infty}$ are solved recursively: having solved for B_0/P_0 and P_0, obtain R_0 from the monetary policy rule in table 2, and derive the nomimal value of debt. Then use (47) redated at $t = 1$ to obtain equilibrium B_1/P_1 and the government budget constraint at $t = 1$ to solve for P_1 using (48) redated at $t = 1$, and so forth.

The equilibrium price level has the same features as it does under the passive monetary/active tax regime in section 3.2. This is because forward-looking agents know that higher current or expected transfers are not backed in present-value terms by expected taxes. This, in turn, raises household wealth which increases the demand for goods and drives up the price level (reducing the value of debt to an equilibrium value). Similarities between this equilibrium and that in section 3.2 stem from the fact that price-level determination is driven by beliefs about policy *in the long run*. From T on, this economy is identical to the fixed-regime passive monetary/active fiscal policies economy and it is beliefs about long-run policies that determine the price level. Alternatively, one may think of price level determination in this economy as coming from agents learning about (43), along the lines of Eusepi and Preston (2011). In such an economy, agents coordinate beliefs on long-run policies and the equilibrium would be one in which fiscal policy is active and monetary policy is passive. Of course, before the fiscal limit the two economies are quite different and the behavior of the price level will also be different.

In this environment, monetary policy continues to determine expected inflation rate while fiscal policy determines realizations. Combining (4) with the monetary policy rule in table 2, we obtain an expression in expected inflation

$$E_t\left(\frac{P_t}{P_{t+1}} - \frac{1}{\pi^*}\right) = \frac{\alpha}{\beta}\left(\frac{P_{t-1}}{P_t} - \frac{1}{\pi^*}\right) \tag{49}$$

As argued above, the equilibrium price level sequence, $\{P_t\}_{t=0}^{\infty}$ is determined by versions of (47) and (48) for each date t, so (49) describes the evolution of expected inflation. Given equilibrium P_0 from (48) and an arbitrary P_{-1}—arbitrary because the economy starts at $t = 0$ and cannot possibly determine P_{-1}, regardless of policy behavior—(49) shows that $E_0(P_0/P_1)$ grows relative to the initial inflation rate. In fact, throughout the active monetary policy/passive fiscal policy phase, for $t = 0, 1, \ldots, T-1$, expected inflation grows at the rate $\alpha\beta^{-1} > 1$. In periods $t \geq T$ monetary policy pegs the nominal interest rate at R^*, and expected inflation is constant: $E_t(P_t/P_{t+1}) = (R^*\beta)^{-1} = 1/\pi^*$.

The implications of the equilibrium laid out in equations (47), (48), and (49) for government debt, inflation, and the anchoring of expectations on the target values (b^*, π^*) are most clearly seen in a simulation of the equilibrium. Figure 2 contrasts the paths of the debt-GDP ratio from two models: the fixed passive monetary/active tax regime in section 3.2—dashed line—and the present model in which an active monetary/passive tax regime is in place until the economy hits the fiscal limit at date T, when policies switch permanently to a passive

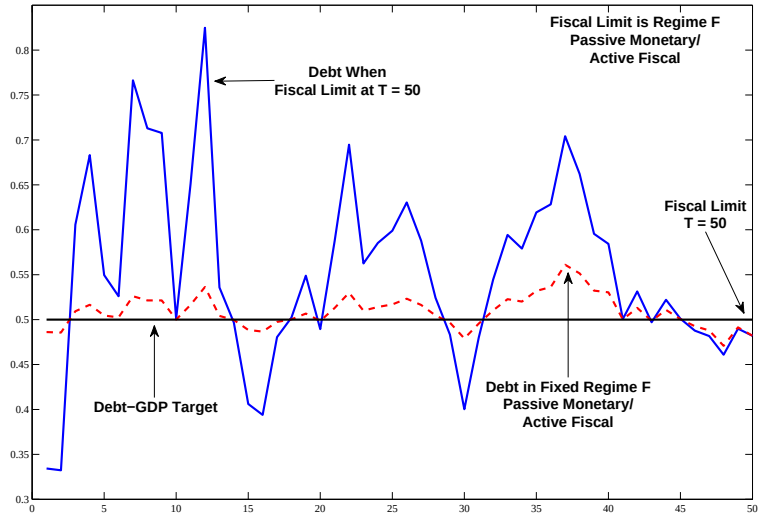

FIGURE 2: Debt-GDP ratios for a realization of transfers for two models: fixed passive monetary/active tax regime in section 3.2—dashed line—and model in which an active monetary/passive tax regime is in place until the economy hits the fiscal limit at date T, when policies switch permanently to passive monetary/active tax—solid line.

monetary/active tax combination—solid line.[20] The fixed regime displays stable fluctuations of real debt around the 50 percent steady state debt-GDP, which, of course, the other model also produces once it hits the fiscal limit. Leading up to the fiscal limit, however, it is clear that the active monetary/passive tax policy combination does not keep debt as close to target.

Expected inflation evolves according to (49). Since leading up the fiscal limit monetary policy is active, with $\alpha > 1/\beta$, there is no tendency for expected inflation to be anchored on the inflation target. Figure 3 plots the inflation rate from the fixed-regime model in section 3.2—dashed line—and from the present model—solid line—along with expected inflation from the present model—dotted dashed line. Inflation in the fixed regime fluctuates around π^* and, of course, with the pegged nominal interest rate, expected inflation is anchored on target. But in the period leading up to the fiscal limit, the price level is being determined primarily by fluctuations in the real value of debt, which as figure 2 shows, deviates wildly from b^*. Expected inflation in that period, though not *independent* of the inflation target, is certainly not anchored by the target. Instead, under active monetary policy, the deviation of expected inflation from target grows with the deviation of actual inflation from target in the previous period. The figure shows how equation (49) makes expected inflation *follow* actual inflation, with active monetary policy amplifying movements in expected inflation.

