

Inflation and Unemployment:
Lagos-Wright meets Mortensen-Pissarides*

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Abstract

Inflation and unemployment are central issues in macroeconomics. While much progress has been made on these issues by incorporating frictions using search theory, existing models analyze either unemployment *or* inflation. We develop a framework to analyze unemployment *and* inflation together. This makes contributions to disparate literatures, and provides a unified model for theory, policy, and quantitative analysis. We discuss optimal fiscal and monetary policy. We calibrate the model, and discuss the extent to which it can account for salient aspects of a half century's experience with inflation, unemployment, interest rates, and velocity.

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There is a natural rate of unemployment at any time determined by real factors. This natural rate will tend to be attained when expectations are on average realized. The same real situation is consistent with any absolute level of prices or of price change, provided allowance is made for the effect of price change on the real cost of holding money balances. Friedman (1977).

1 Introduction

Inflation, unemployment, and relation between the two are central policy concerns and classic topics for macroeconomic analysis. In recent years, much progress has been made studying unemployment and inflation using theories that incorporate frictions explicitly using search theory.¹ It is not surprising that models with frictions are useful for understanding dynamic labor markets and hence unemployment, as well as for understanding the role of money and hence inflation. However, existing models along these lines analyze either unemployment or inflation. The goal of this project is to integrate and extend these disparate theories, in order to develop a new framework that can be used to analyze unemployment and inflation together.

We think this makes contributions to two different literatures. Thus, we learn a lot by extending the standard labor market model to include the exchange of commodities, even without cash playing a role, but perhaps especially when cash does play a role. Similarly, we learn a lot by extending the standard model of monetary exchange to have a more interesting labor market. Our model provides a unified framework for theory, policy, and quantitative analysis. We analyze optimal fiscal and monetary policy. We also calibrate the model, and discuss the

¹In terms of unemployment, we have in mind search-based macro models of the labor market along the lines of Mortensen and Pissarides (1994), but also going back to work by Diamond (1982), Mortensen (1982), and Pissarides (1990), and continuing up to recent contributions by Shimer (2005), Hall (2005), Hagedorn and Manovskii (2006) and others; see Rogerson et al. (2005) for a survey. In terms of inflation, we have in mind search-based models of monetary economies along the lines of Lagos and Wright (2005), but also going back to work by Kiyotaki and Wright (1989,1993), Shi (1995,1997) Trejos and Wright (1995), Kochelakota (1998), Wallace (2001) and many others.

extent to which it can account for salient aspects of inflation, unemployment, interest rate, and velocity behavior.²

2 The Basic Model

Time is discrete and continues forever. Each period, there are three distinct markets where economic activity takes place: a labor market, in the spirit of Mortensen-Pissarides; a goods market, in the spirit of Kiyotaki-Wright; and a general market, in the spirit of Arrow-Debreu. For brevity we call these the MP, KW and AD markets. While it does not especially matter if they meet sequentially, simultaneously, or in some combination, for concreteness we assume here that they meet sequentially.³ There are two basic configurations: after any meeting of the MP market, we can convene either the KW or AD market. It actually does not matter for any interesting results, so we arbitrarily choose the configuration shown in Figure 1. In general, agents can discount between one market and the next at any rate, as shown in the Figure, but since all that matters is $\beta = \beta_1\beta_2\beta_3$, we set $\beta_1 = \beta_2 = 1$ to reduce notation.

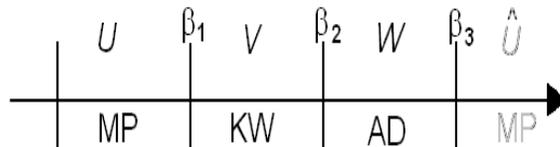


Figure 1: Timing

²Some recent attempts to pursue similar ideas include Farmer (2005), Blanchard and Gali (2005), and Gertler and Trigari (2006), but they all take a very different tact – they make nominal or real wages sticky. In our framework, we do not need stickiness to generate interesting feedback from money to real variables such as unemployment. Moreover, in this project, we are more interested in intermediate-to-long-run phenomena, at which frequency we find wage or price stickiness less compelling. Lehmann (2006), Lehmann and van der Linden (2006), and Kumar (2006) are recent attempts more in line with our approach, although the details are different. Rocheteau, Rupert and Wright (2006) also integrate modern monetary theory into an alternative model of unemployment – Rogerson’s (1988) indivisible labor model.

³See e.g. Williamson (2005) for a model in which a search-based market (where money is essential) and a perfectly competitive market meet simultaneously.

There are two types of private agents (plus government). What one calls them depends on which market – or which literature – one looks at; e.g. they could be called firms and workers in the MP market, or buyers and sellers in the KW market. We call them *firms* and *households*, and index them by f and h . The set of households is $[0, 1]$; the set of potential firms has arbitrarily large measure, although not all will be active at any point in time. Households work, consume and enjoy utility; firms maximize profit and pay out dividends to households.⁴ As in the standard MP model, a household and a firm can combine to create a job that produces output y . Let e index employment status: $e = 1$ indicates that a household (firm) is matched with a firm (household); $e = 0$ indicates otherwise. As seen in Figure 1, we introduce three value functions for the three markets, U_e^i , V_e^i and W_e^i , which generally depend on type $i \in \{h, f\}$, employment status $e \in \{0, 1\}$, and possibly other state variables as specified below; note \hat{U}_f^i in the figure is the MP value function next period (in our notation, \hat{a} is the value of any variable a next period).

2.1 Households

Let us analyze one round of three markets, starting with h in the AD market with money holdings m . He chooses a vector of consumption goods \mathbf{x} and money for next period \hat{m} to solve

$$\begin{aligned} W_e^h(m) &= \max_{\mathbf{x}, \hat{m}} \left\{ \Upsilon_e(\mathbf{x}) + \beta \hat{U}_e^h(\hat{m}) \right\} \\ \text{st } \mathbf{p}\mathbf{x} &= \mathbf{p}\bar{\mathbf{x}} + ew_n(1 - \tau) + (1 - e)b_n + \Delta - T + m - \hat{m}, \end{aligned} \quad (1)$$

where Υ_e is instantaneous utility conditional on e , \mathbf{p} is the price vector, $\bar{\mathbf{x}}$ is the endowment vector, w_n is the (nominal) wage, b_n is unemployment income, Δ is dividend income, T is a lump sum tax, and τ is a wage tax. We assume

⁴This is different from the textbook MP model, where firms are interpreted as consuming profits (but see Merz 1995, Andolfatto 1996, or Fang and Rogerson 2006, e.g.). This is not important here, and everything interesting goes through if firms are consumers.

quasi-linear utility – i.e. $\Upsilon_e(\mathbf{x}) = x + \tilde{\Upsilon}(\tilde{\mathbf{x}})$ is linear in x , where $\tilde{\mathbf{x}}$ is the vector of goods other than x . Although this is not necessary for the theory as long as one is willing to proceed numerically, quasi-linear utility allows us to make a lot of progress analytically. As a benchmark, we often assume $\tilde{\Upsilon}_e(\tilde{\mathbf{x}}) = 0$ – i.e., one generic consumption good.

