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Bankruptcy: Is It Enough to Forgive or Must We Also Forget?*

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Abstract

In many countries, lenders are not permitted to use information about past defaults after a specified period of time has elapsed. We model this provision and determine conditions under which it is optimal.

We develop a model in which entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. We show that forgetting a default makes incentives worse, ex-ante, because it reduces the punishment for failure. However, following a default it is generally good to forget, because pooling riskier agents with safer ones makes exerting high effort to preserve their reputation more attractive.

Our key result is that if agents are sufficiently patient, and low effort is not too inefficient, then the optimal law would prescribe some amount of forgetting — that is, it would not permit lenders to fully utilize past information. We also show that such a law must be enforced by the government — no lender would willingly agree to forget. Finally, we also use our model to examine the policy debate that arose during the adoption of these rules.

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I Introduction

In studying the "fresh start" provisions of personal bankruptcy law, economists typically focus on the *forgiveness* of debts. However, another important feature is the *forgetting* of past defaults. In many countries, lenders are not permitted to use information about past defaults after a specified period of time has elapsed; for example, in the United States information about a bankruptcy cannot be used after 10 years.

In this paper we model this provision and determine conditions under which it is optimal. We develop a model in which entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. In this model, reputation effects encourage agents to exert high effort; however, it is typically the case that reputation is not efficacious until agents have accumulated a long enough credit history to make default unattractive.

We show that in our model forgetting a default makes incentives worse, ex-ante, because it reduces the punishment from failure; as a result, it delays the salutary impact of reputation. On the other hand, however, following a default it can be good to forget, because pooling riskier agents with safer ones makes exerting high effort to preserve their (undeservedly good) reputation more attractive. Our key result is that if agents are sufficiently patient, and low effort is not too inefficient, then the optimal law would prescribe some amount of forgetting — that is, it would not permit lenders to fully utilize past information. We also argue that this law must be enforced by the government — no lender would willingly agree to forget.

The focus of this paper is the effect of laws governing bankruptcy on investment. We thus have in mind a world of small entrepreneurs who finance their business ventures with loans for which they are personally liable. Data from the 1993 National Survey of Small Business Finance (NSSBF) suggests that the majority of small businesses do indeed finance themselves with some sort of personal loan or guarantee; see also Berger and Udell (1995). These entrepreneurs are also three times as likely to file for personal bankruptcy as their counterparts in the general population — see Sullivan, Warren, and Westbrook (1989). In such a setting we are then naturally led to explore the incentive effects of these laws; this seems to be where the greatest economic impact should be found.[1] An alternative approach, however, might be to focus on the risk-sharing and redistributive impact of these laws on

[1]Indeed, the debate surrounding the enactment of the Fair Credit Reporting Act (see U.S. House, 1970) also refers to the impact of these laws on small businesses.

consumers.

In the United States, the Fair Credit Reporting Act (FCRA) prescribes that a personal bankruptcy filing may be reported by credit bureaus for up to ten years, after which it must be removed from the records made available to lenders (other derogatory information can be reported for a maximum of seven years). Similar provisions exist in many other countries — in the European Union the median time that a bankruptcy can stay on records is even shorter — only six years (see Jentzsch 2006, for a summary of international regulations). Another feature of some European laws is that they sometimes restrict disclosure to only *negative* information (such as defaults), rather than "positive" (such as the account balance); this is studied by Padilla and Pagano (2000). It should also be noted that information release is also restricted after a certain time in other contexts, such as motor vehicle records (Lemaire) and juvenile delinquency records (Funk, 1996).

Musto (2004) finds that these restrictions are binding. He shows that for those bankrupts who are more creditworthy, access to credit increases significantly when the bankruptcy "flag" is dropped from their credit file (after 10 years); for the less creditworthy bankrupts dropping the flag has little impact, because they have many other derogatory indicators in their file. He also finds that those individuals who obtain new credit are subsequently likelier than average to default on this new credit; he interprets this as evidence that these laws are suboptimal. Our model will be consistent with his findings, although we will argue that it need not be evidence of inefficiency.

In the Congressional debate surrounding the adoption of the FCRA (U.S. House, 1970 and U.S. Senate, 1969), the following arguments were put forward in favor of forgetting past defaults (or other negative information): (1) limited computer storage capacity, (2) old information might be less reliable or salient, and (3) if information was not erased the stigmatized individual would not obtain a "fresh start" and so would be unable to continue as a productive member of society. On the other hand, the arguments raised against forgetting this information were (1) it forces honest borrowers to subsidize the dishonest ones, (2) it discourages borrowers from repaying their debts by reducing the penalty of failure,[2] (3) it could lead to a tightening of credit policies (which would affect the worst risks disproportionately), and finally, (4) that it increases the chance of costly fraud or other crimes by making it harder to identify (and exclude) seriously bad risks.[3]

[2] See also Staten (1993), who ties the increase in bankruptcy rates to the increasing availability of post-bankruptcy credit.

[3] Some of these arguments have also appeared in the criminology literature, particularly

The paper is organized as follows. In the next section we briefly discuss the relevant literature. In section II we present the model and characterize the equilibrium; we then verify our equilibrium in section B. In section IV we present our main results — the conditions under which forgetting a default can be optimal — and relate them to the empirical evidence and the policy debate surrounding the adoption of the FCRA. Section V provides examples that illustrate these results.

A Previous Literature

There is, of course, a large literature on the potential inefficiency of public revealing information, dating back at least to Hirshleifer (1971). Hirshleifer argues that revealing information may be socially inefficient, because it can preclude insurance opportunities.[4] Our paper is closer to another strand of the literature, one that examines the effect that revealing information can have on the punishments a principal can impose on an agent. Crémer (1995) shows that using an inefficient monitoring technology can sometimes improve incentives because by monitoring less efficiently a principal ensures that he will have less information about a default and can thereby commit to punish more severely (because he will not have enough information to renegotiate punishments).[5] In a context closer to ours, Vercammen (1995) presents an example[6] in which a socially optimal policy involves restricting the memory of a credit bureau; in his example the benefit comes from forgetting past *successes*, since if an agent has many successes — and so is believed to be good with high probability — then he will have little incentive to exert high effort because the current period's outcome will have little impact on his reputation.[7]

Our paper is different, in that for us forgetting actually weakens the punishment incurred by a defaulter; as such it generally makes incentives worse, ex-ante. However, ex-post having a weaker punishment can be better and the optimal policy trades off these two effects. This tradeoff between current and future incentives is the key focus of our paper. We believe that such a model is more appropriate as a description of a credit market in that incentives are worst at the beginning of an agent's life, rather than

in the case of juvenile delinquency records; see Funk (1996).

[4] See also Marin and Rahi (2000).

[5] The idea that restricting renegotiation can be beneficial has also been used to motivate corporate bankruptcy laws — see Berkovitch, Israel, and Zender (1998).

[6] Although he does not fully analyze his model.

[7] In this sense his model resembles that of Holmstrom (1982).

at the end; it is well known, for example that older borrowers, as well as those with longer credit histories, are less likely to default (see Elul and Subramanian, 2002, and Fair Isaac and Co., 2003).

Another related literature is that examining the outcomes when lenders can punish defaulting agents by varying degrees of exclusion from financial markets. For example, Kehoe and Levine (1993) study the outcomes in a model in which repayment is induced by the threat of exclusion from all future credit markets. More recent work examines the optimality of this assumption; for example, Bond and Krishnamurthy (2004) develop a model in which an optimal law would only exclude defaulters until they repay their original debt.

There is also an interesting literature that studies the impact of the introduction of a credit bureau on borrower and lender behavior; see Brown and Zehnder (2006) for an experimental study and de Janvry, McIntosh, and Zadoulet (2006) for an empirical examination of the Guatemalan case.

Finally, our basic model shares many aspects with other models of reputation, such as Diamond (1989), Mailath and Samuelson (2001), and Fishman and Rob (2005). In these models, principals and agents also interact repeatedly under conditions of both adverse selection and moral hazard. The equilibrium in our model shares many similarities with the ones in these papers; in particular, the outcome is characterized by a period of low effort followed by one in which risky agents exert high effort. More to the point, however, our goals are of course quite different; we are interested in analyzing the effect of government intervention, an issue that is not addressed in the previous literature.

II The Model

Consider an economy made up of a continuum (of unit mass) of risk-neutral *entrepreneurs*, who live forever and discount the future at the rate $\beta \leq 1$. In each period $t = 0, 1, \ldots$ an entrepreneur receives a new project, which requires one unit of financing in order to be undertaken. This project yields either R (success) or 0 (failure). Output is non-storable, so entrepreneurs must seek external financing in each period.

We assume that there are two types of entrepreneurs. There is a set of measure $p_0 \in (0, 1)$ of *riskless* agents, whose projects always succeed (i.e., their return is R with probability one), and a set of *risky* agents, with measure $1 - p_0$, for whom the project may fail with some positive probability.[8] The returns on the risky agents' projects are independently and identically distributed among them. The success probability of a risky agent depends on his effort choice. He may choose to exert high effort (h), at a cost $c(h) \equiv c > 0$ (in units of the consumption good), in which case the success probability will be $\pi_h \in (0,1)$. Alternatively, he may choose to exert low effort. Low effort (l) is costless ($c(l) = 0$), but the success probability under low effort is only $\pi_l \in (0, \pi_h)$.

We assume:

Assumption 1. $\pi_h R - 1 > c$, $\pi_l R < 1$;

i.e., the project has a positive NPV if high effort is exerted (even when the cost of exerting high effort is taken into account), while it has a negative NPV under low effort.

In addition, we require the cost of effort c to be sufficiently high, which will imply that entrepreneurs face a nontrivial incentive problem. More specifically, as we will see, this condition implies that high effort cannot be implemented in the absence of reputational incentives (e.g. in a static framework) when the entrepreneur is known for certain to be risky.[9]

Assumption 2. $\frac{c}{\pi_h - \pi_l} > R - 1/\pi_h$

Finally, we introduce one further parameter restriction, requiring that π_h and π_l not be too far apart. This condition will be used only in some parts of the analysis below (and then only to ensure that a stronger notion of equilibrium obtains):

Assumption 3. $\pi_h^2 \leq \pi_l$

In addition to entrepreneurs, there are lenders, who provide external funding to entrepreneurs in the loan market. More specifically, we assume that in each period there are N lenders (where N should be thought as large) who compete among themselves on the terms of the contracts offered to borrowers with the objective of maximizing expected profits. Each lender lives only a single period, and is replaced by a new lender in the following period.

[8] We discuss the role that this assumption plays below.

[9] Assumption 2 is weaker than the analogous restriction in Diamond (1989), which is that $\frac{c}{\pi_h - \pi_l} > R - 1$. The latter implies that high effort cannot be implemented in a static setup even when the entrepreneur is believed by lenders to be a safe type with arbitrarily high probability.

Since lenders live only a single period, it is immediate that they cannot write long-term contracts. This seems broadly consistent with actual practice in U.S. credit markets, where borrowers often switch between lenders. Moreover, it would be difficult to sensibly model "forgetting" in the presence of long-term relationships with a single lender.[10]

A contract is then simply described by the interest rate r at which an entrepreneur is offered one unit of financing at the beginning of a period (if the entrepreneur is not offered financing in this period then we set $r = \emptyset$). If the project succeeds, the entrepreneur makes the required interest payment r to the lender. On the other hand, if the project fails, the entrepreneur is unable to make any payment and we assume that the debt that was incurred is forgiven - or discharged. Since borrowers have no funds to repay lenders other than the proceeds from their project in this period, with no loss of generality r can be taken to lie in $[0, R] \cup \emptyset$.

We assume that an entrepreneur's type, as well as the effort he undertakes, is his private information. The loan market is hence characterized both by the presence of adverse selection and moral hazard. Since under Assumption 1 it is only profitable to lend to a risky agent if he exerts high effort, this creates an incentive problem: a risky entrepreneur may in fact prefer to exert low effort even though the total surplus in that case is lower (indeed negative).

At the same time, in a dynamic framework such as the one we consider, the history of past financing decisions and past outcomes of the projects of an agent may convey some information regarding the agent's type. Let σ_t^i denote the *credit history* of agent $i \in [0, 1]$ at date t, describing for each previous period $\tau < t$ whether the agent's project was funded and if so, whether it succeeded or failed. Hence, denoting by S a success, F a failure, and $\emptyset$ the event where the project is not funded (either because the agent is not offered financing or because he does not accept any offers), σ_t^i is given by a sequence of elements out of $\{S, F, \emptyset\}$: $\sigma_t^i \in \Sigma_t \equiv \{S, F, \emptyset\}^t$. Since lenders do not live beyond the current period, we assume that there is a *credit bureau* that records this information in every period and makes it available to future lenders.

As discussed earlier, the focus of our paper is the effect of restrictions on the transmission of credit bureau records. We model the *forgetting policy* in this economy as follows: when an entrepreneur's project fails, with probability q the credit bureau ignores the failure and updates the entrepreneur's

[10] As argued below, in the set-up considered, allowing for long term contracts would lead to rather extreme properties of the contracts and the equilibrium outcome.

record as if his project succeeded in that period. That is, S now represents either a success or a failure that is forgotten, and F represents a failure that has not been forgotten. The parameter $q \in [0, 1]$ then describes the forgetting policy in the economy. Note that we take q as being fixed over time, which is in line with existing laws.

We adopt this representation of forgetting to make the analysis simpler, though it is somewhat different from existing institutions.[11] As we have discussed above, in the United States the Fair Credit Reporting Act (FCRA) prescribes that a personal bankruptcy filing may be reported by credit bureaus for up to 10 years, after which it must be removed from records made available to lenders (other derogatory information can be reported for a maximum of seven years). Note that, as in our model, U.S. law only proscribes reporting negative information. We intend to argue however that the effects on borrowers' incentives and access to credit are similar; in particular, that the consequences of higher values of q are analogous to those of allowing for a shorter period until negative information is forgotten. This is indeed exactly so for the polar cases of $q = 0$, which implies that a failure is remembered forever, and $q = 1$, which is equivalent to forgetting immediately, i.e., not keeping any record of failures.

We show below that only pooling equilibria can exist in this economy; that is, lenders are unable to separate borrowers by offering a menu of contracts to entrepreneurs with the same credit history. Note, however, that they may optimally choose to differentiate the terms of the contracts offered on the basis of entrepreneurs' credit histories. Hence, without loss of generality we can focus our attention on the case where a lender offers a single contract $r(\sigma_t)$ to borrowers with a given credit history σ_t. We let $\mathcal{C}(\sigma_t)$ denote the set of contracts offered at date t by the N lenders to entrepreneurs with credit history σ_t, and let $\mathbb{C}_t \equiv \cup_{t,\sigma_t \in \Sigma_t} \mathcal{C}(\sigma_t)$ be the set of contracts offered by lenders for any possible history up to date t.

Another important feature to be specified is the information available to lenders regarding the contracts offered and adopted in the past. We assume that while lenders present at date t know $\mathbb{C}_t$, i.e., the set of contracts which were *offered* to borrowers in the past, they do not know the particular contracts which were *chosen* by an individual borrower. This in line with actual practice; while credit bureaus do not report the actual contracts adopted by individual borrowers, the set of contracts available is in fact provided by databases such as Comperemedia®.

The timeline of a single period is then as follows. Each entrepreneur

[11] A similar approach is also taken by Padilla and Pagano (2000).

must obtain a loan of 1 unit in the market in order to undertake his project. Lenders simultaneously post the rate at which they are willing to lend in this period to an agent with a given credit history, and do this for all possible credit histories at that date. If an entrepreneur is offered financing, and if he chooses to be financed, he undertakes the project (funds lent cannot be diverted to consumption), and if he is risky he also chooses his effort level. The outcome of the project is then realized: if the project succeeds the entrepreneur uses the revenue R of the project to make the required payment r to the lender, while if the project fails the entrepreneur defaults and makes no payment (since his default is forgiven). Note that — purely for convenience — we assume that entrepreneurs repay at the end of the same period in which they borrow.

The credit history of the entrepreneur is then updated by adding a $\emptyset$ to the sequence if the project was not financed and, if it was financed, S if the project succeeded in the period or, with probability q, if it failed, and an F otherwise. This timeline is illustrated in the following figure for the case of high and low effort (when $q = 0$).

