## The Reversal Interest Rate

## Joseph Abadi

Federal Reserve Bank of Philadelphia Research Department

## Markus Brunnermeier

Princeton University
PUBLISHED

Yann Koby
Brown University

## ISSN: 1962-5361

Disclaimer: This Philadelphia Fed working paper represents preliminary research that is being
circulated for discussion purposes. The views expressed in these papers are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. Any errors or omissions are the responsibility of the authors. Philadelphia Fed working papers are free to download at: https://philadelphiafed.org/research-

# The Reversal Interest Rate ${ }^{*}$ 

Joseph Abadi ${ }^{\ddagger} \quad$ Markus Brunnermeier ${ }^{\S} \quad$ Yann Koby ${ }^{\S}$

June 2022


#### Abstract

The reversal interest rate is the rate at which accommodative monetary policy reverses and becomes contractionary for lending. We theoretically demonstrate its existence in a macroeconomic model featuring imperfectly competitive banks that face financial frictions. When interest rates are cut too low, further monetary stimulus cuts into banks' profit margins, depressing their net worth and curtailing their credit supply. Similarly, when interest rates are low for too long, the persistent drag on bank profitability eventually outweighs banks' initial capital gains, also stifling credit supply. We quantify the importance of this mechanism within a calibrated New Keynesian model.


Keywords: Monetary Policy, Lower Bound, Negative Rates, Banking

JEL Codes: E43, E44, E52, G21

[^0]
## 1 Introduction

In most New Keynesian models, the economy can enter a liquidity trap because of an exogenously assumed zero lower bound. This assumption has been called into question since a growing number of central banks - the Swedish Riksbank, the Danish Nationalbank, the Swiss National Bank, the European Central Bank, and the Bank of Japan - have led money market rates into negative territory as a response to the Great Recession. In addition to going negative, these rates have been kept low for a long period.

This motivates the question: what is the effective lower bound on monetary policy? We suggest in this paper that it is given by the reversal interest rate, the rate at which accommodative monetary policy reverses its effect and becomes contractionary for bank lending. A monetary policy rate decrease below the reversal interest rate depresses rather than stimulates the economy.

Importantly, the reversal interest rate is not necessarily zero, as commonly assumed. In our model, when the reversal interest rate is positive, say $1 \%$, a policy rate cut from $1 \%$ to $0.9 \%$ is already contractionary. On the other hand, if the reversal interest rate is $-1 \%$, policy rate cuts remain expansionary up to that point, even if their effectiveness might be impaired.

To study the emergence of a reversal rate, we develop an infinite-horizon New Keynesian macroeconomic model with a banking sector. The model features two key frictions: banks have market power to set deposit rates, and bank lending is constrained by net worth. In order to highlight the mechanism that gives rise to a reversal rate, we begin by theoretically analyzing the transmission of monetary policy to bank credit supply in partial equilibrium. Following an interest rate cut, two opposing forces affect banks' net worth. On the one hand, banks make capital gains on longterm assets with fixed-rate coupon payments (the capital gains channel). On the other hand, as interest rates head lower, the pass-through from the policy rate to deposit rates declines, e.g., due to the presence of cash, compressing banks' profit margins (the net interest income channel). We theoretically demonstrate that the reversal interest rate is precisely the rate below which the net interest income effect of further interest rate cuts outweighs the capital gains effect. A reversal rate is guaranteed to exist when banks' capital gains from maturity mismatch are sufficiently small. We show that our main results depend on two empirically verifiable properties of the model: first, that banks' net interest income falls following an interest rate cut, and second, that this downturn in banks' profitability causes them to reduce lending.

We apply our theoretical framework to study the effects of "low-for-long" monetary policies in which the central bank promises to keep interest rates low for a prolonged period of time. Banks' net interest income losses cumulate every period, but the initial revaluation of their long-term assets eventually fades out as those assets mature. Consequently, a promise to keep rates low might initially stimulate bank lending but later become contractionary: as banks' net worth is drained over time, they cut back on lending due to financial constraints. We precisely characterize the conditions under which extending the period of low interest rates is bound to eventually become counterproductive.

The economics behind our results carry through in general equilibrium with sticky prices. After
calibrating the model to the Euro area, we compute its full non-linear response to monetary shocks. We find in our calibration that the monetary authority's ability to stimulate bank lending on impact declines with the size of the monetary shock and reverses at an interest rate close to $-1 \%$. Given the persistence of the monetary shock, the negative effects are even more pronounced on bank lending one or two years ahead: banks' capital gains shield them from rate cuts on impact, but not later. Once the reversal rate for bank lending is crossed, the economy's reliance on bank credit - the share of firms that are bank-dependent - dictates the aggregate implications for investment and output. The reversal interest rate for aggregate output is lower, as other channels through which monetary policy operates - non-bank-dependent firms' funding costs and the inter-temporal substitution channel - remain active.

The calibrated model provides an ideal setting to study the determinants of the reversal rate for bank lending. We find that its primary determinants are (1) the maturity of banks' fixedincome assets, (2) the tightness of banks' net worth constraints, and (3) the share of bank profits attributable to deposit issuance. A larger maturity mismatch on banks' balance sheets results in a greater asset revaluation following interest rate cuts, decreasing the reversal interest rate. Tighter net worth constraints imply that banks are forced to cut back on lending sooner following the drop in profitability caused by rate cuts, ceteris paribus. A greater reliance of bank profits on deposit spreads implies a steeper deterioration in profitability once rates enter negative territory, resulting in a higher reversal rate.

In an application of our theoretical results, we also study the consequences of "low-for-long" policies in general equilibrium. In standard New Keynesian models, it is well-known that promises to hold interest rates low for prolonged periods result in implausibly large economic stimulus (the "forward guidance puzzle"). In our model, by contrast, the stimulative effects of such announcements are significantly smaller than in a standard model without banking frictions. Upon the announcement of the interest rate cut, agents foresee that the cut will put downward pressure on bank profits and lead to an eventual decline in lending, investment, and output; thus, the initial response of demand is much weaker than in a standard model. In this sense, the reversal rate mechanism can mitigate the forward guidance puzzle.

The rest of the paper is organized as follows. Section 2 develops a full New Keynesian model with banking frictions. In Section 3, we derive analytical results by studying the partial equilibrium response of bank credit supply to monetary shocks. Section 4 calibrates the model, solves for the general equilibrium in response to monetary shocks, and presents our main quantitative results regarding the level of the reversal rate, comparative statics, and the power of forward guidance. Section 5 studies the robustness of the main quantitative results to alternative assumptions on parameters and the sources of interest rate shocks. Section 6 concludes. Proofs and supporting results are in the Online Appendix.

Related Literature. A long-standing literature developed the concepts of the "balance sheet" and "bank lending" channels of monetary policy, emphasizing the importance of the balance sheet structure and the net worth of intermediaries for the transmission of monetary policy (Bernanke
and Blinder, 1988; Bernanke and Gertler, 1995; van den Heuvel, 2007). In our model, these objects are key determinants of the transmission of monetary policy. From a theoretical standpoint, our microeconomic modeling of banks stands on the shoulders of a literature formally started by Klein (1971) and Monti (1972) ${ }^{1}$ who emphasize the importance of market power when modeling banks.

Our paper also relates to the growing literature that studies the transmission of monetary policy through banks in low-interest rate environments. Closest to our paper, in contemporaneous and independent work, Eggertsson et al. (2019) and Ulate (2021) present models in which monetary policy is weakened when rates cross into negative territory, due to the reduction in banks' profit margins and the resulting decline in their net worth. Other work, such as Drechsler et al. (2017), Wang (2022), and Wang et al. (2022), studies the reduced pass-through of monetary policy to bank lending when rates are low but positive. Our work differs from those papers in two key respects. First, our model highlights that interest rate cuts may not become contractionary until rates enter substantially negative territory - we theoretically demonstrate the existence of a reversal rate and quantitatively estimate it, whereas previous work estimates the effectiveness of interest rate cuts near the lower bound on deposit rates. Second, our theoretical framework permits us to characterize the full dynamic response of banks' credit supply to monetary shocks, allowing us to demonstrate how "low-for-long" policies can be detrimental and address the forward guidance puzzle.

Banks' market power on their funding sources materializes in the impaired transmission of money market rates to bank deposit rates, affecting banks' interest rate risk exposure. Saunders and Schumacher (2000) and Maudos and Fernandez de Guevara (2004), among others, document this fact empirically. Importantly, this impaired transmission suggests a wedge between contractual and effective maturity of deposits, a fact long recognized by regulators when assessing banks' interest rate risk (Hoffmann et al., 2019). The interest rate exposure on banks' liabilities due to market power is balanced by banks' long-term fixed-income assets. Begenau et al. (2015) and Gomez et al. (2021) document that banks' assets are exposed to interest rate risk, which banks do not fully hedge in derivative markets (Abad et al., 2016). Drechsler et al. (2021) show that this asset exposure is rationalized by banks' market power on deposits as it provides a natural hedge to the resulting interest rate risk. Due to this hedge, realized net interest income varies little with the level of policy rates: banks optimally choose a maturity mismatch in order to hedge the interest rate risk created by their market power on the liability side of their balance sheets.

We view our study as complementary: we argue that the long-lasting low/negative interest rate period was largely unexpected and hence not hedged, and we study the implications of an unexpected shock. Di Tella and Kurlat (2021) offer an alternative rationalization with a similar result. In Brunnermeier and Sannikov (2016), banks also hold interest rate risk since appropriate monetary policy provides a "stealth recapitalization" in downturns. Importantly, these hedging strategies work in expectation. Hence, upon unusual interest rate realizations, the valuations of banks can fluctuate significantly, as in our model.

A recent empirical literature has shown real effects of low/negative rate environments on banks'

[^1]profitability. English et al. (2018), Ampudia and van den Heuvel (2022), Claessens et al. (2018), Eisenschmidt and Smets (2019), and Wang et al. (2022) provide evidence that banks' net interest income and equity valuations vary with the level of interest rates, possibly in a nonlinear way. In particular, Claessens et al. (2018) find that a $1 \%$ policy rate drop implies, on average, a net interest margin decline of 8 basis points, but that this magnitude grows as rates move lower. This effect carries through to bank profitability. Moreover, for each additional year of low rates, margins and profitability fall further. Altavilla et al. (2018) document that the European Central Bank's introduction of negative interest rates was significantly detrimental to banks' net interest income, although increased intermediation activity as well as an improvement in the risk profile of banks' assets helped sustain returns on assets. Evidence provided by Ampudia and van den Heuvel (2022) suggests that banks' profitability response to interest rate cuts is non-monotonic: in normal times, interest rate cuts increase banks' valuations, although this does not hold in low-rate environments.

Finally, we rely on a literature showing that the profitability of banks impacts their lending activities and hence the level of intermediation in the economy. Brunnermeier and Sannikov (2014) provide a theoretical foundation where intermediaries' profitability is key for the economy to function properly. Cavallino and Sandri (2018) obtain contractionary monetary easing in their theoretical model and explore the implications in an open economy context. Empirically, Chodorow-Reich (2014) shows real effects of bank lending frictions on firm employment. Heider et al. (2019) employ a difference-in-difference analysis using syndicated loans in the Euro area to document that banks with a high deposit base decreased their lending relative to low-deposit, wholesale-funded banks following the ECB's decision to implement negative interest rates 2 Importantly, Gropp et al. (2018) show that banks exposed to higher capital requirements decrease their risk-weighted assets instead of recapitalizing, as in our model.

## 2 Model

We consider a New Keynesian economy in discrete time, $t \in\{0,1,2, \ldots\}$. The main players are households that work and consume, intermediate goods firms that employ capital and labor to produce output, and a continuum of banks $j \in[0,1]$ that intermediate funds from households to intermediate producers. Importantly, banks have market power in setting deposit and loan rates (and hence make profits that are later paid out to households). The presence of financial frictions will imply that bank net worth matters for lending.

As in most sticky-price models, there are also monopolistic retailers $k \in[0,1]$ that use intermediate goods to produce differentiated varieties, which are then sold to final goods producers that aggregate those varieties to produce consumption goods and capital. Both firms and banks are owned by households. A central bank implements monetary policy by setting the nominal interest rate $i_{t}$, and there is a government that issues risk-free, long-term bonds and sets taxes and transfers.

[^2]The economy begins at its steady state. We study the effects of an unanticipated monetary policy announcement at $t=0$, after which point the economy evolves deterministically.

In this section, we first set up a general model. In Section 3, we impose specific assumptions on the model's parameters that permit us to prove analytical results regarding the response of bank loan supply to monetary policy shocks. Section 4 performs a calibration that allows us to verify the quantitative relevance of these theoretical mechanisms and provide a realistic estimate of the reversal rate.