It is useful to provide a few results about the AD market before specifying the rest of the model. First, substitute x from the budget equation into the objective function in (1) and rearrange to write

$$W_e^h(m) = \frac{I_e + m}{p} + \max_{\tilde{\mathbf{x}}} \left\{ \tilde{\Upsilon}_e(\tilde{\mathbf{x}}) - \frac{\tilde{\mathbf{p}}\tilde{\mathbf{x}}}{p} \right\} + \max_{\hat{m}} \left\{ -\frac{\hat{m}}{p} + \beta \hat{U}_e^h(\hat{m}) \right\}, \quad (2)$$

where p is the price of x , $\tilde{\mathbf{p}}$ is the price vector for other goods, and nominal income given e is $I_e \equiv \mathbf{p}\bar{\mathbf{x}} + ew_n(1 - \tau) + (1 - e)b_n + \Delta - T$. Although the continuation value $\hat{U}_e^h(\hat{m})$ depends on e , it turns out the derivative does not, and hence the choice of \hat{m} is independent of e , m and I_e (see below). So as long as this choice is unique, all households exit the AD market and enter the next period with the same \hat{m} . Moreover, notice that W_e^h is linear: $\partial W_e^h / \partial m = 1/p$.⁵

We now move back to the KW market, where a commodity q different from those in the vector \mathbf{x} is traded bilaterally between firms and households. We assume that households are anonymous in this market, as is standard in monetary theory, in order to generate an essential role for a medium of exchange. See Kocherlakota (1997), Wallace (2001) Corbae, Temzildes and Wright (2003) and Aliprantis, Camera and Puzzello (2006) for formal discussions of anonymity and essentiality, but to convey the flavor of the idea, suppose firms cannot identify households by name. Then any h that asks f for q now with a promise to pay later (in the next AD market, say) could renege without fear of repercussions. Hence, f must insist on quid pro quo. If we assume consumption goods are not

⁵Analogous results (the AD value function is linear in m and all households exit with the same \hat{m}) hold in Lagos-Wright (2005), and are what make the analysis so tractable. This all assumes the solution is interior for x , of course, which can be guaranteed here simply by assuming $\tilde{\mathbf{x}}$ is big enough.

storable by households, then fiat money will step into the role of medium of exchange.⁶

Given this, for h with m dollars and employment status e in the KW market,

$$V_e^h(m) = \alpha_h [v(q) + W_e^h(m - d)] + (1 - \alpha_h)W_e^h(m), \quad (3)$$

where α_h is the probability of trading and v is a standard utility function. The terms of trade are given by the quantity q and dollars d , as discussed below. The probability of trade is determined by a matching function, $\alpha_h = \mathcal{M}(B, S)/B$, where B is the number of buyers and S the number of sellers in the market, and \mathcal{M} satisfies the standard assumptions, including constant returns. Hence, $\alpha_h = \mathcal{M}(Q, 1)/Q$, where $Q = B/S$ is the *queue length* or *market tightness* in the KW market. Since all households participate in the this market $B = 1$; since only firms with $e = 1$ participate $S = 1 - u$, where u denotes unemployment, and so $\alpha_h = \mathcal{M}(1, 1 - u)$. This particular functional relation depends of course on the details of the model, but the idea that it is better to be a buyer when there are more sellers seems quite general.

In the MP market,

$$U_1^h(m) = \delta V_0^h(m) + (1 - \delta)V_1^h(m) \quad (4)$$

$$U_0^h(m) = \lambda_h V_1^h(m) + (1 - \lambda_h)V_0^h(m), \quad (5)$$

where δ is the exogenous rate at which matches are destroyed, and λ_h the endogenous rate at which they are created. The latter is determined by another standard matching function, $\lambda_h = \mathcal{N}(u, v)/u$, where u is the number of unemployed workers and v the number of vacancies posted by firms. By constant

⁶The modern monetary literature goes into considerable detail about specialization and other features of the environment that give rise to a role for a medium of exchange, and we see no need to repeat all of that here. A more subtle issue is why claims to real assets, like shares in firms, are not used for this purpose. One response is to say that agents may not (always) recognize counterfeit claims in the KW market, even if they can be authenticated in the AD market, but they can (relatively easily) recognize money; see Lester, Postlewaite and Wright (2006) for details.

returns, $\lambda_h = \mathcal{N}(1, v/u)$, where v/u is labor market tightness. We assume that wages are determined when firms and households meet in the MP market, as discussed below, even though they are not paid until the next AD market, as (1) indicates.⁷

This completes the specification of our household problem. Before moving on to firms, we point out that although there are some advantages to having a value function for each market – e.g. it makes for easier comparisons with other literatures – this is by no means necessary. Substituting (3) into (4), e.g., we get

$$\begin{aligned} U_1^h(m) &= \delta [\alpha_h v(q) + \alpha_h W_0^h(m-d) + (1-\alpha_h)W_0^h(m)] \\ &\quad + (1-\delta) [\alpha_h v(q) + \alpha_h W_1^h(m-d) + (1-\alpha_h)W_1^h(m)] \\ &= \alpha_h \left[v(q) - \frac{d}{p} \right] + \frac{m}{p} + \delta W_0^h(0) + (1-\delta)W_1^h(0), \end{aligned}$$

using the linearity of W_e^h . Something similar can be done for U_0^h . Inserting these into (2), the AD problem can be written

$$\begin{aligned} W_e^h(m) &= \frac{I_e + m}{p} + \max_{\tilde{\mathbf{x}}} \left\{ \tilde{\Upsilon}_e(\tilde{\mathbf{x}}) - \frac{\tilde{\mathbf{p}}\tilde{\mathbf{x}}}{p} \right\} \\ &\quad + \max_{\hat{m}} \left\{ -\frac{\hat{m}}{p} + \beta \hat{\alpha}_h \left[v(\hat{q}) - \frac{\hat{d}}{\hat{p}} \right] + \beta \frac{\hat{m}}{\hat{p}} \right\} + \beta \mathbb{E} \hat{W}_{\hat{e}}^h(0), \end{aligned} \quad (6)$$

where the expectation \mathbb{E} is with respect to next period's employment status \hat{e} .

A nice property of (6) is that it makes clear \hat{m} does not depend on e , I_e or m , at least as long as (q, d) do not, as we verify below. Hence, we get a degenerate distribution of money holdings across households in the KW market. This is, of course, exactly what the model is designed to deliver.⁸

⁷This is merely for convenience: paying workers in the AD market allows us to avoid specifying whether they are paid in dollars, goods, etc., since all that matters in this market is the implied purchasing power.

⁸Dispensing with quasi-linear utility, or adding a cost to access the AD market, as in Molico (2006) or Chiu and Molico (2006), would generate a model in the same spirit, but with a nondegenerate distribution, and we could only proceed numerically. For an extension with a nondegenerate yet tractable distribution, we can simply assume $v_e(q)$ depends on e and switch the timing of the MP and AD markets; then \hat{m} would depend on one's employment status, but not on others aspect of history.

2.2 Firms

First, since firms do not need money in the KW market, they obviously choose $\hat{m} = 0$. Then, in the MP market, we have

$$\begin{aligned}U_1^f &= \delta V_0^f + (1 - \delta)V_1^f \\U_0^f &= \lambda_f V_1^f + (1 - \lambda_f)V_0^f\end{aligned}$$

where $\lambda_f = \mathcal{N}(u, v)/v$, as is completely standard. However, here we depart from the textbook MP model, as follows: rather than having f and h each consume some share of their output when they are matched, in our setup, f takes y to the KW market and tries to sell it to some other household. The idea that should be uncontroversial is that agents do not necessarily want to always consume what they made at work that day (it is obviously only for ease of presentation that we assume that they never consume what they made at work).

Trade in the KW market is in general probabilistic. If q is the random amount f sells in this market, we assume that remainder $y - q$ is transformed into generic consumption goods in the next AD market according to the technology $x = \gamma(y - q)$, where as long as $\gamma > 0$ we can set it to 1 without loss in generality.⁹ Although the case $\gamma = 0$ (full depreciation) is also fine, we like the idea of giving f an opportunity cost to selling output in the KW market. In principle, we could say that $y - q$ is carried forward to the next KW market, but then we need to track the distribution of inventories across firms over time. Having an AD market where firms can liquidate unsold inventories allows us to capture opportunity cost in the KW market while avoiding this technical problem, just like it allows us to avoid tracking the distribution \hat{m} across households over time.

⁹This is really just a choice of units, given that q enters utility according to general function $v(q)$. Formally, the calibration procedure discussed below cannot identify γ , but only the ratio of γ to other parameters.