[20]Figures 2 through 5 use the following calibration. Leading up to the fiscal limit, $\alpha = 1.50$ and $\gamma = 0.10$ and at the limit and in the fixed-regime model, $\alpha = \gamma = 0.0$. We assume steady state values $\tau^* = 0.19$, $z^* = 0.17$, $\pi^* = 1.02$ (gross inflation rate) and we assume $1/\beta = 1.04$ so that $b^* = 0.50$. The transfers process has $\rho = 0.90$ and $\sigma = 0.003$. Identical realizations of transfers were used in all the figures.

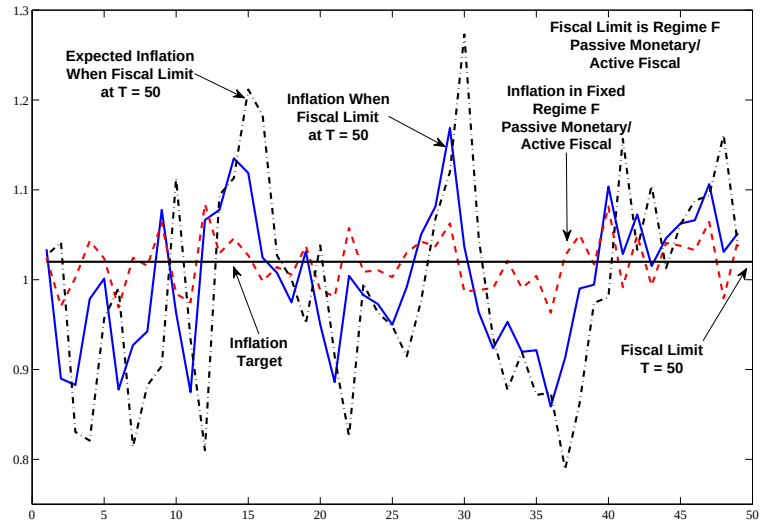

FIGURE 3: Inflation for a realization of transfers for two models: fixed passive monetary/active tax regime in section 3.2—dashed line—and model in which an active monetary/passive tax regime is in place until the economy hits the fiscal limit at date T, when policies switch permanently to passive monetary/active tax—solid line; expectation of inflation from present model—dotted dashed line.

To underscore the extent to which inflation is unhinged from monetary policy, even in the active monetary/passive tax regime before the fiscal limit, suppose that tax policy reacts more aggressively to debt. A higher value of γ can have unexpected consequences. Expression (47) makes clear that raising γ, which in a fixed active monetary/passive tax regime would stabilize debt more quickly, *amplifies* the effects of transfers shocks on debt. A more volatile value of debt, in turn, translates into more volatile actual and expected inflation. Figures 4 and 5 show this result by repeating the previous figures, but with a passive tax policy that responds more strongly to debt (γ is raised from 0.10 to 0.15).

Figures 4 and 5 also illustrate a general phenomenon: as the economy approaches the fiscal limit at time T, the equilibrium with different tax policies converge. As we also see in figures 2 and 3, of course, as time approaches T, the equilibrium also converges to the fixed-regime economy.

An analogous exercise for monetary policy illustrates its impotence when there is a fiscal limit. A more hawkish monetary policy stance, higher α, has no effect whatsoever on the value of debt and inflation: α does not appear in expression (47) for real debt or expression (48) for the price level. More hawkish monetary policy does, however, *amplify* the volatility of expected inflation, as the evolution of expected inflation, equation (49), shows.

Because monetary policy loses control of inflation after the fiscal limit is reached, forward-looking behavior implies it also loses control of inflation before the fiscal limit is hit. By extension, changes in fiscal behavior in the period leading up to the limit affects both the equilibrium inflation process and the process for expected inflation.

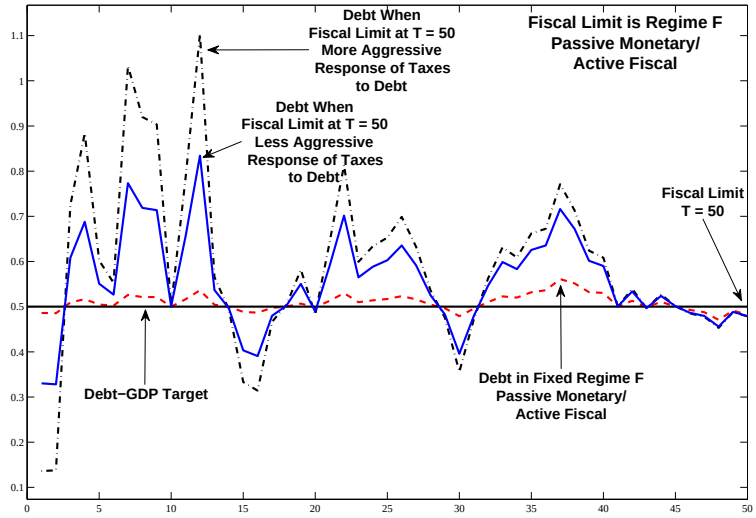

FIGURE 4: Debt-GDP ratios for two settings of tax policy: fixed passive monetary/active fiscal regime in section 3.2—dashed line—the active monetary/passive fiscal regime before the fiscal limit at date T with weaker response of taxes to debt ($\gamma = 0.10$)—solid line—the active monetary/passive fiscal regime before the fiscal limit at date T with stronger response of taxes to debt ($\gamma = 0.15$)—dotted dashed line.

4.2. **Risky Sovereign Debt and Inflation.** Bi, Leeper, and Leith (2010) explore how the possibility of sovereign debt default can further complicate the central bank's efforts to control inflation. Here we show this basic result in a simple example.

