Redistributive Shocks and Productivity Shocks

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Abstract

We document the cyclical properties of the labor share: it is quite volatile; an innovation to productivity produces an initial reduction of labor share, making it countercyclical, but it also produces a long-lasting subsequent increase that peaks five years later at a level larger in absolute terms than the initial drop. We pose and estimate a bivariate shock to the production function that, under the assumption of competition in factor markets, simultaneously accounts for movements in the Solow residual and in the factor shares of production. We then incorporate this bivariate process into an otherwise standard real business cycle model and compare the outcomes with those that result from the specification of a univariate productivity shock that matches the properties of the Solow residual. The volatility of hours worked in the bivariate shock economy is a lot smaller than that in the standard univariate shock economy (about 33% of the standard deviation or 11% of the variance), with productivity innovations in the bivariate economy generating 6% of the variance of hours displayed in the univariate economy. The effect of the productivity innovation on labor share reduces dramatically the incentives to work now relative to later both because wages will increase later and because the rate of return will go down. This behavior can be described in terms of a very strong positive wealth effect in the bivariate shock economy relative to the univariate shock economy, while the implied substitution effects tend to delay, first intra- and then intertemporally, the response of hours and not to mitigate them. Our results hold independently of the Frischian elasticity of labor supply. We conclude that understanding the cyclical movements of labor share, and hence constructing a theory of its particular movements should become an important piece of business cycle research.

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1 Introduction

Models driven by productivity shocks have been successful in accounting for a broad set of business cycle phenomena, in particular, the cyclical volatility of output and hours, and the majority of the movements in hours worked can be attributed to such shocks (Prescott (1986), Kydland and Prescott (1991), Prescott (2006)). The gist of this research is to pose optimizing agents that respond optimally to changes in the environment: a productivity shock is an opportunity to produce more than normal and agents take advantage of it by increasing hours worked and investment as well as consumption. Over the past 25 years these results have survived a wide range of abstractions of the standard real business cycle model via the introduction of complementary sources of fluctuations and alternative propagation mechanisms (see a comprehensive review in King and Rebelo (1999) and Rebelo (2005)). Yet, an ingredient common to (almost all) real business cycle models is the assumption that the functional distribution of income is constant at all frequencies, a direct implication of Cobb-Douglas technology and competitive factor pricing. Implicit therein is the premise of unimportant implications for the business cycle of the seemingly small fluctuations we observe in the factor shares of income (which move within a range of 5-6%, U.S. 1954.I-2004.IV).

In this paper, we investigate whether the interaction of the Solow residual and the movements in the factor shares matters for our understanding of the business cycle and we find that it does. First, we document in some detail the cyclical behavior of labor share over the period 1954.I-2004.IV. We find that 1) labor share is quite volatile (its standard deviation is 43% that of output (18% of the variance),[1] and 81% that of the Solow residual (66% of the variance)); 2) it is countercyclical (a correlation of -.24); 3) it is highly persistent (first order autocorrelation of .78); 4) it lags output (its correlation with output four quarters before is .47); and 5) it overshoots, this is, while an innovation to productivity lowers labor share on impact, there is a subsequent increase over time, making labor share peak five years later with a positive deviation that is higher than the original negative one.

Second, we impose these properties on an otherwise absolutely standard real business cycle model and find that the size of the fluctuations of HP-filtered hours falls dramatically to about

[1]While it is customary to describe what models account for in terms of percentage of the standard deviation in the model relative to that in the data, we also report the percentage of the variance. The reason is that variances are additive while standard deviations are not. In this sense if one mechanism accounts for 70% percent of of the standard deviation, what other (orthogonal) mechanisms have to account for is not 30% as one might think, but 71%. This is not the case for the variance: if one mechanism accounts for 70% of the variance then other (orthogonal) mechanisms account for the remaining 30%.

one-third in terms of the standard deviation (one-tenth of the variance); the volatility of HP-filtered output falls to about two-thirds in terms of the standard deviation (half in terms of the variance); and the correlation of hours with output drops to one-fifth. Moreover, these results are not sensitive to the Frischian elasticity of labor supply.

To implement cyclical movements in labor share that replicate those in the U.S. economy we pose a bivariate shock to a Cobb-Douglas production function that, when factors markets behave competitively, can reproduce the joint cyclical movements of the productivity residual and the labor share of income. That is, we extend the standard univariate shock real business cycle model by incorporating an additional source of fluctuation that reproduces the cyclical properties of the labor share. The two sources of fluctuations in our model are a productivity shock that is essentially identical to the Solow residual, and a redistributive shock constructed from the deviations of the labor share with respect to its mean. Moreover, we preserve the productivity residual as the main driving force of the business cycle and treat innovations in labor share as purely redistributive in nature, that is, without productivity level effects. We estimate a joint process for these two shocks, which we feed back into the model. We find that in such an environment, the response of hours and output is dramatically reduced: The variance of hours drops to 11% of its univariate counterpart. Moreover, the specific contribution of productivity innovations alone is about 6% of its univariate counterpart because almost one-half of the volatility of hours is due to redistributive innovations.

The reason for the decline in the volatility of hours can be described with the standard tools of wealth and substitution effects: When the labor share interacts with the productivity residual, the response of factor prices to productivity innovations generates a larger wealth effect than in the standard univariate shock economy that induces a very small response of hours. The differential substitution effects have a small impact tending to delay, rather than reduce, the response of hours worked, first through the relative price of the labor input and then through the intertemporal price of consumption. This finding is independent of the Frisch elasticity of labor, since it also holds in economies with Hansen-Rogerson indivisible-labor environments. In terms of the joint dynamics of the productivity residual and the labor share, the crucial feature that shapes the smaller volatility of output is the large positive effect of the current productivity shock in next period's labor share.

While there have been a few papers concerned with the cyclicality of the factor shares our contribution is to explore the implications for business cycles of its dynamics relative to productivity. Our paper is related to Castañeda, Díaz-Giménez, and Ríos-Rull (1998), who posed an exogenous process for the labor share coefficient that moves one to one with the productivity shock in its study of the cyclical behavior of income distribution. However, its modelization of labor share misses the dynamic interactions with the productivity residual and in any case, the paper abstracts from movements in hours. Our paper is also related to Young (2004), who introduces a sole univariate process for the coefficients in the Cobb-Douglas production function and abstracts from productivity shocks in an otherwise standard real business cycle model.[2] He obtains a countercyclical labor share with a correlation coefficient of -.99. The cyclical behavior of the real variables in his model is, however, sensitive to the capital-labor ratio (and in turn to the definition of the labor share). As we discuss below, whenever the capital-labor ratio is not equal to one, shocks to the labor share introduce level effects whose magnitude depends on the units in which the labor input is defined.

Some other papers look at endogenous cyclical movements in labor share. A first set of these papers builds on the cyclical allocation of risk and optimal labor contracts. Gomme and Greenwood (1995) study a complete markets economy with workers and entrepreneurs that insure against business cycle income losses through the structure of the firm. They use two different financial arrangements that yield the same real allocations: first, workers' Arrow securities are directly included in the wage bill, and second, workers buy bonds issued by the entrepreneurs and only the insurance component net of workers' savings is added to the wage bill. Either wedge counterbalances the procyclical marginal product of labor and generates a countercyclical labor share of income.[3] Importantly, in this model the labor choice is not affected by movements in the labor share. Boldrin and Horvath (1995) use contract theory in a model with workers and entrepreneurs where workers are not allowed to self-insure through savings and are more risk averse than entrepreneurs. The optimal contract trades a provision of insurance from entrepreneurs to workers for a more flexible labor supply. They find a negative correlation of the labor share with GNP.[4] Notice that precluding the worker's ability to smooth consumption alters not only factor prices but also equilibrium allocations. In particular, they find that hours tend to move more (by a factor of 1.08) in their model than in its complete markets counterpart. Donaldson, Danthine, and Siconolfi (2005) analyze stylized financial business cycle facts in a risk-sharing model where

[2] This formalizes a broad idea of biased technical change. Notice that here the elasticity of substitution between capital and labor remains one in all periods.

[3] The former yields a labor share that is highly negatively correlated with output, while the latter attains a correlation more in line with the data but with a persistence of the labor share very close to zero. Also, in both cases the volatility of the labor share exceeds that of their observed data by a factor of 1.6 and 2, respectively. See Tables 1 and 2 in Gomme and Greenwood (1995).

[4] This correlation is 2.75 times higher and the volatility .49 lower than what they observe in the data.

risk-averse workers cannot trade financial assets and shareholders are risk neutral. They introduce a distribution of risk calibrated to generate the cyclical variation of the factor shares observed in the data. However, the model is silent about the allocation of hours because agents in this model supply labor inelastically.

A second set of papers that consider endogenous cyclical movements in labor share use models with binding capacity constraints. Hansen and Prescott (2005) introduce variable capacity utilization and idle resources in a real business cycle model to study asymmetries generated by *occasionally* binding capacity constraints. In this model small plants face decreasing returns to scale and operate if they satisfy a minimum labor input requirement.[5] Aggregate output is then determined by labor, capital, and 'location' capital (which, in equilibrium, is the number of operative plants - all using the same input mix). At full capacity the labor share of income is lower than when some plants remain idle because, in the latter case, the 'location' capital is not a scarce factor and does not earn income. Since the capacity constraint binds in expansions, the model obtains a countercyclical labor share of income (of -.51). The changes in the cyclical behavior of the real variables are minor with respect to the standard model; in particular, hours are 90% of that of the standard (Hansen-Rogerson) real business cycle model. This is also the case when capacity constraints *always* bind; see Cooley, Hansen, and Prescott (1995), whose results yield a negative correlation between output and labor share of -.91.

A third strand of the literature that deals with endogenous cyclical variations in the factor shares is that which includes an explicit role for markups. With increasing returns to scale, a fixed number of firms in monopolistic competition, and a constant markup, Hornstein (1993) obtains a labor share that is half as volatile as what is observed in the data and perfectly and negatively correlated with output. Noteworthy is that in his model the volatility of hours drops to 27% of that of the standard real business cycles model (see his Table 2 column 3). This is due to a positive overhead cost[6] that creates a negative relation between employment and productivity near the steady state (see his expression (24)). Ambler and Cardia (1998) allow for the (not simultaneous) entry and exit of firms and obtain a labor share that co-moves with output, similar to the data, while its volatility is 28% of that of the data.[7]

[5] With decreasing returns to scale increases in output are generated by new operating firms if maximum capacity has not been reached. The labor requirement sets an upper bound for the number of operative plants.

[6] This overhead cost is a common feature of these models and sets the long-run pure profits to zero.

[7] To deliver cyclical movements of the labor share these models of imperfect competition require that equilibrium profits not be zero in the short run. Hornstein (1993) achieves this by completely preventing the entry and exit of firms, and Ambler and Cardia (1998) achieve it by building entries and exits that do not

That the functional distribution of income and its movements are very relevant to understand the macroeconomy, beyond its implications for business cycle fluctuations, has also been recently highlighted by Blanchard (1997) and Caballero and Hammour (1998). Blanchard (1997) studies the medium-run interactions between the decline of the labor share and the rise of the unemployment rate that continental European countries have experienced from late 1970s and early 1980s. Blanchard's explanation for this phenomenon are adverse "labor demand shifts": 1) a rise in markups (i.e., the ratio marginal productivity-wages) via diminished union density or lessened labor hoarding and, 2) the implementation of labor saving technologies. With a calibrated version of his model, he obtains these both suspects to be similarly plausible. Closely related, Caballero and Hammour (1998) analyze the interaction between the appropriability of specific quasi-rents (that imply distributional shifts from labor to capital income) and factor substitution, a link that helps us to understand the recent paucity of growth in jobs in Europe. We find our analysis complementary to these papers in pointing out that the labor share of income can significantly help us to shed light on a wide-range of currently debated economic issues.

Christiano and Eichenbaum (1992) have argued that the behavior of wages (uncorrelated with output) is problematic for the notion that productivity shocks are the main mechanism in generating volatility in hours worked. The reason is that in the model wages and hours move procyclically, while this does not hold in the data. The standard response to this comment is that productivity shocks account for only a fraction of hours' volatility and, given decreasing returns to labor, anything else that moves hours (preference shocks, policy shocks, foreign shocks) should push real wages toward countercyclical behavior. Our concerns may seem closely related to those, but we think they are not. In our environment it is the productivity shocks themselves that generate wages and rate of return movements that are not conducive to a large response in hours.

Recently, there has been a new discussion about the role of productivity shocks in generating business cycles. Gali (1999) identifies technology innovations as the only ones that can have a permanent impact on average labor productivity and finds that under this identifying assumption (the growth rate of) hours do not respond positively to productivity innovations. Using refinements of the Solow residual as productivity shocks, Basu, Fernald, and Kimball (2004) attain similar results. However, looking at the industry level and using the Solow residual as productivity shocks, Chang and Hong (2006) find that (for most industries) non-stationary hours rise in response to a productivity shock. More recently, Fisher (2006) follows an identification strategy

occur simultaneously.

similar to Gali's building from a model where only neutral and investment-specific technological shocks can have a long-run impact on labor productivity. Fisher finds that non-stationary hours rise in response to productivity shocks. This discussion, while important in shaping our understanding of productivity shocks, is complementary to ours. We do not identify productivity shocks with its impact on long-run labor productivity, instead, we note the central role played by the cyclical movements in factor shares associated with the productivity innovations in determining the volatility of hours, and we call for the specification of models where labor share moves endogenously.

The reminder of the paper is structured as follows: We begin in Section 2 by describing the cyclical properties of labor share. Section 3 describes how we construct the stochastic process that creates both shocks to productivity and shocks to labor share (redistributive shocks), and in Section 3.3 we estimate these shocks. In section 4 we incorporate the bivariate shock into the standard real business cycle model to derive our results and report our findings. Section 5 discusses why such a small change in the model has such dramatic implications, while Section 6 concludes. In the Appendix we lay out in detail how we construct labor share (Section A); we replicate our analysis of the dynamic properties under alternative definitions of the labor share (Section B); we explore the sensitivity to an alternative identification scheme of the joint dynamics of productivity and the labor share (Section C); and we derive the Slutsky decomposition of hours (and consumption) that we use to discuss our results (Section D).

2 The Cyclical Behavior of Labor Share

The ratio of all payments to labor relative to output is labor share. Its exact value depends on the details of the definition of output and its partition into payments to labor and payments to capital. Perhaps the more standard definition of labor share, which is the one we take as the baseline, is that proposed by Cooley and Prescott (1995), which assumes that the ratio of ambiguous labor income to ambiguous income is the same as the ratio of unambiguous labor income to unambiguous income. Alternative definitions we explore expand capital stock and capital services to include durables and also add government afterward, while a fourth definition sets labor share equal to the ratio of compensation of employees (CE) to gross national product (GNP), which renders all ambiguous income to capital. A detailed analysis of the construction of labor share of income data series is given in Appendix A.2.

The baseline definition of labor share for the period 1954.I-2004.IV is plotted in Figure 1. It

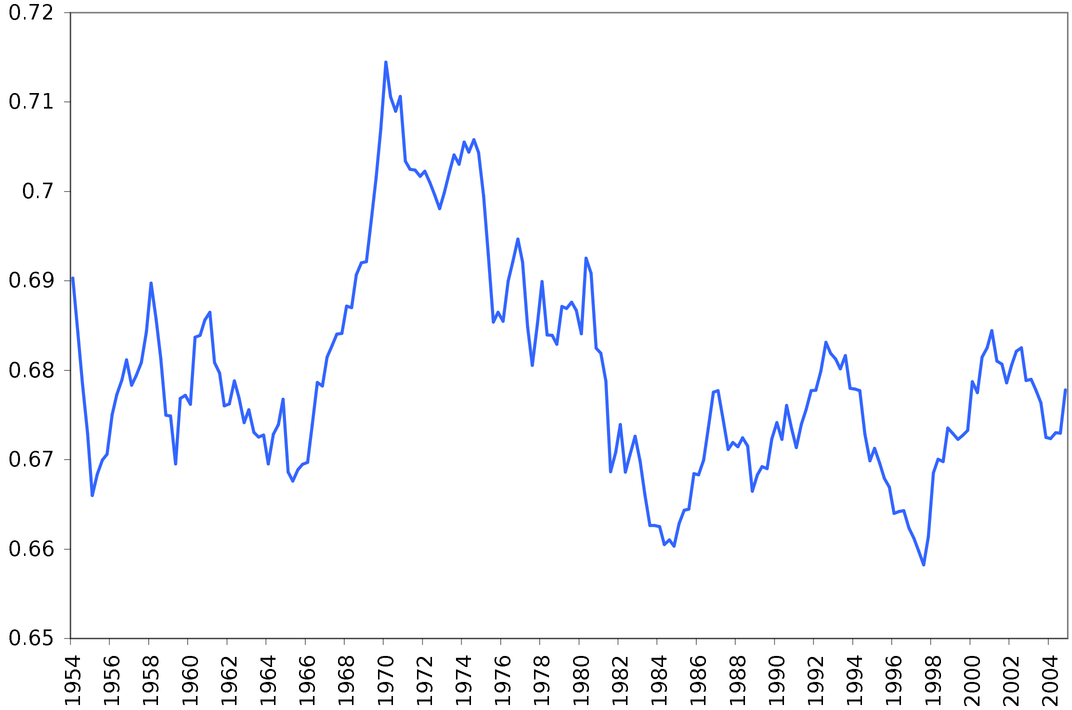

Figure 1: Labor Share, U.S. 1954.I-2004.IV

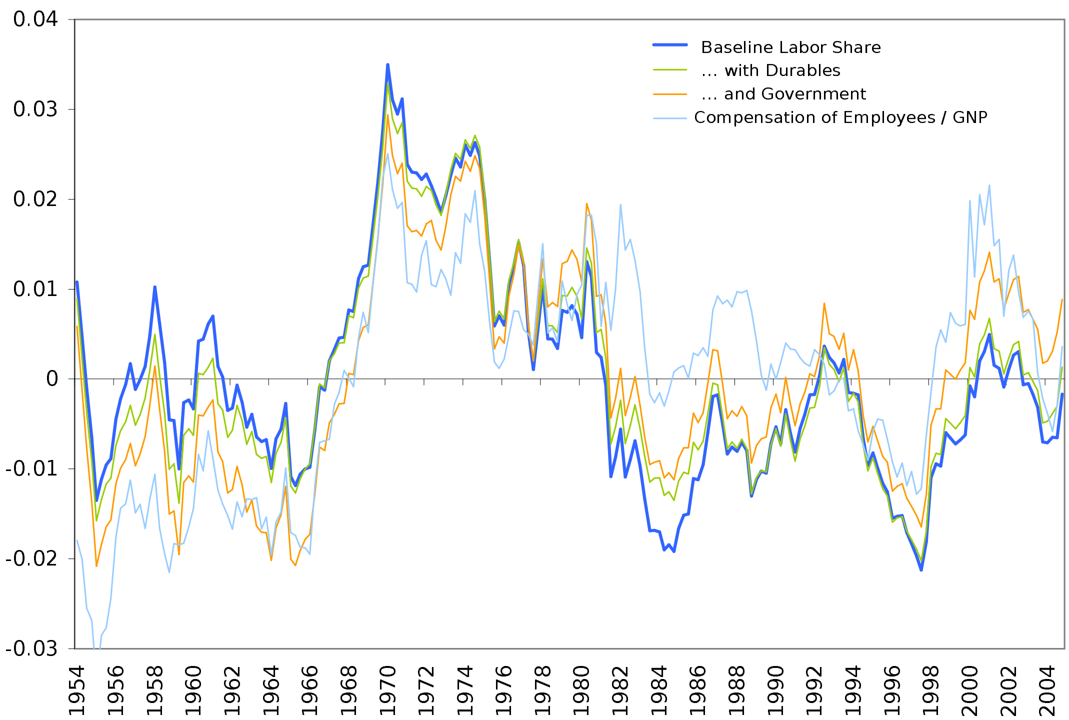

Figure 2: Deviations from Average Labor Share, U.S. 1954.I-2004.IV

oscillates between a maximum value slightly above 0.71 in 1970 and a minimum value about 0.66 in 1997 without discernible trend. The other definitions, while differing on their average have very similar properties. This can be seen in Figure 2, which plots their deviations with respect to the mean.