### 2.1 Households

A household's lifetime utility is given by:

$$
\begin{equation*}
\sum_{t=0}^{\infty} \beta^{t}\left(u\left(C_{t}, C_{t-1}, H_{t}\right)+\zeta \Phi\left(\mathcal{L}_{t}\right)\right) . \tag{1}
\end{equation*}
$$

In the utility function $u, C_{t}$ denotes consumption, $H_{t}$ denotes hours worked, and $\beta \in(0,1)$ is households' discount factor. We permit $C_{t-1}$ to enter the utility function in order to accommodate habits in consumption. We assume

$$
u\left(C_{t}, C_{t-1}, H_{t}\right)=\frac{\left(C_{t}-h C_{t-1}\right)^{1-\sigma}}{1-\sigma}-\chi \frac{H_{t}^{1+\varphi}}{1+\varphi}
$$

As in Feenstra (1986) or Poterba and Rotemberg (1986), liquid asset holdings enter directly into households' utility ${ }^{3}$ The function $\Phi$ specifies the utility that households derive from holding liquid savings $\mathcal{L}_{t}$, which in turn are an aggregate of deposits and cash, $\mathcal{L}_{t}=\mathcal{L}\left(D_{t}, M_{t}\right)$, where $D_{t}$ and $M_{t}$ denote real deposit and cash holdings. The aggregator $\mathcal{L}$ is assumed to be concave, differentiable, and weakly increasing in both of its arguments. In order to allow for the possibility of negative interest rates, we assume that the function $\Phi$ has a satiation point $\mathcal{L}^{*}$. Parameter $\zeta>0$ determines the scale of liquid asset demand ${ }^{4}$

In each period, households choose their consumption $C_{t}$, labor supply $H_{t}$ (taking the nominal wage $W_{t}$ as given), and savings, which are allocated across three types of assets: deposits $D_{t}$, cash holdings $M_{t}$, and risk-free bonds $B_{t}$. Each household is matched with a single bank $j$ and may deposit funds at the nominal rate $1+i_{j t}^{D}$ set by that bank. However, the household may not deposit at other banks; that is, banks have market power in setting deposit rates. All banks behave identically in equilibrium, so we drop the subscript $j$ in what follows. Cash earns a net return of zero, and bonds pay the policy rate $i_{t}$. Hence, households' budget constraint is

$$
\begin{equation*}
C_{t}+D_{t}+M_{t}+B_{t} \leq \frac{W_{t}}{P_{t}} H_{t}+\frac{1+i_{t-1}}{1+\pi_{t}} B_{t-1}+\frac{1+i_{t-1}^{D}}{1+\pi_{t}} D_{t-1}+M_{t-1}+\Pi_{t}^{F}+\Pi_{t}^{B}+T_{t}, \tag{2}
\end{equation*}
$$

[^3]where $P_{t}$ is the price level at $t, 1+\pi_{t}=\frac{P_{t-1}}{P_{t}}$ is inflation from $t-1$ to $t, \Pi_{t}^{F}\left(\Pi_{t}^{B}\right)$ denotes dividends paid out to the household by firms (banks), and $T_{t}$ denotes lump-sum transfers from the government. Households' problem is to maximize (1) subject to (2) and the non-negativity constraints $B_{t}, D_{t}, M_{t} \geq 0$. The solution to households' problem will determine the deposit demand function $D_{t}\left(i_{t}^{D}, i_{t}\right)$ taken as given by each bank $j$ in period $t .5$

### 2.2 Intermediate goods firms

Intermediate goods firms are set up without funds of their own and operate for two periods. They produce goods that are sold to monopolistic retailers $k \in[0,1]$ at a competitive price $P_{t}^{I}$. There are two types of firms: bank-dependent (fraction $\xi$ ) and non-bank-dependent (fraction $1-\xi$ ). Like households, each bank-dependent firm is matched with a single bank $j$ and may borrow only from that bank (at nominal rate $1+i_{j t}^{L}$ ). Again, in equilibrium all banks will set the same rate $i_{j t}^{L}$, so we henceforth drop the $j$ subscript. Non-bank-dependent firms instead borrow by issuing safe one-period bonds directly to households at the policy rate $1+i_{t}$. The two types of firms operate distinct types of capital that trade at competitive real prices $Q_{t}^{K, b}$ and $Q_{t}^{K, n b}$ (respectively), but they hire from a single labor market with competitive wage $W_{t}$.

When an intermediate producer is born, it borrows in order to buy capital. In the second period it produces and sells output, sells back undepreciated capital, repays its debt and closes shop. Firms operate a decreasing-returns-to-scale technology using capital $K_{t}$ and labor $H_{t}$, with a productivity parameter that differs across firm types. The production function for a firm of type $z \in\{b, n b\}$ is then $Y_{t}=A^{z}\left(K_{t}^{\alpha} H_{t}^{1-\alpha}\right)^{\nu}$, with $\alpha \in(0,1)$ (so that $\alpha$ is the capital share) and $\nu \in(0,1)$ (capturing decreasing returns to scale) ${ }^{6}$

The problem faced by bank-dependent firms is therefore

$$
\begin{equation*}
\max _{K_{t}, H_{t}} P_{t}^{I} \cdot A^{b}\left(K_{t}^{\alpha} H_{t}^{1-\alpha}\right)^{\nu}+(1-\delta) P_{t} Q_{t}^{K, b} K_{t}-\left(1+i_{t-1}^{L}\right) P_{t-1} Q_{t-1}^{K, b} K_{t}-W_{t} H_{t} \tag{3}
\end{equation*}
$$

where $\delta \in(0,1)$ denotes the depreciation rate of capital. Firms' first-order conditions imply that their loan demand $L_{t}\left(i_{t}^{L}\right)$ takes the form

$$
\begin{equation*}
L_{t}\left(i_{t}^{L}\right)=\left(\left(\frac{\nu(1-\alpha) A^{b} P_{t}^{I}}{W_{t}^{(1-\alpha) \nu}}\right)^{\frac{1}{1-\nu(1-\alpha)}} \frac{1-\nu(1-\alpha)}{\left(1+i_{t-1}^{L}\right) P_{t-1} Q_{t-1}^{K, b}-(1-\delta) P_{t} Q_{t}^{K, b}}\right)^{\frac{1-\nu(1-\alpha)}{1-\nu}} . \tag{4}
\end{equation*}
$$

This loan demand schedule is taken as given by banks.
The problem of non-bank-dependent firms is identical to that of bank-dependent firms except for the fact that their productivity is $A^{n b}$, they borrow directly from households by issuing one-period risk-free bonds at the nominal rate $1+i_{t}$, and they trade capital at price $Q_{t}^{K, n b}$.

[^4]
### 2.3 Banks

Banks extend loans $L_{t}$ and purchase safe, long-term government bonds $B_{t}^{L}$ using their net worth $N_{t}$ as well as by issuing deposits $D_{t}$. Their balance sheet constraint is

$$
\begin{equation*}
L_{t}+Q_{t}^{B} B_{t}^{L}=D_{t}+N_{t} . \tag{5}
\end{equation*}
$$

Long-term bonds are modeled as in Hatchondo and Martinez (2009) or Chatterjee and Eyigungor (2012): a bond matures with probability $\frac{1}{\tau}$ each period (so that the expected maturity is $\tau$ ) and yields a nominal payoff of 1 at maturity $]^{7}$ The (real) bond price in period $t$ is denoted $Q_{t}^{B}$. Noarbitrage implies that from $t=0$ forward, $\frac{1+i_{t}}{1+\pi_{t+1}} Q_{t}^{B}=\left(1-\frac{1}{\tau}\right) Q_{t+1}^{B}+\frac{1}{\tau} \frac{1}{1+\pi_{t+1}}$. That is, bond returns must equal the risk-free rate.

Banks have market power in setting deposit and loan rates, since each household and bankdependent firm is constrained to deal with a single bank. In each period, a bank sets a deposit rate $i_{t}^{D}$, a loan rate $i_{t}^{L}$, and bond holdings $B_{t}^{L}$, taking as given households' deposit demand $D_{t}\left(i_{t}^{D}, i_{t}\right)$, intermediate firms' loan demand $L_{t}\left(i_{t}^{L}\right)$, and the bond price $Q_{t}^{B}$.

Banks face two frictions in choosing the composition of their balance sheets. First, bank lending is constrained by net worth. A bank with net worth $N_{t}$ that issues loans $L_{t}$ incurs a cost $\Psi^{L}\left(N_{t}, L_{t}\right)$ that is homogeneous (of degree one), decreasing in net worth $N_{t}$, and increasing in loans $L_{t}$. The assumption that $\Psi^{L}$ is homogeneous implies that loan spreads will depend on banks' loan-to-net worth ratio $\frac{L_{t}}{N_{t}}$. This cost is a smooth approximation of the types of net worth constraints typically present in macro-finance models such as Kiyotaki and Moore (1997) or Bernanke, Gertler, and Gilchrist (1999). Second, it is costly for banks to lack a buffer of safe, liquid assets held against their deposits. We model this motive for banks to hold safe assets as cost $\Psi^{D}\left(Q_{t}^{B} B_{t}^{L}, D_{t}\right)$ that is homogeneous, decreasing in bond holdings $Q_{t}^{B} B_{t}^{L}$, and increasing in deposit issuance $D_{t}$. This type of cost may stem from regulation requiring banks to hold a buffer of safe assets against their liabilities, or it may come from banks' need to mitigate fire sales of loans when facing an unexpected outflow of deposits.

In each period, banks pay out a fixed fraction $\gamma \in\left(\beta^{-1}-1,1\right)$ of their net worth as dividends $\|_{8}^{8}$ so from $t=0$ forward, they accumulate net worth according to

$$
\begin{align*}
N_{t+1}=(1-\gamma)\left(\frac{1+i_{t}}{1+\pi_{t+1}} Q_{t}^{B} B_{t}^{L}\right. & \frac{1+i_{t}^{L}}{1+\pi_{t+1}} L_{t}\left(i_{t}^{L}\right)-\frac{1+i_{t}^{D}}{1+\pi_{t+1}} D_{t}\left(i_{t}^{D}, i_{t}\right)  \tag{6}\\
& \left.-\Psi^{L}\left(N_{t}, L_{t}\right)-\Psi^{D}\left(Q_{t}^{B} B_{t}^{L}, D_{t}\right)\right)
\end{align*}
$$

[^5]Banks' objective is to maximize the discounted stream of dividends,

$$
\begin{equation*}
\max _{B_{t}^{L}, i_{t}^{L}, i_{t}^{D}} \sum_{t=0}^{\infty} \beta^{t} \Lambda_{t} \gamma N_{t} \text { s.t. (5), (6), } \tag{7}
\end{equation*}
$$

where $\Lambda_{t}$ denotes the household's marginal utility of consumption, $\Lambda_{t}=\frac{\partial U\left(C_{t}, C_{t-1}, H_{t}\right)}{\partial C_{t}}$.
Banks enter $t=0$ with a maturity-mismatched position, consisting of loans $L^{*}$, a quantity $B^{L *}$ of long-term bonds, and outstanding short-term deposits $D^{*}$ (which are the quantities of loans, bonds, and deposits in the economy's long-run steady state, before the unanticipated $t=0$ monetary policy announcement). Due to the maturity mismatch, banks experience capital gains or losses at $t=0$, and their initial net worth is revalued to

$$
\begin{equation*}
N_{0}=N^{*}+(1-\gamma)\left(1-\frac{1}{\tau}\right)\left(Q_{0}^{B}-Q^{B *}\right) B^{L *} \tag{8}
\end{equation*}
$$

where $N^{*}$ is their steady-state net worth and $Q^{B *}$ is the steady-state bond price.

### 2.4 Final goods and capital producers

There is a representative final goods producer that aggregates differentiated varieties supplied by monopolistic retailers $k \in[0,1]$ (at prices $P_{t}^{k}$ ) in order to produce output, which it sells at a competitive price $P_{t}$. Its production function is $Y_{t}=\left(\int_{0}^{1} Y_{t}^{\frac{\varepsilon-1}{\varepsilon}} d k\right)^{\frac{\varepsilon}{\varepsilon-1}}$, where $\epsilon>1$ is the elasticity of substitution. Accordingly, demand for a variety $k$ with price $P_{t}^{k}$ is $\left(\frac{P_{t}^{k}}{P_{t}}\right)^{-\varepsilon} Y_{t}$, where $P_{t}$ is the usual CES price index.

There are representative capital goods producers $z \in\{b, n b\}$ that produce capital employed by bank-dependent and non-bank-dependent firms, respectively. Capital producers use the output of the final goods producer as an input. Capital in sector $z$ sells at (real) price $Q_{t}^{K, z}$ in competitive markets. The capital accumulation equation reads

$$
K_{t+1}^{z}=(1-\delta) K_{t}^{z}+I_{t+1}^{z}\left(1-\Xi\left(\frac{I_{t+1}^{z}}{I_{t}^{z}}\right)\right)
$$

where $I_{t}^{z}$ denotes investment at time $t$ in sector $z$ and $\Xi\left(\frac{I_{t+1}^{z}}{I_{t}^{z}}\right)$ is a function capturing costs in adjusting the rate of investment. The problem faced by capital producers is

$$
\begin{equation*}
\max _{I_{t}^{z}} \sum_{t=0}^{\infty} \beta^{t} \Lambda_{t}\left(Q_{t}^{K, z} I_{t+1}^{z}\left(1-\Xi\left(\frac{I_{t+1}^{z}}{I_{t}^{z}}\right)\right)-I_{t+1}^{z}\right) . \tag{9}
\end{equation*}
$$

### 2.5 Retailers

As in most New Keynesian models, there is a continuum of monopolistic retailers $k \in[0,1]$ that produce using intermediate goods purchased in competitive markets (using a production function $Y_{t}^{k}=X_{t}^{k}$, where $X_{t}^{k}$ denotes the quantity of intermediate inputs). Retailers set their prices
$P_{t}^{k}$ subject to Rotemberg (1982) adjustment costs (parameterized by $\theta>0$ ), taking final goods producers' demand $Y_{t}^{k}=\left(\frac{P_{t}^{k}}{P_{t}}\right)^{-\epsilon} Y_{t}$ as given. Their problem is

$$
\max _{P_{t}^{k}} \sum_{t=0}^{\infty} \beta^{t} \Lambda_{t}\left(\left(\frac{P_{t}^{k}}{P_{t}}\right)^{1-\epsilon} Y_{t}-P_{t}^{I}\left(\frac{P_{t}^{k}}{P_{t}}\right)^{-\epsilon} Y_{t}-\frac{\theta}{2}\left(\frac{P_{t}^{k}}{P_{t-1}^{k}}-1\right)^{2} Y_{t}\right) .
$$

### 2.6 Government and monetary policy

The government supplies long-term bonds elastically and sets lump-sum transfers (or taxes) $T_{t}$ in each period in order to be able to repay interest to bondholders. The central bank makes an unanticipated announcement of its monetary policy at $t=0$, under which it sets the nominal rate according to a rule

$$
\begin{equation*}
i_{t}=i\left(Y_{t}, \pi_{t}, i_{t-1}, \epsilon_{t}^{m p}\right), \tag{10}
\end{equation*}
$$

where $\left\{\epsilon_{t}^{m p}\right\}$ is an unanticipated sequence of monetary shocks whose value is realized at $t=0$ (since we assume perfect foresight). For our theoretical results, we will assume that the central bank employs a simple monetary policy rule meant to capture a negative shock to the interest rate. In our calibration, by contrast, we assume that the central bank follows a conventional Taylor rule. Both policy rules can be written in the general form 10 .

## 3 Bank loan supply and the reversal rate

In this section, we present analytical results demonstrating the existence of a reversal rate for loan supply in our model. Specifically, we hold the loan and deposit demand functions fixed at their steady-state values, and we characterize the impulse response of bank credit supply for a given interest rate cut at $t=0$. We provide conditions under which an interest rate cut beyond a certain point can be contractionary for credit supply at the margin. Finally, we apply our results to study the dynamic effects of monetary shocks and the effects of "low-for-long" monetary policies. All results are proven in Online Appendix B.