Consider a constant endowment, cashless economy in which the equilibrium real interest rate, $1/\beta$, is also constant. Government default is the sole source of uncertainty and for the current purposes, the decision to default by the fraction $\delta_t \in [0, 1]$ on outstanding debt carried into period t is exogenous and follows a known stochastic process. Let R_t be the gross risky rate of return on nominal government debt and $\pi_t = P_t/P_{t-1}$ be the inflation rate. Household optimization yields the Fisher relation

$$\frac{1}{R_t} = \beta E_t \left[\frac{1 - \delta_{t+1}}{\pi_{t+1}} \right] \tag{50}$$

while trade in risk-free bonds (assumed to be in zero net supply) gives an analogous relation for the risk-free interest rate, R_t^f,

$$\frac{1}{R_t^f} = \beta E_t \left[\frac{1}{\pi_{t+1}} \right] \tag{51}$$

The government's budget constraint is

$$\frac{B_t}{P_t} + s_t = \frac{(1 - \delta_t)}{\pi_t} R_{t-1} \frac{B_{t-1}}{P_{t-1}} \tag{52}$$

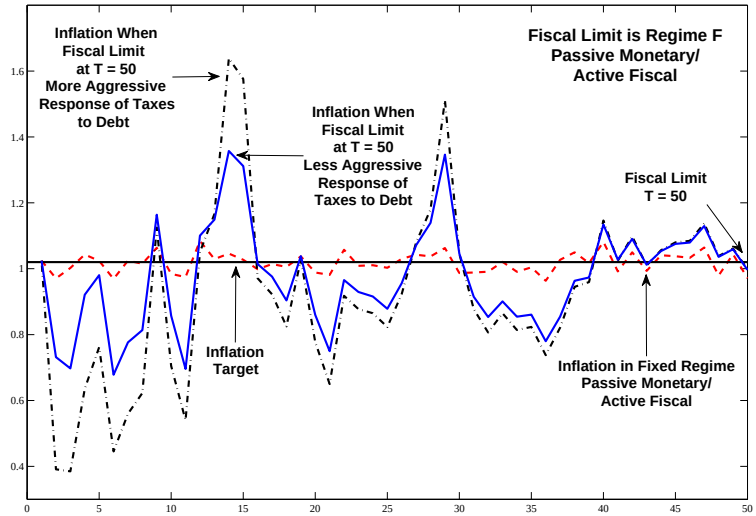

FIGURE 5: Inflation for two settings of tax policy: actual inflation in fixed passive monetary/active fiscal regime in section 3.2—dashed line—expected inflation in the active monetary/passive fiscal regime before the fiscal limit at date T with weaker response of taxes to debt ($\gamma = 0.10$)—solid line—expected inflation in the active monetary/passive fiscal regime before the fiscal limit at date T with stronger response of taxes to debt ($\gamma = 0.15$)—dotted dashed line.

where s_t is the primary surplus. Write this constraint at $t+1$, take expectations conditional on information at t, impose the Euler equation $\beta^{-1} = E_t(1-\delta_{t+1})R_t/\pi_{t+1}$, and solve for B_t/P_t to yield

$$\frac{B_t}{P_t} = \beta E_t \frac{B_{t+1}}{P_{t+1}} + \beta E_t s_{t+1} \tag{53}$$

When the real interest rate is fixed, both the nominal rate and the inflation rate reflect default, so that the expected default rate drops out once expectations are taken. This implies that only surprises in default directly affect the evolution of real government debt in this flexible-price endowment economy. In light of this, we obtain, by iterating on (53) and imposing the household's transversality condition

$$\frac{B_t}{P_t} = \sum_{j=1}^{\infty} \beta^j E_t s_{t+j} \tag{54}$$

Expression (54) is the usual intertemporal equilibrium condition that equates the value of government debt to the expected present value of "cash flows," which are primary surpluses.

Fiscal policy sets the surplus in order to stabilize the post-default value of government debt

$$s_t - s^* = \gamma \left[(1-\delta_t)\frac{B_{t-1}}{P_{t-1}} - b^* \right] \tag{55}$$

where s^* and b^* are target and steady state values for the surplus and real debt and $b_{t-1} = B_{t-1}/P_{t-1}$.

Substituting (55) into (52) and taking expectations at time t yields the evolution of expected debt

$$E_t b_{t+1} + (s^* - \gamma b^*) = [\beta^{-1} - \gamma(1 - E_t \delta_{t+1})] b_t \tag{56}$$

One result that emerges immediately from (56) is that stability of the debt process in the face of debt default requires that

$$\gamma > \frac{\beta^{-1} - 1}{1 - E_t \delta_{t+1}} \tag{57}$$

a condition that potentially is far more demanding than the usual one that $\gamma > \beta^{-1} - 1$, particularly when substantial default rates are possible. Provided this condition is fulfilled, however, fiscal policy remains passive and capable of stabilizing the real value of government debt.

Following Uribe (2006) and Schabert (2010), we assume that monetary policy sets the rate on short-term government debt, the risky nominal interest rate, R_t, according to a simple Taylor rule

$$\frac{1}{R_t} = \frac{1}{R^*} + \alpha \left(\frac{1}{\pi_t} - \frac{1}{\pi^*} \right) \tag{58}$$

Monetary policy targets inflation by setting $\alpha/\beta > 1$. Aside from being the dominant rule in the literature, in the context of our cashless model it is natural for monetary policy to be implemented by varying the contractual interest rate on government debt, rather than the risk-free interest rate on private debt, over which the government has no direct control and which is in zero net supply in equilibrium. More generally, in the transmission from the very short-term rates targeted through open market operations to the wider economy and, ultimately inflation, the central bank would expect to see a significant degree of pass through to the contractual interest rates employed throughout the economy.[21] Indeed, since government bonds typically form the collateral for the repo contracts undertaken by central banks, it is inevitable that without an offsetting policy adjustment, the policy rates pick up some of the default risk.[22]

Combining the policy rule defined in terms of the risk-free interest rate with the Fisher relation, (51), yields the dynamic equation for inflation

$$\frac{1}{\pi_t} - \frac{1}{\pi^*} = \frac{\beta}{\alpha} E_t \left(\frac{1}{\pi_{t+1}} - \frac{1}{\pi^*} \right) \tag{59}$$

which implies monetary policy hits its target inflation rate, provided the policy behavior is sufficiently active, $\beta/\alpha < 1$.[23] Although default can weaken the passivity of a fiscal rule defined in terms of the post-default level of debt, provided it satisfies (57), fiscal policy

[21]Empirical evidence suggests that the rate at which policy interest rates pass through to bank interest rates is quite high—about 90 percent within a quarter [Gambacorta (2008)]. We are implicitly assuming similarly high rates of pass through to government bond yields.