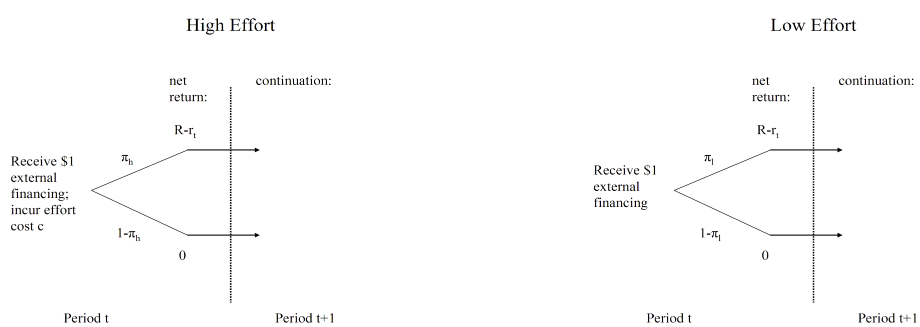

Figure 1: Timeline: $q = 0$

Next period, the same sequence is repeated: for each updated credit history, lenders choose the contracts they will offer, each entrepreneur then freely chooses the best contract among the ones he is offered,[12] and so on

[12]We assume therefore that entrepreneurs are unable to commit to any future choice of

for every t.

Since the updated credit history may affect the contracts the agent will get in the future, and hence his future expected utility, and since for a risky agent such history is, at least partly, affected by his current effort choice, this will affect the agent's incentives to choose high versus low effort. In particular, the agent may care for having a good credit history (i.e., a good reputation), as this might improve his future funding prospects, and this may strengthen the agent's incentives with respect to the case of a static contracting problem. Under assumption 2, as we will show in what follows, incentives may be sufficiently poor that we need reputational effects to elicit high effort (and as a result financing cannot take place at all nodes).

To summarize, a lender's strategy consists in the choice of the contracts to offer to entrepreneurs at any given date, for any possible credit history. The strategy of an entrepreneur then describes, in every period and for every possible credit history, the choice of the contract among the ones he is offered and, if the entrepreneur is risky, also his choice of effort. To evaluate the expected profit of a loan offered to an entrepreneur with credit history σ_t an important role is played by the lender's belief, $p(\sigma_t)$, that the entrepreneur is a risky type. At the initial date such belief is given by the prior probability p_0; the belief is then updated over time on the basis of the knowledge of the credit history σ_t as well as of the contracts $\mathbb{C}_t$ offered up to such date, and of the entrepreneurs' borrowing and effort strategies. We will often refer to $p(\sigma_t)$ as an entrepreneur's *credit score*.[13]

III Characterization of the Equilibria

A Characterization

In what follows we will restrict attention to *Markov Perfect Equilibria* (MPE) in which players' strategies depend on past events only through *credit scores* — i.e., through the belief p, entertained by lenders, that an entrepreneur with a given credit history is of the safe type. A key appeal of such equilibria is not only that players' strategies are simpler, but also that they resemble actual practice in consumer credit markets, where many lending decisions are conditioned on credit scores, most notably the "FICO score" developed

contract.

[13] We will sometimes drop the reference to the borrower's credit history and refer simply to p.

by Fair Isaac and Company. In addition, we will discuss below the differences between MPE and other equilibria and argue that the latter exhibit properties of players' behavior that we consider less plausible.[14]

We can now describe players' strategies more formally for the Markov Perfect Equilibria that we consider. Let $r^n(p) \in [0, R] \cup \emptyset$ denote the strategy of lender n, i.e., the contract offered (or, when $r = \emptyset$, the fact that no contract is offered) to entrepreneurs with credit score p; then let $\mathcal{C}(\cdot)$ denote the collection of the strategies of all lenders.

The strategy of an entrepreneur, whatever his type, consists in the choice, for every credit score p he may have, and given that he is offered a set of contracts $\mathcal{C}'$, whether or not to accept any of the loan contracts offered, and if so, which one to accept. In addition, a risky entrepreneur has to choose the effort level he exerts. We will allow for mixed strategies with regard to effort and hence denote the effort level by $e \in [0, 1]$, where e signifies the probability with which the entrepreneur exerts high effort.[15] Thus $e = 1$ corresponds to a pure strategy of high (h) effort, $e = 0$ to a pure strategy of low (l) effort, and $e = 1/2$ corresponds to mixing between high and low effort with equal probability. The entrepreneur's choices are based on the evaluation of both his immediate payoff, which depends on the level of the interest rate on the contracts presently offered to him and his effort choice, as well as on his anticipation of the contracts he will be offered in the future conditional on the outcome of his project, which in turn depend on the lenders' strategies and on how they update their beliefs concerning the agent's type in light of the outcome of the current project.

In particular, we will establish the existence and analyze the properties of *symmetric, sequential MPE*, where all agents of a given type (i.e., all lenders, or all safe entrepreneurs, or all risky entrepreneurs with the same credit score) optimally choose the same strategies. Let $p^S(p, \mathcal{C}')$ specify how lenders update their beliefs in case of success (or forgotten failure) of the project of a borrower with credit score p and facing current contracts $\mathcal{C}'$. Analogously, $p^F(p, \mathcal{C}')$ denotes the updated belief in case of a failure (which is not forgotten) and $p^{\emptyset}(p, \mathcal{C}')$ when the entrepreneur is not financed.[16] The updated beliefs will be computed according to Bayes' rule whenever possible; when this is not possible they will be required to be *consistent* in the Sequential Perfect Equilibrium sense.

[14] Restricting attention to MPE to rule out implausible equilibria is common in the analysis of reputation games; see Mailath and Samuelson (2001), for example.

[15] This is the only form of mixed strategies that we allow; we demonstrate (see below) that mixing only occurs for at most a single period along the equilibrium path.

[16] We will sometimes omit the arguments and write simply $p^S, p^F, p^{\emptyset}$.

Observation 1. Since only risky agents can fail, we must have $p^F(p, \mathcal{C}') = 0$ for any p and $\mathcal{C}'$.

Furthermore, when entrepreneurs are not offered any loan, $\mathcal{C}' = \emptyset$, and hence are not financed, it is immediate to verify that beliefs remain unchanged: $p^\emptyset(p, \emptyset) = p$ for all p.

We are now ready to write the formal choice problem for the entrepreneurs. For this it is convenient to use $v^r(p, \mathcal{C}')$ to denote the maximal discounted expected utility that a risky entrepreneur with credit score p, facing a set of contracts $\mathcal{C}'$, can get, given the lenders' updating rules $p^S(\cdot), p^F(\cdot), p^\emptyset(\cdot)$ and their strategies $\mathcal{C}(\cdot)$, determining future offers of contracts (to simplify the notation we do not make the dependence of v^r on these terms explicit). Observe that $v^r(\cdot)$ is recursively defined as the solution to the following problem:

$$v^r(p, \mathcal{C}') =$$

$$\max_{e \in [0,1], r \in \mathcal{C}' \cup \emptyset} \begin{cases} (e\pi_h + (1-e)\pi_l)(R-r) - ec \\ +\beta[e(\pi_h + (1-\pi_h)q) + (1-e)(\pi_l + (1-\pi_l)q)]v^r(p^S, \mathcal{C}(p^S)) \\ +\beta[(e(1-\pi_h) + (1-e)(1-\pi_l)][1-q]v^r(0, \mathcal{C}(0)), \text{ if } \ r \neq \emptyset; \\ \\ \beta v^r(p^\emptyset, \mathcal{C}(p^\emptyset)), \text{ if } r = \emptyset. \end{cases} \tag{1}$$

Note that in writing this expression we have used the fact that, by observation 1, $p^F(\cdot) = 0$. Let us denote the solution of problem (1) by $e^r(p, \mathcal{C}'), r^r(p, \mathcal{C}')$, which describes the risky entrepreneur's strategy as p and $\mathcal{C}'$ vary.

Analogously, letting $v^s(p, \mathcal{C}')$ be the maximal discounted expected utility for a safe entrepreneur, we have:

$$v^s(p, \mathcal{C}') = \max_{r \in \mathcal{C}' \cup \emptyset} \begin{cases} R - r + \beta v^s(p^S, \mathcal{C}(p^S)) \text{ if } \ r \neq \emptyset; \\ \beta v^s(p^\emptyset, \mathcal{C}(p^\emptyset)), \text{ if } r = \emptyset. \end{cases} \tag{2}$$

The solution to this problem is denoted by $r^s(p, \mathcal{C}')$; once again this describes the safe entrepreneur's strategy as p and $\mathcal{C}'$ vary.

Since lenders cannot observe the specific contract chosen by an individual borrower in any given period, but only whether or not he was financed, and if so the outcome of his project, we have:

Observation 2. Whenever an entrepreneur accepts financing, he will choose the contract with the lowest interest rate: i.e., for all $p, \mathcal{C}'$ we have $r^j(p, \mathcal{C}') \in \min(\mathcal{C}') \cup \emptyset$, for $j = s, r$.

Next, we determine the expected profits for an arbitrary lender n from a loan with interest rate r to an entrepreneur with credit score p, given the entrepreneurs' strategies, $r^s(\cdot), r^r(\cdot)$, and $e^r(\cdot)$, and the contracts $\mathcal{C}^{-n}$ offered by the *other* lenders. If r is strictly lower than any rate offered by the other lenders, and if both types of entrepreneurs choose to accept financing, the per capita expected profits of n are $(p + (1-p)\pi_h)r$ if the risky entrepreneurs exert high effort and $(p + (1-p)\pi_l)r$ if they exert low effort (and a convex combination of the two when risky entrepreneurs mix over effort levels). When r is still lower than any rate offered by the other lenders, but only the safe entrepreneurs accept financing, expected profits are given by the first term of the above expressions, pr, whereas if only the risky entrepreneurs accept, they are $(1-p)\pi_h r$ or $(1-p)\pi_l r$ depending on the effort strategy of the risky borrower. In the same situations, if the rate is equal to the lowest one offered by other lenders, i.e., $r = \min \mathcal{C}^{-n}$, the market is equally shared among all lenders offering the minimal rate and hence the above expressions have to be divided by the number of lenders offering the minimal rate. Finally, if lender n offers no loan contract, or a rate above the lowest one offered by the other lenders ($r > \min \mathcal{C}^{-n}$), or if all borrowers reject financing, then expected profits are 0. Thus we have:

$$\Pi(r, p, \mathcal{C}^{-n}, r^S(\cdot), r^r(\cdot), e^r(\cdot)) =$$

$$\begin{cases} \{p + (1-p)\left[e(p, \mathcal{C}^{-n} \cup r)\pi_h + (1 - e(p, \mathcal{C}^{-n} \cup r))\pi_l\right]\} r / [1 + \#(r^n \in \mathcal{C}^{-n} \text{ s.t. } r^n = r)], \\ \quad \text{if } r \leq \min(\mathcal{C}^{-n}), \text{ and } r^S(p, \mathcal{C}^{-n} \cup r) \neq \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) \neq \emptyset \\ (1-p)\{e(p, \mathcal{C}^{-n} \cup r)\pi_h + (1 - e(p, \mathcal{C}^{-n} \cup r))\pi_l\} r / [1 + \#(r^n \in \mathcal{C}^{-n} \text{ s.t. } r^n = r)], \\ \quad \text{if } r \leq \min(\mathcal{C}^{-n}) \text{ and } r^s(p, \mathcal{C}^{-n} \cup r) = \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) \neq \emptyset \\ pr / [1 + \#(r^n \in \mathcal{C}^{-n} \text{ s.t. } r^n = r)], \\ \quad \text{if } r \leq \min(\mathcal{C}^{-n}), r^s(p, \mathcal{C}^{-n} \cup r) \neq \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) = \emptyset \\ 0, \text{ if either } r > \min(\mathcal{C}^{-n}), \text{ or } r = \emptyset, \text{ or } r^s(p, \mathcal{C}^{-n} \cup r) = \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) = \emptyset. \end{cases} \tag{3}$$

Notice that in the above expression (3) for lenders' profits we used observation 2 that entrepreneurs never choose a rate above the lowest interest rate offered in $\mathcal{C}^{-n} \cup r$.

Since a lender lives only a single period, his objective is simply to choose r so as to maximize his expected profits given by (3). Given our focus on symmetric MPE, we can denote the solution simply by $r(p)$.

We are now ready to give a formal definition of a MPE:

Definition 1. A symmetric, sequential Markov Perfect Equilibrium is a collection of strategies $(r(\cdot), r^s(\cdot), r^r(\cdot), e^r(\cdot))$ and beliefs $p(\cdot)$, such that:

- Lenders maximize profits, given $r^s(\cdot), r^r(\cdot), e^r(\cdot)$: for every p, $r = r(p)$

maximizes (3), when $\mathcal{C}^{-n} = r(p)$;

- Entrepreneurs' strategies are sequentially rational. That is,
  - for all $p, \mathcal{C}'$, $(r^r(p, \mathcal{C}'), e^r(p, \mathcal{C}'))$ solves (1) when $\mathcal{C}(p) = r(p)$.
  - for all $p, \mathcal{C}'$, $r^s(p, \mathcal{C}')$ solves (2) when $\mathcal{C}(p) = r(p)$.
- Beliefs are computed via Bayes' Rule whenever possible and are consistent otherwise.

Observe that along the equilibrium path, strategies and beliefs can be written solely as functions of the credit score p, i.e., $r(p), r^r(p), r^S(p), \mathcal{C}(p)$ and $\{p^S(p), p^F(p), p^{\emptyset}(p)\}$. Similarly, entrepreneurs' discounted expected utility can be written as $v^s(p), v^r(p)$.

It will also be useful to have the notation $r_{zp}(p, e)$ to denote the lowest interest rate consistent with lenders' expected profits being non-negative on a loan to entrepreneurs with credit score p, when risky entrepreneurs exert effort e, and all agents accept financing at this rate. That is,

$$r_{zp}(p, e) \equiv \frac{1}{p + (1-p)(e\pi_h + (1-e)\pi_l)}. \tag{4}$$

The following proposition demonstrates that a Markov Perfect Equilibrium exists, and characterizes its properties. We demonstrate in Proposition 2 that this equilibrium is in fact the most efficient MPE.

Proposition 1. *Under assumptions 1-3, a (symmetric, sequential) Markov Perfect Equilibrium always exists with the following properties:*

i. Entrepreneurs never refuse financing and always take the contract with the lowest interest rate offered to them: $r^s(p, \mathcal{C}') = r^r(p, \mathcal{C}') = \min(\mathcal{C}')$, *whenever* $\mathcal{C}' \neq \emptyset$.

ii. Lenders never offer financing to entrepreneurs known to be risky with probability 1: $r(0) = \emptyset$, *and so* $v^r(0) = 0$.

iii. $p^{\emptyset}(p, \mathcal{C}') = p$ *whenever* $\mathcal{C}' \neq \emptyset$. *That is, if a borrower refuses financing, which only happens off-the-equilibrium path, a consistent belief for lenders is that the probability remains unchanged at* p. *On the other hand, lenders' beliefs after financing and success are always updated via Bayes' rule as follows:*

$$p^S(p, \mathcal{C}') = \frac{p}{p + (1-p)[e^r(p, \mathcal{C}')(\pi_h + (1-\pi_h)q) + (1 - e^r(p, \mathcal{C}'))(\pi_l + (1-\pi_l)q)]},$$

for all $p, \mathcal{C}' \neq \emptyset$.