From the point of view of the study of business cycles, what matters is not whether labor share moves but whether it does so in any systematic way with respect to the main macroeconomic aggregates. Our analysis of the cyclical behavior of labor share U.S. 1954.I-2004.IV attains 5 main properties that we now describe and document:

	σ_x	σ_x/σ_{GNP}	$\rho(x, GNP)$	$\rho(x, s^0)$	$\rho(x_t, x_{t-1})$
GNP	1.59	1.00	1.00	$.74^a$	$.85^a$
Solow Residual: s^0	.85	.53	$.74^a$	1.00	$.71^a$
Baseline Labor Share	.68	.43	$-.24^a$	$-.47^a$	$.78^a$
... with Durables	.71	.45	$-.21^a$	$-.44^a$	$.77^a$
... and Government	.84	.52	$-.26^b$	$-.43^a$	$.78^a$
CE/GNP	.81	.50	$-.23^a$	$-.61^a$	$.71^a$

Note: All variables are *logged* and HP-filtered. Let a and b denote significance at 1% and 5%, respectively

Table 1: Standard Deviation and Correlation of Labor Share with Output, U.S. 1954.I-2004.IV

1. **Labor share is quite volatile**. Table 1 displays the main business cycle statistics of output, the Solow residual, and the various definitions of labor share (all variables are *logged* and HP-filtered).[8] As we can see, the standard deviation of the baseline definition of labor share is 43% of that of output (65% of the variance) and 80% of that of the Solow residual (89% of the variance). The values for the alternative definitions are even larger.

2. **Labor share is countercyclical**. The (baseline) labor share is negatively correlated with output with a coefficient of -.24. Similar figures are attained under alternative definitions of the labor share. Moreover, this negative correlation is much larger with respect to the Solow residual where the value is -.47.

3. **Labor share is highly persistent**. The first order autocorrelation coefficient of (baseline) labor share is .78; that is, labor share displays almost as much persistence as output and slightly more than the Solow residual.

[8]As in Gomme and Greenwood (1995) and Young (2004), we *log* the labor share.

	Cross-correlation of GNP_t with										
	x_{t-5}	x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}	x_{t+5}
Baseline Labor Share	-.20	-.26	-.32	-.34	-.33	-.24	.03	.25	.40	.47	.44
... with Durables	-.21	-.26	-.32	-.33	-.20	-.21	.07	.28	.41	.47	.42
... and Government	-.20	-.25	-.31	-.34	-.33	-.26	.03	.27	.42	.48	.44
CE/GNP	-.24	-.30	-.35	-.38	-.31	-.23	.09	.31	.47	.49	.46

Table 2: Phase-Shift of Labor Share, U.S. 1954.I-2004.IV

4. **Labor share lags output.** To see this, Table 2 shows the phase-shift of labor share with respect to output. We observe that while labor share is negatively correlated with the peak of output before and during the peak, the correlation takes large positive values over .4 in quarters 3 through 5 after the peak. In fewer words, labor share lags output by one year or so.

5. **Labor share *overshoots*.** This is a property of the impulse response of labor share with respect to the cycle. Figure 3 shows labor's share response to (orthogonalized) output innovations in the left panel and to (orthogonalized) Solow residual innovations[9] in the right panel within two asymptotic (analytic) standard errors. Both panels display a similar behavior: After falling below -.2% from average upon impact, labor share continuously rises in a concave fashion, it *overshoots* its long-run average after 6 quarters, and it peaks at the 5th year (about .27% above average), after which it slowly returns to average.

3 The Specification of the Shocks

Recall that we want to construct a stochastic process of shocks capable of generating the cyclical properties of labor share that we have described and feed them to a standard business cycle model. Consequently, both productivity and labor share have to be directly affected by the stochastic process. To do so, we start by recalling how in a standard business cycle model the Solow residual is given a structural interpretation as a shock in Section 3.1. We then turn to our specification of a joint process that yields both labor share and a residual as a bivariate process in Section 3.2. We estimate the process for the standard univariate productivity shock as well as for our bivariate

[9] To compute labor share's responses in Figure 3 we run two bivariate vector AR(n) where the first endogenous variable is either real GNP or the Solow residual, and the second endogenous variable is labor share. Lag length order criteria suggest $n = 1$ for both systems. The impulse responses are obtained with an identification scheme that assumes that redistributive innovations affect only labor share.

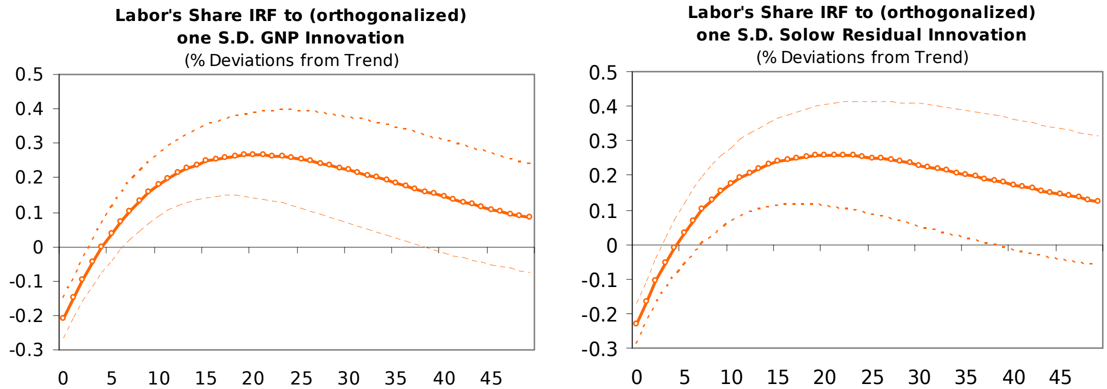

Figure 3: Labor's Share Impulse Response Functions to GNP (left panel) and Solow Residual (right panel) Innovations

process in Section 3.3.

3.1 The Standard Specification: Solow Residuals as Shocks

The Solow residual, which we denote S_t^0, is computed from time series of real output Y_t, real capital K_t, and labor input N_t [10] (see Kydland and Prescott (1993) or King and Rebelo (1999)):

$$\ln S_t^0 = \ln Y_t - \zeta \; \ln K_t - (1-\zeta) \; \ln N_t \tag{1}$$

where ζ is a relative input share parameter chosen to match the long-run average of the capital share of income.

But S_t^0 has trend and we want a trendless object. Consider now a detrending procedure that uses the following linear regression

$$\ln X_t = \chi_x + g_x t + \widetilde{x}_t. \tag{2}$$

where X_t is any economic variable, and where χ_x and g_x are the mean and trend parameters and $\widetilde{x}_t$ are the residuals. To detrend the Solow residual we can either apply (2) directly to (1), or equivalently, we can apply (2) to the series of output, capital and labor input, to obtain the

[10] Real output is obtained from NIPA-BEA Table 1.7.6. The construction of the real capital (extended with durables and government capital services) and labor input (employment times hours per worker) series is explained in Appendix A.

following detrended Solow residual:

$$s_t^0 = \widetilde{y}_t - \zeta\, \widetilde{k}_t - (1-\zeta)\, \widetilde{n}_t \tag{3}$$

3.1.1 A structural interpretation of the Solow residual

To see that in the standard business cycle model the Solow residual has a structural interpretation consider the following Cobb-Douglas technology with constant coefficients and multiplicative shocks to productivity,

$$Y_t = e^{z_t^0} A\, K_t^{\theta} \left[(1+\lambda)^t\, \mu\, N_t\right]^{1-\theta} = e^{z_t^0} A\, K_t^{\theta} \left[(1+\lambda)^t\, (1+\eta)^t\, \mu\, h_t\right]^{1-\theta} \tag{4}$$

where z_t^0 represents a shock that follows a univariate process, and λ is the rate of labor-augmenting (Harrod-neutral) technological change. The labor input, N_t, is the product of the number of agents in the economy, L_t, and the fraction of time agents devote to market activities, $0 \leq h_t \leq 1$. Population grows deterministically according to $L_t = (1+\eta)^t$. Parameters A and μ are just unit parameters (it will be clear later why we are posing two different unit parameters).

Note that in the balanced growth path, output Y_t and capital K_t grow at rate (approximately) $\gamma \approx \lambda + \eta$, and that if preferences are CRRA, the model economy generates paths for capital and output that can be written as $K_t = (1+\lambda)^t(1+\eta)^t\, k_t$ and $Y_t = (1+\lambda)^t(1+\eta)^t\, y_t$ where both k_t and y_t are stationary. Denote steady-state values by x^* and let lower-case-hat variables be log deviations from steady state, *i.e.* $\widehat{x}_t = \log(\frac{x_t}{x^*})$. Then we can write the equilibrium paths as

$$Y_t = (1+\lambda)^t(1+\eta)^t\, y^*\, e^{\widehat{y}_t}, \tag{5}$$

$$K_t = (1+\lambda)^t(1+\eta)^t\, k^*\, e^{\widehat{k}_t}, \tag{6}$$

$$N_t = (1+\eta)^t\ h^*\, e^{\widehat{h}_t} \tag{7}$$

If we plug these paths (5)-(7) into the production function (4), cancel trend terms, take logs, and rearrange variables, we yield:

$$z_t^0 = \widehat{y}_t -\ \theta\widehat{k}_t -\ (1-\theta)\widehat{h}_t + \ln \frac{y^*}{Ak^{*\theta}\,(\mu h^*)^{1-\theta}}\ =\ \widehat{y}_t -\ \theta\widehat{k}_t -\ (1-\theta)\widehat{h}_t \tag{8}$$

where the second equality follows directly from the fact that the denominator of the third term is steady-state output. But this is exactly the Solow residual as calculated in (3), allowing us to interpret the Solow residual generated by the data as the multiplicative shock to the production function.

3.2 The Bivariate Specification: Redistributive Shocks and Productivity Shocks

We now specify a bivariate stochastic process for the labor share and a productivity residual (slightly different from the Solow residual) that explicitly considers the fact that factor input shares change over time. We provide these two data series with a structural interpretation in Section 3.2.1

Labor share. Labor share is a unitless ratio. Rather than in the level of the labor share, we are interested in deviations from its mean, that are

$$s_t^2 = \zeta - \zeta_t \tag{9}$$

where the average of labor share is $1 - \zeta = \sum_t \frac{1-\zeta_t}{T}$. The data series s_t^2 extracted from various definitions of labor share are depicted above in Figure 2.

Productivity residual. This productivity residual is different from the Solow residual in Section 3.1 only in one regard: we now use the time-varying relative input share , ζ_t, instead of a constant share parameter, ζ. We define this productivity residual as[11]

$$\ln S_t^1 = \ln Y_t - \zeta_t \; \ln K_t - (1 - \zeta_t) \; \ln N_t \tag{10}$$

which we detrend as,

$$s_t^1 = \widetilde{y}_t - (1 - \zeta_t) \, \widetilde{k}_t - \zeta_t \, \widetilde{n}_t \tag{11}$$

where as above $\widetilde{y}_t$, $\widetilde{k}_t$ and $\widetilde{n}_t$ are the corresponding residuals of a fitted linear trend to the logged original series of output, capital, and labor.

In addition, we note that the residual s_t^1 is extremely similar to s_t^0; see Figure 4. This can also be seen by noting that we can write an expression that links the two residuals s_t^0 and s_t^1 as

[11]Interestingly, the original residual constructed in Solow (1957) uses a time series for the factor shares of income, as we do in our specification (11).

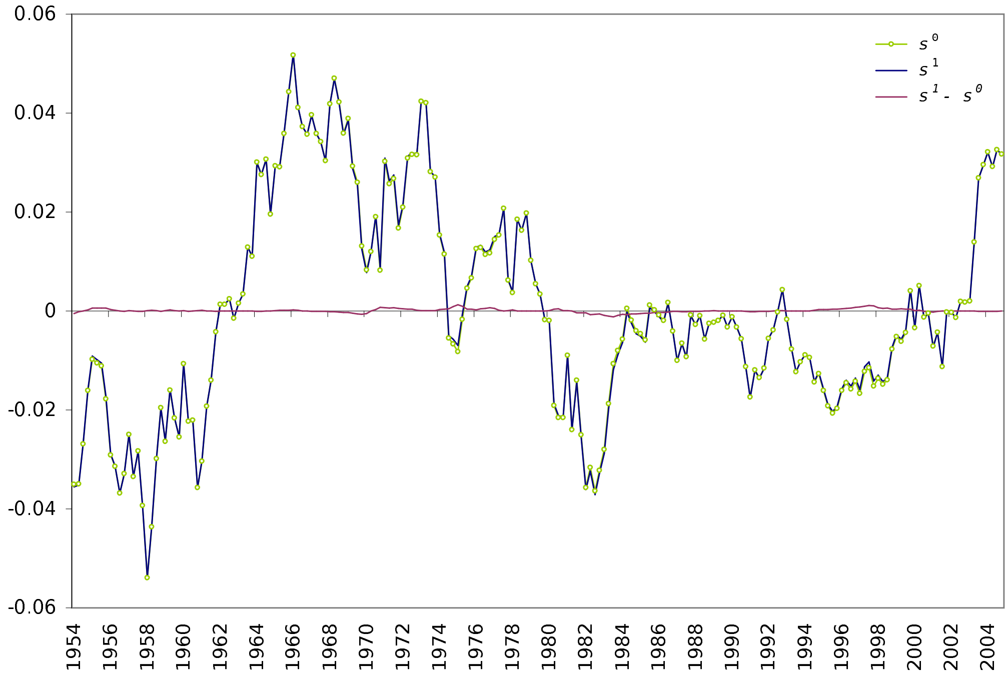

Figure 4: Productivity Residuals s_t^0 and s_t^1, U.S. 1954.I-2004.IV

follows,

$$s_t^1 = s_t^0 + s_t^2(\widetilde{k}_t - \widetilde{n}_t)$$

and that the last term, $s_t^2(\widetilde{k}_t - \widetilde{n}_t)$, is very small.

3.2.1 A structural interpretation of labor share and associated productivity residual

We now pose a production function with stochastic factor shares, which is otherwise the standard Cobb-Douglas technology,

$$Y_t = e^{z_t^1}\ A\ K_t^{\theta - z_t^2}\ \left[(1+\lambda)^t\ (1+\eta)^t\ \mu\ h_t\right]^{1-\theta+z_t^2} \tag{12}$$

where z_t^1 and z_t^2 are the two elements of a bivariate stochastic process and we refer to them as the productivity and the redistributive shock, respectively. We use parameters A and μ to determine the units of effective labor and to normalize output to one. However, unlike in the standard specification, μ now plays an important role.

Under competitive markets, labor share of income in the model is given by

$$\frac{\frac{\partial Y_t}{\partial N_t} N_t}{Y_t} = (1-\theta) + z_t^2 \tag{13}$$

But this implies that with the choice $\theta = 1 - \zeta$ the redistributive shock in the model is the deviation from mean labor share in the data: $z_t^2 = s_t^2$.

We now turn to the model counterpart of the residual (11). Similarly to what we did in Section 3.1.1, we look at the production function along the equilibrium path. Divide both sides of (12) by $(1+\lambda)^t (1+\eta)^t$, take logs, and rearrange to get

$$z_t^1 = \widehat{y}_t - (\theta - z_t^2)\widehat{k}_t - (1 - \theta + z_t^2)\widehat{h}_t + z_t^2 \ln\left(\frac{k^*}{\mu h^*}\right) \tag{14}$$

where we have used $y^* = Ak^{*\theta}\left(\mu h^*\right)^{1-\theta}$.