### 3.1 Setting

In order to prove sharp analytical results, we impose assumptions on some of our model's parameters. First, we assume that prices are fully rigid (i.e., we take the limit as price adjustment $\operatorname{costs} \theta$ go to infinity). This permits the central bank to control the real rate, which we assume is set according to the policy

$$
i_{t}=\left\{\begin{array}{cc}
i & t \leq T  \tag{11}\\
i^{*} & t>T
\end{array}\right.
$$

where $i^{*}=\beta^{-1}-1$ denotes the economy's long-run natural rate and $i<i^{*}$. That is, the central bank announces an unanticipated interest rate cut at time $t=0$ until time $T$, at which point the interest rate returns to its natural level. Given the announced sequence of interest rates, bond
prices satisfy

$$
\begin{equation*}
Q_{t}^{B}=\frac{1}{\tau} \sum_{s=0}^{\infty}\left(\prod_{r=0}^{s} \frac{1}{1+i_{t+r}}\right)\left(1-\frac{1}{\tau}\right)^{s} \text { and } Q^{B *}=\frac{1}{\tau} \frac{1+i^{*}}{1+\tau i^{*}} . \tag{12}
\end{equation*}
$$

Second, we assume that banks face a simple capital constraint on their lending and a liquidity constraint on their deposit issuance. Their costs of loan and deposit issuance take the form

$$
\Psi^{L}\left(N_{t}, L_{t}\right)=\left\{\begin{array}{cc}
0 & L_{t} \leq \psi^{L} N_{t}  \tag{13}\\
\infty & L_{t}>\psi^{L} N_{t}
\end{array}, \Psi^{D}\left(Q_{t}^{B} B_{t}^{L}, D_{t}\right)=\left\{\begin{array}{cc}
0 & Q_{t}^{B} B_{t}^{L} \geq \psi^{D} D_{t} \\
\infty & Q_{t}^{B} B_{t}^{L}<\psi^{D} D_{t}
\end{array}\right.\right.
$$

When financial frictions take this form, banks effectively face the constraints $L_{t} \leq \psi^{L} N_{t}$ and $Q_{t}^{B} B_{t}^{L} \geq \psi^{D} D_{t}$. A bank's lending cannot exceed a multiple $\psi^{L}$ of its net worth. Such constraints, which resemble capital requirements implemented in practice, are typical in models where bankers face moral hazard problems (e.g., Kiyotaki and Moore 1997, Gertler and Karadi 2011). Moreover, a bank's holdings of safe, liquid bonds must exceed a fraction $\psi^{D}$ of its deposit issuance. These types of constraints are usually present in models where banks face random deposit outflows (e.g., Drechsler et al. 2018, Bianchi and Bigio 2022) but can reflect regulatory requirements as well.

Third, we fix the loan and deposit demand curves at their steady-state values. Formally, in the loan demand equation (4), we fix the price level $P_{t}=1$ (since prices are fully rigid) and hold real wages $W_{t}$, the price of intermediate goods $P_{t}^{I}$, and the price of capital $Q_{t}^{K, b}$ at their steady-state values $W^{*}, P^{I *}$, and $Q^{K, b *}$, yielding a typical downward-sloping loan demand curve $L^{*}\left(i_{t}^{L}\right)$ that is time-invariant. Loan demand depends only on the loan rate; that is, bank-dependent firms do not substitute from loans to bonds when loan rates rise. Instead, in our model, this substitution takes place at the aggregate level: in general equilibrium, an increase in loan rates (all else equal) causes bank-dependent firms to reduce investment, putting downward pressure on the price of capital and allowing non-bank-dependent firms to profitably invest more.

We proceed analogously for deposit demand by fixing households' deposit demand curve $D^{*}\left(i_{t}^{D}, i_{t}\right)$, which depends on the deposit rate and the policy rate, at its steady-state level. In order to suppress general equilibrium effects, we hold households' total savings constant and derive deposit demand from households' optimal portfolio allocation across bonds, deposits, and cash given interest rates $i_{t}^{D}$ and $i_{t}$. Deposit demand in the steady state can be derived as the solution to the problem ${ }^{9}$

$$
\begin{gather*}
\max _{B_{t}, D_{t}, M_{t}} \zeta \Phi\left(\mathcal{L}\left(D_{t}, M_{t}\right)\right)+\beta \Lambda^{*}\left(\left(1+i_{t}\right) B_{t}+\left(1+i_{t}^{D}\right) D_{t}+M_{t}\right)  \tag{14}\\
\text { s.t. } B_{t}+D_{t}+M_{t} \leq S^{*}, B_{t}, D_{t}, M_{t} \geq 0,
\end{gather*}
$$

where $\Lambda^{*}$ denotes the household's marginal utility of consumption in steady state and $S^{*}$ denotes its steady-state savings. In this formulation, deposit demand depends separately on the deposit rate and the policy rate (rather than just on the deposit spread $i_{t}-i_{t}^{D}$ ) because of the presence of cash. This feature permits our model to capture the "deposits channel" of monetary policy studied

[^6]by Drechsler et al. (2017): the pass-through of the policy rate to the deposit rate is imperfect, so banks' profits from deposit issuance vary with the level of $i_{t}$.

Banks solve (7) with the household's discount factor held constant at its steady-state level $\Lambda^{*}$, taking as given the loan and deposit demand curves $L^{*}\left(i_{t}^{L}\right)$ and $D^{*}\left(i_{t}^{D}, i_{t}\right)$, the sequence of interest rates 11), and bond prices $Q_{t}^{B}$. Banks' initial net worth $N_{0}$ is determined according to (8), with the initial bond price $Q_{0}^{B}$ and the steady-state bond price $Q^{B *}$ given by 12 .

### 3.2 Definition of the reversal rate

In this setting, we will be interested in studying the impulse response of lending (i.e., credit supply) to the policy rate cut (11). Until noted otherwise, we hold the length of the policy shock $T$ fixed and consider the response of bank lending to different initial interest rates $i$. We let $L_{t}(i)$ denote bank lending at time $t$ (in partial equilibrium) when the interest rate is cut to $i<i^{*}$ at $t=0$. We define the time-t reversal rate to be the interest rate below which further cuts are contractionary for lending at time $t$ (given the fixed length $T$ of the policy shock).

Definition 1. A time-t reversal rate is the highest interest rate $i_{t}^{R R}$ such that $L_{t}(i)$ is increasing in $i$ for all $i<i_{t}^{R R}$
The reversal rate relates to the marginal effect of an interest rate cut on lending. If $i_{t}^{R R}$ is the time- $t$ reversal rate, then if the central bank cuts rates to some $i<i_{t}^{R R}$, lending at time $t$ is less than it would have been if the bank had cut rates only to $i_{t}^{R R}$. However, it may still be the case that time- $t$ lending $L_{t}(i)$ under this policy is greater than lending in the economy's long-run steady state, so that the total effect of the rate cut on lending is still positive.

Note that the reversal rate is permitted to depend on the horizon $t$ of the impulse response function. That is, in principle, it is possible that decreasing the policy rate below $i$ may be contractionary for lending at some time $t$ while at the same time stimulating lending at some other time $s$. In Section 3.4, we will provide conditions under which the reversal rate is increasing as a function of the horizon, so that if an interest rate cut is contractionary for time- $t$ lending at the margin, then it is also contractionary for lending at all times $s>t$.

### 3.3 The bank's problem

We now solve the bank's problem in the setting of Section 3.1. By holding loan and deposit demand constant, we will be able to isolate the effect of the monetary policy shock (11) on bank credit supply and theoretically provide conditions under which a reversal rate exists. In our calibration exercise, we demonstrate that these effects are strong enough to survive in general equilibrium, giving rise to a situation in which an interest rate cut causes a contraction in aggregate output.

The bank's problem (7) reduces to choosing long-term bond holdings $B_{t}^{L}$ as well as loan and deposit rates $i_{t}^{L}, i_{t}^{D}$ to maximize net interest income period-by-period ${ }^{11}$ In this environment, given

[^7]that the loan and deposit demand curves are static, a bank's net interest income in a period depends only on its net worth $N_{t}$ and the policy rate $i_{t}$.
\[

$$
\begin{align*}
\operatorname{NII}\left(N_{t}, i_{t}\right)= & \max _{B_{t}^{L}, i_{t}^{L}, i_{t}^{D}} i_{t} Q_{t}^{B} B_{t}^{L}+i_{t}^{L} L^{*}\left(i_{t}^{L}\right)-i_{t}^{D} D^{*}\left(i_{t}^{D}, i_{t}\right)  \tag{15}\\
& \text { s.t. (5), } L^{*}\left(i_{t}^{L}\right) \leq \psi^{L} N_{t}, Q_{t}^{B} B_{t}^{L} \geq \psi^{D} D^{*}\left(i_{t}^{D}, i_{t}\right)
\end{align*}
$$
\]

The solution to the bank's problem 15 can be written in compact form as

$$
\begin{gather*}
i_{t}^{L}=\underbrace{i_{t}}_{\text {Marginal cost }}+\underbrace{\frac{1}{\varepsilon_{t}^{L}}}_{\text {Mark-up }}+\underbrace{\lambda_{t}+\mu_{t}}_{\text {Constraints }}  \tag{16}\\
i_{t}^{D}=\underbrace{i_{t}}_{\text {Marginal benefit }}-\underbrace{\frac{1}{\varepsilon_{t}^{D}}}_{\text {Mark-down }}+\underbrace{\left(1-\psi^{D}\right) \mu_{t}}_{\text {Liq. constraint }} \tag{17}
\end{gather*}
$$

Here, $\varepsilon_{t}^{L}$ and $\varepsilon_{t}^{D}$ denote the semi-elasticities of loan and deposit demand with respect to $i_{t}^{L}$ and $i_{t}^{D}$, respectively ${ }^{12}$ The terms $\lambda_{t}$ and $\mu_{t}$ represent the Lagrange multipliers on the bank's net worth and liquidity constraints, respectively. When the constraints are slack, the Lagrange multipliers are simply zero and loan (deposit) rates are set to the policy rate $i_{t}$ plus a mark-up (minus a markdown); when they bind, their values are defined by the FOCs, and actual rates are determined by the constraints.

When banks are unconstrained, (16) and (17) illustrate that monetary policy is transmitted to credit supply through the standard bank lending channel of monetary policy. A decrease in $i_{t}$, the return on bonds, reduces banks' opportunity cost of lending, so they offer lower loan rates to borrowers, increase their lending activity, and finance this lending by setting lower deposit rates.

We will be particularly interested in the case in which banks' net worth constraints bind. In this regime, their lending is fully determined by their net worth, $L_{t}(i)=\psi^{L} N_{t}(i)$. The standard bank lending channel of monetary policy is therefore shut down. Instead, monetary policy is transmitted to banks' credit supply through two channels by which interest rate cuts affect banks' net worth: a capital gains channel and a net interest income channel. A rate cut announced at $t=0$ leads to a revaluation of assets initially held on bank balance sheets. Since banks enter with a maturity mismatch, an interest rate cut has an unambiguous positive effect on bank net worth through the capital gains channel. On the other hand, it is possible for a policy rate cut to depress banks' net interest income, thereby reducing their net worth. For instance, when rates are in negative territory, it is difficult for banks to fully pass on rate cuts to their depositors, who have the option to hold cash 13

The case in which banks are liquidity-constrained is quite similar and less empirically relevant-

[^8]banks have tended to hold excess liquidity even in countries with negative rates. We therefore postpone discussion of this regime to Online Appendix B.1.

### 3.4 Existence of the reversal rate

The discussion above suggests that when banks are constrained, an interest rate cut can trigger a reversal in bank lending if the deterioration of banks' interest income outweighs the effect of capital gains, $\frac{d N_{t}(i)}{d i}<0$. Figure 1 illustrates that under these conditions, we can expect that a reversal rate will exist: interest rate cuts eventually depress banks' net worth enough to constrain their lending. Until banks become constrained, rate cuts stimulate lending. Past that point, further cuts reduce net worth and credit supply. The following proposition provides a characterization of the reversal rate in line with this logic.

Proposition 1 (Characterization). Suppose $i$ is the highest interest rate satisfying the following two properties:

1. Either the net worth constraint or the liquidity constraint binds at for all $i^{\prime} \leq i$;
2. Time-t net worth is increasing in the interest rate, $\frac{d N_{t}(i)}{d i}>0$, for all $i^{\prime}<i$.

Then $i$ is the time- $t$ reversal rate $i_{t}^{R R}$.
It is worth pointing out that this characterization of the reversal rate says nothing about whether the reversal rate should be positive or negative. Indeed, in our calibration, we find that even though the reversal rate for short horizons is negative, it is possible for the reversal rate at long horizons to be slightly positive.

Applying the Envelope Theorem to banks' problem (15), the effect of a change in interest rates on net interest income $N I I_{t}$ at time $t$, holding net worth fixed, is

$$
\begin{equation*}
\frac{\partial N I I\left(N_{t}, i_{t}\right)}{\partial i_{t}}=Q_{t}^{B} B_{t}^{L}+\left(i_{t}-i_{t}^{D}\right) \frac{\partial D^{*}\left(i_{t}^{D}, i_{t}\right)}{\partial i_{t}} \tag{18}
\end{equation*}
$$

Intuitively, an interest rate cut reduces the interest income that banks receive from their bond holdings. However, it can also generate an inflow of deposits: bonds become less attractive relative to deposits, so households are encouraged to deposit at the bank $\left(\frac{\partial D^{*}\left(i_{t}^{D}, i_{t}\right)}{\partial i_{t}}<0\right)$. The bank can then decrease its deposit rate without fearing an erosion of its deposit base, allowing it to maintain its interest margins. When this substitution effect is weak enough, a reduction in $i$ (holding net worth fixed) is guaranteed to reduce bank net interest income, meaning that rate cuts exert downward pressure on bank net worth through the net interest income channel.

In fact, in our setting, the presence of cash implies that the substitution from bonds to deposits is shut down when the policy rate is sufficiently low.

Lemma 1. There exists $\underline{i}$ such that deposit demand is independent of the policy rate, $\frac{\partial D^{*}\left(i_{t}^{D}, i_{t}\right)}{\partial i_{t}}=0$, for all $i_{t} \leq \underline{i}$.


Figure 1: An illustration of how a reversal may take place. In the left panel, the policy rate is cut from $i=0.03$ to $i^{\prime}=0.01$. Bank net worth is reduced from $N$ to $N^{\prime}$, but banks remain unconstrained, so the interest rate cut stimulates credit supply. In the right panel, the policy rate is cut further from $i^{\prime}=0.01$ to $i^{\prime \prime}=-0.01$. Bank net worth is again reduced from $N^{\prime}$ to $N^{\prime \prime}$, and banks become constrained, reducing the supply of credit.