Furthermore, along the equilibrium path, the value functions $v^r(p)$ *and* $v^s(p)$ *are weakly increasing and the players' strategies are as follows:*

a. *if* $\frac{c}{\pi_h - \pi_l} \geq \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ *then:*

  - *when* $p \geq p_{NF} \equiv \frac{1-\pi_l R}{(1-\pi_l)R}$, *the agent will be financed at the rate* $r(p) = r_{zp}(p,0)$, *and if risky exerts low effort* ($e^r(p) = 0$).
  - *when* $p < p_{NF}$ *the agent is not financed* ($r(p) = \emptyset$).

b. *if* $\frac{c}{\pi_h - \pi_l} \leq \frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ *then there is financing at the rate* $r(p) = r_{zp}(p,1)$, *for all* $p > 0$, *and risky agents exert high effort.*

c. *if* $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h - \pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ *then there exists* $0 < p_l \leq p_m \leq p_h < 1$ *such that:*

  - *if* $p \geq p_h$ *then the agent is financed at* $r(p) = r_{zp}(p,1)$ *and if risky exerts high effort;*
  - *if* $p \in [p_m, p_h)$, *risky agents mix over high and low effort with probability* $e^r(p) > 0$, *increasing in* p, *and a loan is offered at the rate* $r(p) = r_{zp}(p, e^r(p))$;
  - *if* $p \in [p_l, p_m)$ *the entrepreneurs are financed at the rate* $r(p) = r_{zp}(p,0)$ *and if risky exert low effort;*
  - *if* $p < p_l$ *there is no financing* ($r(p) = \emptyset$).

Figure 2 illustrates the equilibrium outcomes as p varies for the case $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h - \pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$. Note that the low-effort and mixing regions may be empty, while the high-effort and no-financing regions must always exist for this case. That is, we must have $0 < p_l \leq p_m \leq p_h < 1$.

We first prove property ii. of Proposition 1, that entrepreneurs who are known to be risky are never financed, and show that this is actually a general property of Markov equilibria.

Lemma 1. *Under assumptions 1 and 2, any Markov Perfect Equilibrium is characterized by no financing when* $p = 0$: *i.e.,* $r(0) = \emptyset$ *and hence* $v^r(0) = 0$.

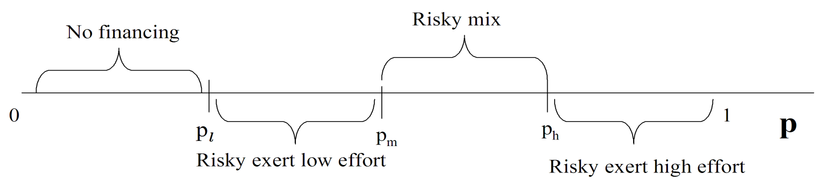

Figure 2: MPE when $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$

Proof. If $p = 0$, since only risky agents fail, we must have $p^S(p, C') = p^F(p, C') = 0$ whatever C', i.e., the agent will be known to be risky in the future as well.

Furthermore, under assumption 1, if the agent is known to be the risky type, he can only be financed in a given period if he exerts high effort with some probability, as otherwise lenders cannot break even. But for high effort (or mixing) to be incentive compatible, the utility from high effort must be no less than that from low effort, i.e., if the interest rate r which is offered must be such that

$$\pi_h(R-r) - c + (\pi_h + (1-\pi_h)q)\beta v^r(p^S(p)) + (1-\pi_h)(1-q)\beta v^r(p^F(p)) \geq$$
$$\pi_l(R-r) + (\pi_l + (1-\pi_l)q)\beta v^r(p^S(p)) + (1-\pi_l)(1-q)\beta v^r(p^F(p)),$$

or

$$\frac{c}{\pi_h - \pi_l} \leq R - r + \beta(1-q)[v^r(p^S(p)) - v^r(p^F(p))]. \tag{5}$$

But since, as argued, when $p = 0$ we have $p^S(p) = p^F(p) = 0$, given the Markov property of the equilibrium we have $v^r(p^S(p)) = v^r(p^F(p))$, and so (5) becomes

$$c \leq (\pi_h - \pi_l)[R - r]$$

By assumption 2, this can only be satisfied if $r < 1/\pi_h$, in which case lenders cannot break even. Thus the agent cannot be financed in equilibrium if he is known to be risky. Finally, since this agent is never financed, it is immediate that $v^r(0) = 0$. ∎

Combining the lemma with observation 1 we get:

Corollary 1. *If an agent fails and this failure is not forgotten, he can no longer obtain financing in any MPE.*

Remark. As this corollary makes clear, when an entrepreneur fails in our set-up he is identified as risky and — because of our parameter restrictions – can no longer obtain financing (since he would always exert low effort, whatever r is). In practice, although it is indeed the case that those borrowers with a bankruptcy in their credit record do find it considerably more difficult to obtain credit, both Staten (1993) and Musto (2004) point out that some post-bankruptcy credit is in fact available.

The assumption that only risky agents can fail obviously simplifies the analysis.[17] If both types could fail, it would no longer be the case that a single failure would automatically exclude an agent from financing (although a long enough string of failures would). We conjecture that our finding — that forgetting a default can sometimes be beneficial — should nevertheless carry through in this more general environment, and indeed, numerical simulations suggest that this is the case.

Remark. To properly understand the differences between MPE as defined and other Perfect Bayesian equilibria of the game considered, it is important to observe that the Markov property of players' strategies only bind at nodes where the entrepreneur is not financed. This is because when the agent is financed the updated belief at the end of the period will necessarily be different from the initial one and, as we will see in what follows, will always be higher in the event of success, when financing continues. Hence p never hits the same value twice, so that on this path the Markov restriction is not binding. Where it is binding is at the nodes where the agent is denied financing, i.e., when p equals zero after a failure or in cases a. and c. of the Proposition when p is sufficiently low. There the Markov property prescribes that lenders' behavior has to remain the same in the following period as well, since p remains unchanged, and so on, so that entrepreneurs are always denied financing.

[17]It also avoids the existence problems that can arise in models where both types can fail (see Mailath and Samuelson, 2001, for a discussion).

By contrast, at non-Markov equilibria, lenders' strategy may entail both financing and non financing when $p = 0$. For instance, we can have financing when p first hits 0, i.e., after a first failure, as well as at any successor node as long as the agent's project succeeds, and denial of financing after a second failure and forever after. Such strategies might be part of an equilibrium[18] since the threat of exclusion after two failures could be enough to induce high effort and hence to make lending profitable for lenders. But the strategies are clearly not Markov, since after an agent fails there is no further uncertainty about his type, p remains always equal to zero and so at an MPE an entrepreneur should be treated always in the same way. The fact that these strategies imply that the entrepreneur is treated differently at the initial nodes after p hits zero and at later nodes, requires some coordination among lenders to ensure that financing is offered in the initial periods and as long as future projects succeed, but after another failure all lenders deny financing and will keep doing that at all later periods; we could argue this is somewhat fragile, being open to the possibility of breakdowns in such coordination, or to renegotiation, which is not true for the MPE we consider.

B Proof of Proposition 1

We will prove in what follows the remaining properties, i. and iii. of MPE stated in Proposition 1 and the characterization of MPE in the various parameter regions provided in a.,b.,c. of that Proposition.

a. We show first that when $\frac{c}{\pi_h-\pi_l} \geq \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ an MPE exists where, as long as $p \geq p_{\text{NF}}$, entrepreneurs are always financed, $e^r(p) = 0$, $r(p) = r_{zp}(p,0)$ and $p^S(p) = \frac{p}{p+(1-p)(\pi_l+(1-\pi_l)q)}$.

   To show that such strategies constitute an MPE in this case, we need to demonstrate that (i) low effort is incentive compatible for $p \geq p_{\text{NF}}$; (ii) that $r_{zp}(p,0) \leq R$ for $p \geq p_{\text{NF}}$, i.e., that the contract is admissible; and (iii) that there are no profitable deviations for any player.

   a-i. Given the above strategies and beliefs, from (1) we get:

$$v^r(p) = \pi_l(R - r_{zp}(p,0)) + (\pi_l + (1-\pi_l)q)\beta v^r(p^S(p)). \qquad (6)$$

   By analogy with (5) above, for low effort to be incentive compatible we need to show that:

$$\frac{c}{\pi_h - \pi_l} \geq R - r(p,0) + \beta(1-q)v^r(p^S(p)), \qquad (7)$$

[18] In particular this is the case when $\frac{c}{\pi_h-\pi_l} \leq \frac{R-1/\pi_h}{1-\beta\pi_l}$.

since from lemma 1 we necessarily have $v^r(p^F(p)) = v^r(0) = 0$.

Since $r_{zp}(p,0) \geq r_{zp}(1,0) = 1$ for all p, $v^r(p) < \frac{\pi_l(R-1)}{1-\beta(\pi_l+(1-\pi_l)q)}$ for all $p < 1$, where the term on the right-hand side is the present discounted utility of a risky entrepreneur who is financed in every period (until there is a failure which is not forgotten) at $r = 1$. So for any $p \in (p_{\mathrm{NF}}, 1)$, we have

$$\begin{aligned} R - r_{zp}(p,0) + \beta(1-q)v^r(p^S(p)) &< R - 1 + \beta(1-q)\tfrac{\pi_l(R-1)}{1-\beta(\pi_l+(1-\pi_l)q)} \\ &= \tfrac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} \end{aligned}$$

Since in the parameter region under consideration $\frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} \leq \frac{c}{\pi_h-\pi_l}$, we obtain:

$$R - r_{zp}(p,0) + \beta(1-q)v^r(p^S(p)) < \frac{c}{\pi_h - \pi_l},$$

thereby verifying (7).

a-ii. Note that $r_{zp}(p,0) \leq R$ if and only if $\frac{1}{p+(1-p)\pi_l} \leq R$, or equivalently $p \geq p_{\mathrm{NF}}$.

a-iii. Finally, we show that there can be no profitable deviation from this equilibrium.

First consider possible deviations by a borrower. We have shown above that at the contract $r(p)$ the borrower never wants to switch from low to high effort; also from observation 2 it follows that borrowers would never choose a rate above $r(p)$ when $r(p)$ is also offered. It remains then to consider a deviation consisting of refusing an offer of financing.

To this end we establish first that the first part of property iii. of the Proposition also holds in this case, i.e., that a consistent belief for lenders is to keep their beliefs unchanged if an entrepreneur refuses financing. To see this, simply let both safe and risky borrowers refuse financing at some node with probability $\varepsilon > 0$, and let $\varepsilon \to 0$. Consistency of the above belief can then be readily verified using Bayes' Rule. The second part of property iii. follows immediately given entrepreneurs' equilibrium strategy.

Since the following result will also be used in other parts of the proof, we state it as a lemma:

Lemma 2. *When property iii. of Proposition 1 holds, as long as $v^s(p)$ and $v^r(p)$ are weakly increasing it is never optimal for an entrepreneur to refuse financing.*

Proof. Consider first a safe entrepreneur with credit score p.

Let $\mathcal{C}' \neq \emptyset$ denote the set of contracts offered, either on or off-the-equilibrium path, and let r' be the lowest rate in this set. Recall that $r' \in [0, R]$ and $p^S(p, \mathcal{C}')$ describes lenders' beliefs when the project is financed and the agent succeeds. From the second part of property iii., these beliefs are such that $p^S(p, \mathcal{C}') > p$.

So if the entrepreneur accepts r' his expected discounted utility will be $R - r' + \beta v^s(p^S(p, \mathcal{C}'))$. Conversely, if he deviates and instead refuses financing, his expected discounted utility will be $\beta v^s(p)$ from property iii. By the weak monotonicity of $v^s(\cdot)$, $v^s(p^S(p, \mathcal{C}')) \geq \beta v^s(p)$ because $p^S(p, \mathcal{C}') > p$ and so, since $r' \leq R$, accepting financing must be (weakly) better.

The same argument applies to risky entrepreneurs. ∎

Since $r(p) \equiv r_{zp}(p, 0)$ is strictly decreasing and $p^S(p)$ is increasing, it is immediate from (6) above to verify that the discounted expected utility $v^r(p)$ is weakly increasing in p in this case. The same argument also applies to the safe agents' expected discounted utility $v^s(p)$. On the basis of the above Lemma, this implies that refusing financing is never profitable for any borrower, which establishes that property i. holds.

Next consider a deviation by a lender. Since $r(p) = r_{zp}(p, 0)$, lenders always break even when they offer financing at $r(p)$, and so they would not be able to increase their profits by refusing to offer financing when $p \geq p_{\text{NF}}$.

Consider then the alternative deviation consisting in the offer of a different contract, with interest rate r'. Without loss of generality we can restrict attention to deviations in which $r' < r(p)$, if $r(p) \neq \emptyset$, since otherwise entrepreneurs would not accept the new offer, and $r' > 1$, since otherwise the deviation would not be profitable for the lender.

Let the new set of contracts (which includes the deviation r') be $\mathcal{C}'$. Observe that by property iii., $p^S(p, \mathcal{C}') < 1$ whenever $p < 1$. But then by the same argument as in a-i. above we can show that the optimal effort choice for risky entrepreneurs is to exert low effort, i.e., $e(p, \mathcal{C}') = 0$. When $p \geq p_{\text{NF}}$, since $r' < r(p) = r_{zp}(p, 0)$, this makes the deviation unprofitable. Alternatively when $p < p_{\text{NF}}$, since $r(p) = \emptyset$, for the deviation to be profitable under low effort we would need $r' > r_{zp}(p, 0)$; however, when $p < p_{\text{NF}}$ this implies $r' > R$, i.e., that the contract is not admissible.

b. Next, we show that for values of c such that $\frac{c}{\pi_h - \pi_l} \leq \frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ an MPE exists where, for all $p > 0$ entrepreneurs are always financed, $e^r(p) = 1$, $r(p) = r_{zp}(p, 1)$ and $p^S(p) = \frac{p}{p+(1-p)(\pi_h+(1-\pi_h)q)}$.

As above, to show that such strategies constitute an MPE for the above values of c, we have to show that (i) risky entrepreneurs indeed prefer to exert high rather than low effort for all $p > 0$ and (ii) there are no profitable deviations for any player. Note that by assumption 1 $r_{zp}(p, 1) \leq R$ for all $p > 0$, so $r(p) = r_{zp}(p, 1)$ is always admissible.

b-i. To show that high effort is IC for all $p > 0$, given the above strategies, we need to show that

$$\frac{c}{\pi_h - \pi_l} \leq R - r(p) + \beta(1-q)v^r(p^S(p)) \tag{8}$$

for any $p > 0$.

We first argue that, for any $p > 0$, a lower bound for $v^r(p)$ is given by $\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+q(1-\pi_h))}$, which is the present discounted utility for a risky entrepreneur financed in every period (until a failure that is not forgotten) at $r = 1/\pi_h$ and exerting high effort. This follows immediately from the fact that $v^r(p)$ is the present discounted utility under the same circumstances except that the interest rate is $r(p) = r_{zp}(p, 1) < 1/\pi_h$ for all $p > 0$.

Thus since $p^S(p) > 0$ for all $p > 0$, we have

$$R-r(p)+\beta(1-q)v^r(p^S(p)) > R-1/\pi_h+\beta(1-q)\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+(1-\pi_h)q)}.$$

So to verify (8) it suffices to show that

$$R - 1/\pi_h + \beta(1-q)\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+(1-\pi_h)q)} \geq \frac{c}{\pi_h - \pi_l}.$$

But multiplying both sides of this inequality by $(\pi_h - \pi_l)(1 - \beta(\pi_h + (1-\pi_h)q))$ and then simplifying, this is equivalent to showing that

$$\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} \geq \frac{c}{\pi_h-\pi_l},$$

which is precisely the condition on c that defines case b.

b-ii. Now we show that there can be no profitable deviations.

First, note that lenders cannot profitably deviate. To see this, simply observe that, given property i., for any $r' < r(p) = r_{zp}(p,1)$ a lender would make negative profits, regardless of the risky entrepreneurs' effort choices, and hence r' cannot be a profitable deviation.

Next consider possible deviations by a borrower. In light of observation 2 we can limit our attention to deviations in which an entrepreneur refuses financing. Now, by essentially the same argument as in a-iii. above, we can show that property iii. holds for this case as well. In addition, since high effort is exerted for $p > 0$, $r(p)$ is strictly decreasing in p and so from (1) and (2) it is easy to see that $v^r(p)$ and $v^s(p)$ are (strictly) increasing in p. Thus from lemma 2 above, property i. must hold in this case as well, i.e., refusing financing must be unprofitable.

c. Finally, we show that for intermediate values of c, $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, an MPE exists characterized by $0 < p_l \leq p_m \leq p_h < 1$ such that: for $p \geq p_l$ entrepreneurs are always financed, $e^r(p) = 1$ for $p \geq p_h$, $e^r(p) \in (0,1)$ and is (strictly) increasing in p for $p \in [p_m, p_h)$, $e^r(p) = 0$ for $p \in [p_l, p_m)$ and $r(p) = r_{zp}(p, e^r(p))$.