Thus, for a firm with $e = 1$ in the KW market,

$$V_1^f = \alpha_f W_1^f(y - q, d) + (1 - \alpha_f) W_1^f(y, 0), \quad (7)$$

where $\alpha_f = \mathcal{M}(B, S)/S$ and $W_1^f(x, m)$ is the value of entering the AD market with x goods in inventory and m dollars in cash receipts. The latter is given by

$$W_1^f(x, m) = x + m/p - w + \beta \hat{U}_1^f, \quad (8)$$

where $w = w_n/p$ is the real wage, which as we said above is paid to workers in the AD market. Collapsing (7) and (8), we have

$$V_1^f = R - w + \beta \left[\delta \hat{V}_0^f + (1 - \delta) \hat{V}_1^f \right], \quad (9)$$

where

$$R = \alpha_f(y - q + d/p) + (1 - \alpha_f)y = y + \alpha_f(d/p - q) \quad (10)$$

is expected real revenue entering the KW market.¹⁰

Expected real profit for firms with $e = 1$ is $R - w$. As in the standard model, a firm with $e = 0$ has no current revenue or wage obligations, but if it pays a real cost k in the current AD market, it enters the next MP market with a vacancy that may or may not match with a worker. Thus,

$$W_0^f = \max \left\{ 0, -k + \beta \lambda_f \hat{V}_1^f + \beta(1 - \lambda_f) \hat{V}_0^f \right\},$$

where $\hat{V}_0^f = \hat{W}_0^f = 0$ if we make the usual free entry assumption. In steady state $k = \beta \lambda_f V_1^f$, which by (9) can be written

$$k = \frac{\beta \lambda_f (R - w)}{1 - \beta(1 - \delta)}. \quad (11)$$

Average real profit across all firms is $(1 - u)(R - w) - vk$. As we said, our firms pay out profit as dividends to the households. If we assume the representative

¹⁰We assume $q \leq y$, since the firm cannot sell more than it has. It is easy to put conditions on primitives to guarantee this is true in equilibrium, since as we shall see, in equilibrium, $q \leq q^*$ where $v'(q^*) = 1$.

household holds a representative portfolio of shares, then the real dividend is $\Delta/p = (1 - u)(R - w) - vk$.¹¹

2.3 Government

Government consumes G , levies lump sum and proportional taxes T and τ , and prints money at rate π so that $\hat{M} = (1 + \pi)M$. It also pays out a UI (unemployment insurance) benefit to households with $e = 0$. Hence, the nominal value of unemployment in (1) satisfies $b_n = pb(1 - \tau) + p\ell$, where b is the real value of UI, which is taxed, and ℓ is the real value of leisure plus home production, which is not; we distinguish between b and ℓ because of the way they enter the calibration and welfare calculations. Assuming G and T denote nominal quantities, the government budget constraint is $G + bu = T + \tau w(1 - u) + \pi M$. We usually describe monetary policy in terms of the nominal interest rate i . This is equivalent in steady state to setting the growth rate of the money supply π , because the Fisher equation implies $1 + i = (1 + \pi)/\beta$.¹² We assume $i > 0$, although we do consider the limit as $i \rightarrow 0$.

3 Equilibrium

Various assumptions can be made concerning price determination in the different markets, including (Walrasian) price taking, bargaining, and price posting with or without directed search. We think the most reasonable scenario is the following: price taking in the AD market, bargaining in the MP market, and posting with directed search in the KW market. We like price taking in the AD market because it is simple, and in any case the AD market is not the prime focus of our analysis. In the MP market, which is a key part of the theory, we opt

¹¹One can imagine households holding shares in a mutual fund that owns all of the firms. In any case, dividend income matters little here due to quasi-linear utility.

¹²The Fisher equation is an arbitrage condition that implies $1 + i = (1 + r)\hat{p}/p$, where r is the real interest rate between AD markets, which is pinned down by $1 + r = 1/\beta$, while inflation is given by $\hat{p}/p = 1 + \pi$ in steady state.

for bargaining because it seems realistic for many labor markets and because it is standard in the related literature. The choice is less clear for the KW market, and we actually start with a bargaining version because it is perhaps better known and slightly easier to present; we soon switch to posting with directed search, however, which is our preferred approach for the KW market for several reasons that we now discuss.

First, posting with directed search – also known as competitive search equilibrium, after Moen (1997) and Shimer (1996) – is fairly convenient in terms of both methods and results, at least after some initial set-up cost. Second, directed search should seem like a big step forward to those like Howitt (2005) who criticize monetary theory with random matching for the assumption of randomness *per se*.¹³ It should also appease those who don't like modern monetary theory simply because they don't like bargaining, such as Phelan (2005). More seriously, models with bargaining typically need the unpalatable assumption that agents can see each others' money holdings to avoid the technical difficulties inherent with bargaining under private information. Finally, using competitive search eliminates (i.e. endogenizes) bargaining power as a free parameter, which helps in calibration.

Hence, competitive search deflects several critiques, gives convenient results, and seems fairly realistic. But having said all this, again, we start with bargaining in the KW market before introducing competitive search. Generally, we break the analysis into three parts. We first describe the determination of the value of money q , taking unemployment u as given, following the usual ap-

¹³As Howitt (2005) puts it, “In contrast to what happens in search models, exchanges in actual market economies are organized by specialist traders, who mitigate search costs by providing facilities that are easy to locate. Thus, when people wish to buy shoes they go to a shoe store; when hungry they go to a grocer; when desiring to sell their labor services they go to firms known to offer employment. Few people would think of planning their economic lives on the basis of random encounters.” (p. 405). Previous directed search models of monetary exchange, which were in part motivated by this critique, include e.g. Corbae et al. (2003) and Rocheteau and Wright (2005).

proach in the Lagos-Wright (2005) model. We then determine u , taking q as given, as in the standard Mortensen-Pissarides (1996) model. It is convenient to depict these two relationships graphically in (u, q) space by what we call *the LW curve* and *the MP curve*. The intersection of these two curves determines the equilibrium unemployment rate and value of money (u, q) , from which all of the other endogenous variables easily follow.

3.1 The LW Curve

As we said, we start by assuming firms and households meet at random and bargain over the terms of trade in the KW market, subject to the constraint $d \leq m$, where m is the money holdings of the household. We use the generalized Nash bargaining solution with threat points equal to continuation values from not trading, and let θ denote the buyer's bargaining power.¹⁴ The surplus for a household with employment status e and money m is

$$v(q) + W_e^h(m - d) - W_e^h(m) = v(q) - d/p,$$

by the linearity of $W_e^h(m)$. Similarly, the surplus for a firm, which must have $e = 1$ to be a seller and in equilibrium brings no money to the KW market, is

$$W_1^f(y - q, d) - W_1^f(y, 0) = d/p - q.$$

It is easy to show the solution implies $d = m$ and q solves¹⁵

$$\frac{m}{p} = g(q, \theta) \equiv \frac{\theta v'(q)q + (1 - \theta)v(q)}{\theta v'(q) + 1 - \theta}. \quad (12)$$

Now recall the problem in (6), which in terms of the choice of \hat{m} is summarized by

$$\max_{\hat{m}} \left\{ -\frac{\hat{m}}{p} + \beta \hat{\alpha}_h \left[v(\hat{q}) - \frac{\hat{m}}{\hat{p}} \right] + \beta \frac{\hat{m}}{\hat{p}} \right\},$$

¹⁴Rocheteau and Waller (2004) discuss several alternative bargaining solutions for monetary models.

¹⁵See Lagos-Wright (2005) for details, but basically $d = m$ follows from a simple arbitrage condition, and (12) follows from the first order condition from the bargaining problem. Notice (q, d) depends on the household's current m via the constraint $d \leq m$, but not on previous values, nor on e or I_e , as claimed above.

where it is understood that \hat{q} is a function of \hat{m} , given implicitly by (12). By virtue of the Fisher equation, $1 + i = \hat{p}/p\beta$, this is equivalent to

$$\max_{\hat{m}} \left\{ v(\hat{q}) - \frac{\hat{m}}{\hat{p}} \frac{i + \hat{\alpha}_h}{\hat{\alpha}_h} \right\}. \quad (13)$$

The first order condition for an interior solution is $v'(\hat{q})\partial\hat{q}/\partial\hat{m} = (i + \hat{\alpha}_h)/\hat{\alpha}_h\hat{p}$ (second order conditions are discussed below). Inserting $\partial\hat{q}/\partial\hat{m} = 1/\hat{p}g_1(\hat{q}, \theta)$, which we get by differentiating (12), as well as $\hat{\alpha}_h = \mathcal{M}(1, 1 - \hat{u})$, and then imposing steady state, this can be written

$$\frac{v'(q)}{g_1(q, \theta)} - 1 = \frac{i}{\mathcal{M}(1, 1 - u)}. \quad (14)$$