[22]Sims (2008) emphasizes that the unconventional operations of many central banks—particularly the Fed and the ECB—in recent years have made the central banks' balance sheets riskier. If foreign reserves are an important component of the bank's assets, as for the ECB, then surprise appreciation of the euro devalues its assets relative to its liabilities. The Fed's increased holdings of long-term Treasuries expose its balance sheet to more interest-rate risk than normal. Riskiness is exacerbated if the central bank is not assured that the fiscal authority will back it in times of large declines is asset values.

[23]Throughout this paper, we restrict attention to locally bounded solutions.

remains passive, and an active monetary policy can successfully target inflation when the central bank's instrument is the risk-free nominal rate.

But if monetary policy controls the risky interest rate, R_t, default influences the ability of the monetary authority to target inflation, even if fiscal policy remains passive and monetary policy is active. To see this, combine the monetary policy rule in (58) with the Fisher relation to yield the dynamic equation for inflation

$$\frac{1}{\pi_t} - \frac{1}{\pi^*} = \frac{\beta}{\alpha} E_t \left(\frac{1-\delta_{t+1}}{\pi_{t+1}} - \frac{1}{\pi^*} \right) \tag{60}$$

which now depends on the expected default rate.

Active monetary policy implies that the unique locally bounded solution for inflation is

$$\frac{1}{\pi_t} = \frac{1}{\pi^*} \left(1 - \frac{\beta}{\alpha}\right) \left\{ 1 + E_t \sum_{i=1}^{\infty} \left(\frac{\beta}{\alpha}\right)^i \prod_{j=1}^{i} (1-\delta_{t+j}) \right\} \tag{61}$$

In the absence of default, $\delta_t \equiv 0$, monetary policy achieves its inflation target exactly, $\pi_t = \pi^*$. Higher expected default rates in the future raise current inflation. The farther into the future default is expected, the more it is discounted by $\beta/\alpha < 1$, and the smaller is its impact on inflation at time t. Notice also that if the default rate is constant, $\delta_t \equiv \delta \in [0,1]$, then more aggressive monetary policy enhances the central bank's control of inflation. A constant default rate yields the solution for inflation

$$\pi_t = \pi^* \left[\frac{1-(1-\delta)\frac{\beta}{\alpha}}{1-\frac{\beta}{\alpha}} \right] \tag{62}$$

so that $\pi_t \to \pi^*$ as $\alpha \to \infty$. A more aggressive monetary policy response to inflation reduces the inflationary consequences of default.

Finally, consider a stylized experiment. At time t news arrives that raises the expected default rate at $t+1$, $E_t\delta_{t+1} > 0$, but all subsequent expected default rates are zero, $E_t\delta_{t+j} = 0$ for $j > 1$. Then (61) reduces to

$$\pi_t = \pi^* \left[\frac{1}{1-\frac{\beta}{\alpha}E_t(\delta_{t+1})} \right] > \pi^* \tag{63}$$

and again we see that higher expected default raises inflation, but the extent to which it does so is mitigated by a more aggressive monetary response to inflation in the form of a higher α.

The source of this inflationary response to default can be seen in contrasting the interest rate rules when defined in terms of risky and risk-free interest rates. A risk-free rule, coupled with a passive fiscal policy, can successfully target inflation. To see why the rule defined in terms of the risky-rate cannot, it is helpful to return to the simple case where the default rate is constant, $\delta_t \equiv \delta \in [0,1]$, so that $\frac{1}{R_t} = \frac{1-\delta}{R_t^f}$. Rewrite (58) in terms of the risk-free rate as

$$\frac{1}{R_t^f} = \frac{1}{R^*} + \frac{\alpha}{1-\delta} \left[\frac{1}{\pi_t} - \left(\frac{1}{\pi^*} - \frac{\delta}{\alpha R^*} \right) \right] \tag{64}$$

The monetary policy rule defined in terms of the risky rate of interest can be transformed into a rule of the same form as that defined in terms of the risk-free rate, but with two important differences. First, default does not make monetary policy less active; in fact, it

raises the coefficient on excess inflation, $\frac{\alpha}{1-\delta} > \alpha$. Second, default raises the effective inflation target from π^* to $\frac{\pi^*}{1-\delta\beta/\alpha}$. Intuitively, a higher rate of default creates partial monetary policy accommodation: in the presence of default, the monetary authority must allow the risky rate of interest to rise to induce bondholders to continue holding the stock of government bonds. Given the monetary policy rule, the monetary authority will not raise interest rates without a rise in inflation. Bondholders attempt to sell bonds, increasing aggregate demand as they try to increase their consumption paths. This behavior pushes up the price level until bondholders are being compensated for their default risk and inflation and interest rates are consistent with the monetary rule. Stronger responsiveness of policy to inflation, higher α, reduces the effective rise in the inflation target needed to achieve the rise in interest rates desired by bondholders.

As a general proposition, the possibility of default can undermine the central bank's control of inflation: there is a tight connection between expected default rates and inflation, as in Uribe (2006), but the mechanism differs from Uribe's. Uribe obtains his result through a standard fiscal theory of the price level mechanism by coupling an active monetary policy rule like (58) with an active fiscal rule akin to setting $\gamma = 0$ in (55), just as in Loyo (1999) and, more recently, Sims (2011). Such analyses echo the logic of Sargent and Wallace's (1981) unpleasant arithmetic, where the fiscal consequences of a tight monetary policy can ultimately generate a worsening inflation situation because fiscal policy does not adjust to stabilize government debt. In contrast, our results stem from the monetary policy response to default, but where the policy rule remains active and fiscal policy passive. Although we also find a positive link between default and inflation, that link differs in crucial aspects. For example, in Uribe (2006) delaying default supports unstable inflation dynamics for longer, making it more difficult for the monetary authorities to hit their inflation target. In our active monetary/passive fiscal regime, though, the impact of future default on prices is discounted so that delaying default reduces the immediate inflationary consequences of default. Furthermore, in Uribe (2006) raising α and making monetary policy more active further destabilizes inflation dynamics and moves the economy farther from its inflation target. More active monetary policy in our environment reduces deviations from the inflation target due to default.