We begin by characterizing the values of (i) p_h, (ii) p_m and (iii) p_l, showing that the above effort choice of the risky entrepreneurs is optimal. We then show (iv) that there are no profitable deviations.

c-i. We first determine the lower bound p_h on the high effort region. Let $\tilde{p}^S(p,e) \equiv \frac{p}{p+(1-p)[e(\pi_h+(1-\pi_h)q)+(1-e)(\pi_l+(1-\pi_l)q)]}$; this is the posterior belief, following a success, that an entrepreneur is risky, when the prior belief is $p \in (0,1)$ and the effort undertaken if risky is e. That is, $\tilde{p}^S(p,e)$ is calculated via Bayes' Rule as in property iii. of the Proposition, but assuming that the risky agents' effort is e. Also, let $\tilde{v}^r(p,1)$ denote the discounted expected utility for a risky entrepreneur with credit score p when he is financed in every period until experiencing a failure that is not forgotten, he exerts high effort ($e = 1$), beliefs are updated according to $\tilde{p}^S(p,1)$ and the interest rate is $r_{zp}(p', 1)$ for all $p' \geq p$; $\tilde{v}^r(p, 1)$ then satisfies the following equation:

$$\tilde{v}^r(p,1) = \pi_h(R - r_{zp}(p,1)) - c + \beta(\pi_h + (1-\pi_h)q)\tilde{v}^r(\tilde{p}^S(p,1),1). \tag{9}$$

Note that while $\tilde{v}^r(p, 1)$ and $\tilde{p}^S(p, e)$ are well defined for all $p \in (0, 1)$, they coincide with the equilibrium values $v^r(p)$ and $p^S(p)$ only for, respectively, $p \geq p_h$ and $e = e^r(p)$.

We then define p_h as the value of p that satisfies the following equality:

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p_h, 1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h, 1), 1) \tag{10}$$

Observe that, since $\tilde{p}^S(p, 1)$ is strictly increasing in p, and $r_{zp}(p, 1)$ is strictly decreasing, $\tilde{v}^r(p, 1)$ is strictly increasing in p. Thus the term on the right-hand side of (10) is increasing in p, and so (10) has at most one solution.

We now show that a solution $p_h \in (0, 1)$ to (10) always exists. Since $\tilde{p}^S(p, 1)$ and $r_{zp}(p, 1)$ are both continuous for all $p \in (0, 1)$, $\tilde{v}^r(p, 1)$ is also continuous. As $p \to 1^-$, $r_{zp}(p, 1) \to 1$ and $\tilde{p}^S(p, 1) \to 1$, and so $\tilde{v}^r(p, 1) \to \frac{\pi_h(R-1)-c}{1-\beta(\pi_h+(1-\pi_h)q)}$. Thus as $p \to 1^-$, we have

$$R - r_{zp}(p,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p,1),1) \to R - 1 + \beta(1-q)\frac{\pi_h(R-1)-c}{1-\beta(\pi_h+(1-\pi_h)q)}.$$

For the values of c in the region under consideration it is easy to verify[19] that $R - 1 + \beta(1-q)\frac{\pi_h(R-1)-c}{1-\beta(\pi_h+(1-\pi_h)q)} > \frac{c}{\pi_h-\pi_l}$, and so as $p \to 1^-$, we have

$$R - r_{zp}(p,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p,1),1) > \frac{c}{\pi_h - \pi_l}.$$

Conversely, as $p \to 0^+$ it is immediate to see that $R - r_{zp}(p, 1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p, 1), 1) \to R - 1/\pi_h + \beta(1-q)\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+q(1-\pi_h))}$.

[19]Suppose this were not the case, so that $R - 1 + \beta(1-q)\frac{\pi_h(R-1)-c}{1-\beta(\pi_h+(1-\pi_h)q)} \leq \frac{c}{\pi_h-\pi_l}$. If we multiply both sides of this inequality by $(\pi_h - \pi_l)(1 - \beta(\pi_h + (1-\pi_h)q))$ and then simplify, this becomes $\frac{c}{\pi_h-\pi_l} \geq \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1\pi_l)q)}$, contradicting the condition on c defining case c.

By the same argument as the one in the last part of b-i. above, we can show that under the condition on c defining case c., $R - 1/\pi_h + \beta(1-q)\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+q(1-\pi_h))} < \frac{c}{\pi_h-\pi_l}$, and so as $p \to 0^+$,

$$R - r_{zp}(p,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p,1),1) < \frac{c}{\pi_h - \pi_l}.$$

Thus by the continuity and monotonicity of $\tilde{v}^r(\cdot,1)$, there must be a unique solution $p_h \in (0,1)$ to (10). It is then immediate to see, given the monotonicity of the term on the right-hand side of (10), that for all $p \geq p_h$ the incentive compatibility constraint for high effort (5) is satisfied.

c-ii. Next, we find p_m, the lower bound of the region where risky entrepreneurs mix over high and low effort, and establish the properties of the equilibrium in this mixing region.

For mixing to be an equilibrium strategy at p, risky entrepreneurs must be indifferent between high and low effort, i.e.,

$$R - r_{zp}(p,e) + \beta(1-q)v^r(\tilde{p}^S(p,e)) = \frac{c}{\pi_h - \pi_l} \tag{11}$$

for some $e \in [0,1]$. Now, let $\left(\tilde{p}^S\right)^{-1}(p_h,1)$ denote the preimage of p_h of the map $\tilde{p}^S(p,1)$, i.e., $\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h,1),1\right) = p_h$.[20] We define p_m to be the lowest value of $p \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$ for which a solution of (11) can be found for some e. Observe that by the construction of p_h, $e = 1$ is a solution to (11) when $p = p_h$, and so $p_m \leq p_h$.

Now, for any $p \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$ we have $\tilde{p}^S(p,e) \geq p_h$ for all e (this follows from the fact that, as observed above, $\tilde{p}^S(p,e)$ is increasing in p, whatever e is and, given p, is decreasing in e[21]). Thus for any $p \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$, using our results from c-i., we have $v^r(\tilde{p}^S(p,e)) = \tilde{v}^r(\tilde{p}^S(p,e),1)$ for any e. By the continuity of $\tilde{v}^r(p,1)$ and $r_{zp}(p,e)$ it follows that the minimum value p_m must exist.

[20] That is, the posterior belief of lenders, after observing a success, is equal to p_h when the prior belief was $\left(\tilde{p}^S\right)^{-1}(p_h,1)$ and the entrepreneur exerts high effort if risky.

[21] This property can be easily verified from the expression of $\tilde{p}^S(p,e)$ and can be understood as follows: for any given p, the lower the probability e that an entrepreneur if risky exerts high effort, the stronger success is a signal that the entrepreneur is a safe type.

We can also show that $p_m > \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, which will imply that there is only a single period of mixing along the equilibrium path. To see this, note that by assumption 3, we have $\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h, 1), e\right) \leq \tilde{p}^S(p_h, 1)$ for any e.[22] That is, no matter what effort level risky entrepreneurs exert when lenders' prior belief is $\left(\tilde{p}^S\right)^{-1}(p_h, 1)$, the posterior belief of lenders following a success will be lower than when their prior belief is p_h (in which case entrepreneurs exert high effort). Therefore, since $r_{zp}(\left(\tilde{p}^S\right)^{-1}(p_h, 1), e) > r_{zp}(p_h, 1)$ for any e and $\tilde{v}^r(p, 1)$ is strictly increasing in p, we must have

$$\begin{aligned} & R - r_{zp}\left(\left(\tilde{p}^S\right)^{-1}(p_h, 1), e\right) + \beta(1-q)\tilde{v}^r(\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h, 1), e\right), 1) \\ < \; & R - r_{zp}(p_h, 1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h, 1), 1) = \frac{c}{\pi_h - \pi_l} \end{aligned}$$

for any e. From our previous findings we also have $\tilde{v}^r(\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h, 1), e\right), 1) = v^r(\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h, 1), e\right))$. We conclude therefore that (11) has no solution for e at $\left(\tilde{p}^S\right)^{-1}(p_h, 1)$ and so we must have $p_m > \left(\tilde{p}^S\right)^{-1}(p_h, 1)$.

To conclude this part, it remains to establish that for all $p \in [p_m, p_h]$ we can indeed find a value of e satisfying (11), and moreover that such value is strictly increasing with respect to p.

Suppose a solution to (11) with respect to e exists for some $p \in [p_m, p_h]$; since we can always take $p = p_m$, this is always possible. Let $e(p)$ denote this solution (if there is more than one solution, pick the highest one):

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p, e(p)) + \beta(1-q)v^r(\tilde{p}^S(p, e(p))).$$

[22] Even without assumption 3, it would still be true that we have only a single period of mixing along the equilibrium path, although the proof would be longer. Hence the characterization would remain essentially unchanged. To see this, suppose it were not the case and that we mixed both at p and its successor $p^S(p)$. Since, as shown below in the text, $e(p) < e(p^S(p))$, we have $r(p) > r(p^S(p))$, so for mixing to be incentive compatible, i.e., for equation (11) to hold both at p and $p^S(p)$, we would need $v^r(p^S(p)) > v^r(p^S(p^S(p)))$. But this is impossible, since from (14) we have $v^r(p) = v^r(p^S(p)) = v^r(p_h)$, whereas $v^r(p^S(p^S(p)) \geq v^r(p^S(p))$ since $p^S(p^S(p)) \geq p^S(p)$.

As we discuss below in footnote 24, assumption 3 is only strictly necessary in order to be able to associate an equilibrium response consistent with Bayes' Rule to any deviation.

To prove the claim it suffices to show that for all $p' \in (p, p_h)$ a solution $e(p')$ of (11) also exists, and $e(p') > e(p)$.

Having established above that $p_m > \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, since $p \geq p_m$ we have $\tilde{p}^S(p, e(p)) > p_h$ and, for all $p' > p$, $\tilde{p}^S(p', e') > p_h$ whatever e' is. Hence $v^r(\tilde{p}^S(p, e(p))) = \tilde{v}^r(\tilde{p}^S(p, e(p)), 1)$ and $v^r(\tilde{p}^S(p', e')) = \tilde{v}^r(\tilde{p}^S(p', e'), 1)$ for all e'. Since, as we showed, $\tilde{v}^r(\cdot, 1)$, as well as $\tilde{p}^S(\cdot, e)$, are strictly increasing while $r_{zp}(\cdot, e)$ is strictly decreasing (whatever e is), when $p' > p$ we must have

$$\frac{c}{\pi_h - \pi_l} < R - r_{zp}(p', e(p)) + \beta(1-q)v^r(\tilde{p}^S(p', e(p))).$$

By the same properties, since $p' < p_h$ we also have

$$\frac{c}{\pi_h - \pi_l} > R - r_{zp}(p', 1) + \beta(1-q)v^r(\tilde{p}^S(p', 1)).$$

Hence, by the continuity of $\tilde{v}^r(\cdot, 1)$ there must be a solution $e' \in (e(p), 1)$ to (11) at p'.

c-iii. We still have to determine p_l, the lower bound on the financing region.

If $p_m \geq p_{\text{NF}}$, set $p_l = p_{\text{NF}}$. By construction, $r_{zp}(p, 0) \leq R$ for all $p \geq p_{\text{NF}}$; hence the contract $r_{zp}(p, e^r(p))$, with $e^r(p) = 0$ for $p \in [p_l, p_m)$, $e^r(p) = e(p)$ for $p \in [p_m, p_h)$ and $e^r(p) = 1$ for $p \geq p_h$ is admissible for all $p \geq p_{\text{NF}}$.

Alternatively, if $p_m < p_{\text{NF}}$ set p_l to be the lowest value of $p \geq p_m$ such that the contract $r_{zp}(p, e(p))$ is admissible (i.e., not greater than R). Note that since $r_{zp}(p, e)$ is decreasing in e, we have $r_{zp}(p, e(p)) \leq r_{zp}(p, 0)$ for all $p \in [p_m, p_{\text{NF}}]$, so this will imply that $p_l \leq p_{\text{NF}}$. In this case we also redefine p_m, with some abuse of notation, to be equal to p_l; following this redefinition the low effort region $[p_l, p_m)$ is then empty in this case.

Observe that in either case we have $p_l > 0$. Furthermore, $p_l \leq p_{\text{NF}}$, which implies that $r_{zp}(p, 0) > R$ for $p < p_l$. Furthermore, $p_l \leq p_m$, with p_m as defined in the preceding paragraphs.

It remains thus to show that $e^r(p) = 0$ for $p \in [p_l, p_m)$, i.e., that low effort is optimal in this region. We prove this in what follows, together with the property that $v^r(p)$ and $v^s(p)$ are (weakly) increasing for all p, which will also be used in part c-iv. of the proof.

Solving the recursive expression (1) for $v^r(p^S(p))$ and substituting into the different expressions of the IC constraint for the three regions of values of p, we obtain:[23]

$$\begin{aligned}
v^r(p) \geq & \frac{c(\pi_l + q/(1-q))}{\pi_h - \pi_l} - (R-r)\frac{q}{1-q}, \text{ if } e^r(p) = 1; && (12)\\
v^r(p) \leq & \frac{c(\pi_l + q/(1-q))}{\pi_h - \pi_l} - (R-r)\frac{q}{1-q}, \text{ if } e^r(p) = 0; && (13)\\
v^r(p) = & \frac{c(\pi_l + q/(1-q))}{\pi_h - \pi_l} - (R-r)\frac{q}{1-q}, \text{ if (11) holds (mixing).} && (14)
\end{aligned}$$

As established in c-i. above, when $p \geq p_h$ we have $e^r(p) = 1$, in which case $v^r(p) = \tilde{v}^r(p,1)$, which we have shown is strictly increasing. From (14) we also find that $v^r(p)$ is constant for all $p \in [p_m, p_h]$ and hence, using (12), that it is weakly increasing for all $p \geq p_m$.