When $i<0$, at least, households no longer hold bonds, since cash delivers a higher return. Households then do not substitute into deposits from bonds following a policy rate cut. Therefore, when interest rates are low enough, a further interest rate cut is guaranteed to decrease banks' net interest income: banks' income from their bond holdings is reduced, but they cannot pass lower rates on to depositors without losing some of their deposit base ${ }^{14}$

Lemma 2. Banks' net interest income is increasing in the policy rate $i_{t}$ (holding net worth $N_{t}$ fixed) when rates are sufficiently low: $\frac{\partial N I I\left(N_{t}, i_{t}\right)}{\partial i_{t}}>0$ for all $N_{t}$ and $i_{t} \leq \underline{i}^{15}$

We can then decompose the effect of an interest rate cut on bank net worth into the part attributable to the capital gains channel and the part attributable to the interest income channel. To this end, define $N_{t}\left(N_{0}, i\right)$ to be bank net worth at time $t$ when the bank's initial net worth is $N_{0}$ and the policy rate is cut to $i i^{16}$ Initial net worth $N_{0}$ is itself a function of $i$, determined by (8) and (12). Then

$$
\begin{equation*}
\frac{d N_{t}}{d i}=\underbrace{\frac{\partial N_{t}}{\partial N_{0}} \frac{d N_{0}}{d i}}_{\text {Capital gains }}+\underbrace{\frac{\partial N_{t}}{\partial i}}_{\text {NII }} . \tag{19}
\end{equation*}
$$

With this decomposition, it is possible to prove two simple facts.
Lemma 3. The strength of the capital gains channel, $\frac{\partial N_{t}}{\partial N_{0}} \frac{d N_{0}}{d i}$, approaches zero as either:

[^9]- The horizon $t \rightarrow \infty$, or
- Steady-state bond holdings $B^{L *} \rightarrow 0$ (holding steady-state net worth $N^{*}$ fixed) 17

This result captures the fact that the capital gains channel is less relevant (1) when considering bank net worth far in the future, or (2) when banks' initial bond holdings are small. Banks' initial capital gains are largely irrelevant in determining their net worth far in the future, and when banks do not initially have a significant maturity mismatch, the revaluation of their assets triggered by an interest rate cut is limited.

On the other hand, when an interest rate cut decreases banks' interest income, then the effect of an interest rate cut through the net interest income channel is to reduce bank profitability and net worth at all future dates.

Lemma 4. The effect captured by the interest income channel, $\frac{\partial N_{t}}{\partial i}$, is positive whenever $i<\underline{i}$ (defining $\underline{i}$ as in Lemma 1) and $t<T$.

These two results can be used to establish sufficient conditions for the existence of a reversal rate.

Proposition 2 (Existence for small maturity mismatch). There exists $\underline{B}^{L}$ such that whenever banks' steady-state bond holdings are $B^{L *} \leq \underline{B}^{L}$, a time-t reversal rate exists for all $0<t \leq T$.

Proposition 3 (Existence for long horizons). There exists $T$ such that when the length of the policy shock (11) is $T \geq \underline{T}$, a time-t reversal rate exists for all $t \in[\underline{T}, T]$.

That is, there exists a time- $t$ reversal rate whenever banks' initial bond holdings are sufficiently small or when the horizon $t$ considered is sufficiently long. From Lemmas 3 and 4 it is clear why this should be the case. As interest rates are cut into negative territory, intermediation booms and banks lever up to their constraints. However, their profit margins are compressed. For a small initial maturity mismatch, or long horizons, the capital gains channel is weak. The interest income channel dominates, so a further interest rate cut drags down bank net worth at time $t$, which then causes a reduction in aggregate lending.

Of course, these results provide somewhat weak guarantees: the existence of a reversal rate is not guaranteed at every horizon, nor is it guaranteed when banks hold large quantities of safe bonds. However, in our quantitative application, we show that for a realistic calibration, it is indeed the case that a reversal rate exists at all horizons, even in general equilibrium.

The logic underlying our existence results also suggests something about the dynamic response of bank credit supply to monetary shocks. Since the capital gains channel becomes weak relative to the net interest income channel at long horizons, an interest rate cut can stimulate lending in the short run while causing a contraction in lending in the long run. In our model, this can occur if the time- $t$ reversal rate $i_{t}^{R R}$ is increasing in $t$, e.g. if $i_{t}^{R R}<i<i_{t+s}^{R R}$.

[^10]Of course, this logic holds only if banks' net worth constraints actually bind. Hence, we assume liquidity demand is large enough that banks are capital-constrained rather than liquidityconstrained: deposit demand is sufficient to ensure banks always have ample funds to invest in safe bonds.

Lemma 5. There exists $\underline{\zeta}$ such that if the liquidity demand parameter $\zeta \geq \underline{\zeta}$, then banks' liquidity constraints are slack in each period $t$ for any interest rate $i$ announced by the central bank at $t=0$.

As long as reversals in lending are triggered by banks' net worth constraints, the time- $t$ reversal rate is increasing as a function of $t$.

Proposition 4 (Dynamics of the reversal rate). Suppose $\zeta \geq \zeta$. If a time-t reversal rate $i_{t}^{R R}<\underline{i}$ exists for some $t<T$, then $i_{t+1}^{R R}$ exists and is (weakly) greater than $i_{t}^{R R}$.

Proposition 4 implies that a central bank that attempts to stimulate lending in the long run cannot infer the success of its policy from the short-term response of bank credit: an initially stimulative interest rate cut can eventually backfire, reducing total lending over the horizon considered. In Section 4, we demonstrate that this result is borne out quantitatively as well.

## 3.5 "Low-for-long" monetary policies

A key question facing a central bank attempting to stimulate the economy is whether to implement "low-for-long" monetary policies, promising extended periods of low interest rates. This consideration has become especially relevant in the past decade, as inflation and demand have at times remained stubbornly below target. In this section, we study the implications of low-for-long interest rate environments for bank credit supply and provide conditions under which they can be contractionary in the long run.

We again consider monetary policies of the form (11). Up until this point, we have held the length of the shock $T$ fixed and considered comparative statics with respect to the level of the policy rate $i$. In this section, instead, we consider comparative statics with respect to $T$ with $i$ held fixed. We now make the dependence on $T$ explicit and let $L_{t}(i, T)$ denote lending at time $t$ when the central bank announces that it will set interest rate $i$ until time $T$. Our interest is in characterizing how the impulse response of bank credit supply, $L_{t}(i, T)$, depends on $T$.

In our model, a low-for-long policy can compress banks' interest margins and drain their net worth. Therefore, if interest rates are held at $i$, banks' lending can eventually contract if their net interest income at that level of rates is insufficient to permanently sustain the steady-state level of lending $L^{*}$, as implied by the following inequality:

$$
\begin{equation*}
N I I(N, i) \leq \frac{\gamma}{\psi^{L}} L^{*} \forall N \leq N^{*} . \tag{20}
\end{equation*}
$$

This inequality is guaranteed to hold if banks' net worth constraints are sufficiently tight ( $\psi^{L}$ is low), or if their payout rate $\gamma$ is sufficiently high. Moreover, it will tend to hold when banks' profitability is low, e.g., if loan and deposit demand are elastic enough.

Our main result on the effect of low-for-long policies is then:
Proposition 5 (The effects of low-for-long rates). Fix $i<i^{*}$, and suppose that (20) holds when the policy rate is $i$. Then there exists $\underline{T}$ such that if $T>\underline{T}, L_{t}(i, T)<L^{*}$ for all $t \in[\underline{T}, T]$.

Simply put, as the horizon of the interest rate cut is extended, eventually there comes a point at which bank lending contracts. Unlike our previous results, which were about the marginal effects of monetary stimulus, Proposition 5 implies that the total effect of the stimulus is contractionary (in the sense that lending eventually falls below its steady-state level). ${ }^{18}$

The intuition is straightforward: extending the length of the interest rate cut further and further has diminishing returns in terms of capital gains at $t=0$. By contrast, extending the interest rate cut decreases banks' net worth at future dates through the net interest income channel (and this effect does not weaken with the horizon $T$ ). Therefore, at some point, the loss of interest income outweighs the initial capital gains, causing banks to become constrained and resulting in a contraction of lending. When banks face financial constraints, "low-for-long" policies are bound to eventually become counterproductive.

In our calibration, inequality (20) holds for low values of $i$, so low-for-long policies indeed eventually become contractionary for bank lending. General equilibrium effects imply that such policies will lead to a recession and a reduction in aggregate investment as well. This prediction stands in stark contrast to standard New Keynesian models, in which promises to keep interest rates low for extended periods provide implausibly strong stimulus (the forward guidance puzzle). In Section 4.6, we therefore demonstrate that when embedded in a quantitative model, our mechanism greatly dampens the power of forward guidance.

### 3.6 Discussion of assumptions

Our model embeds several particular assumptions about the form of loan and deposit demand, so it is natural to wonder the extent to which the results generalize beyond the specific setting considered here.

In order to address this question, we consider the partial equilibrium problem of a monopolistic bank (15) that faces arbitrary loan and deposit demand functions of the form $L^{*}\left(i^{L}, i\right)$ and $D^{*}\left(i^{D}, i\right)$ (rather than the ones implied by our model). We assume that

- Deposit demand $D^{*}\left(i^{D}, i\right)$ is continuous, differentiable, increasing in its first argument, and weakly decreasing in its second argument;
- Loan demand $L^{*}\left(i^{L}, i\right)$ is continuous, differentiable, decreasing in its first argument, and weakly increasing in its second argument. Moreover, $L^{*}\left(i^{L}, i\right) \rightarrow \infty$ as $i^{L} \rightarrow-\delta$.

[^11]These conditions simply guarantee that banks' optimization problem is well-defined and that loan demand becomes arbitrarily large as the user cost of capital approaches zero (as in standard macroeconomic models).

All of our main analytical results follow from imposing two additional properties on top of these basic assumptions.

Property 1. When the policy rate is low enough, further interest rate cuts decrease banks' net interest income. There exists $\underline{i}$ such that for all $N$ and $i<\underline{i}, \frac{\partial N I I(N, i)}{\partial i}>0$.
Property 2. Banks' net worth imposes a constraint on their lending, $\psi^{L} \in(0, \infty)$.
Note that the liquidity constraint, $\psi^{D}>0$, is not essential. In our model, it aids only in proving Property 1, since a reduction in $i$ reduces banks' interest income from their bond holdings.

As we demonstrate in Online Appendix B.2, if the loan and deposit demand functions satisfy Property 1 and there is a net worth constraint on bank lending (Property 2), then analogues of Propositions 2 and 3 hold: a reversal rate exists for sufficiently long horizons or when banks' bond holdings are small enough. That is, the existence of the reversal rate depends only on the fact that interest rate cuts in negative territory reduce bank profits and therefore lending as well. These properties also suffice to prove an analogue of our low-for-long result, Proposition 5 . For these reasons, our model's main results should be interpreted as providing predictions that could be expected to hold in a much more general class of economies that are consistent with the observed behavior of bank profits and lending. Proposition 4 additionally requires a condition on deposit demand that we describe in further detail in the Online Appendix, which implies banks are not liquidity-constrained in equilibrium.

Importantly, both of the key properties have empirical support. There is broad agreement in the literature that, at least during the recent period of low rates in Europe, decreases in interest rates have had adverse effects on bank' net interest margins (Alessandri and Nelson 2015, Claessens et al. 2018, Borio et al. 2017). More consequential is the assumption that bank net worth is an important determinant of lending, which has been the subject of some debate. Heider et al. (2019), for instance, demonstrate that when rates entered negative territory, banks that were more reliant on deposits (and therefore experienced a greater decline in profits) reduced their lending volumes relative to other banks, but they also increased their risk-taking. Bittner et al. (2022) find that net worth was a key driver of lending for Portuguese banks during the period of negative rates as well. Other papers, such as Bräuning and Wu (2017) and Wang (2022) have found that lending volumes respond positively to further monetary accommodation when rates are low, but these results are not inconsistent with our proposed mechanism: in the model, interest rate cuts can increase bank net worth even in negative territory through the capital gains channel.

## 4 Quantitative Evaluation

In this section, we investigate the quantitative relevance of our theoretical mechanisms and demonstrate that the reversal rate is also present in general equilibrium. We begin by describing

Table 1: Conventional DSGE parameters.

| Parameter | Description | Value |
| :---: | :---: | :---: |
| $\sigma$ | IES parameter | 1 |
| $h$ | Habit formation | 0.62 |
| $\varphi$ | Inverse Frisch elasticity | 2 |
| $\delta$ | Capital depreciation | 0.025 |
| $\alpha$ | Capital share | 0.36 |
| $\varepsilon$ | Retail price elasticity | 3.9 |
| $\theta$ | Rotemberg cost | 70.7 |
| $\phi^{\pi}$ | Taylor rule inflation coefficient | 2.74 |
| $\rho^{m p}$ | Taylor rule persistence | 0.93 |

our calibration strategy. Our theoretical model is somewhat stylized, so in this section we also outline the adjustments to our model needed to better match the data. Then, we analyze the economy's response to monetary policy shocks in order to provide an estimate of the reversal rate. Finally, we analyze the sensitivity of our results to the model's key parameters and illustrate the implications of our "low-for-long" result for the power of forward guidance.

### 4.1 Solution concept

We solve the model under perfect foresight, following an unanticipated monetary policy shock at $t=0$. Our solution algorithm (implemented in Dynare) finds the full nonlinear solution of the corresponding system of equations and thus does not rely on perturbation methods. This is important because our economy inherently features large non-linearities and non-monotonicities.

### 4.2 Calibration strategy

We calibrate our model to the Euro area, where negative interest rates were first implemented in 2014. We set the length of a period to one quarter.

Conventional parameters: Several of the parameters in our model have conventional values in the DSGE literature. These are the preference parameters $(\sigma, h, \varphi)$, technology parameters ( $\delta, \alpha$ ), the parameters $(\varepsilon, \theta)$ describing the elasticity of substitution across monopolistic retailers' goods and the cost of adjusting their prices, and the parameters of the Taylor rule we will specify. The parameter values are summarized in Table 1, and Online Appendix C provides sources for the value of each parameter.

Households: It remains to calibrate two of households' conventional preference parameters: the subjective discount factor $\beta$ and the disutility of labor $\chi$. We set $\beta=0.995$ to match a real interest rate of $2 \%$ per annum, and we set $\chi=0.41$ as a normalization so that households work a quarter of their available time in the economy's steady state.