This is the LW curve, determining q for a given u , exactly as in the baseline LW Model. An increase in u affects q via $\alpha_h = \mathcal{M}(1, 1 - u)$ for the following reason: more unemployment is equivalent to fewer sellers, which makes it less attractive to be a buyer, which reduces the value of money. From known results we can easily deduce the properties of this curve. First, it is not automatic that the LHS of (14) is decreasing in q , but one can impose conditions to guarantee monotonicity, and hence to guarantee a unique $q > 0$ solving this condition for any $u \in (0, 1)$, with $\partial q/\partial u < 0$.¹⁶ But even if v'/g_1 is not globally decreasing, it is decreasing at any q such that the second-order condition is satisfied; hence, whenever the first- and second-order conditions hold, $\partial q/\partial u < 0$. Also, letting q^* be the efficient quantity, given by $v'(q^*) = 1$, q is bounded by q^* for any u . Also, $u = 1$ implies $q = 0$. Summarizing:

Proposition 1 *For all $i > 0$ the LW curve slopes downward in (u, q) space, with $u = 0$ implying $q \in (0, q^*)$ and $u = 1$ implying $q = 0$. It shifts down with i and up with θ . In the limit as $i \rightarrow 0$, $q \rightarrow q_0$ for all $u < 1$, where q_0 is independent of u , and $q_0 \leq q^*$ with $q_0 < q^*$ unless $\theta = 1$.*

¹⁶Examples of conditions from Lagos-Wright (2005) that can be used to make v'/g_1 globally decreasing in q are: 1. u' is log-concave; or 2. $\theta \approx 1$.

3.2 The MP Curve

When unmatched firms and households meet, they negotiate wages according to the generalized Nash bargaining solution, with threat points equal to the continuation values from remaining unmatched and η denoting the firm's bargaining power. The household's surplus is $S^h = V_1^h(m) - V_0^h(m) = W_1^h(0) - W_0^h(0)$, due to the linearity of W_e^h . Inserting W_e^h from (6) and simplifying, we get

$$S^h = (w - b)(1 - \tau) - \ell + \beta(1 - \delta - \lambda_h) \hat{S}^h.$$

The firm's surplus is $S^f = V_1^f - V_0^f = R - w + \beta(1 - \delta) \hat{S}^f$, by virtue of (7) and free entry. To solve the bargaining problem, first maximize the Nash product given future surpluses \hat{S}^i . Then insert the steady state results

$$S^h = \frac{(w - b)(1 - \tau) - \ell}{1 - \beta(1 - \delta - \lambda_h)} \text{ and } S^f = \frac{R - w}{1 - \beta(1 - \delta)},$$

and solve for w .

The solution is

$$w = \frac{\eta[1 - \beta(1 - \delta)]z + (1 - \eta)[1 - \beta(1 - \delta - \lambda_h)]R}{1 - \beta(1 - \delta) + (1 - \eta)\beta\lambda_h}, \quad (15)$$

where $z = b + \ell/(1 - \tau)$ is the value to the household of not working, adjusted for taxes.¹⁷ If we substitute w from (15) and R from (10) into the free entry condition (11), we have

$$k = \frac{\lambda_f \eta [y - z + \alpha_f (d/p - q)]}{r + \delta + (1 - \eta)\beta\lambda_h},$$

where $r = 1/\beta - 1$. To reduce this to one equation in (u, q) we do three things:

1. insert the arrival rates from the matching functions $\lambda_f = \mathcal{N}(u, v)/v$, $\lambda_h = \mathcal{N}(u, v)/u$ and $\alpha_f = \mathcal{M}(1, 1 - u)/(1 - u)$; 2. insert

$$\frac{d}{p} - q = g(q, \theta) - q = \frac{(1 - \theta)[u(q) - q]}{\theta u'(q) + 1 - \theta}$$

¹⁷In other words, $z(1 - \tau) = b(1 - \tau) + \ell$ is the after-tax benefit of not working. Except for notation, this is the same as the after-tax real wage in the standard MP model.

from the bargaining solution (12); and 3. use the steady state version of the law of motion for unemployment,

$$\hat{u} = u + (1 - u)\delta - \mathcal{N}(u, v) = 0, \quad (16)$$

to solve for and insert $v = v(u)$. The final answer is

$$k = \frac{\mathcal{N}[u, v(u)]}{v(u)} \eta \left\{ y - z + \frac{\mathcal{M}(1, 1-u)}{1-u} \frac{(1-\theta)[u(q)-q]}{\theta u'(q) + 1 - \theta} \right\}. \quad (17)$$

This is the *MP curve*, determining u exactly as in the standard MP model, except in the standard model we effectively have $q = 0$ (i.e. the final term in braces does not appear). It is a matter of routine calculation to show that this curve is downward sloping. Intuitively, there are three effects from an increase in u , two from the usual MP model and one that is new, all of which encourage entry: 1. it is easier for firms to hire; 2. it is harder for households to get hired, which lowers w ; 3. it is easier for firms to compete in the KW market, which is the new effect. Again, these three effects go in the same direction, so the MP curve slopes downward. Deriving other properties is easy, including showing how the MP curve shifts with changes in parameters. Summarizing:

Proposition 2 *The MP curve slopes downward in (u, q) space. It shifts in with y, η or θ , and out with k, r, δ or $z = b + \ell/(1 - \tau)$.*

3.3 LW meets MP

Propositions 1 and 2 imply both LW and MP slope downward in the box $[0, 1] \times [0, q^*]$ in (u, q) space, as show in Figure 2. The former enters the box from the upper left when $u = 0$ at $q_0 \leq q^*$, and exits at $(0, 0)$. The latter enters the box when $q = q^*$ at some $\underline{u} > 0$, with $\underline{u} < 1$ iff k is not too big, and exits by either hitting the horizontal axis at some $u_0 \in (0, 1)$, or hitting the vertical axis at some $q_1 \in (0, q^*)$. It is easy to check MP hits the horizontal axis $u_0 \in (0, 1)$,

as shown by the curve labeled 1 in the Figure, iff $\eta(y - z) > k(r + \delta)$. This inequality simply says there would be entry into the MP market even if we shut down the KW market. In this case, there exists a nonmonetary steady state equilibrium at $(u_0, 0)$, which is exactly the standard MP equilibrium, and there exists at least one monetary steady state with $q > 0$ and $u < u_0$.

The Figure also shows two cases, labeled 2 and 3, where the MP curve intersects the vertical axis at some q_1 . In these case, there either exist multiple monetary steady states, as shown by curve 2, or there exist no monetary steady states, as shown by curve 3. In either case there also exists a nonmonetary steady state at $u = 1$ and $q = 0$. In these nonmonetary equilibria, which occur iff $\eta(y - z) < k(r + \delta)$, the KW market shuts down, and this means the MP market also shuts down. In case 3, this is the only possible equilibria; in case 2, however, there are also monetary equilibria with the KW and MP markets both open and $u < 1$. In case 1, recall, even if $q = 0$ and the KW market shuts down, there is still the standard MP equilibrium with $u < 1$.

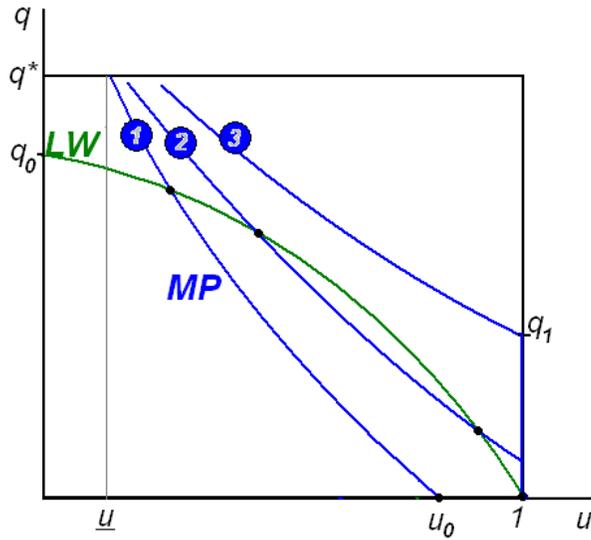


Figure 2: LW meets MP

To understand which case is more likely to occur, simply look at the results in Propositions 1 and 2 concerning the effects on the curves of changes in parameters. In every possible case we have established the existence of steady state equilibrium. We do not generally get uniqueness, as is clear from the Figure, but it is possible for the monetary steady state to be unique, as shown with curve 1. If there exists any steady state with $u < 1$ then there exists a monetary steady state. We also know that a sufficient condition for a steady state with $u < 1$, and hence for a monetary steady state, is $\eta(y - z) > k(r + \delta)$, which is also required for a steady state with $u < 1$ in the standard MP model. Given (u, q) , we can recover all of the other endogenous variables, including vacancies v , the arrival rates α_j and λ_j , real balances $m/p = g(q, \theta)$, and the nominal price level $p = M/g(q, \theta)$.¹⁸