To prove that $e^r(p) = 0$ for $p \in [p_l, p_m)$ it suffices to consider the case $p_l = p_{\text{NF}}$ (when $p_l < p_{\text{NF}}$, we showed above that $p_l = p_m$). First consider $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(p_h, 1)], p_m\right)$. We will show that the contract $r_{zp}(p,0)$, together with lenders' beliefs in case of success $p^S(p) = \tilde{p}^S(p,0)$, satisfy the IC constraint for low effort. Suppose this were not true, i.e.,

$$R - r_{zp}(p,0) + \beta(1-q)v^r(\tilde{p}^S(p,0)) > \frac{c}{\pi_h - \pi_l}.$$

We will prove this leads to a contradiction. Note that for p in the above interval $p \geq \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, hence $\tilde{p}^S(p,1) \geq p_h$ and $v^r(\tilde{p}^S(p,1)) = \tilde{v}^r(\tilde{p}^S(p,1),1)$. Also note that $p < p_h$, and so we have $\tilde{p}^S(p,1) < \tilde{p}^S(p_h,1)$. Thus from the monotonicity of $r_{zp}(\cdot,1)$ and $\tilde{v}^r(\cdot,1)$, we have

$$\begin{aligned}
& R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,1)) = R - r_{zp}(p,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p,1),1)\\
& < R - r_{zp}(p_h,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h,1),1) = \tfrac{c}{\pi_h - \pi_l},
\end{aligned}$$

[23] When $e^r(p) = 1$ then (1) reduces to $v^r(p) = \pi_h(R - r(p)) - c + \beta(\pi_h + (1-\pi_h)q)v^r(p^S(p))$, and hence we get $\beta(1-q)v^r(p^S(p)) = \frac{(v^r(p)+c-\pi_h(R-r(p)))(1-q)}{\pi_h+(1-\pi_h)q}$. The expression of the IC constraint for high effort (5) can then be rewritten as follows: $\beta(1-q)v^r(p^S(p)) \geq \frac{c}{\pi_h-\pi_l} - (R-r(p))$. Substituting for $v^r(p^S(p))$ from the above equation, yields $\frac{(v^r(p)+c-\pi_h(R-r(p)))(1-q)}{\pi_h+(1-\pi_h)q} \geq \frac{c}{\pi_h-\pi_l} - (R-r(p))$, or $(v^r(p)+c-\pi_h(R-r(p)))(1-q) \geq \left(\frac{c}{\pi_h-\pi_l} - (R-r(p))\right)(\pi_h + (1-\pi_h)q)$. Simplifying, we get (12). The other expressions are similarly obtained.

where the latter equality follows from the construction of p_h.
Since $\tilde{p}^S(p,e) \geq \tilde{p}^S(p,1)$ for all e, so that we also have $v^r(\tilde{p}^S(p,e)) = \tilde{v}^r(\tilde{p}^S(p,e),1)$, and $\tilde{v}^r(\cdot,1)$ is continuous, the two inequalities above imply that there must be a solution $\tilde{e}$ to (11) at p, which contradicts the construction of p_m as minimal in $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(p_h,1)], p_h\right]$.
So we conclude that $e^r(p) = 0$ for $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(p_h,1)], p_m\right)$.
By the argument in a. this also implies that $v^r(p)$ is strictly increasing in $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(p_h,1)], p_m\right)$ and, using (12) - (14) above, that it is increasing for all $p \geq \max[p_l, \left(\tilde{p}^S\right)^{-1}(p_h,1)]$.
If $p_l \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$ we are done. Otherwise, we extend the result by induction. It is convenient to use here the shorthand $\tilde{p}^{S^{-1}}$ to denote the term $\left(\tilde{p}^S\right)^{-1}(p_h,1)]$. We will now demonstrate that (i) $e^r(p) = 0$ for $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}},0)], \tilde{p}^{S^{-1}}\right)$ and (ii) $v^r(p)$ is increasing for $p \geq \max[p_l, \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}},0)]$.
Consider $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}},0)], \tilde{p}^{S^{-1}}\right)$. To show (i), first note that since $p < \tilde{p}^{S^{-1}} < p_m$, we must have $p^S(p) = \tilde{p}^S(p,0)$, since lenders update via Bayes' Rule and the equilibrium prescribes low effort ($e^r(p) = 0$) in this region. So

$$R - r(p) + \beta(1-q)v^r(p^S(p)) = R - r_{zp}(p,0) + \beta(1-q)v^r(\tilde{p}^S(p,0)).$$

In addition, since $p < \tilde{p}^{S^{-1}} = \left(\tilde{p}^S\right)^{-1}(p_h,1)$, by assumption 3 we must also have $p^S(p) = \tilde{p}^S(p,0) \leq \tilde{p}^S(p_h,1) = p^S(p_h)$. Thus $v^r(p^S(p)) \leq v^r(p^S(p_h))$, since $v^r(\cdot)$ is increasing above $\tilde{p}^{S^{-1}}$, as shown in the previous paragraph, and $p^S(p) \geq \tilde{p}^{S^{-1}}$. Then using the fact that $r(p) = r_{zp}(p,0) > r_{zp}(p,1) > r_{zp}(p_h,1) = r(p_h)$, since $r_{zp}(\cdot,\cdot)$ is strictly decreasing, yields:

$$R - r_{zp}(p,0) + \beta(1-q)v^r(\tilde{p}^S(p,0)) < R - r(p_h) + \beta(1-q)v^r(p^S(p_h)) = \frac{c}{\pi_h - \pi_l},$$

with the latter equality following from the construction of p_h. Thus $e^r(p) = 0$, i.e., low effort is IC at p. The same argument as above can then be used to establish that $v^r(p)$ is increasing for $p \geq \max[p_l, \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}},0)]$.
Now, if $p_l \geq \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}},1)$ we are done. Otherwise, iterate the same argument over the interval $\left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(\left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}},1),0)], \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}},1)\right)$ and continue doing so until we reach p_l.

Finally, to show that $v^s(p)$ is increasing, simply note that $e^r(p)$ is (weakly) increasing, and so $r(p) = r_{zp}(p, e^r(p))$ is decreasing. The result then follows from the definition of $v^s(p)$ in (2).

c-iv. Finally, we show that there are no profitable deviations.

First consider the possibility of deviations by entrepreneurs, in particular, the rejection of an offer of financing (as argued before in a-iii. this is the only deviation we need to consider). By the same argument as in part a-iii. above, property iii. of the Proposition can be established also for this case. We showed in part c-iii. above that $v^r(p)$ and $v^s(p)$ are (weakly) increasing, so we can use lemma 2 to verify property i., i.e., that it is never optimal for entrepreneurs to refuse financing.

Next consider deviations by lenders. By the same argument as in a-iii., refusing to finance an agent when $p \geq p_l$ is not profitable for lenders. So it suffices to consider deviations consisting, when $p \geq p_l$, in the offer of a contract $r' < r(p) \equiv r_{zp}(p, e(p))$, and when $p < p_l$ in the offer of a contract $r' \leq R$. Without loss of generality we can also restrict our attention to the region $p < p_h$, since there can be no profitable deviations when the contract offered in equilibrium supports high effort (as in that case $r(p) = r_{zp}(p, 1) \leq r_{zp}(p, e)$ for all e).

In the statement of the Proposition we did not describe the risky entrepreneurs' effort strategy $e^r(p, r')$ off the equilibrium path. We do this here and show that $e^r(p, r')$ renders any possible deviation r' described in the previous paragraph unprofitable.

First consider the case $p \in \left(\left(\tilde{p}^S\right)^{-1}(p_h, 1), p_h\right)$. Now if

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p, 0)) \leq \frac{c}{\pi_h - \pi_l}, \tag{15}$$

then $e^r(p, e') = 0$ is an optimal effort choice of entrepreneurs when they are offered the rate r' and lenders' belief is that they exert low effort. If in addition $p \geq p_l$ then $r' < r(p) \leq r_{zp}(p, 0)$ and so the deviation is unprofitable. If $p < p_l$, from c-iii. above we know that $r_{zp}(p, 0) > R$, while the admissibility of the contract requires $r' \leq R$, so that $r' < r_{zp}(p, 0)$, i.e., the deviation is unprofitable in this case as well.

Alternatively, suppose the reverse inequality to (15) holds. Then since $\tilde{p}^S(p, e)$ is decreasing with respect to e and $v^r(\cdot)$ is weakly increasing, we either have[24]

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p,e')) \geq \frac{c}{\pi_h - \pi_l}, \text{ for } e' = 1 \qquad (16)$$

or

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p,e')) = \frac{c}{\pi_h - \pi_l} \text{ for some } e' \in (0,1). \quad (17)$$

so that the optimal effort choice of risky entrepreneurs when offered rate r' is $e^r(p,r') = e'$. Suppose $r' > r_{zp}(p,e')$; we will prove in what follows that this implies a contradiction, thus establishing that $r' \leq r_{zp}(p,e')$, i.e., that again the deviation to r' is unprofitable.

When $e' = 1$, $r' > r_{zp}(p,e') = r_{zp}(p,1)$ together with (16) imply $R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,1)) \geq \frac{c}{\pi_h - \pi_l}$. But since, as we argued, $v^r(\cdot)$ is increasing and $r_{zp}(\cdot,1)$ strictly decreasing, this would imply that $R - r_{zp}(p_h,1) + \beta(1-q)v^r(\tilde{p}^S(p_h,1)) > \frac{c}{\pi_h - \pi_l}$, contradicting the construction of p_h in (10).

So consider instead $e' < 1$. Then from $r' > r_{zp}(p,e')$ and equation (17) we get

$$R - r_{zp}(p,e') + \beta(1-q)v^r(\tilde{p}^S(p,e')) > \frac{c}{\pi_h - \pi_l}.$$

Recall that $p \in \left(\left(\tilde{p}^S\right)^{-1}(p_h,1), p_h\right)$, so that $v^r(\tilde{p}^S(p,1)) = \tilde{v}^r(\tilde{p}^S(p,1),1)$ and, from the definition of p_h,

$$R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,1)) < \frac{c}{\pi_h - \pi_l}.$$

Since, as we argued, $v^r(\tilde{p}^S(p,e)) = \tilde{v}^r(\tilde{p}^S(p,e),1)$ for any e, by the continuity of $\tilde{v}^r(p,1)$ it follows that there must be a solution $\tilde{e} \in (e',1)$ to (11). If $p < p_m$ the existence of such a solution contradicts the construction of p_m as the minimal value of p for which a solution $e(p)$ to (11) with $r_{zp}(p,e(p)) \leq R$ exists in the region $p \in [\left(\tilde{p}^S\right)^{-1}(p_h,1), p_h]$, since $r_{zp}(p,\tilde{e}) < r_{zp}(p,e') < r' <$

[24] By assumption 3, when $p \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$, we have $v^r(\tilde{p}^S(p,\tilde{e})) = \tilde{v}^r(\tilde{p}^S(p,\tilde{e}),1)$, and so $v^r(\cdot)$ is continuous in $\tilde{e}$.

This is the only point in the proof where assumption 3 proves strictly necessary. Without this assumption we could still prove the existence of an MPE, but the off-equilibrium path beliefs would not necessarily be consistent everywhere with Bayes' Rule and hence we would not be able to claim that our equilibrium is also a sequential MPE.

$r(p)$. Alternatively, consider $p \geq p_m$. If $\tilde{e} > e(p)$ this contradicts the construction of $e(p)$ as the highest solution of (11) at p; if $\tilde{e} \leq e(p)$, this implies $e' < e(p)$, and thus $r' > r_{zp}(p, e') > r_{zp}(p, e(p))$, another contradiction.

We have thus shown that $r' \leq r_{zp}(p, e')$ when $p \in \left(\left(\tilde{p}^S\right)^{-1}(p_h, 1), p_h\right)$, so that the deviation r' will not be profitable given the risky entrepreneurs' optimal response $e^r(p, r') = e'$.

It remains only to consider the case $p \in \left(0, \left(\tilde{p}^S\right)^{-1}(p_h, 1)\right)$. We restrict attention to deviations $r' > r_{zp}(p, 1)$; this is without loss of generality, since if this were not the case the deviation could never be profitable, regardless of the risky entrepreneurs' effort choice. We can show that in this case $e' = 0$ is an equilibrium response to r' for the risky borrowers. To see this, note that since $r' > r_{zp}(p, 1)$ and $p < \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, by assumption 3 it must be that $\tilde{p}^S(p, 0) \leq \tilde{p}^S(p_h, 1)$, and so $v^r(\tilde{p}^S(p, 0)) \leq v^r(\tilde{p}^S(p_h, 1))$ by the monotonicity of $v^r(\cdot)$ (established in c-iii. above). Using this property and the fact that $r' > r_{zp}(p, 1)$ and $r_{zp}(p, 1) > r_{zp}(p_h, 1)$ (by the monotonicity of $r_{zp}(\cdot, 1)$), yields

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p, 0)) < R - r_{zp}(p, 1) + \beta(1-q)v^r(\tilde{p}^S(p, 0))$$
$$< R - r_{zp}(p, 1) + \beta(1-q)v^r(\tilde{p}^S(p_h, 1)) = \frac{c}{\pi_h - \pi_l},$$

with the latter equality following from the definition of p_h. This establishes that $e' = 0$ is indeed the risky entrepreneurs' optimal effort choice when offered r'.

We now argue that this renders the deviation unprofitable. First, if $p \geq p_l$ in equilibrium there is financing and so the offer of r' will only be accepted if $r' < r(p)$. But $r(p) = r_{zp}(p, 0)$ in this region, which means that the deviation will be unprofitable. Alternatively, if $p \in (0, p_l)$ when r' is admissible ($r' \leq R$) we necessarily have $r' < r_{zp}(p, 0)$ since, as shown above, $p_l \leq p_{NF}$, which again makes the deviation unprofitable.

C Efficiency of Equilibrium

We now demonstrate that the equilibrium characterized in Proposition 2 above is the most efficient MPE. We do this by first showing that only pooling equilibria exist, and then demonstrating that our equilibrium is the most efficient pooling equilibrium. For simplicity we restrict attention to the case $q = 0$ (no forgetting); the argument for general values of q is exactly the same.

The most efficient equilibrium will be the one which maximizes the ex-ante utility of all of the agents in the economy, including lenders.

If the ex-ante probability that an entrepreneuer is the safe type is given by p_0, then welfare is given by the total surplus accruing from the agents' projects:

$$\mathcal{W}(p_0) = p_0\mathcal{W}^s(p_0) + (1-p_0)\mathcal{W}^r(p_0), \tag{18}$$

where $\mathcal{W}^s(\cdot)$ and $\mathcal{W}^r(\cdot)$ can be computed recursively:

$$\mathcal{W}^s(p) = \begin{cases} 0; \text{ if } e^s(p) = \emptyset \\ R - 1 + \beta\mathcal{W}^s(p^S(p)); \text{otherwise,} \end{cases} \tag{19}$$

and

$$\mathcal{W}^r(p) = \begin{cases} 0; \text{ if } e^r(p) = \emptyset \\ (e^r(p)\pi_h + (1-e^r(p))\pi_l)R - 1 - e^r(p)c + \beta(e^r(p)\pi_h + (1-e^r(p))\pi_l)\mathcal{W}^r(p^S(p)); \text{otherwise,} \end{cases} \tag{20}$$

where we have used the fact that the only consistent belief following failure is that the agent is risky, and that by lemma 1 there can be no financing when the agent is known to be risky.[25]

We then have the following:

Proposition 2. *In the economy under consideration:*

1. *only pooling MPE exist;*

2. *the equilibrium constructed in Proposition 1 is the most efficient MPE.*

Proof. 1. We first demonstrate that any symmetric, sequential MPE must be a pooling equilibrium.

Suppose this is not the case; consider a candidate separating equilibrium. Let r^s be the contracts offered to the safe types and r^r those offered to the risky in such an equilibrium. From lemma 1 we know that in a separating MPE we must have $r^r = \emptyset$ for all nodes along the equilibrium path, and so the risky types would receive $v^r = 0$.

We will first show that this implies that the safe types must also receive $v^s = 0$. We then show this cannot be an equilibrium.

[25] We have asssumed that both types of entrepreneuers receive the same rate at p, i.e. that the equilibrium is pooling. This is demonstrated immediately below.

Now, if either $r^s = \emptyset$ and/or $r^s = R$ for all nodes along the equilibrium path then we have shown $v^s = 0$ as desired.

Alternatively, suppose $r^s(\sigma_t) \in [0, R)$ for some node σ_t along the equilibrium path. Then it would be profitable for a risky entrepreneur to deviate and pretend to be safe (taking $r^s(\sigma_t)$ instead), and exert low effort whenever financed; this would give him utility above 0, since the risky entrepreneur visits the same nodes as the safe entrepreneurs with positive probability.

So in order for this deviation not to be profitable we must have $v^s = 0$, i.e., either $r^s = \emptyset$ or $r^s = R$ for all nodes along the equilibrium path. We now show this cannot be an equilibrium.

Now if $r^s = \emptyset$ for all nodes along the equilibrium path, then we in fact have a pooling equilibrium since both types receive the same contracts (i.e., no financing in any node).

Suppose however that $r^s(\sigma_t) = R$ for some σ_t along the equilibrium path. Then in the successor node $\{\sigma_t, S\}$ following this node, there would be a profitable deviation in which a lender offers $r^s = R - \epsilon$. This would be profitable for the safe entrepreneurs, and since none of the risky would be in the pool (since they were not financed in the previous period in the separating equilibrium), it would also be profitable for the lender when ϵ is small.

So any MPE must be pooling.

2. We now show that our equilibrium is the most efficient MPE; as shown immediately above without loss of generality we can restrict attention to pooling equilibria. That is, we will show that for any p, our equilibrium maximizes $\mathcal{W}(p)$ as defined above.

   We begin by showing that, as constructed in Proposition 1, our equilibrium maximizes $e^r(p)$, the effort exerted by the risky entrepreneurs for any p.

   Lemma 3. *The equilibrium constructed in Proposition 1 maximizes the risky entrepreneurs' effort $e^r(p)$ and utility $v^r(p)$ across all symmetric sequential MPE.*

   Proof. To see this, first note that this property is immediate for $p \geq p_h$. Since our equilibrium gives all of the surplus to the borrowers, this then implies that $v^r(p)$ is maximal for our equilibrium when $p \geq p_h$. Also recall that $v^r(p)$ was shown above to be increasing for our equilibrium.