We must then specify the payoff $\Phi(\mathcal{L})$ that households derive from liquid asset holdings, the liquidity aggregator $\mathcal{L}(D, M)$, and the parameter $\zeta$ that scales their demand for liquid assets. We
assume deposits and cash are perfect substitutes, so liquid asset holdings can be written as

$$
\mathcal{L}(D, M)=D+M
$$

We make this assumption to ensure that the deposit rate behaves as it does in the data: when the policy rate $i_{t}$ is positive, the deposit spread $i_{t}-i_{t}^{D}$ is positive as well, but when $i_{t}$ goes negative, the deposit rate remains stuck at zero. The utility from liquid assets $\Phi$ has a satiation point $\mathcal{L}^{*}$,

$$
\zeta \Phi(\mathcal{L})=-\frac{1}{2} \zeta\left(\mathcal{L}^{*}-\min \left\{\mathcal{L}, \mathcal{L}^{*}\right\}\right)^{2} .
$$

The parameters $\zeta$ and $\mathcal{L}^{*}$ will be calibrated to match banking data, so we specify their values when we describe the calibration of the banking sector ${ }^{19}$

Finally, we make one change to the specification of households' portfolio allocation problem. The goal of our model is to analyze a novel transmission channel of monetary policy through bank lending. In order to isolate this channel, aggregate demand and the investment of non-bankdependent firms should respond to monetary stimulus exactly as they would in a conventional model. Therefore, we would like households to discount at the policy rate (which, in turn, causes non-bank-dependent firms to discount at the policy rate, since they issue bonds directly to households).

In our benchmark model, the issue is that households may invest unlimited quantities in liquid assets, so they do not hold bonds when interest rates are in negative territory: holding cash strictly dominates investing in bonds. In order to allow the aggregate demand channel to operate when rates are in negative territory, we assume that households face an additional constraint: they may not invest more than a quantity $\overline{\mathcal{L}}$ of funds in liquid assets, so we add the constraint $D_{t}+M_{t} \leq \overline{\mathcal{L}}$ to households' problem. This constraint can be motivated by the fact that (1) it is costly to hold cash in large quantities, and (2) retail depositors are typically limited in the quantities they can deposit. Alternatively, this constraint could be viewed as a reduced-form representation of the fact that some investors have a "preferred habitat" and save in bonds rather than deposits or cash 20 With this constraint, when rates are in negative territory, at the margin households will decide between consumption and investment in bonds, so they indeed discount at the rate $1+i_{t}$. For convenience, we set the limit $\overline{\mathcal{L}}$ on liquid assets equal to the satiation point $\mathcal{L}^{*}$, so that there is not a discontinuous jump in deposit holdings when the policy rate goes into negative territory.

Intermediate goods firms: The curvature parameter $\nu$ in firms' production function is set to 0.85 to match a steady-state consumption-investment ratio of 2.7 , close to the value reported by Coenen et al. (2019). We identify bank-dependent firms with small and medium enterprises

[^12](SMEs). In Eurostat data, such firms comprise $99.8 \%$ of the total universe of firms and account for $55.8 \%$ of output. Hence, we set the fraction of bank-dependent firms $\xi=0.998$ and the relative productivity of bank-dependent firms $\frac{A^{b}}{A^{n b}}=0.42$ to match these two targets. Since such a large proportion of firms are bank-dependent, their productivity must be significantly lower to account for the fact that non-bank-dependent firms produce a large share of aggregate output. In our sensitivity analysis in Online Appendix C, we show that what matters for our results is the share of output produced by bank-dependent firms rather than the fraction $\xi$ of firms that are bank-dependent. Finally, the productivity of non-bank-dependent firms $A^{n b}$ is normalized to one.

Banks: We begin by making an adjustment to our model of banks. In our benchmark model, we assumed for analytical convenience that banks are monopolists in both the deposit and loan markets. However, when the parameters determining aggregate loan and deposit demand are calibrated realistically, the resulting demand curves are relatively inelastic, resulting in spreads that are quantitatively too large if we maintain the assumption of complete monopoly. In order to match spreads, in our quantitative model we instead assume that banks engage in monopolistic competition: loans and deposits provided by different banks are imperfect substitutes, as in Gerali et al. (2010). The household has a constant elasticity of substitution across deposits issued by different banks, and they can substitute towards cash when deposit rates go negative. The household's demand for bank $j$ 's deposits is

$$
D_{j t}=\left\{\begin{array}{cc}
\left(\frac{1+i_{j t}^{D}}{1+i_{t}^{D}}\right)^{-\varepsilon^{D}} D_{t} & i_{j t}^{D} \geq 0  \tag{21}\\
0 & \\
i_{j t}^{D}<0
\end{array}\right.
$$

where $D_{t}$ is aggregate deposit demand at $t, \varepsilon^{D}<-1$ is the elasticity of substitution across deposits provided by different banks, and $1+i_{t}^{D}$ is the usual CES price index given the rates set by individual banks. We derive this demand curve explicitly in Online Appendix D. The important implication of this deposit demand curve is that aggregate deposit rates cannot go negative, since cash is a perfect substitute.

We also make one additional change to banks' problem: banks earn a marginal benefit $\mu^{D}$ per deposit issued. This income is meant to represent benefits banks receive from issuing deposits, e.g. fees charged to depositors or the benefits of using a relatively stable source of financing. This assumption helps our model to rationalize the fact that banks continue to take deposits even when rates are deep in negative territory: by issuing deposits, banks receive the benefit $\mu^{D}$ and can lend the proceeds to firms, making an additional spread $i^{L}-i$. We set $\mu^{D}=50 \mathrm{bp}$ per annum to match the fees charged by German banks during the recent period of low rates ${ }^{21}$ Under this calibration, banks are always willing to take deposits for the interest rate cuts we consider, and equilibrium

[^13]deposit rates are set according to
\[

$$
\begin{equation*}
1+i_{t}^{D}=\max \left\{\frac{\varepsilon^{D}}{\varepsilon^{D}-1}\left(1+i_{t}+\mu^{D}\right), 1\right\} . \tag{22}
\end{equation*}
$$

\]

Similarly, a firm's demand for loans provided by bank $j$ is given by $L_{j t}=\left(\frac{1+i_{j t}^{L}}{1+i_{t}^{L}}\right)^{-\varepsilon^{L}} L_{t}$ instead of (4). Here, $\varepsilon^{L}>1$ is the elasticity of substitution across loans provided by different banks (which is the key parameter we calibrate to match loan spreads), $1+i_{t}^{L}$ is the usual CES price index, and $L_{t}$ is aggregate loan demand at $t$. When banks engage in monopolistic competition in the loan market, equilibrium loan rates are set according to

$$
\begin{equation*}
1+i_{t}^{L}=\frac{\varepsilon^{L}}{\varepsilon^{L}-1}\left(1+i_{t}+\frac{\partial \Psi^{L}}{\partial L_{t}}\right) . \tag{23}
\end{equation*}
$$

These specifications of loan and deposit demand can be micro-founded either by (1) directly assuming that the total funds raised by a firm are a CES aggregate of the quantity borrowed from different banks or by (2) assuming that firms face a discrete choice problem and face random taste shocks that affect their costs of borrowing from each bank (as in Anderson, De Palma, and Thisse 1989). Similarly, the specification of deposit demand can be derived by directly assuming a CES utility over different banks' deposits or through a discrete choice framework. Online Appendix D discusses this issue further. We set $\varepsilon^{L}=200$ to target a loan spread of $2 \%$ in steady state, as reported by Freriks and Kakes (2021), and we set $\varepsilon^{D}=-275$ to target a steady-state deposit spread of $1 \%$, computed from the ECB's MIR database.

We now calibrate the remaining parameters determining households' deposit demand and banks' balance sheet composition. First, we set banks' dividend payout ratio $\gamma$, the deposit demand parameter $\zeta$, and the maturity of long-term bonds $\tau$ to match three targets: a steady-state net worth-to-loan ratio of 0.155 (corresponding to an average Tier-1 capitalization ratio of $15.5 \%$ reported by Altavilla et al. 2018), a steady-state loan-to-bond ratio on bank balance sheets of 3.6, and an average maturity of bank bond holdings of 3.4 years (both documented by Hoffmann et al. 2019). Finally, we set the satiation point of liquid asset demand equal to $\mathcal{L}^{*}=6.93$ in order to match the increase in the deposit-to-GDP ratio reported in the ECB's MFI data from 2000 until 2014, which is when interest rates first went negative.

Next, we parameterize the cost functions $\Psi^{L}(N, L)$ and $\Psi^{D}\left(Q^{B} B^{L}, D\right)$ faced by banks, corresponding to their capital and liquidity constraints, respectively. We assume that the function $\Psi^{L}$ is such that banks' marginal cost of lending, $\frac{\partial \Psi^{L}}{\partial L}$, is a convex function of the loan-to-equity ratio $\frac{L_{t}}{N_{t}}$,

$$
\frac{\partial \Psi^{L}\left(N_{t}, L_{t}\right)}{\partial L_{t}}=\kappa^{L}\left(\max \left\{\frac{L_{t}}{N_{t}}-\frac{L^{*}}{N^{*}}, 0\right\}\right)^{2},
$$

where $\frac{L^{*}}{N^{*}}$ is the loan-to-equity ratio in steady state ${ }^{22}$ Under this specification, banks have a loan-

[^14]to-equity target ratio $\frac{L^{*}}{N^{*}}$, and they pay a convex marginal cost for deviating from that target. The parameter $\kappa^{L}$ is set to 0.018 so that a 1 percentage point increase in banks' target capitalization ratio $\frac{L^{*}}{N^{*}}$ results in a 28bp increase in loan rates, as estimated by Macroeconomic Assessment Group (2010). ${ }^{23}$ Parameter $\kappa^{L}$ will be key in our calibration, since it modulates the strength of banks' capital constraints. We set $\Psi^{D}$ as in (13) and assume $\psi^{D}=0$. Quantitatively, this parameter is irrelevant because in the model (as in the data), banks actually issue more deposits and therefore hold more bonds as rates decline below zero.

For our analytical results, we assumed that banks' net worth followed a particularly simple process: they simply paid out a fixed fraction of their net worth as dividends each period. To obtain more realistic dynamics of bank net worth, we make an adjustment to the process followed by bank equity. We assume that after paying out a fraction $\gamma$ of their net worth as dividends, banks additionally receive a fixed quantity of new funds $\hat{N}$ from the household at the beginning of each period ${ }^{24}$ This assumption allows us to separate banks' net worth-to-asset ratio (which will be governed by the parameter $\gamma$ ) from the persistence of their net worth, which would otherwise be too sluggish to recover from downturns. Our assumption is therefore conservative in the sense that it prevents the model from overstating the negative consequences of interest rate cuts for bank net worth in the long run. We interpret the funds $\hat{N}$ injected into banks each period as new equity issuance, so we set $\hat{N}=0.06$ to match a $1 \%$ annual equity issuance-to-asset ratio on bank balance sheets (consistent with the ECB's MFI data).

Capital goods producers: Capital goods producers solve Problem (9). As is typically assumed, adjustment costs take a quadratic form,

$$
\Xi\left(\frac{I_{t+1}^{z}}{I_{t}^{z}}\right)=\frac{\kappa^{I}}{2}\left(\frac{I_{t+1}^{z}}{I_{t}^{z}}-1\right)^{2}
$$

for $z \in\{b, n b\}$. We set $\kappa^{I}=5$ so that the elasticity of investment to a shock to the price of capital is $\frac{1}{\kappa^{I}}=0.2$, as estimated by Smets and Wouters (2003) for the Euro area.

Monetary policy: Unlike in our stylized theoretical model, for our calibration exercise we assume that monetary policy follows a conventional Taylor rule with inertia. There is an unanticipated monetary shock at $t=0$, but from that point forward, the economy is deterministic. Hence, the monetary policy rule can be written as

$$
\begin{equation*}
\frac{1+i_{t}}{1+i^{*}}=\left(\frac{1+i_{t-1}}{1+i^{*}}\right)^{\rho^{m p}}\left(\frac{1+\pi_{t}}{1+\pi^{*}}\right)^{\phi^{\pi}\left(1-\rho^{m p}\right)} \exp \left(\epsilon_{t}^{m p}\right) \tag{24}
\end{equation*}
$$

ratio.
${ }^{23}$ Specifically, let $\frac{N^{*}}{L^{*}}=0.155$ denote our estimated value of banks' steady-state capitalization ratio. We choose $\kappa^{L}$ so that $0.0028=\kappa^{L}\left(\frac{L^{*}}{N^{*}}-\frac{L^{*}}{N^{*}+0.01 L^{*}}\right)^{2}$, since a $1 \%$ higher target capitalization ratio would be $\frac{N^{*}+0.01 L^{*}}{L^{*}}$.
${ }^{24}$ The process followed by bank net worth in our model is a reduced-form version of that in Gertler and Karadi (2011) or Gertler, Kiyotaki, and Prestipino (2020): in those models, a fraction of "bankers" die in each period, returning their net worth to the household, and a fraction of "workers" become bankers and bring a fixed quantity of funds into the bank. This formulation permits us to calibrate the persistence of downturns in bank net worth separately from banks' long-run leverage ratios. We show in Online Appendix E how to micro-found our specification in that way.

Table 2: Calibrated parameters.

| Parameter | Description | Value | Target |
| :---: | :---: | :---: | :---: |
| $\beta$ | Time rate of preference | 0.995 | Real interest rate |
| $\chi$ | Disutility of labor | 0.41 | Labor hours |
| $\nu$ | Scale parameter | 0.85 | Consumption-investment ratio |
| $\xi$ | Fraction of bank-dependent firms | 0.998 | Fraction of SMEs |
| $A^{n b}$ | Non-bank-dependent firm productivity | 1 | Normalization |
| $\frac{A^{b}}{A^{n b}}$ | Bank-dependent firm productivity | 0.43 | SME output share |
| $\varepsilon^{L}$ | Elasticity of loan demand | 200 | Loan spread |
| $\varepsilon^{D}$ | Elasticity of deposit demand | -275 | Deposit spread |
| $\zeta$ | Liquid asset demand | 0.0045 | Loan-to-bond ratio $\frac{L^{*}}{B^{L *}}$ |
| $\mathcal{L}^{*}$ | Liquid asset satiation point | 6.93 | 2014 deposit-GDP ratio |
| $\gamma$ | Bank dividend payout rate | 0.06 | Capitalization ratio $\frac{N^{*}}{L^{*}}$ |
| $\hat{N}$ | Bank equity injection | 0.016 | Equity issuance-to-asset ratio |
| $\tau$ | Long-term bond maturity | 13.6 | Bank asset maturity |
| $\kappa^{L}$ | Leverage cost parameter | 0.018 | Elasticity of $i^{L}$ to $\frac{L^{*}}{N^{*}}$ |
| $\mu^{D}$ | Deposit issuance benefit | 12.5 bp | Deposit fees |
| $\kappa^{I}$ | Capital adjustment cost | 5 | Elasticity of $I_{t}$ to $Q_{t}$ |

where $i^{*}$ is the steady-state policy rate (and similarly for $\pi^{*}$ ), $\rho^{m p}=0.93$ is the persistence of the nominal rate, $\phi^{\pi}=2.74$ is the Taylor rule coefficient on inflation, and $\epsilon_{t}^{m p}$ is the time- $t$ monetary shock ${ }^{25}$ There is no uncertainty after $t=0$, so agents learn the full sequence of shocks $\epsilon_{t}^{m p}$ at $t=0$. Our benchmark results will consider only a monetary shock $\epsilon_{0}^{m p}$ occurring at $t=0$, but in Section 5.1 we will also consider other types of shocks that could cause an initial interest rate displacement: a productivity shock and a demand shock entering through the household's discount factor $\beta$.