4.3. **Monetary Union.** The example in section 4.1 shows that the inability of policy makers to commit to a particular policy stance in the future has repercussions today. We now provide an example of an economy in which fiscal authorities in two countries in a monetary union are unable (or unwilling) to commit to passive fiscal behavior. It turns out that it takes only one country to deviate in order for the fiscal theory of the price level to emerge in the monetary union. The exposition simplifies the setup in Bergin (2000).

Consider two symmetric countries in a monetary union. One simplification of Bergin is to consider a cashless economy and another is to assume a constant world endowment of goods, $y_t = y_{1,t} + y_{2,t} = y$ for all t. A representative household in country j maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{j,t})$$

subject to

$$c_{j,t} + \frac{B_{j,t}}{P_t} + \tau_{j,t} = y_{j,t} + z_{j,t} + \frac{R_{t-1}B_{j,t-1}}{P_t} \tag{65}$$

Note that countries retain fiscal sovereignty in the sense that they set taxes, $\tau_{j,t}$, and transfers, $z_{j,t}$, independently. But there is a common price level, P_t, and one-period nominal interest rate, R_t, across the economies. Below we describe how the single central bank sets R_t each period.

Country j's government chooses policies to satisfy the flow budget constraint

$$\frac{D_{j,t}}{P_t} + \tau_{j,t} + v_{j,t} = z_{j,t} + \frac{R_{t-1}D_{j,t-1}}{P_t} \tag{66}$$

where $v_{j,t}$ is lump-sum transfers received from the common central bank.

The central bank buys and sells bonds, $B_{m,t}$, in order to implement its interest rate policies. The bank does not levy taxes or issue debt. Interest earnings from its portfolio holdings, $v_{1,t}$ and $v_{2,t}$, are rebated to the countries' national governments. The central bank's budget constraint is

$$\frac{B_{m,t}}{P_t} + v_{1,t} + v_{2,t} = \frac{R_{t-1}B_{m,t-1}}{P_t} \tag{67}$$

The Euler equation from household j's optimization is

$$u'(c_{j,t}) = \beta R_t E_t \frac{P_t}{P_{t+1}} u'(c_{j,t+1}) \tag{68}$$

Households also have the transversality condition

$$\lim_{T\to\infty} \beta^T E_t u'(c_{j,t+T}) \frac{B_{j,t+T}}{P_{t+T}} = 0 \tag{69}$$

Goods and bond market clearing conditions are

$$c_{1,t} + c_{2,t} = y_{1,t} + y_{2,t} = y$$
$$B_{1,t} + B_{2,t} + B_{m,t} = D_{1,t} + D_{2,t}$$

Although not strictly necessary for an equilibrium, we follow Sims (1997) and Bergin (2000) in imposing that each individual government must choose policies that are consistent with individual solvency.[24]

Assume that preferences are quadratic, as in Bergin (2000): $u(c_{j,t}) = c_{j,t} - \frac{a}{2}c_{j,t}^2$ for each $j = 1, 2$. Then with a constant worldwide endowment of goods, the Euler equations (68) imply the simple Fisher relation

$$\frac{1}{R_t} = \beta E_t \frac{P_t}{P_{t+1}} \tag{70}$$

and applying (68) to each j, country-specific consumptions are random walks

$$c_{1,t} = E_t c_{1,t+1}$$
$$c_{2,t} = E_t c_{2,t+1}$$

[24] Woodford (1998) observes that private optimizing behavior imposes only that the sum $D_{1,t} + D_{2,t}$ would satisfy transversality. But Sims (1997) points out that any effort to rationalize government policies would lead immediately to corresponding transversality conditions for $D_{j,t}$ individually. An analogous argument applies to rule out overaccumulation of debt by the central bank.

Imposing equilibrium, the Fisher relation, and government flow budget constraints on iterated versions of (66) yields two country-specific intertemporal equilibrium conditions

$$\frac{R_{t-1}D_{1,t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \left[\tau_{1,t+j} + v_{1,t+j} - z_{1,t+j}\right] \tag{71}$$

$$\frac{R_{t-1}D_{2,t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \left[\tau_{2,t+j} + v_{2,t+j} - z_{2,t+j}\right] \tag{72}$$

and an analogous intertemporal equilibrium that stems from private and central bank behavior

$$\frac{R_{t-1}B_{m,t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \left[v_{1,t+j} + v_{2,t+j}\right] \tag{73}$$

Consider a mix of monetary and fiscal policies in which the central bank pegs the nominal interest rate at $R_t = R^*$ for all t, while country 1 sets the primary surplus, $x_{1,t} = \{\tau_{1,t} - z_{1,t}\}$, exogenously and country 2 makes its primary surplus, $x_{2,t}$, strongly responsive to the state of its government debt

$$x_{2,t} - x_2^* = \gamma \left(\frac{D_{2,t-1}}{P_{t-1}} - b_2^*\right) \tag{74}$$

where x_2^* is the steady state primary surplus and b_2^* is the steady state value of government debt in country 2. By setting $\gamma > 1/\beta - 1$, the government in country 2 adjusts future surpluses in response to deviations of debt from b_2^* by enough to retire debt back to steady state.

Two results immediately emerge. First, if $\{x_{1,t}\}$ is exogenous and rebates from the central bank to the government, $\{v_{1,t}\}$, are independent of the state of government debt in country 1, then the worldwide price level, P_t, is determined by equilibrium condition (71). At time t, $R_{t-1}D_{1,t-1}$ is predetermined and the expected present value of primary surpluses plus rebates are independent of P_t, so the price level must adjust to ensure that (71) holds. News of lower taxes or rebates or of higher transfers payments, reduces the value of country 1's debt, inducing agents in country 1 to substitute out of bonds and into consumption goods. This higher demand for goods raises the price level until agents are content to buy their initial consumption baskets.

In turn, a higher price level reduces the value of country 2's debt and, via the surplus rule in (74), reduces expected surpluses in that country. Thus, fiscal disturbances in country 1 spill over to country 2 through general equilibrium effects on the price level. The quantitative importance of these spillover effects depend upon the size of the tax cut or transfer payment in country 1.