Using the model-generated data we can compute the productivity residual s_t^1 as:

$$s_t^1 = \widehat{y}_t - (\theta - z_t^2)\widehat{k}_t - (1 - \theta + z_t^2)\widehat{h}_t = z_t^1 - z_t^2 \ln\left(\frac{k^*}{\mu h^*}\right) \tag{15}$$

which means that the units matter: If the units in the model are chosen so that the ratio of capital to effective labor in the steady state is one, then the residual $s_t^1 = z_t^1$, *i.e.*, the productivity residual, is the productivity shock. This is what we do.

Another way of seeing the role of the choice of units is that if $k^* \neq \mu h^*$, then shocks to factor shares also have implications for productivity. We want to distinguish pure redistributive shocks that we associate with z_t^2 from productivity shocks that we associate with z_t^1, and the suitable choice of units allows us to do so.

We now turn to estimating a parameterization to represent the univariate process z_t^0 and another one for the bivariate process $\{z_t^1, z_t^2\}$.

3.3 Estimation of a Process for the Shocks

We start by discussing a univariate process for the Solow residual in Section 3.3.1 and then we move to a bivariate process for the Solow residual and labor share in Section 3.3.2.

3.3.1 A univariate process for the Solow residual

While a univariate representation of the Solow residual z_t^0 is one of the most widely used processes, there are very few actual estimations of it, and most authors just use the calculations in Prescott (1986). We assume the Solow residual follows an AR(1) process with normally distributed innovations. For the whole sample 1954.I-2004.IV the *full* maximum-likelihood estimation delivers,[12]

$$z_t^0 = \underset{(.020)}{.954}\, z_{t-1}^0 \;+\; \epsilon_t^0, \qquad \epsilon_t^0 \sim N\,(0, \underset{(.000)}{.00668})$$

Notice that the volatility of the innovations is lower than the value of .00763 originally estimated in Prescott (1986) or the value of .007 used in Cooley and Prescott (1995). This is due to the sample period. There has been a reduction in volatility recently.[13]

3.3.2 A bivariate process for the Solow residual and labor share

We now pose a statistical model to find an underlying stochastic process that generates the joint distribution of z_t^1 and z_t^2 described in Section 3 using the residuals obtained. In particular, we aim at capturing the volatility and persistence of each series and their observed contemporaneous correlation. We assume the processes to be weakly covariance stationary so that classical estimation and inference procedures apply.

For estimation purposes we specify a vector AR(n) model. Thus, we express each variable z_t^1 and z_t^2 as a linear combination of n-lags of itself and n-lags of the other variable. Information criteria (AIC, SBIC and HQIC) suggest that the correct specification is a vector AR(1), which we write compactly as

$$z_t = \Gamma\, z_{t-1} \;+\; \epsilon_t, \qquad \epsilon_t \sim N\,(0, \Sigma) \tag{16}$$

where $z_t = (z_t^1, z_t^2)'$ and Γ is a 2-by-2 square matrix with generic element γ_{ij}. The innovations $\epsilon_t = (\epsilon_t^1, \epsilon_t^2)'$ are serially uncorrelated and follow a bivariate Gaussian distribution with unconditional

[12] The OLS estimation yields a (biased) regressor coefficient of .947 and a standard deviation of .00667. Despite the high persistence of the process, we do not find substantial differences between these estimates and the full maximum likelihood estimates in terms of the equilibrium fluctuations in the real business cycle model.

[13] For instance, using a similar sample (1955.III-2003.II), Arias, Hansen, and Ohanian (2006) obtain an autocorrelation coefficient of 0.95 and a volatility of the innovations of .0065.

mean zero and a symmetric positive definite variance-covariance matrix Σ. Thus, this specification has seven parameters: the four coefficient regressors in Γ, and the variances and covariance in Σ.

The regressors of the endogenous variables z_t^1 and z_t^2 are the same; thus, we can separately apply the OLS method to each vector AR equation and yield consistent and efficient estimates. Also, with normally distributed innovations, these OLS estimates are equivalent to the *conditional* maximum likelihood estimates. Using the whole quarterly 1954.I-2004.IV sample, the estimated parameters associated with the baseline labor share are

$$\widehat{\Gamma} = \begin{pmatrix} .946 & .001 \\ \text{(.023)} & \text{(.042)} \\ .050 & .930 \\ \text{(.010)} & \text{(.019)} \end{pmatrix}, \qquad \widehat{\Sigma} = \begin{pmatrix} .00668^2 & -.1045E-04 \\ -.1045E-04 & .00304^2 \end{pmatrix}$$

This generates a negative contemporaneous correlation between innovations ϵ_t of -.51. Notice that all parameters except γ_{12} are statistically significant. We also reject the joint null hypothesis that $\gamma_{12} = \gamma_{21} = 0$. We will use the unrestricted statistical model to feed our economic model.

To get a better idea of dynamics of the vector AR system we use impulse response functions and forecast error variance decompositions. First, we check that the estimated vector AR is stable with eigenvalues .951 and .925 so that we can have a moving average representation of it. Second, since our innovations ϵ_t are contemporaneously correlated, we transform ϵ_t to a set of uncorrelated components u_t according to $\epsilon_t = \Omega\, u_t$, where Ω is an invertible square matrix with generic element ω_{ij}, such that

$$\widehat{\Sigma} = \frac{1}{n}\sum_t \epsilon_t \epsilon_t^{'} = \Omega \left(\frac{1}{n}\sum_t u_t u_t^{'}\right)\Omega^{'} = \Omega\Omega' \tag{17}$$

and we have normalized u_t to have unit variance. Notice that while $\widehat{\Sigma}$ has three parameters, the matrix Ω has four: there are many such matrices. We further impose the constraint that u_t^2 have a contemporaneous effect on z_t^2 but not on z_t^1; that is, we set Ω to be a lower triangular matrix.[14]This choice follows from the fact that we aim to treat z_t^2 as purely redistributive shocks

[14]Because $\widehat{\Sigma}$ is positive definite symmetric, it has a unique representation of the form $\widehat{\Sigma} = ADA'$ where A is a lower triangular matrix with diagonal elements equal to one and D is a diagonal matrix. A particularization of this is to set $\Omega = AD^{1/2}$, as we do, which is the Cholesky factorization.

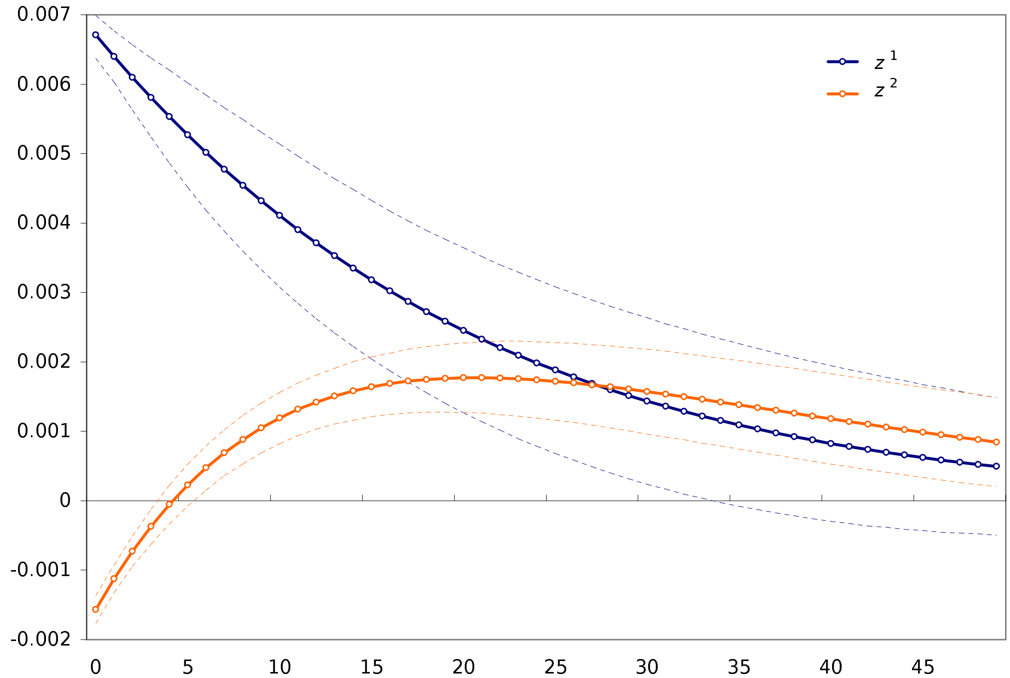

Figure 5: Impulse Response Functions to Orthogonalized Productivity Innovations ϵ^1

with no influence on productivity.[15] Our factorization of $\widehat{\Sigma}$ results in

$$\begin{pmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{pmatrix} = \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \omega_{22} \end{pmatrix} \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix} = \begin{pmatrix} .00668 & .0 \\ -.00156 & .00260 \end{pmatrix} \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix}$$

where $\omega_{11} = \sigma_{\epsilon^1}$, $\omega_{21} = E[\epsilon_t^2 | \epsilon_t^1]$, and ω_{22} is the standard error of the regression of ϵ_t^2 on ϵ_t^1.

Figure 5 illustrates the consequences for z_t^1 and z_t^2 within a band of one asymptotic (analytic) standard error if u_t^1 were to increase by one at $t = 0$ and be set to zero afterward. We find that z_t^1 reacts promptly and positively to this perturbation in its own innovations and that it dies slowly out afterward, very similar (if not exactly) to the univariate process z_t^0 in response to a one-time one-standard-deviation of ϵ_t^0. More interestingly, we find that the labor share of income immediately drops at $t = 0$ by -.156%, from where it rises to above average after the fifth quarter, reaching a maximum after 5 years and approaching monotonically to its unconditional mean afterward.

[15] Our vector AR system allows for the reverse ordering. That is, we can alternatively implement an identification scheme that lets the contemporaneous innovations to the factor shares of income affect productivity while not the opposite. In this case factor share innovations are not purely redistributive. We explore the resulting dynamics under either identification assumption and find similar responses of the endogenous variables in our economic model; see Appendix C. In any case, note that the equilibrium business cycle moments of the economic model (the total effects), which is what we are interested in, remain exactly the same.

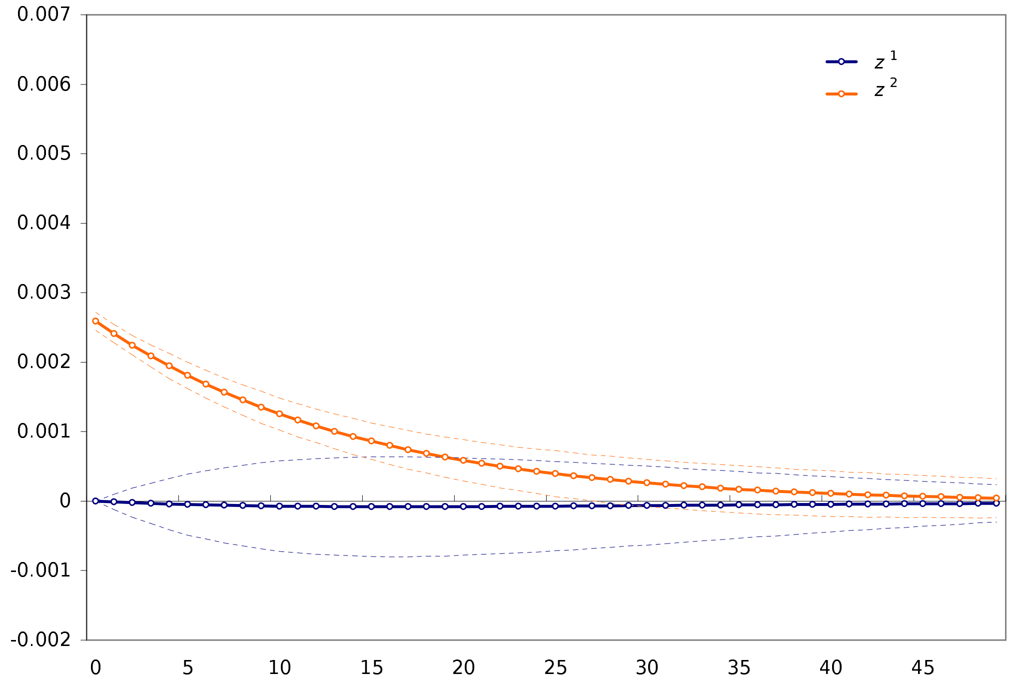

Figure 6: Impulse Response Functions to Orthogonalized Redistributive Innovations ϵ^2

We learn the time-path of z_t^1 and z_t^2 derived from a one-time shock $u_0^2 = 1$ in Figure 6. This perturbation results in a labor share above average that monotonically decreases from a maximum attained at $t = 0$. The assumptions made on the purely redistributive nature of z_t^2 and u_t^2 make the response of z_t^1 to redistributive innovations negligible.

Finally, we decompose the variance of z_t^1 and z_t^2 and find with a long-run horizon that the fluctuations in z_t^1 are 100% due to its own innovations, u_t^1, while 64.6% of the variation in z_t^2 is due to innovations in u_t^1 and 36.4% to its own innovations u_t^2.

4 The Implications of the Specification of the Shocks for Output and Labor Fluctuations

In this section we explore the implications of the two alternative specifications of shocks to the production function for the behavior of standard real business cycle models. Since it is well known that the answer to how important productivity shocks are in generating business cycle fluctuations depends on labor elasticity, we explore two different sets of preferences with different values for this elasticity. We start by specifying the model economies in Section 4.1

4.1 The Model Economies

The economy is populated by a large number of identical infinitely lived households with the following preferences

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \; L_t \; u(c_t, 1-h_t) \right\} \tag{18}$$

where c_t is per capita consumption and h_t denotes the proportion of time devoted to work. Population grows at rate η, $L_t = (1+\eta)^t$. Agents discount future with a factor β, and E_0 is the expectations operator conditioned by the initial information. We choose standard momentary utility functions $u(.,.)$ that imply balanced growth paths. One parameterization that fulfills this requirement is the log-log utility function used in Cooley and Prescott (1995).

$$U(c_t, 1-h_t) = (1-\alpha)\log(c_t) + \alpha \; \log(1-h_t) \tag{19}$$

This specification has a Frisch labor elasticity of 2.2 given that we set the fraction of substitutable time working to .31. The other utility function we use is the Rogerson (1988) log-linear utility function popularized by Hansen (1985): $U(c_t, 1-h_t) = \log(c_t) + \kappa \; (1-h_t)$, where the linearity in leisure arises from nondivisibilities and the use of lotteries and it generates a very high aggregate labor elasticity (in fact, its Frisch labor elasticity is infinity). This is a closed economy where output Y_t, is used either for consumption or for investment I_t. The aggregate stock of capital K_t evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t = (1-\delta)K_t + Y_t - C_t \tag{20}$$

where δ is the geometric depreciation rate.

The production function is as described in Section 3: Cobb-Douglas with labor-augmenting technical progress where we consider model economies with univariate shocks z_t^0 and model economies with bivariate shocks z_t^1 and z_t^2. The specification we posed to obtain the Solow residual as a univariate process with both productivity and population growth was

$$Y_t = e^{z_t^0} A \; K_t^{\theta} \left[(1+\lambda)^t \; (1+\eta)^t \; \mu \; h_t \right]^{1-\theta} \tag{21}$$

In this model economy the units are irrelevant. Still for consistency across models we choose A

and μ so that steady-state output is one and the ratio of steady-state capital k^* to steady-state effective labor μh^* is also set to one.

The production we posed to model the bivariate process with productivity and redistributive shocks is

$$Y_t = e^{z_t^1} A K_t^{\theta - z_t^2} \left[(1+\lambda)^t (1+\eta)^t \ \mu \ h_t\right]^{1-\theta+z_t^2} \tag{22}$$

As we saw in Section 3.2 the units matter for this specification. Again, we set A and μ so that both steady-state output and the steady-state capital to effective labor ratio are one. In this fashion, z_t^2 does not have implications for productivity, since it is a pure redistributive shock.

We can ensure stationarity in the model economies by taking into account population and technological growth. As before, we use small-case letters to denote detrended variables, and we use small-case hat variables to denote detrended log deviations from steady state. With log-log utility, in the transformed economy the planner's problem is to solve[16]

$$\max_{\{c_t, k_{t+1}, h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \ (1+\eta)^t \left[(1-\alpha)\log(c_t) + \alpha \log(1-h_t)\right] \tag{23}$$

subject to

$$c_t + k_{t+1}(1+\eta)(1+\lambda) \ = \ y_t + (1-\delta) k_t \tag{24}$$

and either

$$y_t \ = \ e^{z_t^0} \ A \ k_t^{\theta} \ (\mu \ h_t)^{1-\theta} \tag{25}$$

or

$$y_t \ = \ e^{z_t^1} \ A \ k_t^{\theta - z_t^2} \ (\mu \ h_t)^{1-\theta+z_t^2} \tag{26}$$

The aggregate shocks, either z_t^0 or $\{z_t^1, z_t^2\}$, follow the processes described in Section 3.3.

[16]In our economies the welfare theorems hold so we can use the planner's problem in lieu of solving for the competitive equilibrium.