### 4.3 Main results

In our main results, we seek to answer the following question: given an initial level of the interest rate, how does the economy respond to additional monetary stimulus? Specifically, we first study the effects of a marginal - minus 10 basis point - innovation $\epsilon_{0}^{m p}$ to the Taylor rule around the economy's steady state and report the impulse response functions. Then, we generate initial innovations of increasingly larger sizes and study impulse responses to an additional 10-basis point innovation to the Taylor rule. In other words, we compute three impulse responses: the IRF to a small 10-basis point shock in the vicinity of the steady state, the IRF to a large shock, and then the IRF to that large shock plus 10 basis points. We then compare the difference between the last two IRFs to the first IRF.

If we were to solve our model using a log-linear approximation, the economy's response to a 10 -basis point Taylor rule innovation would be independent of the initial interest rate. Our solution method, by contrast, allows us to highlight the non-linear (and possibly non-monotonic) response

[^15]of aggregates to shocks as well as the dependence of the response on the level of interest rates.
Before describing our results, we point out two subtleties of our analysis. First, a reversal within our experiment necessarily applies to a particular variable at a particular horizon. For instance, our theoretical results suggest that the reversal rate for bank lending increases with the horizon $t$. Differently from our theoretical results, the calibrated general equilibrium model permits us to study the reversal rate for any aggregate quantity at any horizon. Second, given that our economy's constraints are smooth - in contrast to the constraints faced by banks in the stylized theoretical model - the economic mechanisms we highlight have consequences before aggregate variables display a full reversal, serving to dampen the effectiveness of monetary policy.

Figure 2 displays our main result. It depicts the impulse responses of bank lending and aggregate investment to an additional 10-basis point Taylor rule innovation for various initial interest rates. When the initial interest rate is greater than roughly $-1 \%$, additional monetary stimulus increases both bank lending and investment on impact, as in standard models. However, once the initial interest rate is below about $-1 \%$, this effect is reversed: monetary stimulus is contractionary for bank lending and investment. In fact, we find that the reversal rate (on impact) is $-0.8 \%$ for aggregate investment and $-1.2 \%$ for bank lending.

The reversal in bank lending and investment transmits to aggregate output as well. Figure 3 displays the marginal response of output to a 10-basis point Taylor rule innovation in two economies: a baseline economy in which the shock occurs in the vicinity of the steady state and an economy in which the shock occurs on top of an innovation to the Taylor rule that, on its own, would have depressed the policy rate to $-1 \%$. When the economy begins at its steady state, the impulse response to a monetary shock is similar to that in a model without banking frictions. Once the reversal rate has been reached, however, the response of output changes substantially. Unlike investment and bank lending, the reversal in output occurs only with a four-quarter lag. An interest rate cut initially stimulates aggregate demand, boosting output. However, the reversal in investment gives rise to a gradual decline in the capital stock, reducing the economy's productive capacity and eventually depressing output.

Our quantitative results up until this point have demonstrated that even when a reversal does not occur at the impact of a monetary shock, it may still occur in the future. For instance, in Figure 2, when the initial interest rate is $0 \%$, an interest rate cut increases investment on impact but reduces it at longer horizons. Similarly, at longer horizons output experiences a reversal (Figure 3). Thus, the dynamic response predicted by Proposition 4 is also relevant in general equilibrium, and it can provide some guidance for the conduct of monetary policy. The main implication of this result is that even if an interest rate cut is initially successful at stimulating lending and investment, the effect may reverse later on.

### 4.4 The main mechanism

Near the reversal rate for bank lending, the economy's response to monetary shocks is driven by a persistent decline in bank profits and a corresponding increase in their lending costs and


Figure 2: Marginal responses of bank lending (left panel) and aggregate investment (right panel) to a 10 -basis point Taylor rule innovation for various initial levels of the interest rate $i$. The path plotted for each variable is the difference between two impulse responses: the impulse response of a Taylor rule innovation that would reduce the time-0 interest rate to $i$, and the impulse response to a Taylor rule innovation that is greater by 10 basis points.


Figure 3: Marginal response of output to a 10-basis point Taylor rule innovation in two economies. In the baseline economy (blue line), the shock occurs when the economy is at its steady state. In the alternative economy (green line), the shock occurs when the initial interest rate is equal to $-1 \%$.
loan rates. Figure 4 illustrates the impulse response of banks' marginal cost of leverage and the corresponding increase in the one-year real loan rate, $R_{t, t+4}^{L}=\prod_{s=1}^{4} \frac{1+i_{t+s-1}^{L}}{1+\pi_{t+s}}$. Near the steady state, a negative innovation to the Taylor rule results in lower long-term loan rates, and the change in banks' leverage costs is negligible. However, near the reversal rate, the same shock actually increases loan rates due to the increase in banks' leverage costs. Due to the higher borrowing rates they face, bank-dependent firms demand less capital, reducing investment in that sector.

Bank lending $L_{t}=Q_{t}^{K, b} K_{t}^{b}$ can even decline at the impact of the shock, before any disinvestment has occurred, due to a reduction in the price of capital $Q_{t}^{K, b}$. To understand this result, it is conceptually useful to write bank-dependent firms' capital demand condition as an asset pricing equation,

$$
\begin{equation*}
Q_{t}^{K, b}=\frac{1}{R_{t}^{L}}\left(M P K_{t+1}+(1-\delta) Q_{t+1}^{K, b}\right)=\sum_{s=0}^{\infty} \frac{(1-\delta)^{s}}{\prod_{r=0}^{s} R_{t+r}^{L}} M P K_{t+s+1}, \tag{25}
\end{equation*}
$$

where $M P K_{t}$ is the marginal product of capital for bank-dependent firms at time $t$ and $R_{t}^{L} \equiv \frac{1+i_{t}^{L}}{1+\pi_{t+1}}$ is the real loan rate. That is, the price of capital is equal to the discounted value of $M P K_{t}$, using the real loan rate as the discount rate. The higher discount rate on capital puts downward pressure on its price.

Figure 5 illustrates that the reversal in aggregate investment can be attributed to a reversal in the investment of bank-dependent firms. As their investment declines, lending shifts towards the non-bank-dependent sector, and monetary policy continues to stimulate the investment of non-bank-dependent firms. Indeed, near the reversal rate, a further interest rate cut stimulates their investment more than it would near the economy's steady state (due to the substitution of investment from the bank-dependent sector to the non-bank-dependent sector). Nevertheless, the net effect of an interest rate cut on aggregate investment remains negative.

It may seem surprising that despite the substitution towards investment in the non-bankdependent sector, the reversal rate for aggregate investment is actually higher than the reversal rate for bank lending. The reason is that bank lending and loan rates both respond gradually over time to shocks as the capital stock and banks' net worth adjust, whereas investment is forward-looking: due to the presence of adjustment costs, it is optimal to smooth disinvestment over time. Indeed, it is clear from Figure 5 that the reversal rate for bank-dependent investment is much lower than the reversal rate for aggregate investment.

### 4.5 The determinants of the reversal rate

The calibrated model provides an ideal laboratory to study the primary determinants of the reversal rate. In this section, we study the reversal rate's dependence on the model's key banking sector parameters and elaborate on the intuition behind our results. In these comparative statics experiments, we vary the value of one parameter while holding the parameters in Table 1 constant and re-calibrating the parameters in Table 2 to match the specified targets. We study how changing


Figure 4: Marginal response of one-year loan rates (left panel) and bank leverage costs (right panel) following a minus 10-basis point Taylor rule innovation in two economies: a baseline economy in which the shock occurs when the economy is in steady state, and an alternative economy in which the shock occurs on top of an innovation to the Taylor rule that would, on its own, cause the interest rate to fall to $-1 \%$.


Figure 5: Marginal responses of investment for bank-dependent firms (left panel) and non-bankdependent firms (right panel) to a 10 -basis point Taylor rule innovation in two economies: one in which the economy is initially near its steady state and another in which the innovation occurs on top of one that would have reduced the initial interest rate to $-1 \%$ (as described in Figure 3).
each parameter value affects the reversal rate for aggregate investment.
One of the parameters most closely related to the theoretical mechanisms that give rise to the reversal rate is the maturity $\tau$ of bonds held on bank balance sheets. The left panel of Figure 6 demonstrates that the reversal rate is decreasing in the maturity of bonds held by banks. When banks have a greater maturity mismatch, they experience greater asset revaluation upon the announcement of an interest rate cut. Therefore, the capital gains channel of monetary policy, by which interest rate cuts shore up bank net worth, is stronger relative to the net interest income channel, by which interest rate cuts in negative territory reduce bank net worth. Overall, greater maturity mismatch then has the effect of lowering the interest rate at which further cuts become contractionary.

The right panel of the figure shows the dependence of the reversal rate on bank-dependent firms' share of output, which is the target in the data used to calibrate their relative productivity $\frac{A^{b}}{A^{n b}}$. This parameter is important for determining the strength of the reversal rate mechanism in general equilibrium: one of the main countervailing forces in the model is the substitution of lending from the bank-dependent sector to the non-bank-dependent sector. As expected, the reversal rate is increasing in bank-dependent firms' share of output: when most output is produced by non-bankdependent firms, a large negative response of bank lending to an interest rate cut is required before a reversal of aggregate investment occurs. Our results indicate, in fact, that the reversal rate is likely to be substantially lower in countries where investment is far less bank-dependent (e.g., the U.S.).

The composition of banks' profits also has important implications for the reversal rate. Figure 7 depicts the dependence of the reversal rate on the elasticity of loan demand $\varepsilon^{L}$ and the elasticity of deposit demand $\left|\varepsilon^{D}\right|$. Since we re-calibrate the model to keep banks' capitalization ratio $\frac{N^{*}}{L^{*}}$ constant, these comparative statics should be interpreted as changing the shares of bank profits accounted for by loans and deposits, respectively. The reversal rate is increasing in the elasticity of loan demand but decreasing in the elasticity of deposit demand. The key mechanism underlying this result is that when loan markets are relatively competitive, or when deposit markets are relatively uncompetitive, then bank profitability is highly reliant on deposit spreads. Interest rate cuts into negative territory are then significantly detrimental to bank net worth, leading to a higher reversal rate. One would expect a relationship between the reversal rate and the share of banks' profits attributable to deposit market power, but there is not necessarily a relationship between the overall concentration of the banking sector and the reversal rate.

### 4.6 The power of forward guidance

Following the Global Financial Crisis of 2008, when short-term interest rates were driven to zero, central banks have begun to experiment with forward guidance (i.e., promises of low rates in the future) to provide additional monetary stimulus without resorting to negative interest rates. The literature on forward guidance has encountered a major puzzle: from a theoretical perspective, promises of interest rate cuts further and further in the future have explosive stimulative effects


Figure 6: Dependence of the reversal rate on the bond maturity parameter $\tau$ (left panel) and the relative productivity of bank-dependent firms $\frac{A^{b}}{A^{n b}}$ (right panel). The values of $\frac{A^{b}}{A^{n b}}$ are reported as the corresponding share of output produced by bank-dependent firms. In each panel, all other parameter values are set as in our benchmark calibration.


Figure 7: Dependence of the reversal rate on the values of the loan demand elasticity parameter $\varepsilon^{L}$ (left panel) and the absolute value of the deposit demand elasticity parameter $\varepsilon^{D}$ (right panel). In each panel, all other parameter values are set as in our benchmark calibration.
on output in the present (Del Negro et al., 2012). On the other hand, the empirical evidence of the effectiveness of forward guidance has been mixed, and there has been no clear indication that promises of interest rate cuts in the long term are more effective than promises of stimulus in the present.

Our calibrated model allows us to reassess the forward guidance puzzle and the effectiveness of "low-for-long" monetary policies. Intuitively, forward guidance should not be expected to be as effective in our model as it would be in the textbook New Keynesian model, in light of Proposition 5. our main theoretical result regarding such policies. That is, our theoretical results have shown that keeping rates low for a sufficiently long period of time will eventually have a contractionary effect on bank lending and investment. Thus, a policy that promises to hold interest rates down for an extended period of time should not necessarily be expected to initially produce a large economic boom, since agents will anticipate depressed investment and output in the future.

A typical forward guidance policy involves a commitment to keep the policy rate fixed at a certain level $i$ until some date $T$, at which point the central bank returns to a Taylor rule. Therefore, to evaluate the power of forward guidance, we study policy rules of the form

$$
\frac{1+i_{t}}{1+i^{*}}=\left\{\begin{array}{cl} 
& t \leq T  \tag{26}\\
\left(\frac{1+i_{t-1}}{1+i^{*}}\right)^{\rho^{m p}}\left(\frac{1+i}{1+i^{*}}\right. & \left.\frac{1+\pi_{t}}{1+\pi^{*}}\right)^{\phi^{\pi}\left(1-\rho^{m p}\right)} \\
t>T
\end{array},\right.
$$

where, again, $i^{*}>i$ denotes the steady-state policy rate. We fix the promised interest rate $i$ at $-1 \%$, close to the reversal rate that we estimate in our benchmark results, for $T=8$ quarters.