Second, if the central bank determines rebates to member countries as a function of each country's fiscal stance—the value of outstanding debt—then (71) no longer imposes any restrictions on the equilibrium price level, even if country 1 continues to maintain exogenous primary surpluses. To uniquely determine the price level, the central bank must shift from pegging the nominal interest rate to targeting the inflation rate. It can do this by setting the

nominal rate according to

$$\frac{1}{R_t} = \frac{1}{R^*} + \alpha \left(\frac{P_{t-1}}{P_t} - \frac{1}{\pi^*} \right) \tag{75}$$

where π^* is the inflation target and $\alpha > 1/\beta$ to ensure a unique, stable inflation process.

Although this policy mix delivers a unique equilibrium, it carries an important distributional message. Efforts by the central bank to reduce inflation will translate into higher values of debt in each country—conditions (71) and (72). Country 2, which is following the surplus rule in (74), will need to raise future surpluses. Country 1, which continues to set primary surpluses exogenously, now requires a relatively larger rebate from the central bank. As condition (73) makes clear, a higher rebate to country 1 may require a *lower* rebate to county 2, forcing country 2 to raise taxes or cut transfer payments still further.

5. Empirical Aspects of Policy Interactions

Given the differences in the equilibria described above, it might seem straightforward to distinguish an equilibrium time series generated by active monetary/passive fiscal policies from a time series generated by passive monetary/active fiscal policy. Unfortunately, subtle observational equivalence results may make it difficult to identify which regime is "active" and which regime is "passive." In this section we highlight two identification challenges—one in which observational equivalence exists between determinant and indeterminant equilibrium, which follows Cochrane (2011a), and another that demonstrates the challenges in distinguishing between regimes M and F from empirical observation. We view these results as provocative but incomplete: further study is needed in order to determine the extent to which these results generalize to more sophisticated setups. One implication flows even from the simple experiments conducted here: empirically testing for the interactions between monetary and fiscal policy by examining simple correlations in the data will lead to spurious results and potentially false conclusions. This suggests that existing efforts to "test" for the fiscal theory may be more challenging than originally believed [Bohn (1998), Canzoneri, Cumby, and Diba (2001), Cochrane (1999, 2005), Woodford (1999, 2001), Leeper (1991), Sims (2011)].

5.1. Indeterminacy and Observational Equivalence.

For the simple model described in section 3, there is a straightforward observational equivalence due to Cochrane (2010, 2011a) in which indeterminant equilibria can generate time series that are indistinguishable (same covariance generating process) from determinant ones.

Proposition 1. (Cochrane) *For any stationary time series process for $\{R_t, \pi_t\}$ that solves*[25]

$$E_t \pi_{t+1} = \alpha \pi_t + x_t \tag{76}$$

and for any α, one can construct an x_t process that generates the same process for the observables $\{R_t, \pi_t\}$ as a solution to (76) using the alternative α. If $\alpha > 1$, the observables are generated as the unique bounded forward-looking solution. Given an assumed α and the process $\pi_t = a(L)\varepsilon_{x,t}$, where $a(L)$ is a polynomial in the lag operator L, we can construct $x_t = b(L)\varepsilon_{x,t}$ with

$$b_j = a_{j+1} - \alpha a_j$$

[25] Expression (76) comes from linearizing an Euler equation and an interest rate rule for monetary policy in models like those examined above.

or

$$b(L) = (L^{-1} - \alpha)a(L) - a(0)L^{-1} \tag{77}$$

Proof. To prove the proposition note that for $\alpha > 1$ and $x_t = b(L)\varepsilon_{x,t}$, the unique π_t is given by

$$\pi_t = \left(\frac{Lb(L) - \alpha^{-1}b(\alpha^{-1})}{1 - \alpha L}\right)\varepsilon_{x,t} = a(L)\varepsilon_{x,t} \tag{78}$$

For $\alpha < 1$, the equilibrium will not be uniquely determined and one may construct a π_t solved "backward" to obtain, $\pi_t = x_t/(1 - \alpha L)$. Specifying $b(L)$ as (77) and substituting into (78) gives $\pi_t = x_t/(1 - \alpha L)$. Under this restriction, the inflation process generated by $\alpha < 1$ will be identical to the inflation process generated by $\alpha > 1$. □

The proposition illustrates that important identifying restrictions are imposed on the model through the specification of the exogenous processes. The cross-equation restrictions of (78) make clear the tight relationship between exogenous and endogenous variables. As Cochrane (2011a) emphasizes, for an exogenous process given by (77), it is impossible to tell if observed time series are generated by a determinant or an indeterminate equilibrium.

Proposition 1 relies on the indeterminant equilibria taking a very particular form. But by definition, there are an infinite number of indeterminant equilibria. We now show that a form of observational equivalence, similar in spirit to proposition 1, holds for *unique* equilibria emerging from models with decoupled determinacy regions. The notion of a decoupled determinacy region will be defined more formally below, but for the present discussion note that the two regimes described in section 3 arise from decoupled determinacy regions. In fact, many of the linear rational expectation models that researchers and policy institutions use to study monetary-fiscal interactions yield the decoupled determinacy regions that emerge from the simplest models of interactions, as in Leeper (1991). Examining the dynamic properties of the two equilibria for general exogenous processes delivers an equivalence between the two unique rational expectations equilibria, as opposed to an equivalence between determinant and indeterminant equilibria.

We derive these results in a generic and simple rational expectations model so that the issues can be exposited analytically. The point of this section is to establish that observational equivalence results *can* emerge when examining fiscal and monetary interactions. We do not provide a rigorous treatment of the issues here; a careful treatment would require more than a few pages and is beyond the scope of the current paper. We leave it to future research to establish the robustness and full implications of these results. However, we do believe that the simple model is sufficient to signal a note of caution when examining the empirical aspects of monetary-fiscal interactions.

Let the rational expectations model be given by

$$E_t y_{1t+1} - \alpha y_{1t} = x_{1t} \tag{79}$$

$$y_{2t} - \gamma y_{2t-1} + y_{1t} = x_{2t} \tag{80}$$

$$x_{1t} = A_1(L)\varepsilon_{1t}, \qquad x_{2t} = A_2(L)\varepsilon_{2t} \tag{81}$$

where $A_i(L)$ is a polynomial in the lag operator L and the shocks ε_{it} are mutually and serially uncorrelated. The only restriction imposed on the exogenous processes is square-summability (covariance stationarity) of the coefficients $\sum_{j=0}^{\infty} A_{ij}^2 < \infty$ for $i = 1, 2$. The Wold representation theorem allows for such a general representation.