A convenient result from Propositions 1 and 2 is that changes in i shift the LW curve, while holding i fixed changes in y, η, r, k, δ or z shift the MP curve; only θ shifts both. This makes it easy to analyze most changes in parameters. For example, an increase in the nominal interest rate i shifts the LW curve in toward the origin, reducing q and u if equilibrium is unique (or, in the ‘natural’ equilibria if we do not have uniqueness). The result $\partial q/\partial i < 0$ holds in the standard LW model, with a fixed α_h , but now there is a general equilibrium via on u that reduces α_h , which further reduces q . Similarly, an increase in z shifts the MP curve out, increasing u and reducing q if equilibrium is unique (or, in

¹⁸Given these variables, the AD budget equation yields x for every individual as a function of the m with which he enters. In the general case where there is a vector \tilde{x} of other AD goods, maximization determines an individual demand as a function of employment status and $\tilde{\mathbf{p}}$ (plus p which has already been determined). Write this as $\tilde{\mathbf{x}} = \mathbf{D}_e(\tilde{\mathbf{p}})$. Market demand is $\mathbf{D}(\tilde{\mathbf{p}}) = u\mathbf{D}_0(\tilde{\mathbf{p}}) + (1 - u)\mathbf{D}_1(\tilde{\mathbf{p}})$. Equating this to the endowment vector yields a standard system of general equilibrium equations $\mathbf{D}(\tilde{\mathbf{p}}) = \tilde{\mathbf{x}}$ that solve for $\tilde{\mathbf{p}}$. The model displays classical neutrality: if we change M , we can change p and $\tilde{\mathbf{p}}$ proportionally without affecting the AD equilibrium conditions or the values of the other real variables (q, u, v) . This of course does not mean monetary policy does not matter: a change in i (or, equivalently, π) shifts the LW curve, which affects q, u , and the rest of the system. When AD utility Υ does not depend on e , however, neither does demand, so $\mathbf{D}(\tilde{\mathbf{p}})$ is independent of u and hence $\tilde{\mathbf{x}}$ is independent of monetary policy – a version of the neoclassical dichotomy.

the ‘natural’ equilibria). The result in $\partial u/\partial z > 0$ holds in the standard MP model with fixed R , but now there is an effect via q that reduces R and further increases u . Other experiments can be analyzed similarly.

Proposition 3 *Steady state equilibrium always exist. One steady state is a nonmonetary equilibrium, which has $u < 1$ iff $\eta(y - z) > k(r + \delta)$. If this inequality holds, there also exists a monetary steady state. Assuming the monetary steady state is unique: a rise in i decreases q and increases u ; holding i constant a rise in y or η , or a fall in k , r , δ or z , increases q and decreases u .*

4 Competitive Search

As we said, while things work fine with random matching and bargaining, we want to consider directed search with price posting. There are several ways to formalize competitive search. One is to have sellers first post the terms of trade, then have each buyer direct his search to the most favorable seller, taking into account that he only gets served probabilistically (say, because more buyers might show up than a seller has capacity to serve). Or we can have buyers post the terms of trade to attract sellers. Or we can imagine market makers who set up submarkets, to which they try to attract buyers and sellers by posting the terms of trade (so they can charge them an entrance fee, although this fee is 0 in equilibrium by free entry into market-making), and then in each submarket buyers and sellers match randomly according to a general $\mathcal{M}(B, S)$ but are bound by the posted terms. It is known in the literature than these different stories all lead to the same set of equilibrium conditions.¹⁹

¹⁹Papers that model different versions of these stories, in addition to Moen (1997) and Shimer (1996), include Acemoglu and Shimer (1999), Burdett, Shi and Wright (2001), Mortensen and Wright (2002), and Julien, Kennes and King (2000). There is one detail. While models formalizing these different stories are equivalent in nonmonetary economies, Faig and Huangfu (2005) show that a monetary economy can do better with market makers than with sellers or buyers posting. The idea is that market makers can in principle charge buyers but pay sellers (i.e. charge them a negative fee) to enter their submarket, and then have them trade if they meet at price 0. Effectively, this insures agents against the possibility of not trading,

Given this, we proceed by assuming sellers post the terms of trade, but trade is still probabilistic, in the sense that if a group of B buyers direct their search towards a group of S sellers, the number of meetings is given by $\mathcal{M}(B, S)$. Hence, agents need to know the queue length $Q = B/S$ to determine the relevant probabilities, $\alpha_f = \mathcal{M}(Q, 1)$ and $\alpha_h = \mathcal{M}(Q, 1)/Q$. We imagine the firm posting in the AD market the following message: “If I have anything to sell (i.e. if $e = 1$) next period, then I commit to sell q units for d dollars in the KW market, but I can serve at most one customer, and you should expect a queue length Q .” Formally, the problem of a firm is

$$\begin{aligned} & \max_{q, d, Q} \mathcal{M}(Q, 1) \left(\frac{d}{p} - q \right) \\ \text{st } & \frac{\mathcal{M}(Q, 1)}{Q} \left[v(q) - \frac{d}{p} \right] - i \frac{d}{p} = \hat{Z} \text{ if } Q > 0, \end{aligned} \quad (18)$$

where \hat{Z} is the terms offered by the best alternative seller.²⁰

Assume for now that $\hat{Z} \in (0, \bar{Z})$ where \bar{Z} is not too big (see below), so that the firm wants some buyers to show up: $Q > 0$. Then use the constraint to eliminate d and rewrite (18) as

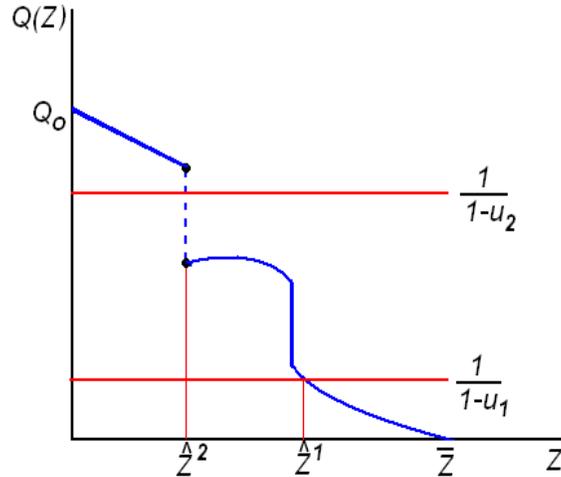
$$\max_{q, Q} \mathcal{M}(Q, 1) \left[\frac{\mathcal{M}(Q, 1)v(q) - \hat{Z}Q}{\mathcal{M}(Q, 1) + iQ} - q \right]. \quad (19)$$

One can show this problem has a solution with $Q > 0$. Indeed, for generic \hat{Z} there will be a unique solution, which means that over the interval $(0, \bar{Z})$ there is at most a finite number of values for \hat{Z} with multiple solutions, as shown in Figure ???. To make life simpler, we assume that the solution $Q(\hat{Z})$ is convex valued – i.e. we rule out the situation at \hat{Z}_2 in the Figure. To ensure this

and allows buyers to each bring less money. However, as in Rocheteau and Wright (2005), one can assume market makers cannot tell if an agent is a buyer or a seller, which precludes differential fees, and recovers the equivalence.

²⁰The objective function is expected revenue net of opportunity cost, ignoring some constant terms that do not affect the maximization; the constraint says that to get any buyers to show up, the seller has to offer them at least \hat{Z} . It is simple to derive this problem from the underlying dynamic programming equations.

is valid, below we give sufficient conditions for $Q(\hat{Z})$ to be single-valued, and strictly decreasing.