Figure 8 displays the impulse responses of aggregate investment and output to a monetary policy of the form (26) in two economies: our benchmark model and an alternative "frictionless" economy in which banks do not face leverage costs (i.e., a version of our benchmark model with $\kappa^{L}=0$ ) ${ }^{26}$ The impulse response in the frictionless economy highlights the dynamics that are typically observed in standard New Keynesian models: a promise to hold interest rates down for eight quarters leads to an implausibly large boom in investment and output. At their peaks, investment and output both reach a level equal to roughly twice their steady-state values. By contrast, in our economy, these responses are only about half as large. Moreover, the forward guidance policy causes a reversal in investment to occur by the eighth quarter after its announcement. As a consequence, the cumulative response of investment to this forward guidance policy (i.e., the change in the capital stock) eventually becomes negative.

In Online Appendix C, we show that the reduction in the power of forward guidance carries over to bank lending, consumption, and inflation as well. As expected, all variables respond explosively even to eight quarters of forward guidance in the frictionless model, whereas in our benchmark model, their responses are much smaller.

Hence, our model's novel channel of monetary policy transmission provides a mechanism to blunt

[^16]

Figure 8: The effect of forward guidance on bank lending in two economies: our benchmark model (solid blue lines) and a "frictionless" economy in which banks do not face leverage costs, $\kappa^{L}=0$ (dashed red lines). We consider policies in which the central bank promises to hold interest rates at $-1 \%$ for eight periods before returning to a Taylor rule. We plot the impulse responses for aggregate investment (left panel) and output (right panel) in both economies.
the unreasonable power of forward guidance predicted by standard models. The persistent drain on bank net worth caused by "low-for-long" policies decreases investment, asset prices, and output in the long run. In turn, the anticipation of these negative long-run effects dampens the initial stimulation of the economy and the response of inflation. This mechanism should be highly relevant for central banks considering how long to hold interest rates down, since it paints a qualitatively different picture of such policies' effectiveness: in contrast to standard theory, our model predicts that promising to hold interest rates down for longer periods can eventually turn counterproductive.

It is worth noting that the muted power of forward guidance in our model is entirely dependent on the reversal rate mechanism: it arises only because of the deterioration in bank profits, and the corresponding increase in leverage costs, when rates are cut low enough. In Online Appendix C, we show that for smaller prolonged interest rate cuts (e.g. from $2 \%$ to $1.5 \%$ ), forward guidance in our model is just as powerful as in a standard model.

## 5 Robustness and Sensitivity

In this section, we address the robustness of our preferred estimate of the reversal rate ( $-0.8 \%$ for aggregate investment in the benchmark model). Given that we have so far focused solely on monetary shocks, we first extend our model by incorporating other types of shocks that can depress interest rates on impact. Then, we analyze the sensitivity of our estimates to the model's key parameters.


Figure 9: Initial interest rate plotted against the marginal response of investment to a 10 -basis point Taylor rule innovation. The left panel depicts the case in which the level of the initial interest rate is determined by a discount factor shock occurring at $t=0$. The right panel depicts the case in which the level of the initial interest rate is determined by a productivity shock occurring at $t=0$. In both cases, the model is solved under perfect foresight.

### 5.1 Alternative types of shocks

The goal of our calibration is to estimate the level of interest rates at which additional interest rate cuts become contractionary for bank lending and investment. In the benchmark model, we answer this question by first hitting the economy with a large monetary shock that reduces the policy rate to a low level and then adding an additional small monetary shock that reduces the policy rate by an additional 10 basis points. We then compute the marginal response of bank lending and investment to this 10 -basis point shock. There is no particular reason that the initial large shock has to be a monetary shock, however - it suffices to consider any shock that would reduce interest rates.

Therefore, we examine two additional types of large shocks in this section: a shock to the discount factor $\beta$ that makes agents more patient (as in Eggertsson and Woodford, 2003), and a shock to firm productivity $\left(A_{t}^{b}, A_{t}^{n b}\right)$ that reduces the natural rate. For the discount factor shock, we assume that agents' subjective discount factor $\beta_{t}=e^{\epsilon_{t}^{\beta}} \beta$, where $\epsilon_{t}^{\beta}$ is the shock to the discount factor. We assume an "MIT" shock: agents learn $\epsilon_{0}^{\beta}$ at $t=0$, and there are no additional shocks to the discount factor thereafter, $\epsilon_{t}^{\beta}=0$ for $t>0$. When we consider shocks to productivity, we impose an exactly analogous process for $\left(A_{t}^{b}, A_{t}^{n b}\right)$. In each case, the shock does not persist after the first period. Nevertheless, since the Taylor rule followed by the central bank has inertia, interest rates are just as persistent as in our benchmark model.

Figure 9 plots the response of investment at the impact of the 10 -basis point Taylor rule shock against the interest rate after the initial large shock. In both cases, the estimated reversal rate is reasonably close to our benchmark estimate of $-0.8 \%$ : in the case of a productivity shock it is $-1.1 \%$, whereas in the case of a discount factor shock it is $-0.6 \%$.

It is sensible that in these cases, the reversal rate does not depend too heavily on the source of the initial interest rate displacement. The theoretical model showed that in partial equilibrium, banks' initial net worth, the tightness of the leverage constraint, and the dependence of banks' profits on the interest rate were the main determinants of the reversal rate. The qualitative dependence of bank profits and initial capital gains on the sequence of interest rates is relatively unchanged by the type of shock considered. Therefore, the reversal rate differs across these two cases mostly via general equilibrium effects that affect loan demand and deposit demand through changes in prices.

Note, however, that the reversal rate would be significantly higher than $-0.8 \%$ following any shock that both reduces interest rates and directly decreases bank net worth on impact. For example, suppose bank net worth were to receive a substantial negative shock at the same time as a negative shock to demand (captured by a discount factor shock). This situation could be interpreted as a financial crisis coupled with a demand recession. In this scenario, banks are initially much more constrained than they would be following a demand shock only. Hence, further interest rate cuts are particularly detrimental to bank lending in this type of recession, so a reversal in aggregate investment should be expected to occur at a higher interest rate.

### 5.2 Sensitivity analysis

A natural question is the extent to which our quantitative results regarding the value of the reversal rate are driven by our calibration strategy. In this section, we discuss the reversal rate's dependence on the values of our model's parameters. In particular, for several parameters in our calibration, we report how the reversal rate (for aggregate investment on impact) changes when we alter the value of that parameter, holding the other parameters fixed as in Section 4.5.

First, we consider the parameters with conventional values in the literature (reported in Table 1). As we document in Online Appendix C, the reversal rate on impact is not very sensitive to any of these choices. For each parameter, we take a range of values considered in the literature and plot the reversal rate for each value in that range. In each case, the reversal rate remains within a range close to our preferred estimate of $-0.8 \%$.

More important are the parameters that we calibrate to match moments in the data (reported in Table 22). Several parameters are uniquely identified by easily measurable bank balance sheet and interest rate moments. ${ }^{27}$ The remaining parameters are not unambiguously identified by bank balance sheet data alone, so we report sensitivity results for those parameters as well. For illustrative purposes, here we describe the results for two important parameters: banks' leverage adjustment cost parameter $\kappa^{L}$ and the investment adjustment cost parameter $\kappa^{I}$. We report results for the remainder of the parameters in Online Appendix C, since they are less directly related to the reversal rate mechanism ${ }^{28}$

It is natural that the reversal rate should be sensitive to $\kappa^{L}$, since that parameter modulates the

[^17]

Figure 10: Dependence of the reversal rate on the values of the leverage cost parameter $\kappa^{L}$ (left panel) and the investment adjustment cost parameter $\kappa^{I}$ (right panel). The values of $\kappa^{L}$ are reported as the corresponding change in loan rates following a $1 \%$ increase in banks' capitalization ratio target, since in our benchmark calibration, $\kappa^{L}$ is set to target a 28 bp increase in loan rates. The values of $\kappa^{I}$ are reported as the corresponding elasticity of investment to changes in the price of capital $\frac{1}{\kappa^{T}}$.
magnitude of the leverage costs that cause a reversal to occur at low interest rates. The importance of $\kappa^{I}$ is also straightforward: when adjustment costs are small, non-bank-dependent firms can easily expand investment when bank lending declines, undoing the effect of the reversal in bank lending (see Koby and Wolf, 2020). We estimated $\kappa^{L}$ from the response of lending rates to increases in capitalization targets reported by Macroeconomic Assessment Group (2010), and $\kappa^{I}$ was estimated by fixing the elasticity of investment to changes in the price of capital at the value of 0.2 reported by Smets and Wouters (2003). Such adjustment cost parameters are notoriously difficult to identify, so Figure 10 plots a range of values for each target against the corresponding reversal rate.

The left panel of Figure 10 shows the dependence of the reversal rate on the increase in loan rates resulting from a $1 \%$ change in banks' capitalization target $\frac{N^{*}}{L^{*}}$, which is the target in the data used to calibrate $\kappa^{L}$. We permit this value to run from 14 bp to 42 bp (i.e., half to one and a half times the benchmark estimate). This is the most important target in determining the value of the reversal rate: at the lower end of this range the reversal rate is equal to roughly $-1.7 \%$, whereas at the upper end, the estimated reversal rate is approximately $-0.5 \%$. The right panel of Figure 10 displays the reversal rate on impact as the elasticity of investment to capital prices ranges from 0.1 (half our preferred estimate) to 0.3 (one and a half times our preferred estimate). The corresponding estimate of the reversal rate ranges from $-0.6 \%$ to $-1.1 \%$.

This sensitivity analysis allows us to draw two conclusions. First, for a reasonable range of parameter estimates, a reversal rate always exists (although it may be below $-1 \%$ for the most optimistic parameter estimates). Second, given this same range of parameter estimates, one can be reasonably confident that slightly negative interest rates (e.g. $-0.25 \%$ ) will not cause a reversal to occur - in our framework, a reversal tends to occur only for larger interest rate cuts. Therefore,
our preferred estimate of $-0.8 \%$ provides guidance similar to the picture provided by our sensitivity analysis: a central bank should consider the possibility of a reversal only when rates are already substantially negative.

## 6 Conclusion

We have shown the conditions for the existence of a reversal interest rate, the rate at which monetary policy stimulus reverses its intended effect and becomes contractionary. Its existence relies on the net interest income of banks decreasing faster than recapitalization gains from banks' initial holdings of fixed-income assets. In a calibrated New Keynesian model, the reversal rate for aggregate investment is close to $-1 \%$. The level of the reversal rate depends on the maturity of banks' fixed-income assets, banks' market power, and the economy's overall dependency on bank lending. As banks' fixed-income assets mature, their initial capital gains fade out, so "low-for-long" interest rate environments can eventually end in recessions. Our calibrated model demonstrates that this mechanism significantly dampens the power of forward guidance.

For the sake of tractability, we have omitted other channels through which monetary policy can affect banks as well as the real economy. In particular, policies such as ECB's long-term refinancing operations could have alleviated some of the low rates' effect on bank margins. Moreover, we have omitted the explicit modeling of risk; hence, we have remained agnostic on how low rates change nonperforming loans and the associated responses in provisions. We see these as important quantitative refinements for future research. Finally, we view our results as driven by unusual surprise movements in interest rates: low-for-long and negative rates were largely unforeseen events. It remains a question whether banks can and will adjust to a permanently lower interest rate environment - for example, by increasing their maturity mismatch. The competitive landscape faced by banks could also change, with depositors growing accustomed to the possibility of negative interest rates, hence supporting banks' profitability in negative-rate environments.

## References

Abad, Jorge, Iñaki Aldasoro, Christoph Aymanns, Marco D'Errico, Linda Fache Rousová, Peter Hoffmann, Sam Langfield, Martin Neychev, and Tarik Roukny, "Shedding Light on Dark Markets: First Insights from the New EU-Wide OTC Derivatives Dataset," 2016. ESRB Occasional Paper Series, No. 2016/11.

Alessandri, Piergiorgio and Benjamin Nelson, "Simple Banking: Profitability and the Yield Curve," Journal of Money, Credit, and Banking, 2015, 47 (1), 143-175.

Altavilla, Carlo, Miguel Boucinha, and Jose-Luis Peydró, "Monetary Policy and Bank Profitability in a Low Interest Rate Environment," Economic Policy, 2018, 33 (96), 531-586.

Ampudia, Miguel and Skander van den Heuvel, "Monetary Policy and Bank Equity Values in a Time of Low and Negative Interest Rates," Journal of Monetary Economics (forthcoming), 2022.

Anderson, Simon, André De Palma, and Jacques-François Thisse, "Demand for Differentiated Products, Discrete Choice Models, and the Characteristics Approach," The Review of Economic Studies, 1989, 56 (1), 21-35.

Bech, Morten and Aytek Malkhozov, "How Have Central Banks Implemented Negative Policy Rates?," BIS Quarterly Review, 2016.

Begenau, Juliane, Monika Piazzesi, and Martin Schneider, "Banks' Risk Exposures," 2015. NBER Working Paper 21334.

Bernanke, Ben and Alan Blinder, "Credit, Money, and Aggregate Demand," American Economic Review, 1988, 78 (2), 435-439. and Mark Gertler, "Inside the Black Box: The Credit Channel of Monetary Policy Transmission," Journal of Economic Perspectives, 1995, 9 (4), 27-48.
__, _ , and Simon Gilchrist, "The Financial Accelerator in a Quantitative Business Cycle Framework," in John Taylor and Michael Woodford, eds., Handbook of Macroeconomics, Vol. 1, North-Holland, 1999, pp. 1341-1393.

Bianchi, Javier and Saki Bigio, "Banks, Liquidity Management, and Monetary Policy," Econometrica, 2022, 90 (1), 391-454.

Bittner, Christian, Diana Bonfim, Carla Soares, Florian Heider, Farzad Saidi, and Glenn Schepens, "The Augmented Bank Balance-Sheet Channel of Monetary Policy," 2022. Working Paper.

Borio, Claudio, Leonardo Gambacorta, and Boris Hoffmann, "The Influence of Monetary Policy on Bank Profitability," International Finance, 2017, 20 (1), 48-63.

Bräuning, Falk and Botau Wu, "ECB Monetary Policy Transmission During Normal and Negative Interest Rate Periods," 2017. Working Paper.

Brunnermeier, Markus and Yuliy Sannikov, "A Macroeconomic Model with a Financial Sector," American Economic Review, 2014, 104 (2), 379-421.
__ and __ , "The I Theory of Money," 2016. NBER Working Paper 22533.
Cavallino, Paolo and Mr. Damiano Sandri, "The Expansionary Lower Bound: Contractionary Monetary Easing and the Trilemma," 2018. IMF Working Paper.