	θ	δ	β	α	A	μ	κ	λ
Baseline Labor Share	.321	.019	.988	.668	.108	29.86	2.92	.00367
... with Durables	.375	.018	.983	.649	.104	31.08	2.69	.00364
... with Government	.42	.014	.981	.632	.089	36.21	2.49	.00352
CE/GNP	.43	.019	.976	.628	.108	29.86	2.45	.00367
Cooley-Prescott (1995)	.40	.012	.987	.640	-	-	-	.00387
Hansen (1985)	.36	.025	.990	-	-	-	2.84	-

Table 3: Calibrated Parameters

4.2 Calibration

Calibration is very simple in this model, since there are only four parameters, θ, δ, β, and α, in addition to the productivity growth rate λ and the population growth rate η, which we choose according to the estimated trends g_y and g_h, respectively 3.29%[17] and 1.79% in annual terms. Again using x^* to denote the steady-state value of x (with the shocks set to zero–their unconditional mean), the equilibrium satisfies a system of four equations

$$\begin{aligned}
(1-\theta)\frac{y^*}{c^*} &= \frac{\alpha}{1-\alpha}\frac{h^*}{1-h^*} && (27)\\
(1+\lambda) &= \beta\left[\left(1-\delta+\theta\frac{y_*}{k_*}\right)\right] && (28)\\
\delta &= \frac{i^*}{k^*}-(1+\eta)(1+\lambda)+1 && (29)\\
1-\theta &= \text{Labor Share}^* && (30)
\end{aligned}$$

that when solved yield the value of the four parameters for four targets of the steady-state values.

The targets we choose are:

1. The fraction of time devoted to market activities: $h^* = 0.31$.

2. The steady-state consumption-output ratio: $c^*/y^* = 0.75$.

3. The capital-output ratio in yearly terms $k^*/y^* = 2.31$.[18]

[17] The measure of output that includes durables grows at an annual rate of 3.28%, and when we also add government capital output grows at an annual rate of 3.23% .

[18] This is the target only for the baseline model economy; it includes only fixed private capital. When we extend measured output with durables, this ratio goes to 2.40, and adding government capital we get 2.81.

4. Labor share = 0.679. [19]

For the Hansen-Rogerson version of the model (with indivisible labor), the only equilibrium condition that changes is (27), which is substituted with

$$(1-\theta)\frac{y^*}{c^*} \;=\; \kappa h^* \tag{31}$$

The implied value of the parameters in quarterly terms is reported in Table 3. For the sake of completion we report the values used in the original sources.

4.3 Findings

We now turn to discussing the main finding of the paper: posing productivity shocks as a bivariate process that affects factor shares implies a striking reduction in the volatility of the business cycle. The volatility of hours is 3 times smaller (9 in terms of the variance) in the bivariate shock economy relative to the univariate shock economy.

4.3.1 Business cycle properties of the model economies

Table 4 reports the business cycle statistics for the main economic variables and factor prices 1954.I-2004.IV in the U.S. and in the model economies with standard log-log preferences. In the univariate model economy, productivity shocks account for 81.8% of the standard deviation (66.8% of the variance) of output in the data. In the bivariate model economy shocks account for 56.6% of the standard deviation (32.0% of the variance).

However, the most important statistic to measure the model's ability to generate fluctuations is the standard deviation of hours, since output moves both because of hours and because of the shocks. In this respect, the univariate model accounts for 41.0% of the standard deviation of the data (16.8% of the variance). The striking finding is that the bivariate model accounts for 13.5% of the standard deviation of hours in the data (1.8% of the variance). Thus the standard deviation of hours in the bivariate model is 32.8% of that in the univariate model (10.8% of the variance), an enormous reduction.

The behavior of hours in the bivariate economy is also very different in terms of its correlation

[19] This is the target only for the baseline model economy. When we extend measured output with durables this share is 0.625, and 0.58 when we also consider the stock of government capital. It is 0.57 when we use the narrowest definition of labor share that includes only compensation of employees as labor income.

	U.S. Data			Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.59**	1.00	.85	**1.30**	1.00	.72	**.90**	1.00	.73
h	**1.56**	**.88**	.89	**.64**	**.98**	.71	**.21**	**.29**	.73
c	1.25	.87	.86	.44	.91	.80	.71	.91	.77
i	7.23	.91	.80	4.05	.99	.71	1.91	.88	.69
r	.08	.74	.78	.05	.96	.71	.06	.68	.70
w	.76	.08	.70	.69	.98	.75	.78	.87	.77
z^0, z^1	.85	.74	.70	.87	.99	.71	.87	.98	.71
ls	.68	-.24	.78	-	-	-	.63	-.27	.72

Notes: Data are obtained from NIPA-BEA: real GNP from Table (1.7.6) and real personal consumption expenditures and real gross private domestic investment from Table (1.1.6). The series of hours uses CES data; see Appendix A. The data series of factor prices are constructed as w = *Labor Share* $\times$ *Output/Hours* and $r = (1 -$ *Labor Share*$) \times$ *Output/Capital*. All variables have been logged (except the rate of return) and HP-filtered.

Table 4: Cyclical Behavior of U.S. Data 1954.I-2004.IV, and Log-Log Utility Real Business Cycle Models with Univariate and Bivariate Shocks

with output. While it is very high in the data (.88) and in the univariate shock economy (.98), it is much lower in the bivariate shock economy (.29).

With respect to the other aggregate variables, the relative volatility of consumption and investment is quite noteworthy. In the data, the standard deviation of consumption is 17.3% that of investment (3.0% in terms of variances), in the univariate shock economy it is 10.9% percent (1.2% in terms of variances), while in the bivariate economy it is 37.2% (13.8% in terms of variances). In fact, consumption moves more in the bivariate shock economy than in the univariate shock economy despite the opposite behavior of output (the ratio of standard deviations is 161.4% while that of variances is 260.4%).

In addition, factor prices are very strongly correlated with output in the univariate model economy and less so in the bivariate model economy. Finally, the behavior of both residuals is very similar, and they are very correlated with output (recall that the residuals are virtually identical across economies, but output is not). While the univariate model economy does not display movements in labor share, the bivariate economy does, and as in the data, they are negatively correlated with output.

Table 5 shows the phase-shift of the variables in the data and in both model economies. The behavior of hours is quite different between the two economies: While in the univariate shock economy hours are very procyclical and they have a slight lead in the cycle, in the bivariate shock

	Cross-correlation of y_t with										
	x_{t-5}	x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}	x_{t+5}
	U.S. Data 1954.I-2004.IV										
y	-.04	.14	.37	.63	.85	1.00	.85	.63	.37	.14	-.04
h	-.22	-.06	.15	.40	.67	.88	.91	.81	.63	.41	.21
c	.14	.33	.51	.70	.84	.87	.71	.50	.26	.03	-.14
i	.05	.20	.39	.60	.79	.91	.75	.51	.24	-.01	-.22
r	.16	.31	.48	.63	.73	.74	.47	.17	-.09	-.29	-.40
w	.18	.19	.19	.17	.10	.08	-.06	-.11	-.15	-.12	-.12
s^0, s^1	.25	.39	.54	.68	.73	.74	.39	.08	-.18	-.33	-.43
ℓ	-.20	-.26	-.32	-.34	-.33	-.24	.03	.25	.40	.47	.44
	Univariate Model $\{z_t^0\}$										
y	-.01	.11	.27	.46	.70	1.00	.70	.46	.27	.11	-.01
h	.08	.20	.34	.52	.73	.98	.63	.35	.14	-.03	-.15
c	-.21	-.09	.07	.29	.56	.91	.77	.63	.50	.37	.26
i	.05	.17	.32	.50	.72	.99	.65	.39	.18	.02	-.10
r	.12	.24	.37	.54	.73	.96	.58	.30	.08	-.09	-.20
w	-.10	.02	.19	.40	.66	.98	.75	.55	.37	.23	.10
z_t^0	.01	.13	.28	.48	.71	1.00	.69	.44	.24	.08	-.04
	Bivariate Model $\{z_t^1, z_t^2\}$										
y	-.01	.12	.28	.47	.72	1.00	.72	.47	.28	.12	-.01
h	-.13	-.09	-.03	.05	.16	.29	.29	.27	.24	.20	.16
c	-.12	.00	.16	.36	.61	.91	.74	.57	.42	.28	.16
i	.11	.22	.35	.50	.68	.88	.53	.26	.05	-.10	-.21
r	.16	.25	.34	.44	.55	.68	.35	.10	-.07	-.20	-.28
w	-.13	-.01	.14	.33	.58	.87	.71	.56	.41	.28	.17
z_t^1	.03	.16	.31	.50	.72	.98	.67	.41	.20	.04	-.08
ls	-.19	-.22	-.24	-.26	-.27	-.27	-.05	.10	.20	.26	.29

Table 5: Phase-Shift of of the U.S. Data 1954.I-2004.IV, and Log-Log Utility Real Business Cycle Models with Univariate and Bivariate Shocks

	U.S. Data			Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.59**	1.00	.85	**1.74**	1.00	.71	**.92**	1.00	.73
h	**1.56**	.88	.89	**1.28**	.98	.70	**.41**	.33	.72
c	1.25	.87	.86	.54	.88	.81	.74	.94	.77
i	7.23	.91	.80	5.58	.99	.70	1.74	.91	.69
r	.08	.74	.78	.06	.95	.70	.06	.61	.70
w	.76	.08	.70	.54	.88	.81	.74	.94	.77
z^0, z^1	.85	.74	.70	.87	.99	.71	.87	.94	.71
ls	.68	-.24	.78	-	-	-	.63	-.13	.72

Table 6: Cyclical Behavior of the U.S. Data 1954.I-2004.IV, and the Hansen-Rogerson Real Business Cycle Models with Univariate and Bivariate Shocks

economy, hours are quite flat and they lag the cycle. In both economies, consumption lags the cycle and investment leads it, although not by much.

The behavior of rates of return is also quite different. In the univariate shock economy they are quite strongly correlated with output, they lead the cycle, and they do not become negative until a year after output peaks. In the bivariate shock economy they are less correlated, they also lead the cycle, but they become negative three quarters after output peaks. The cyclical behavior of wages seems more similar across the two economies than that of the rates of return.

The Hansen-Rogerson Economies Table 6 reports the business cycle statistics for data and the Hansen-Rogerson log-linear preferences with univariate and bivariate shocks. As is well-known, the higher elasticity of hours of this model generates a larger response to the shocks. The economy with univariate shocks displays 82.05% of the standard deviation of hours observed in the data and 109.43% of output (67.32% and 119.75% of the variance, respectively). However when we turn to the cyclicality of the bivariate model economy, the reduction is spectacular. The standard deviation of hours is now 26.28% of that in the data (6.90% of the variance), that is, the bivariate model generates 32.03% of the standard deviation of the univariate model (10.26% of the variance). As in the log-log economy, consumption is more volatile with the bivariate shock than with the univariate shock, and investment is less volatile.

We avoid the cumbersome reporting of all the features of the Hansen-Rogerson economy, but the picture is clear. As is well known, the higher elasticity of hours of these preferences translate to a much higher volatility of hours worked. However, posing the productivity shocks in the bivariate way we are exploring in this paper dramatically dampens the volatility of hours worked. It does so in a similar or more dramatic fashion than in the economy with a lower elasticity of hours worked (the standard deviation of hours in the bivariate model is less than one-third of that in the univariate model) and for similar reasons, which we will explore next.

5 Why Do Hours Move So Little in the Bivariate Economies?

The key question now is: why does such a seemingly small departure from the standard model generate such a large change in the behavior of aggregate hours? We find it useful to decompose the exploration of what happens into four parts: what the actual properties of the response of hours to innovations in both univariate and bivariate economies (Section 5.1) are; how wages and rates of return respond to innovations in both model economies (Section 5.2); how we can interpret the behavior of hours using the implied notions of substitution and wealth effects (Section 5.3); and what the role of the positive effect of the current value of the productivity shock is in the labor share of subsequent periods (Section 5.4).

5.1 Hours Response to Productivity Innovations

Figure 7 shows the impulse response of hours to innovations to all three shocks in percentage deviations from the steady state. A one-standard-deviation innovation to the only shock, e^0, in the univariate model increases hours by .48%, a response that dies out pretty rapidly (this is the standard response). In the bivariate shock economy things are very different. There is barely any immediate response of hours to a current innovation in the productivity shock, u^1; in fact, the little response that there is, a paltry .09%, is delayed dramatically, peaking 18 quarters later. A 1% innovation in the redistributive shock u^2 favoring labor increases hours initially by .16% (about a third of that of the level of a productivity shock in the univariate economy), and its effects die out quite slowly.

Table 7 displays the variance decomposition of the main variables in the bivariate model by the source of the innovation. For most variables innovations to productivity account for the vast majority of the variance (98.9% for output, 95.6% for consumption, 93.2% for wages, and less so on interest rates 72.3%), while for hours both innovations have equal impact on its variance. Note that given our choice for the orthogonalization of the innovations even for the redistributive shock itself z^2, 63.6% of its variance is due to productivity innovations.

We further investigate the contribution of each shock to the cyclical behavior of each series

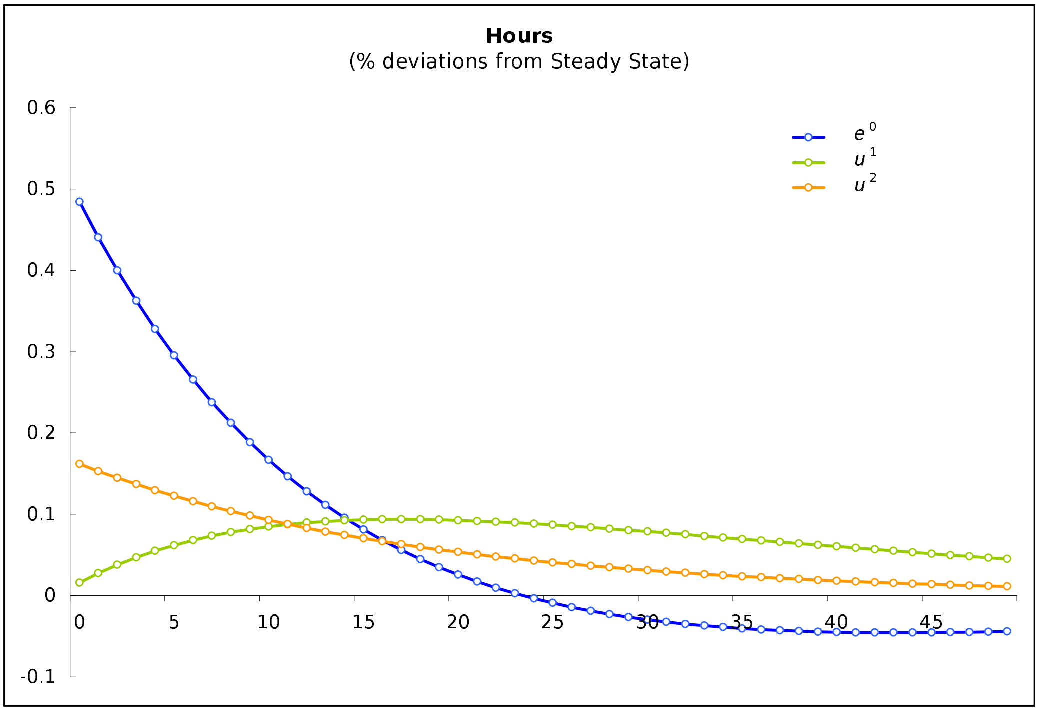

Figure 7: Hours Impulse Response Functions to Innovations to All Shocks

	y	h	c	i	r	w	z^1	z^2
u^1	98.9	54.3	95.6	94.1	72.3	93.2	100.0	63.6
u^2	1.1	45.6	4.5	5.9	27.7	6.8	.0	36.4

Table 7: Forecast Error Variance Decomposition (%)

computing a bivariate economy with productivity innovations alone and a bivariate economy with redistributive innovations alone. In practice, we consider only productivity innovations in the bivariate economy by setting $\omega_{22} = 0$ in the Cholesky factorization of the bivariate shocks, while for a bivariate economy in which only redistributive innovations are at play, we set $\omega_{11} = \omega_{21} = 0$. The business cycle statistics of these economies are reported in Table 8.

	Bivariate $\{u^1, u^2\}$			Bivariate with u_t^1 alone			Bivariate with u_t^2 alone		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**.90**	1.00	.73	**.89**	1.00	.72	**.14**	1.00	.69
h	**.21**	.29	.73	**.06**	.47	.95	**.21**	.99	.70
c	.71	.91	.77	.63	.96	.78	.33	.99	.69
i	1.91	.88	.69	1.86	.95	.68	.43	-.99	.69
r	.06	.68	.70	.06	.88	.69	.03	-.99	.69
w	.78	.87	.77	.65	.94	.79	.42	.99	.70
z^1	.87	.98	.71	.87	.99	.70	.00	.00	.95
ls	.68	-.27	.72	.36	-.70	.74	.48	.99	.69

Table 8: Cyclical Behavior of Log-Log Utility Real Business Cycle Models with the Bivariate Shock with Both Innovations and Isolated Innovations

When the bivariate economy is driven solely by productivity innovations we find that the volatility of hours falls to .06%, a lot less than that of the bivariate model that receives both innovations, and the correlation of hours with output is .47. It is clear then that it is the differential response to a productivity shock that is responsible for the lack of response of hours.

Note that with productivity innovations alone there still are movements in the labor share through ω_{21}. In this case, labor share is less volatile than in the data, and it is highly countercyclical, -.70. The negative impact on z_t^2 through ω_{21} is not counterbalanced by positive redistributive innovations, which strengthens the mechanisms that dampen the volatility of hours. The volatility of the rest of the variables resembles the bivariate model, though they present higher correlation with output. When only redistributive innovations are present in the bivariate economy, the volatility of all real allocations is largely dampened with respect to the bivariate model with both innovations except that of hours and the labor share,[20] and all variables display a high (either positive or negative) correlation with output. In this case, it is noteworthy that the

[20] The model is linear in u^1 and u^2, and hence the variance of the endogenous variables in the bivariate economy with u^1 alone and u^2 alone add up to the variance in the bivariate economy where both innovations are present.

labor share turns highly procyclical.