To indicate where this is all leading, think of Q as the demand for buyers by a seller, and \hat{Z} as the price (in terms of utility) that the seller has to pay in order to get buyers to show up. In equilibrium \hat{Z} will be determined so that the supply of buyers per firm $1/(1-u)$ equals the demand Q as long as $\hat{Z} > 0$; if $\hat{Z} = 0$ then buyers get no surplus in the KW market, and only some of them show up. Let $Q_0 = Q(0)$ and define u_0 by $Q_0 = 1/(1-u_0)$. Then as long as $u < u_0$, there is a $Z > 0$ such that the market clears with $Q(Z) = 1/(1-u)$. When $u \geq u_0$, however, the market equilibrates at $Z = 0$, in which case some buyers stay home and $Q = Q_0 < 1/(1-u)$.²¹ Clearly, the market clears at $Z > 0$ as long as u is not too big. Hence, in equilibrium $Q = Q(u) = \min \{Q_0, 1/(1-u)\}$.

²¹ At $u = u_2$ in the Figure, when there is a discontinuity between two values of Q that solve (19) at $Z = Z_2$, we need to have different sellers posting different Q 's to clear the market. While this is not impossible to deal with, the situation is obviously simpler when $Q(\hat{Z})$ is convex valued.

Given this, take the first order conditions to (18):

$$\frac{\mathcal{M}}{\mathcal{M} + iQ} v'(q) - 1 = 0 \quad (20)$$

$$\mathcal{M}_1 \left[\frac{\mathcal{M}v(q) - \hat{Z}Q}{\mathcal{M} + iQ} - q \right] - \mathcal{M}\mathcal{M}_2 \left[\frac{iv(q) + \hat{Z}}{(\mathcal{M} + iQ)^2} \right] = 0 \quad (21)$$

From (20) we determine q by:

$$v'(q) - 1 = \frac{iQ(u)}{\mathcal{M}[Q(u), 1]} = \begin{cases} \frac{i}{\mathcal{M}(1, 1-u)} & \text{if } u < u_0 \\ \frac{i}{\mathcal{M}(1, Q_0^{-1})} & \text{if } u > u_0 \end{cases} \quad (22)$$

Now eliminate from (18) $\mathcal{M} + iQ$ using (20) and \hat{Z} using the constraint, to rewrite (21) as

$$\frac{d}{p} = \frac{Q\mathcal{M}_1 v'(q)q + \mathcal{M}_2 v(q)}{Q\mathcal{M}_1 v'(q) + \mathcal{M}_2}.$$

Letting $\varepsilon(Q) \equiv Q\mathcal{M}_1(Q, 1)/\mathcal{M}(Q, 1)$ be the elasticity of \mathcal{M} with respect to B , this reduces to

$$\frac{d}{p} = g[q, \varepsilon(Q)], \quad (23)$$

where the function g is defined in (12). Hence, posting and bargaining give similar solutions for real balances, with bargaining power replaced by the elasticity $\varepsilon(Q)$.²²

Let us collect some results. First, substitute (23) into the constraint in (18) and use (20) to write

$$F(q, Q) \equiv \frac{\mathcal{M}(Q, 1)}{Q} \{v(q) - v'(q)g[q, \varepsilon(Q)]\} - Z = 0. \quad (24)$$

Then rewrite (20) as

$$G(q, Q) \equiv v'(q) - 1 - \frac{iQ}{\mathcal{M}(Q, 1)} = 0. \quad (25)$$

Given Z , (q, Q) solves the first order conditions iff it solves (24)-(25). This is a candidate solution to (19), but we need to check the second order conditions.

²²This is standard in competitive search: the Hosios condition holds automatically.

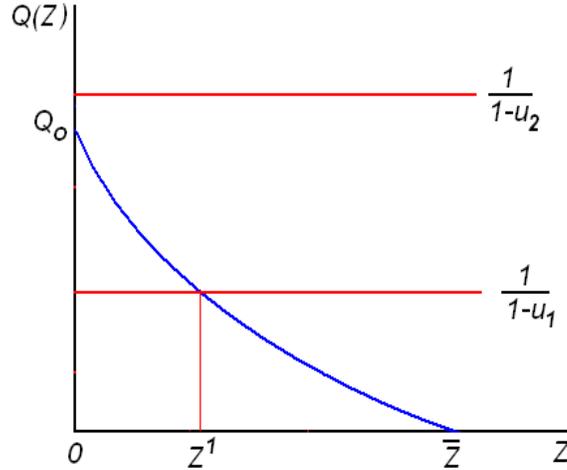
To this end we make the following assumptions.

$$\text{Assumption 1: } \frac{\partial \Phi(q, Q)}{\partial q} \geq 0 \quad (26)$$

$$\text{Assumption 2: } \frac{\partial \varepsilon(Q)}{\partial Q} \leq 0, \quad (27)$$

where $\Phi(q, Q) \equiv v(q) - v'(q)g[q, \varepsilon(Q)]$ is the term in braces in (24). A sufficient condition for (26) is that $\varepsilon(Q)$ is not too small; (27) holds for the matching functions usually used in the literature.

Given (26)-(27), the locus of points in (Q, q) space satisfying (24) is upward sloping. Since the locus satisfying (25) is always downward sloping, there is at most one solution to (24)-(25). Moreover, if a non-zero solution exists, it must be the global maximizer since it yields $\mathcal{M}(Q, 1)[g[q, \varepsilon(Q)] - q] > 0$, and $Q = 0$ or $q = 0$ yields payoff 0. It is easy to check that, at least for i and Z not too big, a non-zero solution (Q, q) does exist. It is also easy to check that $\partial Q/\partial Z < 0$ under (26)-(27). All of this implies we can rule out the complications shown in Figure ??, and the situation is as in Figure ??.



Summarizing, under assumptions (26)-(27), there is a unique solution (q, Q) and it is non-zero iff $Z < \bar{Z}$. Given an unemployment rate $u < u_0$, $Q = 1/(1-u)$,

and q is pinned down by (20). If we increase u , as Figure ?? shows, we reduce Z , which reduces q as one can check by differentiating (24)-(25). This traces out a downward-sloping locus of points in (u, q) space. Intuitively, as u increases the KW queue length Q increases, reducing α_h and q . The only complication is that when we increase u beyond u_0 , there is no $Z > 0$ that equates $Q(Z) = 1/(1-u)$, so in equilibrium $Q = Q_0$ and households get $Z = 0$ in the KW market. For $u > u_0$, increases in u do not change $Q = Q_0$ and hence do not change $q = q_0$.

The LW curve is the value of money q solving (22), shown in Figure 3 for two different levels of i and $i' < i$. From (22), under competitive search with $i = 0$, the LW curve goes through q^* at $u = 0$. This is true under bargaining iff $\theta = 1$, since the denominator on the left hand side of (14) is $g_1(q, \theta)$ while in (22) this does not appear. Also notice that there is a kink in LW at $u = u_0$ and $q = \psi(u_0)$, where we get $\psi(\cdot)$ by solving (24) with $Z = 0$, or $v(q_0) = v'(q_0)g[q_0, \varepsilon(Q_0)]$. Solving this for q_0 and inserting $Q_0 = 1/(1 - u_0)$ implies $q_0 = \psi(u_0)$, with $\psi' \geq 0$ by (27).²³ Therefore, the LW curve here is qualitatively similar to what we derived under bargaining, but slightly simpler because $g_1(q, \theta)$ does not appear, and also slightly more complicated because of the kink.

The MP curve also needs to be modified, as follows. For $u < u_0$ we have

$$k = \frac{\mathcal{N}[u, v(u)]}{v(u)} \frac{\eta \left\{ y - z + \frac{\mathcal{M}(1, 1-u)}{1-u} \frac{(1-\varepsilon)[u(q)-q]}{\varepsilon u'(q) + 1 - \varepsilon} \right\}}{r + \delta + (1 - \eta) \frac{\mathcal{N}[u, v(u)]}{u}}, \quad (28)$$

which is identical to (17), except that we replace bargaining power θ with the elasticity $\varepsilon = \varepsilon(Q) = \varepsilon(\frac{1}{1-u})$. For $u > u_0$, the result is the same except we replace $\frac{\mathcal{M}(1, 1-u)}{1-u}$ with $\mathcal{M}(Q_0, 1)$ and $\varepsilon = \varepsilon(Q_0)$. Hence, the MP curve is still downward sloping, but now has a kink at u_0 , because R is independent of u for $u > u_0$. Notice that in Figure 3, at the low interest rate i' equilibrium occurs at $u < u_0$, while at the higher interest rate i it occurs at $u > u_0$. Despite the small

²³For a Cobb-Douglas matching function $\mathcal{M}(B, S)$, q_0 is actually independent of u_0 – i.e. $\psi(u_0)$ is horizontal.