Substituting (80) into (79) gives the equilibrium as a function of y_{2t} only

$$E_t y_{2t+1} - (\gamma + \alpha) y_{2t} + \gamma\alpha y_{2t-1} = E_t x_{2t+1} - \alpha x_{2t} - x_{1t} \tag{82}$$

The solution, if one exists, will lie in the space spanned by the exogenous processes x_{1t} and x_{2t}. Therefore, guess that the solution has the form $y_{2t} = C_1(L)\varepsilon_{1t} + C_2(L)\varepsilon_{2t}$, and expectations are taken according to the Wiener-Kolmogorov optimal prediction formula $E[y_{2t+1}|\varepsilon^t] = L^{-1}[C_i(L) - C_{i0}]\varepsilon_{it}$ for $i = 1, 2$. Orthogonality of the shocks permits solving the model for ε_{1t} independent of ε_{2t}. Following Whiteman (1983), we may solve the model using z-transforms by invoking the Riesz-Fischer Theorem. The equilibrium is given by

$$z^{-1}[C_1(z) - C_{10}] - (\gamma + \alpha) C_1(z) + \gamma\alpha z C_1(z) = -A_1(L)\varepsilon_{1t}$$

A bit of algebra reveals the conditions necessary for existence and uniqueness of an equilibrium

$$C(z) = \frac{-zA_1(z) + C_{10}}{(1 - \alpha z)(1 - \gamma z)} \tag{83}$$

Expression (83) shows that existence and uniqueness requires either $|\alpha| > 1$, $|\gamma| < 1$ or $|\alpha| < 1$, $|\gamma| > 1$. If both $|\alpha| < 1, |\gamma| < 1$, then (83) is an analytic function and C_{10} cannot be pinned down, which delivers an indeterminant equilibrium. If both $|\alpha| > 1, |\gamma| > 1$, then C_{10} cannot remove both singularities and no equilibrium exists in the ε_t space.

We say that the determinacy region is decoupled if there exists (at least) two regions of the parameter space that deliver a unique equilibrium (e.g., $|\alpha| > 1$, $|\gamma| < 1$ or $|\alpha| < 1$, $|\gamma| > 1$). While the simple model does not line up exactly with the linearized model of section 3, the determinacy regions in that model (and many models with monetary-fiscal interactions) will also be decoupled. We can characterize the two unique equilibria as follows: If $|\alpha| > 1$, $|\gamma| < 1$, then C_{10} must be set to remove the singularity at $z = \alpha^{-1}$, the unique equilibrium in ε_{1t} is then

$$C_1(L)\varepsilon_{1t} = \left(\frac{-LA_1(L) + \alpha^{-1} A_1(\alpha^{-1})}{(1 - \alpha L)(1 - \gamma L)} \right) \varepsilon_{1t}$$

Proceeding in this fashion for ε_{2t} and y_{2t} delivers the equilibrium representation

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} -\frac{LA_1(L) + \alpha^{-1} A_1(\alpha^{-1})}{(1-\alpha L)(1-\gamma L)} & \frac{A_2(L)}{1-\gamma L} \\ \frac{LA_1(L) + \alpha^{-1} A_1(\alpha^{-1})}{(1-\alpha L)} & -A_2(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{84}$$

The other determinacy region is given by $|\gamma| > 1$ and $|\alpha| < 1$, and using the same methodology delivers the alternative unique equilibrium representation

$$\begin{bmatrix} \tilde{y}_{1t} \\ \tilde{y}_{2t} \end{bmatrix} = \begin{bmatrix} -\frac{LA_1(L) + \gamma^{-1} A_1(\gamma^{-1})}{(1-\alpha L)(1-\gamma L)} & \frac{A_2(L)(1-\alpha L) - A_2(\gamma^{-1})(1-\alpha\gamma^{-1})}{(1-\gamma L)(1-\alpha L)} \\ \frac{LA_1(L) - \gamma^{-1} A_1(\gamma^{-1})}{(1-\gamma L)} & \frac{A_2(L)(1-\alpha L) - A_2(\gamma^{-1})(1-\alpha\gamma^{-1})}{(1-\gamma L)} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{85}$$

Representations (84) and (85) are the unique rational expectations equilibria for the two different parameter regions. These representations are quite general in that they hold for any

covariance stationary process for x_{1t} and x_{2t}. When the exogenous processes are unrestricted it is easy to see how an observational equivalence result could emerge. For example, assume $x_{it} = \sigma^2_{\varepsilon_i}$ for $i = 1, 2$ then the covariance generating function for (84) is

$$y_{1t} = \frac{\alpha_M^{-2}\sigma^2_{\varepsilon_1}}{(1-\gamma_M z)(1-\gamma_M z^{-1})} + \frac{(1-\gamma_M^2)\sigma^2_{\varepsilon_2}}{(1-\gamma_M z)(1-\gamma_M z^{-1})}, \qquad y_{2t} = \alpha_M^{-2}\sigma^2_{\varepsilon_1} + \alpha_M^2\gamma_M^{-2}\sigma^2_{\varepsilon_2} \quad (86)$$

and for (85)

$$\tilde{y}_{1t} = \frac{\gamma_F^{-2}\sigma^2_{\varepsilon_1}}{(1-\alpha_F z)(1-\alpha_F z^{-1})} + \frac{\alpha_F^2\gamma_F^{-2}\sigma^2_{\varepsilon_2}}{(1-\alpha_F z)(1-\alpha_F z^{-1})}, \qquad \tilde{y}_{2t} = \gamma_F^{-2}\sigma^2_{\varepsilon_1} + \sigma^2_{\varepsilon_2} \quad (87)$$

where we have relabeled the parameters so that regime M is given by $|\alpha| > 1$, $|\gamma| < 1$, while regime F is $|\alpha| < 1$, $|\gamma| > 1$.