Consequently, in the bivariate economy productivity shocks have very different effects than in the univariate economy, and we first analyze the effects on prices and then see how prices affect household choices.

5.2 The Response of Wages and Rates of Return

Figures 8 and 9 respectively plot the impulse response functions of real wages and the interest rate (actually, tomorrow's rate of return) to productivity innovations and redistributive innovations in percentage deviations from the steady state. The response of wages to productivity innovations displays a clear hump-shaped pattern in the bivariate economy, while it is much less pronounced in the univariate economy. After a productivity innovation, wages in the bivariate economy continuously rise for the following 9 quarters from an initial deviation of .43% to .60% after, that is, 1.38 times the original deviation. In the univariate economy, however, wages remain almost flat for the first three years; they respond initially by deviating by .51% and barely increase to .55% after one year and a half. The rate of return increases initially in response to productivity innovations by .033% in the bivariate economy and .030% in the univariate economy, but it declines more steeply in the former. The rate of return falls below steady state about one year earlier in the bivariate economy (after the 9th quarter).

Wages respond to redistributive innovations positively: they initially jump by .33% and die out monotonically afterward. The rate of return remains always below its steady state, initially dropping to -.022%.

5.3 Implications of Wages and Rates of Return for Hours

In a growth model, there are many prices: the relative prices of labor and consumption within each period (intratemporal substitution effects) and the relative prices of consumption across periods (intertemporal substitution effects). All of these prices are different in the two economies, and they may also affect whether certain allocations are feasible (wealth effects). Denote by $a\{w^i, r^j, T^\ell\}$ the allocation chosen with the intratemporal prices of shock i, intertemporal prices of shock j, and total resources in the amount required to acquire the choice made by the household in response to shock ℓ, for $i, j, \ell \in \{0, 1\}$, and where 0 and 1 respectively stand for the univariate and bivariate economies. We use the Slutsky transfer compensation to compute the correct measure of total

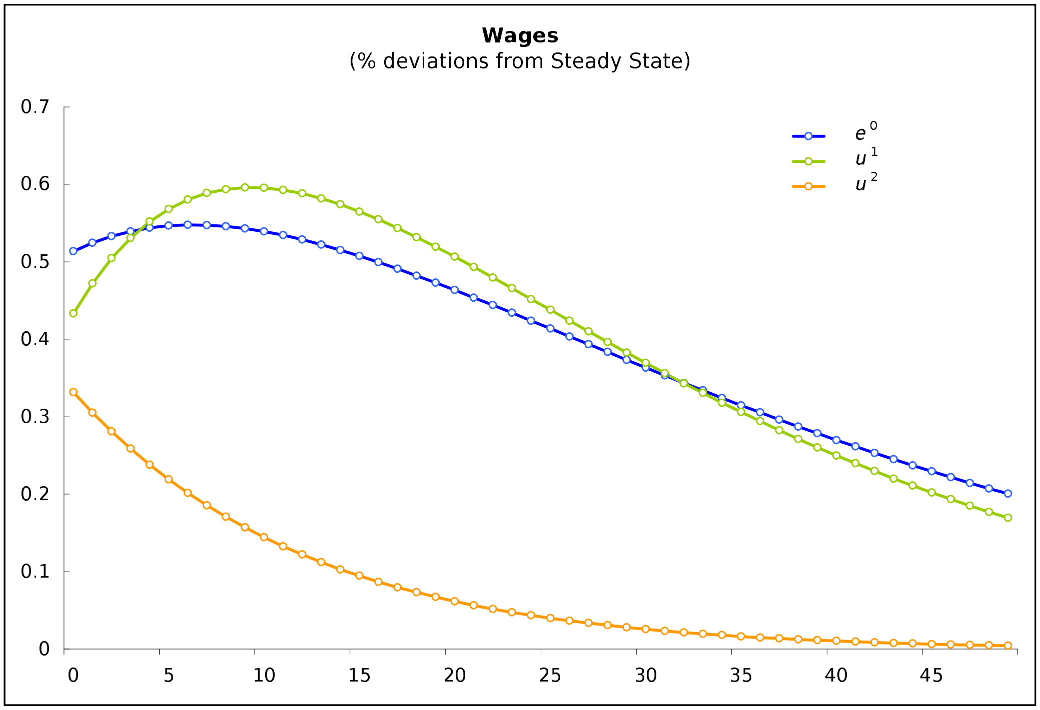

Figure 8: Wage Impulse Response Functions to Innovations to All Shocks

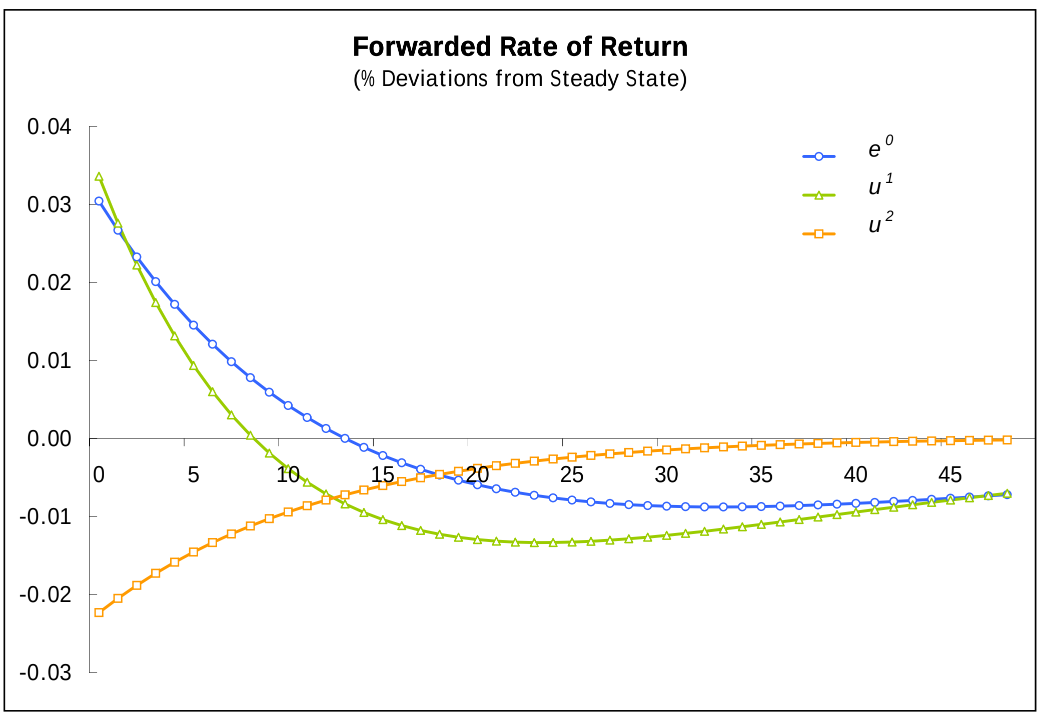

Figure 9: Forwarded Rate of Return Impulse Response Functions to Innovations to All Shocks

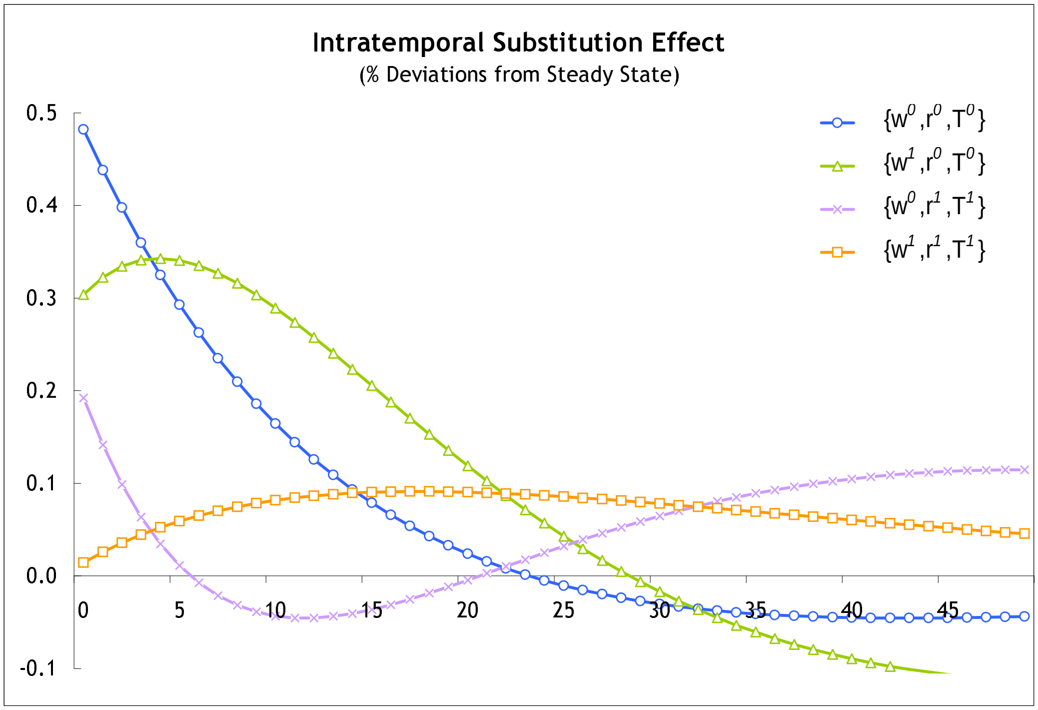

Figure 10: Hours Intratemporal Substitution Effects

resources.[21]

Intratemporal substitution effects. Figure 10 shows the intratemporal substitution effects by plotting the effects of the relative price of labor alone. First, let's compare the response of hours in the univariate economy to those that would occur in an economy that had the same intertemporal relative prices and total resources but had the relative price of labor of the productivity shock in the bivariate economy. Upon impact the effect of the bivariate wages is to dampen hours' movement by more than one-third, but within a year, this effect disappears and the movement of hours is, in fact, larger.

If instead we compare the response of hours in the bivariate economy with those that would prevail in an economy where we substitute its relative prices of labor with those of the univariate economy, we see again that in the latter there is a larger response upon impact, but a few periods later the effect is reversed.

Intertemporal substitution effects. Figure 11 shows the intertemporal substitution effects. Using the intertemporal prices from the bivariate productivity shock in the otherwise univariate

[21]For a detailed derivation of the Slutsky decomposition of hours (and consumption), see Appendix D. Alternatively, King (1991) and King and Rebelo (1999) use a Hicksian decomposition that compensates agents by placing them back on their original indifference curve.

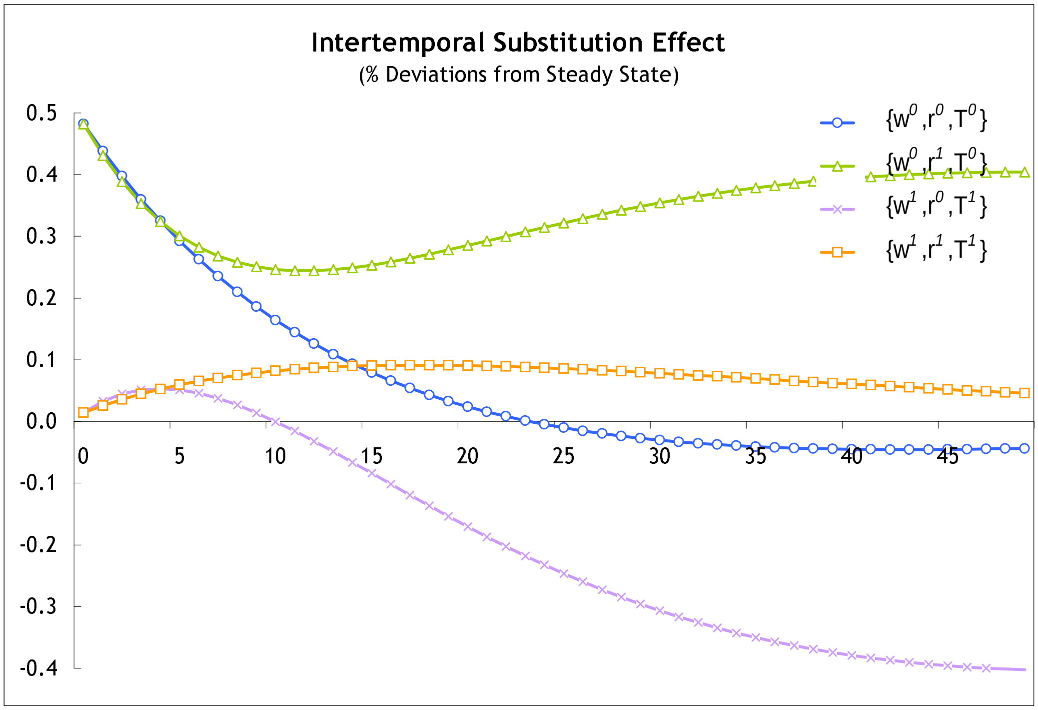

Figure 11: Hours Intertemporal Substitution Effects

economy induces almost no differences for the first 5 or 6 quarters. After that it becomes more expensive to place future leisure (and consumption) into the present in the bivariate economy, which largely raises the level of hours worked. The symmetric exercise (using the intertemporal prices from the univariate shock in the response to the bivariate shock) has a very similar flavor: no difference in hours worked for 6 quarters and then a dramatic reduction of hours worked.

Wealth effect. The wealth effects depicted in Figure 12 are very clear. The changes induced by the productivity shock of the bivariate economy have a substantially more positive wealth effect (.31%) than those of the univariate economy. Consequently, agents choose a lot more leisure when having access to the resources implied by the bivariate shock.

To see the reason for the wealth effect, we can look at Figure 13 where we plot the total wealth change across economies. Let the sum of total resources generated by the factor prices $\{w^i, r^j\}$ for $i, j \in \{0, 1\}$ up to period t be $T_t(w^i, r^j)$. Then $T_t(w^1, r^1) - T_t(w^0, r^0)$ is the total wealth change up to period t, $T_t(w^1, r^0) - T_t(w^0, r^0)$ is the change due to the relative price of the labor input, and $T_t(w^0, r^1) - T_t(w^0, r^0)$ is the wealth change due the intertemporal prices. We can see that it is the lower rate of return after a few periods in the bivariate economy that makes the difference. The higher present value of future units of consumption is responsible for the rise in total wealth in the bivariate economy.

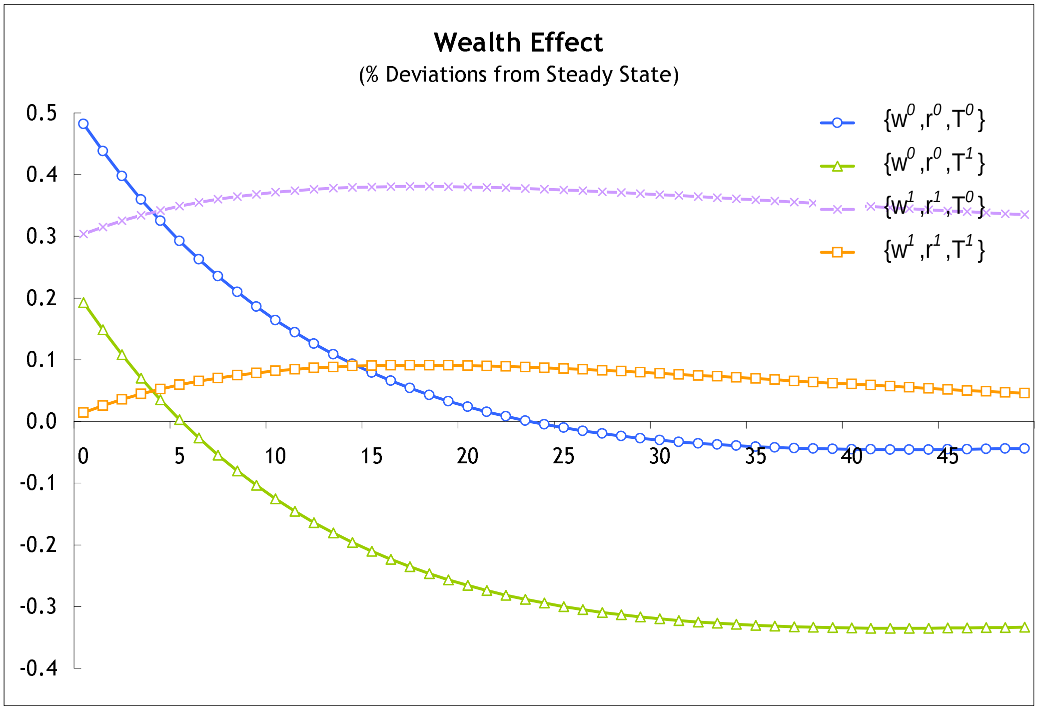

Figure 12: Hours Wealth Effects

Summary of wealth and substitution effects. Perhaps the way to summarize why the bivariate economy induces a very small response of hours is to say that the substitution effects, first intra- and then intertemporally, induce a delay in the response of hours but not the overall reduction, which is due to the wealth effects. This can be seen by looking again at Figure 12 and seeing how to decompose the differences between the univariate shock effects ($\{w^0, r^0, T^0\}$) and ($\{w^1, r^1, T^1\}$) into the substitution effects ($\{w^1, r^1, T^0\}$) and the wealth effect ($\{w^0, r^0, T^1\}$).

5.4 Current Productivity Increases Future Labor Share

One of the most important features of the joint behavior of productivity and the labor share we have documented is that, while the productivity shock is negatively correlated with contemporaneous labor share, the labor share starts increasing, subsequently peaking about 20 periods later with a value that is indeed larger than the initial drop. This surfaces in our representation of the joint process by having a positive value of γ_{21}. To analyze the effect of this channel, we pose a model economy where this effect is absent by setting $\gamma_{21} = \gamma_{12} = 0$. In this economy hours and output are .90% as volatile as the univariate economy, and the correlation of hours and output is also high, .98. Figure 14 displays the impulse response of hours to an innovation in productivity for various alternatives. The findings are clear: the effects of productivity in subsequent labor share is the crucial feature in shaping the smaller volatility of hours.