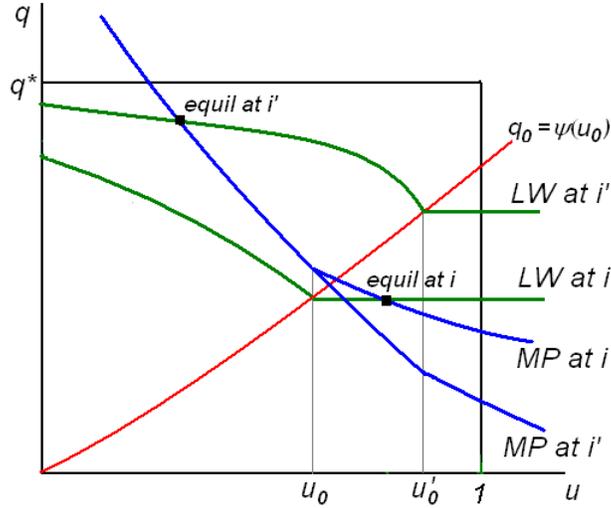


Figure 3: LW and MP with Competitive Search

modifications, all the properties of the model in terms of existence, uniqueness vs. multiplicity, the results of parameter changes, and so on are qualitatively the same as those derived using bargaining.

5 Efficiency

Consider a planner who seeks to maximize the welfare (expected utility) of the representative household, subject to several constraints. First, output y is produced in employment relations that form in the first subperiod, subject to the law of motion for u . Second, if firms and households meet in the second subperiod, which occurs according to the technology $\mathcal{M}(\cdot)$, the former can transfer q to the latter for a payoff of $v(q)$, and bring the remainder to the third subperiod, where it can be allocated to any household for a payoff of $y - q$. Each period, the planner takes unemployment u as a state variable, and chooses how many vacancies v to post, plus the transfer q for meetings in the the second subperiod. He also chooses how to allocate remaining output in the

third subperiod, but because of quasi-linear utility, this does not effect average welfare.²⁴

To reduce the notation, let $s(q) \equiv v(q) - q$, and set the value of leisure and home production to $\ell = 0$. Then we have the following dynamic programming problem:

$$J(u) = \max_{q,v} \left\{ -vk + \mathcal{M}(1, 1-u) s(q) + (1-u)y + \beta \tilde{J}(\hat{u}) \right\}$$

$$st \hat{u} = u + (1-u)\delta - \mathcal{N}(u, v)$$

The instantaneous return subtracts the vacancy creation cost vk from flow utility, computed as follows. There are $1-u$ firms with output \cdot . Each such firm meets a household with probability $\mathcal{M}\left(\frac{1}{1-u}, 1\right)$, which yields utility $v(q)+y-q$, and does not meet a household with probability $1 - \mathcal{M}\left(\frac{1}{1-u}, 1\right)$, which yields utility y . The rest is algebra. It is not hard to show this is a well-behaved problem using standard methods, and in particular J is concave.

The FOC for q is $s'(q) = 0$, which means that $q = q^*$ at every date. The FOC for v is $-k - \beta \tilde{J}'(\hat{u}) \mathcal{N}_2(u, v) = 0$. This together with the law of motion $\hat{u} = u + (1-u)\delta - \mathcal{N}(u, v)$ generates a decision rule for v as a function of u , determining the path for (u, v) .²⁵ To characterize the steady state of this dynamic system, first take the envelope condition

$$J'(u) = -\mathcal{M}_2(1, 1-u) s(q) - y + \beta \tilde{J}'(\hat{u}) [1 - \delta - \mathcal{N}_1(u, v)].$$

Then use the FOC for v to eliminate $J'(u)$ and $\tilde{J}'(\hat{u})$ to get

$$\frac{k}{\beta \mathcal{N}_2(u, v)} = y + \mathcal{M}_2(1, 1-\hat{u}) s(\hat{q}) + \frac{k [1 - \delta - \mathcal{N}_1(\hat{u}, \hat{v})]}{\mathcal{N}_2(\hat{u}, \hat{v})}.$$

²⁴To ease the presentation we assume there are no goods $\bar{\mathbf{x}}$ in the third subperiod other than \mathbf{x} , but if there were, the allocation would satisfy the obvious additional marginal conditions.

²⁵This model is more intricate than the standard MP model, where v/u is independent of u . This is not true here because u enters $\mathcal{M}(1, 1-u)$, which implies more interesting dynamics (for both the planner problem and for equilibrium). In the interest of space, we relegate dynamic analysis to a companion paper.

Hence, the steady state unemployment rate solves

$$k = \frac{\mathcal{N}_2 [u^*, v(u^*)] [y + \mathcal{M}_2 (1, 1 - u^*) s(q^*)]}{r + \delta + \mathcal{N}_1 [u^*, v(u^*)]}, \quad (29)$$

where we inserted $v = v(u)$ as the solution to $(1 - u)\delta = \mathcal{N}(u, v)$.

We want to compare this to the outcome that emerges in competitive search equilibrium, as described by the MP and LW curves derived the previous section.²⁶ To facilitate the comparison, we rewrite the MP curve (28) after inserting the elasticity $\sigma \equiv v\mathcal{N}_2/\mathcal{N}$ (which generally depends on u and v) and simplifying $\mathcal{M}[g(q, \varepsilon) - c(q)] = \frac{\mathcal{M}_2 s(q)}{\varepsilon s'(q) + 1 - \varepsilon}$:

$$k = \frac{\frac{\eta}{\sigma} \mathcal{N}_2 [u, v(u)] \left[y - B + \mathcal{M}_2 (1, 1 - u) \frac{s(q)}{\varepsilon s'(q) + 1 - \varepsilon} \right]}{r + \delta + \frac{1 - \eta}{1 - \sigma} \mathcal{N}_1 [u, v(u)]} \quad (30)$$

We also assume for now that $u < u_0$ (we are to the left of the kink), and we reintroduce $B = b/(1 - \tau)$, since although $\ell = 0$ in the planner problem, the equilibrium is still impacted by UI and taxes.

First observe from the LW curve that with competitive search we get $q = q^*$ iff $i = 0$. Then, given q^* , we can compare (29) and (30) to see when we get $u = u^*$. If the Hosios condition $\eta = \sigma$ holds, then $u = u^*$ exactly when $B = 0$ – as is standard, this condition implies the labor market is efficient without intervention. If $\eta \neq \sigma$, however, continuing to assume $i = 0$ so that $q = q^*$, we

²⁶We can also consider the hybrid problem where v is chosen directly but q is determined as a monetary equilibrium in the KW market. One interpretation is that actions are observable in the first and third but not in the second subperiod. Then a firm that is supposed to transfer q in second-subperiod meetings could renege and keep q for itself (i.e. its owners) to enjoy in the third subperiod without fear of punishment. In this scenario money becomes essential – heuristically, to a firm that only brings $y - q$ to the table in the third subperiod, claiming to have transferred q in the previous subperiod, a planner can say, “show me the money.” Since we can achieve any $q \in (0, q^*)$ as a competitive search equilibrium by varying $i \in (0, \infty)$, and i does not directly affect any variable other than q , we can achieve the same set of outcomes as choosing v and q directly. This indicates that any distortions we get in this section are the result of either labor market distortions that imply entry is inefficient, or constraints on policy. That would not be true if we assumed bargaining equilibrium, as then the KW market has its own well-known inefficiencies (holdup problems). This is one reason for focusing on competitive search here.

get $u = u^*$ iff $B = B^*$ where

$$B^* = [y + \mathcal{M}_2(1, 1 - u^*)s(q^*)] \left[1 - \frac{\sigma r + \delta + \frac{1-\eta}{1-\sigma} \mathcal{N}_1(u^*, v^*)}{\eta r + \delta + \mathcal{N}_1(u^*, v^*)} \right], \quad (31)$$

where we write $v^* = v(u^*)$. Notice $B^* > 0$ iff $\eta > \sigma$. We conclude that when policy is unconstrained, we can always achieve efficiency by running the Friedman rule to get q^* and then setting labor market policy to get u^* .²⁷

Suppose now that there is a restriction, for whatever reason, that $B = \hat{B}$ where $\hat{B} < B^*$. Then we claim that monetary policy should compensate with $i > 0$. To verify this, notice that if $i > 0$ is small then the LW curve implies there is only a second order welfare loss (the envelope theorem), but the MP curve implies a first order gain through the term $\frac{s(q)}{\varepsilon s'(q) + 1 - \varepsilon}$, which is strictly increasing at $q = q^*$. This shows that when labor market policy is constrained to $B < B^*$, monetary policy should compensate with inflation above the Friedman rule. The intuition is clear: $B < B^*$ implies excessive entry; given we are restricted in terms of directly taxing market or subsidizing non-market activity, the next best alternative is to inflate and tax market activity indirectly. Although this generates a welfare loss in terms of q , the net gain due to the reduction in entry is unambiguously positive.