Letting $\alpha_M \approx \gamma_F$ and $\alpha_F \approx \gamma_M$ nearly delivers observational equivalence in *both* y_{1t} and y_{2t}. But this result is due entirely to the symmetry of the model. The equilibrium representations look the same in either regime, a feature which is uncommon in most macroeconomic DSGE models.

A more robust, yet partial observational equivalence result emerges in that the distribution of y_{1t} *or* y_{2t} can be matched exactly once one allows for the variances of the shocks to be different across the two regimes. That is, for $\alpha_M \approx \gamma_F$ and $\alpha_F \approx \gamma_M$, we can find an exogenous process in regime M, $x_{M,t}$, and regime F, $x_{F,t}$, such that the covariance generating function of y_{it} equals that of $\tilde{y}_{it}$ for any i. We state this as a proposition.

Proposition 2. *For any stationary time series process for $\{y_{1t},\ y_{2t}\}$ that solves (82) for $\alpha_M > 1$ and $\gamma_M > 1$ in Regime M, and given $\{x_{M,t}\}$ processes, one can construct $\{x_{F,t}\}$ processes that generate the same process for the observables $\{y_{1t}\}$ or $\{y_{2t}\}$ using Regime F parameters.*

Proof. The proof is by inspection. Notice that for $\alpha_M = \gamma_F$ and $\alpha_F = \gamma_M$ the covariance generating function for ε_{1t} is identical in (86) and (87). Setting the covariance generating functions equal for ε_{2t} reveals two equations ($y_{1t} = \tilde{y}_{1t}$, $y_{2t} = \tilde{y}_{2t}$) and two unknowns (shock variances). Solving for these equations delivers the desired result. □

This result has important implications for the econometrics of fiscal and monetary policy interactions. To understand monetary-fiscal interactions, we are interested in the joint distribution of monetary and fiscal variables, as well as endogenous outcomes like interest rates and inflation. Propositions 1 and 2 spring from the fact that there are two ubiquitous *equilibrium* conditions in any dynamic monetary model: the Fisher relation and the intertemporal equilibrium condition. "Tests" that rely solely on these two conditions cannot inform about the policy behavior underlying observed time series. Because exogenous driving processes are unobserved, achieving identification from restrictions on such processes—for example, assuming all exogenous processes are first-order autoregressive—is suspect. Because the two regimes can generate identical processes for inflation or interest rates, simple summary statistics—like the correlation between debt and inflation—are uninformative about which regime generated the data; by extension, *any* reduced-form statistics will be uninformative.

As the proposition makes clear, the observational equivalence result hinges critically on choice of the exogenous driving processes. Here we have derived the equivalence for an *i.i.d.*

process, but there is an equivalent proposition for more general assumptions about x_{it}. In many models of inflation determination, the exogenous processes (81) are determined through the setting of policy rules. Therefore, taking a stand on the exogenous processes is equivalent to taking a stand on the policy rules. This suggests that one potential solution to these identification issues is careful specification of policy behavior. One such example is to model both long- and short-term debt. As shown above, adding longer maturities has important equilibrium effects and will change the policy rules in obvious ways. Once an empirically-motivated policy rule is in place, one can systematically attack the identification problems mentioned here.

It is important to note the limitations of proposition 2. This proposition is merely suggestive of lurking identification problems. One can easily construct monetary models in which determinacy regions are not decoupled (and ignoring fiscal policy altogether is not a viable way of achieving decoupling, in our view). For example, a Blanchard (1985) model with a large probability of death, which generates substantial wealth effects, modifies the determinacy regions so that it is no longer tenable to maintain the distinctions between monetary and fiscal policy. It is also not clear if these identification problems extend to more general setups. The more sophisticated the model and policy rule, the greater the likelihood that the identification problems discussed here become less severe. This can be seen by noting that proposition 2 relies on some degree of symmetry between fiscal and monetary policy rules.

Scant attention has been paid to these identification issues in the literature [but see Sims (2011) for an exception]. Many authors have attempted to discern whether equilibrium data were generated by Regime M or Regime F. Many of these attempts have been undertaken in reduced-form models in which policy behavior is not identified. Instead, authors have sought to use the restrictions imposed by the government's budget constraint to identify the regime. These efforts cannot work: the government's budget constraint and the associated intertemporal equilibrium condition *must* be satisfied in *any* equilibrium, regardless of the underlying policy regimes.

6. Concluding Remarks

Policy makers need a broad understanding of the factors that determine inflation. The conventional view, what we call Regime M, proposes that monetary policy can control inflation. A requirement of this view is that fiscal policy must reliably adjust surpluses to ensure that government debt is stable. When governments issue nominal debt, an alternative mix of policies—Regime F—reverses the roles of the two macro policies, with fiscal policy determining inflation and monetary policy stabilizing debt.

If current and projected fiscal stress in advanced economies continues unresolved, economic agents will grow more uncertain that the fiscal adjustments that Regime M requires will occur. And central bank behavior in recent years has shown people that monetary policy does not always aggressively lean against inflation—at times, other concerns are paramount. As beliefs become increasingly centered on Regime F, monetary policy loses its ability to control inflation and influence economic activity in the usual ways. Because these developments are driven primarily by fiscal behavior, there is little that independent central bankers can do to anchor expectations on Regime M policies.

Regime M and F produce equilibria in which monetary and fiscal disturbances have very different effects on macroeconomic time series. Despite these differences, we have shown that in a simple model it is impossible to determine which regime generated observed data without making auxiliary assumptions about unobserved driving processes.

This conclusion may seem iconoclastic or even depressing. But if observational equivalence extends to more general classes of models, such as those that policy institutions employ, then it points toward two constructive conclusions for policy modeling. First, policy modelers could adopt more general driving processes and be aware that they achieve identification through arbitrary assumptions about observables. Second, to the extent that simple *ad hoc* specifications of policy rules are integral to interpretations of data, these specifications can be varied to admit more general interpretations.

There is also a message in these results for policy makers themselves. Because two very different understandings of inflation can be equally consistent with observed data, it would be prudent to broaden the perspective on inflation determination beyond the single, conventional view that dominates policy thinking.

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