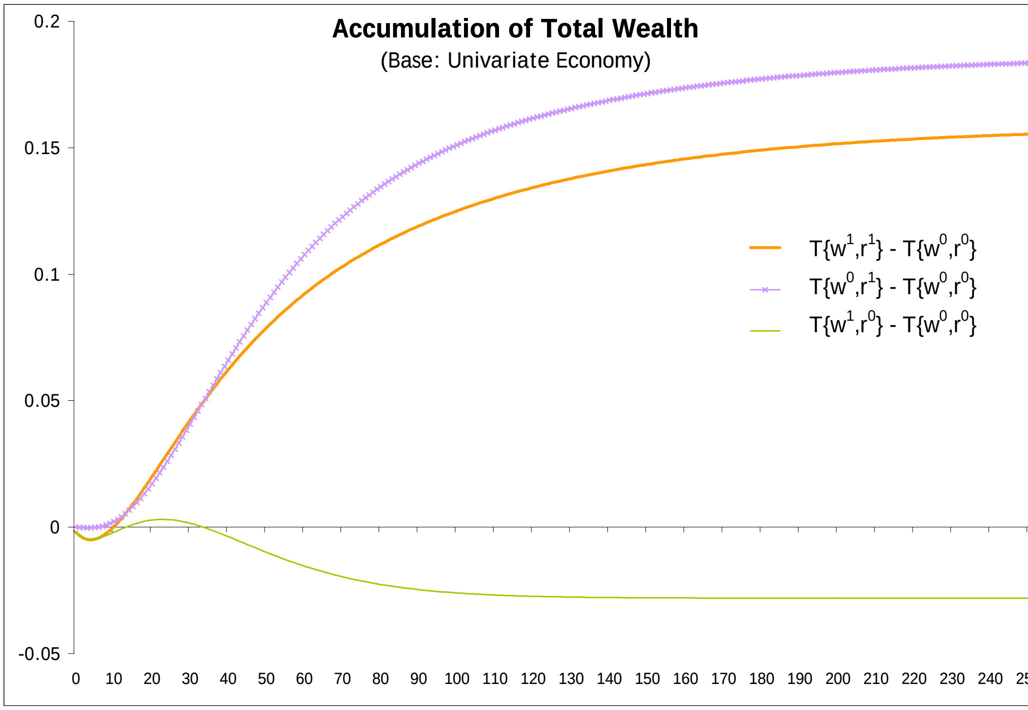

Figure 13: Accumulation of Total Wealth

6 Conclusion

We have documented the cyclical behavior of labor share and made a first step to study its implications for the properties of business cycles. We posed and estimated a bivariate shock process to the production function that under competition in factor markets simultaneously accounts for movements in the productivity residual and in the factor shares of production and for their dynamics. We have shown how confronting agents in a standard real business cycle economy with these shocks entails a much smaller response, 32.8% of the standard deviation of hours relative to that when the standard modelization of the shocks is used (10.8% of the variance). Moreover, half of the movements in hours is due to a redistributive shock and not to the productivity shock itself. Our findings point to the need to understand jointly the cyclical behavior of productivity and factor shares.

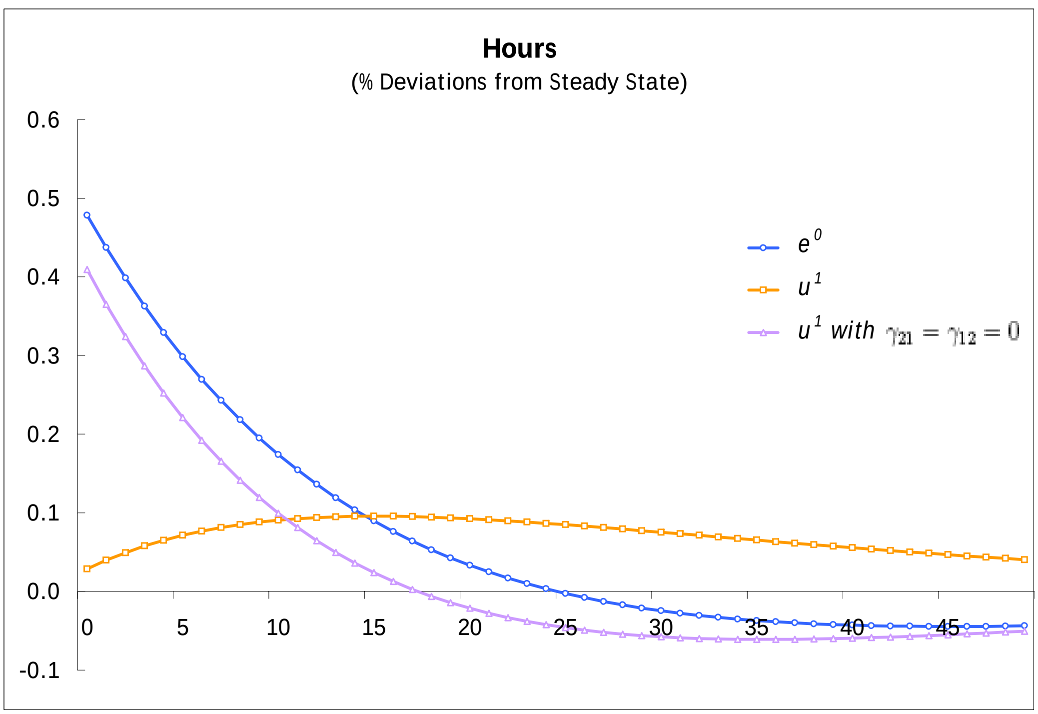

Figure 14: Hours IRFs to Orthogonalized Productivity Innovations

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A Data Construction

A.1 Raw Data Series:

All raw data series were retrieved from the the Bureau of Economic Analysis (BEA; www.bea.gov) and the Bureau of Labor Statistics (BLS; www.bls.gov) for the period 1954.I-2004.IV. To save on notation we drop the period subindex in all series.

National Income and Product Accounts (NIPA-BEA).

1. Table 1.7.5: Gross National Product (GNP), Consumption of Fixed Capital (DEP),[22] Statistical Discrepancy (SDis)[23]
2. Table 1.12: Compensation of Employees (CE), Proprietor's Income (PI), Rental Income (RI), Corporate Profits (CP), Net Interests (NI), Taxes on Production (Tax), Subsidies (Sub), Business Current Transfer Payments (BCTP), Current Surplus of Government Enterprises (GE).
3. Table 5.7.5: Private Inventories (Inv)

Fixed Asset Tables (FAT-BEA).

1. Tables 1.1 and 1.2: Private Fixed Assets (KP), Government Fixed Assets (KG), Consumer Durable Goods (KD).
2. Tables 1.3: Depreciation of Private Fixed Assets (DepKP), Depreciation of Government Fixed Assets (DepKG), Depreciation of Consumer Durable Goods (DepKD).

Current Establishment Survey[24] (CES-BLS).

1. Employment (E) : Series ID CES0000000081
2. Average Weekly Hours (AWH): Series ID CES0500000082, Series ID EEU00500005

[22] This amounts to the difference between Gross National Product and Net National Product.

[23] The Statistical Discrepancy corrects the difference between Net National Product and National Income.

[24] The primary sources of employment and average weekly hours series are the Current Establishment Survey (CES) and Current Population Survey (CPS), which existed in some form since 1947. Our choice of the CES data set is driven by comparison's with Cooley and Prescott (1995).

A.2 Constructed Data Series:

Labor share. The labor share of income is defined as one minus capital income divided by output. Several sources of income, mainly proprietor's income, cannot be unambiguously allocated to labor or capital income. To deal with this we proceed similar to Cooley and Prescott (1995) by assuming that the proportion of ambiguous capital income to ambiguous income is the same as the proportion of unambiguous capital income to unambiguous income, and we compute these series as follows:[25]

1. Unambiguous Capital Income (UCI) = RI + CP + NI + GE
2. Unambiguous Income (UI) = UCI + DEP + CE
3. Proportion of Unambiguous Capital Income to Unambiguous Income: $\theta_P = \frac{\text{UCI+DEP}}{\text{UI}}$

   Then we can use θ_P to compute the amount of ambiguous capital income in ambiguous income,
4. Ambiguous Income (AI) = PI + Tax - Sub + BCTP + SDis
5. Ambiguous Capital Income (ACI) = $\theta_P \times$ AI

Then, capital income (service flows of private fixed capital), Y_{KP}, is computed as the sum of unambiguous capital income, depreciation, and ambiguous capital income, that is,

$$Y_{KP} = \text{UCI} + \text{DEP} + \text{ACI} \tag{32}$$

which we use to construct our baseline labor share[26] as

$$\text{Labor Share} = 1 - \frac{\text{UCI} + \text{DEP} + \text{ACI}}{\text{GNP}} = 1 - \frac{Y_{KP}}{\text{GNP}} = 1 - \theta_P \tag{33}$$

To see the equivalence with Cooley and Prescott (1995) notice that

$$Y_{KP} = \text{UCI} + \text{DEP} + \text{ACI} = \theta_P \text{UI} + \theta_P \text{AI} = \theta_P \text{GNP} \tag{34}$$

Assuming that the return on capital is the same for fixed private capital, consumer durables and government stock we can extend the measure of output, capital income and the labor share to include service flows from consumer durables and government stock as follows:

[25] The labor share is a ratio and we use nominal series to compute it. Notice that unless the same price index is applied to all nominal variables the use of real variables will not yield identical results.

[26] Our computation of the labor share differs from that in Cooley and Prescott (1995) in three regards: we add GE to UCI and Tax - Sub + BCTP to AI, so that UI + AI = GNP; we do not include the stock of land as private fixed assets; and we compute the depreciation rates of consumer durables and government stock differently as we discuss below.

First, we determine the return on capital, i, by solving the following equation that relates capital income to capital stock[27]

$$Y_{KP} = i \times (\mathsf{KP} + \mathsf{Inv}) + \mathsf{DEP} \tag{35}$$

Second, the depreciation rates of consumer durables and government stock are computed as[28]

$$\delta_D = \frac{\mathsf{DepKD}}{\mathsf{KD}} \qquad\qquad \delta_G = \frac{\mathsf{DepKG}}{\mathsf{KG}} \tag{36}$$

This way, the flow of services from consumer durable goods and government capital can be derived as

$$Y_{KD} = (i + \delta_D) \times \mathsf{KD} \qquad\qquad Y_{KG} = (i + \delta_G) \times \mathsf{KG} \tag{37}$$

Finally, the labor share with durables that extends measured output and capital income with flow services from consumer durables is

$$1 - \frac{Y_{KP} + Y_{KD}}{\mathsf{GNP} + Y_{KD}} \tag{38}$$

and the labor share with durables and government that also includes flow services of government stock is

$$1 - \frac{Y_{KP} + Y_{KD} + Y_{KG}}{\mathsf{GNP} + Y_{KD} + Y_{KG}} \tag{39}$$

Our last measure of the labor share is defined as the compensation of employees divided by GNP; that is, we consider labor income the only source that we can unambiguously allocate to labor and add all ambiguous income to capital income.

Aggregate hours. We construct the series of aggregate hours by multiplying the series of employment and average weekly hours:[29] Hours = E $\times$ AWH.[30]

[27]We transform the annual capital stock and depreciation series provided by FAT-BEA to a quarterly series by interpolation.

[28]Cooley and Prescott (1995) use the perpetual inventory method and investment series to pin down δ_D and δ_G. Instead, we use the depreciation series for consumer durables and government stock reported in FAT-BEA, Table 1.3, and operate following (36). We find that our values for $\delta_D = .19$ and $\delta_G = .04$. are similar to those reported in Cooley and Prescott (1995), respectively, .21 and .05 - here notice that we also have a different sample period; theirs runs from 1954 to 1992.

[29]The series of average weekly hours CES0500000082 is available from 1964.I onward. For the period before 1964 we retrieve the annual observations from the series EEU00500005, which we use as quarterly observations. This way, we attribute all quarterly variation in hours before 1964 to employment.

[30]Alternatively, the Productivity and Costs program office at the BLS also provides a quarterly index of aggregate hours since 1947, series ID PRS85006033, which is composed of CES and CPS data and has cyclical properties that are very similar to those of our constructed series of hours in terms of correlation

Real capital. To construct the series of real capital we use the chain-type quantity index from Table 1.2 in FAT-BEA and the current-cost net stock in 2000 from Table 1.1 in FAT-BEA.

B Alternative Definitions of Labor Share

We explore the sensitivity of our results to alternative definitions of labor share in model economies with log-log preferences and Hansen-Rogerson preferences. The results herein confirm our findings discussed in Section 4.3.

B.1 Univariate and Bivariate Estimation

To be consistent in our computations of the Solow residual under each definition of the labor share we take the corresponding extended measures of (deflated) output and extend the measure of the real capital stock series accordingly. This way, when the labor share includes consumer durables (and government stock) the real output and real capital series used to compute the Solow residual are respectively defined as (deflated) GNP + Y_{KD} (+ Y_{KG}) and KP + KD (+ KG). The series of the labor input remains the same in all computations. Table 9 reports the univariate estimation of the Solow residual for the four definitions of the labor share and Table 10 the bivariate estimation[31] of the modified Solow residual and the labor share.

Our estimations show a high persistence of the Solow residual and the labor share, larger volatility of the productivity innovations when government stock is included, larger volatility of the redistributive innovations in our narrowest definition of the labor share, a negative covariance between the productivity and redistributive innovations which is largest under our narrowest definition of the labor share, and negligible (statistically non-significant) marginal effects of z^2_{t-1} on z^1_t under all labor share definitions. The impulse response functions depicted in Figures 15 and 16 show properties very similar to our baseline labor share studied in Section 3.3.2.

B.2 Cyclical Behavior

In Tables 11-13 we report the business cycle statistics of a real business cycle model with log-log preferences when we extend the labor share to include durable goods and government stock, and also when we define the labor share as compensation of employees divided by GNP. With the baseline labor share, aggregate hours in the bivariate model are 32% less volatile than in its univariate counterpart. When we include durable goods, hours move 48% less in the bivariate model, and when we include government, 53% less. Averaging over these three definitions of the labor share we yield a reduction of 44% in the volatility of hours. When we use CE/GNP the drop in σ_h is 59%. A decomposition exercise shows similar values for the contribution of each innovation to the variance of the endogenous variables under all definitions of the labor share, see Table 14. At the same time, in all bivariate models the correlation of hours with output decreases

with output (.88) but slightly more volatile (1.77).

[31] Although we do not report it here, information criteria suggest the use of a vector AR(1) for the bivariate estimation under all definitions of the labor share, as in our baseline case.

	ρ	σ
Baseline Labor Share	.954	.00668
	(.020)	(.000)
... with Durables	.951	.00667
	(.022)	(.000)
... and Government	.937	.00726
	(.019)	(.000)
CE/GNP	.951	.00685
	(.021)	(.000)

Table 9: Univariate Estimation of the Solow Residual, z_t^0

with respect to the univariate case. This is best seen with the impulse response functions of output and hours in Figures 17 and 18. While hours display a clear hump-shape response to u^1, output does not.

Under Hansen-Rogerson preferences we find a very similar reduction in the volatility of hours. With these preferences the bivariate model displays an average σ_h that is 47% less than its standard univariate counterpart; see Tables 15-18.