We now proceed by induction. Having established that $v(p)$ is maximal for all $p \geq p^*$ (where we begin the induction with $p^* = p_h$), we consider $p^{**} \equiv \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, the pre-image of p^* under high effort. In order for another equilibrium to implement a higher level of effort at some $p \in [p^{**}, p^*)$, the continuation utility under success would need to be higher in the other equilibrium than in ours, so as to satisfy the incentive-compatibility condition. That is, if we let $\bar{v}^S$ denote this continuation utility for the other equilibrium, and also let $\bar{p}^S(p)$ denote the posterior under the other equilibrium, we must have $\bar{v}^S > v^S(p) = v(p^S(p))$.[26] But if the other equilibrium implements higher effort at p, then we must have $\bar{p}^S(p) < p^S(p)$, and so from the monotonicity of $v(\cdot)$, we have $v(\bar{p}^S(p)) \leq v(p^S(p))$. But since $\bar{p}^S(p) \geq p^*$, we must have $\bar{v}^S \leq v(p^S(p))$, as $v(p)$ has been established to be maximal for $p \geq p^*$. This implies that our equilibrium implements the highest effort, and hence that $v(p)$ is maximal, for $p \geq p^{**}$. We then set $p^* = p^{**}$ and continue the induction, thereby establishing the desired result for arbitrary p. ∎

The following corollary is also immediate, since for lenders to break even when $p < p_l$ would require a higher level of effort than our equilibrium, which we have shown is impossible.

Corollary 2. *No MPE equilibrium can implement financing when $p < p_l$.*

It is now easy to demonstrate that our equilibrium is the most efficient MPE. From above, this has already been established for $p < p_l$, since we have shown that there cannot be financing in any MPE, and hence $\mathcal{W}(p) = 0$ in this region.

Next we consider $p \geq p_l$. For these values there is always financing in our equilibrium as long as the agent doesn't fail.

Note that $\mathcal{W}^s(p) = \frac{R-1}{1-\beta}$ for all $p \geq p_l$, and $\mathcal{W}^r(p) = \frac{\pi_h R - 1 - c}{1 - \pi_h \beta}$ for $p \geq p_h$ in our equilibrium.

Given lemma 1, this implies that each of these is is clearly maximal across all possible equilibria when $p \geq p_h$, and so our equilibrium maximizes $\mathcal{W}(p)$ for $p \geq p_h$.

[26] More precisely, if we are in the mixing region (in which the risky agents are indifferent between high and low effort) then the other equilibrium could implement higher effort even when $\bar{v}^S = v(p^S(p))$. But recall that we chose effort in the mixing region to be maximal in constructing our equilibrium.

Next consider $p \in [p_l, p_h)$. We will show that our equilibrium also maximizes $\mathcal{W}(p)$. Suppose that this is not the case, and that there exists another equilibrium that implements a higher level of welfare at some p in this interval. We will demonstrate a contradiction. Let $\bar{\mathcal{W}}^r(p)$ refer to the surplus from the risky agents' projects in this other equilibrium.

Now, note that in our equilibrium we always have $\mathcal{W}(p') > 0$ when $p' \geq p_l$. So WLOG we can assume that this other equilibrium does not implement exclusion at p (since this would yield a welfare of 0).

From above, since $\mathcal{W}^s(p')$ is maximal in our equilibrium for $p' \geq p_l$, this other equilibrium must implement higher welfare at p from the risky agents if it is more efficient. Moreover, since our equilibrium implements the maximal effort level at p, from the recursive definition (20) of $\mathcal{W}^r(p)$, it is clear that this other equilibrium can only implement higher welfare at p through the continuation welfare. That is, we must have $\bar{\mathcal{W}}^r(\bar{p}^S(p)) > \mathcal{W}^r(p^S(p))$.

We will establish that this cannot be the case.

First consider $p \in [p_m, p_h)$, where p_m is the mixing cutoff in our equilibrium (recall that if there is no mixing in our equilibrium then $p_m \equiv p_h$ and then this case is empty). Now, since the other equilibrium cannnot implement higher effort at p, we must have $\bar{p}^S(p) \geq p^S(p) \geq p_h$. From above this then implies that $\bar{\mathcal{W}}^r(\bar{p}^S(p)) \leq \mathcal{W}^r(p_h) = \mathcal{W}^r(p^S(p))$, thus establishing that $\mathcal{W}^r(p) \geq \bar{\mathcal{W}}^r(p)$.

The rest of the argument follows by induction. We will show that if $\mathcal{W}^r(p) \geq \bar{\mathcal{W}}^r(p)$ for $p \geq p^*$, where $p^* \in (p_l, p_m]$, then this must also be the case for $p \geq p^{**} = \max[p_l, p^{-1}(p^*, 0)]$, where $p^{-1}(p^*, 0)$ is the pre-image of p^* under the equilibrium effort choice. Observe that this condition has already been established above for $p^* = p_m$.

To demonstrate this, note that $p^{**} < p_m$. Thus since our equilibrium implements low effort for $p \in [p^{**}, p_m)$, the other equilibrium must do so as well (since we have ruled out exclusion, WLOG). This then implies that $\bar{p}^S(p) = p^S(p)$. As above, it suffices to show that $\bar{\mathcal{W}}^r(\bar{p}^S(p)) \equiv \bar{\mathcal{W}}^r(p^S(p)) \leq \mathcal{W}^r(p^S(p))$. But since $p^S(p) \geq p^*$, this has already been established.

We then take $p^* = p^{**}$, and continue the induction until we reach p_l. ∎

IV Optimal Forgetting

A Results

In this section we derive conditions under which forgetting entrepreneurs' failures is a socially optimal policy; that is when, in the equilibria characterized in Proposition 1, $q > 0$ dominates $q = 0$. The welfare criterion we use is the ex-ante utility of all of the agents in the economy (i.e., before the type of each entrepreneur is realized). Since lenders break even in equilibrium, it suffices to consider the discounted expected utility of the entrepreneurs.

We first derive the expression of such utility, highlighting the dependence of the equilibrium variables on the forgetting policy $q \in [0, 1]$. We will write then $p_h(q)$, $p_m(q)$ and $p_l(q)$ to denote the level of lenders' posterior beliefs at which in equilibrium risky entrepreneurs, respectively, switch to high effort, to mixing or start getting financed. It is then convenient to introduce some new notation. Let $n(q, p_0)$ denote the number of periods — or consecutive successes — until, in equilibrium, risky entrepreneurs start exerting high effort (i.e., until lenders' posterior belief is greater or equal than $p_h(q)$). Let $G = \pi_h R - 1 - c$ denote the NPV (net of the effort cost) of the project under high effort and $B \equiv \pi_l R - 1$ the NPV under low effort. Note that, by assumption 1, $B < 0 < G$.

Note that in the parameter region a. of Proposition 1, i.e., for high values of c (where we have low effort for all $p \geq p_{\text{NF}}$), $n(q, p_0) = \infty$ and we then set $p_l(q) = p_{\text{NF}}$ and $p_h(q) = 1$ for all q, p_0; in region b., for low values of c (where we have high effort for all p) $n(q, p_0) = 0$ and we can set $p_l(q) = p_h(q) = 0$ again for all q, p_0. In the intermediate region c., when $p_h(q) = p_m(q)$ (i.e., risky entrepreneurs do not mix in equilibrium) and $p_0 \in (p_l(q), p_h(q))$, $n(q, p_0)$ is the smallest integer for which

$$\frac{p_0}{p_0 + (1 - p_0)[\pi_l + (1 - \pi_l)q]^{n(q,p_0)}} \geq p_h(q)$$

That is, $n(q, p_0)$ is given by

$$\begin{aligned} n(q, p_0) &= \lceil n^*(q, p_0) \rceil \text{, where} \\ n^*(q, p_0) &\equiv \frac{\log \left[\frac{p_0\,(1 - p_h(q))}{(1 - p_0)\,p_h(q)} \right]}{\log \left[\pi_l + (1 - \pi_l)q \right]} \end{aligned} \tag{21}$$

It will also be useful to express $p_h(q)$ in terms of $n(q, p_0)$, inverting (21); since $n(q, p_0)$ is restricted to be an integer, doing so gives us bounds for

$p_h(q)$:

$$\frac{p_0}{p_0 + (1-p_0)[\pi_l + (1-\pi_l)q]^{n(q,p_0)-1}} \leq p_h(q) \leq \frac{p_0}{p_0 + (1-p_0)[\pi_l + (1-\pi_l)q]^{n(q,p_0)}} \tag{22}$$

We can now write the expression of the ex-ante expected discounted utility of entrepreneurs at an MPE (with no mixing). Since lenders break even, this is equivalent to the expected discounted NPV of the projects undertaken by the entrepreneurs:

$$\mathcal{W}(q,p_0) = \begin{cases} 0; \text{ if } p_0 < p_l(q) \\ p_0 \sum_{n=0}^{\infty} \beta^n (R-1) + (1-p_0) \left[\sum_{n=0}^{n(q,p_0)-1} (\pi_l + (1-\pi_l)q)^n \beta^n B + \right. \\ \left. + (\pi_l + (1-\pi_l)q)^{n(q,p_0)} \beta^{n(q,p_0)} \sum_{n=0}^{\infty} (\pi_h + (1-\pi_h)q)^n \beta^n G \right]; \text{if } p_0 \geq p_l(q). \end{cases} \tag{23}$$

This is a generalization of (18) above for arbitrary q.

Note that the first term in the second expression above, which describes the expected discounted NPV of the safe entrepreneurs' projects that are financed, is independent of q since the safe entrepreneurs never fail. When $p_0 \geq p_h(q)$ it thus suffices to restrict attention to the second term, describing the expected discounted NPV of the risky entrepreneurs' projects that are financed, which we can denote by $\mathcal{W}^r(q,p_0)$. This can be rewritten as follows:

$$\mathcal{W}^r(q,p_0) = \frac{B(1-((\pi_l + (1-\pi_l)q)\beta)^{n(q,p_0)})}{1-(\pi_l + (1-\pi_l)q)\beta} + \frac{G(\pi_l + (1-\pi_l)q)\beta)^{n(q,p_0)}}{1-(\pi_h + (1-\pi_h)q)\beta} \tag{24}$$

When $n(q,p_0) = 0$ this simplifies to:

$$\mathcal{W}^r(q,p_0) = \frac{G}{1-(\pi_h + (1-\pi_h)q)\beta}, \tag{25}$$

which is strictly positive, whereas when $n(q,p_0) = \infty$ it reduces to:

$$\mathcal{W}^r(q,p_0) = \frac{B}{1-(\pi_l + (1-\pi_l)q)\beta}, \tag{26}$$

which is strictly negative.

When there is mixing in equilibrium, i.e., $p_m(q) < p_h(q)$, we show first that the expression (21) for $n(q,p_0)$ and the bounds (22) for $p_h(q)$ given above also hold. To see this, note that with mixing the probability of success is greater or equal than when low effort is exerted, and so the posterior is $\tilde{p}^S(p,e(p)) \leq \tilde{p}^S(p,0)$; hence $n(q,p_0)$ will be greater or equal than the expression given in (21) for the case without mixing. But $n(q, p_0)$ cannot be strictly greater, as this would imply that we mix for more than a single period, which we have shown (in the proof of Proposition 1) cannot happen.

The exact expression of the discounted utility of risky entrepreneurs in this case depends on the equilibrium level of effort exerted in the mixing region. Since there can be at most only a single period of mixing in equilibrium, an upper and lower bound for such utility can be found that is independent of the mixing probability:

$$\begin{aligned}
&\frac{B(1-((\pi_l+(1-\pi_l)q)\beta)^{n(q,p_0)})}{1-(\pi_l+(1-\pi_l)q)\beta} + \frac{G((\pi_l+(1-\pi_l)q)\beta)^{n(q,p_0)}}{1-(\pi_h+(1-\pi_h)q)\beta} \\
&\leq \mathcal{W}^r(q, p_0) \\
&\leq \frac{B(1-((\pi_l+(1-\pi_l)q)\beta)^{n(q,p_0)-1})}{1-(\pi_l+(1-\pi_l)q)\beta} + \frac{G((\pi_l+(1-\pi_l)q)\beta)^{n(q,p_0)-1}}{1-(\pi_h+(1-\pi_h)q)\beta}
\end{aligned} \tag{27}$$

We analyze in what follows the effects of changing q on the level of entrepreneurs' welfare in equilibrium; we will denote then by $q(p_0)$ the welfare maximizing level of q. When the parameters are in regions a. or b. of Proposition 1 the only effect of raising q is that the probability that a risky entrepreneur will be excluded from financing will be lower; the failure of the project in fact implies exclusion only with probability $1 - q$. The entrepreneurs' effort choice in these regions is in fact unaffected by changes of q. In region a., where the cost of effort c is sufficiently low entrepreneurs always exert high effort if financed; hence raising q will increase welfare, since the NPV of another period of financing with high effort is $G > 0$. On the other hand, in region b., for high values of c, risky entrepreneurs always exert low effort if financed; in this case increasing q lowers welfare, since the NPV of financing under low effort is $B < 0$. This gives us the following Proposition:

Proposition 3. *The welfare maximizing forgetting policy respectively for high and low values of c is as follows:*

1. *If $\frac{c}{\pi_h-\pi_l} \geq \frac{R-1}{1-\beta\pi_l}$, no forgetting is optimal for all p_0: $q(p_0) = 0$.*

2. *If $\frac{c}{\pi_h-\pi_l} < \frac{R-1/\pi_h}{1-\beta\pi_l}$, for any $p_0 > 0$ the maximal welfare is attained at some forgetting policy $q(p_0) > 0$.*

Proof. Consider the first case. When $\frac{c}{\pi_h-\pi_l} \geq \frac{R-1}{1-\pi_l\beta}$, since $\frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ is decreasing in q, the condition defining region a. in Proposition 1 is satisfied for all q. It then follows that at the MPE there is financing only when $p_0 \geq p_{\text{NF}}$ and risky entrepreneurs never exert high effort, regardless of the value of q.

Hence if $p_0 \geq p_{\text{NF}}$, $\mathcal{W}^r(q, p_0) = \frac{B}{1-(\pi_l+(1-\pi_l)q)\beta}$, as in (26), which is strictly decreasing in q since $B < 0$. Thus $q = 0$ is optimal. If on the other hand $p_0 < p_{\text{NF}}$, $\mathcal{W}(q, p_0) = 0$ for all q, and so $q = 0$ is also (weakly) optimal.

Consider now the second case. By the same argument as above observe that $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ is again decreasing in q. Thus when $\frac{c}{\pi_h-\pi_l} < \frac{R-1/\pi_h}{1-\beta\pi_l}$, the condition defining region b. of Proposition 1 is satisfied for all $q \in [0, q^*]$, where $q^* = \frac{(R-1/\pi_h)-\frac{c}{\pi_h-\pi_l}(1-\beta\pi_l)}{\beta\left((R-1/\pi_h)-\frac{c}{\pi_h-\pi_l}(1-\pi_l)\right)} > 0$. Hence at the MPE there is always financing whatever p_0 is, and for all $q \in [0, q^*]$, and risky entrepreneurs always exert high effort. That is, for $q \in [0, q^*]$, we have $n(q, p_0) = 0$, and so

$$\mathcal{W}^r(q, p_0) = \frac{G}{1-(\pi_h+(1-\pi_h)q)\beta}.$$

Now this is increasing in q since $G > 0$. Thus any $q \in (0, q^*]$ dominates $q = 0$ and the optimal value will be $q(p_0) \geq q^*$.[27] ∎

From the argument in the proof it also follows that when q is increased region a. (for which low effort is implemented whenever there is financing) becomes larger, while region b. (where high effort is implemented for all p) becomes smaller.

The more interesting case is when the parameters lie in region c. of Proposition 1, i.e., for intermediate values of c, so that $0 < n(q, p_0) < \infty$. When $p_0 > p_h(0)$, i.e., $n(0, p_0) = 0$, an analogous argument to that used to prove case 2. of Proposition 3 establishes that the socially optimal level of q is above 0.