Chatterjee, Satyajit and Burcu Eyigungor, "Maturity, Indebtedness, and Default Risk," American Economic Review, 2012, 102 (6), 2674-2699.

Chodorow-Reich, Gabriel, "The Employment Effects of Credit Market Disruptions: Firm-Level Evidence from the 2008-09 Financial Crisis," Quarterly Journal of Economics, 2014, 129 (1), 1-59.

Claessens, Stijn, Nicholas Coleman, and Michael Donnelly, "Low-for-Long' Interest Rates and Banks' Interest Margins and Profitability: Cross-Country Evidence," Journal of Financial Intermediation, 2018, 35, 1-16.

Coenen, Günter, Peter Karadi, Sebastian Schmidt, and Anders Warne, "The New AreaWide Model II: An Extended Version of the ECB's Micro-Founded Model for Forecasting and Policy Analysis with a Financial Sector," 2019. ECB Working Paper 2200.

Di Tella, Sebastian and Pablo Kurlat, "Why Are Banks Exposed to Monetary Policy?," American Economic Journal: Macroeconomics, 2021, 13 (4), 295-340.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, "The Deposits Channel of Monetary Policy," Quarterly Journal of Economics, 2017, 132 (4), 1819-1876.
__, ___ , and __ , "A Model of Monetary Policy and Risk Premia," Journal of Finance, 2018, 73 (1), 317-373.
__ , __ , and __ , "Banking on Deposits: Maturity Transformation Without Interest Rate Risk," Journal of Finance, 2021, 76 (3), 1091-1143.

Eggertsson, Gauti and Michael Woodford, "Optimal Monetary Policy in a Liquidity Trap," 2003. NBER Working Paper 9968.
_ , Ragnar Juelsrud, Lawrence Summers, and Ella Getz Wold, "Negative Nominal Interest Rates and the Bank Lending Channel," 2019. NBER Working Paper 25416.

Eisenshmidt, Jens and Frank Smets, "Negative Interest Rates: Lessons from the Euro Area," in Alvaro Aguirre, Markus Brunnermeier, and Diego Saravia, eds., Monetary Policy and Financial Stability: Transmission Mechanisms and Policy Implications, Central Bank of Chile, 2019.

English, William, Skander van den Heuvel, and Egon Zakrajsek, "Interest Rate Risk and Bank Equity Valuations," Journal of Monetary Economics, 2018, 98, 80-97.

Feenstra, Robert, "Functional Equivalence Between Liquidity Costs and the Utility of Money," Journal of Monetary Economics, 1986, 17 (2), 271-291.

Freriks, Jorien and Jan Kakes, "Bank Interest Rate Margins in a Negative Interest Rate Environment," 2021. De Nederlandsche Bank Working Paper No. 721.

Gerali, Andrea, Stefano Neri, Luca Sessa, and Federico Signoretti, "Credit and Banking in a DSGE Model of the Euro Area," Journal of Money, Credit, and Banking, 2010, 42 (1), 107-141.

Gertler, Mark and Peter Karadi, "A Model of Unconventional Monetary Policy," Journal of Monetary Economics, 2011, 58 (1), 17-34.
__ , Nobuhiro Kiyotaki, and Andrea Prestipino, "A Macroeconomic Model with Financial Panics," Review of Economic Studies, 2020, 87 (1), 240-288.

Gomez, Matthieu, Augustin Landier, David Sraer, and David Thesmar, "Banks' Exposures to Interest Rate Risk and the Transmission of Monetary Policy," Journal of Monetary Economics, 2021, 117, 543-570.

Gropp, Reint, Thomas Mosk, Steven Ongena, and Carlo Wix, "Banks' Response to Higher Capital Requirements: Evidence from a Quasi-Natural Experiment," Review of Financial Studies, 2018, 32 (1), 266-299.

Hatchondo, Juan Carlos and Leonardo Martinez, "Long-Duration Bonds and Sovereign Defaults," Journal of International Economics, 2009, 79 (1), 117-125.

Heider, Florian, Farzad Saidi, and Glenn Schepens, "Life Below Zero: Bank Lending Under Negative Policy Rates," Review of Financial Studies, 2019, 32 (10), 3728-3761.

Hoffmann, Peter, Sam Langfield, Federico Pierobon, and Guillaume Vuillemey, "Who Bears Interest Rate Risk?," Review of Financial Studies, 2019, 32 (8), 2921-2954.

Kiyotaki, Nobuhiro and John Moore, "Credit Cycles," Journal of Political Economy, 1997, 105 (2), 211-248.

Klein, Michael, "A Theory of the Banking Firm," Journal of Money, Credit, and Banking, 1971, 3 (2), 205-218.

Koby, Yann and Christian Wolf, "Aggregation in Heterogeneous-Firm Models: Theory and Measurement," 2020. Working Paper.

Macroeconomic Assessment Group, "Assessing the Macroeconomic Impact of the Transition to Stronger Capital and Liquidity Requirements," Technical Report, Financial Stability Board and the Basel Committee on Banking Supervision 2010.

Maudos, Joaquin and Juan Fernández de Guevara, "Factors Explaining the Interest Rate Margin in the Banking Sectors of the European Union," Journal of Banking and Finance, 2004, 28 (9), 2259-2281.

Monti, Mario, "Deposit, Credit, and Interest Rate Determination Under Alternative Bank Objectives," in Gabor Szëgo and Karl Shell, eds., Mathematical Methods in Investment and Finance, North-Holland, 1972.

Negro, Marco Del, Marc Giannoni, and Christina Patterson, "The Forward Guidance Puzzle," 2012. FRBNY Staff Report No. 574.

Poterba, James and Julio Rotemberg, "Money in the Utility Function: An Empirical Implementation," 1986. NBER Working Paper 1796.

Repullo, Rafael, "The Reversal Interest Rate: A Critical Review," 2020. CEMFI Working Paper No. 2021.

Rotemberg, Julio, "Sticky Prices in the United States," Journal of Political Economy, 1982, 90 (6), 1187-1211.

Santomero, Anthony, "Modeling the Banking Firm: A Survey," Journal of Money, Credit, and Banking, 1984, 16 (4), 576-602.

Saunders, Anthony and Liliana Schumacher, "The Determinants of Bank Interest Rate Margins: An International Study," Journal of International Money and Finance, 2000, 19 (6), 813-832.

Smets, Frank and Rafael Wouters, "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," Journal of the European Economic Association, 2003, 1 (5), 11231175.

Ulate, Mauricio, "Going Negative at the Zero Lower Bound: The Effects of Negative Nominal Interest Rates," American Economic Review, 2021, 111 (1), 1-40.
van den Heuvel, Skander, "The Bank Capital Channel of Monetary Policy," 2007. The Wharton School, University of Pennsylvania Working Paper.

Wang, Olivier, "Banks, Low Interest Rates, and Monetary Policy Transmission," 2022. NYU Stern School of Business Working Paper.

Wang, Yifei, Toni Whited, Yufeng Wu, and Kairong Xiao, "Bank Market Power and Monetary Policy Transmission: Evidence from a Structural Estimation," Journal of Finance (forthcoming), 2022.


[^0]:    *We are grateful for comments by Julien Bengui, Emmanuel Farhi, Lunyang Huang, Florian Heider, Sam Langfield, Cesaire Meh, Helene Rey, Philipp Schnabl, Frank Smets, Adi Sunderam, Skander van den Heuvel, Christian Wolf, and seminar participants at the European Central Bank, the Bank for International Settlements, Princeton University, the Federal Reserve Board, Banque de France, the Danish Central Bank, the Finnish Central Bank, the London School of Economics, ETH Zurich, the Swiss Economists Abroad meetings, the Swiss National Bank, the 2018 AEA meetings, Humboldt University, the Philadelphia Macro Workshop, the International Research Forum on Monetary Policy, the University of Chicago, and the Bank of Canada. Koby acknowledges support previously received from the Macro Financial Modeling Project.
    ${ }^{\dagger}$ Disclaimer: This Philadelphia Fed working paper represents preliminary research that is being circulated for discussion purposes. The views expressed in this paper are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. Any errors or omissions are the responsibility of the authors. No statements here should be treated as legal advice. Philadelphia Fed working papers are free to download at https://philadelphiafed.org/research-and-data/publications/working-papers.
    ${ }^{\ddagger}$ Federal Reserve Bank of Philadelphia, joseph.abadi@phil.frb.org
    ${ }^{\text {§ }}$ Princeton University, markus@princeton.edu
    ${ }^{1}$ Brown University, yann_koby@brown.edu

[^1]:    ${ }^{1}$ Santomero (1984) provides an excellent survey of this early theoretical literature.

[^2]:    ${ }^{2}$ Gomez et al. (2021) offer similar evidence, by studying two groups differentially exposed to interest rate risk. The group whose profitability is affected negatively (in relative terms) by a change in aggregate interest rates decreases its lending.

[^3]:    ${ }^{3}$ More recently, Drechsler et al. (2017) and Di Tella and Kurlat (2021) have adopted similar formulations.
    ${ }^{4}$ We explicitly specify the parameter $\zeta$ (instead of subsuming it in the function $\Phi$ ) because later on, we will study comparative statics with respect to $\zeta$, holding $\Phi$ fixed.

[^4]:    ${ }^{5}$ Due to the presence of risk-free bonds, deposit demand depends on both the deposit rate set by bank $j$ and the policy rate, as demonstrated by the first-order conditions (B.7) and (B.8) in the Online Appendix.
    ${ }^{6}$ With constant returns to scale, frictions in bank lending would become irrelevant. Any shortfall in investment by bank-dependent firms would be undone by non-bank-dependent firms in equilibrium (Koby and Wolf, 2020).

[^5]:    ${ }^{7}$ We assume $\tau<\frac{1}{\delta}$ to ensure that the price of bonds remains finite even as interest rates approach $-\delta$, which is the lowest value they can attain. (Otherwise, the user cost of capital goes to zero, and loan demand diverges.) We find that this parametric restriction holds in our calibration.
    ${ }^{8}$ The fixed-dividend assumption makes certain that banks do not drive leverage costs to zero by borrowing from households. This is consistent with the empirical evidence in Gropp et al. (2018). Repullo (2020) shows that our results would change if banks could flexibly issue equity: in the calibration section, although we do not allow for flexible issuance, we do assume banks receive periodic equity injections.

[^6]:    ${ }^{9}$ Here we use the fact that with fixed prices, inflation is equal to zero.

[^7]:    ${ }^{10}$ Formally, $i_{t}^{R R}=\sup \left\{\tilde{i}: L_{t}(i)<L_{t}\left(i^{\prime}\right) \forall i<i^{\prime} \leq \tilde{i}\right\}$. Note that under our definition, if a time- $t$ reversal rate exists, then it is unique.
    ${ }^{11}$ We prove this fact in Online Appendix B.1.

[^8]:    ${ }^{12}$ Specifically, $\varepsilon_{t}^{L}=-\left.\frac{L^{*^{\prime}}\left({ }_{i}^{L}\right)}{L^{*}\left(i^{L}\right)}\right|_{i_{t}^{L}}$ and $\varepsilon_{t}^{D}=\left.\frac{1}{D^{*}\left(i^{D}, i\right)} \frac{\partial D^{*}\left({ }_{i}{ }^{D}, i\right)}{\partial i^{D}}\right|_{\left(i_{t}^{D}, i_{t}\right)}$.
    ${ }^{13}$ See, for instance, Bech and Malkhozov (2016), Claessens et al. (2018), or Eggertsson et al. (2019), who document a collapse in pass-through to deposit rates when the policy rate enters negative territory.

[^9]:    ${ }^{14}$ If firms could substitute from loans to bond funding, there would be an additional effect: a decrease in the policy rate would cause a substitution away from loans and into bonds, decreasing bank profitability.
    ${ }^{15}$ The threshold $\underline{i}$ is defined in Lemma 1
    ${ }^{16}$ Formally, $N_{t}\left(\bar{N}_{0}, i\right)$ is the net worth at time $t$ of a bank that begins at $t=0$ with net worth $N_{0}$ and solves (15) in each period, taking as given the interest rate sequence 11, the sequence of bond prices $Q_{t}^{B}$ given by 12, and the loan and deposit demand schedules $L^{*}\left(i^{L}\right), D^{*}\left(i^{D}, i\right)$.

[^10]:    ${ }^{17}$ Specifically, we consider a fixed value of steady-state net worth $N^{*}$ and take $B^{L *}$ to zero in (8), which determines banks' initial net worth $N_{0}$.

[^11]:    ${ }^{18}$ We characterize the policy's effect on total lending because the marginal effects of extending the length $T$ of the policy rate cut vary with the horizon $t$ considered: increasing $T$ always increases lending at sufficiently short horizons through the capital gains channel, whereas it can decrease lending at longer horizons.

[^12]:    ${ }^{19}$ Our assumption of a satiation point at $\mathcal{L}^{*}$, rather than negative returns to liquidity for $\mathcal{L}>\mathcal{L}^{*}$ (e.g. $\Phi(\mathcal{L})=$ $\left.-\frac{1}{2}\left(\mathcal{L}^{*}-\mathcal{L}\right)^{2}\right)$, is conservative. If there were negative returns to liquid assets for $\mathcal{L}>\mathcal{L}^{*}$, households would continue to scale up their deposit holdings even as rates head deep into negative territory, putting a further drag on bank profitability.
    ${ }^{20}$ It is possible to micro-found this constraint by assuming that households have two types of members: "depositors" who can invest in liquid assets and "savers" who cannot.

[^13]:    ${ }^{21}$ See, for instance, https://www.wsj.com/articles/banks-in-germany-tell-customers-to-take-deposits-e lsewhere-11614594601

[^14]:    ${ }^{22}$ Ulate (2021) uses a similar specification for leverage costs that is approximately quadratic around a target leverage

[^15]:    ${ }^{25}$ The values of $\rho^{m p}$ and $\phi^{\pi}$ are set to those estimated by the New Area-Wide Model II (Coenen et al. 2019).

[^16]:    ${ }^{26}$ We plot the total effect of such policies (rather than the marginal effect of increasing $T$ ) to compare the quantitative results with Proposition 5

[^17]:    ${ }^{27}$ In particular, parameters $\gamma, \zeta$, and $\tau$ are pinned down in this way.
    ${ }^{28}$ Specifically, we report results for parameters $\beta, \mu^{D}, \xi, \nu, \mathcal{L}^{*}$, and $\hat{N}$. Importantly, the level of the reversal rate is essentially independent of the fraction of bank-dependent firms $\xi$. Section 4.5 reports results for the remaining parameters not considered in Online Appendix C.