	γ_{11}	γ_{12}	γ_{21}	γ_{22}	σ_1	σ_2	σ_{12}
Baseline Labor Share	.946	.001	.050	.930	.00668	.00303	-.1045E-04
	(.023)	(.042)	(.010)	(.019)			
... with Durables	.941	-.012	.055	.930	.00665	.00287	-.1001E-04
	(.024)	(.043)	(.010)	(.019)			
... and Government	.927	-.041	.058	.953	.00723	.00313	-.139E-04
	(.025)	(.044)	(.011)	(.019)			
CE/GNP	.948	-.025	.051	.937	.00685	.00345	-.1696E-04
	(.023)	(.040)	(.011)	(.020)			

Table 10: Bivariate Estimation of the Solow Residual, z_t^1, and Labor Share Deviations, z_t^2

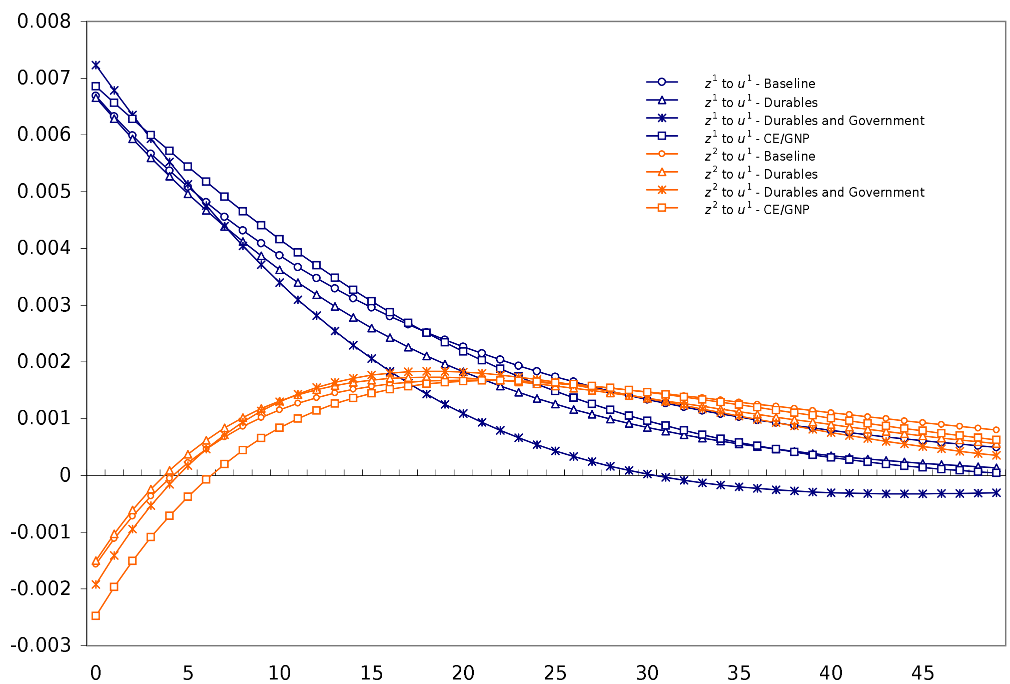

Figure 15: Impulse Response Functions to Productivity Innovations u^1, All Labor Share Definitions

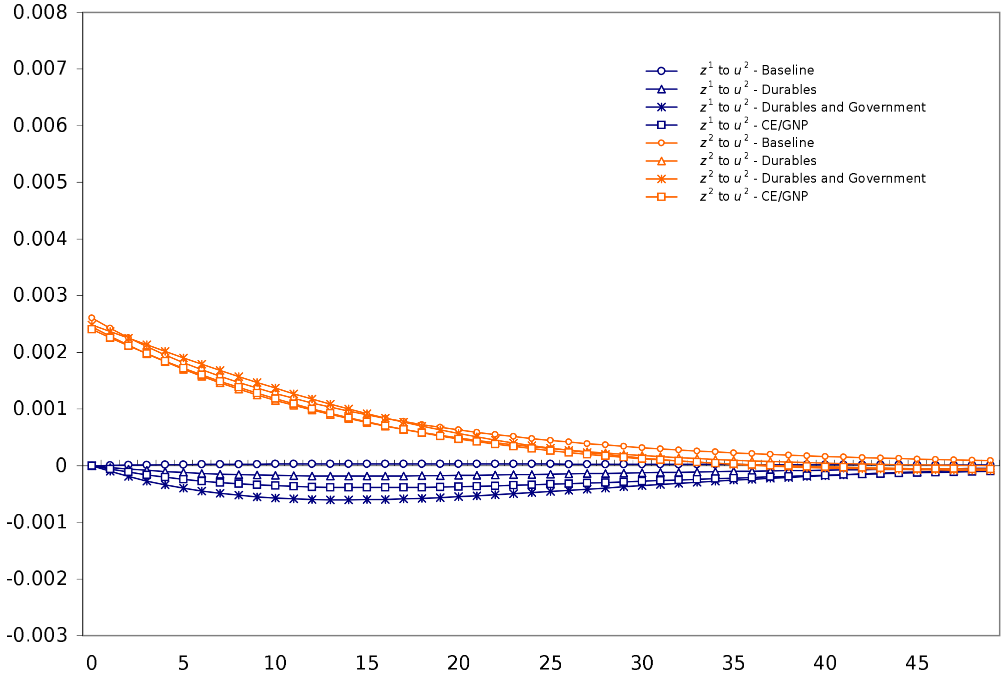

Figure 16: Impulse Response Functions to Distributive Innovations u^2, All Labor Share Definitions

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.26**	1.00	.72	**.93**	1.00	.73
h	**.62**	.98	.71	**.30**	.38	.74
c	.43	.89	.81	.67	.92	.79
i	3.94	.99	.71	2.01	.93	.71
r	.05	.96	.71	.07	.70	.71
w	.67	.98	.75	.77	.88	.79
y/h	.67	.98	.75	.87	.95	.72
z^0, z^1	.87	.99	.71	.87	.97	.72
z^2	-	-	-	.41	-.22	.73

Table 11: Labor Share with Durables and Log-Log Preferences

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.37**	1.00	.72	**1.05**	1.00	.74
h	**.73**	.98	.70	**.39**	.45	.76
c	.39	.85	.83	.61	.91	.80
i	4.52	.99	.71	2.65	.95	.72
r	.05	.97	.70	.07	.76	.72
w	.67	.98	.74	.74	.84	.80
y/h	.67	.98	.74	1.04	.93	.72
z^0, z^1	.94	.99	.71	.96	.96	.72
z^2	-	-	-	.45	-.31	.74

Table 12: Labor Share with Durables and Government and Log-Log Preferences

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	1.25	1.00	.72	.90	1.00	.75
h	.61	.97	.71	.36	.03	.74
c	.44	.88	.83	.60	.90	.82
i	3.89	.99	.71	2.12	.93	.72
r	.06	.96	.71	.08	.75	.71
w	.67	.98	.76	.68	.79	.82
y/h	.67	.98	.76	.96	.92	.72
z^0, z^1	.89	.99	.71	.91	.96	.72
z^2	-	-	-	.48	-.40	.72

Table 13: Compensation of Employees divided by GNP and Log-Log Preferences

		y	h	c	i	r	w	y/h	z^1	z^2
Baseline Labor Share	u^1	98.9	54.3	95.6	94.1	72.3	93.2	98.7	100.0	63.6
	u^2	1.1	45.6	4.5	5.9	27.7	6.8	1.3	.0	36.4
.... with Durables	u^1	98.8	64.1	97.3	99.4	79.5	95.0	97.3	99.6	67.8
	u^2	1.2	35.9	2.7	.6	20.5	5.0	2.7	.4	32.2
... and Government	u^1	98.1	62.3	96.2	98.5	81.2	93.3	92.0	97.7	63.1
	u^2	1.9	37.7	3.8	1.5	18.8	6.7	8.0	2.3	36.9
CE/GNP	u^1	98.7	59.5	97.5	99.0	85.2	95.3	96.1	99.0	68.8
	u^2	1.3	40.5	2.5	1.0	14.8	4.7	3.9	1.0	31.2

Table 14: Forecast Error Variance Decomposition (%), Log-Log Preferences

C Alternative Identification Scheme

Our identification scheme treats innovations to factor shares as purely redistributive, that is, without contemporaneous effects on productivity. Alternatively, we can reverse the order of the vector AR system to orthogonalize the innovations ϵ_t as

$$\begin{pmatrix} \epsilon_t^2 \\ \epsilon_t^1 \end{pmatrix} = \begin{pmatrix} .00304 & .0 \\ -.00349 & .00577 \end{pmatrix} \begin{pmatrix} u_t^2 \\ u_t^1 \end{pmatrix}$$

where $\sigma_{\epsilon^2} = .00304$, $E[\epsilon_t^1 | \epsilon_t^2] = -.00349$, and the standard error of the regression of ϵ_t^1 on ϵ_t^2 is $.00577$. This orthogonalization has the identifying assumption that while innovations to the factor shares have a contemporaneous effect on productivity, productivity innovations do not alter the distribution of income at prompt.

The responses of z_t^1 and z_t^2 to productivity and labor share innovations are depicted in Figures 19 and 20. Under the alternative identification scheme, after a productivity innovation the labor share does not react at prompt, but it starts to continuously rise at $t = 1$ and for the next 4 years or so, after which it slowly decreases dying out toward its steady state. In this case, productivity responds to its own innovations similarly to our previous identification but in a lesser magnitude. With the alternative identification, innovations to the labor share drop productivity below its steady state at all periods, it drops at prompt and monotonically rises back to the steady state. An innovation to the labor share with the alternative identification assumption initially raises the labor share but it starts to decline immediately, falling below its unconditional mean after 3 years and reaching a minimum 8 years after the impulse.

We find that the response of hours and consumption to productivity innovations and the

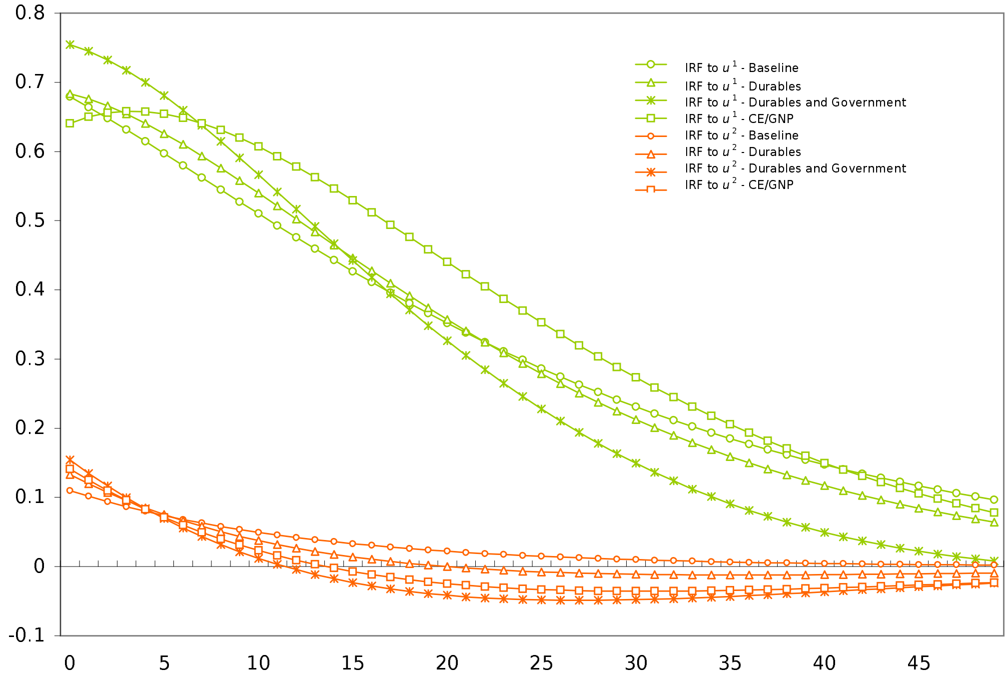

Figure 17: Impulse Response Functions of Output (% Deviations from Steady State), Log-Log Preferences and All Labor Share Definitions

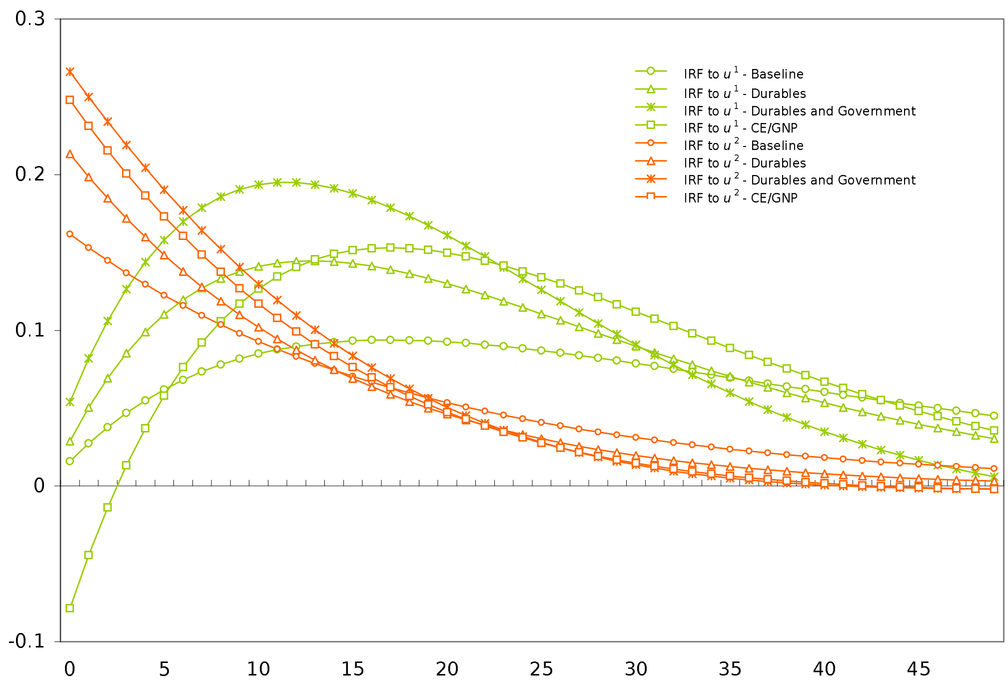

Figure 18: Impulse Response Functions of Hours (% Deviations from Steady State), Log-Log Preferences and All Labor Share Definitions

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.61**	1.00	.72	**.97**	1.00	.75
h	**1.19**	.97	.70	**.55**	.45	.74
c	.51	.87	.82	.70	.96	.78
i	5.17	.99	.70	1.94	.96	.72
r	.07	.96	.70	.06	.64	.71
w	.51	.87	.82	.70	.96	.78
y/h	.51	.87	.82	.87	.82	.71
z^0, z^1	.87	.99	.71	.87	.92	.72
z^2	-	-	-	.41	-.06	.73

Table 15: Labor Share with Durables and Hansen-Rogerson Preferences

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.74**	1.00	.71	**1.11**	1.00	.75
h	**1.38**	.98	.70	**.71**	.53	.76
c	.47	.83	.84	.65	.94	.80
i	5.85	.99	.70	2.69	.97	.73
r	.07	.97	.70	.07	.72	.72
w	.47	.83	.84	.65	.94	.80
y/h	.47	.83	.84	.95	.77	.71
z^0, z^1	.94	.99	.71	.96	.91	.72
z^2	-	-	-	.45	-.16	.74

Table 16: Labor Share with Durables and Government and Hansen-Rogerson Preferences

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y, x)$	$\rho(x_{t-1}, x_t)$	σ_x	$\rho(y, x)$	$\rho(x_{t-1}, x_t)$
y	1.53	1.00	.72	.89	1.00	.76
h	1.12	.97	.70	.67	.17	.73
c	.51	.86	.83	.61	.96	.81
i	4.89	.99	.71	1.87	.96	.73
r	.07	.96	.70	.08	.68	.72
w	.51	.86	.83	.62	.95	.81
y/h	.51	.86	.83	1.02	.76	.71
z^0, z^1	.89	.99	.71	.91	.88	.72
z^2	-	-	-	.48	-.22	.72

Table 17: Compensation of Employees Divided by GNP and Hansen-Rogerson Preferences

		y	h	c	i	r	w	y/h	z^1	z^2
Baseline Labor Share	u^1	96.2	52.0	95.2	99.5	76.3	95.2	98.3	100.0	66.7
	u^2	3.8	48.0	4.8	.5	23.7	4.8	1.7	.0	33.3
.... with Durables	u^1	96.2	60.0	96.2	83.4	96.2	96.1	99.6	99.6	67.8
	u^2	3.8	40.0	3.8	16.6	3.8	3.9	.4	.4	32.2
... and Government	u^1	95.9	59.5	95.7	96.6	85.4	95.7	89.1	97.7	63.1
	u^2	4.1	40.5	4.3	3.4	14.6	4.3	10.9	2.3	36.9
CE/GNP	u^1	96.8	57.1	96.8	97.3	87.8	96.8	95.0	99.0	68.9
	u^2	3.2	42.9	3.2	2.7	12.2	3.2	5.0	1.0	31.1

Table 18: Forecast Error Variance Decomposition (%), Hansen-Rogerson Preferences

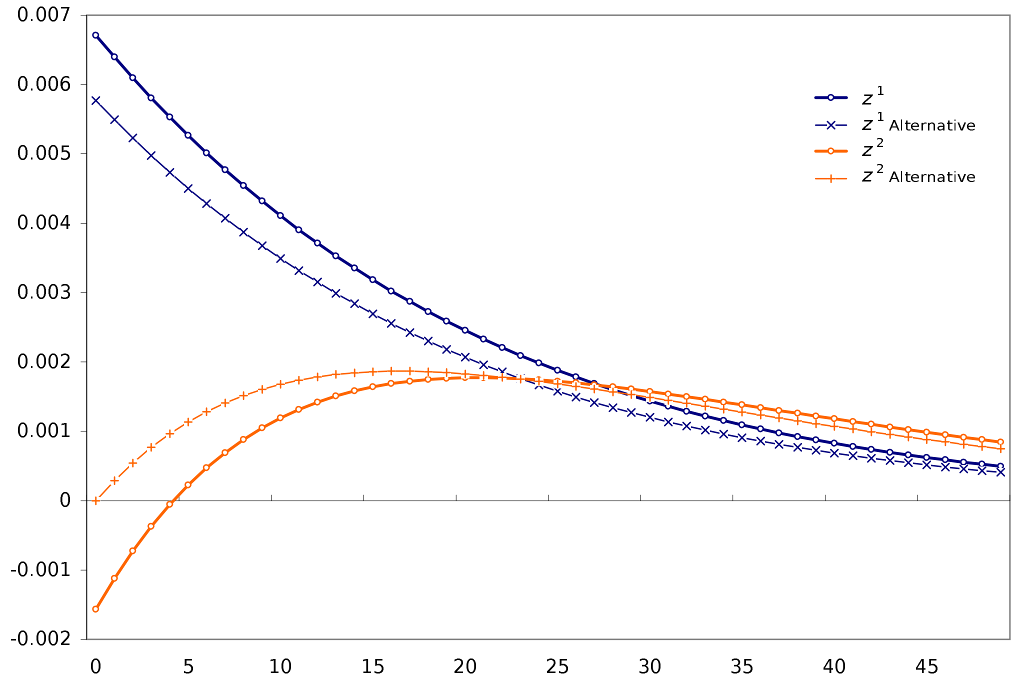

Figure 19: Impulse Response Functions to Orthogonalized Productivity Innovations, Alternative Identification

response of hours to innovations in the labor share are similar under both identification schemes as depicted in Figures 21 and 22. While consumption rises initially to slowly move toward its steady state in response to redistributive innovations, consumption drops below the steady state, following a U-shaped pattern when innovations to the labor share are not purely redistributive.

D Slutsky Decomposition of Hours and Consumption

Productivity innovations alter the relative reward of the labor input (intratemporal substitution effects), introduces intertemporal substitution effects through the (inverse of the) rate of return that households use to discount the future, and also alters the total resources of the agents (wealth effects). Here, we isolate the contribution of each of these effects by means of a Slutsky decomposition of hours and consumption. This involves a lump-sum transfer to agents at $t = 0$ in order to control for the wealth effects by keeping the original equilibrium allocations just feasible at the new prices.