On the other hand, when $p_0 \leq p_h(0)$ then raising q will not necessarily increase welfare: as we see from (24) we face a tradeoff between the positive effect of a lower probability of exclusion when high effort is exerted (i.e., when $p > p_h(0)$) and the negative effect this has when low effort is exerted (when $p < p_h(0)$). We will show in what follows that the first, positive, effect prevails over the second when (i) $|B|$ is sufficiently small relative to G and (ii) agents are sufficiently patient (β close to 1), since the positive effect follows the negative one on the equilibrium path. Note however that raising q not only affects the probability of exclusion in any given period, but also

[27] The optimal value of q could be higher than q^*, which would push us out of region b., into region c.

changes the switching point from low to high effort, i.e., affects $n(q, p_0)$, and this effect also needs to be taken into account.

We will see that increasing q may sometimes lead to an increase in $n(q, p_0)$. This can occur for two reasons. First of all, even if $p_h(q)$ does not change, a higher value of q "slows down" the updating, i.e., $p^S(p)$ will be closer to p when q is higher, so that a longer string of successes is required to reach $p_h(q)$. In addition, we will see that it is possible that $p_h(q)$ increases as well, since the fact that failures are less costly may weaken incentives.[28] In order to find conditions ensuring that we have a higher welfare with $q > 0$ than with $q = 0$, we will therefore need to bound the potential increase in $n(q, p_0)$, as we show in the proof below.

We therefore have:

Proposition 4. *For intermediate values of c,* $\frac{R-1/\pi_h}{1-\beta\pi_l} \leq \frac{c}{\pi_h - \pi_l} < \frac{R-1}{1-\pi_l\beta}$*, the optimal policy also exhibits forgetting. More precisely:*

1. *If* $p_0 > p_h(0)$*, welfare is always maximized at* $q(p_0) > 0$*.*
2. *If* $p_0 \in [p_{NF}, p_h(0)]$ *and* $-\frac{B}{G} < \frac{p_0(1-p_h(0))(1-\pi_l)}{p_h(0)((1-\pi_h)(1+(1-p_0)\pi_h))+\pi_h^2 - p_0(1-\pi_h+\pi_h^2)}$*, then for* β *sufficiently close to 1 we also have* $q(p_0) > 0$*.*

Proof. In case 1 ($p_0 > p_h(0)$), notice that the high effort incentive compatibility constraint is slack when $q = 0$. The same is true for q in a neighborhood of 0, so we have $\mathcal{W}^r(q, p_0) = \frac{G}{1-(\pi_h+(1-\pi_h)q)\beta}$, which is increasing in q. Hence for all $q > 0$ in such a neighborhood the welfare is higher and the maximal welfare is attained at the highest q in such a neighborhood.

Now consider case 2. Since $p_0 \geq p_{\mathrm{NF}}$, the agent will always be financed at p_0. We will determine some conditions under which there exists $\bar{q} > 0$ for which $\mathcal{W}^r(\bar{q}, p_0) > \mathcal{W}^r(0, p_0)$. For this to be the case, it must be that $p_h(q)$ (or, equivalently, $n(q, p_0)$) does not increase too rapidly in q.

We proceed as follows. For any $q > 0$ we find first an upper bound for the threshold $p_h(q)$ above which we have high effort, which we denote as $\tilde{p}_h(q)$, such that *if* $p_h(q) < \tilde{p}_h(q)$ welfare is higher at q than at 0. We then find some parameter restrictions that ensure the existence of $\bar{q} > 0$ such that $p_h(\bar{q}) \leq \tilde{p}_h(q)$.

From the bounds on $\mathcal{W}^r(q, p_0)$ found in (27) for the case where there might be mixing in equilibrium, we have

$$\mathcal{W}^r(0, p_0) \leq \frac{B(1-(\pi_l\beta)^{n(0,p_0)-1})}{1-\pi_l\beta} + \frac{G(\pi_l\beta)^{n(0,p_0)-1}}{1-\pi_h\beta}$$

[28] This, however, may not always be the case, since a higher value of q also increases the continuation utility upon success.

and

$$\mathcal{W}^r(q,p_0) \geq \frac{B(1-((\pi_l+(1-\pi_l)q)\beta)^{n(q,p_0)})}{1-(\pi_l+(1-\pi_l)q)\beta} + \frac{G(\pi_l+(1-\pi_l)q)\beta)^{n(q,p_0)}}{1-(\pi_h+(1-\pi_h)q)\beta}.$$

So to show that $\mathcal{W}^r(q,p_0) > \mathcal{W}^r(0,p_0)$, it suffices to show that we can find $q>0$ such that

$$\frac{B(1-(\pi_l\beta)^{n(0,p_0)-1})}{1-\pi_l\beta}+\frac{G(\pi_l\beta)^{n(0,p_0)-1}}{1-\pi_h\beta} < \frac{B(1-((\pi_l+(1-\pi_l)q)\beta)^{n(q,p_0)})}{1-(\pi_l+(1-\pi_l)q)\beta}+\frac{G(\pi_l+(1-\pi_l)q)\beta)^{n(q,p_0)}}{1-(\pi_h+(1-\pi_h)q)\beta}.$$

Letting $\beta \to 1$ and simplifying, we obtain:

$$\frac{\frac{B}{G}}{1-\pi_l}+\frac{\pi_l^{n(0,p_0)-1}}{(1-\pi_l)(1-\pi_h)}\left[(1-\pi_l)-\frac{B}{G}(1-\pi_h)\right]$$
$$< \frac{\frac{B}{G}}{(1-\pi_l)(1-q)}+\frac{(\pi_l+(1-\pi_l)q)^{n(q,p_0)}}{(1-q)(1-\pi_l)(1-\pi_h)}\left[(1-\pi_l)-\frac{B}{G}(1-\pi_h)\right],$$

since $1-(\pi_l+(1-\pi_l)q) = (1-\pi_l)(1-q)$ and $1-(\pi_h+(1-\pi_h)q) = (1-\pi_h)(1-q)$. Or, equivalently,

$$\pi_l^{n(0,p_0)-1}(1-q) - \frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l)-\frac{B}{G}(1-\pi_h)} < (\pi_l+(1-\pi_l)q)^{n(q,p_0)} \qquad (28)$$

It will be useful to rewrite (28) in terms of a condition on $p_h(q)$ and $p_h(0)$. Now, from (22) we have $\pi_l^{n(0,p_0)} \leq \frac{p_0}{1-p_0}\left(\frac{1}{p_h(0)}-1\right)$, so to satisfy (28) it suffices to show that:

$$\frac{1}{\pi_l}\frac{p_0}{p_h(0)}\left(\frac{1-p_h(0)}{1-p_0}\right)(1-q)-\frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l)-\frac{B}{G}(1-\pi_h)} < (\pi_l+(1-\pi_l)q)^{n(q,p_0)}$$

Furthermore, we also know from (22) that $(\pi_l+(1-\pi_l)q)^{n(q,p_0)-1} \geq \frac{p_0}{1-p_0}\left(\frac{1}{p_h(q)}-1\right)$, so a sufficient condition for the above inequality (and in turn for (28)) to hold is:

$$\frac{1}{\pi_l}\frac{p_0}{p_h(0)}\left(\frac{1-p_h(0)}{1-p_0}\right)(1-q)-\frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l)-\frac{B}{G}(1-\pi_h)} < (\pi_l+(1-\pi_l)q)\frac{p_0}{p_h(q)}\left(\frac{1-p_h(q)}{1-p_0}\right).$$

Simplifying, we have the following sufficient condition for q to implement a welfare improvment as $\beta \to 1$ is given by:

$$p_h(q) < \tilde{p}_h(q) \equiv \frac{p_0(\pi_l+(1-\pi_l)q)}{p_0(\pi_l+(1-\pi_l)q)+(1-p_0)\left[\frac{1}{\pi_l}\frac{p_0}{p_h(0)}\left(\frac{1-p_h(0)}{1-p_0}\right)(1-q)-\frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l)-\frac{B}{G}(1-\pi_h)}\right]} \qquad (29)$$

We now derive parameter restrictions that ensure that we can find $\bar{q} > 0$ such that $p_h(\bar{q})$ satisfies (29) and so we can achieve a welfare improvement. We begin by providing a convenient upper bound for $p_h(q)$.

From equation (10) above, we know that for intermediate values of c, lying in region c. when $q = 0$, $p_h(0)$ belongs to $(0, 1)$ and satisfies the following equality:

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p_h(0), 1) + \beta \tilde{v}^r(\tilde{p}^S(p_h(0), 1), 1; 0), \tag{30}$$

where we use $\tilde{v}^r(p, 1; q)$ to denote the discounted expected utility of a risky entrepreneur with credit score p, when he exerts high effort for all $p' > p$ and the contracts offered are $r_{zp}(p, 1)$, highlighting the dependence of the utility on the forgetting policy q. It is then easy to see from the definition of region c. in Proposition 1 that for any $q > 0$, c will remain in region c. when β is sufficiently close to 1. So for β close to 1 we will also have $p_h(q) \in (0, 1)$ and thus

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p_h(q), 1) + \beta(1 - q)\tilde{v}^r(\tilde{p}^S(p_h(q), 1), 1; q),$$

Comparing this to (30) we obtain:

$$-r_{zp}(p_h(0), 1) + \beta \tilde{v}^r(\tilde{p}^S(p_h(0), 1), 1; 0) = -r_{zp}(p_h(q), 1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h(q), 1), 1; q). \tag{31}$$

By a similar argument to that made in the proof of part a. of Proposition 1, a (strict) upper bound for $\tilde{v}^r(\tilde{p}_h^S(p_h(0), 1), 1; 0)$ is given by the utility of being financed in every period until a failure occurs at the constant rate $r = 1$ while exerting high effort, i.e., by $\frac{\pi_h(R-1)-c}{1-\beta\pi_h}$. Conversely, when the forgetting policy is q, a (strict) lower bound for $\tilde{v}^r(\tilde{p}^S(p_h(q), 1), 1; q)$ is given by $\frac{\pi_h(R-r_{zp}(p_h(q),1)-c}{1-\beta(\pi_h+(1-\pi_h)q)}$, that is, the utility of a risky agent when financed at the constant rate $r_{zp}(p_h(q), 1)$ until he experiences a failure that is not forgotten, still exerting high effort. Together with (31) this then implies that:

$$-r_{zp}(p_h(0), 1) + \beta \frac{\pi_h(R-1) - c}{1 - \beta\pi_h} > -r_{zp}(p_h(q), 1) + \beta(1-q)\frac{\pi_h(R - r_{zp}(p_h(q), 1) - c}{1 - \beta(\pi_h + (1 - \pi_h)q)}.$$

Note that, as $\beta \to 1$, $1 - \beta(\pi_h + (1 - \pi_h)q) \to (1 - \pi_h)(1 - q)$ and so the above inequality becomes

$$-r_{zp}(p_h(0), 1) + \frac{\pi_h(R-1) - c}{1 - \pi_h} > -r_{zp}(p_h(q), 1) + \frac{\pi_h(R - r_{zp}(p_h(q), 1) - c}{1 - \pi_h},$$

or, simplifying,

$$r_{zp}(p_h(q),1) > (1-\pi_h)r_{zp}(p_h(0),1)+\pi_h$$

Substituting from the definition of $r_{zp}(\cdot,\cdot)$ in (4), for β close to 1 we have

$$\frac{1}{p_h(q)+(1-p_h(q))\pi_h} > (1-\pi_h)\frac{1}{p_h(0)+(1-p_h(0))\pi_h}+\pi_h,$$

So

$$p_h(0)+(1-p_h(0))\pi_h > (1-\pi_h)[p_h(q)+(1-p_h(q))\pi_h]+\pi_h[p_h(q)+(1-p_h(q))\pi_h][p_h(0)+(1-p_h(0))\pi_h], \tag{32}$$

Noting that the right-hand side of this inequality can be simplified as follows:

$$\begin{aligned}
(1-\pi_h)[p_h(q)+(1-p_h(q))\pi_h]+\pi_h[p_h(q)+(1-p_h(q))\pi_h][p_h(0)+(1-p_h(0))\pi_h] \\
= [p_h(q)+(1-p_h(q))\pi_h]\left[(1-\pi_h)+\pi_h[p_h(0)+(1-p_h(0))\pi_h]\right] \\
= [p_h(q)+(1-p_h(q))\pi_h]\left[(1-\pi_h)+\pi_h[1-(1-\pi_h)(1-p_h(0)]\right] \\
= [p_h(q)+(1-p_h(q))\pi_h]\left[1-\pi_h(1-\pi_h)(1-p_h(0)\right],
\end{aligned}$$

(32) becomes:

$$p_h(0)+(1-p_h(0))\pi_h > [p_h(q)+(1-p_h(q))\pi_h]\left[1-\pi_h(1-\pi_h)(1-p_h(0))\right].$$

Or

$$p_h(0)(1-\pi_h)+\pi_h > [p_h(q)(1-\pi_h)+\pi_h]\left[1-\pi_h(1-\pi_h)(1-p_h(0))\right],$$

i.e.,

$$\frac{p_h(0)(1-\pi_h)+\pi_h}{[1-\pi_h(1-\pi_h)(1-p_h(0))]} > [p_h(q)(1-\pi_h)+\pi_h].$$

This implies that when β is close to 1 we have:

$$p_h(q) < \bar{p}_h \equiv \frac{p_h(0)(1-\pi_h^2)+\pi_h^2}{[1-\pi_h(1-\pi_h)(1-p_h(0))]}. \tag{33}$$

We now derive the parameter restrictions stated in the Proposition that ensure that there exists $\bar{q}>0$ for which $\bar{p}_h < \tilde{p}_h(\bar{q})$, thus implying that $\bar{q}$ yields a welfare improvement over $q=0$. Note that for q close to 1, $\tilde{p}_h(q)$ is close to $p_0\frac{(1-\pi_l)-\frac{B}{G}(1-\pi_h)}{p_0(1-\pi_l)-\frac{B}{G}(1-\pi_h)}$. Comparing this to the above expression for $\bar{p}_h$ (noticing that $\bar{p}_h$ is independent of q) gives the stated condition on $\frac{B}{G}$. ∎

Remark. While the above results demonstrate that it is possible to achieve an improvement in ex-ante welfare by forgetting past failures, it is useful to distinguish the impact of forgetting across the two types of entrepreneurs. It is easy to see that — if forgetting leads to an improvement in social welfare — this necessarily benefits the risky entrepreneurs, since the improvement arises precisely because rather than being excluded from financing after failing, the risky entrepreneurs are permitted to re-enter the pool of agents who receive financing with some probability. By contrast, since forgetting slows down the updating, this generally hurts the safe types; they subsidize the risky entrepreneurs' projects because they always repay and hence lower the interest rate paid. The only way in which forgetting might possibly benefit these safe types is through its effect on the high-effort cutoff $p_h(q)$ (or, analogously, $p_m(q)$ and $p_l(q)$). If $p_h(q)$ is decreasing in q then it would benefit the safe types as well, since the interest rate will be lower when high effort is implemented. We will see that this is *not* the case for the examples presented in section V below; so in those cases forgetting, while socially optimal, hurts the safe types.

Remark. As we discussed above, the social benefit of forgetting failures arises from the additional periods of financing under high effort which it permits. In light of this, we can also understand the importance of our assumption that the risky entrepreneur can fail even when he exerts high effort, i.e., that $\pi_h < 1$. When this is not the case and we have $\pi_h = 1$ (as, for example, is the case in Diamond, 1989) then high effort ensures success, and there is no benefit from forgetting a failure, since such failures only result from low effort.

In particular, it is easy to see that for $\pi_h = 1$ the middle region of Proposition 1 would disappear and we would be left only with the two extremes, where either $n(q, p_0) = \infty$ or else $n(q, p_0) = 0$, for all p_0. In the former case the same argument as made in the proof of case 1. of Proposition 3 also implies that $q = 0$ is optimal when $\pi_h = 1$. When $n(q, p_0) = 0$, however, then when $\pi_h = 1$ the expression for welfare becomes $\mathcal{W}^r(q, p_0) = \frac{R-1-c}{1-\beta}$. Since this is now independent of q, $q = 0$ is (weakly) optimal here as well, in contrast to case 2. of Proposition 3 above.[29]

[29]The model in Diamond (1989) also has a third type of agent, who always exerts low effort, so in contrast to our model the middle region does not disappear in his model even though $\pi_h = 1$. Nevertheless, we believe that forgetting would also provide no benefit in this case, since $p_h(q)$ would be increasing in q when $\pi_h = 1$ because forgetting makes incentives worse. The reasons for this are (i) that the continuation utility $\tilde{v}^r(p^S(p), 1; q)$ would be decreasing in q since forgetting failures allows this third, bad type to remain in the pool for longer (thereby raising interest rates for the risky type even when he

B Discussion — Empirical Evidence and Policy Implications

In this section we discuss the empirical and policy implications of our results. We begin with an appraisal of the policy debate surrounding the adoption of the FCRA in light of our theoretical findings. We then discuss Musto's (2004) empirical results. Finally, we suggest that the forgetting policy must be determined by the government, rather than by individual lenders.