For completeness, we also mention what happens in the situation where we are constrained to set $i = \hat{i} > 0$. Although this cannot achieve q^* , one can ask how equilibrium does compared to the second-best solution, say (q^i, u^i) . It turns out that, after some algebra, we get the second-best $u = u^i$ iff $B = B^i$

²⁷There is some indeterminacy, of course, in the sense that any combination of b and τ that generates $b/(1 - \tau) = B^*$ does the job; i.e. when $\eta \neq \sigma$, and entry is distorted, there is generally more than one combination of the tax on market activity τ and subsidy to nonmarket activity b that restores efficiency.

where

$$B^i = [y + \mathcal{M}_2(1, 1 - u^i)s(q^i)] \left[1 - \frac{\sigma}{\eta} \frac{r + \delta + \frac{1-\eta}{1-\sigma}\mathcal{N}_1(u^i, v^i)}{r + \delta + \mathcal{N}_1(u^i, v^i)} \right] \\ + \mathcal{M}_2 s'(q^i) \left[\frac{\varepsilon s(q^i)}{\varepsilon s'(q^i) + 1} - \frac{\sigma}{\eta} \frac{r + \delta + \frac{1-\eta}{1-\sigma}\mathcal{N}_1(u^i, v^i)}{r + \delta + \mathcal{N}_1(u^i, v^i)} \right].$$

Notice that if $q^i = q^*$ then we are back to $B^i = B^*$. More generally, one can check $B^i > 0$ iff $\eta > \eta^i$ where η^i solves a simple formula, and $\eta^i > \sigma$. Intuitively, at $i = 0$ we showed earlier that we should set $B > 0$ to reduce entry iff $\eta > \sigma$; now, when we are forced to set $i > 0$, there is a higher threshold for η before we set $B > 0$, since inflation already constitutes a tax on firms.

To summarize, we have the following results.

Proposition 4 *Given no constraints on policy, the optimum is $i = 0$ and $B = B^*$, where $B^* \geq 0$ as $\eta \geq \sigma$. Given no constraints on i and a restriction $B \leq \hat{B} < B^*$ the optimal constrained policy is $i > 0$ and $B = \hat{B}$. Given no constraints on B and a restriction $i \geq \hat{i} > 0$ the optimal constrained policy is $i = \hat{i}$ and $B = B^i$ where $B^i > 0$ iff $\eta > \eta^i$ and $\eta^i > \sigma$.*

We emphasize that in the first case the equilibrium coincides with the solution to the planner problem ; intuitively, monetary policy i is enough to deal with q and labor market policy b and τ are more than enough to deal with u . In the second case u and q are both below the solution to the planner problem, since labor market policy is not allowed to deal with excessive entry and monetary policy compensates with inflation above the Friedman rule. In the third case monetary policy is not set right, which impacts on labor market policy. We think these policy spillovers are interesting and worth further exploration.

6 Calibration

In terms of functional forms, we use the standard utility function $u(q) = Aq^{1-\alpha}/(1-\alpha)$. Following most of the labor search literature, we assume a

Cobb-Douglas matching function in the MP market $N(u, v) = Zu^\sigma v^{1-\sigma}$ (which is actually a good local approximation to any CRS matching function). We assume that in the KW market matching frictions arise exclusively from the coordination friction, which endogenously generates an urn ball matching function $M(B, S) = B(1 - e^{-S/B})$ (see e.g. Burdett, Shi and Wright 2001). In addition to the parameters in the above functions (A, α, Z, σ) , we have to determine two household additional preference parameters (β, b) , three technology parameters (y, δ, k) , and the firm's bargaining power in the MP market η . Without loss of generality, we can set $y = 1$ (this amounts to choosing units for output) and $Z = 1$ (this amounts to choosing units for vacancies). This leaves 8 parameters.

Following Shimer (2005), we set the elasticity $1 - \sigma$ of $N(u, v)$ with respect to v equal to the coefficient from the regression of the detrended job-finding rate on the detrended v/u ratio. In addition, we target the sample average of the monthly job-finding rate 0.37 and the unemployment rate 0.057. We target the average of M/PY , and the correlation between $\log(M/PY)$ and $\log(i)$, after purging high-frequency movements from the data using the HP filter; some version of this procedure, which basically tries to fit the level and curvature of the money demand curve, is used in monetary models from Cooley and Hansen (1989) to Aruoba et al. (2006). We find a correlation coefficient ranging between -0.4 and -0.7 depending on the choice of sample. In our benchmark calibration, we target a model correlation of -0.5 . Finally, we target an annual real interest rate of 3.5 percentage.

The targets described above leave two free parameters: the value of leisure b and the firm's bargaining power η . It is well known that these two parameters are hard to calibrate. Shimer (2005) interprets b as unemployment insurance and targets a replacement rate b/w of $1/2$. He chooses the firm's bargaining power η to equal the elasticity of the matching function with respect to vacancies. In the basic Mortensen-Pissarides model, this guarantees that the entry of firms

in the labor market is efficient. Hagedorn and Manovskii (2006) interpret b as the sum of unemployment benefits, the value of leisure, and home production. They use empirical evidence to on recruitment costs and the cyclical volatility of wages to identify b and η . In their preferred calibration, they set $b = 0.955$ and $\eta = 0.94$. Given these differences of opinion, we choose to remain agnostic about b and η and report our findings for a range of parameter values.

Here is the way we implement our calibration procedure in more detail. Let $\rho = b/w$ denote the replacement rate, and $\mu = R/w$ the gross mark-up. Pick values for ρ and μ .

(1) Average market tightness v/u is such that the steady-state job finding rate associated with $i = 7.2$ percent matches the sample mean

$$\left(\frac{v}{u}\right)^{1-\sigma} = \lambda_h = 0.37.$$

(2) The job destruction rate δ is such that the steady-state unemployment rate associated with $i = 7.2$ percent matches the sample mean

$$\frac{\delta}{\delta + \lambda_h} = u = 0.057.$$

(3) Firm bargaining power η is such that the average replacement ratio is equal to ρ

$$\eta = \frac{(\mu - 1)(1 - \beta(1 - \delta - \lambda_h))}{\mu(1 - \beta(1 - \delta - \lambda_h)) - \rho(1 - \beta(1 - \delta)) - \beta\lambda_h}.$$

(4) Conjecture a value for the coefficient α of the utility function.

(5) The coefficient A in the utility function is such that the average monthly money demand matches the sample mean

$$\frac{g(q, \varepsilon)}{(1/3)M(1, 1 - u)[g(q, \varepsilon) - c(q)] + (1 - u)y} = \frac{M}{PY} = \frac{4}{5}.$$

(6) Compute k from the free entry condition

$$k = \frac{\beta\eta N(\frac{u}{v}, 1) \{y + M(1/(1 - u), 1)[g(q, \varepsilon) - c(q)]\} \left(\frac{\mu - \rho}{\mu}\right)}{1 - \beta(1 - \delta) + (1 - \eta)\beta N(1, v/u)}.$$

(7) Simulate the model and compute the correlation between money demand and interest rate.

(8) Repeat steps (4)-(7) for different values of α to minimize the distance between the simulated and empirical correlation between M/PY and i .

7 Quantitative Results

To be added.

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