D.1 Hours

To investigate these effects on hours we find it convenient to write out the labor supply function explicitly in terms of present and future wages and interest rates. To derive the labor supply function we first consolidate the budget constraint at $t = 0$,

$$\sum_{t=0}^{\infty} \frac{(1+\gamma)^t c_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t (1-h_t)}{\prod_{s=1}^{t}(1+r_s-\delta)} = \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + (1+r_0-\delta)k_0 \quad (40)$$

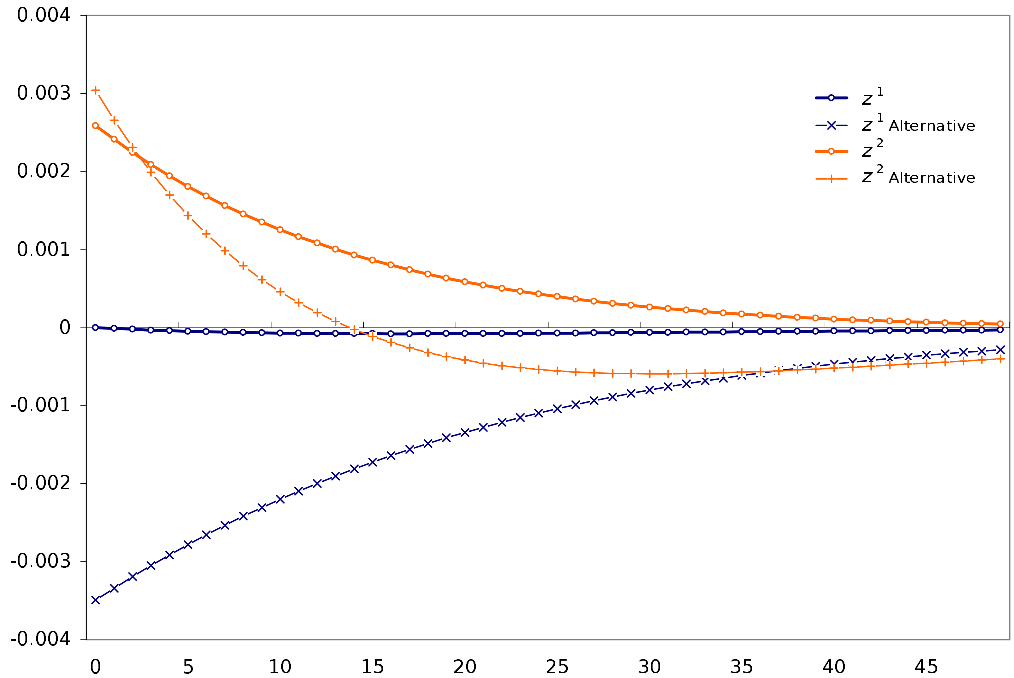

Figure 20: Impulse Response Functions to Orthogonalized Labor Share Innovations, Alternative Identification

where we have used the transversality condition, $\lim_{T\to\infty} \frac{k_T}{\prod_{s=t}^{T}(1+r_s-\delta)} = 0$. The left-hand side is the present value of all future expenditures on consumption and leisure and the right-hand side is the present value of total resources (wealth) accumulated from period $t = 0$ onward. Total resources are composed by the sum of the human wealth and the initial capital income evaluated in units of $t = 0$ consumption. We use the first order condition for labor to substitute out consumption c_t in the left-hand side of (40), and then we use the Euler equation to rewrite the present value of expenditures as

$$\frac{1}{\alpha}\sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t (1-h_t)}{\prod_{s=1}^{t}(1+r_s-\delta)} = \frac{1}{\alpha}\sum_{t=0}^{\infty} \beta^{t-1} w_0 (1-h_0) = \frac{w_0(1-h_0)}{\alpha(1-\beta)} \tag{41}$$

Now, we can plug (41) into (40) and rearrange to find the initial response of leisure for a given forecast of wages and interest rates, $w_0(1-h_0) = \alpha\,(1-\beta)\,\left(\sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + (1+r_0-\delta)k_0\right)$, and using the Euler equation we can recursively find

$$\frac{(1+\gamma)^t w_t (1-h_t)}{\beta^t \prod_{s=1}^{t}(1+r_s-\delta)} = \alpha(1-\beta)\left(\sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + (1+r_0-\delta)k_0\right) \tag{42}$$

That is, the present value of the expenditure on leisure at period t is a constant share of the present value of total resources. This constant share is the marginal propensity to consume leisure, α, and per period, $1-\beta$.

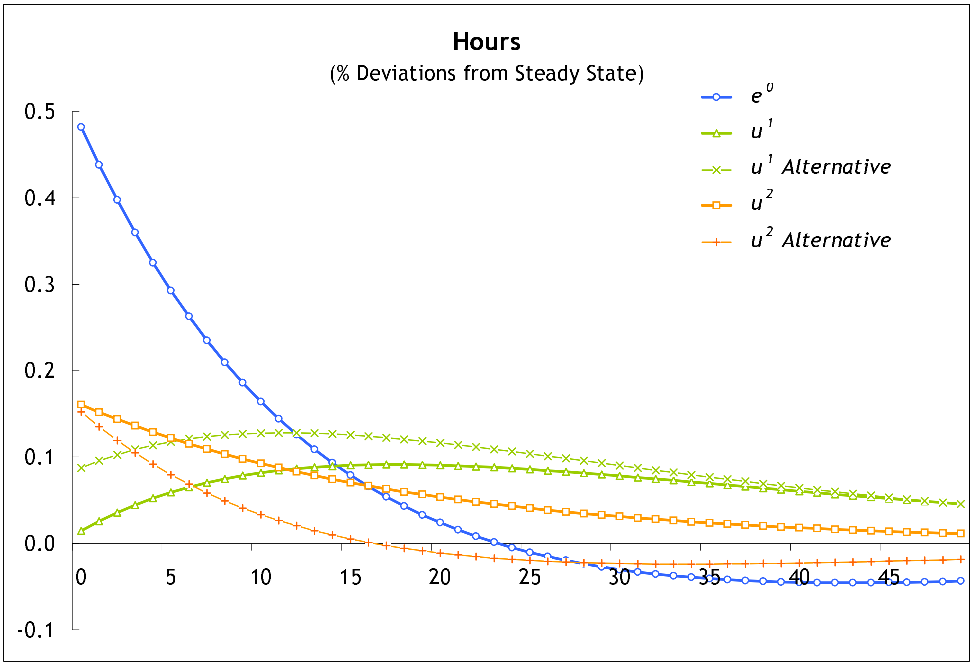

Figure 21: Impulse Response Functions of Hours to All Innovations, Alternative Identification

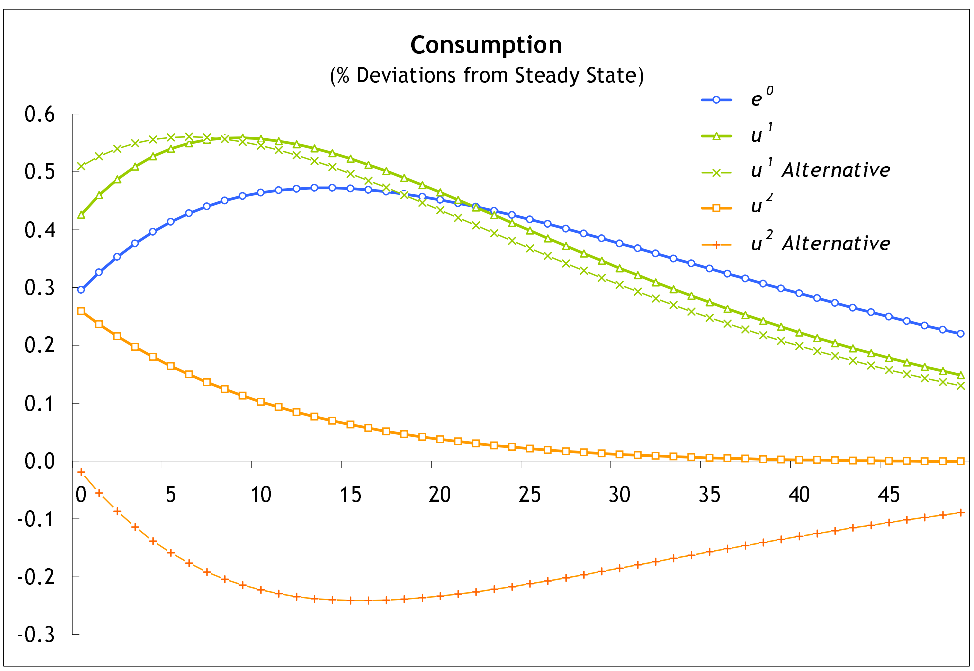

Figure 22: Impulse Response Functions of Consumption to All Innovations, Alternative Identification

If we log-linearize (42) around the steady state we find that the deviation of period-t hours from the steady state can be decomposed as a linear combination of the deviations of period-t wages, the present value of one unit of period-t consumption and the present value of total resources[32]:

$$\widehat{h}_t = \left(\frac{1-h^*}{h^*}\right)\left[\widehat{w}_t + \left(\widehat{\frac{1}{\prod_{s=1}^{t}(1+r_s-\delta)}}\right) - \left(\widehat{\sum_{t=0}^{\infty}\frac{(1+\gamma)^t w_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + (1+r_0-\delta)k_0}\right)\right] \tag{43}$$

where the constant $\frac{1-h^*}{h^*} = 2.2$ is the Frischian elasticity of labor supply. The expression (43) (which is identical for both univariate and bivariate economies) decomposes the overall response of hours to all innovations into intratemporal substitution (wage) effects, intertemporal substitution (rate of return) effects, and wealth (total resources) effects with respect to the steady state. Next, we discuss how we obtain these effects when they arise from crossing the prices between the univariate and bivariate economies.

Intratemporal substitution effect. To see how bivariate wages change the response of hours in an otherwise univariate economy we keep the univariate rate of return and add a Slutsky transfer compensation that sets agents' total resources equal to those generated by the univariate shock, T^0. This transfer compensation is

$$\Psi(w_t^1, r_t^0, T^0) = \sum_{t=0}^{\infty}\frac{(1+\gamma)^t(w_t^1 - w_t^0)}{\prod_{s=1}^{t}(1+r_s^0-\delta)}$$

If we provide agents with this transfer at $t = 0$ we obtain the allocations for an economy with bivariate wages and univariate rate of return and wealth, $a\{w^1, r^0, T^0\}$, which we plot in Figure 10. A symmetric procedure yields $a\{w^0, r^1, T^1\}$.

Intertemporal substitution effect. The introduction of the bivariate interest rate in the univariate economy changes the present value of future units of consumption and, in turn, the present value of the total resources available at $t = 0$. To disentangle these two effects we introduce a Slutsky transfer compensation that keeps total resources unchanged,

$$\Psi(w_t^0, r_t^1, T^0) = \sum_{t=0}^{\infty}\frac{(1+\gamma)^t w_t^0}{\prod_{s=0}^{t}(1+r_s^1-\delta)} + (1+r_0^1-\delta)k^* - \sum_{t=0}^{\infty}\frac{(1+\gamma)^t w_t^0}{\prod_{s=0}^{t}(1+r_s^0-\delta)} - (1+r_0^0-\delta)k^*$$

With this transfer, if we keep univariate wages and introduce the bivariate rate of return we obtain the allocations $a\{w^0, r^1, T^0\}$, depicted in Figure 11. A symmetric procedure yields $a\{w^1, r^0, T^1\}$.

[32] Notice that $(\widehat{1-h_t}) = -\left(\frac{h^*}{1-h^*}\right)\widehat{h}_t$.

Wealth effect. This is a number. The bivariate economy changes the amount of total resources with respect to the univariate economy by

$$\Psi(w_t^0, r_t^0, T^1) = \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w^0}{\prod_{s=0}^{t}(1+r_s^0-\delta)} - (1+r_0^0-\delta)k^* - \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t^1}{\prod_{s=0}^{t}(1+r_s^1-\delta)} + (1+r_0^1-\delta)k^*$$

To measure the wealth effect of the bivariate prices on the univariate hours, we transfer $\Psi(w_t^0, r_t^0, T^1)$ and compute the response of hours to univariate prices. This yields $a\{w^0, r^0, T^1\}$ in Figure 12. A symmetric manipulation yields $a\{w^1, r^1, T^0\}$.

D.2 Consumption

Using the labor supply function and the log-linearization around the steady state of the first order condition for labor we can derive the consumption function as

$$\widehat{c}_t = -\left(\widehat{\frac{1}{\prod_{s=1}^{t}(1+r_s-\delta)}}\right) + \left(\widehat{\sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t}{\prod_{s=0}^{t}(1+r_s-\delta)} + (1+r_0-\delta)\, k_0}\right) \tag{44}$$

The deviations in consumption are driven by the price of future consumption evaluated in present units and the change in the present value of total resources. Figure 23 displays the consumption impulse response functions to all innovations, and Figure 24 shows the intertemporal substitution and wealth effects. Again, productivity innovations cut the price of consumption in the bivariate and univariate economies very similarly during the first year. However, although future units of consumption become expensive more rapidly in the bivariate economy (which would favor a higher consumption in the univariate economy), the important wealth effect in the bivariate economy more than offsets the previous intertemporal substitution effect and sets consumption in the bivariate model above that of the univariate model, which explains the higher volatility of consumption in the bivariate economy.

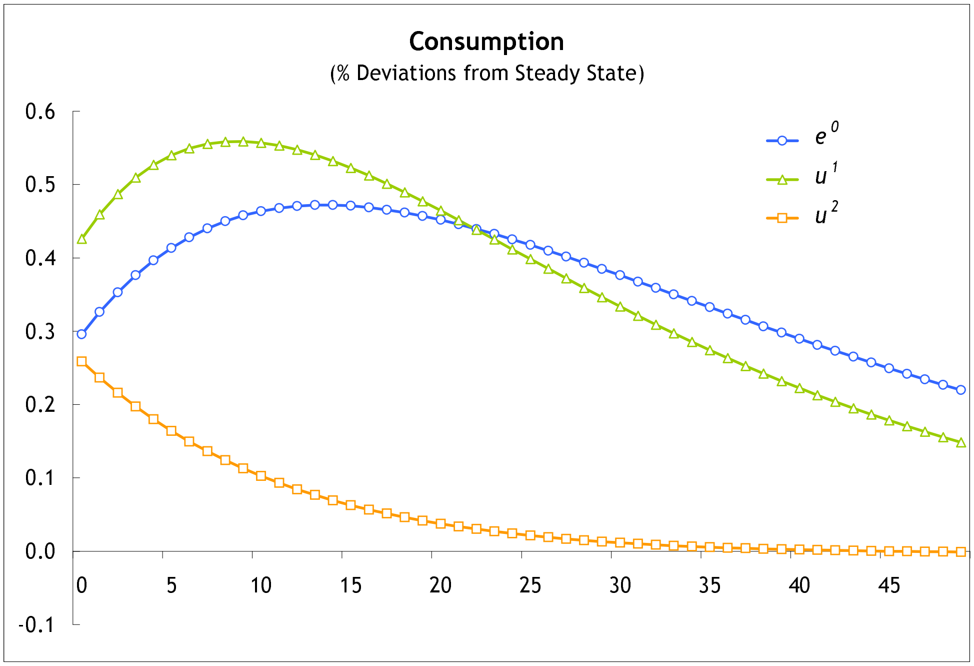

Figure 23: Consumption Impulse Response Functions to Productivity and Redistributive Innovations

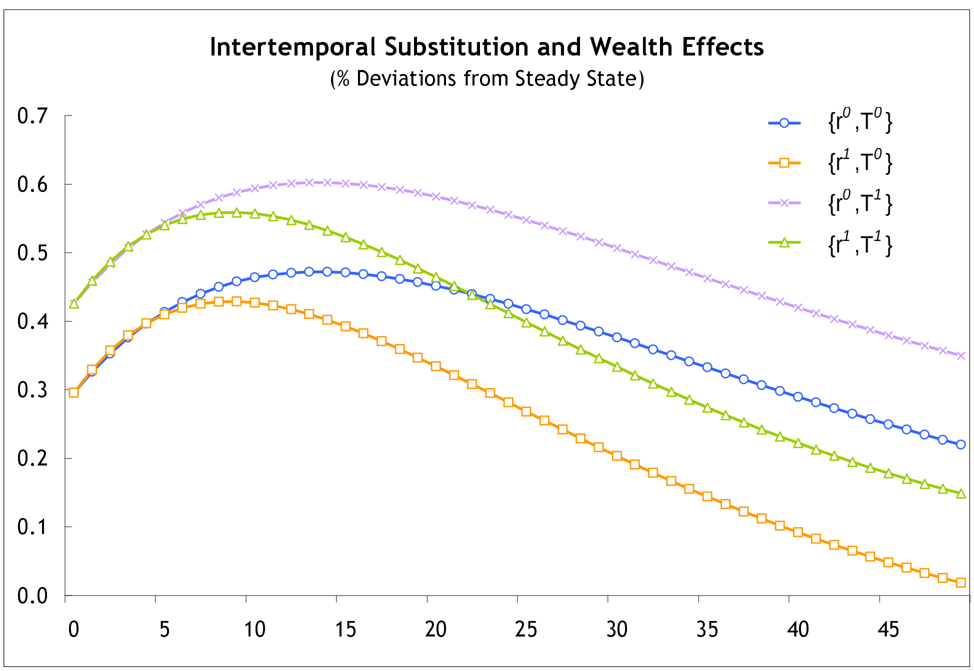

Figure 24: Consumption Intertemporal Substitution and Wealth Effects