Recall that in our discussion of the congressional hearings surrounding the adoption of the FCRA we mentioned the following arguments put forward in favor of forgetting past defaults: (1) if information was not erased then the stigmatized individual would not obtain a "fresh start" and so would not be rehabilitated as a productive member of society, (2) old information might be less reliable or salient, and (3) limited computer storage capacity. On the other hand, the arguments raised against forgetting this information were (1) it discourages borrowers from working to repay their debts by reducing the penalty of failure, (2) it could lead to a tightening of credit policies (which would affect the worst risks disproportionately), (3) it increases the chance of costly fraud or other crimes by making it harder to identify (and exclude) seriously bad risks, and finally, (4) it forces honest borrowers to subsidize the dishonest ones.

From the presentation, it is clear that our model captures the tradeoff between the main arguments in favor and against forgetting. In particular, if an agent does not have his failure forgotten, then in equilibrium he is excluded forever after; from Proposition 4, if exclusion is relatively costly (that is, if the NPV of high effort is sufficiently high), then it is indeed optimal to forget. On the other hand, in our model forgetting can harm incentives ex-ante, precisely as the detractors have argued.

Our model also captures the other arguments against forgetting. First, as suggested in the policy debate, forgetting can indeed lead to a tightening of lending standards in our model. To see this, recall from Proposition 1 that for q sufficiently high, values of c can shift from region b. (in which there is financing for all $p > 0$, to region a., for which there is financing only when $p > p_l$. Just as suggested in the policy debate, these agents who are then excluded from financing are of course the worst risks.

In addition, we also note that if low effort is very costly (i.e., if its NPV is very negative) then Proposition 4 suggests that it might not be optimal to forget. The reason is that reinserting risky agents into the pool will be

exerts high effort), and (ii) because forgetting reduces the cost of low effort for the risky entrepreneurs.

very costly when $p < p_h(q)$ because in such case they will exert low effort. This is consistent with the concern that forgetting — by making it harder to identify and exclude bad risks — increases the risk of costly fraud or other crime.

Regarding the remaining argument against forgetting, observe that while forgetting leads to more pooling, which indeed increases the subsidy from the safe to the risky types, this does not affect aggregate welfare because only the risky types have an incentive problem in our model. If both types were subject to moral hazard, however, then this effect might lead to a reduction in social welfare.

Finally, our model has little to say regarding the remaining two arguments in favor of forgetting. First, old defaults are no less reliable an indicator of a risky type in our model, so lenders have no incentive to forget them. Our model also has no role for limited computer storage or bounded rationality.

We now turn to the question of whether our results are consistent with the empirical evidence in Musto (2004), and whether we concur with his assessment that removing this flag is suboptimal. Recall that Musto (2004) finds that those who receive credit after their default is forgotten are likelier to default in the future and that their credit quality (as measured by their FICO score) declines over time.

This is also a conclusion of our model. Both before and after period $n(q, p_0)$, the only defaulters are risky agents; when they are reinserted into the pool they are always more likely than the average to default in the future (since even when they exert high effort they only succeed with probability $\pi_h < 1$ whereas the riskless agents never fail.). Indeed, they will eventually default with probability 1, since $\pi_h < 1$.

Nevertheless, Propositions 3 and 4 suggest that this is not necessarily suboptimal. The reason is that while these agents are riskier than average, their projects can nevertheless have positive NPV when they are pooled anew, which would not be the case were they separated.

Finally, note that we have modeled the forgetting policy as a choice variable for the social planner, rather than, say, including it in a lenders' strategy sets. While it can be beneficial to forget a past default, it is not difficult to see that lenders would not do this on their own, and that a law is needed to enforce this. The reason is that a lender can always profit by refusing to lend to entrepreneurs whom he knows to be bad (while keeping rates the same); moreover, since we impose a period-by-period break-even constraint on lenders, there is no way for lenders to make up any losses in the current period through future profits. Moreover, any borrower who requested ex-ante that lenders forget a default would be signaling that he is the risky type in our model, and so would not obtain financing (hence no entrepreneur would suggest this); this is reminiscent of Aghion and Hermalin (1990), who argue that restricting the ability of agents to contract privately can sometimes be optimal, because otherwise agents would try to use the contractual form to inefficiently signal that they are of a good type.

V Examples

In this section we present a few examples to illustrate the results of the previous section.

Let $R = 3, \pi_h = 0.5$, and $\pi_l = 0.32$. We must restrict attention to effort costs $c \in (0.18, 0.5)$ in order to satisfy assumptions 1 and 2.

1. First suppose that $c = 0.48$ and $\beta = 0.75$; this corresponds to region a. of Proposition 1, for which the risky entrepreneurs exert low effort whenever financed (regardless of q). So from Proposition 3 the optimal level of forgetting is given by $q(p_0) = 0$ for all p_0. This is illustrated in figure 3; observe that welfare $\mathcal{W}(q, p_0)$ is decreasing in q. Also note that $\mathcal{W}(q, p_0) = 0$ for $p_0 < p_{\text{NF}} = 0.0196$.

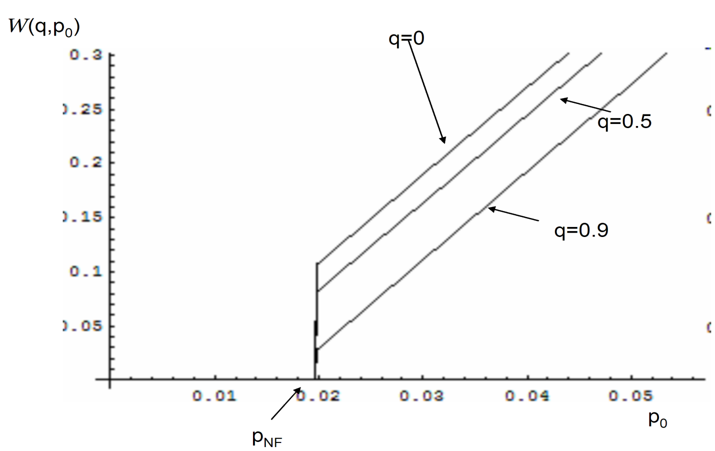

Figure 3: Region a.: $q(p_0) = 0$

2. Now let $c = 0.4$ and $\beta = 0.975$. This corresponds to case c. of Proposition 1, for which high effort is implemented when $p \geq p_h(q)$.

   In order to apply Proposition 4, we must first determine $p_h(0)$. From equation (10), this can be computed as $p_h(0) = 0.241$.

   When $p_0 > p_h(0) = 0.241$, then from Proposition 4 we know that $q(p_0) > 0$ is optimal. This case is analogous to that of case b. (discussed in the next example), in which the risky entrepreneuers continue to exert high effort even for a modest increase in q .

   By contrast, when $p_0 \in [p_{\mathrm{NF}}, p_h(0))$, then a sufficient condition for $q(p_0) > 0$ is that $-\frac{B}{G} < \frac{p_0(1-p_h(0))(1-\pi_l)}{p_h(0)((1-\pi_h)(1+(1-p_0)\pi_h))+\pi_h^2-p_0(1-\pi_h+\pi_h^2)}$, or $p_0 > 0.205$ for these parameters, and that β is sufficiently close to 1.

   For example, consider $p_0 = 0.206$. When $q = 0$ then $p^S(p, 0) = 0.448$ and so low effort is implemented for one period along the equilibrium path, and high effort is implemented thereafter.[30] However, when the forgetting policy is $q > 0$ then it takes longer to implement high effort, both because the updating is slower and because $p_h(q)$ is higher. For example, with $q = 0.735$ we require three periods of low effort until the posterior exceeds $p_h(0.735) = 0.322$.

   We now compare the welfare implication of these forgetting policies. In figure 4 we plot $\mathcal{W}(q, 0.206)$, the NPV of the risky entrepreneur's projects when $p_0 = 0.206$, for various values of q. From this figure one can see that the optimal value is given by $q(0.206) = 0.77$, in which case $\mathcal{W}(0.77, 0.206) = 0.212$.[31]

   We also plot the optimal forgetting policy $q(p_0)$ in figure 5 as a function of the prior probability.[32]

3. Now consider $c = 0.26$ and $\beta = 0.975$. These parameters correspond to region b. of Proposition 1 (for which high effort is implemented for all $p > 0$) when $q \leq 0.359$, and so from Proposition 3 the optimal level of forgetting is $q(p_0) \geq 0.359$ for any p_0.

   Although higher values of q take us out of region b. and into region c., this may nevertheless still be optimal, as discussed in footnote 27 above (this is not surprising, since the previous example shows that it

[30] Note that the risky entrepreneurs do not use a mixed effort-strategy along the equilibrium path in these examples.

[31] We discretize the domain of q in computing these examples.

[32] Although the condition in Proposition 4 is violated for low values of p_0, we can nevertheless still have $q(p_0) > 0$, since the condition is sufficient but not necessary.

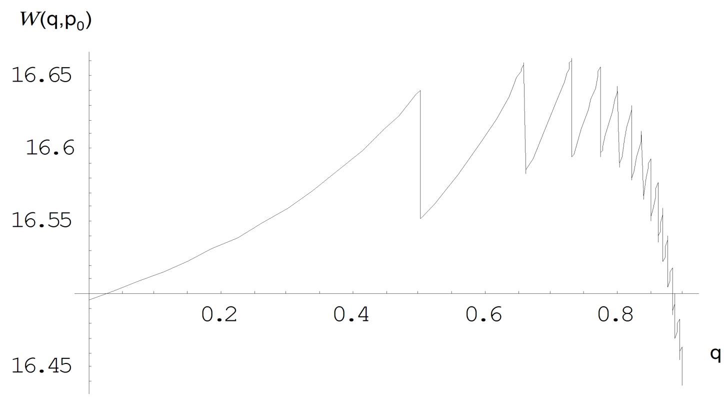

Figure 4: Region c.: social welfare as a function of q ($p_0 = 0.206$)

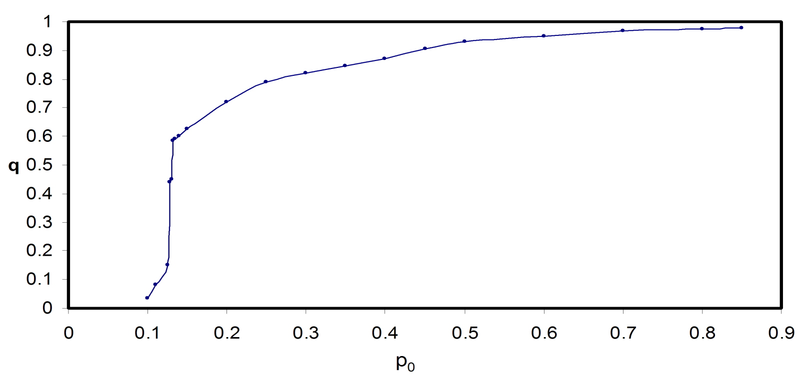

Figure 5: Region c.: welfare-maximizing value of q

may indeed be optimal to forget even in region c.). For example, when $q = 0.975$ we are in region c. (with $p_h(q) = 0.1139$), and from figure 6 one can see that this dominates $q = 0.359$ so long as $p_0 > 0.066$.[33]

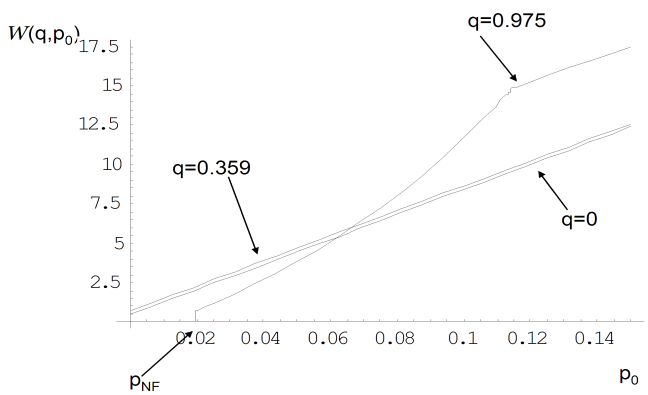

Figure 6: Region b.: $q(p_0) \geq 0.359$ for all p_0

4. Finally, consider $\beta = 0.8$, $\pi_l = 0.3$ and $c = 0.48$. While these parameters are still in region c., the condition in Proposition 4 is violated for all $p_0 < p_h(0) = 0.628659$, and in fact welfare is decreasing in q for p_0 sufficiently low. This is illustrated for the case of $p_0 = 0.2$ in figure 7. To understand why this is the case, we first note that for these parameters $p_h(q)$ is strictly increasing in q, and $n(q, p_0)$ is weakly increasing. So the only possible benefit from raising q could result from a lower probability of exclusion in the high-effort region. But with these parameter values — for which agents are relatively impatient and low effort is relatively inefficient — the cost of less exclusion in the low effort region always dominates the benefit in the high effort region.

[33] Conversely, observe that when $p_0 \leq p_l(0.975) = p_{\text{NF}} = 0.0196$ then there is no financing when $q = 0.975$ and hence $\mathcal{W}(0.975, p_0) = 0$ (and so $q = 0.975$ cannot be optimal for these p_0); this is an instance for which too generous a forgetting policy shuts down the credit market, as discussed in the previous section.

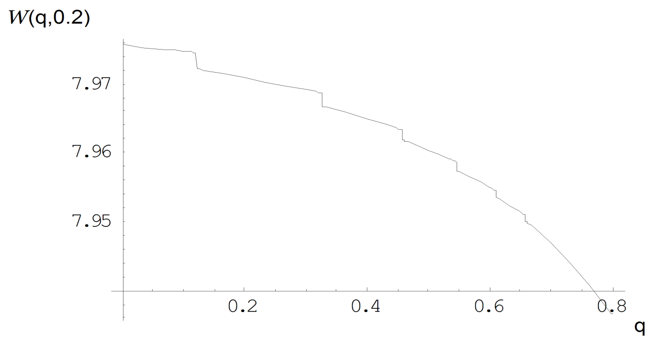

Figure 7: Region c.: condition in Proposition 4 violated ($p_0 = 0.2$)

VI Conclusion

We have developed a model in which entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. We are interested in determining the optimal amount of "forgetting" in this economy; and in particular whether lenders should not be permitted to make use of past defaults. Forgetting a default makes incentives worse, ex-ante, because it reduces the punishment for failure. However, following a default it is generally good to forget, because pooling riskier agents with safer ones makes exerting high effort to preserve their (undeservedly good) reputation more attractive. The optimal policy trades off these effects.

Our key result is that if agents are sufficiently patient, and low effort is not too inefficient, then the optimal law would necessarily prescribe some amount of forgetting — that is, it would not permit lenders to fully utilize past information. We also show that this law must be enforced by the government — no lender would willingly agree to forget.

One direction in which this model might be extended is to explore the robustness to our assumptions. In particular, the assumption that only risky agents can fail means that when a default is observed (and remembered) then the defaulting agent is excluded. This clearly makes the model much more tractable. If the "riskless" agents could also default, then exclusion would no longer follow after the first failure, although experiencing sufficiently many failures would eventually preclude further financing. Nevertheless, we conjecture that the qualitative nature of our results would not be that different — and that for reasonable parameter values forgetting would still be an optimal policy.

Finally, we have noted that there are cross-country differences in laws governing the memory of the credit reporting system; in general, European countries tend to forget defaults more quickly. It would be interesting to study whether this is due to other well-known differences in laws governing bankruptcy or perhaps the economic environment.

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